ANALYSIS OF CONVECTIVE HEAT TRANSFER IN A FLUID FLOW OVER AN IMMERSED AXI-SYMMETRICAL BODY WITH CURVED SURFACES

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DECLARATION

This thesis is my original work and has not been presented for a degree in any other University.

Sign…………………………........   Date……………………………….

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This thesis has been submitted for examination with our approval as the University supervisors.

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JKUAT Kenya

DEDICATION

This thesis is dedicated to my parents Mr. and Mrs.Gathungu Ndung'u, my siblings

Susan, Simon and Dave.

ACKNOWLEDGEMENT

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LIST OF SYMBOLS

ROMAN SYMBOLS

Symbol	Meaning
C_p	Specific heat at constant pressure. $Jkg^{-1} K^{-1}$
L	Reference length ,M
m	Real number
P	Pressure, Pa
E_c	Eckert number
Pr	Prandtl number
Pe	Peclet number
Re	Reynolds number
Q	Quantity of heat added to the system, Joules(J)
$\dot{q}$	Heat generated in the boundary layer, Joules
T	Temperature, K
Ts	Temperature of the body's surface, K
T_∞	Free stream velocity, m/s
h	heat transfer coefficient. $h=q_w(T_w-T_\infty)$,W/m^2K
U	Outer flow fluid velocity in the x-direction, ms^{-1}

V	Reference fluid velocity in the y-direction, ms^{-1}
F_x, F_y	Body forces, Newtons along the x and y directions respectively
x,y,z	Cartesian co-ordinates
i,j,k	Unit vectors in the x,y and z directions respectively
$\frac{D}{Dt}$	Material derivative $(= \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z})$
∇	Gradient operator $(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z})$
∇^2	Laplacian operator $(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2})$

GREEK SYMBOLS

ϑ	Kinematic viscosity, m^2s^{-1} $\vartheta = \frac{\mu}{\rho}$
μ	Absolute viscosity (dynamic viscosity coefficient), kg/ms
ρ	Fluid density, kgm^{-3}
φ	Viscous dissipation function
π	Pi=3.141592654
δ	Boundary layer thickness, m
$\sigma_{i,j}$	Normal stresses, Nm^{-2}
$\tau_{i,j}$	Shear stresses, Nm^{-2}

ABSTRACT

Convective heat transfer in a homogeneous fluid flow Reynolds number of order less than 2000 over an immersed axi-symmetrical body with curved surfaces has been investigated. The fluid flow in consideration was unsteady and of constant density .This study analysed the extent to which convective heat transfer has on drag and lift on bodies submerged in fluid. The different temperature profiles which were as a result of temperature gradients, caused the convective heat transfer. These different temperature profiles were brought about by frictional forces on and within the surface of the body when fluid flowed over it. Velocity variations were also determined and were used to evaluate these temperature profiles. To obtain these profiles, various flow parameters were varied in the equations governing the fluid flow. These equations were non-linear and there exists no analytical method of solving them, hence a suitable numerical method in this case finite difference method was used. Results of the velocity variations and temperature variations were obtained followed by graphical representation of the results. It was however noted that increase in the Reynolds number leads to an increase in the heat dissipation. The heat dissipation increases with increase in surface curvature. These results have major application in designing devices requiring high manoeuvrability and less resistance to the motion e.g. aerofoil, spray atomizers and cooling fans.

CHAPTER ONE

1.0 INTRODUCTION

Convective heat transfer theory is of utmost importance in analysis of thermodynamics of fluid. It involves the free or natural transfer of heat within the fluid. Depending on the conditions under which the fluid flow is occurring, different fluids have different rates of transfer of dissipated heat. This dissipation of heat is brought about by viscosity of the fluid, density gradients and the nature of the surface of the body in the fluid flow region. In this study, under consideration was a fluid whose Reynolds number was of order less than 2000. This fluid was flowing over an axi-symmetrical body with curved surfaces. Studies have been done on fluid flows on curved surfaces with axi-symmetrical orientation which include fluid flow over cylindrical bodies, fluid flow over spheres whereby forces acting on these bodies were investigated. For this study main analysis was done in the boundary layer region in order to investigate how the nature of the surface affected convective heat transfer.

1.1 FLUID

Fluid is a type of matter which under given thermodynamic conditions and in absence of external forces takes the shape of the container. In fluids, the rate of deformation (if the distance between the neighbouring particles changes) is of importance than the deformation, whereby fluid undergoes deformation continuously.

A fluid is considered incompressible if the density is assumed to be invariant otherwise it is compressible if its density is a variable.

In this research, a Newtonian fluid (a fluid which there is a linear relationship between the shear stress and the velocity gradient, $\tau = \mu \frac{\partial u}{\partial y}$) was considered.

A fluid flow is steady if its velocity and the thermodynamic properties at each point in the flow region remains a constant and is independent of time, otherwise unsteady if the flow variables are dependent on time.

Fluid motion may be constrained by geometrical boundaries to be predominantly parallel to the sides. When flow variables (pressure, velocity, density and temperature) at all successive cross sections are identical at any instant, the flow is termed uniform otherwise it is non-uniform.

Fluid flow may be termed as laminar or turbulent. The term laminar is used to refer to a fluid flow in which fluid particles downstream of the leading edge moves in an orderly manner in laminas or layers parallel to the solid boundary as opposed to turbulent whereby fluid velocity components have random turbulent fluctuations imposed upon their mean values. A fluid flow is said to be laminar or turbulent by the velocity and channel configuration or size. Turbulent fluid motion is an irregular condition of flow in which various quantities like velocity and pressure show random variation with time and space. Turbulent flow is also characterised by eddies that causes mixing of layers of the fluid until the layers are no longer distinguishable. This mixing and collision of fluid particles produces heat and the greater the turbulence the larger the amount of heat transfer, as these increased collisions leads to increased dissipation of heat.

A fluid can also be ideal or real, whereby if it is assumed that there exists no frictional effect between the fluid layers and the boundary walls then it is regarded as ideal, otherwise real.

1.2 HEAT TRANSFER

Heat transfer involves energy in transit as a result of temperature gradient in the media or medium. This temperature gradient may arise from various causes such as viscous effects, release of latent heat as fluid vapour condenses and absorption of thermal radiation or radioactivity. This heat transfer takes place mainly in three modes; conduction, convection and radiation. In this study only convective heat transfer over an axi symmetrical body with curved surfaces was of concern.

1.3 CONVECTION

Convection refers to the heat transfer that occurs on a surface and a moving fluid due to temperature gradients between the two. Convective heat transfer is due to the superposition of the energy transport by random molecular motion (diffusion) and by advection (the bulk or macroscopic fluid motion). The contribution due to bulk fluid motion originates from the fact that boundary layer grows as the flow progresses. Convection laws rely on the fundamental principles of both heat transfer and fluid flow which include law of conservation of mass, law of conservation of momentum and law of conservation of energy.

Convective heat transfer depends on viscosity, thermal conductivity, specific heat and the density of the fluid. Viscosity influences the velocity profile of the fluid flow.

Convective heat transfer may be categorized as either free/natural convection or forced convection whereby in forced convection, flow is caused by some external force such as a fan, a pump or atmospheric winds while in free convection or natural convection the flow is induced by buoyancy forces resulting from density gradients as a result of temperature gradients in the fluid. On heating, the density change in the boundary layer will cause the fluid to rise and be replaced by cooler part of the fluid that also will heat and rise. This continues and is a phenomenon called free or natural convection. In free convection the driving force for the fluid motion is the gravity field acting on the density difference. These density gradients are due to temperature and concentration gradients existing in the fluid while the body force is due to the gravitational field. When the body forces act on the fluid there results a buoyancy force that induces free convectional currents. Both forced and natural types of heat convection may occur together a phenomenon termed as mixed convection.

1.4 VISCOSITY

This is the resistance set up due to shear stresses within the fluid particles and the shear stresses between the fluid particles and the solid surface for a fluid flowing around a solid body. As fluid exerts a shear stress on the boundary, the boundary exerts an equal and opposite force on the fluid called shear resistance (frictional drag). Drag coefficient, C_d always depends on the Reynolds number (Re) and the shape of the body. The work done against the viscous effects usually causes fluid flow, consequently the energy spent in doing so is converted to heat. At low values of Reynolds number, the fluid is highly viscous causing deformation drag, the fluid is deformed in a wide zone around the body which brings about pressure force and frictional force. At large values of Reynolds number, the fluid is less viscous for

example in water and air, the viscous effect is limited to the boundary layer thickness. In this case deformation drag is exclusively friction drag. The shear force exerted on the surface of the body due to the formation of boundary layer results into friction drag.

1.5 BOUNDARY LAYER

Boundary layers are thin fluid particles layers adjacent to the surface of a body or solid wall in which viscous forces exist. The fluid particles in contact with the solid body surface attain the velocity of the body. The region outside this layer is called free stream region where the flow is unaffected by viscous forces. Boundary layer thickness theory is of importance in analysing flow problems involving convective transport.

1.5.1 VELOCITY BOUNDARY LAYER

When fluid particles of a real fluid are in contact with a flat surface, their velocities are retarded gradually. These particles then act to retard the motion of the adjoining fluid layer which in turn acts to retard the motion of the particles in the next layer. The process continues until the effect becomes negligible. The velocity boundary layer thickness is defined as the distance away from the plate's surface where the velocity reaches 0.99 that of the free-stream velocity.

1.5.2 THERMAL BOUNDARY LAYER

Thermal boundary layer develops if the temperature of the fluid at the surface of the plate and the free stream temperature differ. Fluid particles that come into contact with the plate attain the same temperature as the temperature of the plate's surface.

In turn these particles exchange heat energy with those in the adjacent fluid layers, and the temperature gradients develops in the fluid. The region in the fluid in which these temperature gradients exist is the thermal boundary layer.

1.5.3 CONCENTRATION BOUNDARY LAYER

When concentration of some species at the surface differs from that of the free stream, a concentration boundary layer will develop; the region of the fluid in which the concentration gradients exist is the concentration boundary layer. Species transfer by convection between the surface and the free stream is governed by the condition of the boundary.

1.6 LIFT AND DRAG

The sum of all the forces on a body that acts perpendicularly to the direction of flow is referred to as lift. This force occurs when fluid moves over a stationary solid body. On the other hand, drag is the force parallel and in opposition to the direction of motion of an object moving in the fluid. Drag takes two forms; form drag or pressure drag which is dependent on the shape of the object moving in the fluid and the other form is skin friction which is dependent on the viscous friction between a surface of a moving solid body and a fluid.

1.7 DIMENSIONAL ANALYSIS

Dimensional analysis is a method which describes a natural phenomenon by a dimensionally correct equation among certain variables which affects the phenomenon. Dimensional analysis is a method used to obtain equation(s) that relates all physical factors of a problem to another. Through this, equations are reduced to non-dimensional

form by using dimensionless groups such as Reynolds number, Eckert's number, Peclet's number, Prandtl's number, Grashoff's number, Nusselt's number e.t.c. It is built on the principle of dimensional homogeneity which states that an equation expressing a physical relationship between quantities must be dimensionally homogeneous. Dimensional analysis gives results which only become quantitative from experimental analysis.

This method has applications in nearly all fields of engineering in particular in fluid mechanics and heat transfer. It is an important tool for analyzing fluid flow problems and very useful in presenting experimental results in a concise form. Through this method dimensions of relevant variables of an appropriate prototype are used in the manufacturing of the actual object.

1.8 REVIEW OF LITERATURE

The theory of convective heat transfer strongly emerged in 20th century. By its nature, convective energy transfer is closely related to fluid particles motion and therefore is a fundamental part of fluid mechanics study. Advancement in research in fluid mechanics (particularly hydrodynamics of non-Newtonian, electric current-conducting and magnetic media, supersonic and hypersonic gas dynamics, dynamics of plasma, fine molecular and heterogeneous flows, the hydro and gas dynamics effects during the theory of heat and transformation) have greatly affected the theory of heat and mass transfer in moving media e.g. air, water and oil.

The relationship between the intensities of turbulent momentum and heat transfer process is one of the subtle problems of heat transfer theory. The determination of the Prandtl number, Pr is paramount. The value of Prandtl number is of order unity beyond

the viscous sub layer, but greater than one in the immediate vicinity of a solid body (at the depth of the viscous sub layer). At Pr>100, the turbulent thermal boundary layer is submerged in the viscous sub layer of the turbulent hydrodynamic boundary layer.

On the formation of the boundary layer in a steady flow (independent of time), Allen (1981) gave evidence to the effect that the location of the transition from laminar to turbulent conditions in the boundary layer might be more closely dependent on the local skin friction coefficient than the Reynolds number.

A German aero dynamist Ludwig Prandtl (1904),established that a flow of large Reynolds number means that it has a low viscosity (frictional forces associated with flow), if the viscosity of the fluid is low then the effects of friction will be confined to a very thin layer known as the boundary layer near the solid body while the region outside the boundary layer can be considered frictionless or ideal i.e. in this region the fluid is assumed to be in viscid or non-viscous .Also for this flow of high Reynolds number the region near the boundary, viscous forces will dominate over the inertia forces and the effects of the viscosity will be very important in this region, as a result shear forces will be very high due to the extremely high velocity gradients at and near the boundary layer.

Barenblatt *et al* (2002),in their study on the model of the turbulent boundary layer with non- zero pressure gradient observed that the turbulent boundary layer at large Reynolds number consist of two separate layers upon which the structure of the vortex fields is different, although both exhibit similar characteristics.

 In the first layer, vertical structure is common to all developed bounded shear flows and the mean flows .The influence of viscosity is transmitted to the main body of flow

via streaks separating the viscous sub layer. The second layer occupies the remaining part of the intermediate region of the boundary layer.

The upper boundary of the boundary layer is covered with statistical regularity by large scale "humps" and the upper layer is influenced by the external flow via the pressure drag of these humps as well as by the shear stress. In their earlier works it is shown that the mean velocity profile is affected by the intermittency of the turbulence and as the humps affects intermittency, the two seeking regions are visible.

On the basis of these considerations, the effective Re, which determines the flow structure in the first layer (and is affected in turn by the viscous sub layer), was identified as one set of such parameters. The other parameters that influence the flow in the upper layer include pressure gradient, $\frac{\partial P}{\partial x}$; dynamic (friction) viscosity , μ; velocity ,u; fluid's kinematic viscosity, ϑ and density,ρ.

In the recent past, many researchers have been interested in solving the boundary layer equations. Smith *et al* (1963) in one of their papers presented a method for solving the complete incompressible laminar boundary layer equations; both for two dimensional and axi symmetrical laminar flow, essentially full generality and with speed.

In their subsequent papers (1970, 1972), they discussed application potential flow and the boundary layer theory in the hydrodynamics, they also provided a solution technique of the laminar boundary layers by means of the differential difference method

Wehrle *et al* (1986) presented an analytical shears for the determination of the separation point in the laminar boundary layer. Unlike conventional approaches the

scheme does not require the full-field solution of the governing partial difference equation, but rather the solution of a first order set of boundary layer equations defined in the neighbourhood of the leading edge.

Continuing interest in flows and heat transfer over flat plate, concave , convex surfaces stems from their possible effects in the turbine blades of jet engines, vehicle aerodynamics ,aircraft wings, submarines, spaceship, cooling plants power plants e.t.c. flow phenomenon are mainly subjected to pressure gradients (favourable or adverse), surface curvature and a wide range of Reynolds number.

There have been many previous investigation of flow and heat transfer on flat plate boundary layers with pressure gradients. Fukagata *et al* (2002) were concerned with transition to turbulent flow and the Reynolds stress distribution. While those of Umur and Karagoz (1999) investigated flow and heat transfer characteristics in laminar flows were investigated with pressure gradients, stream wise distance Reynolds number and wall curvature. Measurements were carried out in a low speed wind tunnel with a dimensionless stream wise pressure gradient parameter of between −4 and 1.0. Results were compared with analytical solutions and numerical predictions and a new empirical equation as a function of k_x. It was found that Stanton numbers augmentation with Reynolds number became more pronounced than concave curvature. Favourable pressure gradients caused heat transfer to increase and adverse pressure gradients to decrease. The results showed that the distribution of Stanton numbers with acceleration has similar trends with analytical solutions and numerical predictions. The proposed equation showed much better agreement with the measured Stanton numbers, in case of both adverse and favourable pressure gradients. Filippova and Hanel (1998) developed a curved boundary treatment using Taylor's series expansion in both space and time for

single particle distribution near the wall. This boundary condition satisfies the no-slip condition to the second order in a space step and preserves the geometrical integrity of the wall boundary.

Mei *et al* (1999) and Bouzidi *et al* (2001) proposed some other boundary treatment methods. In all those methods, the boundary conditions were treated separately for some specific steps when some variations occurs in the specified steps while dealing with curved boundaries, an abrupt change in the single particle mass distribution was caused. In the turbo machinery applications; a variations in the rate of heat transfer due to a small flow disturbance can lead to an increase in the thermal stress and decrease the effective working life span of such a component. On a highly curved wall, the change in heat transfer rate is mainly due to an increase or decrease of the turbulent mixing by effect of streamline curvature. It has been indicated in Von Karman's stability argument (1934) that the convex wall has a stabilizing effect on the fluid particles, while concave wall has a de-stabilizing effect with reference to a flat plate.

The measurement and prediction of the rate of heat transfer for a two dimensional boundary layer on a concave surface have been presented by Mayle *et al* (1979). It was established that the heat transfer on the convex surface was less than a flat surface having the same free stream, Reynolds number and turbulence. Concave surface heat transfer was augmented when compared to the flat surface.

P. Bradshaw *et al* (2006) extended the study on the use of the algebraic analogy to curved shear layers and the effects of the curvature on the apparent mixing length if the shear layers thickness exceeds $\frac{1}{300}$ of the radius of curvature where they concluded that large effects occurred in compressible fluid flows. B.K. Gupta *et al* (2003) investigated

heat transfer on a surface with longitudinal curvature in laminar flow where they investigated the effects of variation of different parameters which included Prandtl number, Nusselt number and Eckert number. They concluded that for any Prandtl number, as the curvature changes from concave to convex, the Nusselt number decreases if the Eckert number is small and it increases if the Eckert number is close to unity. A.B. Khoshevis *et al* (2007) investigated the effects of the concave curvature on turbulent flows using numerical solutions of boundary layer equations on concave surfaces. It was evident that turbulent intensities and turbulent shear stresses are increased on concave walls compared to flat plates under same conditions and they concluded that for the boundary layer on concave surfaces, the destabilizing effects lead to increased turbulent momentum exchange between the fluid particles similar to the way concave curvature causes flows to be destabilized.

One area of practical interest to researchers is on the degradation of aerofoils .Aerofoils form a crucial part of aviation and air conditioning systems.

George O.O *et al* (2009) in their study on the convection heat transfer in a fluid flow over a curved surface established that as fluid flows over an immersed curved surface, some work is done against viscous effects and energy spent is converted into heat and also vortices formed in the boundary layer due to high velocity gradient is swept outwards from the boundary layer. Mugambi K.E *et al* (2008) in their research investigated the forces produced by fluid motion on a submerged finite curved plate. They established the relationship between geometrical shape of the curvature and the variation of drag force of specific velocities of the viscous fluid.

Thus investigation of the effect of using an axi symmetrical body with curved surfaces was done first by getting the velocity profiles and their temperature profiles resulting from temperature variations and velocity variations and eventually heat was dissipated which was transferred by convection.

1.9 STATEMENT OF THE RESEARCH PROBLEM

As fluid of large Reynolds number flowed over an axi-symmetrical body with curved surfaces as shown in the Figure 1, some work was done on the curved surfaces, against viscous forces and energy spent was converted into heat, also whirlwinds and whirlpools were formed in the boundary layer as a result of high velocity gradient outwards from the boundary layer.

The energy converted into heat within the boundary layer was transferred from this boundary layer through convection into the rest of the region of flow. A lot has been done in regard to heat transfer but less has been done in regard to the effect of axi-symmetrical body on the heat generated. In this research convective heat transfer formed the basis of this research which in turn affected lift and drag.
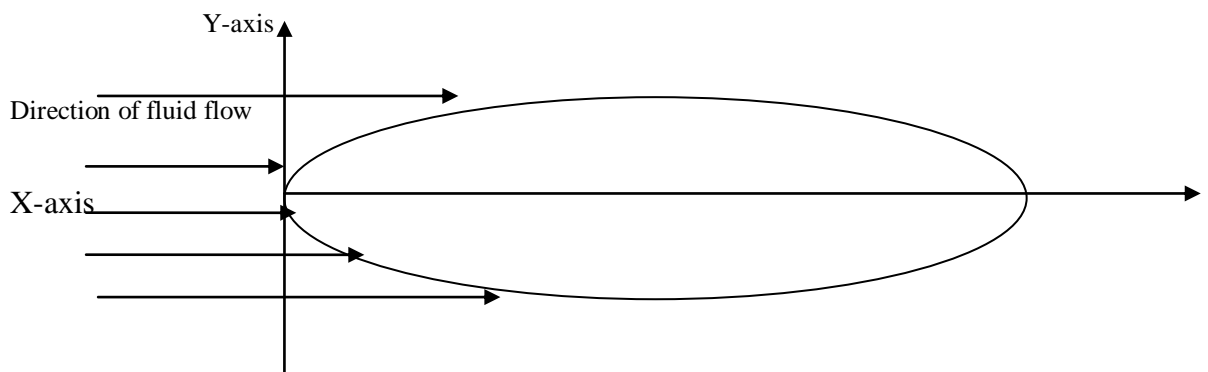

Figure 1: Eppler's aerofoil design, Eppler *et al* (1979).

1.10 OBJECTIVES OF THE STUDY

The objectives of this study were to determine:-

1. The velocity distribution of the fluid flow past an immersed axi-symmetrical body with curved surface.

2. Temperature variation within the thermal boundary layer of the fluid past the immersed axi-symmetrical body with curved surfaces due to velocity variations.

3. The effect of heat generated within the boundary of an immersed axi-symmetrical body with curved surfaces on drag and lift.

1.11 JUSTIFICATION

In our day to day lives we encounter cost of maintenance brought about by degradation of equipment and machines whose parts come in contact with a fluid, this has become a major concern. Heat produced due to viscosity on the body surfaces has led to the degradation of equipment and machines which has led to high cost of maintenance being incurred.

Rise in temperature decreases the viscosity of the fluid and vice versa, thus the need to design bodies that could withstand such variations.

Heat injection or heat withdrawal on immersed curved surfaces enhance velocity variations in the fluid flow thereby improving the manoeuvrability of such bodies in the fluid as in the case of submarines in water, wings of flying planes in the air, weather space ships in the air, fan blades in the air conditioning systems, fan blades in the cooling units of appliances e.g. computer.

1.12 HYPOTHESIS

The null hypothesis of this study was that the presence of curved surfaces would not affect the velocity and temperature.

1.13 THESIS OUTLINE

Chapter one of this thesis contains the introduction and definitions of the fundamental terms as used in the thesis whereby their meaning were expounded further. Also the justification, review of related literature, hypothesis and objectives are contained in this chapter.

Chapter two contains the equations governing the flow and their analysis to suit this particular problem. This analysis includes non-dimensionalising them and re-writing them together with the non-dimensional numbers and writing them in finite difference form for solution using the Crank Nicolson approximation.

Chapter three contains the solution of these differential equations by the use of Visual basic as the programming language. The results obtained from this program by varying various flow parameters are represented graphically followed by an in-depth discussion of the results.

Chapter four is the last chapter, and basically contains the conclusion arising from the analysed results followed by recommendations for future research areas in line with this work. At the end of this chapter references cited in this research are listed in alphabetical order using the MLA formatting.

CHAPTER TWO

2.0 INTRODUCTION

In this chapter, equations governing the flow of an incompressible, Newtonian fluid over an axi symmetrical body with curved surfaces were discussed taking into consideration assumptions and approximations made. The fundamental equations to be considered include mass conservation equation, momentum conservation equation and equation of energy. Also description of the flow and dimensional analysis of equations that govern this fluid flow problem obtained was done in this chapter.

2.1 ASSUMPTIONS AND APPROXIMATIONS

The following assumptions were made for this research problem:-

1. The fluid was Newtonian.

2. The fluid was incompressible (ρ is assumed a constant).

3. The fluid had constant thermal conductivity (ability to conduct heat).

4. The fluid flow velocities were small compared to that of light i.e. ($\frac{q^2}{c^2} <<< 1$).

5. The flow was assumed to be laminar and no slip condition was satisfied.

6. The fluid flow was unsteady (the fluid flow was time dependent).

Boundary layer approximations

1. $\delta \ll L$; The reference length L (m) was large compared to the boundary layer thickness.

2. u ≫ v; The velocity component, u along surface was much larger than the velocity component, v normal to the surface.

3. Radius of curvature, Kr>0.

2.2 EQUATIONS GOVERNING THE FLUID FLOW

The equations governing the fluid flows of any kind are based on general laws of conservation of mass, momentum and energy only they are modified to perfectly suit a particular fluid flow.

2.2.1 Equation of continuity

The equation of continuity is a mathematical statement in any process where the rate at which mass enters a system is equal to the rate at which mass leaves the system. This equation combines the law of mass conservation and that of the transport theorem. This equation arises from the fundamental prepositions that matter is neither created nor destroyed and that the flow is continuous. The general expression representing mass conservation was given by;

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho u) = 0 \qquad (2.1)$$

where ρ and $\vec{u}$ are the fluid's density and the fluid's velocity respectively.

In Cartesian co-ordinate form this equation (2.1) was expressed as;

$$\frac{\partial \rho}{\partial t} + \rho \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) = 0 \qquad (2.2)$$

Now for a incompressible 2-Dimensional fluid flow, w=0 and $\frac{\partial \rho}{\partial t} = 0$ hence (2.2) reduced to;

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2.3)$$

Equation (2.3) was the equation of continuity for single species fluid in the velocity boundary layer under consideration in two dimensions.

2.2.2 Equation of conservation of momentum

The equation of conservation of momentum is derived from the Newton's second law of motion, which states that the time rate change of momentum of a body matter is equal to the net external forces applied to the body. This external force is divided into two types of forces i.e. surface forces (e.g. forces due to static pressure and viscous stresses) and body forces (e.g. gravitational force, centrifugal force, magnetic force or electric fields). The surface forces are due to the interaction between the body and the matter in the immediate contact with it and act on the bounding surfaces. There intensities are expressed in terms of stress and defined as force per unit area. The body forces are defined as the forces which act on a body from a distance and are usually expressed as forces per unit mass. For this particular research problem it was of utmost importance to resolve the forces acting on the curved surfaces. In this case the curved surfaces were both convex and concave surfaces.

The viscous stresses at any point in the velocity boundary layer were resolved in two components;- normal stress which was always perpendicular to the surface and shear stress which was always tangential to the surface in consideration.

Normal stress, $\sigma_{i,j}$

Resultant stress

Shear stress, $\tau_{i,j}$
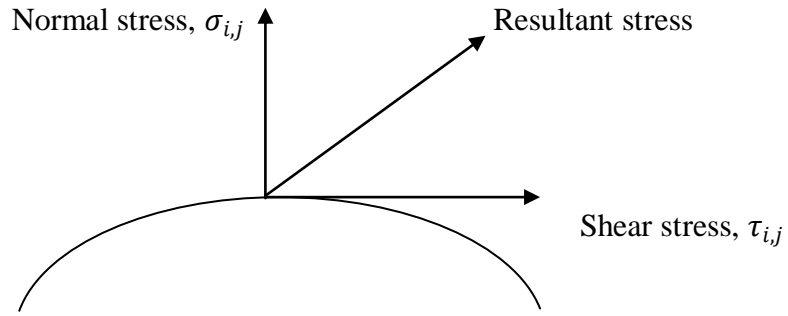
Fig 2: Resolution of surface force acting on an area element into shear and normal stresses.

The double notation of both the shear and the normal stresses was used to identify the components where the first index denoted the direction of the normal to the plane and the second index denoted the direction of the stress component itself. For a two-dimensional flow Fig.2 was resolved as;

σ_{yy}

Y-axis

τ_{yx}

τ_{xy}

X-axis

σ_{xx}

Fig 3: Resolution of forces

The momentum equation along x-axis became;

$$\rho \frac{\partial u}{\partial t} + \rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial P}{\partial x} + \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} \right) + \rho F_x \qquad (2.4)$$

The momentum equation along the y-axis became;

$$\rho \frac{\partial v}{\partial t} + \rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{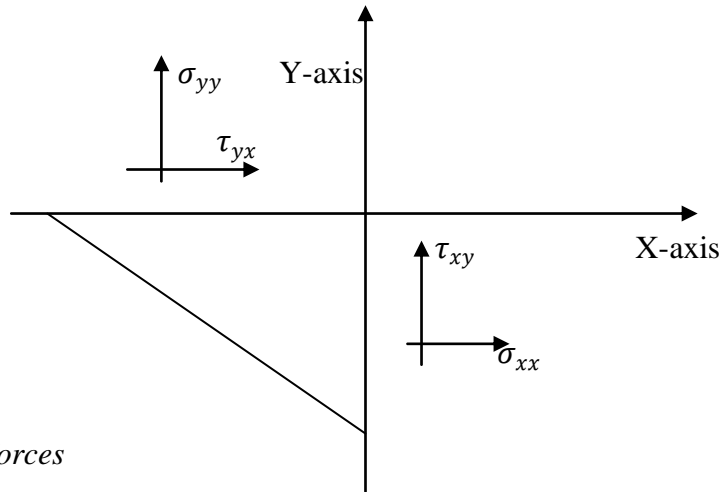\partial P}{\partial y} + \left(\frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} \right) + \rho F_y \qquad (2.5)$$

The viscous stresses and shear stresses in two dimensions were defined by;

$$\sigma_{xx} = 2\mu \frac{\partial u}{\partial x} - \frac{2}{3}\mu \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \qquad (2.5a)$$

$$\sigma_{yy} = 2\mu \frac{\partial v}{\partial y} - \frac{2}{3}\mu \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \qquad (2.5b)$$

$$\tau_{xy} = \tau_{yx} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \qquad (2.5c)$$

Substituting equations (2.5a-c) into equations (2.4) and (2.5) we obtained momentum equation along the x-axis and y-axis as;

Along the x-axis;

$$\rho \frac{\partial u}{\partial t} + \rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right)$$

$$= -\frac{\partial P}{\partial x} + \frac{\partial}{\partial x} \left\{ \mu \left[2\frac{\partial u}{\partial x} - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right] \right\} + \frac{\partial}{\partial y} \left[\mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right]$$

$$+ \rho F_x \qquad (2.6a)$$

Along the y-axis;

$$\rho \frac{\partial v}{\partial t} + \rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right)$$

$$= -\frac{\partial P}{\partial y} + \frac{\partial}{\partial y} \left\{ \mu \left[2\frac{\partial v}{\partial y} - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right] \right\} + \frac{\partial}{\partial x} \left[\mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right]$$

$$+ \rho F_y \qquad (2.6b)$$

since; $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ equations (2.6a) and (2.6b) reduced to

$$\rho \frac{\partial u}{\partial t} + \rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right)$$

$$= -\frac{\partial P}{\partial x} + 2\mu \frac{\partial^2 u}{\partial x^2} + \mu \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y} \right) + \rho F_x \qquad (2.7a)$$

$$\rho \frac{\partial v}{\partial t} + \rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right)$$

$$= -\frac{\partial P}{\partial y} + 2\mu \frac{\partial^2 v}{\partial x^2} + \mu \left(\frac{\partial^2 u}{\partial y \partial x} + \frac{\partial^2 v}{\partial y^2} \right) + \rho F_y \qquad (2.7b)$$

From the boundary layer approximations made earlier, the distance under consideration was very small, boundary layer thickness δ, to the extent that the velocity component in the direction along the surface was much larger than that normal to the surface. Hence the gradients normal to the surface were larger than those along the surface i.e. $\frac{\partial u}{\partial y} \gg$ $\frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial v}{\partial x}$ and also $\frac{\partial v}{\partial t} = 0$. From this approximations (2.7a) and (2.7b) reduced to

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \vartheta \frac{\partial^2 u}{\partial y^2} + F_x \qquad (2.8a)$$

and

$$0 = -\frac{\partial P}{\partial y} + \rho F_y \qquad (2.8b)$$

respectively.

Now from the Bernoulli's equation;

$$P + \frac{1}{2}\rho u^2 = constant \qquad\qquad (2.9)$$

The curved surfaces provided both adverse and favourable pressure gradient (i.e. this is where the pressure decreases in the direction of the flow the physical effect is to accelerate the flow, the boundary layer remains attached to the surface and tends to reduce in thickness and this is termed as favourable pressure gradient) whose tangential component of the velocity of the outer flow reveals a power law dependence on the stream wise x measured along the curved surface boundary as;

$$u = cx^m \qquad\qquad (2.10)$$

where c was a positive velocity coefficient and m was an integer obtained from the angle of inclination. This integer m was given as $m = \dfrac{\theta/\pi}{2 - \theta/\pi}$ where $\dfrac{\theta}{\pi}$ was the angle in radians of the inclination at a given point from the horizontal plane. Let α denote the angle.

Then $= \dfrac{\theta}{\pi}$, hence m= $\dfrac{\alpha}{2 - \alpha}$

Differentiating partially equation (2.9) with respect to x, we obtained

$$\frac{\partial P}{\partial x} + \rho u \frac{\partial u}{\partial x} = 0 \qquad\qquad (2.11)$$

which implied

$$-\frac{1}{\rho}\frac{dP}{dx} = u\frac{du}{dx} \qquad\qquad (2.12)$$

But from the power law dependence

$$u \frac{du}{dx} = c^2 m x^{2m-1} \tag{2.13}$$

hence equation (2.8a) became

$$\frac{\partial u}{\partial t} = P_t + \vartheta \frac{\partial^2 u}{\partial y^2} + F_x \quad , where \; P_t = c^2 m x^{2m-1} \tag{2.14}$$

The body under consideration had both the convex surface and the concave surfaces; there curvature effect on the fluid flow had to be taken into consideration. The concave part of the body brought about an unstable effect which was determined by $\frac{\partial u}{\partial y} \frac{\partial v}{\partial x}$. The curved surface as a curvature was defined by a quadratic equation of the form

$$y = ax^2 + r(x) \tag{2.15}$$

where 0<a<1 was set to ensure surfaces' radius of curvature was large enough and the end points were set at a specific co-ordinates values when solving for a particular case of which length of the plate curvature were determined analytically.

The concave wall exerted a destabilizing influence on the momentum exchange. Prandtl proposed to account for curvature effect by multiplying the length of the concave curved surface by a factor f which was a function of dimensionless curvature parameters, that is

$$f = 1 - \frac{1}{4} \frac{k_r u}{\left(\frac{\partial u}{\partial y} \right)} \tag{2.16}$$

He also deduced that the boundary layer equation on the curved surface was written as

$$\frac{k_r u^2}{h_1} = \frac{1}{\rho} \frac{\partial P}{\partial y} \tag{2.17}$$

where k_r and h_1 are curvature parameters which were defined as

$$k_r(x) = -\frac{1}{r(x)} \tag{2.18}$$

$$h_1 = 1 + k_r y \tag{2.19}$$

where r(x) was the radius of the curved surface.

Equation (2.8b) was rewritten as

$$\frac{1}{\rho}\frac{\partial P}{\partial y} = F_y \tag{2.20}$$

A comparison was done between equations (2.17) and (2.20) which yielded

$$\frac{k_r u^2}{h_1} = F_y \tag{2.21}$$

The body forces under consideration F_x and F_y were purely due to the gravitational pull which was assumed to be a constant in both cases, hence an important assumption that

$$F_x = F_y \tag{2.22}$$

From equations (2.21) and (2.22) it was resolved that

$$\frac{k_r u^2}{h_1} = F_x \tag{2.23}$$

Equation (2.23) was replaced in the equation of conservation of momentum along the x-axis equation (2.14), a result which gave us a generalized equation of conservation of momentum for fluid flow over an axi-symmetrical body with curved surfaces, the equation became

$$\frac{\partial u}{\partial t} = P_t + \vartheta \frac{\partial^2 u}{\partial y^2} + \frac{k_r u^2}{h_1} \tag{2.24}$$

But $h_1 = 1 + k_r y$, hence the term $\frac{k_r u^2}{h_1}$ in equation (2.24) was written in Taylor series as

$$k_r u^2 (1 + k_r y)^{-1} = k_r u^2 (1 - k_r y + k_r^2 y^2 + .. \ ) \tag{2.25}$$

And therefore equation (2.24) yielded

$$\frac{\partial u}{\partial t} = P_t + \vartheta \frac{\partial^2 u}{\partial y^2} + k_r u^2 (1 - k_r y + k_r^2 y^2 + .. \ ) \tag{2.26}$$

Since the flow was along the x-axis $y \cong 0$ and for very small values of k_r equation (2.26) yielded

$$\frac{\partial u}{\partial t} = P_t + \vartheta \frac{\partial^2 u}{\partial y^2} + k_r u^2 \tag{2.27}$$

2.2.3 Equation of conservation of energy

If the flow is not isothermal, it is necessary to analyse the energy equation which draws a balance between heat and mechanical energy and provides a differential equation on the temperature distribution of the fluid flow over an axi-symmetrical body with curved surfaces. The equation of energy was derived from the 1st law of the thermodynamics which stated that the amount of heat added to the a system, dQ was equal to the sum of the change in the internal energy, dE of the system and the external work done dW by the system. Mathematically the law was expressed as

$$dQ = dE + dW \ , \tag{2.28}$$

where dW = PdV = $Pd \left(\frac{1}{\rho} \right)$ for a unit mass.

Equation (2.28) reduced to

$$dQ = dE + Pd\left(\frac{1}{\rho}\right)$$

(2.29)

The 1st law of thermodynamics for a fluid flow with constant thermal conductivity K, zero internal generation and negligible compressibility effect the equation reduced to;

$$\rho C_p \frac{Dh}{Dt} = K\nabla^2 T + \mu\emptyset \ ,$$

(2.30)

where $\mu\emptyset$ was the internal heating due to the viscous dissipation while for an incompressible two-dimensional fluid flow the viscous dissipation function was

$$\emptyset = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2$$

(2.31)

By considering unsteady incompressible flow in a control volume, the standard thermal energy equation for the thermal boundary layer was given by

$$\rho \frac{\partial h}{\partial t} + \rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y}$$

$$= \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \left(u\frac{\partial P}{\partial x} + v\frac{\partial P}{\partial y}\right) + \mu\emptyset + \dot{q} \ ,$$

(2.32)

where h was the enthalpy, $\dot{q}$ was the rate of heat generation.

Now the enthalpy h was given by

$$h = E + P\left(\frac{1}{\rho}\right)$$

(2.33)

then the first order derivative of enthalpy became

$$dh = dE + \frac{1}{\rho}dP + Pd\left(\frac{1}{\rho}\right) \tag{2.34}$$

But, $dQ = dE + dW = dE + Pd\left(\frac{1}{\rho}\right)$ and for a unit mass and a single species

fluid, $dQ = Tds$, therefore

$$dE = Tds - Pd\left(\frac{1}{\rho}\right) \tag{2.35}$$

In view of (2.34), the equation (2.34) became

$$dh = Tds + \frac{1}{\rho}dP + Pd\left(\frac{1}{\rho}\right) - Pd\left(\frac{1}{\rho}\right) \tag{2.36}$$

Hence

$$dh = Tds + \frac{1}{\rho}dP \tag{2.37}$$

Assuming that $u\frac{\partial P}{\partial x}$ and $v\frac{\partial P}{\partial y}$ were negligible and $dh = c_p dT$, the equation (2.32)

reduced to

$$c_p\rho\frac{\partial T}{\partial t} + c_p\rho\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \dot{q} \tag{2.38}$$

Now for fluid flow over a body with curved surfaces the convective heat transfer due to

the viscosity in the thermal boundary layer was modelled to the equation of conservation

of energy. In this case the body in consideration had both convex and concave surfaces.

The convex surface had previously been done by Omboro G.O. *et al* (2009) it remained

the concave surface which had a destabilizing effect on the velocity of the fluid flowing

over it, hence the heat transfer area was the length of the curved surface and increase in

the heat transfer area intensified the natural convective heat transfer. Also increase in the

flow cross sectional area increased the adverse pressure gradient that opposed the buoyancy induced acceleration.

The convection equation was expressed as

$$\dot{q} = kAdT \tag{2.39}$$

where $dT = (T_\infty - T_s)$ was the temperature difference between the surface and the bulk fluid and A was the area of the surface.

In this case the area of the surface was the length of the curved surface and for this concave surface which had a destabilizing effect, the effect of the curved surface was taken into account by multiplying area, A by a dimensionless factor given by equation (2.16) which resulted to

$$\dot{q} = kfAdT, \tag{2.40}$$

where $\dot{q}$ was the heat transferred per unit time.

And on replacing f, equation (2.40) reduced to

$$\dot{q} = k\left(1 - \frac{1}{4}\frac{k_r u}{\left(\frac{\partial u}{\partial y}\right)}\right)A(T_\infty - T_s) \tag{2.41}$$

From Newton's law of cooling the local heat flux was given by

$$q''_s = h(T_\infty - T_s), \tag{2.42}$$

where h was the local convection coefficient.

Since the flow conditions varied from one point to another on the curved surface both q''_s and h also varied along the curved surface.

At any distance x from the leading edge of the curved surface local heat flux q''_s was obtained by applying the Fourier's Law to the fluid at y=0 as

$$q''_s = -k \frac{\partial T}{\partial y} \qquad (2.43)$$

The local convection heat transfer was then expressed as

$$h = \frac{-k \frac{\partial T}{\partial y}}{(T_\infty - T_s)} \qquad (2.44)$$

In the thermal boundary layer the rate of heat conduction along the y-direction was larger than that along the x-axis $\left(i.e. \frac{\partial T}{\partial y} \gg \frac{\partial T}{\partial x} \right)$

Then the equation of 1st law of thermodynamics (2.33) reduced to

$$c_p \rho \frac{\partial T}{\partial t} + c_p \rho \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 + \dot{q} \qquad (2.45)$$

From the above approximations equation (2.45) reduced to

$$c_p \rho \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 + \dot{q} \qquad (2.46)$$

But the value of $\dot{q}$ was replaced with equation (2.41) in order to take of the curvature effects and hence on substituting equation (2.41) in equation (2.46) yielded

$$c_p \rho \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 + k \left(1 - \frac{1}{4} \frac{k_r u}{\left(\frac{\partial u}{\partial y} \right)} \right) A(T_\infty - T_s) \qquad (2.47)$$

Equation (2.47) gave the equation of energy for convective heat transfer over an axi-symmetrical body with curved surfaces.

2.3 DESCRIPTION OF THE FLOW

A two dimensional unsteady flow of a fluid of Reynolds number less than 2000 over an axi-symmetrical body with curved surfaces was studied. The body in consideration had both convex and concave surfaces hence two non-zero pressure gradients existed due to these surfaces. The concave surface was mainly characterized by unstable effect it had on fluid flow while the convex surfaces had a stable effect, hence for this particular body it was prudent to take into account these two effects because they were paramount to the determination of velocity profiles and the temperature profiles. The curved surfaces also had significant effect on the turbulence, where the streamlines in the boundary layer had convex curvature the turbulence were stabilized while the ones with the concave curvature the turbulence were destabilized. The centrifugal force exerted a force normal to the fluid flow direction and was balanced by the pressure gradient. If the fluid was displaced by a disturbance it encountered a pressure gradient larger than that it was accustomed to at the curved surface, hence the fluid was brought back to the surface. The fluid flow over this body was separated into two regimes:-

a) **Convex surface.**

The flow on this surface was characterized by $\frac{\partial P}{\partial x} > 0$ and $\frac{\partial u}{\partial x} < 0$ which implied that the fluid velocity decreased with downstream direction and an adverse pressure gradient was said to exist which was a positive value. In this case the boundary layer increases rapidly and this together with the action of the shear force brought the boundary layer to rest.

b) Concave surface

The flow on this surface was characterized by $\frac{\partial P}{\partial x} < 0$ and $\frac{\partial u}{\partial x} > 0$ which implied that the fluid velocity increased with downstream direction and a favourable pressure gradient was said to exist which was a negative value and this reduced the boundary layer thickness since increased velocity meant there was less retardation of fluid particles adjacent to the surface of the body.

When the fluid flowed over the body, three types of forces acted on it namely; pressure force, viscous forces, drag and lift as shown in the diagrams below.

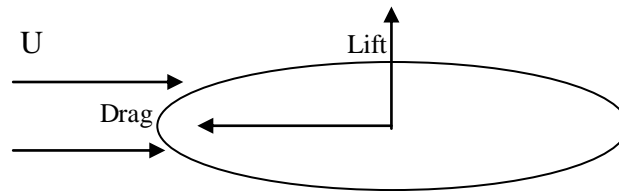a) Resultant forces (drag and lift).

Figure 4: Lift and drag forces.
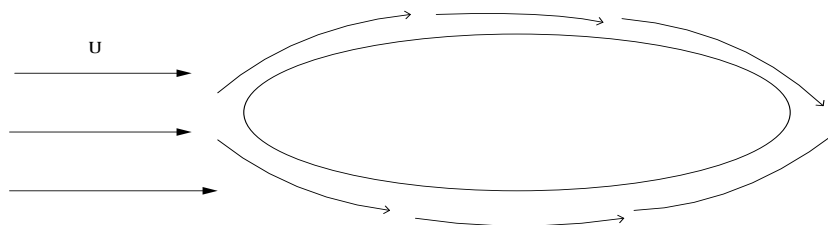
b) Viscous force.

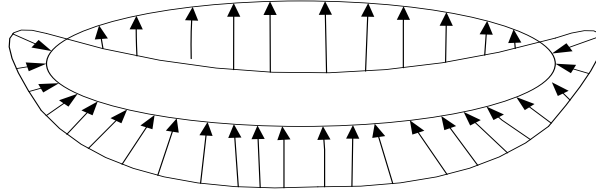Figure 5: Distribution of viscous forces over the curved surfaces.

c) Pressure force.

Figure 6: Pressure force distribution on the body with curved surfaces.

2.4 NON-DIMENSIONALISING THE EQUATIONS GOVERNING THE FLOW

Non dimensionalisation of the equations governing a particular fluid flow falls under a broad area of study known as dimensional analysis. Dimensional analysis is a method which describes a natural phenomenon by a dimensionally correct equation with certain variables which affect the phenomenon.

For this particular problem we let L, V, P and T to be the characteristic length,velocity,pressure and temperature respectively. To non-dimensionalise the equations governing the flow we used the transformations:

$$x = x^*L \ , y = y^*L \ , \qquad u = u^*V \ , \qquad v = v^*V \ , \qquad P = P^*P \ , \ T^* = \frac{T - T_s}{T_\infty - T_s},$$

$$t^* = \frac{tV}{L} \ \ or \ t = \frac{t^*L}{V}$$

2.4.1 EQUATION OF CONTINUITY

For this particular fluid flow the equation of continuity was given by

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \qquad\qquad\qquad (2.48)$$

On non-dimensionalising, the equation of continuity became

$$\frac{\partial(u^*V)}{\partial(x^*L)} + \frac{\partial(u^*V)}{\partial(y^*L)} = 0 \qquad (2.49)$$

or

$$\frac{V}{L}\left(\frac{\partial u^*}{\partial x^*} + \frac{\partial u^*}{\partial y^*}\right) = 0 \qquad (2.50)$$

or

$$\left(\frac{\partial u^*}{\partial x^*} + \frac{\partial u^*}{\partial y^*}\right) = 0 \qquad (2.51)$$

2.4.2 EQUATION OF CONSERVATION OF MOMENTUM

The equation of conservation of momentum for this flow problem was given as;

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = P_t + \vartheta\frac{\partial^2 u}{\partial y^2} + k_r u^2 \qquad (2.52)$$

Since $\frac{\partial u}{\partial x} = 0 \ and \ v = 0$ the equation (2.52) reduced to

$$\frac{\partial u}{\partial t} = P_t + \vartheta\frac{\partial^2 u}{\partial y^2} + k_r u^2 \qquad (2.52.1)$$

On non-dimensionalising the equation became

$$\frac{\partial(u^*V)}{\partial\left(\frac{t^*L}{V}\right)} = P^*P_t + \vartheta\frac{\partial^2(u^*V)}{\partial(y^*L)^2} + k_r(u^*V)^2 \qquad (2.52.2)$$

Hence this equation became

$$\frac{V^2}{L}\frac{\partial u^*}{\partial t^*} = PP_t^* + \frac{\vartheta V}{L^2}\frac{\partial^2 u^*}{\partial y^{*2}} + k_r V^2 U^{*2} \qquad (2.52.3)$$

Dividing this equation throughout by $\frac{V^2}{L}$, we obtained

$$\frac{\partial u^*}{\partial t^*} = \frac{PL}{V^2}P_t^* + \frac{\vartheta}{LV}\frac{\partial^2 u^*}{\partial y^{*2}} + k_r L U^{*2} \qquad (2.52.4)$$

This gave the equation of momentum in non-dimensional form.

2.4.3 EQUATION OF CONSERVATION OF ENERGY

The equation of conservation of energy was given by

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{C_p\rho}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{C_p\rho}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{kA}{C_p\rho}(T_\infty - T_s)\left(1 - \frac{k_r u}{4\left(\frac{\partial u}{\partial y}\right)}\right) \qquad (2.53)$$

From the boundary approximations the above equation reduced to;

$$\frac{\partial T}{\partial t} = \frac{k}{C_p\rho}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{C_p\rho}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{kA}{C_p\rho}(T_\infty - T_s)\left(1 - \frac{k_r u}{4\left(\frac{\partial u}{\partial y}\right)}\right) \qquad (2.53.1)$$

From the non-dimensional form of T, we had

$$T^* = \frac{T - T_s}{T_\infty - T_s}$$

On making T the subject it yielded

$$T = T^*(T_\infty - T_s) + T_s$$

On non-dimensionalising the above equation of energy, we obtained

$$\frac{\partial\big(T^*(T_\infty - T_s) + T_s\big)}{\partial\left(\frac{t^*L}{V}\right)} = \frac{k}{C_p\rho}\frac{\partial^2\big(T^*(T_\infty - T_s) + T_s\big)}{\partial(y^*L)^2} + \frac{\mu}{C_p\rho}\left(\frac{\partial(u^*V)}{\partial(y^*L)}\right)^2 +$$

$$\frac{kA}{C_p\rho}(T_\infty - T_s)\left(1 - \frac{k_r u^* V}{4\left(\frac{\partial(u^*V)}{\partial(y^*L)}\right)}\right) \qquad (2.53.2)$$

This equation became

$$\frac{V(T_\infty - T_s)}{L}\frac{\partial T^*}{\partial t^*} = \frac{k}{C_p\rho}\frac{(T_\infty - T_s)}{L^2}\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\mu V^2}{C_p\rho L^2}\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \frac{kA}{C_p\rho}(T_\infty - T_s)\left(1 - \frac{k_r u^* L}{4\left(\frac{\partial u^*}{\partial y^*}\right)}\right)$$

Dividing above equation throughout by $\frac{V(T_\infty - T_s)}{L}$, we obtained

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{C_p\rho L V}\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\mu V}{C_p\rho L(T_\infty - T_s)}\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \frac{kLA}{C_p\rho V}\left(1 - \frac{k_r u^* L}{4\left(\frac{\partial u^*}{\partial y^*}\right)}\right) \qquad (2.53.3)$$

Multiplying the term $\frac{\mu V}{C_p\rho L(T_\infty - T_s)}\left(\frac{\partial u^*}{\partial y^*}\right)^2$ by V in the numerator and the denominator, we obtained

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{C_p\rho L V}\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\mu V.V}{C_p\rho L.V(T_\infty - T_s)}\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \frac{kLA}{C_p\rho V}\left(1 - \frac{k_r u^* L}{4\left(\frac{\partial u^*}{\partial y^*}\right)}\right) \qquad (2.53.4)$$

Equation (2.59.4) represented the equation of conservation of energy in non-dimensional form.

2.5 NON-DIMENSIONAL NUMBERS

2.5.1 The Prandtl number, *Pr*

This number was named after Ludwig Prandtl a German aerodynamist who was closely associated with the conception of boundary layer theory. It is the parameter which relates the relative thickness of the hydrodynamic and thermal boundary layers. The Prandtl number provided the link between the velocity field and the temperature field. It is expressed as

$$\text{Pr} = \frac{\vartheta}{\alpha} = \frac{\mu\rho}{k\rho\text{Cp}} = \frac{\text{Cp}\mu}{k} \qquad (2.54)$$

This is the ratio of the momentum diffusivity to the thermal diffusivity.

2.5.2 Reynolds number, *Re*

This number was named after a scientist Osborne Reynolds it is defined as the ratio of the inertia forces to the viscous forces. It is given by

$$\text{Re} = \frac{\rho VL}{\mu} = \frac{VL}{\vartheta} = \frac{\text{inertia forces}}{\text{viscous forces}} \tag{2.55}$$

2.5.3 Peclet number, *Pe*

This number was named after a French physicist Jean Claude Peclet, it is defined to be the ratio of advection of a physical quantity by the flow rate of diffusion of the same quantity driven by an appropriate gradient. In context of transport of heat, the Peclet number is equivalent to the product of the Reynolds number and the Prandtl number. This independent heat transfer parameter is defined by

$$\text{Pe} = \text{RePr} = \frac{\rho ULCp}{k} = \frac{VL}{\alpha} \tag{2.56}$$

2.5.4 Eckert number, *Ec*

This is the measure of the kinetic energy of the flow to the boundary layer enthalpy difference across thermal boundary layer, given by;

$$\text{Ec} = \frac{v^2}{Cp(T_\infty - T_S)} \tag{2.57}$$

It plays an important role in high speed flows for which dissipation is significant.

2.6   EQUATIONS GORVERNING THE FLUID FLOW WITH DIMENSIONLESS NUMBERS

2.6.1 Equation of continuity

$$\left(\frac{\partial u^*}{\partial x^*}+\frac{\partial u^*}{\partial y^*}\right)=0 \tag{2.58}$$

2.6.2 Equation of conservation of momentum

$$\frac{\partial u^*}{\partial t^*}=\frac{PL}{V^2}P_t^*+\frac{\vartheta}{LV}\frac{\partial^2 u^*}{\partial y^{*2}}+k_rLU^{*2} \tag{2.59}$$

But, $\text{Re}=\frac{\rho VL}{\mu}=\frac{VL}{\vartheta}$

hence the above equation reduced to;

$$\frac{\partial u^*}{\partial t^*}=\frac{PL}{V^2}P_t^*+\frac{1}{Re}\frac{\partial^2 u^*}{\partial y^{*2}}+k_rLU^{*2} \tag{2.60}$$

2.6.3 Equation of conservation of energy

$$\frac{\partial T^*}{\partial t^*}=\frac{k}{C_p\rho LV}\frac{\partial^2 T^*}{\partial y^{*2}}+\frac{\mu V.V}{C_p\rho L.V(T_\infty-T_s)}\left(\frac{\partial u^*}{\partial y^*}\right)^2+\frac{kLA}{C_p\rho V}\left(1-\frac{k_r u^* L}{4\left(\frac{\partial u^*}{\partial y^*}\right)}\right) \tag{2.61}$$

But , $\text{Pe} = \text{RePr}=\frac{\rho ULCp}{k}=\frac{VL}{\alpha}$,$\text{Re}=\frac{\rho VL}{\mu}=\frac{VL}{\vartheta}$,$\text{Ec}=\frac{V^2}{Cp(T_\infty-T_S)}$ & $\text{Pr}=\frac{\vartheta}{\alpha}=\frac{\mu\rho}{k\rho Cp}=\frac{Cp\mu}{k}$

Hence the equation of conservation of energy reduced to

$$\frac{\partial T^*}{\partial t^*}=\frac{1}{Pe}\frac{\partial^2 T^*}{\partial y^{*2}}+\frac{Ec}{Re}\left(\frac{\partial u^*}{\partial y^*}\right)^2+\frac{L^2 A}{Pe}\left(1-\frac{k_r u^* L}{4\left(\frac{\partial u^*}{\partial y^*}\right)}\right) \tag{2.62}$$

2.7 THE BOUNDARY CONDITIONS

The boundary conditions for the fluid flow over an axi-symmetrical body with curved surfaces, taking into consideration the no-slip condition and negligibility of the effects of viscous forces in the free stream region, were stated below:

2.7.1 Equation of momentum

The equation of conservation of momentum

$$\frac{\partial u^*}{\partial t^*} = \frac{PL}{V^2} P_t^* + \frac{1}{Re} \frac{\partial^2 u^*}{\partial y^{*2}} + k_r L U^{*2}$$

was solved subject to the following boundary and initial conditions

$$U(t,0) = 0$$

$$U(t,\infty) = U_0$$

$$U(0,y) = 0$$

On non-dimensionalising the boundary and initial conditions

$$u^* V \left(\frac{t^* L}{V}, 0 \right) = 0$$

$$u^* V \left(\frac{t^* L}{V}, \infty \right) = U_0$$

$$u^* V (0, y^* L) = 0$$

On simplifying the above boundary and initial conditions we obtained

$$u^*(t^*, 0) = 0$$

$$u^*(t^*, \infty) = 1$$

$$u^*(0, y^*) = 0$$

2.7.2 Equation of conservation of energy

The equation of conservation of energy

$$\frac{\partial T^*}{\partial t^*} = \frac{1}{Pe}\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{Ec}{Re}\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \frac{L^2 A}{Pe}\left(1 - \frac{k_r u^* L}{4\left(\frac{\partial u^*}{\partial y^*}\right)}\right)$$

was solved subject to following boundary and initial conditions

$$T(t, 0) = T_s$$

$$T(t, \infty) = T_\infty$$

$$T(0, y) = 0$$

On non-dimensionalising the boundary and initial conditions;

$$T^*(T_\infty - T_s) + T_s\left(\frac{t^* L}{V}, 0\right) = \frac{T_s - T_s}{T_\infty - T_s}$$

$$T^*(T_\infty - T_s) + T_s\left(\frac{t^* L}{V}, \infty\right) = \frac{T_\infty - T_s}{T_\infty - T_s}$$

$$T^*(T_\infty - T_s) + T_s(0, y^* L) = 0$$

On simplifying the above conditions

$$T^*(t^*, 0) = 0$$

$$T^*(t^*, \infty) = 1$$

$$T^*(0, y^*) = 0$$

2.8 METHOD OF SOLUTION

The proposed method of solving the system of the non-linear equations obtained for this particular flow problem was the numerical approximation method of finite differences. In a finite difference grid to calculate the values at the mesh points, each nodal point was identified by a double index (i, j) that defined its location with respect to t and y as indicated in the figure below. For this particular problem we chose the step value $\Delta y = 0.2$ and $\Delta t = 0.00125$. These step values were so chosen so as to bring about convergence, stability and consistency in the values to be obtained.
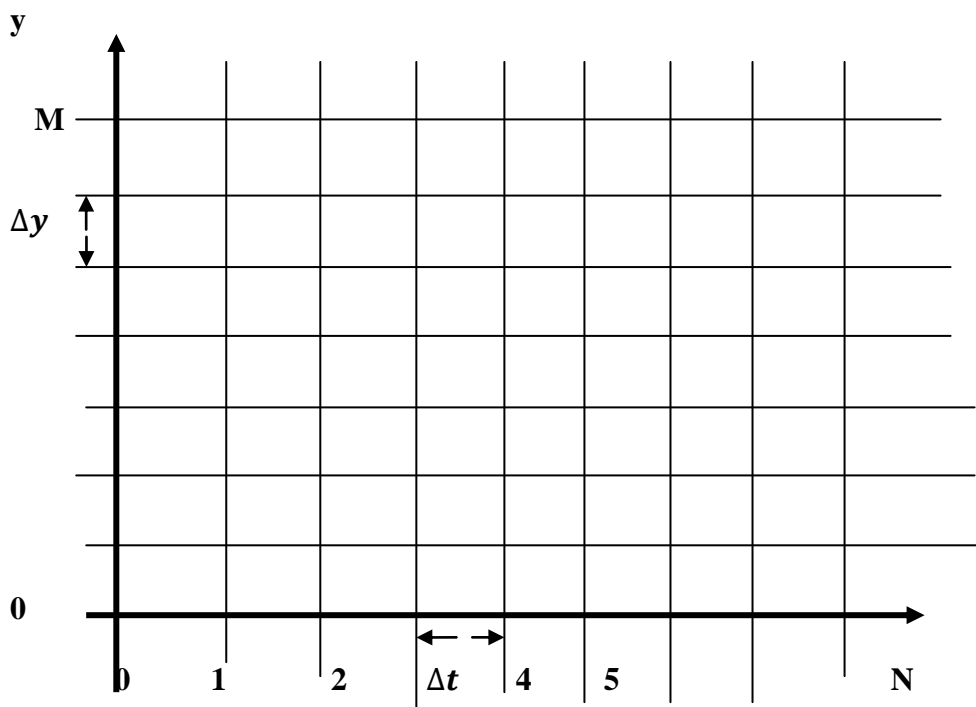
Figure 7: Computational finite difference mesh.

Each corner of the cell forms the mesh or grid point. Considering the y-t plane in the figure 8, it was subdivided into uniform rectangular cells of height of $\Delta y = 0.2$ and width of $\Delta t = 0.00125$. Considering a reference point (i, j) where i and j represent t and y respectively. Using the notation $(i \pm 1)$ for $(t \pm \Delta t)$ and $(j \pm 1)$ for $(y \pm \Delta y)$ we defined the adjacent points to y and t, the points that are i and j units from the reference

point had the co-ordinates $(i\Delta t, j\Delta y)$. In finite difference approximation we replaced the derivatives with the finite differences. If $u = u(t, y)$ and $T = T(t, y)$, their first derivatives with respect to t were written in finite difference form as

$$U_t = \frac{U_{i+1,j} - U_{i,j}}{\Delta t} + 0((\Delta t)^2)$$

$$T_t = \frac{T_{i+1,j} - T_{i,j}}{\Delta t} + 0((\Delta t)^2)$$

Their first order derivatives with respect to y were written in finite difference form as

$$U_y = \frac{U_{i,j+1} - U_{i,j}}{\Delta y} + 0((\Delta y)^2)$$

$$T_y = \frac{T_{i,j+1} - T_{i,j}}{\Delta y} + 0((\Delta y)^2)$$

The second order derivatives with respect to y by forward differencing were;

$$U_{yy} = \frac{U_{i,j-1} - 2U_{i,j} + U_{i,j+1}}{\Delta y^2} + 0((\Delta y)^2)$$

$$T_{yy} = \frac{T_{i,j-1} - 2T_{i,j} + T_{i,j+1}}{\Delta y^2} + 0((\Delta y)^2)$$

The derivatives in the equations governing the flow were replaced by these numerical difference approximations. Values of velocity and temperature were solved iteratively.

The equations governing the fluid flow in finite difference form were written as;

$$u_{i+1,j}^* = \frac{\left[u_{i,j}^* + \frac{\Delta t PL}{v^2} P_{t_i}^* + \frac{\Delta t}{Re} \left[\frac{u_{i+1,j+1}^* + u_{i+1,j-1}^* + u_{i,j+1}^* - 2u_{i,j}^* + u_{i,j-1}^*}{2(\Delta y)^2} \right] + \Delta t k_r L u_{i,j}^{*2} \right]}{\left(1 + \frac{\Delta t}{Re(\Delta y)^2} \right)}$$

$$T^*_{i+1,j} = \left(T^*_{i,j} + \frac{\Delta t}{Pe} \left[\frac{T^*_{i+1,j+1} + T^*_{i+1,j-1} + T^*_{i,j+1} - 2T^*_{i,j} + T^*_{i,j-1}}{2(\Delta y)^2} \right] \right.$$

$$+ \frac{\Delta t Ec}{Re} \left[\frac{u^*_{i+1,j+1} - u^*_{i+1,j} + u^*_{i,j+1} - u^*_{i,j}}{2\Delta y} \right]^2 + \frac{\Delta t L^2 A}{Pe}$$

$$\left. - \frac{\Delta t A L^3 k_r u^*_{i,j}}{4Pe \left[\frac{u^*_{i+1,j+1} - u^*_{i+1,j} + u^*_{i,j+1} - u^*_{i,j}}{2\Delta y} \right]} \right) \div \left(1 + \frac{\Delta t}{Pe(\Delta y)^2} \right)$$

These equations were solved subject to the boundary and initial conditions stated earlier. Visual Basic programming language was used to solve these two equations. In the next chapter, the graphical representation of the results obtained and the analysis was done.

CHAPTER THREE

3.0 INTRODUCTION

In this chapter the equations governing the flow problem in finite difference were reorganized, discretized and computed iteratively. The velocity profiles obtained together with the temperature profiles within the boundary layer are also graphically represented.

3.1 EQUATIONS GOVERNING THE FLUID FLOW IN FINITE DIFFERENCE FORM

The governing equations describing the unsteady, incompressible fluid flow over an axi-symmetrical body with curved surfaces, in finite difference form taking and using the Crank Nicolson method of approximation were given subject to their boundary and initial conditions as:

$$u^*_{i+1,j} = \frac{\left[u^*_{i,j} + \frac{\Delta tPL}{V^2}P^*_{t_i} + \frac{\Delta t}{Re}\left[\frac{u^*_{i+1,j+1} + u^*_{i+1,j-1} + u^*_{i,j+1} - 2u^*_{i,j} + u^*_{i,j-1}}{2(\Delta y)^2} \right] + \Delta t k_r L u^{*2}_{i,j} \right]}{\left(1 + \frac{\Delta t}{Re(\Delta y)^2} \right)}$$

subject to

$$u^*(t^*, 0) = 0$$
$$u^*(t^*, \infty) = 1$$
$$u^*(0, y^*) = 0$$

and

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$$T^*_{i+1,j} = \left(T^*_{i,j} + \frac{\Delta t}{Pe} \left[\frac{T^*_{i+1,j+1} + T^*_{i+1,j-1} + T^*_{i,j+1} - 2T^*_{i,j} + T^*_{i,j-1}}{2(\Delta y)^2} \right] \right.$$

$$+ \frac{\Delta t Ec}{Re} \left[\frac{u^*_{i+1,j+1} - u^*_{i+1,j} + u^*_{i,j+1} - u^*_{i,j}}{2\Delta y} \right]^2 + \frac{\Delta t L^2 A}{Pe}$$

$$\left. - \frac{\Delta t A L^3 k_r u^*_{i,j}}{4Pe \left[\frac{u^*_{i+1,j+1} - u^*_{i+1,j} + u^*_{i,j+1} - u^*_{i,j}}{2\Delta y} \right]} \right) \div \left(1 + \frac{\Delta t}{Pe(\Delta y)^2} \right)$$

subject to;

$$T^*(t^*, 0) = 0$$

$$T^*(t^*, \infty) = 1$$

$$T^*(0, y^*) = 0$$

Values of velocity obtained in the momentum equation were used to compute for temperature values in the energy equation, this was done iteratively and different values were obtained when various flow parameters were varied.

3.2 DATA REPRESENTATION

The following data representation was obtained after solving the equations governing the fluid flow.

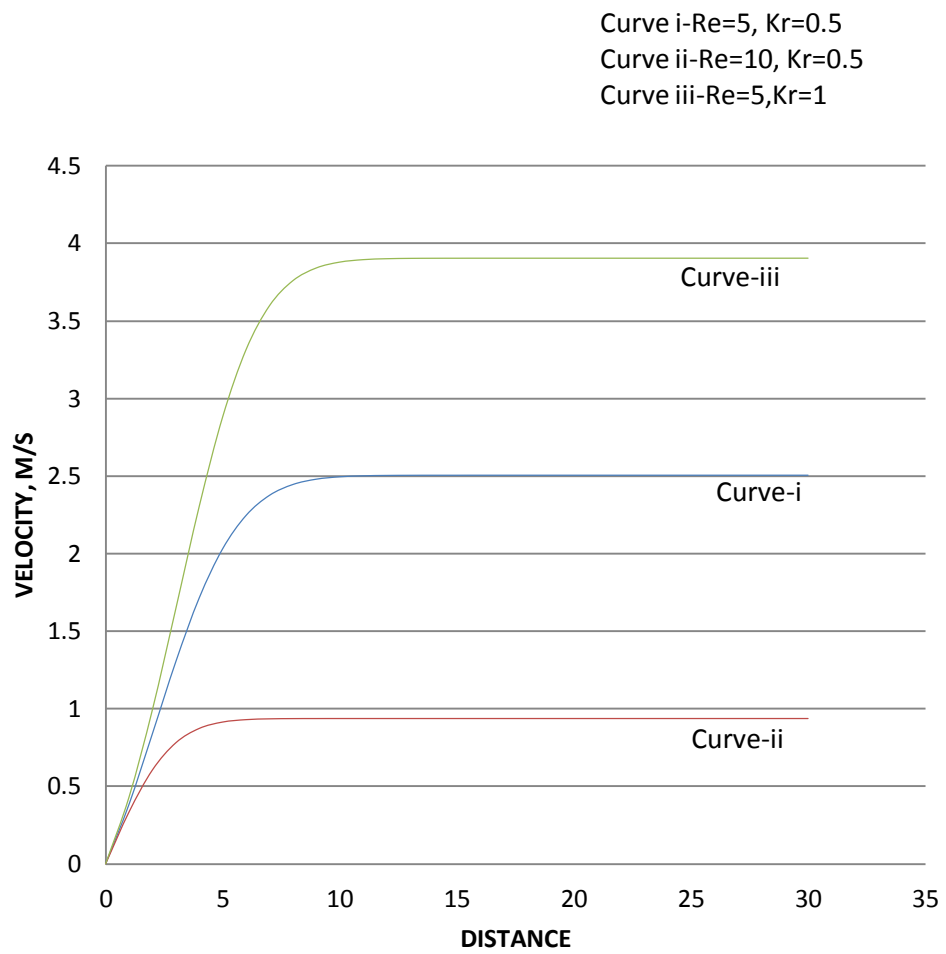**GRAPH OF THE VELOCITY VERSUS THE DISTANCE FROM THE SURFACE WITH Re AND Kr CHANGING**

Curve i-Re=5, Kr=0.5
Curve ii-Re=10, Kr=0.5
Curve iii-Re=5,Kr=1

Figure 8: Velocity profiles for-Ec=1 Pe=2 V=0.5 A=1 L=0.1 P_t=1

GRAPH OF THE TEMPERATURE VERSUS THE DISTANCE FROM THE SURFACE WITH Re AND Kr CHANGING

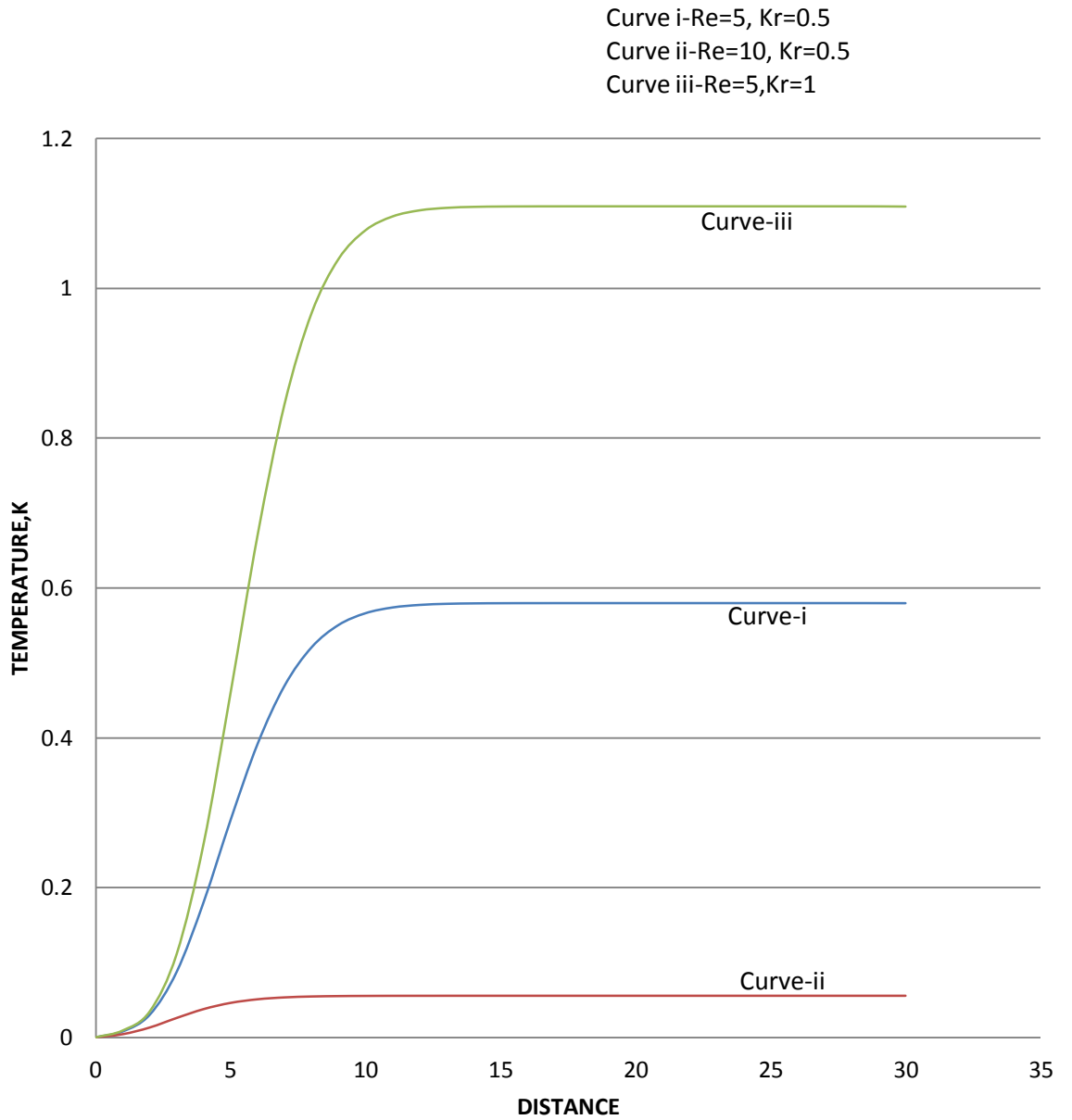Curve i-Re=5, Kr=0.5
Curve ii-Re=10, Kr=0.5
Curve iii-Re=5,Kr=1

Figure 9: Temperature profiles for-Ec=1 Pe=2 V=0.5 A=1 L=0.1 $P_t=1$

GRAPH OF VELOCITY VERSUS THE DISTANCE FROM
THE SURFACE WITH Ec AND Pe CHANGING
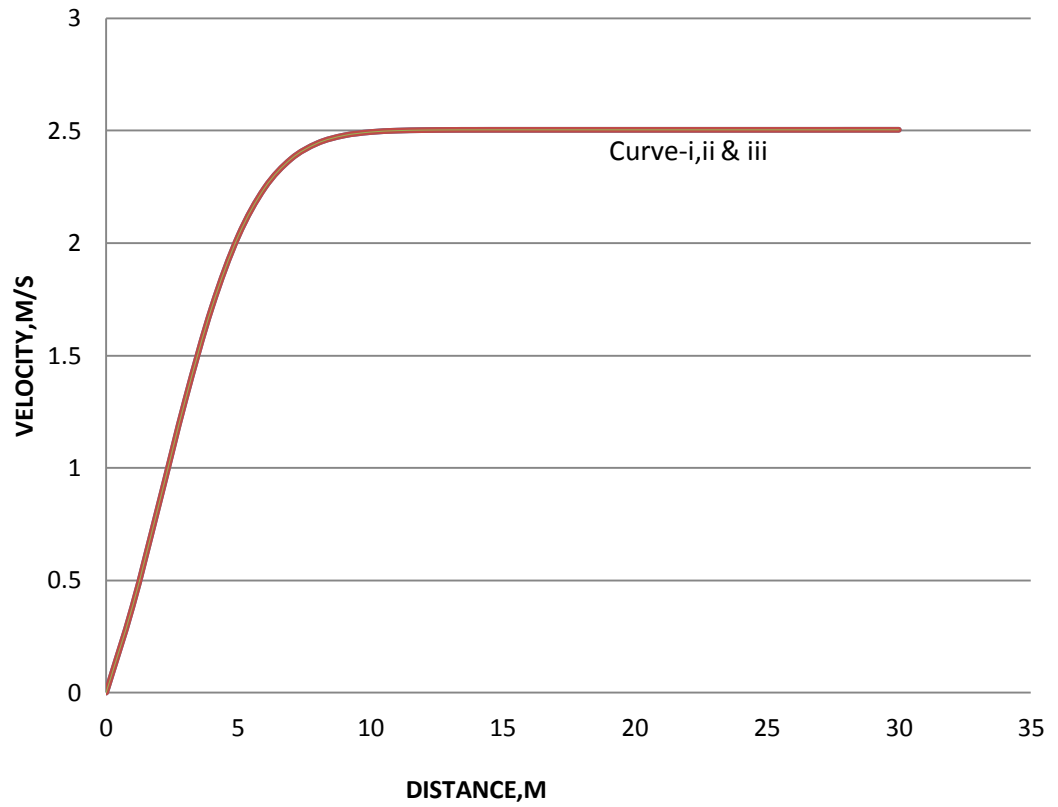
Curve i-Ec=1,Pe=2
Curve ii-Ec=10,Pe=2
Curve iii-Ec=1,Pe=20

Figure 10: Velocity profiles for- Re=5 Kr=0.5 Pt=1 L=0.1 A=1 V=0.5

GRAPH OF TEMPERATURE VERSUS THE DISTANCE FROM
THE SURFACE WITH Ec AND Pe CHANGING
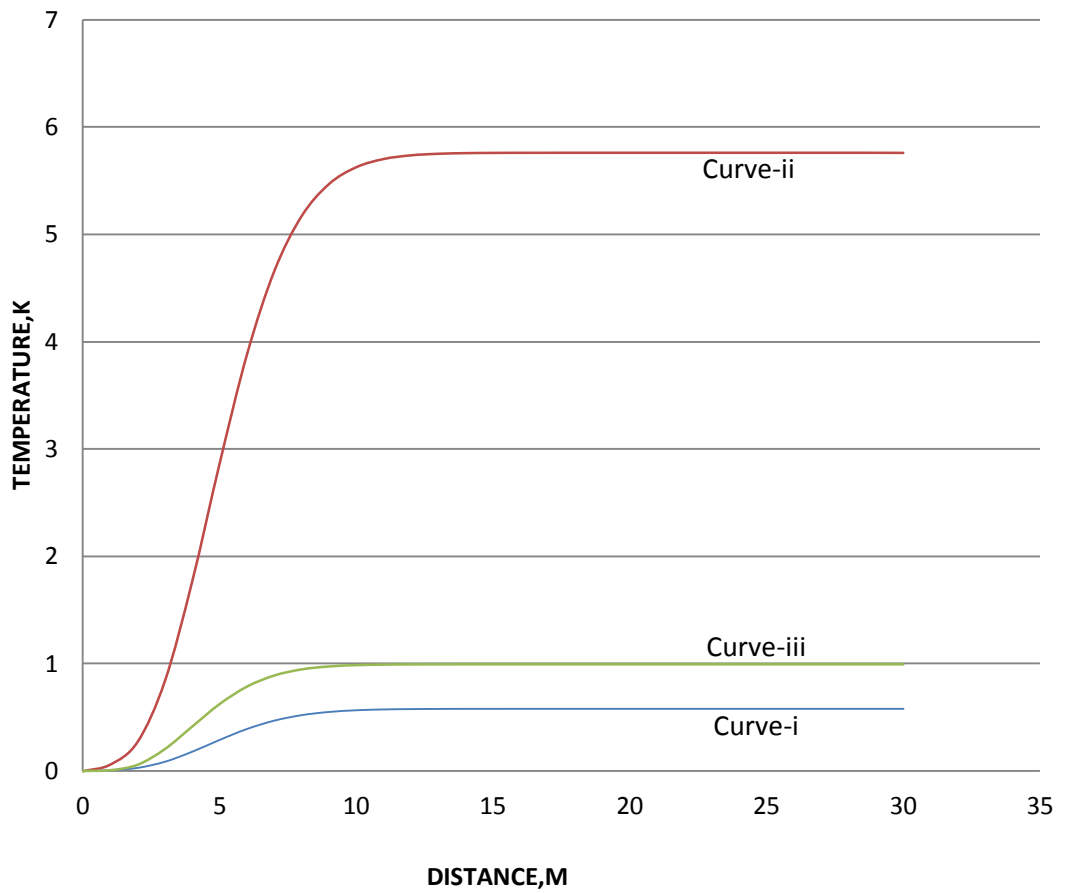
Curve i-Ec=1,Pe=2
Curve ii-Ec=10,Pe=2
Curve iii-Ec=1,Pe=20

Figure 11: Temperature profiles for- Re=5 Kr=0.5 Pt=1 L=0.1 A=1 V=0.5

3.3 Discussion

From figure 8 we noted that:

(i)   When Reynolds number was increased from 5 to 10, we noted that from curve-ii, the free stream velocity of the fluid particles reduced from 2.50423866 m/s to 0.937234433 m/s.

This was because when Re was increased, inertia forces increased and these forces opposed the body from accelerating hence the reduced velocities.

(ii) When the radius of curvature, Kr was increased from 0.5 to 1, we noted that from curve-iii, the free stream velocity of the fluid particles increased from 2.50423866m/s to 3.904712m/s.

This was because increase in curvature increased the velocity gradient. When the curvature of a particular body was increased the velocity gradient also increasesd and when the curvature was reduced the velocity gradient reduced.

From figure 9, we noted that:

(i) When Re was increased from 5 to 10, we noted that from curve-ii, the heat dissipated in the boundary layer reduced from 0.579673 K to 0.0559 K.

This was because when this value of Re was small, it meant that the viscous forces dominated over the inertia forces, these large viscous forces resulted to increased friction between the surface of the body and fluid which brought about increased dissipation of heat within the boundary layer. When Re was large, viscous forces

were minimal and hence the friction between the surface and the fluid was minimal hence resulted to minimal dissipation of heat within the boundary layer.

(ii) When Kr was increased from 0.5 to 1, we noted that from curve-iii, the heat dissipated in the boundary layer increased from 0.579673 K to 1.109492 K.

This was because increasing the curvature resulted in increased velocity gradient, the increased velocity gradient led to increased shear stresses. These shear stresses brought about friction between the fluid and the surface and in turn this friction force led to dissipation of heat within the boundary layer region. This was deduced from the formula

$\tau = \mu \frac{\partial u}{\partial y}$, which implied that, when the velocity gradient was increased, it led to increased shear stress which in turn increased dissipation of heat.

From figure 10, we noted that:

(i) When Ec was increased from 1 to 10, we noted that from curve-iii, the free stream velocity increased from 2.504238662 m/s to 2.504238663 m/s.

This was because when Ec was large; it implied that the kinetic energy dominated the boundary layer enthalpy which meant that the particles or molecules of the fluid had high velocities. When the Ec number was small, it implied that the kinetic energy was small and hence the particles had low velocities, hence when Ec was increased, the velocity also increased.

(ii) When Pe was increased from the 2 to 20, we noted that from curve-ii, the free stream velocity increased from 2.504200941 m/s to 2.504201 m/s.

This was because for large Pe, it meant that rate advection of the fluid dominated the flow rate of diffusion of the same quantity driven by an appropriate gradient, which implied that the fluid particles had large velocities.

From figure 11, we noted that:

(i) When Ec was increased from 1 to 10, we noted that from curve-ii, the heat dissipated increased from 0.576463 K to 5.757413 K.

This was because for large Ec, it implied that the kinetic energy was large and hence the velocities were higher hence when this particles attained high velocities, the vibrations also increased and this led to increased collision of the particles. These increased collisions of particles brought about dissipation of heat in the boundary layer region.

(ii) When Pe was increased from 2 to 20, we noted that from curve-iii, the heat dissipated in the boundary layer increased from 0.576463 K to 0.995561 K.

This was because large Pe led to increased velocities these increased velocities of the fluid particles led to increased collision which in turn led to increased dissipation of heat.

The final part of the discussion was the effect of the convective heat transfer on the drag and lift of the body.

The formula for the drag was given by $F = C_D A \rho \frac{u_0^2}{2}$ where A was the area of the body; C_D was the coefficient of drag, u_0 was the free stream velocity and ρ was the density of the fluid.

The formula for the lift was given by $L = C_L A \rho \frac{u_0^2}{2}$ where A was the area of the body; C_L was the coefficient of lift, u_0 was the free stream velocity and ρ was the density of the fluid.

Particular bodies have specific drag and lift coefficients. For symmetrical bodies the drag coefficient is 0.04 and the lift coefficient is 0.2. The convective heat transfer affected the fluid flowing around the body by varying the velocity of this fluid and hence affected the lift and drag. For this research problem analysis on how Reynolds number affected the lift and drag of a particular body was our main interest. As we have seen earlier the Reynolds number is the ratio between the inertia forces and the viscous forces. When the Re was small it implied that the viscous forces are dominated and hence temperature dissipation in the boundary layer was evident due to increased friction, hence increased drag. As for the lift small Re led to the dissipation in the boundary layer, this resulted in reduced density of the fluid hence reduced lift. When Re was increased the inertia forces dominated over the viscous forces hence this led to reduced velocity in the boundary layer as this forces tended to oppose the body from accelerating, this in turn led to decreased lift. These two analyses on the lift and drag occurred in microscopic increment and decrement.

CHAPTER FOUR

4.0 INTRODUCTION

In this chapter conclusions of the research carried out were outlined and thereafter recommendations for future research work were also made.

4.1 CONCLUSION

Analysis of convective heat transfer over an axi-symmetrical body with curved surfaces has been done and in this chapter, conclusions of the results obtained by varying various parameters that is Reynolds number, Eckert number, Peclet number and the curvature of the surface being investigated were given .The variations of these parameters affected the velocity and the temperature in the boundary layer which was our area of analysis. These variations in turn affected the drag and lift of the body.

It was observed that when Reynolds number, Re was varied, this is the ratio of the inertia forces to the viscous forces say when Re was increased, the boundary layer thickness decreased and inertia force increased. When Re was decreased, the boundary layer thickness increased and inertia force decreased. This matched the theoretical explanation since for increased Re, the viscous forces reduce and the boundary layer thickness reduces and this in turn reduces the dissipation of heat within the boundary layer. Hence when Re was increased, the boundary layer thickness reduced and the temperature also reduced and when Re was reduced, the boundary layer thickness increased and the temperature also increased. Hence for a fluid flow over an axi-symmetrical surface with curved surfaces Reynolds number, Re was inversely proportional to the boundary layer thickness and both the velocity and the temperature.

When curvature of the surface was varied, this led to change in the velocity and also the temperature. When the curvature of the surface was increased, this led to increased velocity and increased temperature. When the curvature of the surface was decreased, this led to decreased velocity and temperature. Hence for a fluid flowing over an axi-symmetrical body with curved surfaces, the curvature was directly proportional to the temperature and the velocity.

When Eckert number was varied this also led to variation in both the temperature and the velocity. When Eckert number was increased, this led to increased velocity and also increased temperature. When Eckert number was decreased, this led to decreased velocity and also decreased velocity. Hence for a fluid flow over an axi-symmetrical surface with curved surfaces, Eckert number was directly proportional to both the velocity and the temperature.

When Peclet number, Pe was varied this also led to variation in both the temperature and the velocity. When Pe number was increased, this led to increased velocity and also increased temperature. When Pe number was decreased, this led to decreased velocity and also decreased temperature. Hence for a fluid flow over an axi-symmetrical surface with curved surfaces, Pe number is directly proportional to both the velocity and the temperature.

Reynolds number Re, affects both lift and drag in that when Re was increased, it led to decreased drag and when Re was decreased, it led to increased drag hence inverse proportionality. When Re is increased, this leads to increased lift and when Re is decreased, it led to decreased lift hence direct proportionality.

4.2 RECOMMENDATIONS

In this thesis results obtained for the analysis of the convective heat transfer of a fluid flow over an axi-symmetrical body with curved surfaces were meaningful to the analysis within the boundary layer; however there remains a lot to be done from this research in order to be able to move closer to the realisation of real life situation and results. In order to achieve this, based on this research the following recommendations will be helpful to assist in further deeper exploration of this topic area;

1. Use of finite element method for solving the problem in order to get more accurate results.

2. Compressible fluid flow over axi-symmetrical body with curved surfaces.

3. For fluid flow of Reynolds number of order greater than 2000 over an axi-symmetrical body with curved surfaces.

4. Flow of an electrically conducting fluid over an axi- symmetrical body with curved surfaces in presence of a magnetic field.

5. Turbulent flow of the same orientation.

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Appendix I: COMPUTER CODE

The following computer program code in Visual Basic was used to solve the equations governing the fluid flow subject to the initial and boundary conditions. The results were obtained by varying various flow parameters notably Reynolds number Re, Eckert number Ec and the surface curvature Kr.

```
Private Sub Command1_Click()

Dim U(0 To 43, 0 To 43) As Double, T(0 To 43, 0 To 43) As Double

Dim ITMAX As Double, Re As Integer, Pt As Integer, Kr As Single, L As Single, Pe As Single, Ec As Single, A As Single

Dim delY As Double, delT As Double, I As Integer, J As Integer, N As Integer

Dim ITCOUNT As Integer

Dim FILENUM As Byte

N = 41:   M = 40 'Grid

delY = 0.2: Re = 20: Pt = 1: Kr = 1: L = 0.1: A = 1: Ec = 1: Pe = 2: V = 0.5

delT = 0.00125

'********************************************************

ITMAX = 43

FILENUM = FreeFile()

Open "C:\Users\duncan\Desktop\MBICHI\researchprogram\profilesTU.txt" For Append As FILENUM

Rem Initial condition

For I = 0 To N

For J = 0 To M

For K = 0 To ITMAX

U(0, J) = 0: T(0, J) = 0:

Next

Next

Rem  Boundary conditions

For I = 1 To N

For J = 1 To ITMAX

T(41, J) = 1: T(41, 0) = 0#
```

```vba
U(41, J) = 1: U(41, 0) = 0#

Next

Next

'Solving for velocities

For I = 1 To ITMAX - 1

 For J = 1 To M - 1

    'calculate u

   U(I + 1, J) = (U(I, J) + delT * ((Pt * L) / (V * V)) * Pt + (delT / (2 * delY * delY * Re)) * (U(I + 1, J +
1) + U(I + 1, J - 1) + U(I, J + 1) - 2 * U(I, J) + U(I, J - 1)) + delT * Kr * U(I, J) * U(I, J)) / (1 + dett / (Re
* delY * delY))

      'calculate v

'Solving for temperatures C

 T(I + 1, J) = (T(I, J) + (delT / (2 * delY * delY * Pe)) * (T(I + 1, J + 1) + T(I + 1, J - 1) + T(I, J + 1) - 2 *
T(I, J) + T(I, J - 1)) + (delT * Ec / (4 * delY * delY * Re)) * (U(I + 1, J + 1) - U(I + 1, J) + U(I, J + 1) -
U(I, J)) ^ 2 + (delT * A * L * L) / (Pe) - (delT * delY * A * L * L * L * Kr * U(I, J)) / (2 * Pe * (U(I + 1,
J + 1) - U(I + 1, J) + U(I, J + 1) - U(I, J)))) / (1 + delT / (Pe * delY * delY))

Next

Next

Print #FILENUM, I

For J = 0 To M

   Print #FILENUM, U(I, J); T(I, J)

     'If I = N Then Print #FILENUM, vbCrLf;

Next

'*********************unsteady values

Close #FILENUM

MsgBox "AM THROUGH SOLVING!!!"

End Sub

Private Sub Command2_Click()

On Error GoTokan

Kill "C:\Users\duncan\Desktop\MBICHI\researchprogram\profilesTU.xlsx"

kan:

Exit Sub

End Sub
```