INVESTIGATIONS OF FLUID FLOWS IN OPEN RECTANGULAR AND TRIANGULAR CHANNELS

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Investigations of fluid flows in open rectangular and triangular channels

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DECLARATION

This thesis is my original work and has not been presented for a degree in any other University.

Signature ____________________________ Date ______________________

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This thesis has been submitted for examination with our approval as university supervisors.

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DEDICATION

I would like to dedicate this thesis to my dear husband David Irungu for his understanding and encouragement during the course of this study. I also dedicate it to my children Patience and Justice and to my house-help Amina for their patience, prayers and understanding during this duration.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF APPENDICES

LIST OF SYMBOLS

SYMBOLS		QUANTITY
Q	-	Discharge (m^3s^{-1})
A	-	Cross-sectional Area (m^2)
n	-	Manning coefficient of roughness ($sm^{-1/3}$)
$\mathbf{S_f}$	-	Friction due to slope
$\mathbf{S_o}$	-	The slope of channel
P	-	The wetted perimeter (m)
b	-	Breadth of the channel floor (m)
Y	-	Depth of the channel (m)
M	-	Mass of the fluid *(Kg)*
t	-	Time (s)
F	-	Force (N)
P	-	Pressure (N/M^2)
C	-	The Chezy's coefficient ($Kgm^{-2}\ s^{-2}$)
V	-	Velocity (ms^{-1})
x	-	Length of the channel (m)
g	-	Acceleration due to gravity (ms^{-2})
R	-	Hydraulic radius (m)
T	-	Channel top width
Z	–	Stage (m)
$\mathbf{D_m}$	-	Hydraulic depth (m)
q	-	Lateral discharge (m^3s^{-1})

τ	-	Shear stress (N)
ρ	-	Density (Kg/m^3)
μ	-	Coefficient of viscosity
α	-	Energy coefficient
γ	-	Specific weight (N)
λ	-	A constant that relates friction, velocity and Kinematic viscosity

ABBREVIATIONS

F.D.M.	Finite Difference Method
P.D.E.	Partial Differential Equation
K.E	Kinetic Energy
P.E	Potential Energy

ABSTRACT

This study is on investigations of open rectangular and triangular channel flows. The aim is to determine the more hydraulically efficient channel between open rectangular and triangular channels. The laws of conservation of mass and momentum have led to partial differential equations which are non-linear. Analytical methods cannot be used to solve such equations hence finite difference method has been used.

Velocity and depth of flow play a major role in determining discharge. Effects of varying various parameters on velocity have been investigated. Variation of velocity of the fluid with depth has also been investigated.

Graphs of velocity profiles obtained by varying parameters such as channel slope, energy coefficient, channel top width and roughness coefficient have been drawn. More graphs of variation of velocity with depth and also for comparison of velocity profiles for both open rectangular and triangular channels have been plotted. It is found out that the velocity of flow increases as depth increases and the velocity becomes maximum slightly below the free surface. Moreover, increase in the channel slope, energy coefficient and top-width leads to an increase in flow velocity whereas increase in roughness coefficient leads to a decrease in flow velocity. It is also found out that for a fixed flow depth and width an open rectangular channel is more hydraulically efficient than an open triangular channel.

This study goes a long way in control of floods, irrigation and in construction of channels such as house gutters.

CHAPTER ONE

1.0 INTRODUCTION

In year 1998 Kenya experienced a heavy rainfall called Elnino. In year 2002, heavy rainfall was again experienced. The rainfall was too heavy that bridges were swept away as rivers flooded. Water would flow from high places towards lowlands where the soil got saturated resulting into excess water remaining stagnant. Such a rainfall becomes a calamity as it is unexpected and the right measures have not been taken. Sometimes when the rainfall just goes above the normal, many places become flooded, houses and other structures get destroyed and such floods become a health hazard to people. This is because handling such unexpected amounts of water is a challenge even to engineers in Kenya. Designing channels that would control such an environmental disaster is very important. Open channels made of earth and concrete have been designed. These channels have been of different cross-sections such as trapezoidal, rectangular and circular. The fact that the flood problem still persists, there is need to come up with a hydraulically efficient channel, that is, a channel that would carry maximum discharge at a given slope, area of flow and roughness coefficient. Such a channel would be used to drive out excess water (floods) from the affected areas.

Kenya is an agricultural nation. Some farmers grow crops by irrigation. Channels that would effectively provide enough water for farming should therefore be put in place.

This study focuses on investigating efficient channels thus solving the problem of floods and also meeting the irrigation demands. Investigations of open rectangular and triangular channels have been carried out and analysis of the results has been done so as to determine which of the two channels is more hydraulically efficient.

1.1 Definitions

In this study several terms will be used extensively and in this section such terms are defined.

1.1.1 Fluids

Matter is said to be a fluid if it undergoes continuous deformation when some external force is applied. It is said to undergo deformation if the distance between any two neighboring molecules change. A fluid has no definite shape but assumes the shape of the container.

Fluids are conventionally classified as liquids and gases. Liquids do not change significantly in volume when subjected to change in pressure and temperature. For this reason they are treated as incompressible fluids. Gases show notable or appreciable volume changes when subjected to change in pressure and temperature. This implies that they are compressible.

1.1.2 Newtonian Fluid

A fluid is said to be a Newtonian fluid if it obeys the Newton's law of viscosity. Supposing that two adjacent layers are at a distance dy apart and the upper layer is faster (moving at a velocity of v+dv) than the lower one (moving at a velocity v).

The upper layer tends to draw the lower layer along with it by means of a force on the lower layer. At the same time, the lower layer tends to retard the faster upper one by an equal and opposite force acting on it. If the force F acts over an area of contact A, the stress τ is given by $\frac{F}{A}$

For straight and parallel motion of a given fluid, the tangential stress between two adjoining layers is proportional to the velocity gradient.

$$\tau = \frac{F}{A} \propto \frac{dv}{dy} \qquad (1.1)$$

or

$$\tau = \mu \frac{dv}{dy} \qquad (1.2)$$

Where μ, is a constant of proportionality known as the coefficient of viscosity of the fluid. The above equation strictly concerns the velocity gradient at a point. The change of velocity considered is that occurring over infinitesimal thickness. A fluid at rest has no tangential or shear stress but in motion the stress develops. The above equation is known as the Newtonian law and any fluid that satisfies it is said to be a Newtonian fluid.

1.1.3 Open Channel Flow

The flow of a liquid, for example, water in a conduit may either be open channel or pipe flow. The two kinds of flows are similar in many ways but differ in one important aspect. Open channel flow is characterized by a free surface whereas pipe flow has none. A free surface is defined as the surface of contact between the liquid and the overlying gaseous fluid. According to Chow (1973), in an open channel water does not fill the conduit completely. Flow in open channels is at the expense of the potential energy (P.E) and hence caused by gravity. Open channel flows are found in large and small scale, for example, a few centimeters in water treatment plants and over 10 m in large rivers. The mean velocity of flow may range from less than 0.01 m/s in tranquil waters to above 50 m/s in high-head spillways. The range of total discharge may extend from $0.001m^3/s$ in chemical plants to greater than 10,000 m^3/s in large rivers or spillways.

1.1.4 Types of Flow

There are several types of flows classified according to change in flow depth with respect to time and space. Flow is said to be steady if the depth of flow at a particular point does not change for the time interval under consideration. A flow in which depth changes with time and space is said to be unsteady. This is the most common type of flow and requires the solution of the energy, momentum and friction equations with time. Open channel flow is said to be uniform if the depth and velocity of flow are the same at every section of the channel. Hence it follows that uniform flow can only occur in prismatic channels. For steady uniform flow, depth and velocity is constant with both time and distance.

This constitutes the fundamental type of flow in an open channel. It occurs when gravitational forces are in equilibrium with resistance forces. A flow in which depth varies with distance but not with time is called steady non-uniform flow. The type of flow may either be gradually varied or rapidly varied. The former requires application of energy and frictional resistance equations.

1.1.5 Types of Channels

There are two groups of open channels namely natural and artificial channels. Artificial channels are channels made by man. They include irrigation canals, navigation canals, spillways, sewers, culverts and drainage ditches. They are usually constructed in a regular cross-section shape throughout and are thus prismatic channels (they do not widen or get narrower along the channel). In the field they are commonly constructed of concrete or earth and have the surface roughness reasonably well defined (although this may change with age). Analysis of flow in such well defined channels will give reasonably accurate results.

Natural channels are not regular or prismatic and their materials of construction can vary widely (although they are mainly of earth they can possess many different properties). The surface roughness will often change with time, distance and elevation. Consequently it becomes more difficult to accurately analyze and obtain satisfactory results for natural channels than it is with manmade ones. This situation may be further complicated if the boundary is not fixed, that is if erosion and deposition of sediments occur. For analysis various geometric properties of the channel cross-sections are required. For artificial channels these can usually be defined using simple geometric equations given the depth of flow.

The commonly needed geometric properties are defined as follows:-

i. Depth (y) is the vertical distance from the lowest point of the channel section to the free surface.
ii. Stage (z) is the vertical distance from the free surface to an arbitrary datum.
iii. Area (A) is the cross-sectional area of flow normal to the direction of flow.
iv. Surface width (T) is the width of channel section at the free surface.

v. Hydraulic radius (R) is the ratio of area to the wetted perimeter ($^A/_P$)

vi. Wetted perimeter (P) is the length of the wetted surface measured normal to the direction of flow.

vii. Hydraulic mean depth (D_m) is the ratio of area to surface width ($^A/_T$)

1.2 LITERATURE REVIEW

The earliest study on open channels was carried out by a French engineer called Chezy in 1768. He discovered the Chezy’s formula and Chezy’s constant. He was concerned with canal flows in his country France (Hamil,1995). Chezy’s formula did not provide results that satisfied engineers.

Manning discovered the Manning formula (Chow, 1973). He identified the coefficient of roughness called the Manning Coefficient, which takes into account the bed materials, degree of channel irregularity, variation in shape and size of the channel and relative effect of channel obstruction, vegetation growing in the channel and meandering, Chadwick (1993).

In 1973, Chow carried out a study on open channel flows and developed many relationships such as velocity formula for open channel flows (Chow,1973)

Sinha and Aggarwal (1980) investigated the development of the laminar flow of a viscous incompressible fluid from the entry to the fully developed situation in a straight circular pipe.

They observed that velocity increases more rapidly during the initial development of the flow in comparison to the downstream flow. It was observed that during the initial stages of the development of the flow, the rate of increase in stream wise velocity is larger and consequently the pressure drop is larger in comparison with their values further downstream. Rantz (1982) studied open channel flows and developed a method of measuring discharge in shallow and deep channels.

Nalluri and Adepoju (1985) analyzed experimental data on resistance to flow in smooth channels of circular cross- section. The results of the tests showed that the measured friction factors are larger than those of a pipe with equivalent diameter.

Crossley (1999) investigated strategies developed for the Euler's equations for application to the Saint Venant equations of open channel flow in order to reduce run times and improve the quality of solutions in the regions of discontinuities. Carlos and Santos (2000) considered an adjoint formulation for the non-linear potential flow equation.

Makhanu (2001) worked on development of simple hydraulic performance model of Sasumua Pipelines of Nairobi. Tuitoek and Hicks(2001) modelled unsteady flow in compound channels with an aim of controlling floods.

Kwanza et al (2007) carried out investigations on the effects of the channel width, slope of the channel and lateral discharge for both rectangular and trapezoidal channels. They noted that discharge increases as the specified parameters are varied upwards and that trapezoidal channels are more hydraulically efficient than rectangular ones.

The current study is on investigation of effects of parameters such as channel slope, energy coefficient, width and roughness coefficient on velocity. Investigation of variation of velocity with depth is also carried out. Investigation on the more hydraulically efficient channel between rectangular and triangular channels has been carried out.

This would solve the problem of flooding during heavy rains as well as the shortage of water for irrigation.

1.3 Statement of the Problem

This study is on investigating fluid flows through triangular and rectangular channels of same depth and width as shown in figure 1.1 below . The study aims at determining the more hydraulically efficient channel between open rectangular and triangular channels. The study investigates the relationship between depth of flow and flow velocity. It also investigates the effects of various parameters such as channel slope, roughness coefficient, energy coefficient and topwidth on flow velocity.

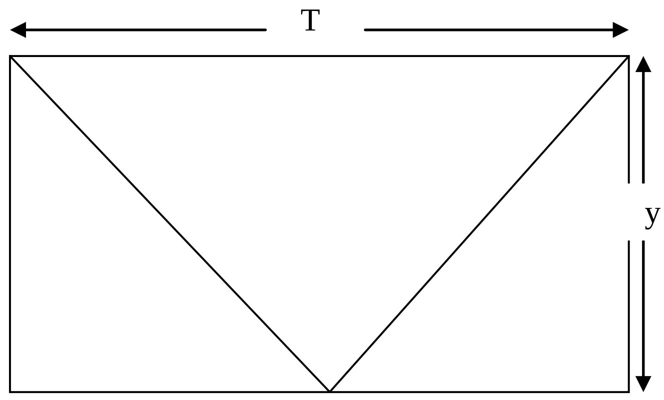

Figure 1.1: Cross section of the conduits under consideration.

1.4 Justification

Flooding has been a problem for many years. It has affected many people especially during heavy rains. Man has tried to drive out this water by constructing canals and waterways (channels) which direct the water to lakes and rivers. Sometimes this has been a big challenge especially in times of heavy rains. It has been a challenge even to the engineers to come up or to design a channel that can convey maximum amount of water efficiently. Therefore a hydraulically efficient channel has to be designed in order to overcome this problem of floods.

Farmers who grow crops by irrigation can also benefit a lot if a channel which can hold maximum discharge is put in place. Therefore this research can help engineers in designing hydraulically efficient open channels.

1.5 Objectives

1. To investigate the effects of channel slope, energy coefficient, width and roughness coefficient on velocity.
2. To investigate the variation of velocity with depth.
3. To investigate the more hydraulically efficient channel between open rectangular and triangular channels

1.6 Null Hypotheses

1. There is no effect of parameters on velocity.
2. There is no variation of velocity with depth.
3. Neither open rectangular nor open triangular channel is more hydraulically efficient than the other.

In this chapter we have generally looked at related literature on open channel flows, statement of the problem and the objectives of the study. In the next chapter the general equations governing open channel flows are outlined.

CHAPTER TWO

2.0 Introduction

In this chapter, we have considered assumptions made on open channel flows. The general governing equations are stated which are used to derive the specific equations for both rectangular and triangular channel flows.

2.1 Assumptions of open channel flows.

In this study, the following assumptions have been made;

i. That sedimentation is negligible.

ii. The roughness coefficient n will be quite small since the walls of the channel are of concrete.

iii. The fluid has constant density.

iv. The flow is unsteady.

2.2 Governing Equations

The law of conservation of mass states that mass can neither be created nor destroyed (Hamil, 1995). In other words mass will always be conserved. From the law of conservation of mass, discharge Q is given by

$$Q = AV \qquad (2.1)$$

From Chezy's formula V = $\sqrt{\left(\frac{\gamma}{K}\right)\left(\frac{A}{P}\right)S_0}$ $\qquad$ *(2.2)*

But $\frac{\gamma}{K} = C$ $\qquad$ *(2.3)*

And from the same equation,

$$\frac{A}{P} = R \qquad (2.4)$$

Substituting (2.3) and (2.4) in (2.2) we obtain,

$$V = C\sqrt{RS} \qquad (2.5)$$

Substituting (2.5) into (2.1) we obtain,

$$Q = AC\sqrt{RS} \qquad (2.6)$$

Also from Robert Manning's formula, we have

$$V = \frac{1}{n} R^{2/3} S^{1/2} \qquad (2.7)$$

Substituting (2.7) in (2.1) we have,

$$Q = A \frac{1}{n} R^{2/3} S^{1/2} \qquad (2.8)$$

From the two formulae mentioned above,

Hydraulic radius, $R = \frac{A}{P}$ *(2.9)*

From (2.9) as P decreases, R increases. Consequently an increase in R leads to an increase in Q.

Therefore for maximum discharge, we need to minimize the wetted perimeter P.

Consider a rectangular channel of width b and depth y.

The wetted perimeter is given by

$$P = b + 2y \qquad (2.10)$$

The cross-section area is given by

$$A = by \qquad (2.11)$$

Expressing width b in terms of area and depth, we have

$$b = \frac{A}{y} \qquad (2.12)$$

Substituting (2.12) into (2.10), it becomes

$$\left.\begin{array}{l} P = \dfrac{A}{y} + 2y \\ or \\ P = Ay^{-1} + 2y \end{array}\right\} \qquad (2.13)$$

For a given value of Area A, surface roughness n and channel slope s, P will be minimum when

$$\frac{dP}{dy} = 0 \qquad (2.14)$$

But $\dfrac{dP}{dy} = -Ay^{-2} + 2$ (2.15)

From (2.14) and (2.15)

$$-Ay^{-2} + 2 = 0 \qquad (2.16)$$

$$\frac{A}{y^2} = 2 \qquad (2.17)$$

$$2y^2 = A \tag{2.18}$$

To justify that p is just a minimum point

$$\frac{d^2p}{dy^2} > 0 \tag{2.19}$$

Differentiating (2.15) again we obtain,

$$\frac{d^2p}{dy^2} = 2Ay^{-3} = \frac{2A}{y^3} \tag{2.20}$$

But A and y are positive values hence

$$\frac{d^2p}{dy^2} > 0 \tag{2.21}$$

Substituting (2.11) into (2.18), it becomes

$$\left.\begin{aligned} 2y^2 &= by \\ 2y &= b \end{aligned}\right\} \tag{2.22}$$

Therefore for maximum discharge the width b should be twice the depth y.

The St. Venant’s equation of continuity governing open channel flows of arbitrary shape is given as

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} - q = 0 \tag{2.23}$$

This equation is obtained from the law of conversation of mass that states that matter can neither be created nor destroyed.

But discharge

$$Q = AV \tag{2.24}$$

Differentiating (2.24) partially with respect to x and substituting in (2.23), it becomes

$$V\frac{\partial A}{\partial x}+A\frac{\partial V}{\partial x}+\frac{\partial A}{\partial t}=q \qquad (2.25)$$

The flow area is assumed to be a known function of the depth; therefore, derivatives of A in (2.25) may be expressed in terms of y as

$$\frac{\partial A}{\partial x}=\frac{dA}{dy}\frac{\partial y}{\partial x}=T\frac{\partial y}{\partial x} \qquad (2.26)$$

$$\frac{\partial A}{\partial t}=\frac{dA}{dy}\frac{\partial y}{\partial t}=T\frac{\partial y}{\partial t} \qquad (2.27)$$

Where T = channel top width and it assumed that T is determined by

$$T=\frac{dA}{dy} \qquad (2.28)$$

Substituting (2.26) and (2.27) in (2.25) , we have

$$VT\frac{\partial y}{\partial x}+A\frac{\partial V}{\partial x}+T\frac{\partial y}{\partial t}=q \qquad (2.29)$$

Dividing (2.29) through by T, it becomes

$$\frac{\partial y}{\partial t}+\frac{A}{T}\frac{\partial V}{\partial x}+V\frac{\partial y}{\partial x}=\frac{q}{T} \qquad (2.30)$$

Equation (2.30) is the general equation of continuity for open channel flows.

The St. Venants equation of momentum for open channels of arbitrary shape is [illegible] (2.31)

This equation is obtained from Newton second law which states that the rate of change of momentum of a body is proportional to the net force and takes place in the direction in which this force acts.

In the next chapter specific governing equations for both open rectangular and triangular channel flows are derived from the general governing equations.

CHAPTER THREE

3.0 Introduction

In this chapter specific governing equations for our problem have been derived from the general governing equations. The derived equations are non linear and are therefore solved by finite difference method. A suitable mesh is defined in order to convert the equations into finite difference form.

Initial and boundary conditions of the flow have been stated. These conditions are same for both rectangular and triangular channels. Variation of velocity with depth has been investigated. Comparison of fluids flows through open rectangular and triangular channels has been carried out. Effects of varying various flow parameters on velocity have been investigated. The varied parameters are:

i. The channel slope (s_0).

ii. The energy coefficient (α).

iii. The top width (T).

iv. The roughness coefficient (n).

Since roughness coefficient is one of the parameters to be varied it is important to put down the Manning's roughness coefficients for open channels. The coefficients are as shown in the table below

Table 3.1 The Manning n for open channels (Gupta, V.1984)

Nature of the channel	**Manning n**
Glass	0.010
Brass	0.011
Steel, smooth	0.012
Steel, painted	0.014
Steel, riveted	0.015
Cast iron	0.013
Concrete, finished	0.012
Concrete, unfinished	0.014
Planed wood	0.012
Clay tile	0.014
Brickwork	0.015
Asphalt	0.016
Corrugated metal	0.022
Rubble masonry	0.025
Gravelly	0.025
Weedy	0.030
Stony, cobbles	0.035

3.1 Specific Equations Governing Open Channel Flows.

3.1.1 Rectangular channels

$$A = by \qquad (2.32)$$

and

$$T = b \qquad (2.33)$$

Substituting (2.32) and (2.33) into (2.30) we obtain

$$\frac{\partial y}{\partial t} + y\frac{\partial V}{\partial x} + V\frac{\partial y}{\partial x} = \frac{q}{b} \qquad (2.34)$$

Equation (2.34) is the specific equation of continuity for open rectangular channel flows.

Again substituting (32) into (31) it becomes

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial y}{\partial x} + g(S_f - S_0) = -\frac{q}{A} \qquad (2.35)$$

Where $A = by$

Equation (2.35) is the specific equation of momentum for an open rectangular channel flow.

3.1.2 Triangular channel

For an open triangular channel

$$A = ½\, Ty \qquad (2.36)$$

where T is the top width of the channel. Substituting equation (2.36) into the general equation of continuity for channels with arbitrary shape that is equation (2.30) we obtain

$$\frac{\partial y}{\partial t} + 0.5y\frac{\partial V}{\partial x} + V\frac{\partial y}{\partial x} = \frac{q}{T} \qquad (2.37)$$

(2.37) is the specific equation of continuity for an open triangular channel flow.

Substituting (2.36) into (2.31) the equation of momentum for a triangular channel flow becomes,

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial y}{\partial x} + g\left(S_f - S_o\right) = -\frac{qV}{A} \qquad (2.38)$$

Where $A = 0.5Ty$

3.2 Method of solution

The specific governing equations above are partial differential equations. They are non linear and cannot be solved by analytical methods. They are approximated by the application of finite difference method. The use of finite difference technique in solving partial differential equations is a three-step process namely:

i. The partial differential equations are approximated by a set of linear equations relating to the values of the functions at each mesh point.
ii. Solving of the set of algebraic equations generated in (i) above
iii. An iteration procedure has to be developed which takes into account the non-linear character of the equation.

3.3 Definition of a Mesh

In order to approximate the partial differential equations by a set of finite difference equations, we first define a suitable mesh. The type of mesh involves subdividing the rectangular region of interest into uniform rectangular elements centered about mesh points whose mesh are denoted by variables i and j. Index i refers to the distance along the channel while j refers to time as shown in the figure 3.1 below

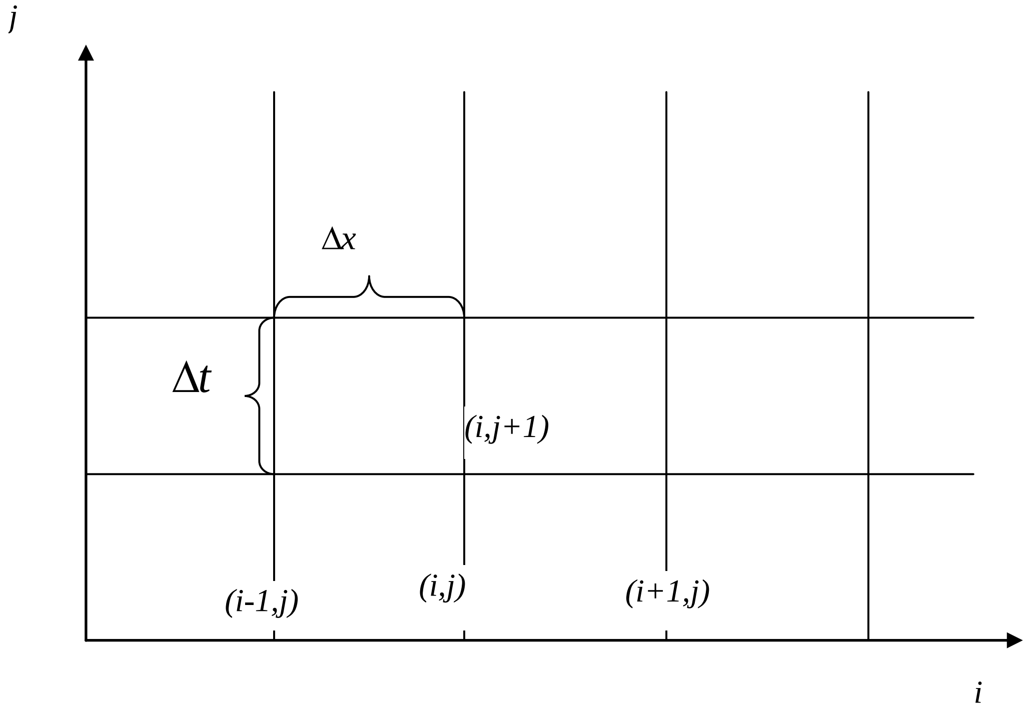

Figure 3.1: Mesh notation of the computation domain

From the mesh above, Δt represents change in t while Δx represents change in x. The partial derivatives in the governing equations can now be replaced by their corresponding finite difference approximations given below:

$$\frac{\partial V}{\partial t} = \frac{V(i,j+1) - V(i,j)}{\Delta t} \qquad (2.39)$$

$$\frac{\partial y}{\partial t} = \frac{y(i,j+1) - 0.5\left(y(i+1,j) + y(i-1,j)\right)}{\Delta t} \qquad (2.40)$$

$$S_f = \frac{\left(\frac{V(i+1,j) + V(i-1,j)}{2}\right)^2 n^2}{R^{4/3}} \qquad (2.41)$$

$$\frac{\partial V}{\partial x} = \frac{V(i+1,j) - V(i-1,j)}{2\Delta x} \qquad (2.42)$$

$$\frac{\partial y}{\partial x} = \frac{y(i+1,j) - y(i-1,j)}{2\Delta x} \qquad (2.43)$$

3.4 Finite Difference Equations

3.4.1 Rectangular channels

The equation of continuity for an open rectangular channel (equation 34) written in finite difference is

$$\frac{y(i,j+1) - 0.5\left(y(i+1,j) + y(i-1,j)\right)}{\Delta t} + y(i,j)\frac{V(i+1,j) - V(i-1,j)}{2\Delta x} + V(i,j)\frac{y(i+1,j) - y(i-1,j)}{2\Delta x} - \frac{q}{b} = 0$$

Making *y (i,j +1)* the subject, we obtain

$$y(i,j+1) = 0.5\left(y(i+1,j) + y(i-1,j)\right) - \Delta t\left\{ y(i,j)\frac{V(i+1,j) - V(i-1,j)}{2\Delta x} + V(i,j)\frac{y(i+1,j) - y(i-1,j)}{2\Delta x} - \frac{q}{b} \right\}$$

(2.45)

The equation of momentum of an open rectangular channel (equation 2.31) in finite difference is

$$\frac{V(i,j+1)-0.5\left(V(i+1,j)+V(i-1,j)\right)}{\Delta t}+V(i,j)\frac{V(i+1,j)-V(i-1,j)}{2\Delta x}+g\frac{y(i+1,j)-y(i-1,j)}{2\Delta x}-g(S_0-S_f)=-\frac{q(i,j)V}{A} \qquad (2.46)$$

Where S_f is as given by equation (2.41).

Making V *(i,j+1)* the subject, (2.46) becomes

$$V(i,j+1)=0.5\left(V(i+1,j)+V(i-1,j)\right)-\Delta t\left\{V(i,j)\frac{V(i+1,j)-V(i-1,j)}{2\Delta x}+g\frac{y(i+1,j)-y(i-1,j)}{2\Delta x}-g(S_0-S_f)+\frac{q(i,j)V}{A}\right\}$$

(2.47)

3.4.2 Triangular channels

The equation of continuity for an open triangular channel (equation 2.37) in finite difference is

$$\frac{y(i,j+1)-0.5\left(y(i+1,j)+y(i-1,j)\right)}{\Delta t}+0.5\,y(i,j)\frac{V(i+1,j)-V(i-1,j)}{2\Delta x}+V(i,j)\frac{y(i+1,j)-y(i-1,j)}{2\Delta x}=\frac{q}{T} \qquad (2.48)$$

Making *y(i,j+1)* the subject (2.48) becomes

$$y(i,j+1)=0.5\left(y(i+1,j)+y(i-1,j)\right)-\Delta t\left\{0.5\,y(i,j)\frac{V(i+1,j)-V(i-1,j)}{2\Delta x}+V(i,j)\frac{y(i+1,j)-y(i-1,j)}{2\Delta x}-\frac{q}{T}\right\} \qquad (2.49)$$

Equation of momentum of an open triangular channel flow (equation 38) in finite difference is

$$\frac{V(i,j+1)-\frac{1}{2}\left[V(i+1,j)+V(i-1,j)\right]}{\Delta t}+V(i,j)\frac{V(i+1,j)-V(i-1,j)}{2\Delta x}+g\frac{y(i+1,j)-y(i-1,j)}{2\Delta x}-g(S_0-S_f)=-\frac{q(i,j)}{A}V \qquad (2.50)$$

Making *V (i,j+1)* the subject (2.50) becomes

$$V(i,j+1)=\frac{1}{2}\left[V(i+1,j)+V(i-1,j)\right]-\Delta t\left\{V(i,j)\frac{V(i+1,j)-V(i-1,j)}{2\Delta x}+g\frac{y(i+1,j)-y(i-1,j)}{2\Delta x}-g(S_0-S_f)+\frac{q(i,j)}{A}V\right\} \qquad (2.51)$$

3.5 Conditions of Flow for both Triangular and Rectangular Channels

The initial conditions are

$$V(x, o) = 14, \qquad y(x, o) = 1.8 \qquad (2.52)$$

And the boundary conditions are

$$\left.\begin{array}{l} V(x_0,t) = 14 \quad y(x_0,t) = 1.8 \\ V(x_n,t) = 14 \quad y(x_n,t) = 1.8 \end{array}\right\} \qquad (2.53)$$

The initial conditions in finite difference are

$$V(i, 0) = 14 \qquad y(i, 0) = 1.8 \qquad (2.54)$$

The boundary conditions in finite difference are

$$\left.\begin{array}{l} V(x_0,j) = 14 \quad y(x_0,j)= 1.8 \\ V(x_n,j)= 14 \quad y(x_n,j)= 1.8 \end{array}\right\} \qquad (2.55)$$

In these equations i refers to the distance along the channel while j refers to time.

X_o and X_n refer to the entry point and exit point respectively of the section of the channel under consideration. In this study the section of the channel under consideration was 4m long.

At the entry point the velocity and depth of flow were assumed to be 14 m/s and 1.8m respectively because the flow is fully developed in the section of the channel under consideration. Therefore reasonable values of velocity and depth were considered.

In our investigation we have considered a uniform mesh in which $\Delta x = 0.1$ *and* $\Delta t = 0.0012$. The convergence is attained by the condition, $\frac{\Delta y}{(\Delta t)2} < \frac{1}{2}$

3.6 Velocity profiles for rectangular and triangular channels

3.6.0 Introduction

Discharge in a channel is directly proportional to velocity of the flow. In order to determine which channel is more hydraulically efficient between open rectangular and triangular channels velocity profiles were plotted as shown in figure 3.1 below. For comparison the dimensions of the two channels were fixed, that is the depth and width. The finite equations for the two channels were subjected to the same initial and boundary conditions and by running the computer programme(visual-basic) values of depth and velocity were obtained. By choosing a point along the length of the channels graphs of velocity against depth for both channels were plotted separately. Finally, the two curves were plotted on the same axes as shown in the figure 3.2 below.

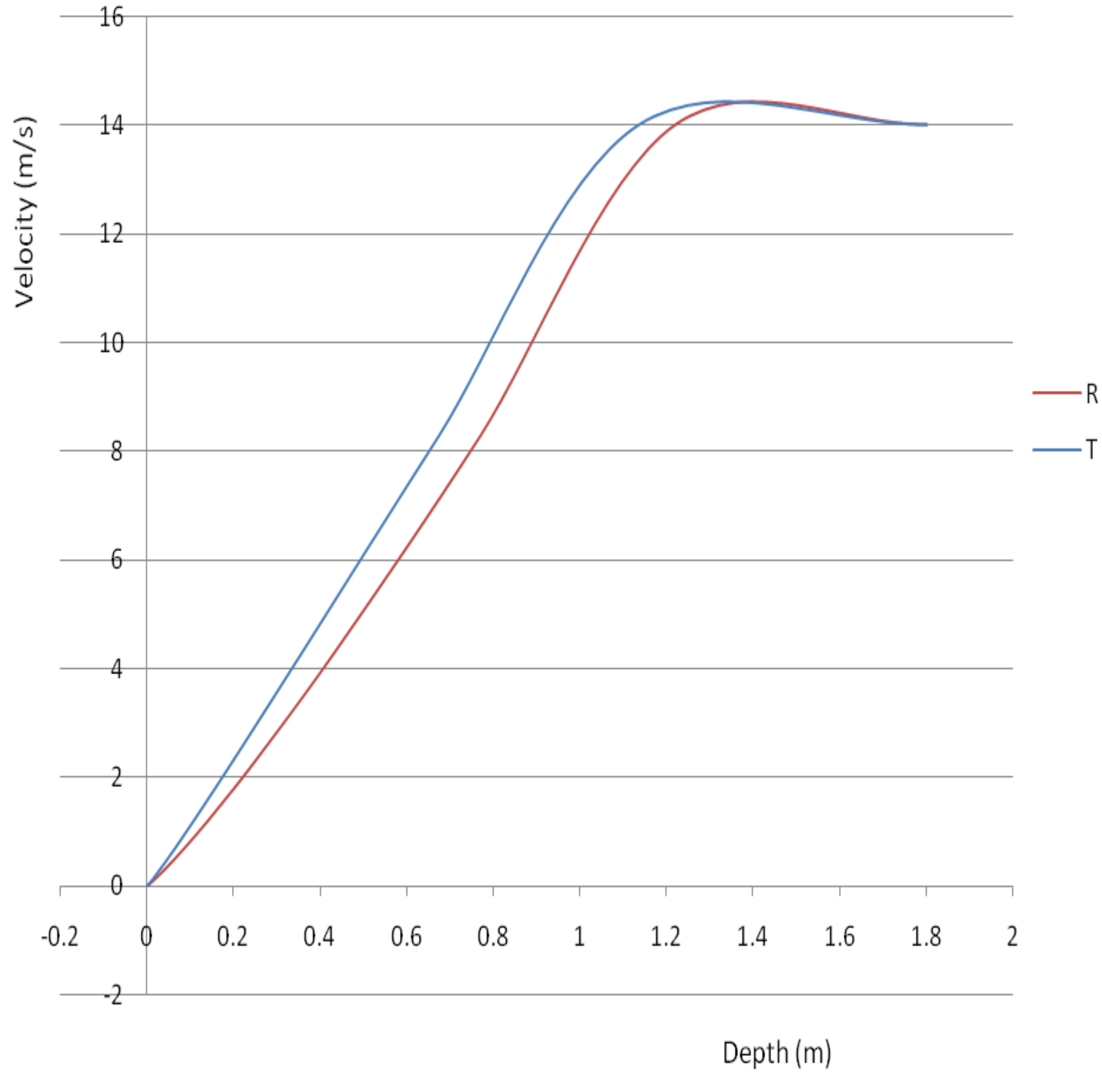

Figure 3.2: Velocity profiles for open rectangular (R) and triangular (T) channels.

3.6.1 Results and discussion

From figure 3.2 it is noted that the velocity of the flow increases to maximum slightly below the free surface and then it starts to decrease. At the bottom of the channel, the velocity of the fluid is zero due to the no-slip condition. Velocity then increases because frictional forces decrease as the vertical distance from bottom increases and becomes maximum slightly below the free surface. Maximum velocity cannot be attained at the free surface due to the effects of surface tension. Air in motion also affects the flow velocity.

From figure 3.2, we observe that for a fixed flow depth and width of the channel, the velocity profiles for a triangular channel are higher than for a rectangular channel.

A large wetted perimeter results in high resistance to the motion of the fluid from the walls of the channel. These resistance forces have an overall effect of reduced velocities. A triangular channel has a less wetted perimeter compared to a rectangular channel which results to less resistance hence high flow velocities.

3.7 Effects of varying channel slope on velocity

3.7.0 Introduction

In this section effects of varying the slope of the channel on velocity of the flow were investigated. Velocity profiles for both rectangular and triangular channels were first obtained when So = 0.0002. The value of So was then varied upwards to 0.002 while the other parameters remained constant and velocity profiles for both channels were again obtained. Finally the four curves were combined on the same axes as shown in figure 3.3 below.

	Rect	Tri
So = 0.002	I	III
So = 0.0002	II	IV

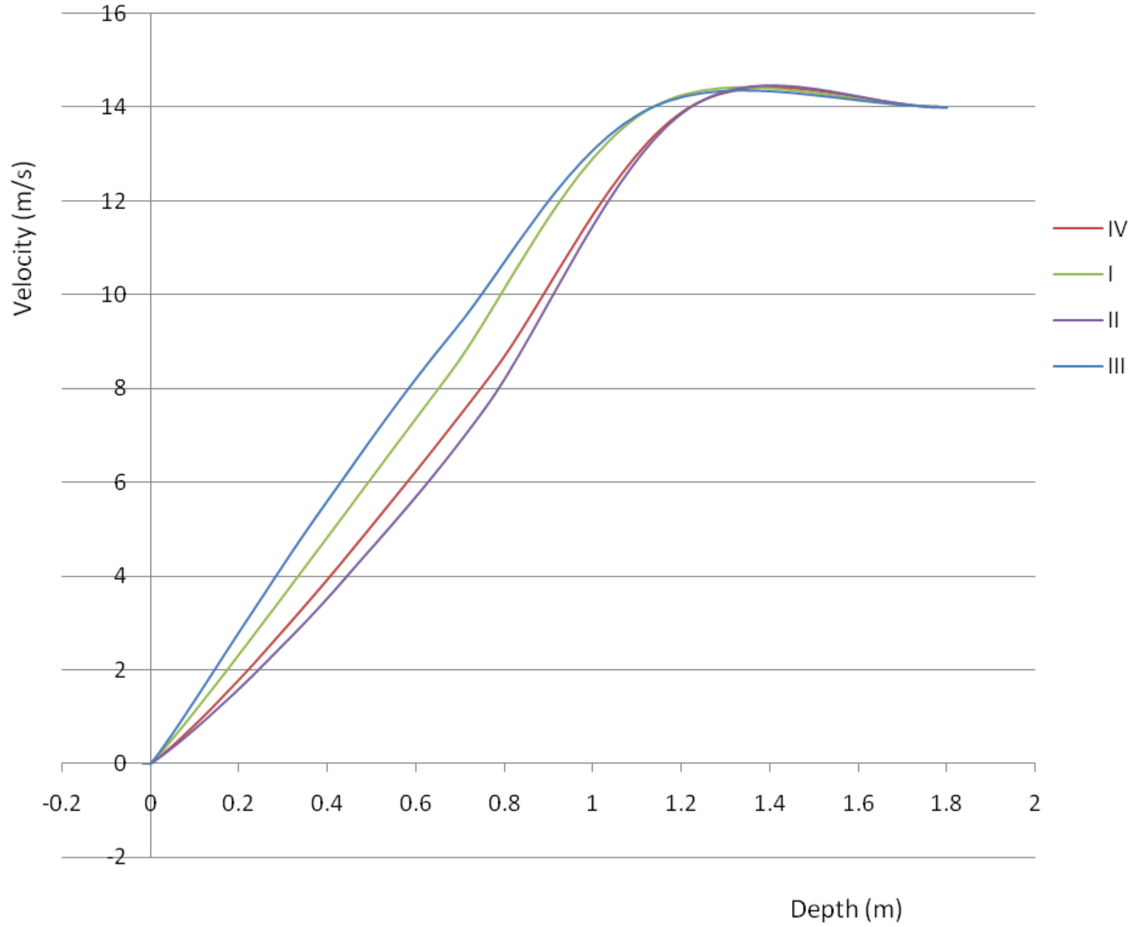

Figure 3.3: Effects of Varying channel Slope on Velocity

3.7.1 Results and discussion

From figure 3.3, for the same depth and width increasing the channel slope from 0.0002 to 0.002 increases the velocity of the flow in both channels. In other words, the velocity values when the slope is 0.002 are higher than when the slope is 0.0002.

According to velocity formula (equation 2.7), velocity is directly proportional to the slope and therefore an increase in slope leads to an increase in velocity.

3.8 Effects of varying energy coefficient on velocity

3.8.0 Introduction

Energy coefficient refers to the ratio of the energy transmitted forward in a flow per unit crest length at a point in shallow and deep water. To investigate the effects of varying energy coefficient velocity profiles for both channels were first obtained at $\propto$ = 2. Then the other parameters,that is,the channel slope, roughness coefficient and top width remained constant as the energy coefficient was varied upwards to 2.5 and other two curves were obtained. The four curves were plotted on the same axes as shown in figure 3.4 below.

	Rect	Tri
α=2	I	III
α=2.5	II	IV

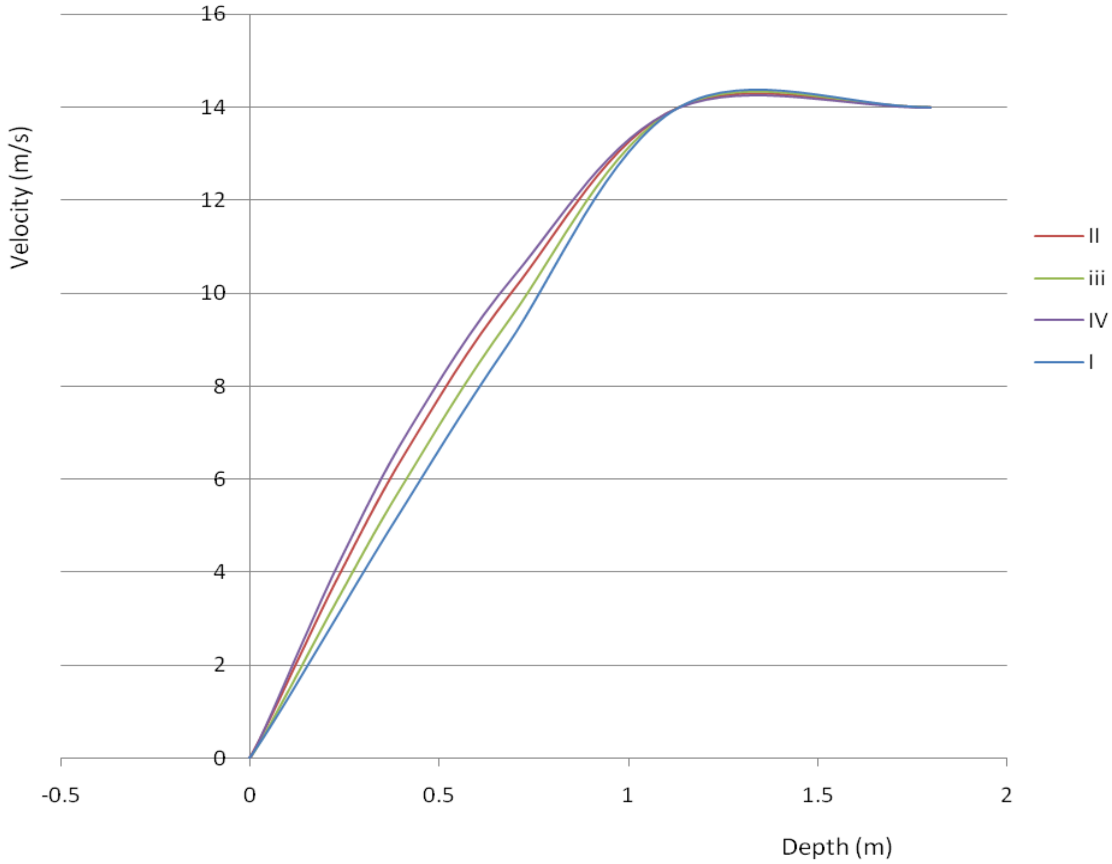

Figure 3.4: Effects of Varying Energy Coefficient on Velocity

3.8.1 Results and discussion

From figure 3.4, we observe that as the energy coefficient increases from 2 to 2.5 velocity increases. According to kinetic theory of matter, fluid molecules possess kinetic energy (energy of motion). If energy of the flow increases, kinetic energy of the particles also increases making the particles to move faster.

3.9 Effects of varying top-width on velocity

3.9.0 Introduction

To investigate the effects of varying the top-width of the channel on velocity of the flow, curves for both channels when T = 1m and when T = 3m were obtained and plotted on the same axes as shown in figure 3.5 below

	Rect	Tri
T = 3	I	III
T= 1	II	IV

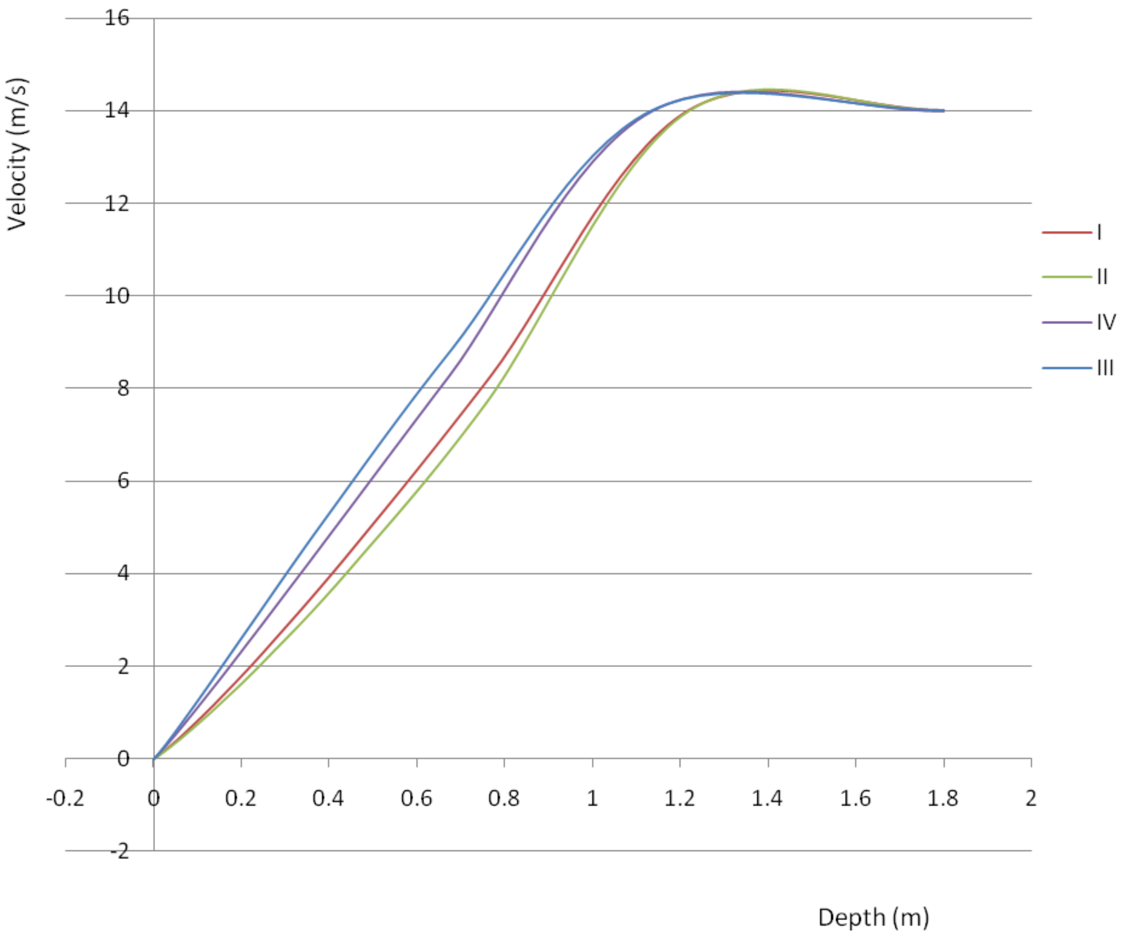

Figure 3.5: Effects of Varying Top-Width on Velocity

3.9.1 Results and discussion

From figure 3.5, we note that for the same depth of flow the velocity profiles for triangular channels are higher compared to the rectangular channel when the top width is 3 metres and 1 metre. This is because for the same depth of flow the triangular channel had a less wetted perimeter compared to the rectangular channel. Less wetted perimeter results in less resistance from the walls of the channel leading to higher velocities.

From figure 3.5 above we also note that for the same depth of flow the velocity profiles for the triangular channel when top width is 3 metres are higher than when top width is 1 metre. This is because when top width is 3 metres the hydraulic radius is higher than when top width is 1 metre. Higher hydraulic radius implies higher velocity as velocity is directly proportional to the hydraulically radius.

3.10 Effects of varying roughness coefficient on velocity

3.10.0 Introduction

Channel walls are different in nature because they are made up of different materials. Its therefore important to investigate how flow velocity is affected by varying roughness coefficient. In this study velocity profiles for both channels were first obtained when n = 0.012

The value of n was then increased to 0.025 while the other parameter remains constant and more curves were obtained. All the four curves were finally plotted on the same axes as shown in the figure 3.6 below.

	Rect	Tri
n = 0.012	I	III
n = 0.025	II	IV

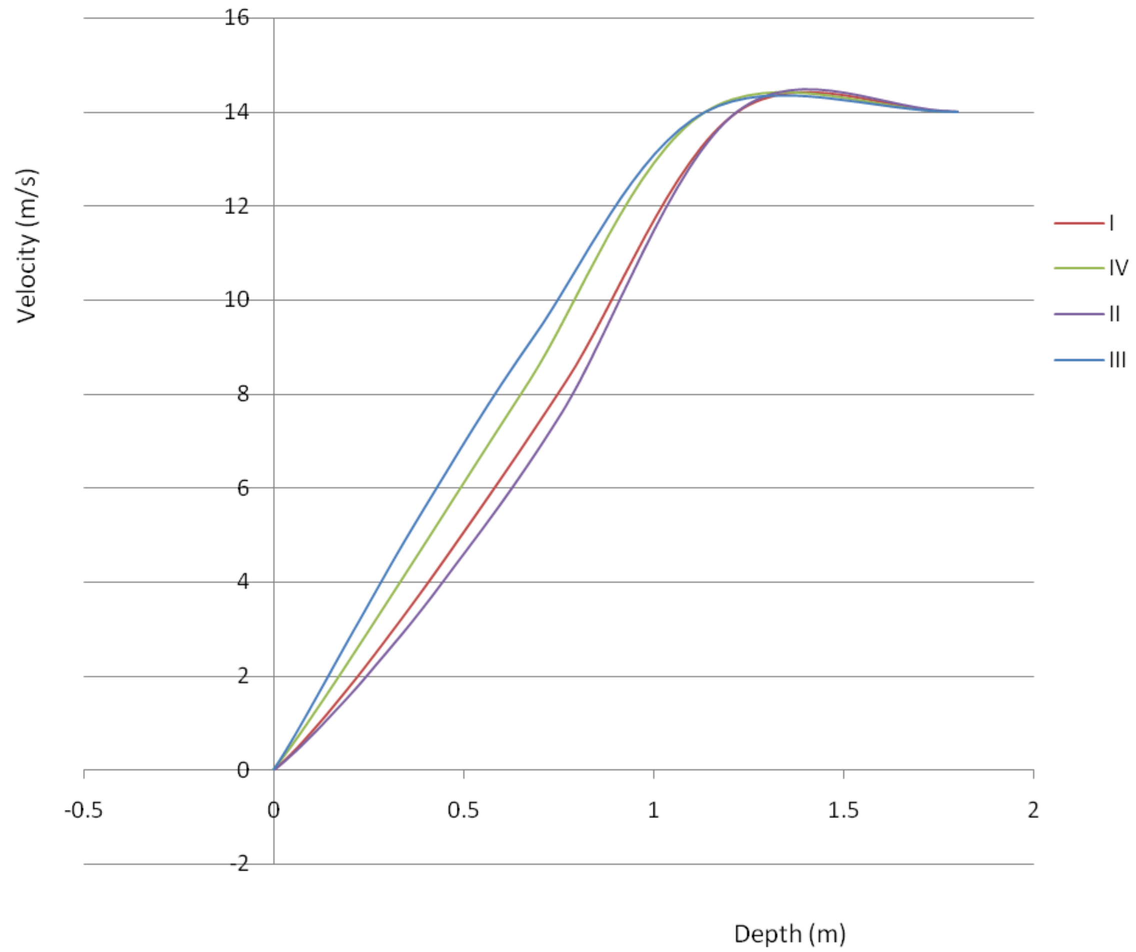

Figure 3.6: Effects of Varying Roughness Coefficient on Velocity

3.10.1 Results and discussion

From figure 3.6, for the same depth and width but varying roughness coefficient the triangular channel exhibits higher velocities compared to rectangular channel. The triangular channel has a less wetted perimeter compared to the rectangular channel and therefore for any material making up the channel walls there will be less resistance from the triangular channel as opposed to the rectangular channel. We also observe that for a triangular channel the velocity profiles are higher when roughness coefficient is 0.012 compared to the velocity profiles when the roughness coefficient is 0.025. An increase in roughness coefficient results to large shear stresses at the sides of the channel. Therefore, the motion of the fluid at or near the surface of the conduit will be reduced. The velocities of the neighbouring molecules will also be lowered due to constant bombardment with the slow moving molecules leading to an overall reduction in the flow velocity.

3.11 Validation of results

From this study when energy coefficient and roughness coefficient are negligible, the results obtained show the same trend as those obtained by Kwanza *et al* (2007)

Results of the study have been discussed in this chapter. It is therefore important to draw conclusions from these results and this has been done in the next chapter. Recommendations of areas on open channel flows that still require further research have also been outlined.

CHAPTER FOUR

4.0 Introduction

In Chapter Three the effects of variation of flow parameters on velocity have been investigated. Investigation of fluid flow in both rectangular and triangular channels and also of variation of velocity with depth has been done. Discussion of the results was also done. This chapter presents conclusion of what was carried out in chapter three and recommendations on areas that require further research.

4.1 Conclusion

1. Analysis of the effects of varying channel slope, top width, energy coefficient and roughness coefficient has been carried out. The conclusion is that for both rectangular and triangular channels increasing energy coefficient, channel slope and top width leads to an increase in flow velocity whereas increasing roughness coefficient leads to decrease in velocity.
2. Velocity profiles for both rectangular and triangular channels indicate that an open rectangular channel is more hydraulically efficient than an open triangular channel considering the same flow depth and width. This is because discharge is a function of area and velocity and for a fixed depth and width, cross-sectional area for a rectangular channel is higher than for a triangular channel.
3. Flow velocity increases with increase in depth and becomes maximum slightly below the free surface.

4.2 Recommendations

The results obtained in this study are theoretical. Therefore it is recommended that an experimental approach to this problem be undertaken taken and compare the results with the theoretical ones. This calls for more research. It is also recommended that further research should be carried out on

i. Effects of lateral inflow on discharge
ii. Effects of lateral outflow on discharge
iii. Fluid flows through elliptic channels
iv. Fluid flows in rectangular and triangular channels and solving the partial differential equations using finite element method or any other numerical technique.

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Appendix I

PROGRAMME FOR AN OPEN TRIANGULAR CHANNEL

```
Private Sub Command1_Click()
Dim Y(0 To 40, 0 To 50), U(0 To 40, 0 To 50), q As Single, Sf As Single, g As Single
Dim delX As Double, delT As Double, I As Integer, J As Integer, ITMAX As Single, N As Integer
Dim ITCOUNT As Integer, theta As Single, So As Single, A As Single, yi As Single, P As Single
Dim FILENUM As Byte, T As Single, alpha As Single, ni As Single
N = 40: delX = 0.1: q = 0: g = 9.81: So = 0.02: alpha = 2.5: T = 2:  yi = 2: ni = 0.012
'N is the no. of subdivisions along the channel.  N and ITMAX should not go beyond 50 and 360 resp.
delT = 0.0012
'********************************************************
ITMAX = 50   'no of subdivisions on the time
FILENUM = FreeFile()
A = 0.5 * T * yi
'yi = A / (0.5 * T)
P = (4 * (yi ^ 2) + (T ^ 2)) ^ 0.5
Open "C:\Documents and Settings\da\Desktop\JANE_2(TRI)\JaneProfiles.txt" For Append As FILENUM
Rem Initial condition
For I = 0 To N
```

```
For J = 0 To ITMAX      'J is time

  'U(I, 0) = 2: Y(I, 0) = 0.1
        U(I, 0) = 14#: Y(I, 0) = 1.8
Next
Next
 '  Boundary conditions
For I = 10 To N
For J = 10 To ITMAX
  U(1, J) = 14: Y(1, J) = 1.8      'entry values
   U(N, J) = 14: Y(N, J) = 1.8     'exit values
Next
Next
'Solving for velocities
For I = 1 To N - 1
  For J = 0 To ITMAX - 1
    'calculate Y
    Y(I, J + 1) = 0.5 * (Y(I - 1, J) + Y(I + 1, J)) - delT * (((A / T) * (U(I + 1, J) - U(I - 1, J)) / (2 * delX)) + (U(I, J) * ((Y(I + 1, J) - Y(I - 1, J)) / (2 * delX))) - q / T)
    'calculate U
    U(I, J + 1) = (0.5 * (U(I - 1, J) + U(I + 1, J))) - delT * ((alpha * U(I, J) * (U(I + 1, J) - U(I - 1, J)) / (2 * delX)) + (g * (Y(I + 1, J) - Y(I - 1, J)) / (2 * delX)) - (g * (So - ((0.5 * ni ^ 2 / (A / P) ^ (4 / 3)) * (U(I - 1, J) ^ 2 + U(I + 1, J) ^ 2)))) + q * U(I, J) /
```

```
A)
Next
Next
'I = 10 'distance along channel
Print #FILENUM, I, "q = " & q



For J = 0 To ITMAX
For I = 0 To N
    Print #FILENUM, U(I, J);
        If I = N Then Print #FILENUM, vbCrLf;
Next
Next
'************************unsteady values
Close #FILENUM
MsgBox "AM THROUGH!!!", vbCritical, "Peter"
End Sub
Private Sub Command2_Click()
On Error GoTo kan
Kill "C:\Documents and Settings\da\Desktop\JANE_2(TRI)\JaneProfiles.txt"
kan:
Exit Sub
End Sub
```

Appendix II

PROGRAMME FOR AN OPEN RECTANGULAR CHANNEL

```
Private Sub Command1_Click()
Dim Y(0 To 40, 0 To 50), U(0 To 40, 0 To 50), q As Single, b As Single, g As Single
Dim delX As Double, delT As Double, I As Integer, J As Integer, ITMAX As Single, N As Integer, ni As Single
Dim ITCOUNT As Integer, So As Single, A As Single, yi As Single, P As Single
Dim FILENUM As Byte, alpha As Single
N = 40: delX = 0.1: q = 0: g = 9.81: So = 0.02: alpha = 1: b = 4: ni = 0.012: yi = 1
'N is the no. of subdivisions along the channel.
delT = 0.0012
'********************************************************
ITMAX = 50   'no of subdivisions on the time
FILENUM = FreeFile()
A = b * yi
P = b + 2 * yi
Open "C:\Documents and Settings\da\Desktop\JANE(RECT)\JaneProfiles.txt" For Append As FILENUM
Rem Initial condition
For I = 0 To N
For J = 0 To ITMAX      'J is time
    'U(I, 0) = 2: Y(I, 0) = 0.1
     U(I, 0) = 14#: Y(I, 0) = 1.8
```

```
Next
Next
 '  Boundary conditions
For I = 0 To N
For J = 0 To ITMAX
  U(0, J) = 14: Y(0, J) = 1.8      'entry values
   U(40, J) = 14: Y(40, J) = 1.8     'exit values
Next
Next
For I = 1 To N - 1
  For J = 0 To ITMAX - 1
   'calculate Y
    Y(I, J + 1) = 0.5 * (Y(I - 1, J) + Y(I + 1, J)) - delT * ((A / b) * (U(I + 1, J) - U(I -
1, J)) / (2 * delX) + U(I, J) * (Y(I + 1, J) - Y(I - 1, J)) / (2 * delX) - (q / b))
    'calculate U
    U(I, J + 1) = 0.5 * (U(I - 1, J) + U(I + 1, J)) - delT * (alpha * U(I, J) * (U(I + 1, J)
- U(I - 1, J)) / (2 * delX) + g * (Y(I + 1, J) - Y(I - 1, J)) / (2 * delX) - g * (So - ((0.5
* ni ^ 2 / (A / P) ^ (4 / 3)) * (U(I - 1, J) ^ 2 + U(I + 1, J) ^ 2))))
Next
Next
Print #FILENUM, I, "q = " & q
For J = 0 To ITMAX
For I = 0 To N
    Print #FILENUM, U(I, J);
```

```
    If I = N Then Print #FILENUM, vbCrLf;
Next
Next
'***********************unsteady values
Close #FILENUM
MsgBox "AM THROUGH!!!", vbCritical, "Peter"
End Sub
Private Sub Command2_Click()
On Error GoTo kan
Kill "C:\Documents and Settings\da\Desktop\JANE(RECT)\JaneProfiles.txt"
kan:
Exit Sub
End Sub
```