HYDROMAGNETIC UNSTEADY FLOW BETWEEN TWO PARALLEL SEMI-INFINITE PLATES WITH CONSTANT SUCTION

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DECLARATION

This thesis is my original work and has not been presented for a degree in any other University.

Signature …………………………………. Date……………..

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This thesis has been submitted for examination with our approval as University Supervisors.

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DEDICATION

This thesis is dedicated to my wife Rose Wawira, my children Nelly Murugi and Billy Kimathi for their patience, encouragement, as well as inspiration, and to my parents for educating me, particularly to my late father Ndwiga Mikaavo, who would have longed to live and see this thesis.

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E. N. Ndwiga

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NOMENCLATURE

ROMAN SYMBOLS

$\boldsymbol{B}$	Magnetic field intensity vector, wbm^{-2}
$\boldsymbol{B_x, B_y, B_z}$	Magnetic field intensity in x, y, z directions respectively, wbm^{-2}
$\boldsymbol{C_p}$	Specific heat at constant pressure, $Jkg^{-1}K^{-1}$
$\boldsymbol{D}$	Electric displacement vector, $C\ m^{-2}$
$\boldsymbol{e}$	Unit electric charge, C
$\boldsymbol{E}$	Electric field intensity vector, vm^{-1}
$\boldsymbol{Ec}$	Eckert number $\left(-\frac{U^2}{C_p \Delta T}\right)$
f	Body force, N
$\boldsymbol{H}$	Magnetic field strength, Wbm^{-2}
$\boldsymbol{i, j, k}$	Unit vectors in the x, y, z directions respectively
$\boldsymbol{J}$	Current density vector, Am^{-2}
$\boldsymbol{K}$	Thermal conductivity, $wm^{-1}k^{-1}$
$\boldsymbol{L}$	Characteristic length, m
$\boldsymbol{P}$	Pressure force vector, Nm^{-2}
$\boldsymbol{q}$	Velocity vector of the fluid, ms^{-1}
$\boldsymbol{P_r}$	Prandtl number $\left(=\frac{C_p \mu}{K}\right)$
$\boldsymbol{R}$	Joule heating parameter $\left(=\frac{\sigma \mu B_0^2}{\rho^2 C_p \nabla T}\right)$
$\boldsymbol{R_e}$	Hydrodynamic Reynolds number $\frac{UL}{v}$

$\boldsymbol{R_h}$	Magnetic pressure number $\left(=\frac{B_0^2}{\rho\mu_e U^2}\right)$
$\boldsymbol{t}$	Time (s) where $\left(t=\frac{Lt^*}{U}\right)$
$\boldsymbol{t^*}$	Non-dimensional time.
$\boldsymbol{T}$	General fluid temperature (K)
$\boldsymbol{T_\infty}$	The fluid temperature in the free stream region (K)
$\boldsymbol{T_w}$	The fixed fluid temperature at the boundary plates (K)
$\boldsymbol{U}$	Characteristic velocity in (ms^{-1})
$\boldsymbol{u^*, v^*, w^*}$	Dimensionless velocity components
$\boldsymbol{V_0}$	Suction parameter (v/U)
$\boldsymbol{x^*, y^*, z^*}$	Dimensionless cartesian co-ordinates

GREEK SYMBOLS

θ	The dimensionless temperature
ρ	Fluid density ($kg\ m^{-3}$)
ρ_e	Electric charge density (Cm^{-2})
τ	Coefficient of viscosity
ν	Kinematic viscosity (m^2s^{-1})
μ_e	Magnetic permeability (Hm^{-1})
σ	Electrical conductivity ($\Omega^{-1}m^{-1}$)
η	The electrical diffusivity of the fluid equal to $\sigma\mu_e$
ϑ_H	Magnetic diffusivity of the fluid where $\vartheta_H = \frac{1}{\sigma\mu_e}$
∇	Gradient operator, $\left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)$
∇^2	Laplacian operator $\left(= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$、
Φ	Viscous dissipation function (s^{-2})

ABBREVIATIONS

MHD	Magnetohydrodynamics
LHS	Left hand side
RHS	Right hand side
Hots	Higher order terms
Fgrand	Magnetic field gradient $\left(\frac{\partial B_y}{\partial x}\right)$

ABSTRACT

In our study we have investigated hydromagnetic unsteady flow of a viscous incompressible fluid between two flat, parallel porous semi-infinite plates with constant suction in the presence of a transverse variable magnetic field. The flow variables such as velocity and thermodynamic properties at every point in the fluid vary with respect to time. The fluid is assumed to be flowing in the positive x direction between two parallel flat plates located at the y = ± L planes. We have particularly investigated the effects of Suction (V_0), Hydrodynamic Reynolds number (R_e), magnetic pressure number (R_h), Eckert number (E_c), Prandtl number (P_r), Variable magnetic field gradient (Fgrand) and magnetic field intensity (B) normal to the direction of flow on the dynamic behavior of the fluid when the lower plate is impulsively started in x direction at constant velocity U, while the upper plate remains stationary. We have employed the finite difference method to solve the coupled non-linear and dimensionless partial differential equations governing this problem. The iterations were performed using a computer program and the results have been presented graphically. Our findings show that both primary and secondary velocity profiles are largely influenced by change in $V_{0,}$ R_e, R_h, B_y, and Fgrand. Temperature profiles are unaffected by magnetic field intensity (By) but influenced by suction, Prandtl number and Eckert number.

CHAPTER ONE

1.0 INTRODUCTION

Fluid flow can be considered to take place in presence of a magnetic field. In the absence of a magnetic field the fluid flow is studied under hydrodynamics. The fluids in this case include liquids and gases. The flow of a conducting fluid in a magnetic field is a branch of fluid flow studied under Magnetofluiddynamics and it combines the flow of both conducting liquids and gases. The fluids which are electrically conducting include liquid metals and ionized gases. The characteristics of the flow of an electrically conducting liquid in presence of a magnetic field are studied under Magnetohydrodynamics (MHD). The principle behind MHD theory and the effect of a transverse magnetic field applied to an electrically conducting fluid is that the interaction between the magnetic field and the induced electric current affects the characteristics of the fluid particle in the flow field. MHD tries to study the variation of these characteristics.

1.1 Definition of terms

1.1.1 Ideal and real fluids

An ideal fluid is one that is incompressible and flows steadily, irrotationally and with no viscosity. According to the non-dissipative nature of ideal fluids, experimental results have shown that fluids such as air and water are not so ideal and that ideal fluids do not actually exist. Real fluids are compressible and their flow exhibits viscous effect. This means that whenever there is a velocity gradient across the real fluid's flow path, frictional forces arise between adjacent fluid particles due to the viscosity μ of the fluid. Ideal fluids obey the Newton's law of viscosity i.e.

$$\tau \propto \frac{du}{dy} \tag{1.1}$$

where τ is the coefficient of viscosity, u is the fluid velocity and y is the transition distance. This means that the shear stress τ in a fluid is proportional to the velocity gradient, which is the rate of change of velocity across the fluid flow path. For a Newtonian fluid, we can express

$$\tau = \mu \frac{du}{dy} \tag{1.2}$$

The constant of proportionality μ is known as the coefficient of viscosity or simply viscosity. For some fluids sometimes known as exotic fluids, the value of μ changes with stress or velocity gradient. The viscosity of a pure Newtonian fluid depends only on temperature and pressure. When viscous fluids flow between stationary solid surfaces, the velocity of fluid particles in contact with the solid boundary is zero. At the solid wall boundary, a type of frictional force called skin friction exists. A boundary layer forms in the fluid flow region close to the solid wall. This is due to the no slip boundary condition. The thickness of the boundary layer will be dependent on the Reynolds number and the local flow properties.

1.1.2 Steady and unsteady flow

Fluid flow can be classified as either steady or unsteady. The flow is said to be steady if the fluid flow variables such as velocity, applied magnetic field and temperature are independent of time. If on the other hand the flow variables are dependent on time the flow is said to be unsteady.

1.1.3 Laminar and turbulent flow

Laminar fluid flow is the motion of the fluid particles in a very orderly manner with all particles moving in straight lines parallel to the boundary walls. The particles do not encounter disturbance on their path. Turbulence in fluid flow occurs when a flowing fluid suddenly encounters a disturbance such as a solid obstruction or a force. As a result the fluid particles move in a disorderly manner with different velocities and energies. The shape of the velocity curve (the velocity profile across any given section of the flow channel) depends upon whether the flow is laminar or turbulent. For turbulent flow in a pipe a fairly flat velocity distribution exists across the section of the flow field, with the result that the entire fluid flows at a given single value. If the flow is laminar the shape is parabolic with the maximum velocity at the centre being about twice the average velocity in the pipe.

1.1.4 The continuum hypothesis

As in ordinary hydrodynamics, the dynamics of the conducting fluid flowing in a trans verse magnetic field obeys theorems expressing the conservation of mass, momentum and energy. These theorems are; matter can neither be created nor destroyed, momentum of a moving body is always conserved and energy can never be destroyed but can be converted from one form to another. These theorems treat the fluid as a continuum. Although this assumption does not generally hold for plasmas, one can gain much insight into magnetohydrodynamics from the continuum approximation. For incompressible fluids, the mean distance between fluid particles remains fairly constant and is not affected by an increase in pressure.

1.1.5 Magnetohydrodynamics

Magnetohydrodynamics (MHD) is a branch of science which concerns the study of the flow of an electrically conducting fluid in the presence of a magnetic field. The fluids can be ionized gases generally called plasmas or liquid metals. Other terminologies used to refer to MHD are hydromagnetics or magnetofluiddynamics. The central point of MHD theory is that magnetic field can induce a current in a moving conductive fluid which ends up creating pondermotive forces on the fluid particles and also change the magnetic field itself. When a conducting fluid moves through the magnetic lines of force the positive and negative charges are each accelerated in such a way that their average motion gives rise to an electric current $j = \sigma(q \times B)$. In accordance with the dynamo rule, the voltage drop or electric field which causes this current is at right angles to the direction of fluid motion and the magnetic field lines; (Verma and Marthur 1968).

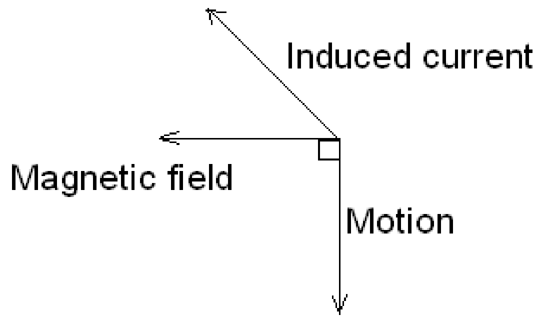

Figure 1.1. Dynamo rule

In the case of a fluid conductor flowing in presence of a transverse magnetic field, the ordinary laws of hydrodynamics can easily be extended to cover the effect of magnetic and electric fields. This is done by adding magnetic force to the momentum conservation equation. To incorporate exhaustively the effects of magnetic and electric fields, the electric heating and work are added to the energy conservation equation. The Lorentz force is in a direction perpendicular to both J and B and is proportional to the magnitude of both J and B and is given by the cross product of J and B i.e.

$$F_e = J \times B \tag{1.3}$$

In MHD this force acts on the fluid particles.

1.1.6 Model and Prototype.

A model is an imitation of the actual object constructed in such a way so as to include

all the technical characteristics of the actual object. For example, if a pump for corrosive liquids is being developed, several models of the actual pump are needed for accelerated life tests with different chemicals. The results of these tests are compared and the desired pump characteristics are compiled and used to construct the actual pump. Such imitations used for testing are called models. In order to analyze the governing equations in MHD flow, a method of developing the flow model is adopted. The fact that the fluid motion in the model and prototype flow can be compared using non-dimensional parameters is an inevitable tool. The non-dimensional parameters are obtained by non-dimensionalising the governing equations. We shall discuss these non-dimensional parameters and the governing equations in chapter two.

1.2 Literature review

The field of MHD fluid flow between parallel plates has attracted the interest of many scientists for a long time. Hartmann and Lazarus (1937) studied the influence of a transverse uniform magnetic field on the flow of a conducting fluid between two stationary, insulated, and parallel infinite plates and discussed the results theoretically and experimentally. The observations made by the above scientists motivated other scholars to do research in this field. Stewartson (1951) studied and analyzed hydromagnetic flow of a viscous incompressible fluid past an impulsively started semi-infinite plate and Rossow (1958) extended the research on the flow of an electrically conducting fluid over a flat plate in the presence of a transverse magnetic field. Jain (1967) concentrated on the effect of wall porosity on the stability of hydromagnetic flow between parallel plates under transverse magnetic field and expressed the idea that the flow is largely influenced by porosity and the flow parameters. Sonju (1968) studied the role of the local acceleration term in MHD momentum equation while Verma and Marthur (1968) researched on MHD laminar flow of an electrically conducting, viscous and incompressible fluid between two wavy walls. The governing equations are taken in the form of Fourier series investigated under the assumption that the coefficient of roughness and the hydrodynamic Reynolds number of the flow are small. Srivastava (1971) studied hydromagnetic couette flow of an electrically conducting viscous and incompressible fluid in presence of a transverse magnetic field when the plates are non-magnetic and non-conducting with variable suction. Bhaskara and Bathaiah (1981) analyzed MHD flow of a viscous, incompressible and slightly conducting fluid between a parallel flat wall and a long wavy wall and evaluated the velocity distribution, the coefficient of skin friction and temperature distribution. Hassanien and Mansour (1990) presented the analysis of a two dimensional unsteady flow of a viscous, incompressible and electrically conducting fluid through a porous medium bounded by two infinite parallel plates under the action of a transverse magnetic field with the lower plate fixed and the other oscillating in its own plane. They discussed the effects of varying magnetic parameter, frequency parameter and permeability of the porous medium. Das and Ahmed (1992) investigated the convective MHD flow past a uniformly moving infinite vertical plate with the magnetic field and the suction applied normal to the plate and Zamm (1996) researched on MHD free convection flow from a vertical semi-infinite flat plate with a step change in magnetic field. Attia (1998) studied transient MHD flow and heat transfer between two parallel plates with temperature dependent viscosity. Nthiarasu (2001) presented numerical work on natural convection in porous medium fluid interface problems where he used the finite difference method to solve the generalized flow governing equations. Attia (2002) studied unsteady MHD flow and heat transfer of a dusty fluid between two parallel plates with variable physical properties. Knaepen *et al*., (2003) analyzed MHD turbulence at moderate magnetic Reynolds number and Singh (2003) presented numerical solution of hydromagnetic unsteady flow past an infinite porous plate. Rajeev and Jain (2004) examined the problem of MHD free convection flow in the presence of a temperature dependent heat source of a viscous incompressible fluid between a long vertical wavy wall and a parallel flat wall with constant heat flux and uniform transverse magnetic field. Chandra (2005) studied a steady hydromagnetic flow of an electrically conducting fluid between two parallel infinite plates and established that the flow profiles are influenced by the variation of the flow parameters. Mittal *et al*., (2005) analyzed buoyancy driven convection flow of liquid metals subjected to a transverse magnetic field and Smolentsev and Moreau (2006) investigated modeling quasi two dimensional turbulence in MHD duct flows in a trans-verse uniform magnetic field where viscous and Ohmic losses occur in the boundary layers at the flow-confining walls perpendicular to the magnetic field. Mittal and Kant (2006) made an analysis of natural convection in liquid metals subjected to an alternating magnetic field. Ramulu *et al*., (2007) investigated the effect of Hall current on MHD flow and heat transfer along a porous flat plate with mass transfer applying numerical methods to obtain the solutions. Ganesh (2007) studied MHD Stokes flow of a viscous fluid between two parallel porous plates in a channel in the presence of a transverse magnetic field when the fluid is being withdrawn through both walls of the channel at the same rate and Jalil and Al-Tae'y (2007) considered MHD turbulent flow of a liquid metal filled in a square enclosure with natural convection.

1.3 Problem statement

In the studies discussed above, none of the researchers has investigated the flow of a conducting fluid between two parallel porous plates when the lower plate is impulsively started at constant velocity and the upper plate stationary when a transverse variable magnetic field is applied in a direction perpendicular to the plates. The present research will be to investigate unsteady flow of an incompressible, viscous and electrically conducting fluid between two parallel semi-infinite porous plates when the lower plate is set impulsively in motion at constant velocity U while the upper plate remains stationary in the presence of a variable magnetic field and constant suction. The fluid under consideration is assumed to be fairly viscous. Semi- infinite implies that the flow field is unbounded in one direction; the Z direction.

1.4 Objectives of the study

I. To determine the velocity and temperature profiles of a conducting fluid flowing between two parallel, porous and semi infinite plates under the influence of a transversely applied inhomogeneous magnetic field when the lower plate is set impulsively in motion at constant velocity in the flow direction while the upper plate is stationary.

II. To analyze the effect of varying the flow parameters; viz Hydrodynamic Reynolds number, Suction, magnetic pressure number, magnetic field intensity, Eckert number and prandtl number on the Velocity and temperature distributions for this flow.

1.5 Justification

Magnetohydrodynamics is a field with a wide range of practical applications particularly in Engineering. Scientific research in electricity and magnetism is on a world wide scale. In many practical engineering applications, we encounter conducting fluids flowing between moving boundaries. Our problem is a particular case of a fluid whose motion is caused by the relative motion of two parallel plates when the lower boundary is impulsively set at constant velocity in the flow direction while the upper plate is stationary. Apart from scientific curiosity about how porosity of the plates affects this type of flow, we intend to consider unsteady flow which has received little attention in previous related research. The application of MHD in engineering structures such as flow of liquid metals, cooling of nuclear reactors, electromagnetic casting, behavior of plasma in fusion reactors, cooling of moving parts in automobile engines, MHD electric current generators gives our study a practical framework. In the next chapter, we present the equations governing an unsteady flow of an incompressible and electrically conducting fluid between two parallel plates in the presence of a transverse magnetic field.

CHAPTER TWO

2.0 GOVERNING EQUATIONS

The assumptions and approximations made in this problem are stated. The governing equations are then subjected to the flow conditions of our problem and simplified. The flow problem is described and the equations non-dimensionalised followed by the discussion of the non-dimensional parameters. A method of solution using finite differences is then discussed and the equations expressed in terms of finite differences.

2.1 Approximations and assumptions

i. The fluid is incompressible, electrically conducting and fairly viscous.
ii. Thermal conductivity, electrical conductivity and coefficient of viscosity are constants.
iii. There is no externally applied electric field and hence E=0.
iv. Compared with the speed of light (c), the fluid velocity $\vec{q}$ is negligible, i.e. $\vec{q}^{\,2} << c^2$.
v. The fluid does not undergo any chemical reaction.
vi. The displacement current is negligible because the fluid is assumed to be fairly conducting and as such, the charge relaxation time is much shorter than the transit time of electromagnetic waves.
vii. The flow is unsteady and there is constant suction at the plates.

viii. The Lorentz force J X B due to magnetic field dominates the force $\rho_e E$ due to the electric field.

ix. The fluid satisfies the continuum hypothesis and the mean free path of the fluid elements is negligible as compared to the distance between the plates.

x. The plates are non-conducting; the upper plate is stationary and the lower plate is set moving in x positive direction at constant velocity.

xi. The permittivity, permeability, and conductivity are all assumed to be isotropic, i.e. D, J and E are in the same direction, and B is in the direction of H.

xii. The structure of the porous plates is not flexible or compressible.

2.2 The governing equations

The principle of the theory of relativity are widely applied in the study of fluid flow by considering that the fluid matter is conserved no matter the medium of particle or molecular interaction. The general equations governing fluid flow between two parallel porous plates under the influence of a transverse inhomogeneous magnetic field include the equation of conservation of mass otherwise called the equation of continuity which guarantees that the mass of the fluid is conserved. This law is based on the preposition that the mass of the fluid is conserved in the flow field and that the mean distance between fluid particles of an incompressible fluid remains fairly constant and that the fluid volume is not affected by an increase in pressure. Hence the continuum hypothesis is an application of the scientific theory of mass conservation and is expressed in the equation of conservation of mass. The momentum equation on the other hand balances the resultant forces affecting the fluid with its consequential accelerations on the basis that the momentum of the fluid particle in motion must be conserved. The acceleration of the particle in this regard is mathematically considered to constitute the temporal and convective terms. Scientists have equally accepted that energy can neither be created nor destroyed but can be transformed from one form to another. There are at least six forms of energy namely mechanical, electrical, chemical, nuclear, and electromagnetic and heat or thermal energy. The energy in the fluid flow system is generally governed by the energy equation which is used to determine the temperature profiles of the electrically conducting fluid in relation to continuously changing kinetic and electro-magnetic energies. The main objective in MHD is to study velocity and magnetic field distributions and their interactions of whose product is an induced electric current which in turn interacts with the magnetic field via Maxwell's electromagnetic laws. A further outcome of this interaction is a complex interference with the fluid flow profiles due to a force called Lorentz force expressed as J X B. Consequently, the fluid flow is governed by a system of highly nonlinear coupled partial differential equations representing the profiles of the local flow variables such as velocity and temperature. The typical equations which govern parallel flow in MHD are presented and discussed in the following sections.

2.2.1 Equation of conservation of mass

The conservation of mass is represented by the equation of continuity and is mathematically expressed as

$$\frac{\partial \rho}{\partial t} + \rho \nabla \vec{q} = 0 \tag{2.1}$$

This law is based on the prepositions that the mass of the fluid is conserved in the flow field and that the mean distance between fluid particles of an incompressible fluid remains fairly constant and that the fluid volume is not affected by an increase in pressure. This is the continuum hypothesis. Since we are considering an incompressible fluid the density ρ is assumed to be constant hence equation **(**2.1**)** takes the form

$$\nabla.\vec{q} = 0 \tag{2.2}$$

2.2.2 The induction equation

The main objective in MHD is to study velocity and magnetic field distributions and their interactions. The basic laws of electricity and magnetism can be summarized in differential form by four equations called the Maxwell's electromagnetic equations (2.3, 2.4, 2.5 & 2.6), (Shercriff, 1965).

- Faraday's law $$\mu \frac{\partial \vec{H}}{\partial t} = -\nabla \times \vec{E} \tag{2.3}$$
- Ampere's law $$\nabla \times \vec{H} = \vec{J} \tag{2.4}$$
- Coulomb's law $$\nabla . D = \rho_e \tag{2.5}$$
- Absence of free magnetic poles $$\nabla . \vec{B} = 0 \tag{2.6}$$

The above equations except Faraday's law were derived from steady state observations. Maxwell spurred on by Faraday's observations that the static equations may not hold unchanged for time dependent fields noted that the equation for Ampere's law was faulty. He noted that the continuity equation could be converted into a vanishing divergence by using Coulomb's law and hence replaced J in Amperes law by its generalization. While $\nabla . J = 0$ is valid for steady state problems, the complete relation is given by the continuity equation for charge and current and can be expressed as

$$\nabla . \vec{J} + \frac{\partial \rho_e}{\partial t} = 0 \tag{2.7}$$

This relation can further be written as

$$\nabla . \vec{J} + \frac{\partial \nabla . D}{\partial t} = 0 \tag{2.8}$$

or

$$\nabla . \left(\vec{J} + \frac{\partial D}{\partial t} \right) = 0 \tag{2.9}$$

This adjustment is necessary considering the fact that most hydromagnetic flows are unsteady and the flowing fluid encounters rapidly fluctuating fields. Below are the set of the revised Maxwell's electromagnetic equations.

- $\mu \frac{\partial \vec{H}}{\partial t} = -\nabla \times \vec{E}$ (2.10)
- $\nabla \times \vec{H} = \vec{J} + \frac{\partial D}{\partial t}$ (2.11)
- $\nabla . D = \rho_e$ (2.12)
- $\nabla . \vec{B} = 0$ (2.13)

Consider an electric charge e moving in an electromagnetic field. It experiences an electric force E and a magnetic force $q \times B$. The resultant force on the charge e is the sum of the two forces and is given by the Lorentz's equation which is expressed as

$$F_e = \vec{E} + \vec{q} \times \vec{B} \tag{2.14}$$

This force acts in a direction normal to both J and B and is proportional to their magnitude. Then we have the generalized Ohm's law in which σ is a material property known as the electrical conductivity;

$$\vec{J} = \sigma\left(\vec{E} + \vec{q} \times \vec{B}\right) + \rho_e \vec{q} \tag{2.15}$$

The displacement current term $\rho_e \vec{q}$ is usually negligible at fluid's local velocity $\vec{q}$ which is usually much less than the speed of light and the law reduces to

$$\vec{J} = \sigma\left(\vec{E} + \vec{q} \times \vec{B}\right) \tag{2.16}$$

Taking the curl of equation (2.16) yields

$$\nabla \times \vec{J} = \sigma\left[\nabla \times \vec{E} + \nabla \times \left(\vec{q} \times \vec{B}\right)\right] \tag{2.17}$$

The displacement current is negligible with respect to J and $\nabla \times \vec{H}$ because we are interested in materials such as molten metal which is sufficiently conducting. Substituting equation (2.10) and (2.11) in (2.17) yields

$$\nabla \times (\nabla \times \vec{\mathrm{H}}) = \sigma \left[-\mu_e \frac{\partial \vec{H}}{\partial t} + \nabla \times (\vec{q} \times \vec{B})\right] \qquad (2.18)$$

Expanding equation (2.18) using the vector triple product rule

$a \times (b \times c) = (a.c)b - (b.c)a$ yields

$$(\nabla.\vec{H})\nabla - (\nabla.\nabla)\vec{H} = -\sigma\mu_e \frac{\partial H}{\partial t} + \sigma\mu_e(\nabla.\vec{H}).\vec{q} - \sigma\mu_e(\vec{q}.\vec{H})\nabla \qquad (2.19)$$

If the divergence of B is zero, the magnetic field is uniform we can deduce that the first term in equation (2.19) vanishes and the equation simplifies to

$$-(\nabla.\nabla)\vec{H} = -\sigma\mu_e \frac{\partial \vec{H}}{\partial t} + \sigma\mu_e \nabla \times (\vec{q} \times \vec{H}) \qquad (2.20)$$

The above equation can be further simplified by substituting a constant $\eta = \sigma\mu_e$ into the form

$$\eta \frac{\partial \vec{H}}{\partial t} = \eta Curl(\vec{q} \times \vec{H}) + \nabla^2 \vec{H} \qquad (2.21)$$

The constant η is called the electrical diffusivity of the fluid. By introducing another constant; the magnetic diffusivity $\vartheta_H = \frac{1}{\eta}$ and simplifying equation (2.21) leads to the form

$$\frac{\partial \vec{H}}{\partial t} = Curl(\vec{q} \times \vec{H}) + \vartheta_H \nabla^2 \vec{H} \qquad (2.22)$$

Equation (2.22) is reffered to as the induction equation and it establishes that the local rate of change of H results from the net effect of convection (the curl term) and diffusion, (the last term). By convection is meant the tendency of a travelling fluid element to have constant characteristics (e.g temperature) in the absence of diffusion.To explain diffusion, consinder a case where convection is supressed by taking vector $\vec{q}$ as zero in the induction equation. Then the evolution of a magnetic field in a stationary conductor is given by

$$\frac{\partial \vec{B}}{\partial t} = \vartheta_H \nabla^2 \vec{B} \tag{2.23}$$

Equation (2.23) can be recognized as similar to the well known heat conduction equation

$$\frac{\partial T}{\partial t} = \propto \nabla^2 T \tag{2.24}$$

in which T is temperature and $\propto$ is thermal diffusivity, the only difference being that B is a vector while T is a scalar. Equations (2.23) and (2.24) express the idea that the relevant quantity B or T cannot change its distribution in the medium instantaneously, but must diffuse at a limited rate. Expanding the convection term in equation (2.22) using vector triple product rule yields

$$\frac{\partial \vec{H}}{\partial t} = \left(\nabla . \vec{H}\right)\vec{q} - (\nabla . \vec{q})\vec{H} + \vartheta_H \nabla^2 \vec{H} \tag{2.25}$$

The vector dot products are commutative i.e. $\nabla.\vec{H} = \vec{H}.\nabla$ and $\nabla.\vec{q} = \vec{q}.\nabla$ and equation (2.25) yields

$$\frac{\partial H}{\partial t} + (\vec{q}.\nabla)\vec{H} - (\vec{H}.\nabla)\vec{q} = \vartheta_H \nabla^2 \vec{H} \qquad (2.26)$$

Equation (2.26) is referred to as the induction equation**.**

2.2.3 Equation of conservation of momentum

The law of conservation of momentum postulates that the sum of all the resultant forces is equal to the rate of change of momentum. The momentum of a body is defined as the product of its mass and velocity. Hence we can conclude from this postulate that on application of a force to an incompressible fluid mass, its velocity changes. The unit of momentum in Standard International units is one kilogram meter per second. When two bodies having different masses are acted upon by the same force for the same time, they attain different velocities but gain equal momentum. This important connection between force and momentum was recognized by Sir Isaac Newton and led to the formulation of the Newton's second law of motion which states that the rate of change of momentum of a body is proportional to the applied force and takes place in the direction in which the force acts. In fluid flow, the rate of change of momentum of a fluid element is equal to the sum of the forces acting on the fluid element. For us to be able to consider all the forces taking effect in hydromagnetic flow, we first discuss electromagnetic force which acts on the fluid particles. The application of a magnetic field (B) to a conducting fluid in motion causes the formation of induced currents (J). The induced currents interact with the externally applied magnetic field resulting in the damping of the flow field by the Lorentz force. An electric charge e moving in an electromagnetic field experiences an electric force E and a magnetic force $\vec{q} \times \vec{B}$. The resultant force on the charge e is the sum of the two forces and is given by Lorentz's equation which is expressed as

$$F_e = \rho_e \vec{E} + \vec{J} \times \vec{B} \tag{2.27}$$

There is no externally applied electric field and hence E=0 and equation (2.27) reduces to

$$F_e = \vec{J} \times \vec{B} \tag{2.28}$$

The momentum equation governing the flow of an electrically conducting fluid can thus be adjusted to include Lorentz force yielding

$$\rho \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}\right] = -\nabla P + \mu \nabla^2 \vec{q} + \vec{J} \times \vec{B} + \rho f \tag{2.29}$$

The first term on the LHS of equation (2.29) represents the temporal acceleration and the second term the convective acceleration. On the RHS, the first term is the pressure gradient force, the second term is the viscous force, and third term is the Lorentz force and lastly the body force. The last two terms are body forces.

2.2.4 Equation of conservation of energy

The energy conservation is expressed in the law of conservation of mater. There are at least six forms of energy namely mechanical, electrical, chemical, nuclear, and electromagnetic and heat or thermal energy. The mathematical formulation of the equation of conservation of thermal energy is derived from the first law of thermodynamics. This law asserts that the amount of heat added to a system dQ is equal to the change in internal energy dE plus the work done dW and is expressed as

$$dQ = dE + dW \tag{2.30}$$

Considering the flow of an incompressible fluid with constant thermal conductivity K, thermal energy equation is expressed as

$$\rho C_p \frac{DT}{Dt} = K\nabla^2 T + \mu\emptyset \tag{2.31}$$

In the spirit of Boussinesq approximation, it is supposed that the fluid has a constant heat capacity per unit volume ρC_p, implying that $\rho C_p \frac{DT}{Dt}$ is equal to the rate of heating per unit volume of a fluid particle. Thermal conductivity K of the fluid is the rate of flow of heat through the fluid per unit cross-sectional area per unit temperature gradient, $\mu\emptyset$ is the internal heating due to viscous dissipation and $\frac{DT}{Dt}$ is the material derivative of the absolute temperature T of the fluid. In addition, Ohmic heating occurs due to the internal resistance of the fluid when a conducting fluid flows in presence of a transverse magnetic field. This leads to an extra term in the energy equation governing hydro-magnetic flow. Granted Ohm's law without Hall effect,

$$\vec{E}.\vec{j} = \frac{\vec{j}^2}{\sigma} - \vec{j}.\left(\vec{q} \times \vec{B}\right) \tag{2.32}$$

Of E.j, the Ohmic heating term $\left(\frac{j^2}{\sigma}\right)$ contributes to an increase in internal energy of the fluid. The remainder part $\vec{j}.\left(\vec{q} \times \vec{B}\right)$ pushes the fluid, either creating kinetic energy or helping to overcome other forces or the reverse if the term is negative. Considering the Ohmic heating, the energy equation takes the form

$$\rho C_p \frac{DT}{Dt} = K\nabla^2 T + \mu\emptyset + \frac{\vec{j}^2}{\rho} \tag{2.33}$$

2.3 Description of the research problem

This study focuses on the effects of a transverse variable magnetic field on the flow of an electrically conducting fluid. An electrically conducting incompressible fluid fills the space between the two non-conducting, non-magnetic, semi infinite parallel porous plates. The two parallel porous plates are located at positions y = - L and y = L respectively, hence their separation is 2L units. The x coordinate axis has been taken along the main flow direction. The system is initially at rest and the lower plate is impulsively set in motion in the positive x direction at constant velocity U while the upper plate remains stationary. We assume that the upper plate attains a temperature T_w while the lower plate is maintained at initial free stream temperature T_∞. The flow is induced by the relative motion of the two solid walls taking into account that the flow of a viscous fluid no matter how little the viscosity must satisfy the no slip condition, i.e. the velocity of the fluid layer adjacent to a solid boundary is equal to the velocity of the solid boundary. For a solid boundary at rest, the fluid in contact with the body has

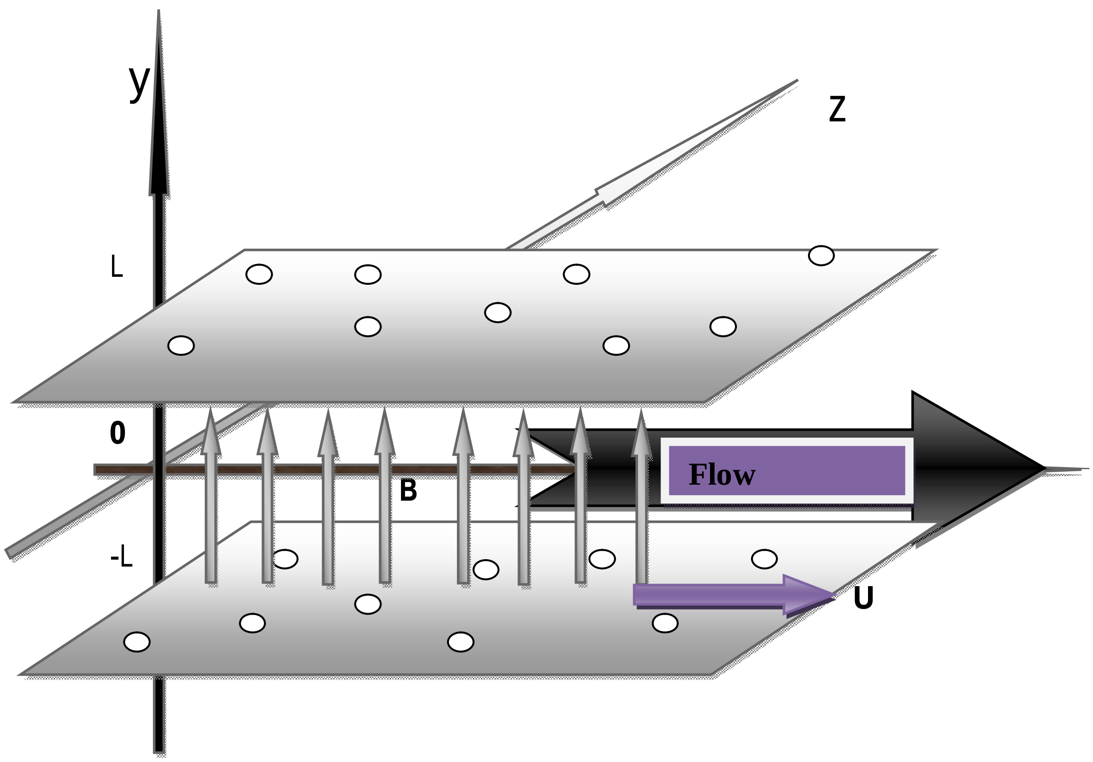

Figure 2.1. The flow configuration of the problem.

velocity zero. In this condition the fluid velocity increases in a direction normal to the boundary. Since the plates are semi infinite, the flow field is unbounded in one direction i.e. the z direction. A variable transverse magnetic field B is applied in the x-direction. The transverse magnetic field induces a voltage drop of magnitude U_xB_y in the z direction leading to a current flow in the same direction. The amount of current depends on the voltage induced since the electrical conductivity σ of the fluid is assumed constant. Electro-magnetic interaction yields Lorentz force which tends to oppose the fluid flow. The magnitude of the Lorentz force depends on the magnetic flux B_y and the magnetic flux gradient $\frac{\partial B_y}{\partial x}$ since the fluid is assumed to have a constant magnetic permeability(μ_e). In this paper, the effect of transverse inhomogeneous magnetic field on the flow behavior will be investigated. The flow configuration describing this system is presented in figure 2.1.

2.4 Flow conditions

The porous plates are electrically non-conducting and the velocity of the fluid particles in contact with the respective plates is equal to that of the plates. This is due to the no slip condition where such particles stick tightly on to the solid plates and are assumed not to slide no matter how little the fluid viscosity is. The initial conditions for incompressible unsteady flows are that everywhere in the solution region, velocity and temperature must be given i.e. the velocity and temperature must be known as a function of position. The velocity, temperature and magnetic field initial and boundary conditions for this flow problem can be stated in summary as follows;

- $t<0$, $u(x, y, 0) = 0$, $w(x, y, 0) = 0$, $T(x, y, 0) = T_\infty$
- $t\geq 0$, $u(x, -L, t) = U$, $w(x, -L, t) = 0$, $T(x, -L, t) = T_\infty$
- $t\geq 0$, $u(x, L, t) = 0$, $w(x, L, t) = 0$, $T(x, L, t) = T_w$
- $t\geq 0$, $u(X, y, t) = 0$, $w(X, y, t) = 0$, $T(X, y, t) = T_\infty$

The flow field considered is arbitrarily within the range $0\leq x\leq X$ where X is the extent along the x axis and $-L\leq y\leq L$ as the plates separation. The applied inhomogeneous magnetic field has the following boundary conditions:

- $t \geq 0$, $B_y = 0$ for $x<0$, $|B_y| = B_y \frac{\partial B_y}{\partial x}$ for $0 \leq x \leq X$

The flow set up is such that the rate at which the magnetic field intensity B_y changes in the defined length $0<x<X$ is constant, implying that the magnetic field gradient $\left(Fgrand = \frac{\partial B_y}{\partial x}\right)$ is constant. Vorticity or dragging effect is generated by the viscosity, and also by the interaction between the resultant electric field, ''Curl B_y which as established in the next section yields $\vec{J} = (0,0,j_z)$'' and the applied magnetic field $\vec{B} = (0, B_y, 0)$ to produce an electromagnetic force (J X B). The mathematical formulation and progressive analysis of this problem are presented in the proceeding subtopics.

2.5 Mathematical formulation

The deduction of the boundary layer equations was perhaps one of the most important advances in ordinary fluid dynamics and MHD. The principles of the boundary layer equations are applied in formulating the equations describing the research problem, ''a two dimensional analysis of unsteady flow of a conducting fluid in presence of a variable transverse magnetic field''. An extra term J X B is included in the momentum equation as mentioned earlier, and the joule heating has been disregarded. Since our interest is largely focused on the effect caused to the flow properties by this variable extra term, the treatment of the boundary layer equations is far more complex due to the time dependent variation of the flow variables. The instantaneous velocity and temperature of a fluid particle is considered to be influenced by the inhomogeneous magnetic field via the instantaneous mean magnetic field strength and its fluctuating part, i.e. $B_y \frac{\partial B_y}{\partial x}$. Therefore, the equation of continuity for a two-dimensional unsteady incompressible flow in Cartesian coordinates is given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.34}$$

The velocity gradient in the y direction far exceeds the tangential velocity gradient hence

$$\frac{\partial v}{\partial y} = 0 \tag{2.35}$$

Integrating this equation yields v=V, a constant representing the suction velocity. The momentum equation requires simplification of the magnetic force to determine the effective component. The variable transverse magnetic field induces a current given by

$$\vec{J} = \nabla \times \vec{H} \tag{2.36}$$

We take that $\vec{B}$ is in the direction of $\vec{H}$ and that $\vec{B} = \mu_e \vec{H}$ yielding

$$\vec{J} = \nabla \times \frac{\vec{B}}{\mu_e} = \frac{1}{\mu_e} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & B_y & 0 \end{vmatrix} \tag{2.37}$$

or

$$\vec{J} = \frac{1}{\mu_e}\left(-\vec{i}\frac{\partial B_y}{\partial z} + \vec{k}\frac{\partial B_y}{\partial x}\right) = -\vec{i}\frac{1}{\mu_e}\frac{\partial B_y}{\partial z} + \vec{k}\frac{1}{\mu_e}\frac{\partial B_y}{\partial x} \tag{2.38}$$

Hence, the induced current in component form can be expressed as $\vec{J} = (\vec{J}_x i + \vec{J}_z k)$

Since the flow is unbounded in the Z direction, $\frac{\partial B_y}{\partial z} = 0$. This implies that $\vec{J} = (0,0,j)$ where the component in the Z direction is given by

$$j_z = \frac{1}{\mu_e}\frac{\partial B_y}{\partial x} \quad (2.39)$$

The Lorentz force obtained by the cross product $\vec{J} \times \vec{B}$ yields

$$\vec{J} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & j_z \\ 0 & B_y & 0 \end{vmatrix} = -\vec{i} B_y j_z \quad (2.40)$$

Substituting J_z in equation (2.40) yields

$$\vec{J} \times \vec{B} = -\vec{i}\frac{B_y}{\mu_e}\frac{\partial B_y}{\partial x} \quad (2.41)$$

The Lorentz force which acts on the fluid particles is in the negative X direction. We take that pressure P does not change in the direction of the flow then $\frac{\partial p}{\partial x} = 0$ and $\frac{\partial p}{\partial y} = 0$. Since the flow is two dimensional, the momentum equation in Cartesian coordinates is given by

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + V\frac{\partial u}{\partial y} = \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{1}{\rho\mu_e}B_y\left(\frac{\partial B_y}{\partial x}\right) \quad (2.42)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + V\frac{\partial w}{\partial y} = \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \quad (2.43)$$

where u and w are the velocity components, ρ is the density and $\nu = \mu/\rho$ is the kinematic viscosity of the fluid at a point. In our work, we have neglected the joule heat dissipation in the energy equation. The viscous dissipation function φ for a three dimensional flow is given by (Jalil 2007).

$$\varphi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)^2 \qquad (2.44)$$

In this problem, the flow is two dimensional and semi-infinite. As the ratio ‘‘typical length scale (L) is to Hartmann distance (δ)’’ becomes very large as compared to dimensionless unity, the rate of change of flow variables in the direction normal to the interface far exceeds that in any tangential direction so that we may neglect tangential derivatives; (Shercriff, 1965). Hence, we neglect derivatives with respect to x and z. Variation in the X direction is small as compared to that in Y direction since the greater variation is in the direction perpendicular to the boundary plate and fluid interface. Consequently, the viscous dissipation function reduces to

$$\varphi = \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 \qquad (2.45)$$

The energy equation in Cartesian coordinates simplifies to

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + V_0\frac{\partial T}{\partial y} = \frac{K}{\rho\, C_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\mu}{\rho C_p}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] \qquad (2.46)$$

In conclusion, the continuity equation is implied by equations (2.34) and (2.35) and the electromagnetic effect is manifested through instantaneous variation of B_y and $\frac{\partial B_y}{\partial x}$. Referring to equation (2.41), the effect of isolated variation of these two terms corresponds to the effect of transverse variable magnetic field on the flow of the conducting fluid without loss of approximate generality. This treatment takes care of the induction equation as well as the Lorentz force term. Hence, we can summarize the governing equations for this research problem as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + V\frac{\partial u}{\partial y} = \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{1}{\rho\mu_e}B_y\left(\frac{\partial B_y}{\partial x}\right) \tag{2.47}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + V\frac{\partial w}{\partial y} = \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \tag{2.48}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + V\frac{\partial T}{\partial y} = \frac{K}{\rho C_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\mu}{\rho C_p}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] \tag{2.49}$$

2.6 Non-dimensional parameters

The non-dimensional parameters allow for the flow field to be bounded. The following non-dimensional numbers have been used in this work.

2.6.1 Hydrodynamic Reynolds number (Re)

This non-dimensional parameter is a ratio of inertial force to viscous force. It gives the relative significance of inertial force to viscous force in a fluid flow problem and is expressed as;

$$R_e = \frac{\rho UL}{\mu} = \frac{UL}{\nu} \qquad (2.50)$$

where U and L are the characteristic velocity and length scales; that is typical measures of how fast the fluid is moving and the size of the system. One can have various values of L and U and of the density (ρ) and investigate the sequence of changes that occurs to the flow pattern as R_e is changed. We note that if the Reynolds number is the same for two geometrically similar situations, then the equations for the non-dimensional variables are the same. Hence they have the same solutions and the same flow patterns occur. If the Reynolds number is different, the equations are different and there is no reason to expect the same flow behavior. Therefore, the condition for dynamic similarity is equality of Reynolds number. Whereas it might seem that one would need to investigate separately the effect of varying each of the quantities L, U, ρ, and μ, in fact one need investigate only the variations with Reynolds number. In reality, Reynolds number can never vanish, or equivalently inertial forces are never zero, but it has been shown that zero Reynolds Number approximation produces an accurate representation of the flow field in the vicinity of small particles.

2.6.2 Magnetic pressure number R_h

The magnetic pressure number is expressed as

$$R_h = \frac{B_0^2}{\rho \mu_e U^2} = \frac{\mu_e H_0^2}{\rho U^2} \qquad (2.51)$$

It is the ratio of magnetic pressure to hydrodynamic pressure and gives the relative significance of the two pressures in MHD fluid flow.

2.6.3 Prandtl number P_r

The aerodynamic boundary layer was first defined by Ludwig (1952) in a paper he presented on August 12, 1904 at the third International Congress of Mathematicians in Heidelberg, Germany In recognition of his contribution in fluid flow research, the ratio of the velocity boundary layer thickness and the thermal boundary layer thickness is governed by a non dimensional parameter called the Prandtl number. If the Prandtl number is 1, the two boundary layers are the same thickness. If the Prandtl number is greater than 1, the thermal boundary layer is thinner than the velocity boundary layer. If the Prandtl number is less than 1, which is the case for air at standard conditions, the thermal boundary layer is thicker than the velocity boundary layer. This non dimensional parameter is a property of the fluid, not of particular flow. Hence, there is a restriction on the transfer of information from experiments with one fluid to those with another. Put in another way it is an approximation to the ratio of momentum diffusivity and thermal diffusivity and is expressed as

$$P_r = \frac{\mu/\rho}{K/\rho C_p} = \frac{C_p \mu}{K} \tag{2.52}$$

Full similarity of forced convection of heat requires that there is equality of both the Reynolds number and Prandtl number.

2.6.4 The Eckert number E_C

This number is expressed as

$$E_c = \frac{U^2}{C_p \Delta T} \tag{2.53}$$

It is the ratio of kinetic energy of the flow to the thermal energy.

2.6.5 Joule parameter R

When a current flows through a conductor, an increase in temperature of the conductor occurs due to its electrical resistance. This phenomenon is called joule heating and is named after the scientist Prescott Joule having been the first scientist to establish Joules law which relates the amount of heat released from an electrical resistor to its resistance and the charge passed through it. This non-dimensional parameter is expressed as

$$\boldsymbol{R} = \frac{\sigma \mu B_0^2}{\rho^2 C_p \nabla T} = \frac{\sigma v B_0^2}{\rho C_p \nabla T} \tag{2.54}$$

2.6.6 Magnetic Reynolds number R_m

The relative strength of resistivity (the reciprocal of conductivity) is measured by a dimensionless number called the Magnetic Reynolds Number R_m which can be thought of as a typical ratio of the advective and the diffusive terms in the induction equation. This parameter is used to decide whether a plasma is diffusion or convection dominated and is expressed as;

$$R_m = \sigma\mu_e UL \tag{2.55}$$

The conductivity is 'infinite' when R_m is large. If R_m is small the magnetic field is not changed appreciably by the flow thus induced magnetic field can be taken to be zero. If it is large, magnetic effects may be expected to be prominent.

2.7 Non-dimensional form of the governing equations

A useful starting point is to emphasize that two similar flow patterns occur when the non-dimensional parameters are the same. Consequently, the governing equations are none dimensionalised with an objective of determining the important parameters necessary in analyzing the flow problem. The following non-dimensional quantities are hereby stated;

$$\left\{x^* = \frac{x}{L}, \quad y^* = \frac{y}{L}, \quad u^* = \frac{u}{U}, \quad w^* = \frac{w}{U}, \quad V_0 = \frac{V}{U}, \quad B_y = B_0 B_y^*, \quad t = \frac{L}{U} t^*, \right\} \tag{2.56}$$

The scale length L is half distance between the parallel plates, U is the constant velocity of the upper plate, B_0 is the characteristic magnetic field intensity, V is the constant suction velocity ρ is the fluid density and $x^*, y^*, u^*, w^*, B_y{}^*$ and t^* are non-dimensional quantities. The momentum equation can be non-dimensionalised as follows;

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + V\frac{\partial u}{\partial y} = \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{1}{\rho\mu_e} B_y \left(\frac{\partial B_y}{\partial x}\right) \tag{2.57}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + V\frac{\partial w}{\partial y} = \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \tag{2.58}$$

- $\frac{U}{L}\frac{\partial}{\partial t^*}\left(Uu^*\vec{i}+Uw^*\vec{j}\right)=\frac{U^2}{L}\left(\frac{\partial u^*}{\partial t^*}\vec{i}+\frac{\partial w^*}{\partial t^*}\vec{j}\right)$ (2.59)
- $Uu^*\left(\frac{\partial Uu^*}{\partial Lx^*}+\frac{\partial Uw^*}{\partial Lx^*}\right)+V\left(\frac{\partial Uu^*}{\partial Ly^*}+\frac{\partial Uw^*}{\partial Ly^*}\right)=\frac{U^2}{L}u^*\left(\frac{\partial u^*}{\partial x^*}+\frac{\partial w^*}{\partial x^*}\right)+\frac{U^2}{L}V\left(\frac{\partial u^*}{\partial y^*}+\frac{\partial w^*}{\partial y^*}\right)$

(2.60)

- $\frac{\mu U}{\rho L^2}\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right)+\frac{\mu U}{\rho L^2}\left(\frac{\partial^2 w^*}{\partial x^{*2}}+\frac{\partial^2 w^*}{\partial y^{*2}}\right)$ (2.61)
- $-\frac{1}{\rho\mu}B_y\frac{\partial B_y}{\partial x}\vec{i}=-\frac{B_0^2}{\rho\mu L}B_y^*\frac{\partial B_y^*}{\partial x^*}\vec{i}$ (2.62)

Substituting (2.59) to (2.62) into the respective equations **(**2.57) and (2.58) yields

$$\frac{\partial u^*}{\partial t^*}+u^*\frac{\partial u^*}{\partial x^*}+\frac{V}{U}\frac{\partial u^*}{\partial y^*}=\frac{\mu}{\rho UL}\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right)-\frac{B_0^2}{\rho\mu U^2}B_y^*\frac{\partial B_y^*}{\partial x^*} \tag{2.63}$$

$$\frac{\partial w^*}{\partial t^*}+u^*\frac{\partial w^*}{\partial x^*}+\frac{V}{U}\frac{\partial w^*}{\partial y^*}=\frac{\mu}{\rho UL}\left(\frac{\partial^2 w^*}{\partial x^{*2}}+\frac{\partial^2 w^*}{\partial y^{*2}}\right) \tag{2.64}$$

Equations (2.63) and (2.64) can be further simplified by substituting the Reynolds number (R_e), suction parameter $\frac{V}{U}=V_0$ and magnetic pressure number R_h yielding the final form of the equations governing the primary and secondary velocity profiles as:

$$\frac{\partial u^*}{\partial t^*}+u^*\frac{\partial u^*}{\partial x^*}+V_0\frac{\partial u^*}{\partial y^*}=\frac{1}{R_e}\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right)-R_hB_y^*\frac{\partial B_y^*}{\partial x^*} \tag{2.65}$$

$$\frac{\partial w^*}{\partial t^*}+u^*\frac{\partial w^*}{\partial x^*}+V_0\frac{\partial w^*}{\partial y^*}=\frac{1}{R_e}\left(\frac{\partial^2 w^*}{\partial x^{*2}}+\frac{\partial^2 w^*}{\partial y^{*2}}\right) \tag{2.66}$$

The energy equation is non-dimensionalised and expressed as follows.

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + V_0\frac{\partial T}{\partial y} = \frac{K}{\rho C_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\mu}{\rho C_p}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] \tag{2.67}$$

- $$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial \theta}\frac{\partial \theta}{\partial t} = (T_w - T_\infty)\frac{\partial \theta}{\partial t} = \frac{U^2}{\nu}(T_w - T_\infty)\frac{\partial \theta}{\partial t^*} \tag{2.68}$$
- $$u\frac{\partial T}{\partial \theta}\frac{\partial \theta}{\partial x} = \frac{U}{L}u^*(T_w - T_\infty)\frac{\partial \theta}{\partial x^*} = \frac{U^2}{\nu}u^*(T_w - T_\infty)\frac{\partial \theta}{\partial x^*} \tag{2.69}$$
- $$V\frac{\partial T}{\partial \theta}\frac{\partial \theta}{\partial y} = \frac{1}{L}V(T_w - T_\infty)\frac{\partial \theta}{\partial y^*} = \frac{U}{\nu}V(T_w - T_\infty)\frac{\partial \theta}{\partial y^*} \tag{2.70}$$
- $$\frac{K}{\rho C_p L^2}(T_w - T_\infty)\left[\frac{\partial^2 \theta}{\partial x^{*2}} + \frac{\partial^2 \theta}{\partial y^{*2}}\right] = \frac{KU^2}{\rho C_p \nu^2}(T_w - T_\infty)\left[\frac{\partial^2 \theta}{\partial x^{*2}} + \frac{\partial^2 \theta}{\partial y^{*2}}\right] \tag{2.71}$$
- $$\frac{\mu}{\rho C_p}\left[\left(\frac{\partial U u^*}{\partial L y^*}\right)^2 + \left(\frac{\partial U w^*}{\partial L y^*}\right)^2\right] = \frac{\mu U^2}{\rho C_p L^2}\left[\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \left(\frac{\partial w^*}{\partial y^*}\right)^2\right] \tag{2.72}$$

Substituting (2.68) to (2.72) and $\Delta T = T_w - T_\infty$ in equation (2.67) and dividing by $\frac{\Delta T U^2}{\nu}$ yields

$$\frac{\partial \theta^*}{\partial t^*} + u^*\frac{\partial \theta}{\partial x^*} + \frac{V}{U}\frac{\partial \theta^*}{\partial y^*} = \frac{K}{\rho C_p \nu}\left(\frac{\partial^2 \theta}{\partial x^{*2}} + \frac{\partial^2 \theta}{\partial y^{*2}}\right) + \frac{\mu\nu}{\rho C_p L^2 \Delta T}\left[\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \left(\frac{\partial w^*}{\partial y^*}\right)^2\right] \tag{2.73}$$

Substituting the suction parameter V_0, Prandtl number (P_r) and the Eckert number (E_c) in equation (2.73) lead to the final form of the energy equation as

$$\frac{\partial \theta^*}{\partial t^*} + u^*\frac{\partial \theta}{\partial x^*} + V_0\frac{\partial \theta^*}{\partial y^*} = \frac{1}{P_r}\left(\frac{\partial^2 \theta}{\partial x^{*2}} + \frac{\partial^2 \theta}{\partial y^{*2}}\right) + E_c\left[\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \left(\frac{\partial w^*}{\partial y^*}\right)^2\right] \tag{2.74}$$

The non-dimensional form of the equations governing this problem can be stated in summary as

- $\frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial x^*} + V_0 \frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \frac{1}{R_e}\left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}}\right) - R_h B_y^* \frac{\partial B_y^*}{\partial x^*}$ (2.75)
- $\frac{\partial w^*}{\partial t^*} + u^* \frac{\partial w^*}{\partial x^*} + V_0 \frac{\partial w^*}{\partial y^*} = -\frac{\partial p^*}{\partial z^*} + \frac{\mu}{\rho U L}\left(\frac{\partial^2 w^*}{\partial x^{*2}} + \frac{\partial^2 w^*}{\partial y^{*2}}\right)$ (2.76)
- $\frac{\partial \theta^*}{\partial t^*} + u^* \frac{\partial \theta}{\partial x^*} + V_0 \frac{\partial \theta^*}{\partial y^*} = \frac{1}{P_r}\left(\frac{\partial^2 \theta}{\partial x^{*2}} + \frac{\partial^2 \theta}{\partial y^{*2}}\right) + E_c\left[\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \left(\frac{\partial w^*}{\partial y^*}\right)^2\right]$ (2.77)

2.8 Non-dimensional boundary conditions

In this problem, the fluid particles in contact with the solid boundaries are assumed to stick tightly and not to slide over the plates, i.e. the fluid satisfies the no-slip condition. This implies that their velocity in time t≥0 is equal to that of the plates. We take the halfway plates separation distance (L) as the characteristic length. The boundary conditions are non-dimensionalised as follows;

$$y^* = \frac{y}{L} \tag{2.78}$$

Substituting the value of y at the upper and lower boundaries respectively in the above equation yields;

$$y^* = \frac{L}{L} = 1 \tag{2.79}$$

$$y^* = \frac{-L}{L} = -1 \tag{2.80}$$

On the fluid boundaries, velocity, magnetic field and temperature must be known. The initial conditions for incompressible unsteady flows are that everywhere in the solution region, velocity and temperature must be given i.e. the velocity and temperature must be known as a function of position. Similarly, taking U as the reference velocity, the non-dimensional boundary conditions on velocity can be determined.

$$u^* = \frac{u}{U} \tag{2.81}$$

The upper plate is stationary and lower plate is impulsively set in motion at constant velocity U. Hence the non-dimensionalised fluid velocities on the solid boundaries at $y^* = -1$ and $y^* = 1$ in time t≥0 are 1 and 0 respectively, i.e.

$$u^* = \frac{U}{U} = 1 \tag{2.82}$$

$$u^* = \frac{0}{U} = 0 \tag{2.83}$$

The temperature T of the fluid and the plates is assumed to be the same everywhere at t<0 and is equal to the free stream temperature T_∞. The temperature of the fluid in contact with the plates is assumed to be equal to that of the respective plate at all times. As a condition at $t^* \geq 0$, the fluid in contact with the plates attain respective boundary temperature T_w, the fixed fluid temperature at the plates. The temperature of the lower plate is assumed to be maintained at $T_w = T_\infty$, which implies that the fluid temperature at the lower plate is non-dimensionalised and expressed as

$$\theta = \frac{T_\infty - T_\infty}{T_w - T_\infty} = 0 \tag{2.84}$$

Similarly, the temperature at the upper boundary is non-dimensionalised and expressed as

$$\theta = \frac{T_w - T_\infty}{T_w - T_\infty} = \mathbf{1} \tag{2.85}$$

In the main flow direction, x*=X/L where X is an arbitrary extent of x beyond which the fluid exhibits free flow conditions and hence, 0≤x*≤ X/L. The numerical value of X/L is determined by the terminal iteration values from the computer program and will be discussed in the next chapter. The non-dimensional initial and boundary conditions on velocity and temperature and magnetic field for this problem can be summarized as shown in the table below.

$t^* < 0$	$u^*(x^*, y^*, 0) = 0$	$w^*(x^*, y^*, 0) = 0$	$\theta(x^*, y^*, 0) = 0$
$t^* \geq 0$	$u^*(x^*, 1, t^*) = 0$	$w^*(x^*, 1, t^*) = 0$	$\theta(x^*, 1, t^*) = 1$
	$u^*(x^*, -\mathbf{1}, t^*) = 1$	$w^*(x^*, -1, t^*) = 0$	$\theta(x^*, -1, t^*) = 0$
	$u^*(X/L, y^*, t^*) = 0$	$w^*(X/L, y^*, t^*) = 0$	$\theta^*(X/L, y^*, t^*) = 0$

Table 2.1. Initial and boundary conditions of the flow system

In our analysis, we have assumed that electromagnetic interaction is initially zero; the magnetic field intensity $\left(B_y^*\right)$ is varied instantaneously from 2.0T to 4.0T, and the magnetic flux gradient $\left(\frac{\partial B_y^*}{\partial x^*}\right)$ similarly from 0.03 to 0.05. The effect of isolated variation of these two terms corresponds to the effect of transverse variable magnetic field on the flow of the conducting fluid without loss of approximate generality. In the referred equation, the non-dimensionalised form of J X B has been simplified to $R_h B_y^* \frac{\partial B_y^*}{\partial x^*}$ where R_h is the magnetic pressure number. This approach renders the induction equation solved *a priori* as well as the Lorentz force term, thus at $t^* \geq 0$, $\left|B_y^*\right| = B_y^* \frac{\partial B_y^*}{\partial x^*}$ for $0 \leq x^* \leq X/L$ and $\left|B_y^*\right| = 0$ elsewhere. In the following chapter, we present the mathematical concepts and the method of solution.

CHAPTER THREE

3.0 MATHEMATICAL CONCEPTS AND METHODOLOGY

Three mathematical concepts are useful in determining the success or otherwise of a numerical solution algorithm. These are convergence, consistence and stability of the method. Convergence is the property of a numerical method to produce a solution which approaches the exact solution as the grid spacing, control volume or element size is reduced to an infinitely small value. Stability is associated with damping of errors as the numerical method proceeds. If a technique is not stable even round off errors in the initial data can cause wild oscillations or divergence. We will use the forward finite difference approximation method to solve our flow's governing equations, the choice of which is based on its convergence and stability property.

3.1 The finite difference approximation

In this study, the velocity and temperature at the end of each time step are indicated via notations u^{k+1}, w^{k+1} and θ^{k+1}. They are computed from respective equations (58), (59) and (60) and their values determined iteratively. Time step variation is indicated via the index k+1 while the coordinate (i, j, k) indicate spatial points in the mesh system with respect to varying x, y and t respectively. To obtain an approximation of the exact partial derivatives appearing in our flow governing equations, we employ Taylor's series expansion of the dependent variable about the grid point (i, j, k). When j is constant, the

Taylor's series expansions of a function f (i, j) via forward and backward grid steps (i+1) and (i-1) respectively when j is constant are given by

$$f(i+1,j) = f(i,j) + f'(i,j)\Delta x + \frac{1}{2}f''(i,j)(\Delta x)^2 + \frac{1}{6}f'''(i,j)(\Delta x)^3 + \cdots \qquad (3.1)$$

$$f(i-1,j) = f(i,j) - f'(i,j)\Delta x + \frac{1}{2}f''(i,j)(\Delta x)^2 - \frac{1}{6}f'''(i,j)(\Delta x)^3 + \cdots \qquad (3.2)$$

If we truncate the Taylor's formulas in equation (3.1) and (3.2) after the first derivative term and combine them by subtraction and represent the higher order terms with the abbreviation 'Hots', the approximation for the first order partial derivative at the grid point (i, j) is

$$f'(i,j) = \frac{f(i+1,j) - f(i-2,j)}{2\Delta x} + Hots \qquad (3.3)$$

Similarly, the Taylor's approximation for the first order derivative with respect to varying y at the grid point (i, j) is

$$f'(i,j) = \frac{f(i,j+1) - f(i,j-1)}{2\Delta y} + Hots \qquad (3.4)$$

If the Taylor's formulas in equation (3.1) and (3.2) are further truncated after the second derivative term and added, the approximation for the second order partial derivatives at the grid point (i,j) are

$$f''_{(i,j)} = \frac{f_{(i+1,j)} - 2f_{(i,j)} + f_{(i-1,j)}}{(\Delta x)^2} + Hots \qquad (3.5)$$

$$f''_{(i,j)} = \frac{f_{(i,j+1)} - 2f_{(i,j)} + f_{(i,j-1)}}{(\Delta y)^2} + Hots \tag{3.6}$$

The truncation errors are of second order i.e. $O(\Delta x)^2$ and $O(\Delta y)^2$ and hence, the central differencing formula is second order accurate. Let the mesh point variable at time step k be denoted by f(i,j,k). The forward difference for the first order derivatives with respect to time t is given by

$$f(i,j,k+1) = \frac{f(i,j,k+1) - f(i,j,k)}{\Delta t} + Hots \tag{3.7}$$

3.2 Convergence of the finite difference method

In the analysis of our flow problem, we have used $\Delta x \approx 0.05$, $\Delta y \approx 0.05$ and $\Delta t \approx 0.00625$. Specific application of these space and time steps in the current paper will be discussed in the next chapter. The choices of Δx, Δy and Δt guarantees the convergence of the finite difference method. According to Leboucher (1999) the solution converges when Δx, Δy and Δt satisfy the following condition;

$$\Delta t \leq \frac{1}{2\left[(\Delta x)^{-2} + (\Delta y)^{-2}\right]} \tag{3.8}$$

3.3 Definition of the mesh

Explicit relation between the partial derivatives and the functional values at the adjacent nodal points are obtained using a uniform mesh system. The rectangular regions are subdivided into smaller and equal square elements whose lengths and widths are Δx by

Δy and the time variation along the vertical axis is represented by Δt. The mesh system can thus be described as shown in figure 3.1 below.

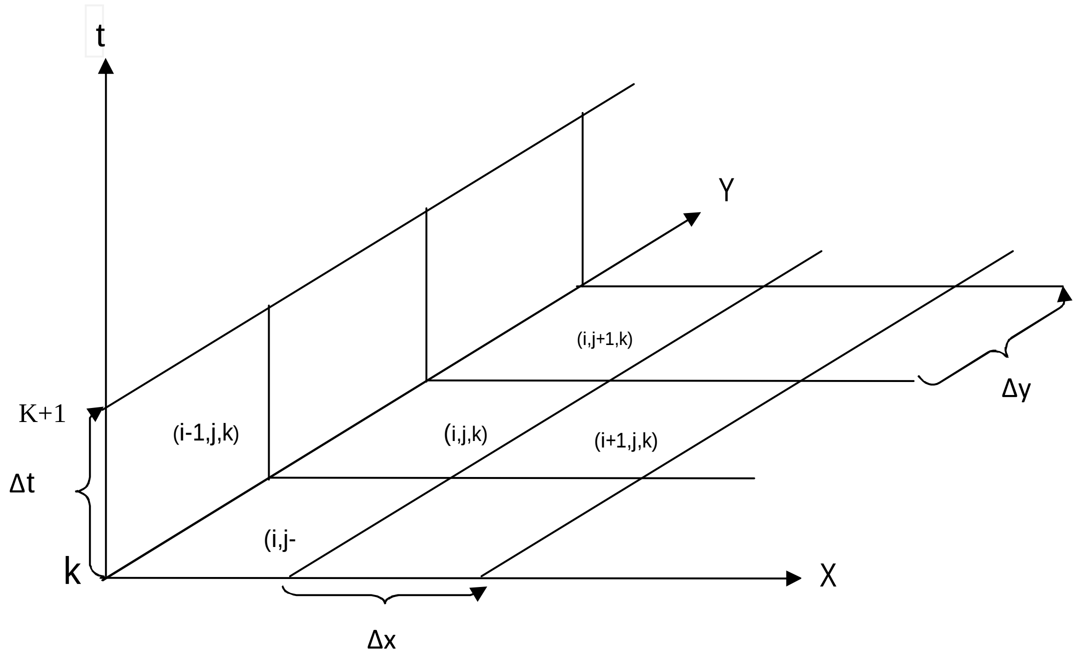

Figure 3.1. The Mesh system

3.4 The finite difference form of the governing equation

The momentum equation can be expressed in finite difference form via time step k+1 yielding the primary velocity profile as

$$\frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial x^*} + V_0^* \frac{\partial u^*}{\partial y^*} = -\frac{\partial P^*}{\partial x^*} + \frac{1}{R_e}\left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}}\right) - R_h B_y^* \frac{\partial B_y^*}{\partial x^*} \quad (3.9)$$

$$\frac{u^{*k+1}(i,j)-u^{*k}(i,j)}{\Delta t}+u^{*k}(i,j)\frac{u^{*k}(i+1,j)-u^{*k}(i,j)}{\Delta x}+V_0\frac{u^{*k}(i,j+1)-u^{*k}(i,j)}{\Delta y}$$
$$=\frac{1}{R_e}\left[\frac{u^{*k}(i+1,j)-2u^{*k}(i,j)+u^{*k}(i-1,j)}{(\Delta x)^2}\right.$$
$$\left.+\frac{u^{*k}(i,j+1)-2u^{*k}(i,j)+u^{*k}(i,j-1)}{(\Delta y)^2}\right]-R_hB^{*k}(i,j)Fgrad$$

(3.10)

The secondary velocity profile is similarly expressed in finite difference form as

$$\frac{\partial w^*}{\partial t^*}+u^*\frac{\partial w^*}{\partial x^*}+V_0^*\frac{\partial w^*}{\partial y^*}=-\frac{\partial p^*}{\partial y^*}+\frac{1}{R_e}\left(\frac{\partial^2 w^*}{\partial x^{*2}}+\frac{\partial^2 w^*}{\partial y^{*2}}\right) \tag{3.11}$$

$$\frac{w^{*k+1}(i,j)-w^{*k}(i,j)}{\Delta t}+u^{*k}(i,j)\frac{w^{*k}(i+1,j)-w^{*k}(i,j)}{\Delta x}$$
$$+V_0\frac{w^{*k}(i,j+1)-w^{*k}(i,j)}{\Delta y}$$
$$=\frac{1}{R_e}\left[\frac{w^{*k}(i+1,j)-2w^{*k}(i,j)+w^{*k}(i-1,j)}{(\Delta x)^2}\right.$$
$$\left.+\frac{w^{*k}(i,j+1)-2w^{*k}(i,j)+w^{*k}(i,j-1)}{(\Delta y)^2}\right]$$

(3.12)

The pressure gradient terms $\left(\frac{\partial p^*}{\partial x^*}\right)$ and $\left(\frac{\partial p^*}{\partial z^*}\right)$ are considered to be constant (zero) since the flow is fully developed and *Fgrand* represents the magnetic flux gradient $\frac{\partial B_y^*}{\partial x^*}$.

The energy equation in finite difference form is expressed as

$$\frac{\partial\theta^*}{\partial t^*}+u^*\frac{\partial\theta^*}{\partial x^*}+V_0^*\frac{\partial\theta^*}{\partial y^*}=\frac{1}{P_r}\left(\frac{\partial^2\theta^*}{\partial x^{*2}}+\frac{\partial^2\theta^*}{\partial y^{*2}}\right)+E_c\left[\left(\frac{\partial u^*}{\partial y^*}\right)^2+\left(\frac{\partial w^*}{\partial y^*}\right)^2\right] \tag{3.13}$$

$$\frac{\theta^{*k+1}(i,j)-\theta^{*k}(i,j)}{\Delta t}+u^{*k}(i,j)\frac{\theta^{*k}(i+1,j)-\theta^{*k}(i,j)}{\Delta x}+V_0\frac{\theta^{*k}(i,j+1)-\theta^{*k}(i,j)}{\Delta y}$$
$$=\frac{1}{P_r}$$
$$+\left[\frac{\theta^{*k}(i+1,j)-2\theta^{*k}(i,j)+\theta^{*k}(i-1,j)}{(\Delta x)^2}\right.$$
$$\left.+\frac{\theta^{*k}(i,j+1)-2\theta^{*k}(i,j)+\theta^{*k}(i,j-1)}{(\Delta y)^2}\right]$$
$$+E_c\left[\left(\frac{u^{*k}(i,j+1)-u^{*k}(i,j)}{\Delta y}\right)^2+\left(\frac{w^{*k}(i+1,j)-w^{*k}(i,j)}{\Delta y}\right)^2\right]$$

(3.14)

The method of solution to these equations involves a step by step substitution of the appropriate nodal values (mesh point values) within a cell grid resolution of 40X40 into the discretised equations governing the flow. Hence, starting with the initial boundary conditions where all the nodal values are known, the solution at each time step has effectively been obtained using a computer program written in Visual C++. The challenges associated with non-linearity of the governing equations and the coupling between the transport equations have been tackled by adopting an iterative strategy. The results of iterations of the flow variables obtained have been presented graphically and discussed.

3.5 Method of solution

The solution to this problem is obtained via an iterative strategy where the variable in question is expressed in terms of its local mesh point values at the previous time step.

We have computed values of velocity and temperature via consecutive expressions of $u^{*k+1}(i,j)$, $w^{*k+1}(i,j)$ and $\theta^{k+1}(i,j)$, i.e.

$$\begin{aligned} u^{*k+1}(i,j) = \Delta t\Bigg[& -u^{*k}(i,j)\frac{u^{*k}(i+1,j)-u^{*k}(i,j)}{\Delta x} - V_0\frac{u^{*k}(i,j+1)-u^{*k}(i,j)}{\Delta y} \\ & + \frac{1}{R_e}\Bigg(\frac{u^{*k}(i+1,j)-2u^{*k}(i,j)+u^{*k}(i-1,j)}{(\Delta x)^2} \\ & + \frac{u^{*k}(i,j+1)-2u^{*k}(i,j)+u^{*k}(i,j-1)}{(\Delta y)^2}\Bigg) - R_h B^{*k}(i,j)Fgrad\Bigg] \\ & + u^{*k}(i,j) \end{aligned} \tag{3.15}$$

$$\begin{aligned} w^{*k+1}(i,j) = \Delta t\Bigg[& -u^{*k}(i,j)\frac{w^{*k}(i+1,j)-w^{*k}(i,j)}{\Delta x} - V_0^*\frac{w^{*k}(i,j+1)-w^{*k}(i,j)}{\Delta y} \\ & + \frac{1}{R_e}\Bigg(\frac{w^{*k}(i+1,j)-2w^{*k}(i,j)+w^{*k}(i1,j)}{(\Delta x)^2} \\ & + \frac{w^{*k}(i,j+1)-2w^{*k}(i,j)+w^{*k}(i,j-1)}{(\Delta y)^2}\Bigg)\Bigg] + w^{*k}(i,j) \end{aligned} \tag{3.16}$$

$$\begin{aligned} \theta^{*k+1}(i,j) = \Delta t\Bigg\{ & -u^{*k}(i,j)\frac{\theta^{*k}(i+1,j)-\theta^{*k}(i,j)}{\Delta x} - V_0\frac{\theta^{*k}(i,j+1)-\theta^{*k}(i,j)}{\Delta y} \\ & + \frac{1}{P_r}\Bigg(\frac{\theta^{*k}(i+1,j)-2\theta^{*k}(i,j)+\theta^{*k}(i-1,j)}{(\Delta x)^2} \\ & + \frac{\theta^{*k}(i,j+1)-2\theta^{*k}(i,j)+\theta^{*k}(i,j-1)}{(\Delta y)^2}\Bigg) \\ & + E_c\Bigg[\left(\frac{u^{*k}(i,j+1)-u^{*k}(i,j)}{\Delta y}\right)^2 + \left(\frac{w^{*k}(i+1,j)-w^{*k}(i,j)}{\Delta y}\right)^2\Bigg]\Bigg\} \\ & + \theta^{*k}(i,j) \end{aligned} \tag{3.17}$$

The computations are performed using small values of Δt, ie constant time steps of $\Delta t= 0.00625$. The range of i and j follows from constant step sizes $\Delta x \approx 0.05$ and $\Delta y \approx 0.05$ within a square mesh framework. Using the expressions $x^*=i\Delta x$ and $y^*=j\Delta y$, the range of j in terms of discrete units is transformed to $-20 \leq j \leq 20$ which is equivalent to $-1 \leq y^* \leq 1$. This follows from the deduction that for $y^* = -1$, $j = -1/0.05 = -20$. Similarly when $y^* = 1, j = 20$. This range is arbitrary and its equivalent whole unit grid steps over the entire mesh system are $0 \leq j \leq 40$. The unit grid steps along the direction of flow are similarly calculated and for a square mesh framework, $0 \leq i \leq 40$. The fluid exhibits free stream profiles in the region defined by $i>40$. The fluid particles in contact with the lower plate at $j=0$ move at the same velocity with the plate due to the no slip condition while the velocity at the upper plate remains zero. The fluid exhibits free stream profiles in the region defined by $i>40$. This procedure ensures that the following conditions apply;

- The flow variables converge after the 300 iterations, i.e. for $k+1=300$. An immediate result for this value of k follows when further iterations do not make any significant change of the flow variable being iterated.
- The fluid exhibits free stream profiles in the region $i>40$ for all j and k, hence $i=41=\infty$.
- The locations of the parallel plates at $y=\pm L$ are arbitrary
- The iterations are carried out for different values of $j=0,1,2,..40$. To cater

for inhomogeneous magnetic field, iterations are carried out for $i=0,1,2,..40$.

In short, the discretised conditions are shown in table 3.1.

Flow Conditions	Primary velocity	Secondary velocity	Temperature
$t < 0$	$u^{*0}(i,j) = 0$	$w^{*0}(i,j) = 0$	$\theta^0(i,j) = 0$
$t \geq 0$	$u^{*k}(i,40) = 0$	$w^{*k}(i,40) = 0$	$\theta^k(i,40) = 1$
	$u^{*k}(i,0) = 1$	$w^{*k}(i,0) = 0$	$\theta^k(i,0) = 0$
	$u^{*k}(40,j) = 0$	$w^{*k}(40,j) = 0$	$\theta^k(40,j) = 0$

TABLE 3.1. Discretised boundary conditions

To be able to cater for the inhomogeneous magnetic field, we have considered the variation of B_y and $\frac{\partial B_y}{\partial x}$. These two terms are as a result of the simplified Lorentz force J×B; ‘‘the cross product of the induced current when the displacement current is neglected in line with Ampere’s law (2.4) and the applied transverse inhomogeneous magnetic field’’. The results have been obtained for B_y=2.0-4.0 T, Fgrand=0.03-0.05 T^2m^{-1}, R_h=1.0-3.0, θ=0-1.0, P_r=0.71-0.74, E_c=0.5-0.7, V_0=0-0.5 and R_e=2000-3000.

In the next chapter, we discuss the results of the velocity and temperature profiles of the flow.

CHAPTER FOUR

4.0 RESULTS AND DISCUSSIONS

4.1 Discussion of results

The computations and results for the primary velocity u and the secondary velocity w were made at constant values of Prandtl number and Eckert number. The temperature computations and results have been obtained at constant values of Reynolds number, magnetic flux gradient and magnetic pressure number. The graphs show the primary velocity profiles, secondary velocity profiles as well as the temperature profiles. The various profiles are distinguished using different dash type line style curves and letter codes.

4.1.1 Primary Velocity Profiles

Figure 4.1 provides the primary velocity profiles obtained with respect to y when the Prandtl number (P_r) and Eckert number (E_c) are fixed and the other parameters varied. From this figure, it is clear that;

(i) Removal of suction i.e. $V_0=0.0$ leads to an increase primary velocity profiles. Suction enhances vertical fluid particle transition which slows its horizontal velocity and vice versa.

(ii) An increase in Reynolds number leads to an increase primary velocity profiles. Since this parameter acts to dampen the viscous effects, inertial forces tend to dominate over the viscous forces and the fluid tends to continue with its state of motion with negligible resistance of frictional forces.

$P_r = 0.71$, $E_c = 0.05$

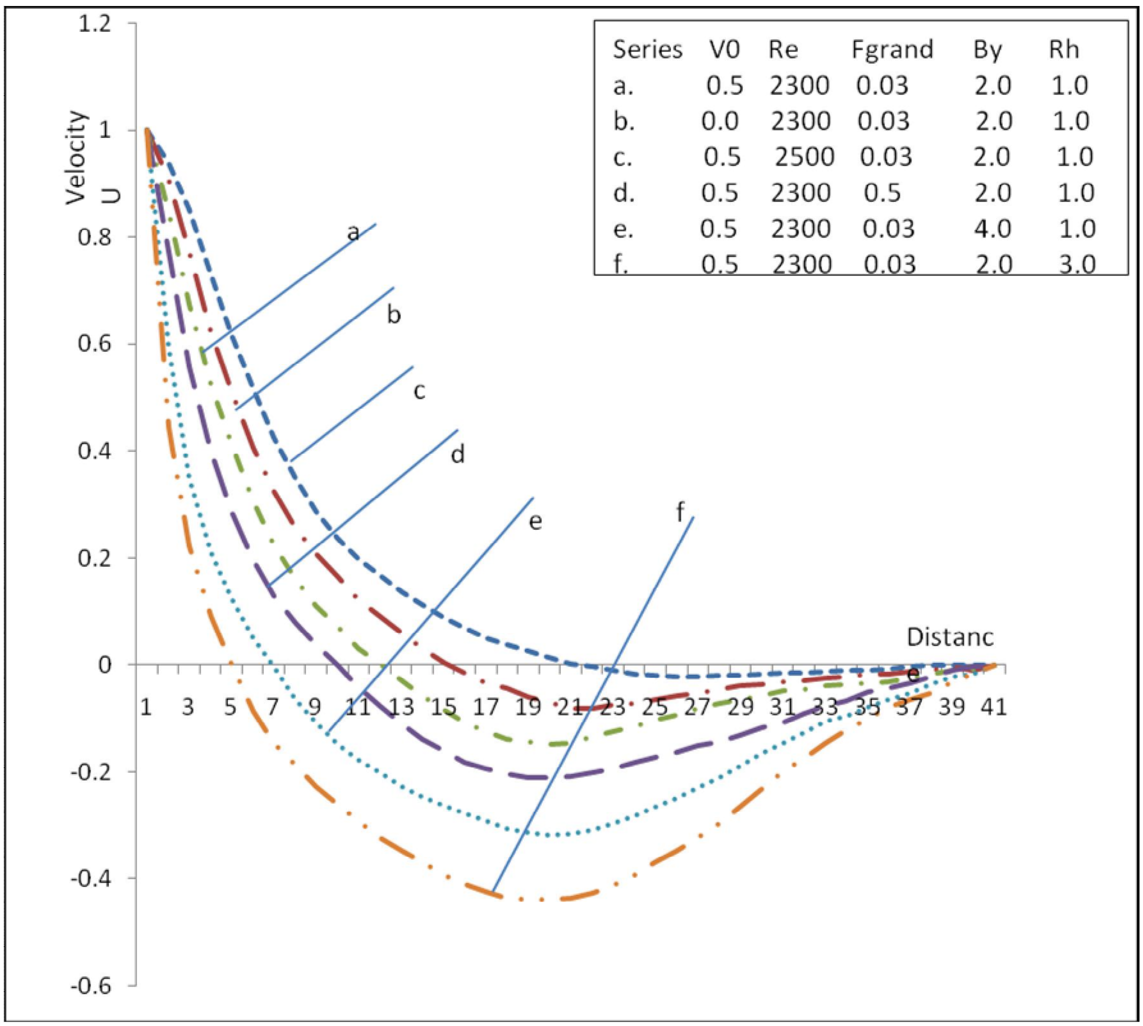

Figure 4.1. Primary Velocity Profiles

(iii) An increase in magnetic field gradient (*Fgrand*) leads to a decrease in primary velocity profiles. The interaction between the magnetic field gradient and the induced current in the fluid generates Lorentz force which opposes the flow thus slowing it down. The greater the flux gradient, the greater is this force implying greater opposition to the flow, yielding to reduced velocity profile.

(iv) An increase in magnetic field intensity (B_y) leads to a decrease in primary velocity profiles. The interaction between the magnetic field gradient and the induced current in the fluid generates Lorentz force which opposes the flow thus slowing it down. This is the effect of the inhomogeneous magnetic flux on the flow.

(v) An increase in magnetic pressure number (R_h) leads to a decrease in primary velocity profiles. This implies that an increase in magnetic pressure number yields an increase in magnetic pressure force. This force acts to oppose the flow and hence slowing it down.

4.1.2 Secondary velocity profiles

Figure 4.2 provides the secondary velocity profiles obtained with respect to y when the Prandtl number P_r and Eckert number E_c are fixed and the other parameters varied as shown in table 4.1. From this figure, it is clear that;

- Removal of suction leads to an increase in the secondary velocity profiles. The Downward suction enhanced by gravitational pull has a counter effect of slowing down the upward transition of fluid particles.
- An increase in Reynolds number leads to a decrease in secondary velocity profiles. Inertia forces dominate over viscous forces and hence the fluid particles

translate dominantly towards the main flow direction as compared to the vertical transition.

- An increase in magnetic field intensity leads to a decrease in secondary velocity profiles. An increase in magnetic field intensity leads to magnetic flux change which yields an increase in Lorentz force and in turn reduces the rate of vertical transition.
- An increase in magnetic field gradient leads to a decrease in secondary velocity profiles. There is an increase in magnetic field strength magnetic flux changes. This leads to an increase in Lorentz and consequently the rate of vertical transition.

Series	V_0	R_e	Fgrand	B_y	R_h
a	0.5	2300	0.03	2.0	1.0
b	0.0	2300	0.03	2.0	1.0
c	0.5	2500	0.03	2.0	1.0
d	0.5	2500	0.05	2.0	1.0
e	0.5	2300	0.03	4.0	1.0
f	0.5	2300	0.03	2.0	3.0

Table 4.1. Variation of the flow parameters

$P_r = 0.71, \quad E_c = 0.05$

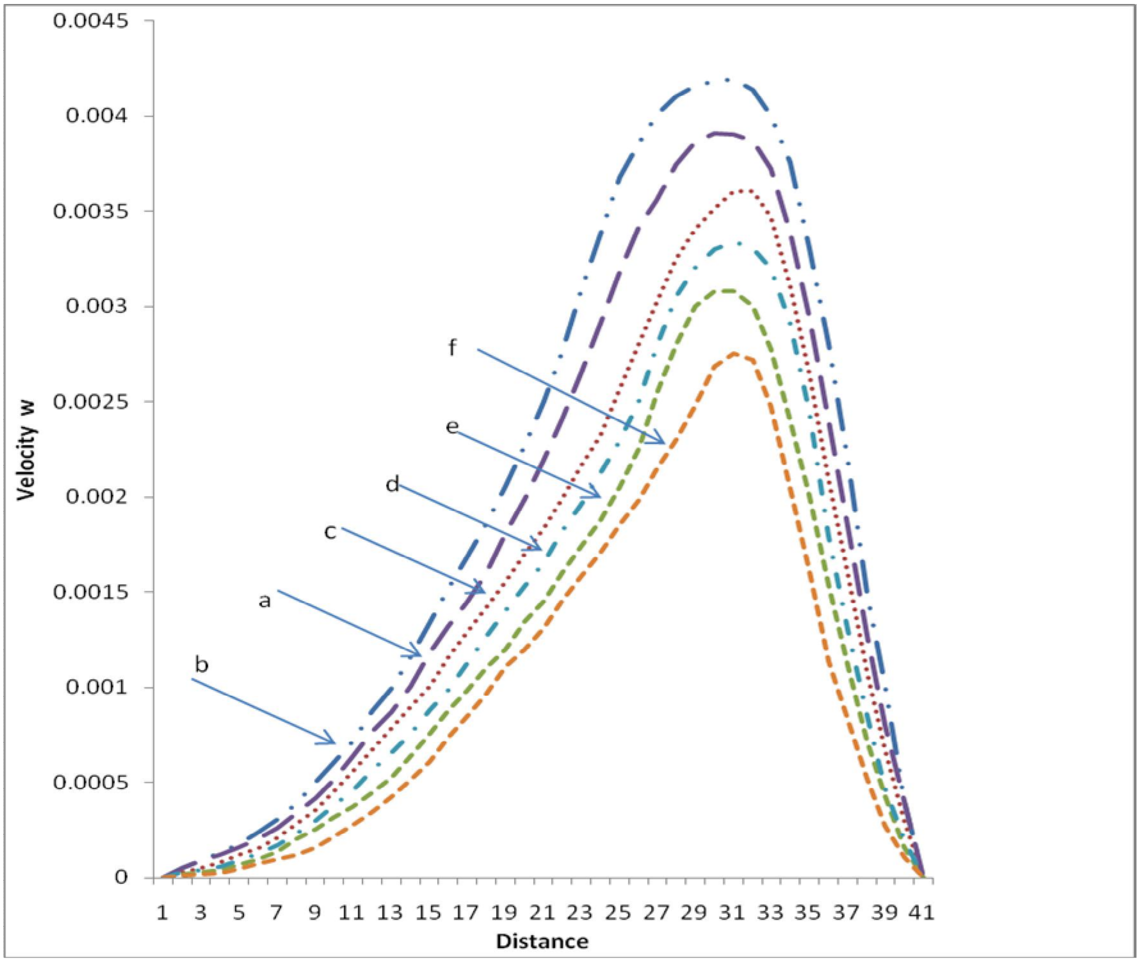

Figure 4.2. Secondary Velocity Profiles

- An increase in magnetic pressure number leads to a decrease in secondary velocity profiles. There is increased magnetic permeability which leads to increased magnetic pressure force. This force opposes the flow of fluid particles.

4.1.3 Temperature profiles

Figure 4.3 provides the secondary velocity profiles obtained with respect to y when the Reynolds number, magnetic flux gradient and magnetic pressure number are fixed and the other parameters varied

R_e=2300, Fgrand=0.03, R_h =3.0

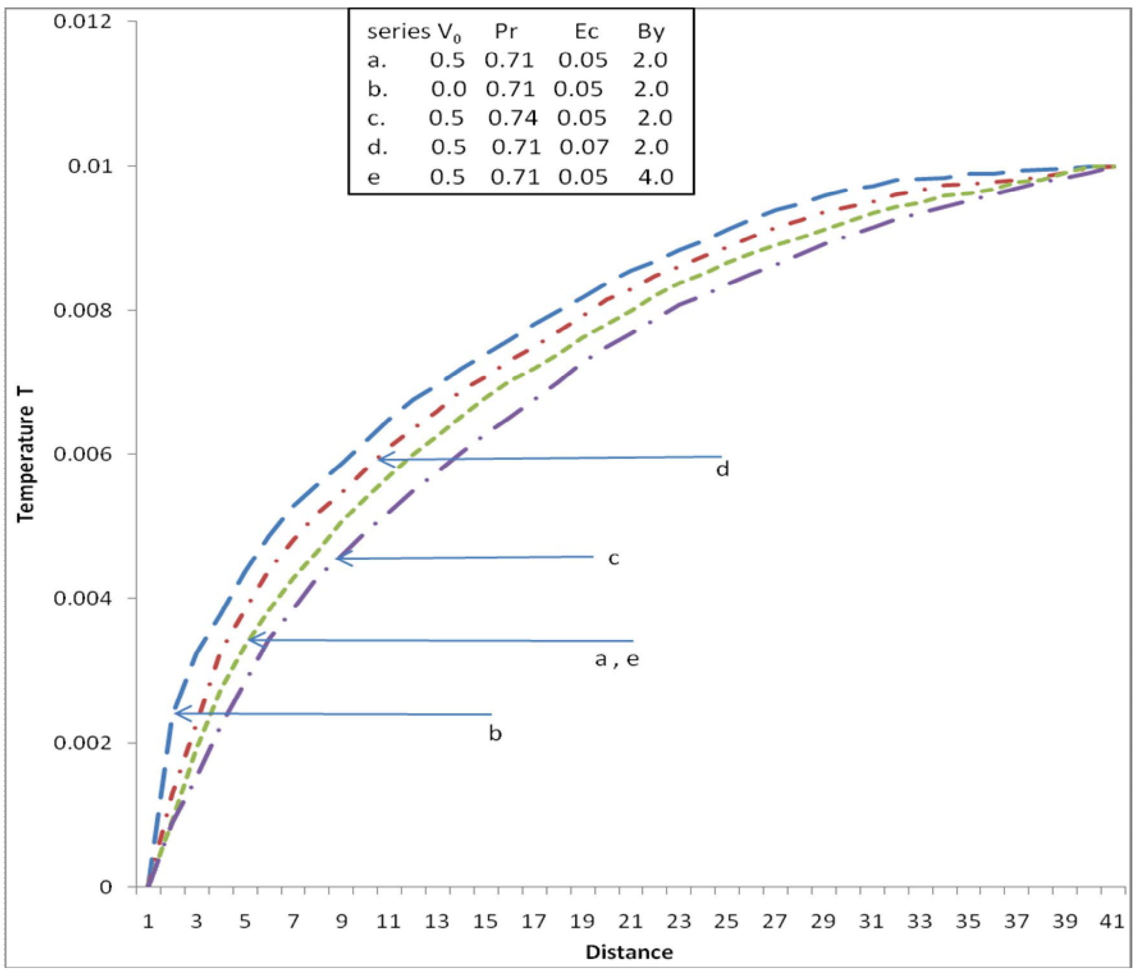

Figure 4.3. Temperature Profiles.

From figure 4.3, it is clear that;

i. Removal of suction leads to an increase in the temperature profiles. Sucking fluid from the boundary layer removes the heat from the flow field otherwise heat is contained.
ii. An increase in Prandtl Number leads to a decrease in temperature profiles. This is because the viscous forces dominate over thermal forces as Prandtl Number is raised.
iii. Variation of magnetic field intensity does not affect the temperature profiles (a and e). The temperatures are influenced by a change in magnetic flux gradient but remain unchanged when the flux gradient is constant
iv. An increase in Eckert Number (E_c) leads to an increase in temperature profiles. Hence the rate at which the fluid loses heat decreases as the Eckert Number is increased. This observation can be attributed to the viscous dissipation which increases with kinetic energy of the fluid particles

A conclusion of the research done and the recommendations for further research are discussed in the next chapter.

CHAPTER FIVE

5.0 CONCLUSION AND RECOMMENDATIONS

In this chapter, a conclusion based on the results obtained and the recommendations for further research are presented.

5.1 Conclusion

We have analyzed the effects of various parameters on MHD fluid flow past two parallel horizontal porous plates with the lower plate set impulsively in motion and the upper one fixed. The applied magnetic field is varied lengthwise in the direction of the flow hence resulting to a magnetic flux gradient. The applied inhomogeneous magnetic field yields an opposing force; the Lorentz force which is due to the interaction between the field and the induced current in the fluid. The equations governing the MHD flow are non-linear and hence we have employed a finite difference scheme in order to obtain the solutions. In this study, our flow problem involves fluids of Reynolds number in the range of 2300 to 3000, and thus is the case for unsteady problem. The velocity at the upper plate was fixed at zero and the lower plate was impulsively started at constant velocity in the direction main flow. The temperature at the lower plate is maintained constant and at the initial fluid temperature by cooling. We have observed that the velocity and temperature of the conducting fluid are significantly influenced by the variation of the respective flow parameters considered in this problem. In conclusion, we can deduce the following from our study;

a. Increase in magnetic field gradient causes a decrease in velocity profile in the direction of flow.
b. An increase in magnetic field intensity causes a decrease in both the primary and secondary velocity profiles.
c. The fluid is slowed down by an increase in magnetic pressure number.
d. An increase in Prandtl number causes a decrease in temperature profiles.
e. The fluid temperature gradient increases with increase in Eckert number.
f. The removal of suction causes an increase in both the velocity and temperature profiles.

5.2 Recommendations

In this thesis, our study of MHD flow between two parallel plates was not exhaustive but can provide a basis for further research while considering the following areas;

a) Fluid flow between two parallel plates moving in opposite directions at constant velocity.
b) Fluid flow between two parallel plates moving in opposite direction to the direction of fluid motion.
c) Fluid flow under the action of a variable magnetic field inclined at an angle to the flow direction.
d) Fluid flow where one or both of the plates are rotating.

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