

**AN EXTREME VALUE THEORY FRAMEWORK FOR
DETERMINING THE EFFECTIVENESS OF MONETARY
POLICY IN ENSURING INTEREST RATES AND
EXCHANGE RATES STABILITY – A CASE FOR KENYA**

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**An Extreme Value Theory Framework for Determining the Effectiveness
of Monetary Policy in Ensuring Interest Rates and Exchange Rates
Stability – A Case for Kenya**

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in Applied Statistics in the Jomo Kenyatta University of Agriculture and
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DECLARATION

This thesis is my original work and has not been presented for a degree in any other University.

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DEDICATION

This thesis is dedicated to my wife Risper and children Richard, Rachael and Ryan for their relentless encouragement and prayers.

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LIST OF ABBREVIATIONS

ADF	Augmented Dickey-Fuller
AR	Auto-Regressive
ARCH	Auto-Regressive Conditional Heteroscedasticity
ARMA	Auto-Regressive Moving Average
CBK	Central Bank of Kenya
GARCH	Generalised Auto-Regressive Conditional Heteroscedasticity
EGARCH	Exponential GARCH
GED	Generalised Error Distribution
GPD	Generalised Pareto Distribution
i.i.d.	Independent and Identically Distributed
Yen	Japanese Yen
Ksh	Kenya Shilling
MLE	Maximum Likelihood Estimation
OMO	Open Market Operations
POT	Peaks Over Threshold
PRSP	Poverty Reduction Strategy Paper
QQ-plot	Quantile-Quantile plot
QML	Quasi Maximum Likelihood
QMLE	Quasi Maximum Likelihood Estimation
Pound	Sterling Pound
USD	US Dollar
Treasury bill rate	91-day Treasury bill rate

ABSTRACT

Interest rates and exchange rates in Kenya have witnessed significant volatility since liberation in 1991 and 1993, respectively. This raises questions on whether the CBK has been meeting its primary objective of formulating and implementing monetary policy to achieve and maintain price stability. This thesis uses the POT approach to derive a model for extremes in interest rates in the period 1991 to 2006, and exchange rates in Kenya in the period 1993 to 2006. The estimated POT models were used to determine the return period of specific extreme exchange rate returns and changes in interest rates, and trends in excess sizes and inter-arrival times of exceedances of respective thresholds. This information is vital in informing the formulation of long term monetary policy.

The analyses revealed three key results for exchange rate returns on the USD, Pound, Yen, and Euro. Exchange rate returns for these currencies are leptokurtic and positively skewed while those exceeding the thresholds 0.5, 1.0, 1.0, and 1.8, respectively, can be modelled by the GPD. The return period for exchange rate returns of 3.12, 2.74, 2.86 and 2.41 on the USD, Pound, Yen and Euro, respectively, is 100 days. Similarly, the return period for exchange rate returns of 7.74, 5.39, 6.18 and 3.73 on the USD, Pound, Yen, and Euro, respectively, is 1000 days. The results also showed a decreasing trend in the excess sizes while the inter-arrival times of the exceedances of the respective thresholds have decreased for returns on the USD and Pound, but increased for the Yen and Euro.

Analyses of the interest rates data revealed that interest rate changes exceeding thresholds of 0.6, 0.2 and 0.4 percent for the interbank rate, repo rate and Treasury bill rate, respectively, can be modelled by the GPD. The return period for interest rate changes of 2.82, 2.56, and 3.19 percent on the interbank, repo, and Treasury bill rates, respectively, are 100 days. Similarly, the return period for interest rate changes of 7.21, 5.09, and 7.39 percent for the interbank, repo and Treasury bill rates, respectively, is 1000 days. In addition, excess sizes of the changes in the repo and Treasury bill rates have been decreasing while there was no significant trend in excess sizes for changes in the interbank rate. However, the inter-arrival times of the exceedances of the respective thresholds have generally decreased for the repo rate, but remained stable for changes in the interbank and Treasury bill rates.

Overall, the results indicate that the volatility in the interest rates and exchange rates as well as occurrences of extreme movements in the data have decreased significantly. Consequently, we conclude that the CBK achieved the objective of ensuring stability in interest rates and exchange rates during the period.

CHAPTER ONE

1.0 GENERAL INTRODUCTION

1.1 Introduction

Price stability is important in mitigating the long term risk of any investment - Choy (2002). It assists in promoting economic growth since investment is the main channel of increasing real output and employment. Both the public and private sectors are affected by financial price risk. According to Smith *et al.* (1990), volatility in prices has implications on the profits and survival of a company.

Interest rates and exchange rates in Kenya have witnessed significant volatility since liberalisation in July 1991 and October 1993, respectively. Unexpected changes in interest rates and exchange rates bring in risks which cannot be ignored in the financial markets. Price fluctuations also raise the question on the effectiveness of the central bank, which is charged with the responsibility of formulation and implementation of the country's monetary policy, to perform its mandate of ensuring price stability.

Extreme fluctuations in interest rates and exchange rates are equivalent to outliers in the respective data series. In standard statistical theory, outliers are influential observations that should be excluded from the analysis when fitting standard statistical distributions to data. The presence of outliers in data can change the distributional properties of the data. However, in insurance or finance, outliers are safely defined as extreme events. Extreme events occur with very low probability, but can lead to heavy losses financially, materially

or even life. Examples of extreme events include major insurance claims, flood levels of rivers, large decreases or increases of stock market prices over a certain period of time, extreme levels of environmental indicators such as ozone or carbon monoxide, wind speed values at a certain site, wave heights during a storm or maximal and minimal performance values of an investment. All these, and many more examples, have a common property in that they concern questions about extreme values of some underlying set of data.

According to the CBK Act (1996), the objectives of the CBK are: (a) to formulate and implement monetary policy directed at achieving and maintaining price stability, (b) to promote solvency and proper functioning of a stable market based financial system, (c) formulate and implement Kenya's foreign exchange policy, and to manage the country's foreign exchange reserves, (d) to perform as a banker, advisor and fiscal agent to the government, (e) promote and facilitate an efficient and effective national payments system, and (f) to issue currency notes and coins. The CBK would be more effective in fulfilling its core objective of formulation and implementation monetary policy if its forecasting models are robust enough to capture extreme movements in the data. Monetary policy is the central bank process of managing money supply to achieve specific goals, such as constraining inflation, and achieving full employment or economic growth, Beardshaw *et al.* (2001). This involves changing certain interest rates, either directly or indirectly through OMO, and setting reserve requirements. An environment of stable prices including interest rates and exchange rates promotes economic growth as it creates certainty for investors.

Overall, the thesis analyses the effect of monetary policy on the stability of interest rates and exchange rates. The research analyses extremes in short term interest rates comprising of the Repo rate, the interbank rate, and Treasury bill rate. Movements in these short terms interest rates are usually transmitted to changes in the average commercial banks lending and deposit interest rates. The Treasury bill rate is mainly determined by government domestic borrowing activities and the level of liquidity in the money market, while the Repo interest rate is mainly determined by the CBK's OMO.

The research also analyses extremes in the Ksh to the USD exchange rate, and the Ksh against the Pound, the Euro, and Yen which are the main bilateral exchange rates.

1.2 Statement of the Problem

The thesis provides an extreme value theory framework for establishing whether the CBK met its primary objective of ensuring price stability, including interest rates and exchange rates stability between 1991 and 2006. The thesis is motivated by the fact that the current models that the CBK has been using to generate forecasts for exchange rates and interest rates are not robust to extremes movements in the data. Consequently, the models have tended to understate the severity of movements in the two variables over time. Furthermore, current methods of determining stability of the two variables which are mainly based on computation of standard deviations or deviations from targets have not been effective in the long run because of frequent distortions.

1.3 Purpose of the Research

The main objective of this thesis is therefore to produce a framework for analysing extremes in interest rates and exchange rates data in Kenya which can be used in evaluating the effectiveness of the core mandate of the CBK, which is ensuring price stability.

The specific objective of the study is therefore to analyse the extreme values in the interest rates and exchange rates data in the period January 1991 to December 2006 in order to establish: (a) their distributions, (b) whether there is an increasing or decreasing trend in the occurrence of extremes, (c) the expected length of time before the occurrence of a specific high interest rate or exchange rate (return period), (d) the probability that the maximum interest rate and exchange rate for the next year exceeds all previous levels, and (e) the probability that the maximum interest rate or exchange rate for the next year will exceed the target or a specific level. The thesis also highlights the shortfalls of the most common statistical tools for threshold determination when fitting the POT model to data, and recommends suitable alternatives.

1.4 Importance of the Research

The research will provide a robust method of evaluating the effectiveness of the CBK's monetary policy, which is directed at achieving stability in prices including interest rates and exchange rates. The estimated POT model could also be used by the CBK to generate

projections of future extreme exchange rates and interest rates and their inter-arrival times, for the purposes of formulating a long-term monetary policy for the country.

Knowing the inter-arrival times of specific extreme exchange rates and interest rates will be useful to the CBK in judging when and how to intervene in the foreign exchange market or conduct OMO in order to influence interest rates. Keeping track of the trends in extreme interest and exchange rates helps detect financial crises. An increasing trend in extremes would prompt the CBK to take measures which would ensure stability of interest rates and exchange rates, without compromising on its monetary policy objectives.

In determining the threshold above which interest rates or exchange rates will be declared large, an indicative target or baseline will emerge for each of the two. This will provide an alternative approach to setting targets for the two macroeconomic variables and will also provide an answer to several questions raised in past on what should be the appropriate or equilibrium exchange rate. The return period for particular large observations will provide information on when to expect specific extreme interest and exchange rates. Projections of extreme exchange rates or interest rates are also important for the equity markets, as they will provide useful information on future trends in the money market.

1.5 Justification of the Research

The priorities and measures necessary for poverty reduction and economic growth in Kenya are outlined in the PRSP (2001). The long term economic growth target of 10 percent envisaged by the Kenya *Vision 2030* development plan, which replaced the PRSP

in 2007, is pegged on achieving and sustaining macroeconomic stability, including price stability. Both *Vision 2030* and PRSP identified distortions in interest rates and exchange rates as some of the main impediments to achieving and sustaining high economic growth rates in Kenya as this creates an environment of uncertainty for investors. The PRSP also identified unemployment as the main cause of poverty in Kenya and that unemployment can be addressed by implementing economic measures which promote growth. In 2006, about half of the Kenyan population were living below the poverty line.

For a country to attract investment opportunities there must be an enabling environment which includes private sector access to credit from commercial banks. Private sector lending from commercial banks is driven mainly by economic activity and low and stable interest rates. If the Treasury bill rate is high, then commercial banks will prefer lending to the Government on short term basis, which they deem risk-less, than to lend to a risky private sector. In such a situation, commercial banks would prefer to lend to the private sector at higher interest rates.

On the other hand, the exchange rate, which is the rate of exchange between international currencies, affects the prices of exports and imports and hence the current account of the balance of payments. The balance of payments is an account of all transactions of all entities living and working in Kenya with the rest of the world. Exporters will continue doing their business as long as the export earnings can sustain their business, while importers will sustain their businesses only if they can afford to buy foreign currency required to purchase goods such as manufacturing inputs. A strong exchange rate also

ensures less external debt repayments for the government. This presents the case for a achieving and maintaining a stable exchange rate.

The rest of this thesis is structured as follows. Chapter 2 presents the literature review. The methodologies used in the analyses to characterise volatility and extreme values in the data are then highlighted in chapter 3. The data and description of the inherent trends is then provided in chapter 4. The empirical results of the thesis are presented and discussed in chapter 5. Lastly, chapter 6 concludes and also provides the policy recommendations and areas of further work from the thesis.

CHAPTER TWO

2.0 LITERATURE REVIEW

2.1 Implications of Exchange Rate Volatility

Exchange rates in Kenya have witnessed significant volatility since liberalisation in October 1993 – Ndung’u *et al.* (2001). Volatility in exchange rates cannot be ignored in the financial markets as both importers and exporters of goods and services are affected by exchange rate risk. Exporters will remain in business as long as their earnings can sustain their trade, while importers will maintain their businesses only if they can afford to buy foreign currency required to purchase goods or services.

Reliable estimations and forecasts of volatility are important for large financial institutions where volatility is used as a direct measure of risk. The ability of macroeconomic models to forecast volatility has been examined in literature. Although macroeconomic factors have some forecasting capabilities, the most important factor in these studies was the lagged endogenous variable – Franke *et al.* (2008). Consequently, recent studies focus mainly on time series models. GARCH models are preferred to ARCH models in describing volatility as they are parsimonious in that fewer lags are required to model the time varying variance – Wang (2008).

Holmes (2003) found that exchange rate depreciation causes inflation although the impact could not prevail over the gains from increased external competitiveness. Depreciation reduces the real value of assets denominated in the local currency and increases the real

value of foreign currency denominated assets. Domestic inflation increases if the first effect dominates the second effect. This underscores the need for reliable models for policy analysis and forecasting of the volatility of the exchange rates to guide the central bank on when to intervene in the foreign exchange rate market.

The factors which affect movements in the exchange rate in Kenya were also studied by Ndung'u *et al.* (2001). The thesis found that an increase in the differential between domestic and foreign interest rates results in an appreciation of the exchange rate by attracting private capital flows. Similarly, improvements in the current account balance and net external inflows leads to an appreciation of the exchange rate. However, although the exchange rate is expected to depreciate with a widening price differential, the thesis found the reverse. Sentiments or key announcements, particularly by development partners, were also found to affect the exchange rate. In other studies, Calderon *et al.* (2001) found that current account balances are significantly correlated to movements in the exchange rate in African countries. Movements in the exchange rate of the Ksh against major currencies also reflect respond to developments in the cross rates of the major currencies. For example, financial crises in Europe tend to strengthen the USD relative to the Euro as the currency would be viewed as a safety haven by investors in the European financial markets.

Exchange rate volatility in any country raises the question on the effectiveness of the central bank to perform its core mandate of ensuring price stability, and management of the country's foreign exchange reserves.

2.2. Implications of Interest Rates Volatility

Volatility of interest rates can affect the survival of financial institutions such as banks and insurance companies since the assets and liabilities of these institutions are strongly correlated with movements in interest rates. When interest rates rise, households are inclined more to saving than consumption since interest payments on existing loans rise. Investment is also reduced as higher interest rates make it expensive for firms to finance investment. A fall in consumption and investment has negative implications on economic growth. A rise in interest rates can also crowd out exports through strengthening of the exchange rate due to short term capital inflows. The role of the CBK in ensuring interest rates stability is therefore important in promoting consumption, investment and economic growth in the country.

The theoretical underpinning of most empirical studies on the behaviour of short term interest rates is the Fisher hypothesis which attributes movements in short-term nominal interest rates primarily to expected inflation. According to the hypothesis, a one percent increase in inflation leads to a one percent increase in the rate of interest. The liquidity of the investment asset has also been found to play a major role in the pricing of securities. Strongin and Tartan (1990), for example, observed that liquidity dominates inflation considerations in explaining the behaviour of short-term interest rates. They observed that there is a positive relationship between maturity and interest rates or returns. This observation was, however, contradicted by Missale and Blanchard (1994) who considered the relationship between return and debt maturity more an outcome of government policy

geared to decreasing the interest burden on government budget. This conclusion followed from the result obtained from their empirical work, which showed a negative relationship between maturity of debt and interest rates as the debt burden (debt to GDP ratio) increased.

Movements in interest rates in Kenya have generally been attributed to high budget deficits funded mainly through domestic borrowing using Treasury bills, level of required reserves held by commercial banks at the CBK which affects liquidity in the interbank market, build-up of Government deposits at the CBK which drains liquidity from commercial banks, tax payments by companies which also drains liquidity from banks, and the level of OMO (repo sales and purchases) which have the effect of injecting or mopping up liquidity from the interbank market - CBK (2000).

Interest rate models are classified into general equilibrium models in which interest rate movements are derived from economic agents who maximize expected utility, and no-arbitrage models which assume that financial markets have no arbitrage opportunities - Franke *et al.* (2008).

2.3. Modelling Volatility in Time Series Data

The ARMA is a standard model of time series analysis – Chatfield (1980). The model explains the observations Y_t in terms of its lagged values (autoregressive part) and moving averages of the innovations ε_t (moving average part) as follows

$$Y_t = \varepsilon_t + \sum_{j=1}^p \gamma_j Y_{t-j} + \sum_{i=1}^q \eta_i \varepsilon_{t-i}, t = 1, 2, \dots$$

Where $\{\varepsilon_t\}$ is a sequence of i.i.d. random variables such that $E(\varepsilon_t) = 0$ and $cor(\varepsilon_t, \varepsilon_s) = 0$ for $t \neq s$, while γ and η are constant parameters .

Despite their wide use in the past, ARMA or traditional time series models cannot explain adequately the properties of financial time series data - Engle (1982). These include volatility clustering and heteroscedasticity which calls for more robust models such as GARCH and ARCH.

The GARCH model introduced by Bollerslav (1986) is a generalisation of the ARCH model of Engle (1982). The GARCH is widely used to model the volatility in the exchange rate returns and is formulated as follows. Let (Z_n) be the sequence of i.i.d. random variables such that $Z_t \sim N(0,1)$. Then, ε_t is the *GARCH*(p, q) process if

$$\varepsilon_t = \sigma_t Z_t, \quad t \in \mathbf{Z} \quad \dots\dots\dots 2.1$$

with

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^q \beta_i \sigma_{t-i}^2, \quad t \in \mathbf{Z}$$

Where σ_t^2 is a non-negative process, and $\alpha_0 > 0$, $\alpha_i \geq 0$ for $i = 1, \dots, p$ while $\beta_i \geq 0$ for $i = 1, \dots, q$. The non-negativity restrictions on the parameters ensure positivity of the variance σ_t^2 .

GARCH models have been criticised in that they do not provide a theoretical explanation of volatility or what information flows are in the volatility generating process. However, Clark (1973) and Lastrapes (1990) suggest a mixture of distribution hypothesis where the variance of returns in a given time interval is proportional to the rate of information arrival. Consequently, volatility clustering could be attributed to the serial correlation of information arrival frequencies. Traders receive new price signals at the same time and they shift to a new equilibrium immediately. This could be due to increased volume of trading in the case of the foreign exchange market.

Consequently, the GARCH model remains the benchmark model for analysing heteroscedastic time series – Bollerslev *et al.* (1992). The next section provides the properties of the GARCH model and conditions under which the estimated model is stationary.

2.3.1 Properties of the GARCH Model

The GARCH model in equation 2.1 captures the stylized features if ε_t are uncorrelated. Bollerslev (1986) showed that the GARCH (1, 1) model has kurtosis greater than 3 as long as $E(\varepsilon_t^4) < \infty$ and Z_t is Gaussian. The general GARCH model is also shown to be heavy tailed in Fan (2003). A necessary and sufficient condition for the weak stationarity of the

GARCH model in 2.1 is $\sum_{i=1}^p \alpha_i + \sum_{j=1}^q \beta_j < 1$ – Bollerslav (1986).

However, strict stationarity of the GARCH (1, 1) model requires that $E(\log(\alpha_1 Z_t^2 + \beta_1)) < 0$, which allows for $\alpha_1 + \beta_1$ being equal to or slightly above 1 – Nelson (1990). In the case of a $GARCH(p, q)$ model, if $r = \max(p, q)$, then α_i and β_j are equal to 0 for $i > p$ and $j > q$. Let $\tau_t = (\alpha_1 Z_{t-1}^2 + \beta_1, \dots, \alpha_{r-1} Z_{t-r+1}^2 + \beta_{r-1}) \in \mathbf{R}^{r-1}$ and the square matrix A_t of size r in block form be

$$A_t = \begin{pmatrix} \tau_t & \alpha_r Z_{t-r}^2 + \beta_r \\ I_{r-1} & 0 \end{pmatrix}$$

where I_{r-1} is the identity matrix of size $r-1$. Let $B_t = (\alpha_0, 0, \dots, 0)^T \in \mathbf{R}^r$ and $X_t = (\sigma_t^2, \sigma_{t-1}^2, \dots, \sigma_{t-r+1}^2)^T \in \mathbf{R}^r$. Therefore, ε_t is a solution of 2.1 if and only if X_t is a solution of

$$X_t = A_t X_{t-1} + B_t \tag{2.2}$$

A necessary and sufficient condition for strict stationarity of autoregressive processes such as 2.2 was established by Bougeral and Picard (1992). This requires that the top Lyapunov exponent associated with (A_t) is negative, i.e. $\gamma(A_t) < 0$, where

$$\gamma(A_t) = \inf_{t \in \mathbf{Z}} \left\{ \frac{1}{t+1} E(\log \|A_0 \dots A_{-t}\|) \right\} < 0 \tag{2.3}$$

Where $\| \cdot \|$ is the matrix norm defined as $\|A\| = \sum |a_{ij}|$. Bougeral and Picard (1992) also showed that $\sum_{i=1}^p \beta_i < 1$ is a necessary condition for strict stationarity of the GARCH model.

A weakly stationary process has time-invariant mean and covariance, while strict stationarity requires the joint distribution to be same with any shift in time. Stationarity of the GARCH model ensures that the behaviour and properties of the estimators do not change over time. In addition, the persistence of shocks is not infinite in a stationary time series. The next section discusses variations of the GARCH model which have been proposed in literature to address the weaknesses in the standard linear GARCH model.

2.3.2 Variations of the GARCH model

The GARCH model has several weaknesses some of which are outlined in Tsay (2005). In particular, the model responds equally to positive and negative shocks. Furthermore, the tail behaviour of GARCH models remains short even with standardised student-t observations. Similarly, the linear GARCH model cannot cope with significantly skewed time series which results in biased estimates. Consequently, other variations of the GARCH model have been proposed to address some of these weaknesses.

The EGARCH allows for asymmetric effects between positive and negative asset returns – Nelson (1991). The EGARCH (1, 1) model is specified as

$$\varepsilon_t = \sigma_t Z_t,$$

$$\ln(\sigma_t^2) = \alpha_0 + \alpha_1 \frac{|\varepsilon_{t-1}| + \delta_1 \varepsilon_{t-1}}{\sigma_{t-1}} + \beta_1 \ln(\sigma_{t-1}^2) \quad \dots\dots\dots 2.4$$

A positive ε_{t-1} contributes $\alpha_1(1 + \delta_1)|Z_{t-1}|$ to the log volatility while a negative ε_{t-1} contributes $\alpha_1(1 - \delta_1)|Z_{t-1}|$ with $Z_{t-1} = \frac{\varepsilon_{t-1}}{\sigma_{t-1}}$. There are no positivity restrictions on the model as in the linear GARCH model. If the estimated leverage parameter $\delta_1 = 0$, then positive and negative return shocks of the same size have the same effect on volatility. However, if $\delta_1 < 0$, positive return shocks have a negative effect on volatility. For $\delta_1 > 0$, a positive return shock increases volatility. Empirical evidence indicates that δ_1 is expected to be negative, implying that positive return shocks generate less volatility compared with negative ones.

Other variations of the GARCH model include the Integrated GARCH Model (IGARCH) model which is appropriate when there are occasional level shifts in volatility. The GARCH in the Mean Model (GARCH-M) is appropriate when the return depends on its volatility. The Threshold GARCH Model (TGARCH) model also addresses leverage effects of positive and negative shocks, Zakořian (1994). The strong persistence which occurs in GARCH models can also be explained by the Regime Switching GARCH model (RS-GARCH), Tsay (2005).

To circumvent the shortfalls of the linear GARCH model, this thesis also considers the EGARCH model. The next section presents the asymptotic properties of the GARCH estimator.

2.3.3 Asymptotic Properties of the GARCH Estimator

The asymptotic properties of an estimator relate to how the estimator behaves with sample sizes. An important issue for consideration in the estimation of the parameters of models is the robustness of the estimators. The estimators must be unbiased, consistent and asymptotically normal. The asymptotic properties of the QMLE for the GARCH (1, 1) model are highlighted in Posedel (2005). The results for the general GARCH (p , q) and ARMA-GARCH (p , q) were studied by Francq and Zakoïan (2004).

We assume that $\{Z_t\}$ is a sequence of i.i.d. random variables with $\mu = 0$ in likelihood function $L_n(\theta)$. The parameter space Θ is a compact subset of $0 \times (0, \infty) \times [0, \infty)^q \times B$,

where $B := \left\{ (\beta_1, \dots, \beta_q)^T \in [0, 1)^q \mid \sum_{i=1}^q \beta_i < 1 \right\}$. The QMLE $\hat{\theta}_n$ maximises $L_n(\theta)$ within the

parameter space Θ . Define polynomials $A_\theta(z) = \sum_{j=1}^p \alpha_j z^j$ and $B_\theta(z) = 1 - \sum_{i=1}^q \beta_i z^i$. The

traditional assumptions include:

- a) Z_t^2 has a non-degenerate distribution with $E(Z_t^2) = 1$.
- b) The true parameter $\theta_0 \in \Theta$.
- c) Under θ_0 , the top Lyapunov exponent γ as defined in equation 2.3 is strictly negative.
- d) The polynomials $A_{\theta_0}(z)$ and $B_{\theta_0}(z)$ have no common root. $A_{\theta_0}(1) \neq 0$, and the true values of α_p and β_q are not zero.
- e) θ_0 is in the interior of Θ .

$$\text{f) } \kappa_Z := E(Z_t^4) < \infty.$$

The assumption in (d) ensures the identifiability of the model parameters. The asymptotic properties of estimators of parameters in GARCH models (proofs can be found in Francq and Zakoïan (2004)), can be stated as follows. Let $(\hat{\theta}_n)$ be a sequence of QMLEs. Under assumptions (a) to (d) stated above;

$$\hat{\theta}_n \rightarrow \theta_0, \text{ almost surely as } n \rightarrow \infty. \quad \dots 2.5$$

If in addition, (e) and (f) are satisfied, then

$$\sqrt{n}(\hat{\theta}_n - \theta_0) \xrightarrow{D} N(0, \Sigma), \text{ almost surely } n \rightarrow \infty \quad \dots 2.6$$

where

$$\Sigma = (\kappa_Z - 1) E_{\theta_0}^{-1} \left(\frac{1}{\sigma_t^2(\theta_0)} \frac{\partial \sigma_t^2(\theta_0)}{\partial \theta} \frac{\partial \sigma_t^2(\theta_0)}{\partial \theta^T} \right).$$

The asymptotic normality of the GARCH estimator ensures that it is consistent. In addition, the speed at which the estimator converges to the true parameter can be deduced. It is also possible to construct a confidence interval around the estimator. The next section provides the background on extreme value theory used in this thesis.

2.4. Extreme Value Theory

The foundation of extreme value theory is the theorem of Fisher and Tippett (1928) which deals with the convergence of maxima. Let $X_1, X_2, \dots, X_n, \dots$ be a sequence of independently and identically distributed exchange rate returns with unknown distribution function F , and the returns are observed at times $T_1 < T_2 < \dots < T_n < \dots$. Let

$M_n = \max(X_1, \dots, X_n)$ be the maximum of the first n returns and that we can find a sequence of normalizing constants a_n and $b_n > 0$ such that

$$\lim_{n \rightarrow \infty} P\{(M_n - a_n)/b_n \leq x\} = \lim_{n \rightarrow \infty} F^n(b_n x + a_n) = H(x)$$

$H(x)$ is a non-degenerate distribution. The extremal type's theorem of Fisher and Tippet (1928) states that $H(x)$ is an extreme value distribution. For particular choices of a_n and b_n , $H(x)$ has the Von Mises parameterisation of the generalised extreme value distribution (GEV)

$$H_\xi(x) = \begin{cases} e^{-(1+\xi x)^{-1/\xi}} & \text{if } \xi \neq 0, \\ e^{-e^{-x}} & \text{if } \xi = 0 \end{cases} \quad \dots\dots\dots 2.7$$

Where $1 + \xi x > 0$ and the case $\xi = 0$ is the limit of the distribution function as $\xi \rightarrow 0$. The three cases $\xi < 0$, $\xi = 0$ and $\xi > 0$ correspond to the Weibull, Gumble and Frechet extreme value distributions respectively. The case $\xi > 0$ corresponds to heavy tailed distributions while the case $\xi = 0$ corresponds to thin tailed distributions. Heavy tailed distributions include, among others, the Pareto, the Student-t, Cauchy and Burr while thin tailed distributions include the normal and exponential distributions.

Since we are interested in the behaviour of large observations which exceed a high threshold u , the distribution of excess values of X over u is defined by

$$F_u(y) = \Pr(X - u \leq y | X > u) = \frac{F(y+u) - F(u)}{1 - F(u)}$$

This is the probability that the value of X exceeds the threshold u by at most an amount $y \geq 0$ given that X exceeds u . Balkema and De Haan (1974), and Pickands (1975) showed that for a sufficiently high threshold, the distribution function of the excesses can be approximated by the GPD. Denoting the threshold by u and the shape parameter by ξ , the conditional distribution of excesses over u converges to the GPD as the threshold gets large.

$$G_{\xi, \beta}(x) = \begin{cases} 1 - \left(1 + \xi \frac{x}{\beta}\right)^{-\frac{1}{\xi}} & \text{if } \xi \neq 0 \\ 1 - e^{-\frac{x}{\beta}} & \text{if } \xi = 0 \end{cases}$$

Where $x \geq 0$ and $\beta > 0$ is the scale parameter. The estimate of the shape parameter ξ determines the weight of the tail. Distributions for which $\xi > 0$, Frechet case, are called heavy-tailed and can be used to model large observations while distributions for which $\xi = 0$ are called thin tailed and correspond to all the common continuous distributions of statistics.

The GPD has finite mean for $0 < \xi < 1$ and finite variance for $0 < \xi < 0.5$. Hosking and Wallis (1987) showed that for $\xi > -0.5$, maximum likelihood regularity conditions are achieved and the maximum likelihood estimates are asymptotically normally distributed.

The approximate standard errors for the estimators of β and ξ can therefore be obtained using i.i.d. from the Fisher Information Matrix.

The GPD model is also used in this thesis to estimate the return period of specific large exchange rate returns. The tail estimator for the GPD model is given by

$$\hat{F}(x) = 1 - \frac{N_u}{n} \left(1 + \frac{\hat{\xi}}{\hat{\beta}} (x - u) \right)^{-\frac{1}{\hat{\xi}}}.$$

Where N_u is the number of exceedances of the threshold u , and n is the sample size. For a given probability p , a quantile x_p at the tail is computed by inverting the tail estimator above,

$$\hat{x}_p = u + \frac{\hat{\beta}}{\hat{\xi}} \left(\left(\frac{n}{N_u} (1 - p) \right)^{-\hat{\xi}} - 1 \right).$$

In extreme value terminology, an estimate of the tail quantile \hat{x}_p , $0 \leq p \leq 1$, gives the return level associated with the return period $\frac{1}{p}$, Coles (1998). This implies that we would wait for about $\frac{1}{p}$ periods to experience an exchange rate return of \hat{x}_p .

2.4.1 Point Processes and the POT Model

The POT model is the point process characterisation of extreme values which is consistent with a Poisson process for the occurrence of exceedances of a high threshold and the GPD for excesses over this threshold. A point process is a random function which assumes point measures as values. In extreme value theory, the point process of exceedances is the most

important, Embrechts *et al.* (1997). If u is a real number and (X_n) a sequence of random variables, the process of exceedances $N_n(\cdot) = \sum_{i=1}^n \varepsilon_{n^{-1}i}(\cdot) I_{\{X_i > u\}}$, $n = 1, 2, \dots$, with state space $E = (0, 1]$ counts the number of exceedances of the threshold u .

2.4.2 Determination of the Appropriate Threshold for the GPD Model

The first task in estimation of the GPD model is the determination of an appropriate threshold u above which the values will be considered to be extreme values. In most cases, graphical techniques such as the QQ-plot, sample mean excess plot, median excess plots, and adhoc methods which impose arbitrary threshold values are used – Embrechts *et al.* (1997). These methods have shortcomings in that they do not use any formal computable method, and that they only give rough estimates of the extreme observations. The sample mean excess over threshold plot is depicted by $[(u, e_n(u)), X_{N:N} < u < X_{1:N}]$, where $X_{N:N}$ and $X_{1:N}$ are the N^{th} and first order statistics of the data sample, and $e_n(u)$ is the sample mean excess function defined by:

$$e_n(u) = \frac{\sum_{i=1}^N (X_i - u)^+}{\sum_{i=1}^N 1_{\{X_i > u\}}}.$$

This is the sum of the excess over the threshold divided by the number of data points, which exceed the threshold u . The sample mean excess function is an empirical estimate of the mean excess function which is defined to be $e(u) = E(X - u \leq y | X > u)$. The mean excess function describes the expected overshoot of a threshold given that an exceedance

occurs. If the empirical plot follows a reasonably straight line with positive gradient above a certain value of the threshold u , then this is an indication that the excesses over this threshold follow a GPD with positive shape parameter – Embrechts *et al.* (1997). The mean excess function for the GPD is expressed as

$$E(X - u \leq y | X > u) = \frac{\beta + \xi u}{1 - \xi}, \text{ where } \xi < 1.$$

Several problems result when one wants to choose an optimal threshold u using the mean excess over threshold plot. A value of the threshold which is too high results in very few exceedances and as a result high variance estimators. For large threshold values, there are few data points to compute the mean excess function which leads to some distortion at the end of the plot. As a result, the upper plotted values are excluded when making inferences on the plot. A value of the threshold which is too low leads to biased estimators. Embrechts *et al.* (1997) indicate that it is possible to choose u asymptotically optimal by quantifying the bias versus variance trade-off. Plots of resulting estimates across different threshold levels are recommended.

2.5. Other Threshold Determination Techniques for the GPD Model

Given the weaknesses of the mean excess over threshold and QQ-plots in determination of the appropriate threshold for estimating the GPD model (see section 2.4.1), this section provides alternative frameworks for selecting the threshold.

Gonzalo and Olmo (2004) consider the threshold as the order statistic which minimises a Kolmogorov-Smirnov (KS) statistic between the empirical distribution of the

corresponding largest observations and the corresponding GPD. In particular, let $F_{\theta,n}$ be the empirical version of the distribution F_{θ} of the largest observations and $GPD_{\hat{\xi}_n^{(ml)}(\theta), \hat{\beta}_n^{(ml)}(\theta)}$ the distribution function of the largest observations with parameters estimated using MLE. The weighted Pickands distance d^{WP} can be defined as:

$$d^{WP}\left(F_{\theta,n}, GPD_{\hat{\xi}_n^{(ml)}(\theta), \hat{\beta}_n^{(ml)}(\theta)}\right) = k^{\gamma} \sup_{0 \leq y \leq \infty} \left| F_{\theta,n}(y) - GPD_{\hat{\xi}_n^{(ml)}(\theta), \hat{\beta}_n^{(ml)}(\theta)}(y) \right|,$$

with $0 \leq \xi \leq \frac{1}{2}$ and $k = \sum_{j=1}^n \mathbf{1}_{\{x_j > \theta\}}$.

The parameter γ determines the weight assigned by the distance d^{WP} to the tail observations defined by the corresponding θ . This distance is the Pickands distance when $\gamma = 0$, and the KS statistic when $\gamma = \frac{1}{2}$. The corresponding threshold choice is therefore the order statistic that minimizes the distance, i.e.

$$u_n^{(WP)} = \arg \min_{\theta} d^{WP}\left(F_{\theta,n}, GPD_{\hat{\xi}_n^{(ml)}(\theta), \hat{\beta}_n^{(ml)}(\theta)}\right)$$

with θ taking values along the ordered sample $x_{(1)} \leq \dots \leq x_{(n)}$. Although the parameter γ allows for different weighting schemes, Gonzalo and Olmo (2004) preferred the case when $\gamma = \frac{1}{2}$, KS statistic.

The Hill plot is also a good instrument to find the optimal threshold for the GPD model – Embrechts *et al.* (1997). Let $X_1 > X_2 > \dots > X_n$ be the order statistics of positive random

variables which are i.i.d. The Hill estimator of the tail index ξ using $k + 1$ order statistics is defined by

$$H_{k,n} = \hat{\xi} = \frac{1}{k} \sum_{j=1}^k \ln \left(\frac{X_{j,n}}{X_{k+1,n}} \right)$$

The Hill plot is defined by the set of points $\{(k, H_{k,n}^{-1}), 1 \leq k \leq n-1\}$. The threshold u is selected from the stable areas of the plot. Andreev *et al.* (2010) highlights the following intuitive ideas on how to choose the optimal threshold using the Hill plot: a) The sequence of turning point is less than $\sim \frac{n}{10}$; b) the Hill estimator in the turning point has a relatively large deviation from the fitted stationary straight line, and c) the turning point is the last sequence of points that satisfies the two conditions above.

Lastly, Smith (2003) proposes the use of the W-statistic - $W_i = \frac{1}{\xi} \log \left(1 + \xi \frac{X_i - u}{\beta + \xi u} \right)$

- to avoid the subjectivity in the threshold selection using the mean excess over threshold plot. If all the assumptions are correct including the selected threshold u and the time span, the W_i 's are independent and exponentially distributed variables with mean 1.

The QQ-plot and sample mean excess plots have been adopted in this thesis to determine the appropriate threshold. As recommended by Embrechts *et al.* (1997), plots of the shape estimate for the GPD over a variety of thresholds are also used in this study to reinforce the judgement in choosing the appropriate thresholds using the QQ-plot and sample mean excess plot. The W-statistics are used to test the robustness of the thresholds selected using QQ-plot and sample mean excess plots. Although the median excess plot is more robust

compared with QQ-plot and sample mean excess plots, is not used in this study as it has a complex form and does not also yield a unique value for the threshold. Lastly, the use of the method proposed by Gonzalo and Olmo (2004) and the Hill plot is left as an area of further work.

CHAPTER THREE

3.0 METHODOLOGIES

3.1 Modelling Exchange Rate Volatility

The GARCH model is used in this thesis to isolate the volatility clustering in the exchange rates returns data. The standardized residuals from the GARCH models are considered to be realizations of a strict white noise process. Extreme value analyses are then applied on the standardized residuals. However, we first outline the theoretical foundations of the GARCH model.

Exchange rate returns are fitted to the GARCH (1, 1) model in equation 2.1. The sizes of the parameters α_1 and β_1 determine the short run dynamics of the resulting volatility process. A large ARCH error coefficient α_1 implies that volatility reacts significantly to market movements. Large GARCH coefficients β_1 indicates that shocks to the conditional variance take a long time to die out (i.e. volatility is persistent). A high α_1 coefficient relative to β_1 indicates that volatility tends to be more extreme.

The lag length in the GARCH model is determined using information criteria. The two main information criterions available for determining the order of an AR model are the Akaike information criterion (AIC) and the (Schwartz) Bayesian information criterion (BIC). The AIC is defined as $AIC = \frac{-2}{T} \ln L + \frac{2}{T}k$ where L is the likelihood function

computed at the maximum likelihood estimates, T is the sample size and k is the number of parameters estimated in the model. The AIC for an AR (p) model with normal errors can be expressed as $AIC = \ln(\sigma_p^2) + \frac{2l}{T}$ where σ_p^2 is the MLE of the variance of the residuals and T is the sample size. To use the AIC to select the order of an AR model, the procedure involves computing the AIC (p) for $p = 1, \dots, q$, where q is a pre-specified positive integer, and selecting the order with minimum AIC value. The BIC for an AR (p) model with normal errors is $BIC = \ln(\sigma_p^2) + \frac{p \ln(T)}{T}$.

Bollerslev *et al.* (1992) found that whilst relatively long lags are required in ARCH models, the GARCH (1, 1) is usually adequate in describing many financial time series. However, higher order GARCH models are required for some time series. The relationship between observations and their conditional variances (characterised by a GARCH model) can be interpreted as returns and risks in financial time series analysis.

3.2 Estimation of the GARCH Model

This section discusses estimation of the GARCH and EGARCH models. Bollerslev (1986) and Nelson (1991) showed that the tails of financial data are much heavier than the tails of the normal distributions due to higher excess kurtosis. Consequently, this thesis uses leptokurtic distributions such as the GED or the Student- t in modelling the errors. The density function for the GED is given by

$$f(\mu, \sigma, \nu; z_t) = \frac{\nu}{2 \left[\frac{(\nu+1)}{\nu} \right] \lambda \left[\frac{1}{\nu} \right]} \exp \left[\frac{-1}{2} \left| \frac{z_t}{\lambda} \right|^\nu \right]$$

where $z_t = \frac{y_t - \mu}{\sigma}$, $\lambda = \left[\frac{2^{(-2/\nu)} \left[\frac{1}{\nu} \right]}{\Gamma(3/\nu)} \right]^{\frac{1}{2}}$, $\Gamma(\cdot)$ is the gamma function, μ is a location parameter, σ is the scale parameter, and ν is a positive shape parameter which measures the thickness of the tails.

The GED yields the normal distribution for $\nu = 2$, and the Laplace or double exponential distribution for $\nu = 1$. The density has heavier tails than the normal distribution for $\nu < 2$, and thinner tails for $\nu > 2$. The log-likelihood function of the GED is

$$L(\mu, \sigma, \nu; y_t) = \ln \left(\frac{\nu}{2} \right) + \frac{1}{2} \ln \left[\frac{\Gamma(3/\nu)}{\Gamma(1/\nu)} \right] - \frac{3}{2} \ln \left[\frac{1}{\nu} \right] - \frac{1}{2} \sum_{t=1}^n \ln \sigma_t^2 - e^{-\left[\frac{\nu}{2} \left(\ln \left[\frac{\Gamma(3/\nu)}{\Gamma(1/\nu)} \right] - \ln \left[\frac{1}{\nu} \right] \right) \right]} \times \sum_{t=1}^n \left| \frac{y_t - \mu}{\sigma} \right|^\nu$$

which gives parameter estimates that are unbiased, consistent, asymptotically normal, and of minimum variance – Nelson (1991).

The GARCH and EGARCH models in this thesis will therefore be estimated by MLE specifying the density of the error term ε as a GED. The next section covers the diagnostic tests for the estimated GARCH models.

3.3 Diagnostic Tests for the GARCH model

To check the adequacy of the estimated GARCH model, diagnostic tests are carried out on the standardised residuals

$$(z_t, \dots, z_{t+n-1}) = \left(\frac{y_t - \hat{\mu}_t}{\hat{\sigma}_t}, \dots, \frac{y_{t+n-1} - \hat{\mu}_{t+n-1}}{\hat{\sigma}_{t+n-1}} \right)$$

These should be i.i.d. if the model is adequate. Following Chatfield (1980), the Ljung-Box statistic, $Q' = T(T+2) \sum_{k=1}^p \frac{r_k^2}{T-k}$, is used to test for the null hypothesis that the residual series is white noise. Where T is the sample size and p the lag length. Both statistics have $\chi^2(p)$ under the null hypothesis of no serial correlation.

The GARCH effects in the model are examined using the correlograms of the squares of the residuals. The Lagrange Multiplier (LM) test ARCH (p) against the no ARCH effects ARCH (0) can also be carried out by computing $\chi^2 = TR^2$ in the regression of ε_t^2 on a constant and q lagged values. Under the null hypothesis of no ARCH effects, the statistic has a limiting $\chi^2(p)$. Values of the computed statistic larger than the critical values give evidence of presence of ARCH and GARCH effects. The Jarque and Bera (1987) test combines prior tests for skewness and kurtosis, and is used to test for normality of data.

The test uses the statistic $JB = \frac{\hat{S}^2(x)}{(6/n)} + \frac{(\hat{K}^2(x) - 3)}{(24/n)} \sim \chi^2(2)$, where $S(x) = E\left(\frac{(X - \mu_x)^3}{\sigma_x^3}\right)$

and $K(x) = E\left(\frac{(X - \mu_x)^4}{\sigma_x^4}\right)$ are the skewness and kurtosis coefficients respectively.

3.4 Modelling Interest Rates Volatility

The thesis also uses extreme value theory to model the tail of the standardized residuals from the interest rate models for the Treasury bill rate, repo rate, and interbank rate to be

estimated in this thesis. As was the case for the exchange rate returns, the trends in the extremes will be established. Interest rate instability will be implied by an increasing trend in the extremes. We will also determine the return period of specific extreme interest rates.

In order to address both the volatility clustering phenomenon and mean reverting aspect of the interest rates data, the thesis adopts a model which combines both the Vasicek (1977) and GARCH models. Let r_t be the current level of the short term interest rate and Δr_t the change in the interest rate at time t , therefore

$$\Delta r_t = \gamma(\theta - r_{t-1}) + \varepsilon_t, \quad \text{where } \varepsilon_t = \sigma_t Z_t, \quad \dots\dots (3.1)$$

$$\text{and } \sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^q \beta_i \sigma_{t-i}^2, \quad t \in \mathbf{Z}$$

Where the parameter θ is the long run interest rate (rate to which the short rate reverts), and γ is the speed of mean reversion or adjustment of the interest rate to its long run level. The mean reversion property ensures that interest rates do not drift permanently upwards as this does not happen in practise. Here, (Z_n) is a sequence of i.i.d. random variables such that $Z_t \sim N(0,1)$.

The restrictions on the coefficients of the variance equation are as stated in the formulation of the GARCH model. The point of departure for the GARCH model in 2.1 from the Vasicek model is that $\varepsilon_t = \sigma dB_t$ in the Vasicek model, where σ is a fixed volatility parameter for the short rate, and B_t is the standard Brownian motion.

3.5 Estimation of the GPD Model

The distribution of excesses can be approximated by the GPD by estimating β and ξ at a high threshold – Embrechts *et al.* (1997). Estimation of the parameters is done using numerical maximum likelihood method or other methods such as probability weighted moments. The likelihood function for the GPD model for the sample $X = (x_1, x_2, \dots, x_n)$ is given by

$$L(\xi, \beta; X) = \begin{cases} -n \ln \beta + \left(\frac{1}{\xi} - 1\right) \sum_{i=1}^n \ln \left(1 - \frac{\xi x_i}{\beta}\right) & \text{if } \xi \neq 0 \\ -n \ln \beta - \frac{1}{\beta} \sum_{i=1}^n x_i & \text{if } \xi = 0 \end{cases}$$

For $\xi \neq 0$, the estimates for the parameters ξ and β are obtained by solving the simultaneous equations:

$$\frac{\partial L(\hat{\xi}, \hat{\beta})}{\partial \hat{\xi}} = \frac{1}{\hat{\xi}} \sum_{i=1}^n \log \left(1 + \hat{\xi} \frac{x_i}{\hat{\beta}}\right) - \left(\frac{1}{\hat{\xi}} + 1\right) \sum_{i=1}^n \left(\frac{x_i}{\hat{\beta} + \hat{\xi} x_i}\right) = 0 \quad \dots 3.2$$

$$\frac{\partial L(\hat{\xi}, \hat{\beta})}{\partial \hat{\beta}} = -\frac{n}{\hat{\beta}} + \left(\frac{1}{\hat{\xi}} + 1\right) \sum_{i=1}^n \frac{\hat{\xi} x_i}{\hat{\beta}^2 + \hat{\beta} \hat{\xi} x_i} = 0 \quad \dots 3.3$$

Embrechts *et al.* (1997) presents the MLE's of $\hat{\xi}_n$ and $\hat{\beta}_n$ based on equation 3.2 and 3.3. It is assumed that the excesses Y_1, \dots, Y_N are distributed as the GPD, where $N = N_u$ is independent of Y_i . The conditional likelihood equations can be solved through the re-parameterisation $(\xi, \beta) \rightarrow (\xi, \tau)$, where $\tau = -\xi / \beta$. The MLEs are

$$\hat{\xi} = \hat{\xi}(\tau) = N^{-1} \sum_{i=1}^N \ln(1 - \tau Y_i) ,$$

where τ satisfies $h(\tau) = \frac{1}{\tau} + \frac{1}{N} \left(1 + \frac{1}{\hat{\xi}(\tau)} \right) \sum_{i=1}^N \frac{Y_i}{1 - \tau Y_i} = 0$, where $h(\tau)$ exists for

$\tau \in (-\infty, \max(Y_1, \dots, Y_N))$ and is continuous at 0. By letting $u = u_n \rightarrow \infty$ the asymptotic

distribution of $(\hat{\xi}_n, \hat{\beta}_n)$ is expressed as $n^{1/2} \left(\hat{\xi}_n - \xi, \frac{\hat{\beta}_n}{\beta} - 1 \right) \xrightarrow{d} N(0, M^{-1})$, $n \rightarrow \infty$,

where $M^{-1} = (1 + \xi) \begin{pmatrix} 1 + \xi & 1 \\ 1 & 2 \end{pmatrix}$ and $N(\mu, \Sigma)$ is the bi-variate normal distribution with mean

vector μ and covariance matrix Σ . In addition, $(\hat{\xi}_n, \hat{\beta}_n)$ satisfy the properties of consistency and asymptotic efficiency.

3.6 The POT Model

The thesis uses extreme value theory to model the tail of the standardized residuals from the GARCH (1, 1) models for the exchange rate returns and changes in interest rates, and determine if there was an increasing trend in the extremes. An increasing trend in the extremes would imply that the CBK was not achieving its objective of maintaining interest rate and exchange rate stability. The return period of specific extreme exchange rate returns and changes in interest rates is also determined. Extreme value theory usually assumes i.i.d. data which is not common with financial data. The standardized residuals are considered to be a realization of a strict white noise process (based on statistical tests for stationarity). The thesis follows the approach of Resnick *et al.* (1996) in which

standardised observations are used to avoid the problem of using dependent data. Smith (2003) as well as McNeil and Frey (2000) also use a two-stage estimator which embeds GARCH (1, 1) volatility estimation in the POT framework to get around the problem of dependent data. However, the theoretical foundations of the extreme value model are highlighted first.

In the POT model, excess sizes and exceedance times of a threshold u are modelled as a two dimensional homogeneous Poisson process – Embrechts *et al.* (1997). The model is formulated as follows:

a) The corresponding excesses over the threshold u are independent and distributed as a GPD. This assumption is checked using crude residuals, as in Smith (2003), defined by the W-statistic. If the GPD assumption is correct, then W_1, W_2, \dots should be i.i.d. exponential random variables with mean 1.

b) Excesses of i.i.d. exchange rate returns over a threshold u occur at times of Poisson process. If the exceedances of the threshold occur at times of a homogeneous Poisson

process with constant intensity which is expressed as $\lambda_u = \left(1 + \xi \frac{u}{\beta}\right)^{-\frac{1}{\xi}}$, then the

scaled inter-arrival times of exceedances given by $Z_k = \lambda \times (T_k - T_{k-1})$, should be independently and identically distributed exponential random variables with mean 1.

Where T_k is the time of the k^{th} exceedance and $T_0 = 0$.

c) Excesses and exceedance times are independent of each other. However, it is important to note that the distribution of the W and Z values is only approximately exponential

since we do not know the true values of the GPD parameters and these have to be estimated.

3.7 Diagnostic Tests for the POT Model

The diagnostic checks for the fitted POT model in this thesis are constructed in line with Smith (2003). Scatter plots of W_i and Z_k values against the order of occurrence are used to check for identical distribution of these values. Any variation of these values with time would suggest a trend in the model. Smaller Z_k values indicate that exchange rate returns are becoming smaller. Plots of ordered W_i and Z_k values against expected exponential quantiles are used to check the exponential assumption on the distribution of these statistics. Approximate linearity of these plots would suggest that the exponential fit is good. To check for independence of these statistics, we will plot their sample correlograms with corresponding 95 percent confidence intervals $\pm \frac{2}{n}$, where n is the number of exceedances of the threshold u . If all autocorrelations fall within the confidence bounds for lags greater than 1, we accept the hypothesis of independence of W_i and Z_k .

However, the use of the sample correlograms to check for independence of the W - values has one weakness in that the GPD has infinite variance for $\xi \geq \frac{1}{2}$. This implies that theoretical autocorrelations do not exist for this range of ξ . The assumption of normal errors in estimating the confidence bands when in fact the distribution of exchange rate returns and changes in interest rates is heavy tailed could also impede the autocorrelation

coefficients to fall to zero. Other tests for independence such as the ADF test can also be undertaken at this stage. The ADF test for the null hypothesis that $\rho = 1$ is based on the regression $\Delta x_t = \mu + \rho x_{t-1} + \sum_{i=1}^p \phi_i \Delta x_{t-i} + \varepsilon_t$, where Δ is the difference operator, x is the time series variable being tested for stationarity, p is the lag length considered, ε is the disturbance term while μ , ϕ and ρ are parameters to be estimated. Failure to reject $\rho = 1$ indicates that x is non-stationary.

CHAPTER FOUR

4.0 TRENDS AND PROPERTIES OF THE DATA

4.1 Data Descriptions and Sources

The exchange rates data used consist of daily data for the Ksh exchange rates against the USD, Euro, Pound and Yen (100). The daily exchange rate is derived by the CBK as an average of buying and selling rates of commercial banks spot exchange rates. The computation of the exchange rates during the study period was based on foreign exchange transactions of nine commercial banks which were the major participants in the foreign exchange market. These are Barclays bank of Kenya, Citibank, Standard Chartered, Stanbic, Co-operative, National bank of Kenya, Bank of Africa, Commercial bank of Africa, and Kenya commercial bank.

Except for the Euro which was introduced in 1999, the rest of the exchange rates are available over the study period (1993 – 2006). Consequently, the exchange rate of the Ksh against the Euro comprises of 1997 observations while those against the USD, Pound and Yen comprise of 3501 observations each.

The choice of the above currencies was based on their relative proportions in the CBK's foreign exchange investment portfolio, and their currency composition of imports. The currency composition of Kenya's imports comprised about 52 percent in USDs in

December 2006 – CBK (2006). The foreign exchange reserves portfolio was held mainly in USDs, Pounds, Euros and Yens during the study period.

The interest rates data consists of daily interbank/overnight rates, daily repo interest rates and weekly Treasury bill rates. Both interest rates are annual rates and are computed by the CBK. The inter-bank rate is a weighted average rate based on the daily transactions in the interbank market. The repo interest rate is a weighted average rate which is based on the daily OMO by the CBK. The Treasury bill is a weighted average of all successful competitive bids for the Treasury bills in the weekly auctions. The daily inter-bank rates data span the period 1995 to 2006 while the repo rates are available from 1996 to 2006. On the other hand, the weekly 91-days Treasury bill rates span the period from 1991 to 2006.

The data was obtained from the CBK database and is also published on a weekly basis in the CBK's weekly bulletin of economic indicators at the CBK's website. Monthly average and end period data on interest rates and exchange rates are also published in the bank's Monthly Economic review and bi-annual Statistical Bulletin.

4.2 Exchange Rates Data

4.2.1 Exchange Rate Policy

The CBK uses foreign exchange market operations as one of the instruments of monetary policy by injecting or withdrawing liquidity through foreign exchange transactions. However, the bank's participation in the foreign exchange market is aimed at stabilization of the exchange rate when there is volatility or speculation, build up of foreign exchange

reserves to the required statutory levels (currently four months of imports cover), and purchasing of foreign exchange to meet government external debt payments.

A fixed exchange rate policy was adopted between 1966 and 1982. It was thought that frequent adjustments of the exchange rate would create uncertainty in the market which could affect investment - CBK (2000). However, the exchange rate was adjusted in 1967, 1975 and 1981 in order to ensure that Kenya's exports remained competitive.

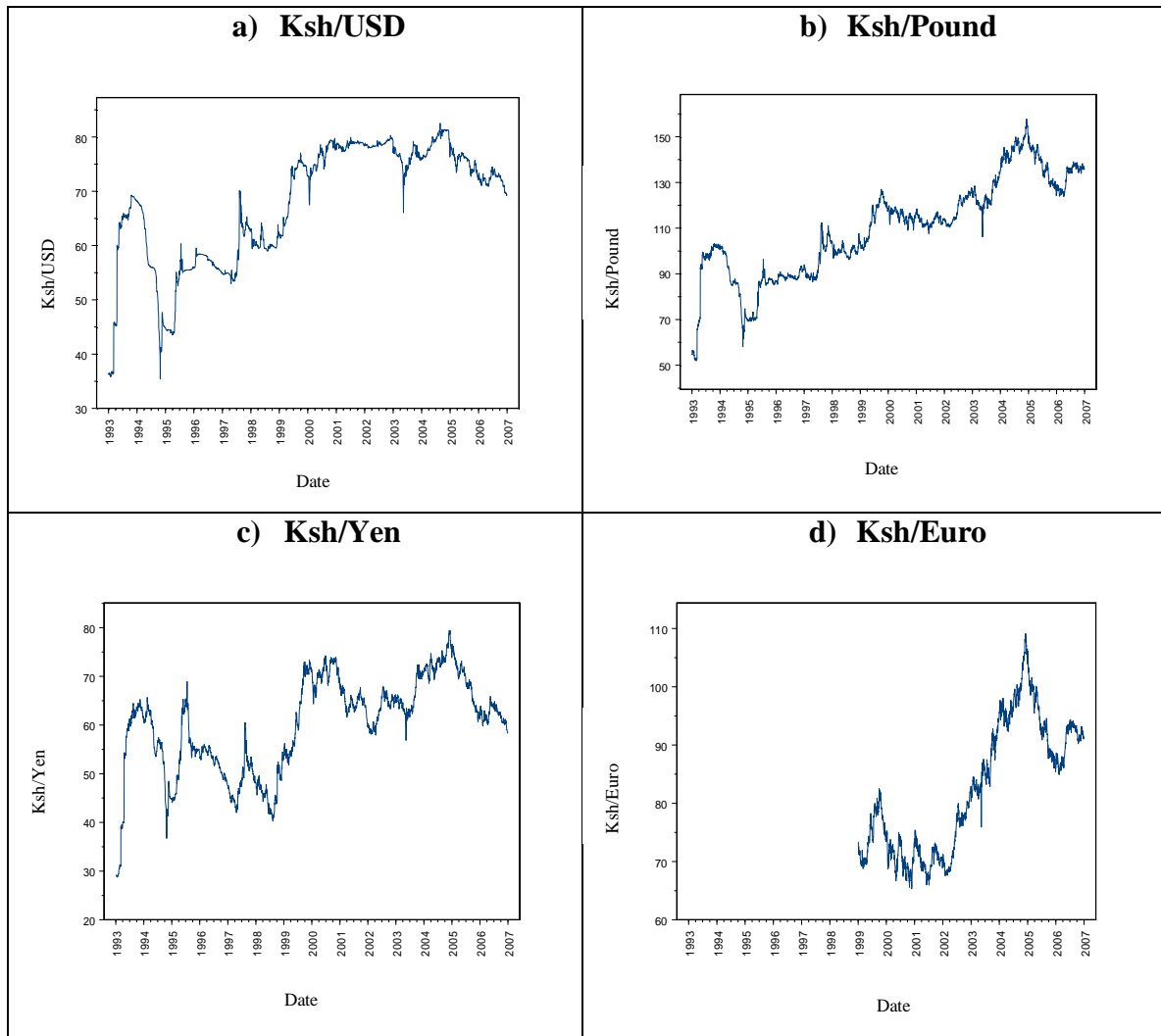
In 1983, the fixed exchange rate regime was replaced by a crawling peg system in which the rate was adjusted daily against a basket of currencies of Kenya's main trading partners based on inflation differentials with these countries. The exchange rate was discretely devalued during the period relative to inflation and external payments conditions.

The foreign exchange market was liberalised further in the 1990s with introduction of foreign exchange bearer certificates by CBK. The introduction of the certificates was aimed at stemming capital flight as well as to attract foreign exchange from outside the domestic banking system. The foreign exchange bearer certificates were discontinued after the official exchange rate (Ksh to USD) was devalued by almost 70 percent between March and May 1993 to avoid capital flight. This was followed by the adoption of the floating exchange rate in October 1993.

4.2.2 Trends in the daily Exchange Rates Data

As shown in Figure 1, there was significant volatility in the exchange rate of the Ksh against the USD, Pound and Yen between 1993 and 1998, and relative stability thereafter.

The Euro was introduced in 1999. Except for the year 2000 and 2003, the Euro has been relatively stable compared with the other currencies.



Source: CBK

Figure 1: Trends in the daily exchange rate of Ksh to the USD, Pound, Yen, and Euro.

The official exchange rate (Ksh/USD) was devalued from 36.6 to 62.2 between March and May 1993 in order to reduce the margin between the official rate and the foreign exchange certificates premium which reflected the market rate. Foreign exchange bearer certificates

were introduced by CBK in December 1991 through the Exchange Control Circular no. 3/90/35 in order to attract foreign currency held abroad by Kenyans.

Despite the introduction of the bearer certificates, the shilling exchange rate remained overvalued in real terms thereby eroding the competitiveness of the country's exports. It was therefore necessary to devalue the exchange rate to restore external competitiveness. Following the need to build foreign exchange reserves and make the exchange rate more competitive, the Ksh/ USD exchange rate was devalued from: 36 to 45 on 9th March 1993, 45 to 60 in April 1993 and 60 to 65 in May 1993.

The relaxation of fiscal policy to accommodate a sharp increase in government expenditure resulted in significant rise in money supply. This coupled with the droughts of 1992 and 1993 exerted pressure on domestic prices with overall month-on-month inflation and the average Ksh to USD exchange rate increasing significantly from 32.1 percent and Ksh 35.9 in January 1993 to 42.1 percent and Ksh 59.9 respectively in April 1993.

In order to control the rising inflation, and also meet the high deficit financing requirements, the CBK increased the amounts in the weekly Treasury bill auctions. As a result, the Treasury bills rate increased from 19.2 percent on 15th March 1993 to 32.4 percent on 22nd March 1993, 44.5 percent on 19th April 1993 to 55.7 percent on 26th April 1993 to reach the highest level of 87.9 percent on 19th July 1993. The differential between Kenyan and foreign interest rates widened thereby attracting speculative capital inflows – Ndung'u *et al.* (2001). This, coupled with elimination of foreign exchange licensing in

April 1993 and liberalisation of offshore borrowing in May 1994, explains the strengthening of the shilling between 1994 and 1995.

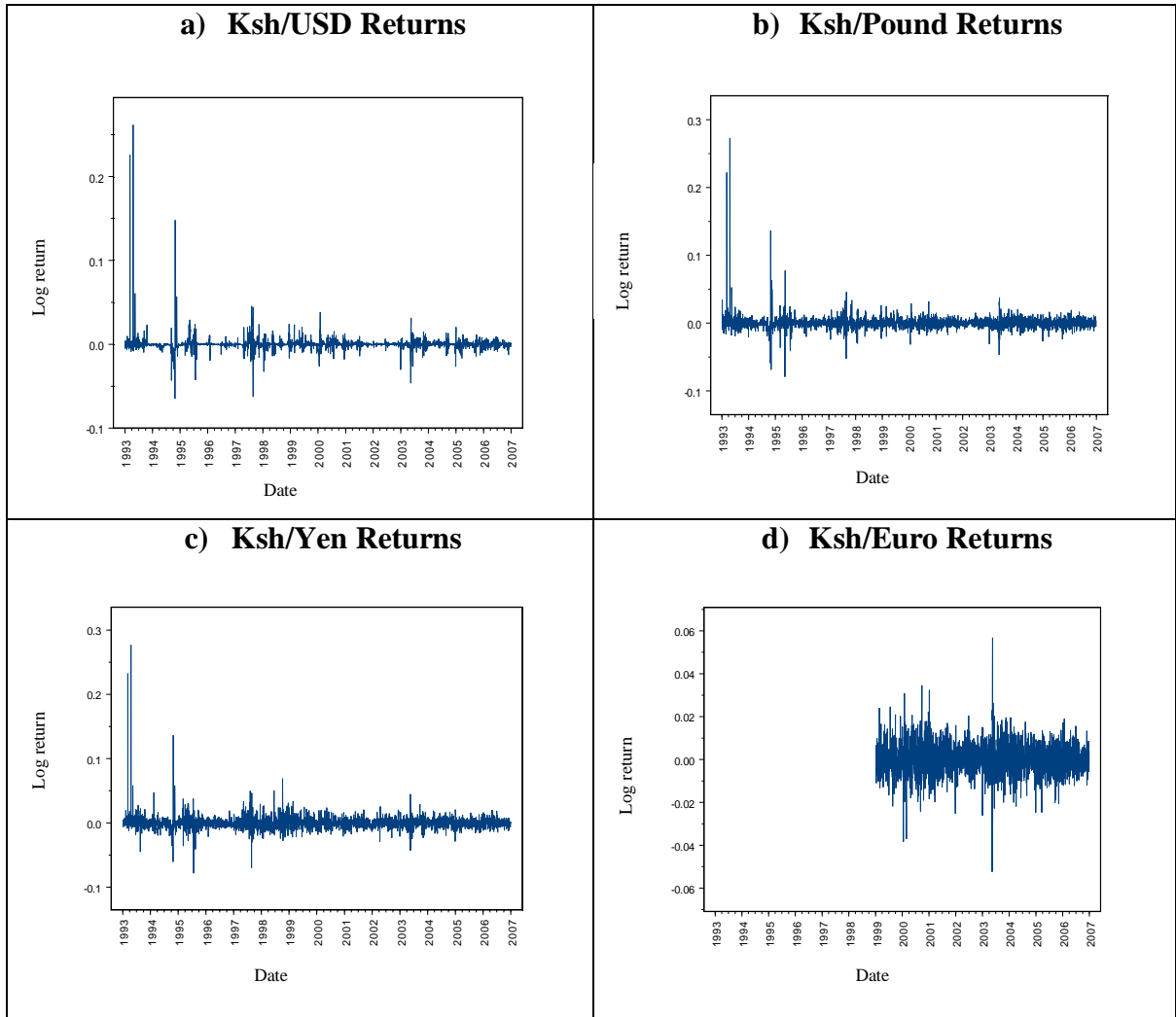
Exchange rate movements in the period were also affected by specific events and expectations. Failure by the government to implement specific expenditure management reforms as a precondition for aid disbursement from development partners in 1995 resulted in an aid freeze expectation and depreciation of the exchange rate. Kenya received a significant level of external aid inflows towards the drought in 1994 which resulted in strengthening of the shilling in September and October 1994. The shilling depreciated significantly in August 1997 following significant private capital outflows attributed to the aid freeze.

According to CBK (2006), the exchange rate remained relatively stable between 1999 and 2006 due to improved economic management, increased foreign exchange earnings from exports of goods and services, mainly tourism, and capital inflows through Diaspora remittances. However, the shilling depreciated sharply from 72.86 per USD on 6th October 2004 to 79.47 per USD on 2nd December 2004 following an announcement by the International Monetary Fund (IMF) that it was delaying an expected disbursement of balance of payments support pending reforms on governance – Ministry of Finance (2006). This was interpreted by some financial market players as an aid freeze and that other development partners would follow suit.

4.2.3 Properties of Exchange Rates Data

Foreign exchange rates are usually transformed and analysed as log-returns. The transformation makes prices independent of their unit and therefore comparable with each other - Mikosch (2001) and Posedel (2005). Unlike prices data which are believed to be non-stationary, the log-returns are thought to evolve as a stationary process.

The plots of the exchange rate returns in Figure 2 reveals a characteristic dependence structure where periods of high returns tend to be followed by high values. This indicates volatility clustering in the data. The structure is consistent with empirical evidence as in Embrechts *et al.* (1997) and Taylor (1986) in which financial data is heavy tailed, has changing volatility, and exhibits serial dependence in the data.



Source: CBK and own computations

Figure 2: Trends in the daily exchange rate returns of the Ksh to USD, Pound, Yen, and Euro.

4.3 Interest Rates Data

4.3.1 Interest Rates Policy

During the first twenty years after independence in 1963, interest rates in Kenya were controlled by the CBK (2000). The CBK pursued a low interest rate policy during the

period to promote investment and also to protect borrowers. The CBK used the interest rate structure as an instrument to direct credit growth and to encourage savings. The interest rates were left largely unchanged during the period as it was felt that changes would create uncertainty which would have a negative impact on investment.

However, from 1983, interest rates assumed a more significant role as an instrument of monetary policy as the rates were adjusted regularly to control inflationary pressures, promote savings and assist in balance of payments management. As part of broad financial sector reforms undertaken by the Government, interest rates were liberalised in July 1991. Consequently, market forces of supply and demand were left to determine the interest rates levels.

The requirement that commercial banks and some financial institutions maintain a certain proportion of their deposit liabilities in cash balances with the CBK is also an effective monetary policy instrument which has had a direct impact on interest rates. A rise in the cash ratio requirement reduces the banks' available cash reserves at CBK thereby limiting their capacity to extend credit to the private sector. The instrument was introduced in late 1971 when the CBK compelled banks to maintain a minimum level of cash balances equivalent to 5 percent of their deposit liabilities.

4.3.2 Trends in the Interest Rates Data

As shown in Figure 3a, the Treasury bill rate had remained low and stable from July 1991 when interest rates were liberalized. The rate started to rise in April 1993 when the CBK

enhanced OMO in order to withdraw excess liquidity in the economy to address inflationary pressures and depreciation of the exchange rate of the Ksh that were getting out of hand. As a result, the investor confidence in the economy was eroded thereby worsening balance of payments position and general slowdown in economic activity.

To address the worsening macroeconomic conditions, the CBK, beginning April 1993, resolved to mop the excess liquidity in the economy by stepping up Treasury bill sales both at the primary auction and through OMO. Consequently, the Treasury bills rate increased from 19.2 percent on 15th March 1993 to 32.4 percent on 22nd March 1993 as the Government increased the amount of Treasury bills in the weekly auctions to Ksh 5,000 million in a bid to curb the rising inflation rates. The Treasury bill rates continued to rise, increasing from 44.5 percent on 19th April 1993 to 55.7 percent on 26th April 1993, and reached the highest level of 87.9 percent on 19th July 1993.

In order to limit money creation by commercial banks, the CBK raised the cash ratio requirement in succession from 8 percent to 10 percent through banking circular no. 6 of April 1993, and further to 12 percent through banking circular no. 11 of July 1993. Banking circulars no. 11 and 13 of 1993 increased the cash ratio rate to 14 percent and further to 16 percent and 20 percent in March and April 1994, respectively. These measures, coupled with increased monetary policy operations, resulted in a drop in the Treasury bills rates from 87.9 percent on 19th July 1993 to 45.5 percent on 27th December 1993. To ensure success in mopping up the excess money in circulation and raising the resources required to finance the domestic debt redemptions in the high inflation

environment, it was necessary to accept bids at high interest rates in order to realize the enhanced Treasury bill sales.

The tightening of control on commercial banks trading with their deposits through increased reserve requirements at CBK also meant reduced income for banks from lending operations in the private sector. In view of the envisaged decline in earnings from lending to the private sector due to enhanced cash ratio, commercial banks as the leading players in the Treasury bills market increased investment in Treasury bills to maintain the desired profits margin by tendering for Treasury bills at high interest rates.

The CBK continued with these measures by raising the cash ratio requirement from 14 percent to 16 percent. In addition, CBK started paying interest at a rate of 20 percent on commercial banks deposits at CBK which were in excess of the required cash ratio. This measure was taken to tap any excess cash held by the commercial banks.

The CBK continued with similar measures to control money supply and restore stability in interest rates. The banking circular no. 7 of 13th April 1994 limited the frequency of access by commercial banks to the overnight lending facility at CBK to a maximum of ten times in a month. Furthermore, no bank was to be allowed to access the facility for four consecutive days in any month. Since non-bank financial institutions were also taking deposits from the public, banking circular no. 2 of 13th June 1994 introduced cash ratio requirements for non-bank financial institutions.

The banking circular no. 15 of 19th August 1994 reduced cash ratio requirements for both banks and non-bank financial institutions from 20 percent to 18 percent. Interest rates had dropped to 22.7 percent on 15th August 1994, and inflation was under control. Overall month on month inflation, which stood at 61.5 percent in January 1994, decreased to single digit level of 8.4 percent in November 1994. Similarly, the shilling to USD exchange rate, which had weakened from an average of Ksh 28.8 in January 1992 to Ksh 69.1 in October 1993 strengthened to Ksh 41.3 in October 1994, contributing significantly to the improvement in foreign exchange reserves from 3.9 months of imports cover in June 1993 to 7.3 months in October 1994.

The evolution of the interbank rate depends mainly on the level of liquidity in the market. Transactions which reduce liquidity in the market include corporate tax payments and payments for primary issues of government securities. However, government payments and maturities of government securities increase market liquidity. Build up of government deposits at the CBK has also been shown to create tight liquidity conditions in the market – hence increase the interbank rate. As shown in Figure 3b, the inter-bank rate is strongly correlated with the Treasury bills rate, and was quite volatile in 1995. The rate dropped from 24.8 percent on 5th September 1995 to 13.0 percent on 6th September 1995; increased from 7.5 percent on 17th November 1995 to 19.0 percent on 20th November 1995; dropped from 22.4 percent on 21st November 1995 to 7.2 percent on 22nd November 1995; increased from 8.2 on 27th November 1995 to 20.5 percent on 28th November 1995; and increased from 14.6 percent on 15th December 1995 to 25.1 percent on 18th December 1995.

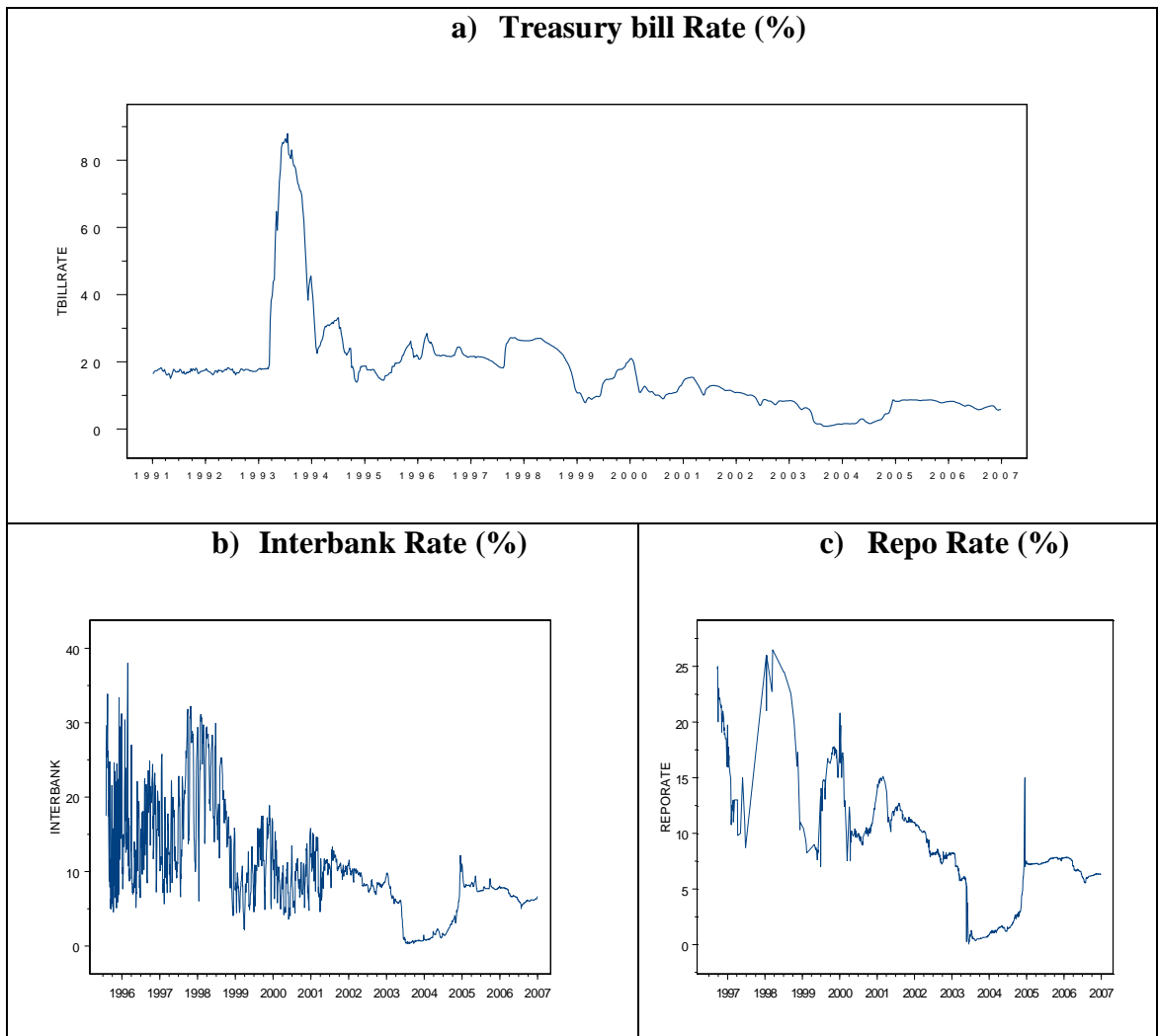
To address the volatility of the interest rates in the interbank market, the CBK issued various banking circulars to address the operations of the interbank market. First, interest payments on cash deposits held at CBK which were in excess of the required cash ratio were discontinued. To encourage the primary market, OMO interest rates were set at lower rates than the primary market rates. In order to increase participation by small investors, average tendering (non-competitive) of Treasury bills was introduced. The market became more transparent as information on the money was disclosed to the media and clearing house. Relative stability returned in the interbank market following implementation of these measures.

Repo Treasury bills were formally introduced through banking circular no.16 of 19th November 1996 to draw a line between Treasury bills used for domestic borrowing and those used as instruments for implementing monetary policy. However, the evolution of the Repo rate is very much correlated with that in the Treasury bills rate (Figure 3c).

Further reductions in the cash ratio requirements were implemented through banking circulars no. 12 of 1998, no. 22 of 1998 and no.17 of 2000. The cash ratio requirements reduced from 14 percent to 10 percent during the period. As a result of these measures, the repo rate dropped from 26 percent on 25th June 1997 to 12.9 percent on 18th December 2000; the interbank rate fell from 28.5 percent on 31st December 1997 to 15.8 percent on 29th December 2000; and the Treasury bills rate dropped from 26.3 percent on 29th December 1997 to 13.5 percent on 25th December 2000. There was also relative stability in the interests during the period compared with earlier periods.

In a move to ensure that commercial banks had adequate liquidity to enable them to lend to the private sector, and to facilitate reduction of commercial banks lending rates, the cash ratio requirement was reduced further to 6 percent in June 2003. As a result, the Treasury bills rate dropped from 8.4 percent on 30th December 2002 to 1.4 percent on 29th December 2003; the Repo rate dropped from 8.3 percent on 30th December 2003; and the interbank rate dropped from 9.7 percent on 31st December 2002 to 1.1 percent on 31st December 2003.

The introduction of the CBK rate in June 2006 through banking circular no.5 of 2006 did not have a significant effect on the stability of interest rates. However, short term interest rates have generally responded to changes in the CBK rate since its introduction. The Treasury bill was the main borrowing instrument used by the central government to raise funds for financing the budget deficit until May 2001 when the government embarked on the bond programme which would ensure public domestic debt was restructured in favour of Treasury bonds - Annual Public Debt Management Report (2006). The lengthening of the maturity profile of domestic debt contributed significantly to the fall in the Treasury bill rate during the period as rollover and market (interest rate) risks which characterize short term domestic borrowing were minimised.



Source: CBK

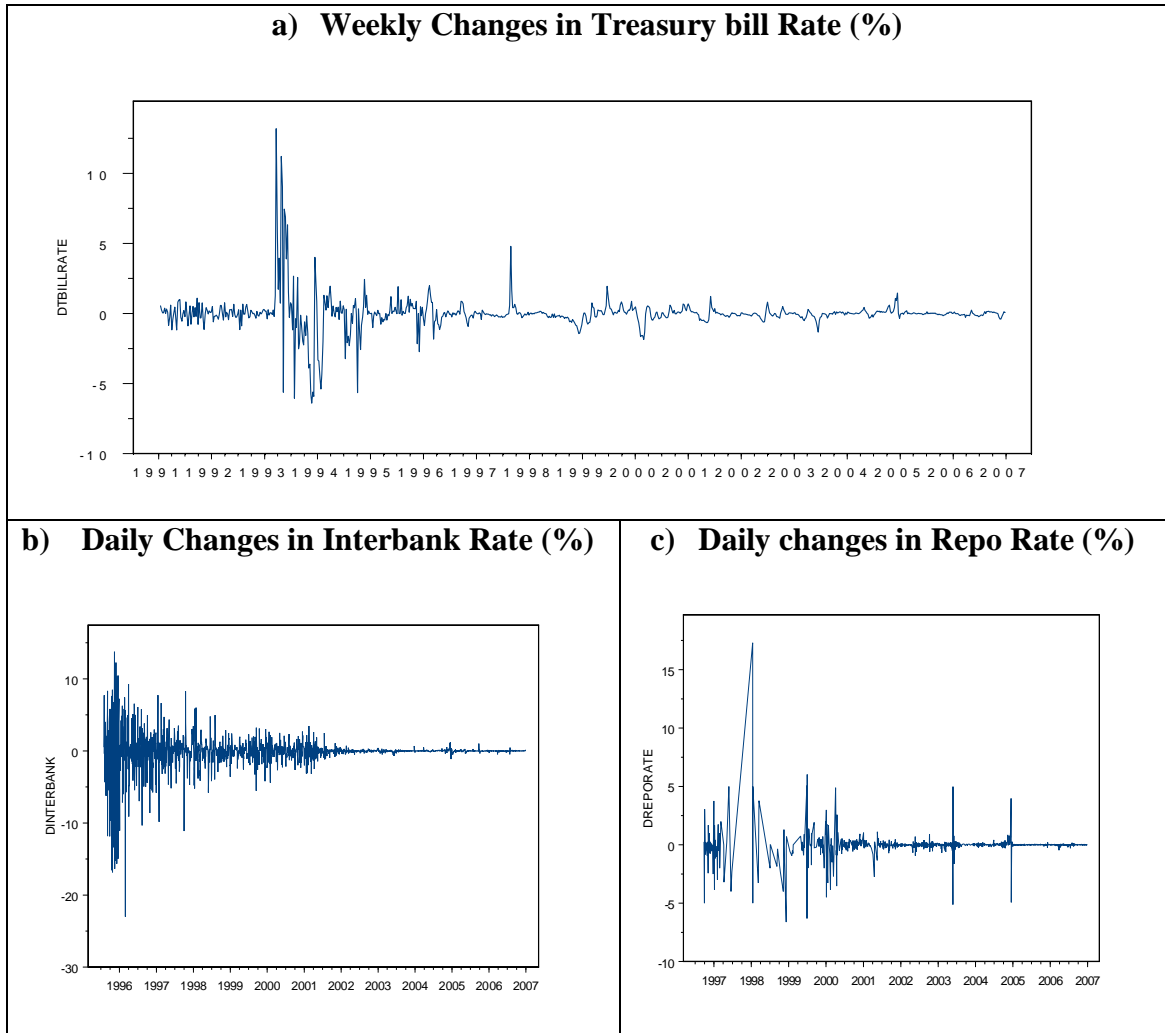
Figure 3: Trends in the weekly Treasury bill rate, daily interbank rate, and daily repo rates

4.3.3 Properties of the Interest Rates Data

The properties of the interest rates data are depicted in Figure 4, which shows that volatility in the interest rates was higher in the mid 1990s. However, the Figures also show that the volatility in the interest rates has generally been falling. The characteristics of movements in interest rates are discussed by Ahlgrim *et al.* (1993). These include:

- a) Mean reversion – increases in interest rates tend to be followed by decreases in the rates, and vice versa.
- b) The volatility of interest rates is proportional to the level of the rates.
- c) The volatility of interest rates on assets of different maturities varies. Short term rates tend to vary more than long term rates.
- d) Interest rates of different maturities are positively correlated.
- e) Interest rates are positive in most cases.

The plots of the movements in interest rates reveal a characteristic dependence structure where periods of high rates tend to be followed by high interest rates. This indicates volatility clustering in the data.



Source: CBK

Figure 4: Trends in changes in 91-days Treasury bill, interbank and repo interest rates

This chapter has documented the reasons behind key movements in the interest rates and exchange rates data since liberalisation. The main conclusion from the analysis of the trends indicates a declining trend in extreme movements in the respect data. However, the data depicts a characteristic dependence structure similar to volatility clustering.

CHAPTER FIVE

5.0 EMPIRICAL RESULTS

5.1 Descriptive Statistics for Exchange Rate Returns

Descriptive statistics for the exchange rate returns are presented in Table 1. The mean exchange rate returns range from 0.026 percent on the Pound to 0.011 percent on the Euro. Both the kurtosis and skewness coefficients are positive and higher than 3 and zero respectively. This indicates that the distribution of the exchange returns is not normal. The positive skewness coefficients indicate that the distribution of the returns is right skewed. This implies that depreciations in the exchange rate occur more often than appreciations.

Kurtosis coefficients of 461.8, 255.4, 6.7 and 200.5 for the log-returns of exchange rate of the shilling against USD, Pound, Euro and Yen are much higher than the 3 for a normal distribution. This indicates that the underlying distributions of the returns are leptokurtic. The Jarque-Bera test for normality indicates that the distribution of exchange rate returns for all the currencies have tails which are heavier than that of the normal distribution.

	Ksh/Pound	Ksh/Euro	Ksh/USD	Ksh/Yen
Mean	0.000260	0.000110	0.000185	0.000199
Median	-0.000040	-0.000010	-0.000040	-0.000140
Maximum	0.272460	0.056780	0.261380	0.276750
Minimum	-0.079400	-0.052380	-0.064780	-0.078230
Standard Deviation	0.009868	0.007587	0.008342	0.010738
Skewness	9.952964	0.039193	15.970640	8.414123
Kurtosis	255.364100	6.725157	461.849200	200.475400
Jarque-Bera Statistic	9348233.0	1155.18	30861733.0	5729931.0
Jarque-Bera Probability	0.000	0.000	0.000	0.000
No. of Observations	3501	1997	3501	3501

Table 1: Summary statistics for exchange rate returns

To check for serial correlation and GARCH type of heteroscedasticity in the exchange rate returns, correlograms for the returns and squared returns are plotted in Appendix 1 and 2, respectively. A notable difficulty in interpreting the correlogram identified by Chatfield (1980) is that it is possible to have one or more autocorrelation coefficients lying outside the confidence bands, even when the time series is random. The hypothesis of no serial correlation at 5 percent significance level is accepted for the returns on the Euro and Pound since the autocorrelations are within their 95 percent confidence bands. However, although the autocorrelations for the returns on the USD and Yen die out at lag 1, the significance of the autocorrelation at lag 1 suggest that the data could adequately be characterised by a first order autoregressive model. The correlograms of squared exchange rate returns for both currencies show presence of serial correlation which is an indication of GARCH type of heteroscedasticity – Franke *et al.* (2008).

5.2 Estimated Volatility Models for Exchange Rate Returns

The GARCH (1, 1) and EGARCH (1, 1) models were fitted to the exchange rate returns of the Ksh to USD, Pound, Euro, and Yen. The parameter estimates and the value of the AIC for the respective models are presented in Tables 2 and 3. The AIC is computed for comparison between the GARCH and EGARCH models. Both models are produced almost similar AIC values for the exchange returns across all currencies and sample periods. In addition, the statistical insignificance of the estimated leverage effect in the EGARCH models across most of the exchange rate returns indicates that the GARCH (1, 1) model is generally adequate in describing the volatility process.

The estimated GARCH (1, 1) models are highly significant at 5 percent significance level. There is a high persistence of shocks in the volatility. The estimated α_1 and β_1 parameters are positive for returns on all currencies while their sum is 0.97 each for GARCH models for returns on the Euro, Pound and Yen which is less than one. However, the sum of the parameters is 1.67 for the GARCH model for returns on the USD which is significantly different from unity. We deduce that the GARCH models for the Pound, Euro and Yen are well specified. The QML estimates (corresponding to the estimated GED parameters) of the exchange rate returns on the USD (0.545), Euro (1.55), Pound (1.004) and Yen (1.024) are highly significant and correspond to distributions with heavier tails than the normal distribution. A significantly higher estimated ARCH coefficient α_1 corresponding to returns on the USD indicates that the volatility is extreme and responds significantly to market movements.

The GARCH (1, 1) models were re-estimated for sample periods 2000-2006 and 2003-2006 to examine the stability of the parameter estimates, and as robustness tests. The sum of the estimated α_1 and β_1 parameters are less than one for all currencies except the USD. Furthermore, parameter estimates for all models except for returns on the USD do not change significantly using different samples. However, the sum of the parameters is almost unity for the shorter samples for returns on the USD, showing a better fit compared with the full sample.

	<i>Sample</i>	α_0	α_1	β_1	<i>GED Parameter</i>	<i>AIC</i>
Ksh/USD	1993 – 2006	0.00000105 (5.899737)	1.076694 (10.92432)	0.590251 (34.36391)	0.545061 (61.42906)	-9.3481
	2000 – 2006	0.00000081 (3.836304)	0.437386 (7.118349)	0.740559 (32.72725)	0.640984 (30.15752)	-9.3963
	2003 – 2006	0.00000378 (3.493522)	0.356550 (4.610119)	0.730243 (19.74466)	0.693861 (18.71917)	-8.98143
Ksh/Euro	1993 – 2006	0.0000169 (2.992535)	0.057368 (5.631162)	0.912865 (53.4694)	1.550046 (27.89108)	-7.0216
	2000 – 2006	0.0000134 (3.046872)	0.058170 (5.84111)	0.917479 (62.58599)	1.586057 (26.05356)	-7.0236
	2003 – 2006	0.0000225 (1.97455)	0.072265 (4.37786)	0.885066 (26.54393)	1.624607 (18.09584)	-7.0726
Ksh/Pound	1993 – 2006	0.0000235 (7.955607)	0.124040 (10.02159)	0.844105 (147.8030)	1.004069 (81.52386)	-7.2559
	2000 – 2006	0.0000128 (2.733988)	0.070897 (5.339601)	0.897343 (44.2896)	1.427107 (26.38942)	-7.3967
	2003 – 2006	0.0000381 (2.18688)	0.076450 (4.183171)	0.836014 (16.56647)	1.554226 (16.46329)	-7.2362
Ksh/Yen	1993 – 2006	0.0000303 (10.44116)	0.090346 (8.567443)	0.874988 (140.6107)	1.024499 (94.81803)	-6.9000
	2000 – 2006	0.0000397 (3.128433)	0.078186 (4.7511)	0.832441 (21.94708)	1.45685 (22.75235)	-7.2357
	2003 – 2006	0.0000355 (2.551353)	0.079279 (4.088599)	0.839489 (19.90056)	1.438015 (17.93582)	-7.2522

Note: t-statistics are in parentheses. AIC indicates Akaike Information Criterion.

Table 2: Estimated GARCH (1, 1) models for exchange rates returns.

The fitted EGARCH (1, 1) models are highly significant at 5 percent significance level, with t-statistics much higher than those for the GARCH (1, 1) model. The estimated α_1 and β_1 parameters are positive for returns on all currencies while their sum is 1.35, 1.10, 1.15 and 1.12 for returns on the USD, Euro, Pound and Yen respectively. The sum of the parameters is closer to one for both returns on both currencies. In particular, there is significant improvement over the fit of the GARCH (1, 1) model for returns on the USD, and deterioration from the fit for returns on the Euro, pound and yen. The QML estimates of the exchange rate returns are highly significant with similar magnitudes as those in the GARCH (1, 1) model.

	<i>Sample</i>	α_0	α_1	δ_1	β_1	<i>GED Parameter</i>	<i>AIC</i>
Ksh/USD	1993 – 2006	-0.646893 (-17.87874)	0.383126 (18.77143)	-0.052651 (-3.731063)	0.9621144 (344.5348)	0.523415 (52.37227)	-9.2770
	2000 – 2006	-0.87151 (-7.435369)	0.529358 (11.7547)	0.049839 (1.486621)	0.953523 (112.9572)	0.641221 (29.12996)	-9.3944
	2003 – 2006	-1.050638 (-4.939496)	0.484099 (6.798754)	0.037387 (0.838241)	0.934808 (59.14176)	0.690890 (19.16782)	-8.9827
Ksh/Euro	1993 – 2006	-0.449203 (-3.662692)	0.136609 (6.600913)	-0.010839 (-0.748975)	0.96510 (83.50108)	1.544579 (27.65084)	-7.0187
	2000 – 2006	-0.402533 (-3.954703)	0.139543 (6.790636)	-0.010873 (-0.724192)	0.970205 (101.8560)	1.575333 (26.34401)	-7.0207
	2003 – 2006	-0.491151 (-2.385804)	0.160189 (4.958176)	-0.021532 (-0.813106)	0.963071 (50.24587)	1.628466 (17.02169)	-7.0695
Ksh/Pound	1993 – 2006	-0.359830 (-8.196257)	0.174879 (12.51631)	-0.003353 (-0.408018)	0.976350 (242.3789)	1.006180 (67.02286)	-7.25328
	2000 – 2006	-0.51297 (-3.761789)	0.160765 (6.033251)	-0.039911 (-2.380713)	0.961799 (76.86947)	1.436644 (26.34424)	-7.3964
	2003 – 2006	-0.865466 (-2.481454)	0.161604 (4.435239)	-0.043758 (-1.878920)	0.926398 (27.89978)	1.549447 (15.10658)	-7.2269
Ksh/Yen	1993 – 2006	-0.337404 (-9.723696)	0.145569 (9.938119)	0.002980 (0.381372)	0.975623 (337.0338)	1.017989 (83.8370)	-6.9033
	2000 – 2006	-0.858558 (-3.373376)	0.168927 (5.426422)	-0.033044 (-1.855601)	0.927544 (38.64576)	1.44318 (22.55336)	-7.2327
	2003 – 2006	-0.793135 (-2.701557)	0.188801 (4.909579)	-0.013573 (-0.595912)	0.935580 (33.81567)	1.420982 (17.58030)	-7.2450

Note: t-statistics are in parentheses. AIC indicates Akaike Information Criterion.

Table 3: Estimated EGARCH (1, 1) models for exchange rates returns

The fitted EGARCH (1, 1) model shows that a leverage effect exists for returns on the USD. The estimated leverage effect δ is negative and statistically significant at 5 percent level for returns on the USD. This indicates that a positive return shock reduces the volatility of the exchange rate of the Ksh to USD. However, the estimated leverage effects for returns on the Euro and Pound are negative although not statistically significant at 5 percent level. The estimated leverage effect for returns on the Yen is not statistically significant, although it is positive. The estimated EGARCH model reveals that positive and negative shocks have the same effect on the volatility of exchange rate returns on the Euro, Pound and Yen.

The EGARCH (1, 1) models were re-estimated for sample periods 2000-2006 and 2003-2006 to examine the stability of the parameter estimates, and as robustness tests. The parameter estimates for all models do not change significantly using the different samples, an indication that the EGARCH model is robust. However, the estimated leverage coefficients for returns on the USD, Euro and Yen are not significant at 5 percent significance level for models based on the shorter samples. The estimated leverage coefficient for returns on the Pound is negative and statistically significant at 5 percent and 10 percent for the shorter periods.

Appendices 5 and 6 show plots of the estimated volatility process and the standardised residuals of the GARCH (1, 1) and EGARCH (1, 1) models in Table 2 and 3, respectively.

The plots reveal decreasing volatility in the exchange rate returns implying general stabilisation in the exchange rate.

Except for the USD and Yen, the probability values for the Ljung-Box Q-statistics of the standardised residuals from the GARCH (1, 1) models at selected lag lengths (Appendix 3) suggest no statistically significant serial correlation at 5 percent level. However, the probability values for Ljung-Box Q-statistics for the squared residuals (Appendix 4) do not suggest any significant conditional heteroscedasticity at 5 percent level for all exchange rate returns. Further, as shown in Table 4, tests for stationarity using the ADF unit root tests however revealed that the standardised residuals from GARCH models of both exchange rate returns are stationary. The lag selection was automatically done through the Schwartz Criterion.

Variable	ADF Statistic	P-Value
Ksh/USD	-55.4397	0.0001
Ksh/Pound	-57.6203	0.0001
Ksh/Euro	-45.1850	0.0001
Ksh/Yen	-56.0299	0.0000

Table 4: ADF tests for standardised residuals from GARCH models of exchange rate returns.

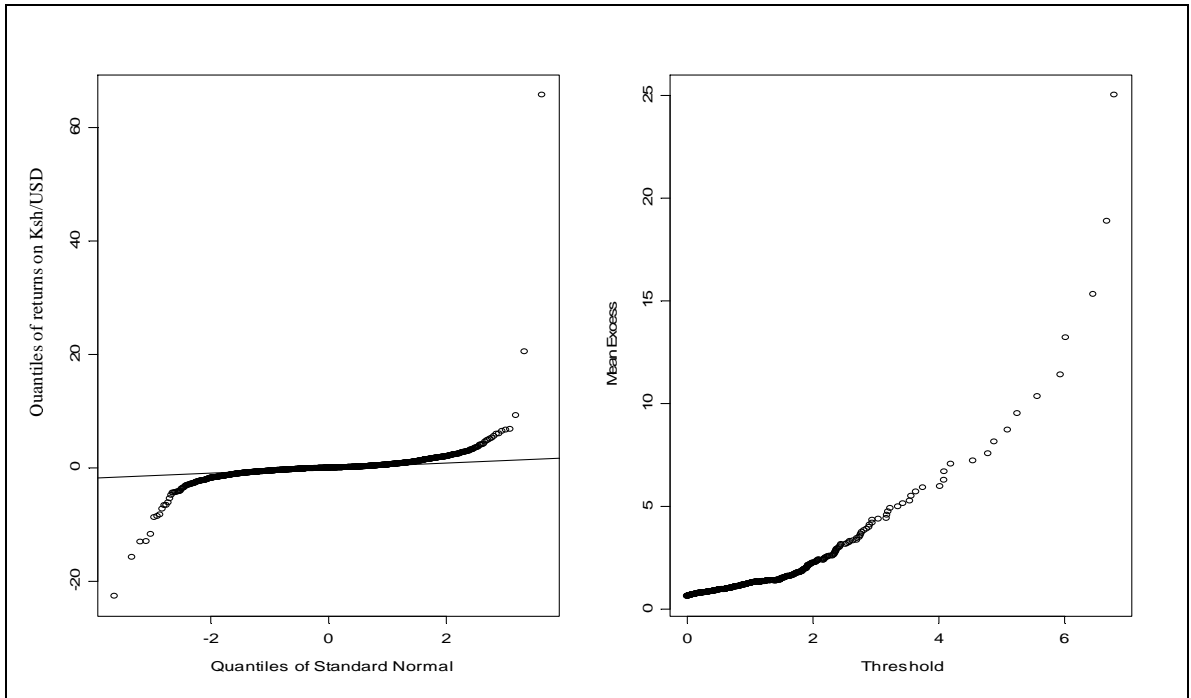
5.3 Extreme Value Models for Exchange Rate Returns

5.3.1 Data Exploration

The QQ-plots and sample mean excess plots are used to determine if the distribution of the standardised residuals from the GARCH (1, 1) models of the exchange rates is heavy tailed, and determine an appropriate threshold above which the residuals can be modelled by a GPD. The QQ-plots of the standardised residuals against the normal distribution in Figures 5 and 6 shows a concave relationship between the quantiles of the empirical and the above distributions, which suggests that the distributions of the standardised residuals of the exchanges rates of the Ksh to USD, Pound, Yens and Euro are fat tailed.

The plots show that the sample points start deviating from linear behaviour and form a concave shape at around 0.5 for the Ksh/USD residuals, 1.0 for Ksh/Pound and Ksh/Yen residuals, respectively, and 1.8 for the Ksh/Euro residuals. Similarly, the sample mean excess plots for the residuals are approximately linear and positively sloped from the above thresholds which indicate heavy tailed behaviour. We therefore deduce that the distributions of excesses of the standardised residuals above these thresholds can be modelled by the GPD.

a) Ksh/USD



b) Ksh/Pound

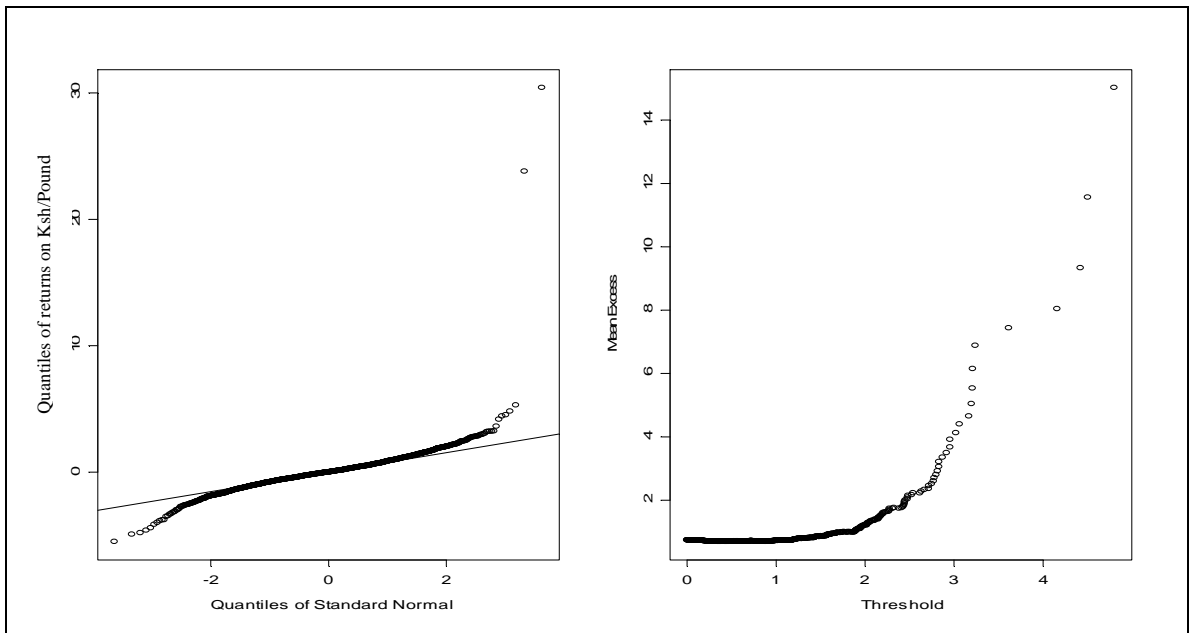
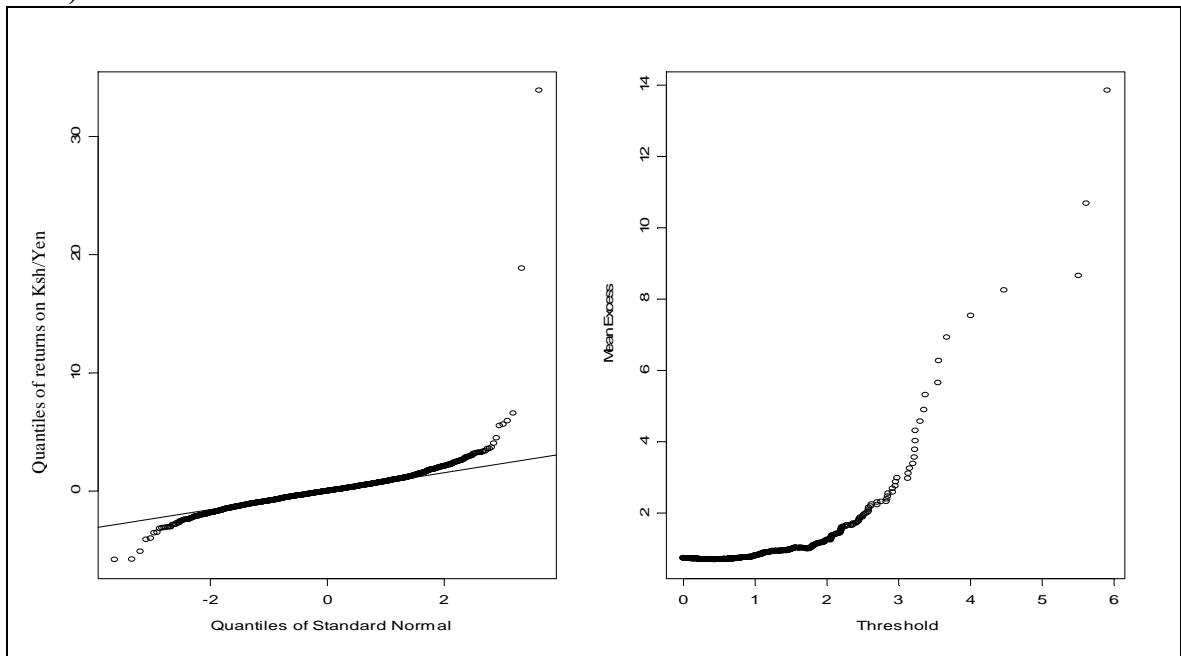


Figure 5: QQ-plot (left panel) and mean excess plot (right panel) for standardised residuals for exchange rate returns of Ksh/USD (top) and Ksh/Pound (bottom)

c) Ksh/ Yen



d) Ksh/Euro

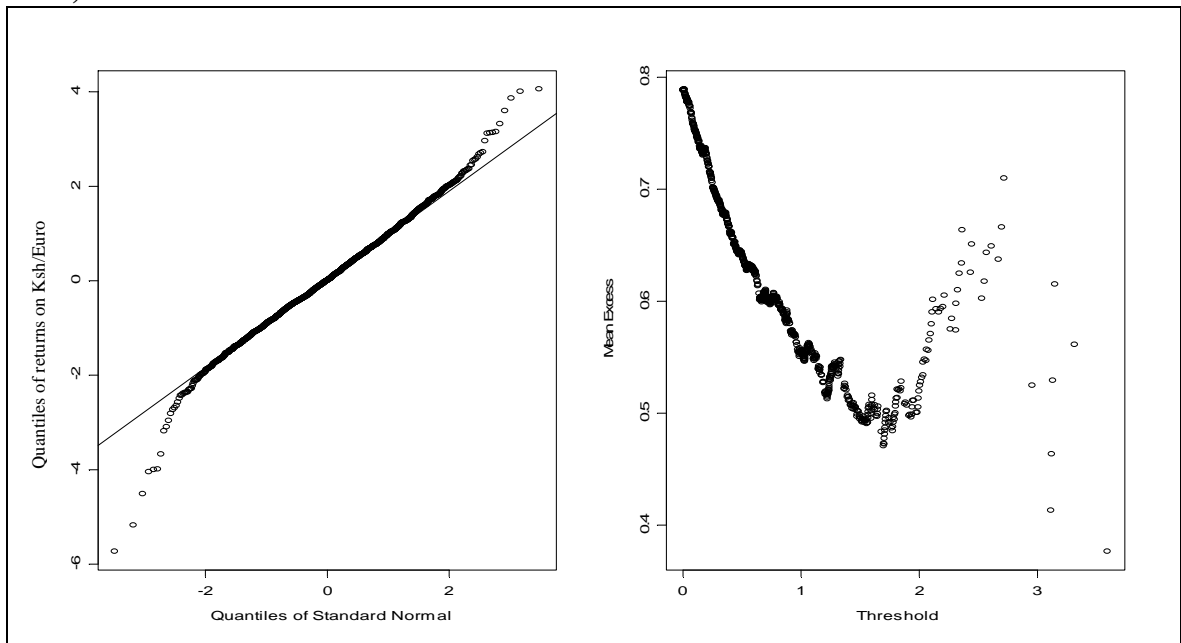


Figure 6: QQ-plot (left panel) and mean excess plot (right panel) for standardised residuals for exchange rate returns of Ksh/Yen (top) and Ksh/Euro (bottom)

5.3.2 Estimated GPD Models

As shown in Table 5, the fit of the GPD model to excesses of the standardized residuals exceeding the respective thresholds for the Ksh to USD, Pound and Yen was highly significant. The estimates for ξ and β were statistically significant at 5 percent significance level. Furthermore, the shape estimates indicate heavy tailed distributions with finite mean and variance. The GPD model was also fitted to the excesses of standardized residuals exceeding a threshold of 1.8 for the Ksh against the Euro. Although the estimated shape parameter is not statistically significant at 5 percent significance level, the estimate of the scale parameter β is highly significant. This implies that the distribution of the standardized residuals for the Ksh to Euro has a medium sized tail.

<i>Variable</i>	<i>Threshold (u)</i>	<i>No. of exceedances of u</i>	$\hat{\xi}$	$\hat{\beta}$
Ksh/ USD	0.5	576	0.3025006 (0.04912275)	0.5946626 (0.03756408)
Ksh/ Pound	1.0	431	0.2112642 (0.04435883)	0.5256734 (0.03400394)
Ksh/ Yen	1.0	395	0.266724 (0.05622015)	0.5468862 (0.04060328)
Ksh/ Euro	1.8	72	0.1069013 (0.1518836)	0.4428747 (0.08495671)

Standard errors for the respective coefficient estimates are in parenthesis

Table 5: Estimated GPD models for standardised residuals of exchange rate returns

The estimates of the shape parameters over various threshold levels with their 95 percent confidence levels are shown in Figures 7 and 8 (left panels). The plots show that the shape estimates are all greater than zero, and quite stable in the ranges [0.3,0.49] for returns of the Ksh to USD, and [0.2,0.4] each for returns of the Ksh to Pound and Yen. The shape

plot for the returns of the Kshs to Euro over various thresholds shows that the estimate is less than zero over most of the plot. The respective GPD models fitted the data quite well in the tails of the distribution - Figures 7 and 8 (right panels). However, the GPD fit for returns of the USD, Pound and Yen do not capture two extreme observations at the end of the plots which suggests that the occurrence of these two events may not have been manifested in the data.

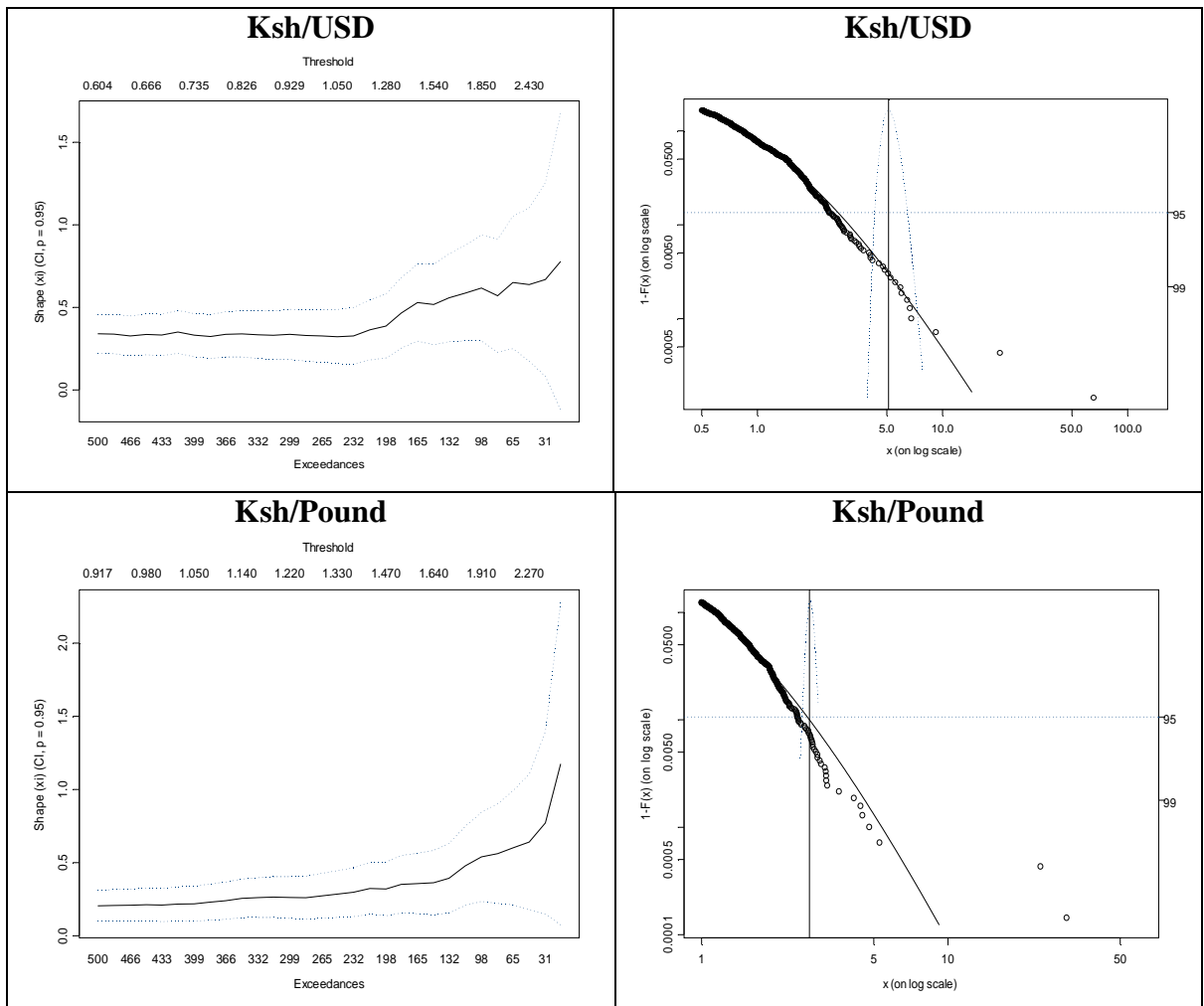


Figure 7: Shape (left panel) and tail (right panel) plots for standardised residuals for exchange rate returns of the Ksh/USD (top) and Ksh/Pound (bottom)

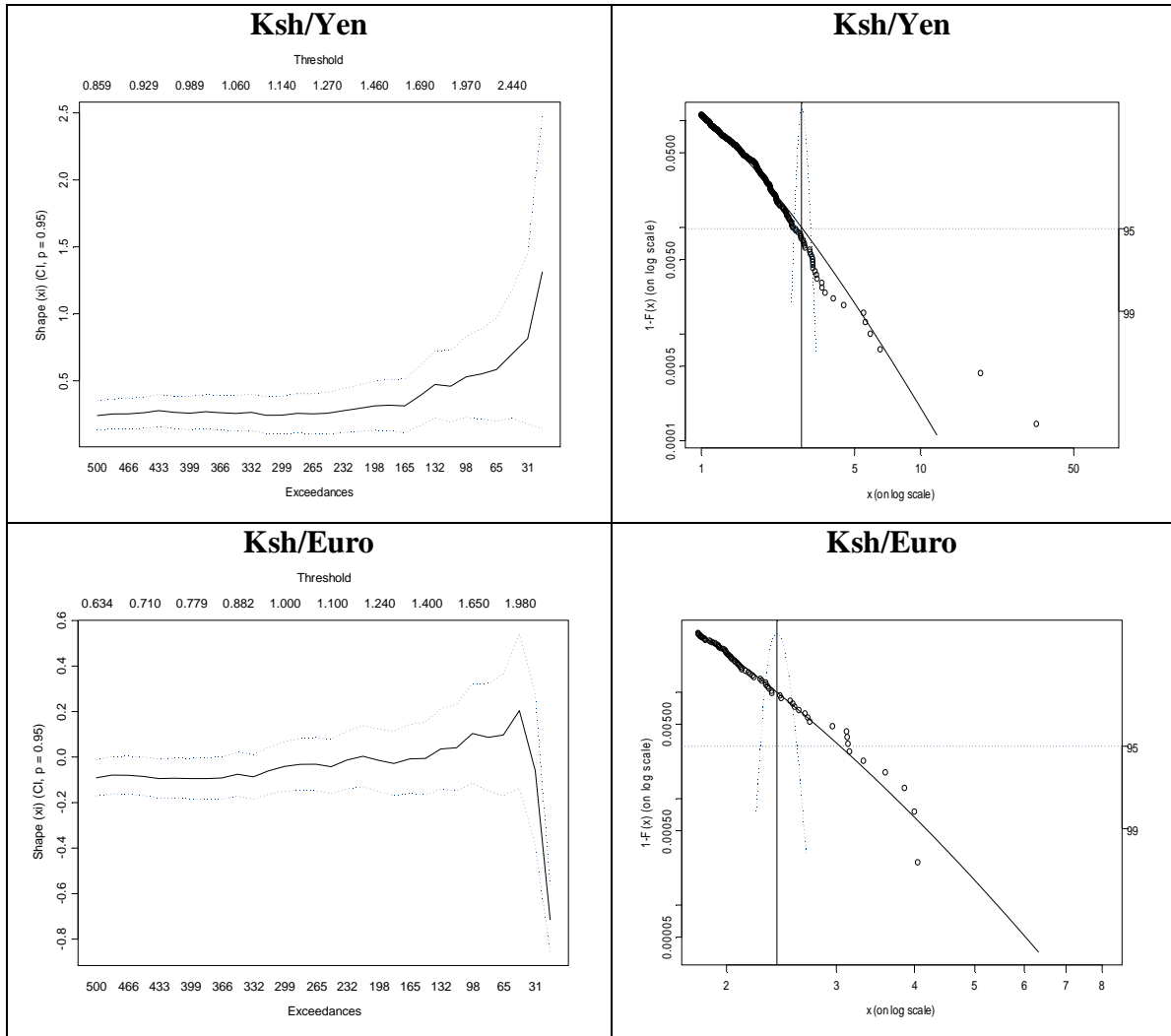


Figure 8: Shape (left panel) and tail (right panel) plots for standardised residuals for exchange rate returns of the Ksh/Yen (top) and Ksh/Euro (bottom)

5.3.3 Return Periods for Specific Extreme Exchange Rate Returns

The estimates of the 99 percent and 99.9 percent tail quantiles for the estimated GPD models in Table 5 are shown in Table 6. The occurrence of these levels of exchange rate returns is expected every 100 days (99 percent quantile) and 1000 days (99.9 percent quantile), respectively. Although, the probability of occurrence of these levels of

exchange returns is small (0.01 and 0.001, respectively), their impact on the economy can be far reaching.

Exchange rate returns for the USD on 9th March 1993, 20th April 1993 and 14th May 1995 are way above the estimated 99.9 percent quantiles for the GPD models. This indicates that exchange rate returns observed during these dates for the USD were not in the nature of the economy when they occurred. Similarly, the exchange rate returns for the Pound and Yen observed on 9th March and 20th April 2009 are much higher than the estimated 99.9 percent quantiles for the estimated GPD models. Exchange rate returns on the Euro observed on 23rd February 1999, 20th July 1999, and 25th September 2000 are also way above the estimated 99.9 percent quantile implying that the occurrence of these events was not manifested in the data.

Probability p	Estimated Quantile \hat{x}_p	Return period in Days
Ksh/USD		
0.990	3.120396	100
0.999	7.737729	1000
Ksh/Pound		
0.990	2.740681	100
0.999	5.390253	1000
Ksh/Yen		
0.990	2.862858	100
0.999	6.181653	1000
Ksh/Euro		
0.990	2.408732	100
0.999	3.734850	1000

Table 6: Estimated quantiles and return period for exchange rate returns

5.3.4 Linear Trends in the Exceedances of the Thresholds

In this section, we analyse the trends in the excesses and exceedance times in the standardised residuals from the GARCH (1, 1) model of the exchange rate returns for the USD, Pound, Yen and Euro over the respective thresholds. Linear regression models of the excesses against their order of occurrence as well as the inter-exceedance times against their order of occurrence are estimated. Let X_t be the exceedances of the threshold against the occurrence time z_t , the linear model is specified as $X_t = a + bz_t + \varepsilon_t$, where $\{\varepsilon_t\}$ is such that $E(\varepsilon_t)=0$ and $\text{cor}(\varepsilon_t, \varepsilon_s)=0$ for $t \neq s$, while a and b are constant parameters. Here b is the trend coefficient.

As shown in Table 7, the linear regressions of excess sizes for exchange rate returns for the USD, Pound and Yen against their order of occurrence show a decreasing trend in the excess sizes. The p -values of 0.019, 0.001 and 0.003 for the USD, Pound and Yen respectively indicate that the regressions are highly significant. Despite depicting a similar trend, the regression for returns on the Euro is marginally significant at 10 percent.

Variable	\hat{a}	\hat{b}	p-value of regression
Ksh/ USD	1.4336 (0.2465)	-0.0017 (0.0007)	0.01902
Ksh/ Pound	1.2171 (0.1765)	-0.0023 (0.0007)	0.00124
Ksh/ Yen	1.3008 (0.1976)	-0.0025 (0.0009)	0.00348
Ksh/ Euro	0.6670 (0.1264)	-0.0047 (0.0030)	0.12140

Standard errors for the respective coefficient estimates are in parenthesis

Table 7: Estimated linear trend models for the excesses of standardised residuals for exchange rate returns against their order of occurrence

Similarly, as shown in Table 8, the linear regressions for the inter-arrival times of the exceedances of the respective threshold against their occurrence times depict a decreasing trend for returns on the USD and Pound although the trend component is only statistically significant at around 10 percent level. However, the trend components for the Euro is positive and statistically significant at 5 percent level suggesting that inter-arrival times for the exceedances is increasing.

Variable	\hat{a}	\hat{b}	p-value of regression
Ksh/ USD	10.4660 (0.9708)	-0.0056 (0.0029)	0.05373
Ksh/ Pound	13.4854 (1.2296)	-0.0077 (0.0049)	0.1201
Ksh/ Yen	11.9688 (1.3950)	0.0045 (0.0061)	0.4573
Ksh/ Euro	16.9448 (10.0554)	0.6145 (0.2394)	0.0124

Standard errors for the respective coefficient estimates are in parenthesis

Table 8: Estimated linear trend models for inter-arrival times of exceedances for the exchange rate returns

The graphical analyses to check whether the excess amounts are i.i.d. from the GPD, and whether the threshold exceedance times occur as a homogeneous Poisson process with constant intensity are shown in Figures a, b, c and d in Appendix 7 for the USD, Pound, Yen and Euro, respectively. The ADF tests for stationarity are provided in Table 9. The scatter plots of the scaled inter-arrival times Z against the order of their occurrence (right hand panel), and with superimposed Lowess (locally weighted scatter plot smoothing) curves to capture the trend, indicate no significant trend for returns on the USD and Pound. The Lowess curve is a smoothed mean value of the data and estimates the reciprocal of the

intensity of the Poisson process – Embrechts *et al.* (1997). However, there seems to be an increasing trend in the inter-arrival times of threshold exceedances for returns on the Yen and Euro. The correlograms of the Z-statistics and further tests using the ADF unit root tests revealed that the inter-arrival times of the threshold exceedances are independent. The QQ-plots of the scaled inter-arrival times reveal that these are exponentially distributed which indicates that the threshold exceedances occur as homogeneous Poisson process.

Scatter plots of the W-statistics against the occurrence time with superimposed smooth curves to capture the trend seem to suggest no significant trend for returns on both currencies (left hand panel). On the other hand, the correlograms of the W-statistics and further tests using the ADF unit root tests did not show any evidence of non-stationarity. We deduce that the excess sizes for the returns on the respective currencies are i.i.d. from the GPD. The QQ-plots of the W-statistics show that they are exponentially distributed which indicates that excesses are i.i.d. as GPD.

Variable	Z-Statistic		W-Statistic	
	ADF Statistic	P-Value	ADF Statistic	P-Value
Ksh/USD	-9.9060	0.0000	-24.7656	0.0000
Ksh/Pound	-20.6750	0.0000	-15.6744	0.0000
Ksh/Euro	-8.2856	0.0000	-9.8426	0.0000
Ksh/Yen	-18.9790	0.0000	-15.9655	0.0000

Table 9: ADF tests for the Z and W-statistics for the POT models of standardised residuals of exchange rate returns

5.4 Descriptive Statistics for the Interest Rates Data

The mean of the changes in interest rates is negative over all the rates suggesting a downward trend in the changes (Table 10). Standard deviations for the changes indicate that movements in the interbank and Treasury bill rates were more volatile than those in the repo rate. Both the kurtosis and skewness coefficients indicate that the distribution of the changes in interest rates is non-normal. The skewness coefficients indicate that increases in the Treasury bill rate and repo rates occur more often while the reverse is true for the interbank rate. Kurtosis coefficients and Jarque-Bera tests of changes in both interest rates are suggest that the underlying distributions are leptokurtic.

	Treasury Bill Rate	Repo Rate	Interbank Rate
Mean	-0.013	-0.013	-0.004
Median	-0.008	0.000	0.005
Maximum	13.188	17.300	13.768
Minimum	-6.397	-6.620	-22.996
Std. Dev.	1.224	0.863	1.825
Skewness	2.602	5.219	-2.183
Kurtosis	39.473	135.637	34.362
Jarque-Bera test	47111.79	1017100.00	119018.60
Probability	0.000	0.000	0.000
Observations	833	1379	2849

Table 10: Descriptive statistics for daily changes in the interbank, repo and Treasury bill interest rates

As shown in the correlograms in Figure 9, both the repo, Treasury bill and interbank interest rates data are non-stationary in first differences. Similarly, the correlograms of squared changes in the interest rates indicate presence of serial correlation which is an indication of GARCH type of heteroscedasticity.

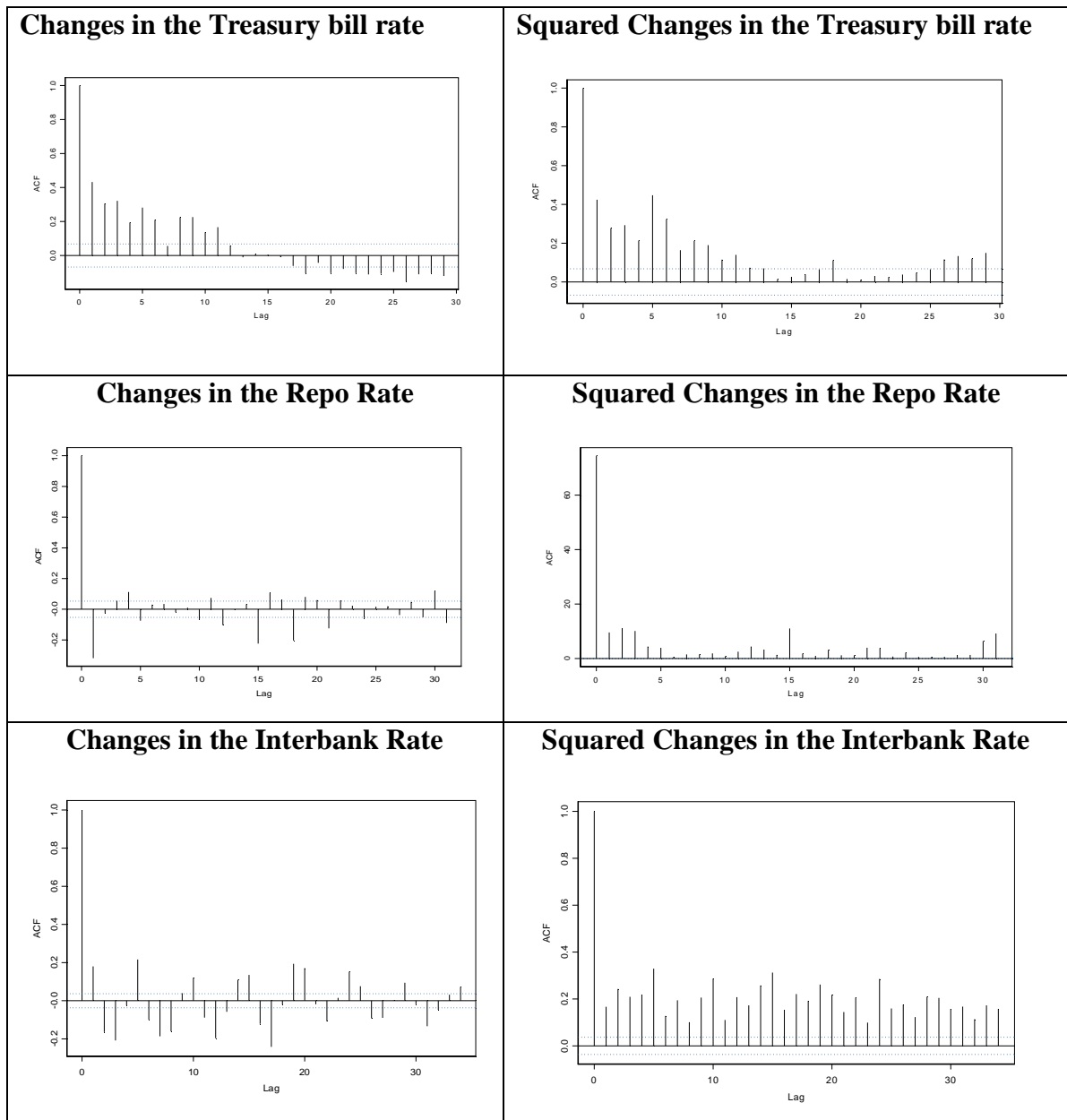


Figure 9: Correlograms of changes in the interbank, repo and Treasury bill interest rates

5.5 Estimated Volatility Models for Changes in Interest Rates

As shown in Table 11, the estimated GARCH (1, 1) models for changes in interest rates are highly significant at 5 percent significance level. The estimated α_1 and β_1 parameters are

positive for all interest rates while their sum is not significantly different from unity for the repo and interbank rate. We deduce that the GARCH models for the changes in the repo and interbank rates are reasonably specified.

The QML estimates (corresponding to the estimated GED parameters) of the changes in interest rates are highly significant and correspond to distributions with heavier tails than the normal distribution. The GARCH (1, 1) models were re-estimated for sample periods 2000-2006 and 2003-2006 to examine the stability of the parameter estimates, and as robustness tests. Parameter estimates for all models do not change significantly using different samples. The sum of the parameters is almost unity for the shorter samples for changes in the Treasury bill rate. A higher estimate for the ARCH coefficient α_1 indicates that volatility tends to be extreme and also responds significantly to market movements.

	Sample	γ	Θ	α_0	α_1	β_1	GED Parameter	AIC
Interbank	1993 – 2006	0.000562 (2.72107)	6.304938 (5.01696)	0.000198 (5.66909)	0.784122 (12.2256)	0.606650 (39.2302)	0.784299 (48.6415)	0.967286
	2000 – 2006	0.000586 (3.18514)	6.083608 (5.92745)	0.000274 (5.60170)	0.940444 (9.73832)	0.526410 (21.9364)	0.782447 (37.82696)	-1.044411
	2003 – 2006	0.000742 (5.20487)	6.311263 (8.82878)	0.000463 (4.75731)	1.067049 (6.13369)	0.424972 (8.66409)	0.677236 (27.6580)	-2.752362
	1993 – 2006	0.001066 (3.7460)	7.199722 (65.5314)	0.000147 (3.48023)	0.782164 (6.43557)	0.620847 (21.4948)	0.610246 (32.3768)	-1.477046
	2000 – 2006	0.001971 (7.26286)	7.199495 (86.6524)	0.000150 (3.62556)	0.740833 (6.29683)	0.617314 (20.2560)	0.629712 (32.5551)	-1.972195
	2003 – 2006	0.001157 (4.15478)	7.199845 (116.062)	0.000231 (3.67658)	0.884928 (4.36315)	0.567540 (12.2333)	0.579213 (27.2535)	-2.888226
Treasury bill rate	1993 – 2006	0.002839 (5.29185)	7.210240 (4.47855)	0.003015 (3.02135)	1.250885 (9.40432)	0.391674 (14.1737)	0.927503 (23.1607)	0.854222
	2000 – 2006	0.002621 (2.06092)	8.413389 (3.96867)	0.003488 (4.91015)	1.298204 (6.42517)	0.097573 (2.22709)	1.530706 (10.1500)	-0.621521
	2003 – 2006	0.004101 (1.83899)	8.117797 (3.66641)	0.003862 (4.26507)	1.161863 (4.39665)	0.063748 (1.34599)	1.486378 (7.25804)	-1.126126

Note: t-statistics are in parentheses. AIC indicates Akaike Information Criterion.

Table 11: GARCH (1, 1) models for changes in the interbank, repo and Treasury bill interest rates

As shown in Table 12, the kurtosis and skewness coefficients indicate that the distributions of the standardised residuals from the GARCH models are non-normal. The Jarque-Bera test for normality indicates that the distribution of the residuals have tails which are heavier than that of the normal distribution.

	<i>Interbank</i>	<i>REPO</i>	<i>Treasury bill</i>
Mean	-0.0009	-0.0467	0.0047
Median	0.0343	0.0020	0.0022
Maximum	21.5781	8.6805	9.0587
Minimum	-16.7052	-22.1683	-6.6345
Std. Dev.	1.2070	1.2746	1.0797
Skewness	1.9085	-6.4664	1.2642
Kurtosis	93.1180	107.8749	17.9421
Jarque-Bera test	965790.0	641580.40	7971.08
Probability	0.0000	0.0000	0.0000
Observations	2849	1379	833

Table 12: Descriptive statistics for standardised residuals from the GARCH models for changes in the interest rates

The volatility processes and standardised residuals from the GARCH (1, 1) models for changes in the interbank, repo and Treasury bill rates are shown in Figure 10. Movements in interest rates were less volatile after the year 2000 compared to that in the 1990s. The period 2000 to 2006 was characterised by improved liquidity conditions coupled with implementation of sound monetary and fiscal policies by the government.

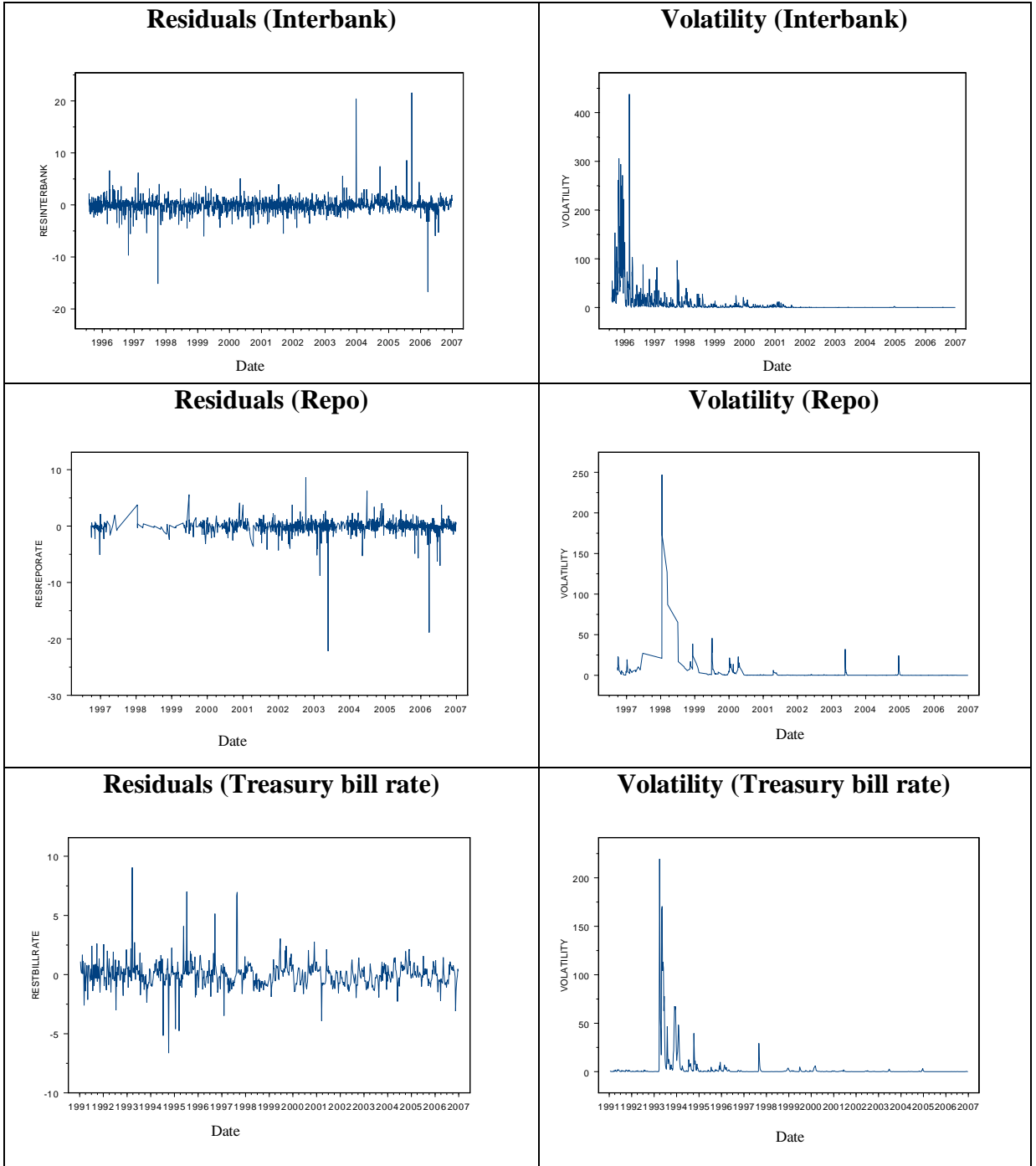


Figure 10: Standardised residuals and volatility process for changes in the interest rates

Table 13 shows the probability values for the Ljung-Box Q-statistics of the standardised residuals from the GARCH (1, 1) models at selected lag lengths. The tests suggest significant serial correlation at 5 percent level. However, the probability values for Ljung-Box Q-statistics for the squared residuals (Table 14) do not suggest any significant conditional heteroscedasticity at 5 percent PAC level for all exchange rate returns. Further tests for stationarity using the ADF unit root tests however revealed that the standardised residuals from both interest rates are stationary (Table 15).

Lag	Interbank				Repo				Treasury bill rate			
	AC	PAC	Q-Stat	Prob	AC	PAC	Q-Stat	Prob	AC	PAC	Q-Stat	Prob
1	0.1820	0.1820	94.9870	0.0000	-0.0770	-0.0770	8.1566	0.0040	0.3350	0.3350	93.9860	0.0000
6	-0.0340	-0.0430	111.5000	0.0000	-0.0020	0.0100	19.5590	0.0030	0.0390	-0.0200	168.6500	0.0000
12	-0.0330	-0.0270	126.0400	0.0000	-0.0190	-0.0190	35.2480	0.0000	0.0680	0.0800	178.0100	0.0000
18	-0.0120	0.0070	142.9700	0.0000	-0.0110	-0.0150	38.6630	0.0030	-0.0220	-0.0100	194.4300	0.0000
24	0.0270	0.0180	175.9800	0.0000	0.0020	0.0010	40.3110	0.0200	0.0280	0.0220	199.8900	0.0000
28	-0.0140	-0.0080	179.3400	0.0000	-0.0210	-0.0210	41.2040	0.0510	-0.0550	-0.0700	202.7600	0.0000
36	-0.0140	-0.0170	187.0300	0.0000	0.0050	0.0130	44.3860	0.1590	-0.0500	-0.0300	218.6300	0.0000

Table 13: Tests for Serial Correlation in the Standardised Residuals for Changes in Interest rates

Lag	Interbank				Repo				Treasury bill rate			
	AC	PAC	Q-Stat	Prob	AC	PAC	Q-Stat	Prob	AC	PAC	Q-Stat	Prob
1	-0.0040	-0.0040	0.0538	0.8170	-0.0050	-0.0050	0.0322	0.8580	0.1620	0.1620	21.9580	0.0000
6	-0.0050	-0.0050	0.6756	0.9950	-0.0050	-0.0060	0.2566	1.0000	-0.0210	-0.0260	23.8760	0.0010
12	-0.0060	-0.0070	1.7107	1.0000	-0.0050	-0.0040	0.7926	1.0000	0.0510	0.0570	27.0560	0.0080
18	-0.0050	-0.0050	2.0464	1.0000	0.0030	0.0030	0.8925	1.0000	0.0200	-0.0020	31.6410	0.0240
24	-0.0050	-0.0060	2.3028	1.0000	-0.0030	-0.0030	1.1236	1.0000	-0.0300	-0.0470	33.7990	0.0880
30	-0.0060	-0.0060	2.7030	1.0000	-0.0040	-0.0040	1.2176	1.0000	-0.0150	-0.0320	35.9060	0.2110
36	0.0000	0.0000	2.9486	1.0000	-0.0030	-0.0030	1.3418	1.0000	0.0300	0.0220	39.6310	0.3110

Table 14: Tests for ARCH in the standardised residuals for changes in interest rates

Variable	ADF Statistic	P-Value
Interbank rate	-44.3544	0.0001
Repo rate	-40.0181	0.0000
Treasury bill rate	-15.14073	0.0000

Table 15: ADF tests for standardised residuals from GARCH models of changes in interest rates

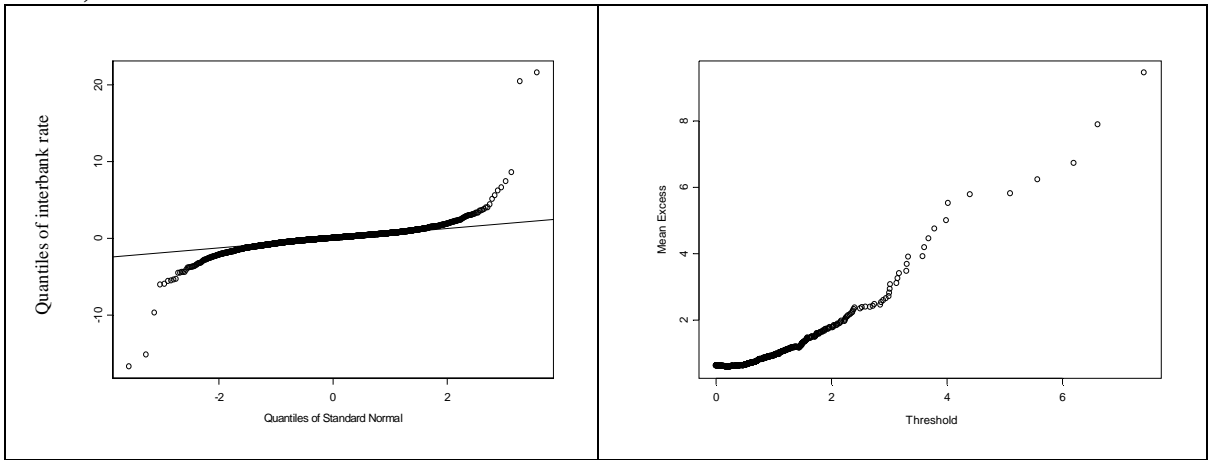
5.6 Extreme Value Models for the Changes in Interest Rates

We apply extreme value theory to model the tail of the standardized residuals from the GARCH (1, 1) models for changes in interest rates. We also determine if there is an increasing trend in the extremes.

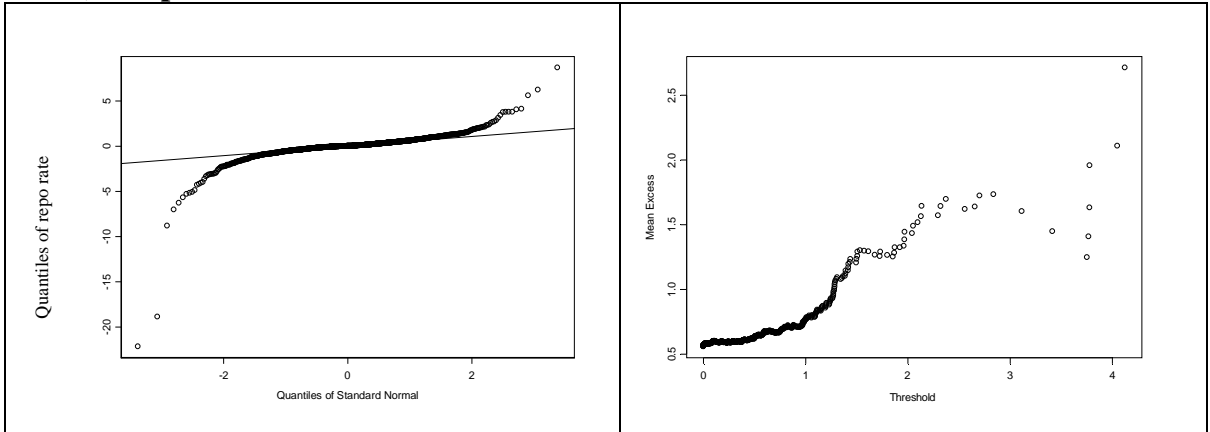
5.6.1 Data Exploration

The QQ-plots of the standardised residuals against the normal distribution in Figure 11 (Left panel) shows a concave relationship between the quantiles of the empirical and the above distributions, which suggests that the distributions of the standardised residuals of the changes in the interbank, repo and Treasury bill rates are fat tailed. The sample points start deviating from linear behaviour and form a concave at around 0.6 for the interbank rate, 0.2 for the repo rate and 0.4 for the Treasury bill rate residuals. Similarly, the sample mean excess plots for the residuals are approximately linear and positively sloped after the above thresholds which indicate heavy tailed behaviour. The distributions of excesses of the standardised residuals above these thresholds can therefore be modelled by the GPD for the respective residuals.

a) Interbank rate



b) Repo rate



c) Treasury bill rate

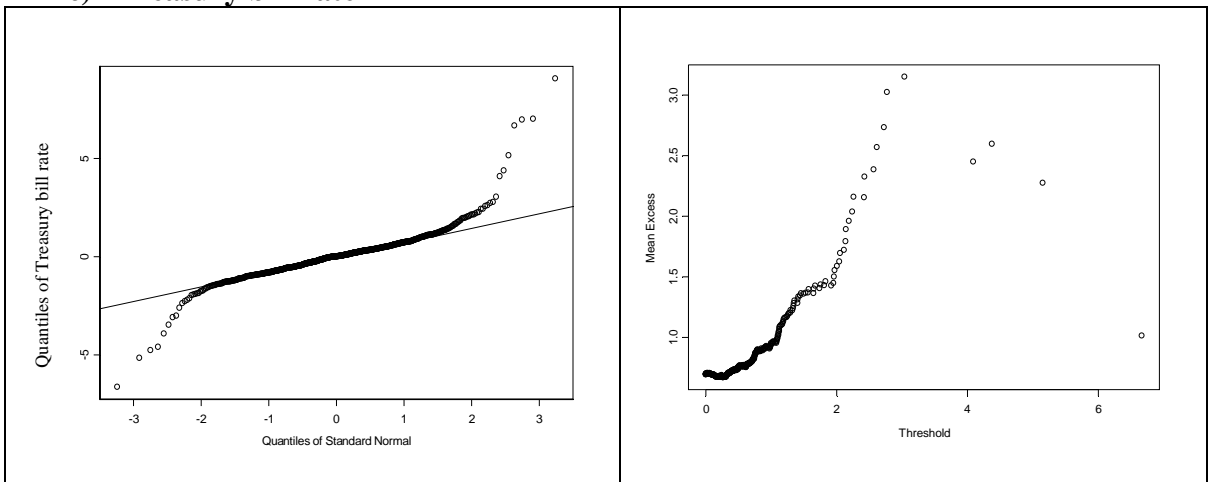


Figure 11: QQ-plot (left panel) and mean excess (right panel) plots for standardised residuals of changes in interbank, repo and Treasury bill interest rates

5.6.2 Estimated GPD Models

The fit of the GPD model to excesses of the standardized residuals exceeding the respective thresholds for the changes in the interbank, repo and Treasury bill rates was highly significant (Table 13). The estimates of the shape and scale parameters ξ and β were statistically significant at 5 percent significance level. Furthermore, the shape estimates indicate heavy tailed distributions with finite mean and variance.

Variable	Threshold (u)	No. of exceedances of u	$\hat{\xi}$	$\hat{\beta}$
Interbank rate	0.6	512	0.3557342 (0.05682667)	0.4409546 (0.03097759)
Repo rate	0.2	475	0.1732691 (0.04782173)	0.4838149 (0.0318326)
Treasury bill rate	0.4	226	0.2815446 (0.07815889)	0.5127191 (0.05173843)

Standard errors for the respective coefficient estimates are in parenthesis

Table 16: Estimated GPD model for the changes in standardised residuals of changes in interest rates

Figures 12 (left panel) show the estimates of the shape parameters over various threshold levels with their 95 percent confidence levels. The shape estimates are all greater than zero, and quite stable in the ranges [0.25,0.5] for residuals of the interbank rate, and [0.1,0.3] each for residuals of the repo and Treasury bill rates. As shown in Figures 12 (right panel), the respective GPD models fitted the data reasonably well in the tails of the distribution. However, the GPD fit for residuals of the interbank rate do not capture two extreme observations at the end of the plots which suggests that the occurrence of these two events may not have been manifested in the data.

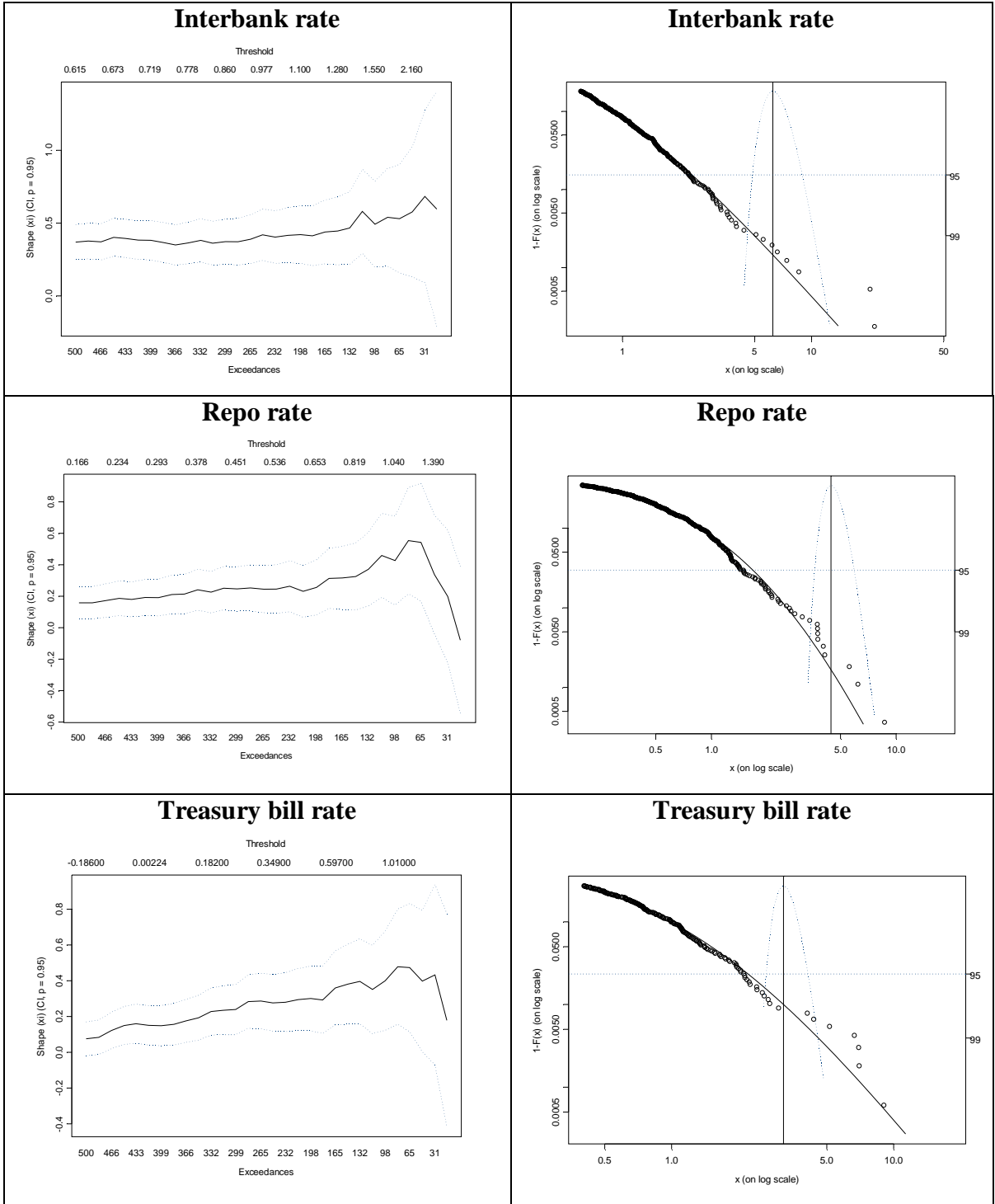


Figure 12: Shape plots (Left Panel) and tail estimates (Right Panel) for standardised residuals of changes in interest rates

5.6.3 Return Periods for Specific Extreme Changes in Interest Rates

The estimates of the 99 percent and 99.9 percent tail quantiles for the estimated GPD models are shown in Table 17. The 99 percent quantiles for the changes in the interbank, repo, and Treasury bill rate, respectively, are expected to occur once in 100 days. Similarly, the 99.9 percent quantiles for the interbank, repo and Treasury bill rates respectively are expected to occur once every 1000 days. These are extreme changes in interest rates which would require the CBK to have adequate instruments for stabilising the rates.

Probability p	Estimated Quantile \hat{x}_p	Return period in Days
Interbank rate		
0.990	2.824330	100
0.999	7.218193	1000
Repo rate		
0.990	2.563515	100
0.999	5.091317	1000
Treasury bill rate		
0.990	3.191192	100
0.999	7.398747	1000

Table 17: Estimated quantiles and return periods for changes in interest rates

5.6.4 Linear Trends in the Exceedances of the Thresholds

The linear regressions (based on the linear regression model in section 5.3.4) of excess sizes for changes in interest rates against their order of occurrence show a decreasing trend in the excess sizes for the repo and Treasury bill rates, and a non-significant trend for the interbank rate (Table 18). The p -values of 0.09 and 0.04 for the repo and Treasury bill rate,

respectively, indicate that the regressions are significant at 10 percent and 5 percent significance levels, respectively.

Variable	\hat{a}	\hat{b}	p-value of regression
Interbank rate	0.5525 (0.1342)	0.0006 (0.0005)	0.1783
Repo rate	0.6939 (0.0718)	-0.0004 (0.0003)	0.09249
Treasury bill rate	0.9820 (0.1444)	-0.0023 (0.0011)	0.03783

Standard errors for the respective coefficient estimates are in parenthesis

Table 18: Estimated linear trend models for the excesses for changes in interest rates

The linear regressions for the inter-arrival times of the exceedances of the respective threshold against their occurrence times suggest no significant trend for the interbank and Treasury bill rates (Table 19). This is an indication of stability in the interest rates. However, the trend component for the repo rate is negative and statistically significant at 5 percent level suggesting that inter-arrival times for the exceedances are decreasing.

Variable	\hat{a}	\hat{b}	p-value of regression
Interbank rate	7.3717 (0.9033)	0.0029 (0.0030)	0.3480
Repo rate	15.8032 (1.6772)	-0.0333 (0.0061)	0.0000
Treasury bill rate	20.4455 (5.5358)	0.0467 (0.0423)	0.2708

Standard errors for the respective coefficient estimates are in parenthesis

Table 19: Estimated linear trend models for inter-arrival times of exceedances for changes in interest rates

Figures a, b, and c in Appendix 8 depict diagnostic checks on whether the excess amounts for changes in interbank, repo and Treasury bill rates are i.i.d. from the GPD. The plots also indicate whether the threshold exceedance times occur as a homogeneous Poisson process with constant intensity. The scatter plots of the scaled inter-arrival times Z against the order of their occurrence (right hand panel), and with superimposed Lowess smoothed curves to capture the trend, indicate no significant trend for the changes in both interest rates. The correlograms of the Z -statistics and further tests using the ADF unit root tests on changes in the interbank rate revealed that the inter-arrival times of the threshold exceedances are independent (Table 20). The QQ-plots of the scaled inter-arrival times reveal that these are exponentially distributed for the changes in the interbank rate which indicates that the threshold exceedances occur as homogeneous Poisson process. However, these assumptions hold only marginally for the changes in the repo and Treasury bill rates. Scatter plots of the W -statistics against the occurrence time with superimposed smooth curves to capture the trend seem to suggest no significant trend for changes in both interest rates (left hand panel). The correlograms of the W -statistics and further tests using the ADF unit root tests did not show any evidence of non-stationarity. We deduce that the excess sizes for the returns on the respective currencies are i.i.d. from the GPD.

Variable	Z-Statistic		W-Statistic	
	ADF Statistic	P-Value	ADF Statistic	P-Value
Interbank rate	-24.4526	0.0000	-22.6491	0.0000
Repo rate	-14.5966	0.0000	-23.1551	0.0000
Treasury bill rate	-15.1480	0.0000	-12.8701	0.0000

Table 20: ADF tests for the Z and W -statistics for the POT models of standardised residuals of changes in interest rates.

CHAPTER SIX

6.0 CONCLUSIONS AND POLICY RECOMMENDATIONS

The thesis formulates an extreme value theory framework for determining the effect of the CBK's monetary policy on the stability of interest rates and exchange rates in Kenya. Since the primary aim of the CBK is to formulate and implement monetary policy directed at achieving and maintaining price stability, this thesis provides an alternative framework for studying the effectiveness of the CBK's core responsibility. In particular, exchange rates and interest rates stability are fundamental to the country's pursuit to achieve and sustain high economic growth rates as a means to reducing poverty levels through promoting investment and employment creation.

The thesis uses the POT approach to derive a model for extremes in interest rates in the period 1991 to 2006, and exchange rates data in Kenya in the period 1993 to 2006. The thesis also employs the GARCH model to characterise and isolate the volatility clustering in the data. The fitted POT models were used to determine the return period of specific extreme interest rates and exchange rates. The information is vital to the CBK to enable it formulate long term monetary policy.

The results revealed three key results for exchange rate returns on the USD, Pound, Yen, and Euro in the study period. First, exchange rate returns for these currencies are

leptokurtic and slightly positively skewed, and that the Ksh to USD, Pound, Yen, and Euro which exceed the thresholds 0.5, 1.0, 1.0, and 1.8, respectively, are i.i.d. from the GPD. The GARCH (1, 1) and EGARCH (1, 1) models were applied in estimating volatility of exchange rates data and found to characterise the volatility process well. These results suggest that exchange rate depreciation was preferred during the period, probably to ensure that Kenya's exports remained competitive. The benefits of maintaining a weaker shilling during the period could have outweighed the implications on the import bill, external debt service, and possible inflationary effects on the economy. In this regard, there is need for the CBK to develop an indicative exchange rate threshold beyond which depreciations would be considered extreme, hence the need for intervention. The threshold would also be important in providing room for shocks on the exchange rate. Second, once every 100 days, we expect to observe exchange rate returns of 3.12, 2.74, 2.86 and 2.41 on the USD, Pound, yen and Euro, respectively. However, the results show that once every 4 years (1000 days), we expect to observe exchange rate returns of 7.74, 5.39, 6.18 and 3.73 in the USD, Pound, Yen, and Euro respectively.

These extreme exchange rate returns relate to extreme depreciations in the above currencies and would therefore require the CBK to have adequate foreign exchange reserves to intervene through injections of appropriate amounts of foreign currencies to stabilise the exchange rate of these currencies. The CBK should however ensure that the desired levels of exchange rates do not come at the expense of maintaining price stability or significant drawdown of foreign exchange reserves. Given that the foreign exchange

reserves of the Central are low and therefore prone to depletion below the statutory level of 4 months of imports during periods of shocks on the economy, the government should implement policies that will promote production of goods and services for export. This will facilitate a sustainable accumulation of foreign exchange reserves which will guarantee success of foreign exchange interventions when required.

The results also showed a decreasing trend in the excess sizes of the exchange rate returns during the period. However, the inter-arrival times of the exchange rate returns exceeding the respective thresholds have generally decreased for the USD and Pound, but increased for the Yen and Euro. This implies that the extremes in the exchange rate returns on the USD and Pound have become more frequent while the reverse is true for returns on the Yen and Euro. This is an indication that the exchange rates for the Yen and Euro were generally more stable than the USD and Pound during the period.

Analyses of changes in interest rates showed that the volatility in the rates could be modelled by the GARCH (1, 1). The analyses of the interest rates data revealed that changes exceeding the thresholds of 0.6, 0.2 and 0.4 percent for the interbank rate, repo rate and Treasury bill rate, respectively, are i.i.d. from the GPD. The role of the CBK would therefore be to implement monetary policy in such a way as to stabilise changes in interbank, repo and Treasury bill rates between zero and the respective thresholds. As interest rates in Kenya are liberalised, these thresholds would act as a guide to the CBK on how much interest rates could change without significantly affecting the growth in private

sector credit. The CBK should therefore ensure that there are adequate and effective monetary policy implementation instruments to ensure that interest rate movements are maintained within the thresholds identified in this thesis.

The return period of specific large changes in interest rates was also analysed. The results indicate that once every 100 days, we expect to observe changes of 2.82, 2.56, and 3.19 percent on the interbank, repo, and Treasury bill rate respectively. However, once every 4 years (1000 days), we expect to observe changes of 7.21, 5.09, and 7.39 percent for the interbank, repo and Treasury bill rates respectively. These extreme movements in interest rates relate to tight liquidity conditions in the market which would require the CBK to have adequate instruments to facilitate injection of liquidity to stabilise the interest rates. The results showed a decreasing trend in the excess sizes of the changes in the repo and Treasury bill rates and no significant trend in excess sizes for changes in the interbank rate. However, the inter-arrival times of the interest rate changes exceeding the respective thresholds have generally decreased for the repo rate, but remained stable for changes in the interbank and Treasury bill rates.

Overall, the results indicate that the amplitude of the volatility in the interest rates and exchange rates analysed as well as extreme movements in the data decreased significantly in the study period. The main conclusion is that the CBK achieved its primary objective of ensuring price stability during the study period.

AREAS OF FURTHER WORK

There are three areas of possible further work resulting from this thesis. The POT models estimated could be improved by considering time varying parameters for the shape and scale parameters. Appropriate computer programmes could be developed to facilitate implementation of the frameworks for choosing the appropriate threshold using and the framework proposed by Gonzalo and Olmo (2004) which uses the weighted Pickands distance with the KS statistic, and the Hill plot.

The exact levels of interventions and level of foreign exchange reserves which would ensure that interest rates and exchange rate movements are maintained within the identified thresholds in this thesis could also be established through different studies.

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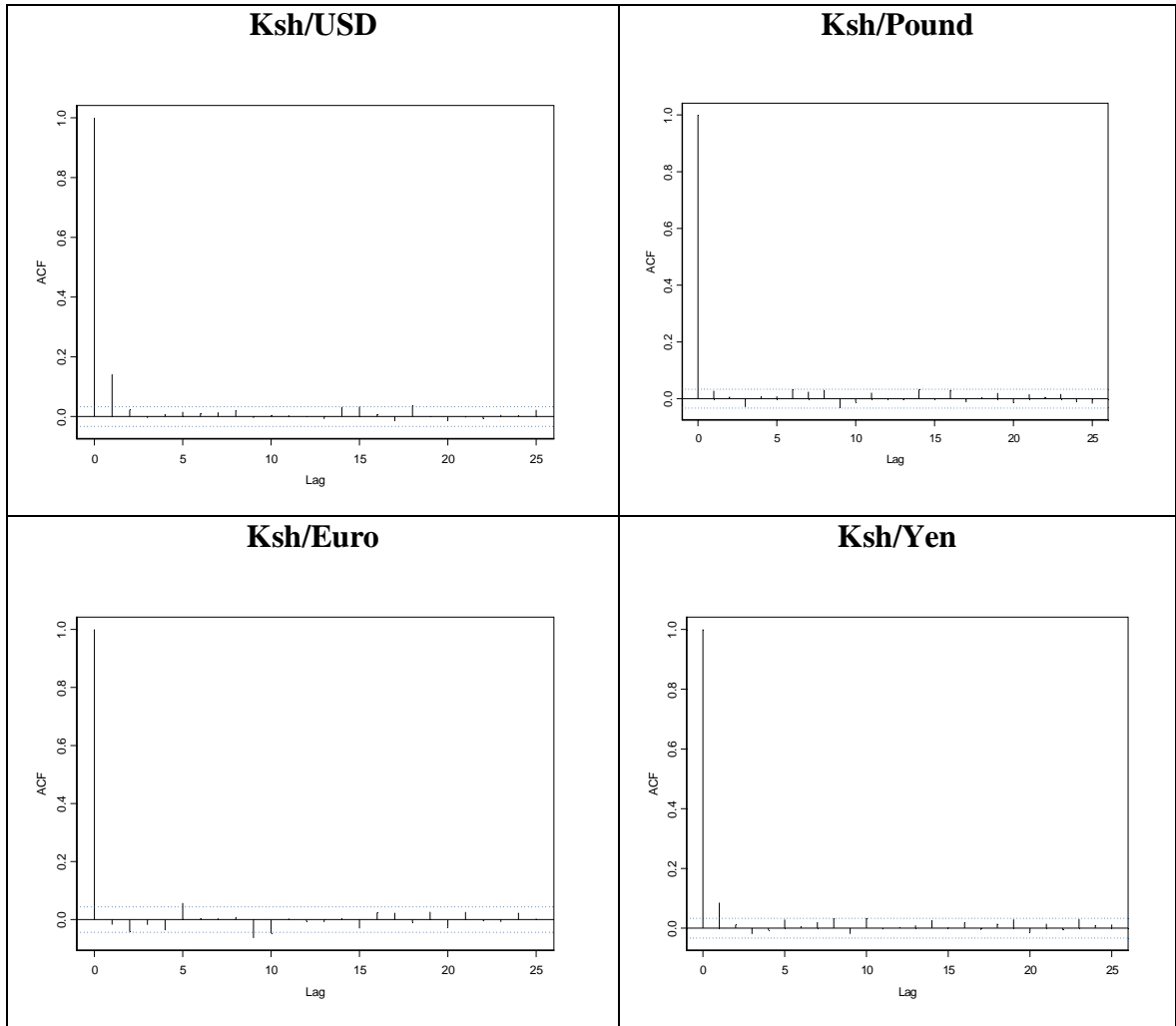
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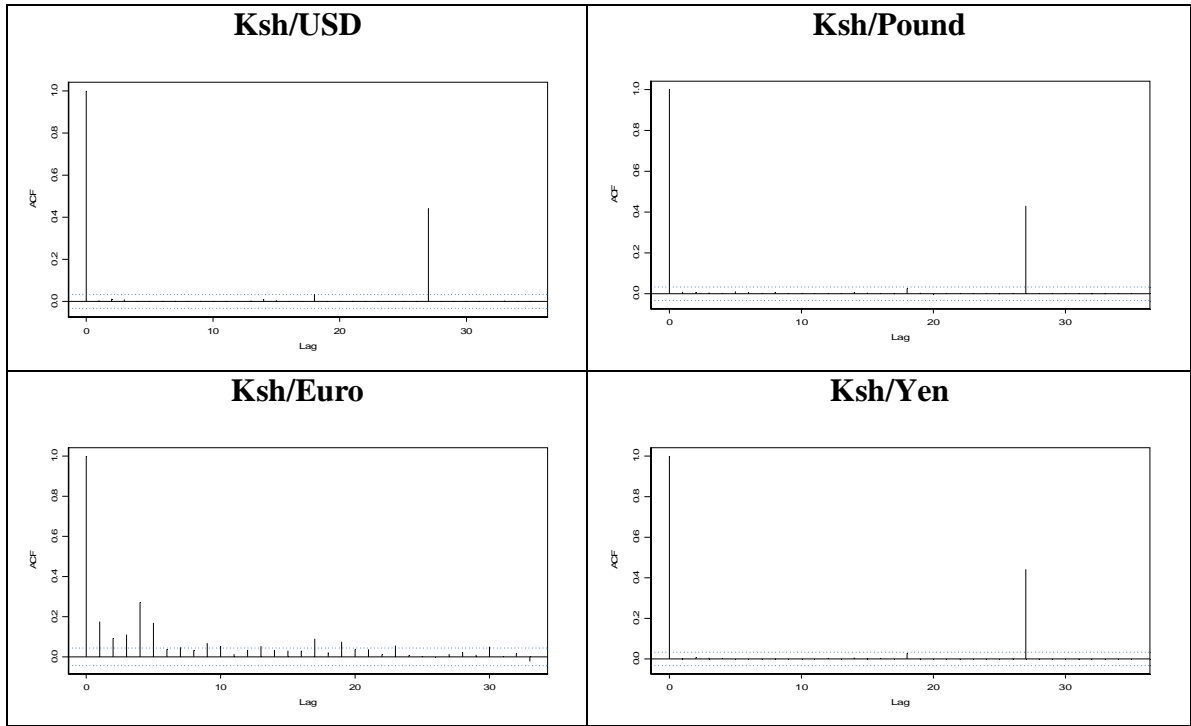
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APPENDICES



Appendix 1: Correlograms of exchange rate returns



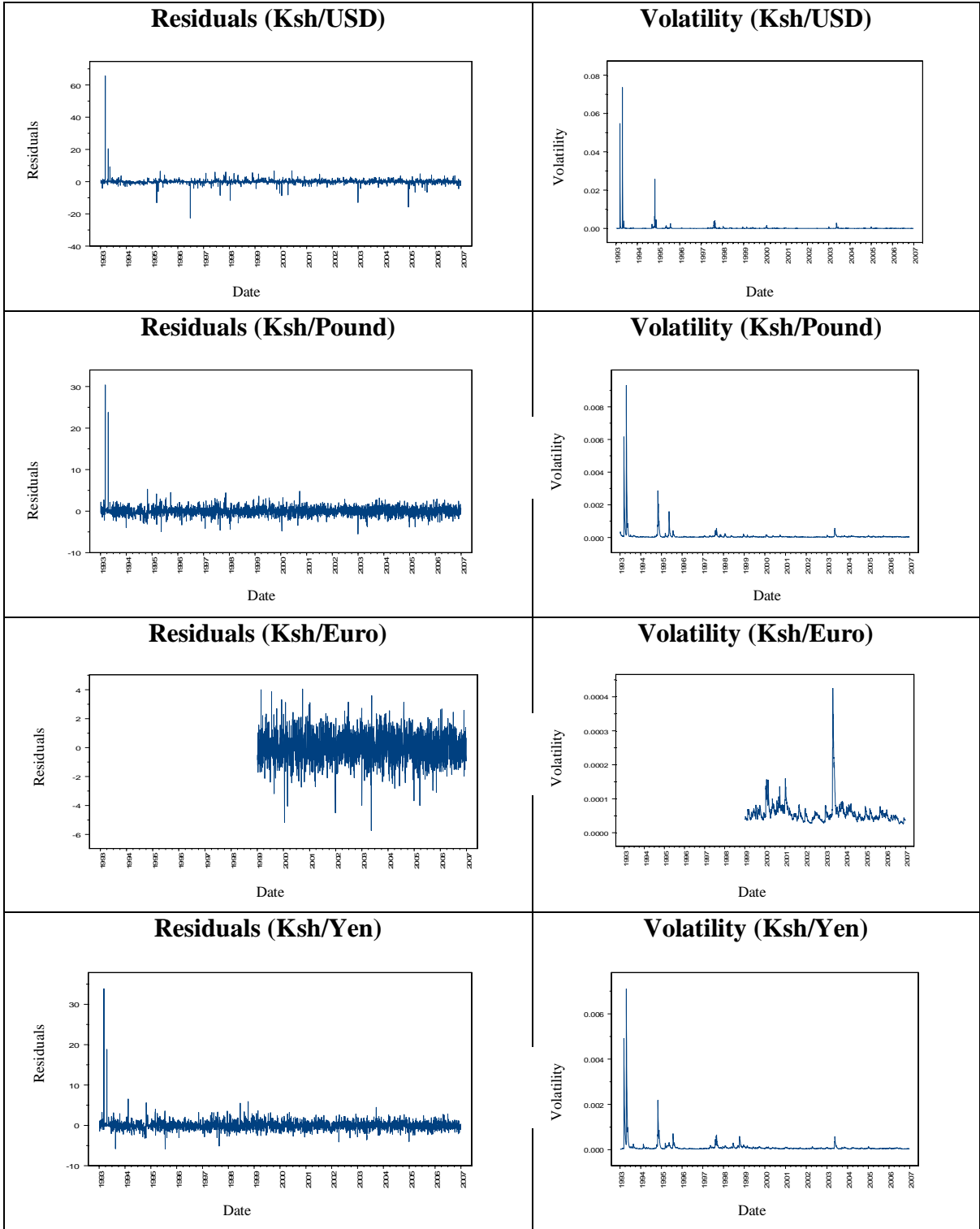
Appendix 2: Correlograms of squared exchange rate returns

Lag	Ksh/USD				Ksh/Euro				Ksh/Pound				Ksh/Yen			
	AC	PAC	Q-Stat	Prob	AC	PAC	Q-Stat	Prob	AC	PAC	Q-Stat	Prob	AC	PAC	Q-Stat	Prob
4	0.016	0.014	19.485	0.001	-0.011	-0.012	3.567	0.468	0.020	0.020	6.323	0.176	0.015	0.015	12.907	0.012
8	0.012	0.008	23.692	0.003	-0.002	-0.001	8.719	0.367	0.001	0.001	12.098	0.147	0.013	0.010	18.470	0.018
12	0.023	0.022	26.474	0.009	0.015	0.008	20.062	0.066	0.008	0.006	15.474	0.217	0.024	0.021	23.528	0.024
16	0.014	0.010	29.038	0.024	0.025	0.026	24.295	0.083	0.027	0.026	20.114	0.215	0.028	0.028	27.486	0.036
20	0.008	0.004	33.615	0.029	-0.012	-0.011	27.528	0.121	-0.001	0.000	21.230	0.384	0.001	-0.003	32.215	0.041
24	0.011	0.006	36.026	0.055	0.028	0.023	29.606	0.198	0.000	-0.001	22.916	0.525	0.010	0.002	37.988	0.035

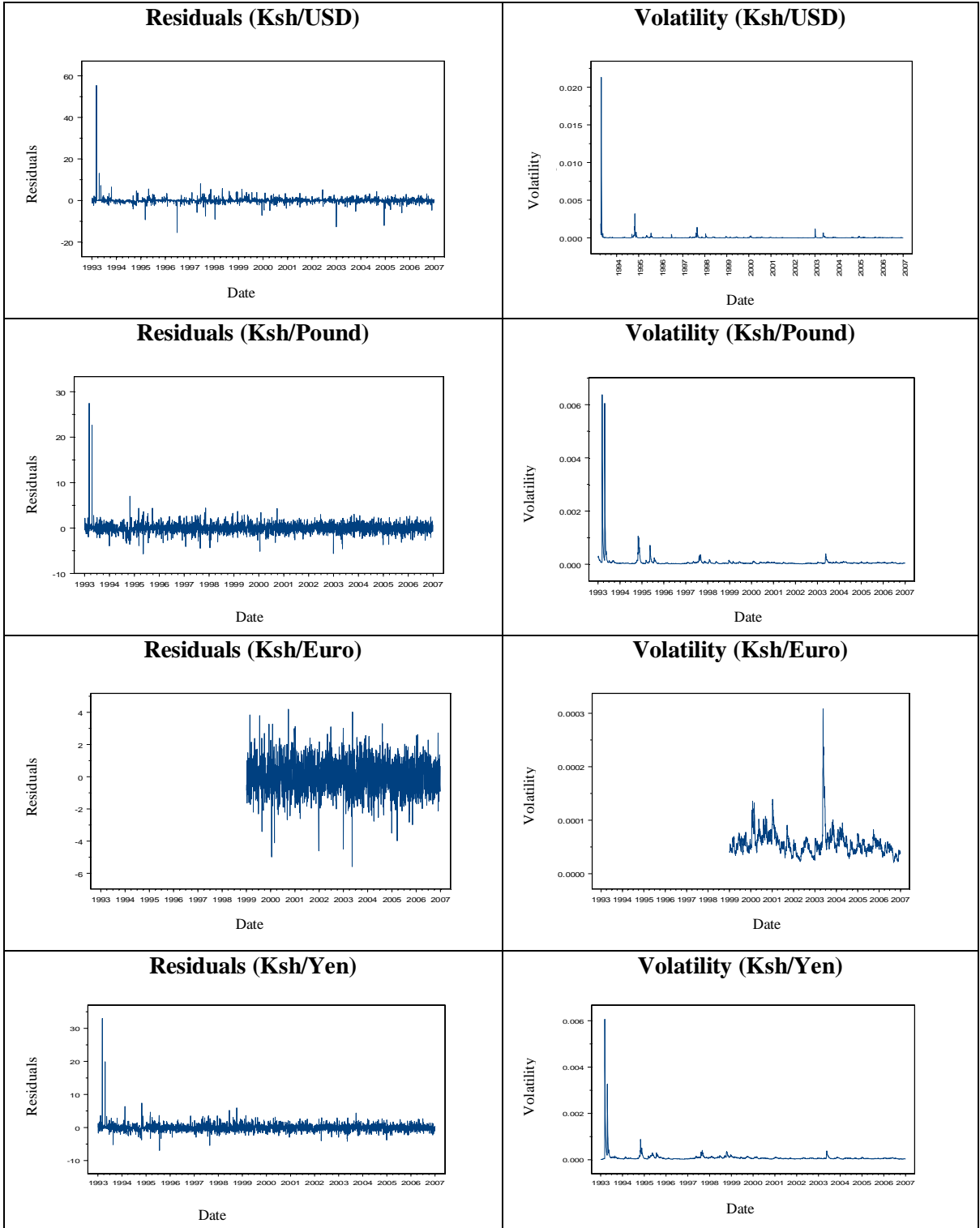
Appendix 3: Test for serial correlation in the standardised residuals from GARCH (1, 1) models for exchange rate returns

Lag	Ksh/USD				Ksh/Euro				Ksh/Pound				Ksh/Yen			
	AC	PAC	Q-Stat	Prob	AC	PAC	Q-Stat	Prob	AC	PAC	Q-Stat	Prob	AC	PAC	Q-Stat	Prob
4	-0.001	-0.001	0.015	1.000	0.030	0.029	4.844	0.304	-0.002	-0.002	0.068	0.999	-0.001	-0.001	0.029	1.000
8	-0.001	-0.001	0.028	1.000	-0.002	-0.004	6.122	0.634	0.001	0.001	0.124	1.000	-0.002	-0.002	0.103	1.000
12	-0.001	-0.001	0.037	1.000	0.003	0.005	7.906	0.792	-0.003	-0.003	0.198	1.000	-0.001	-0.001	0.128	1.000
16	-0.001	-0.001	0.047	1.000	0.004	0.004	8.450	0.934	-0.002	-0.002	0.2967	1.000	-0.002	-0.002	0.176	1.000
20	-0.001	-0.001	0.064	1.000	0.000	-0.003	9.627	0.974	-0.003	-0.003	0.361	1.000	-0.001	-0.001	0.335	1.000
24	0.000	0.000	0.074	1.000	0.000	-0.003	12.827	0.969	-0.002	-0.002	0.426	1.000	-0.002	-0.002	0.383	1.000

Appendix 4: Test for ARCH in the standardised residuals from GARCH (1, 1) models for exchange rate returns



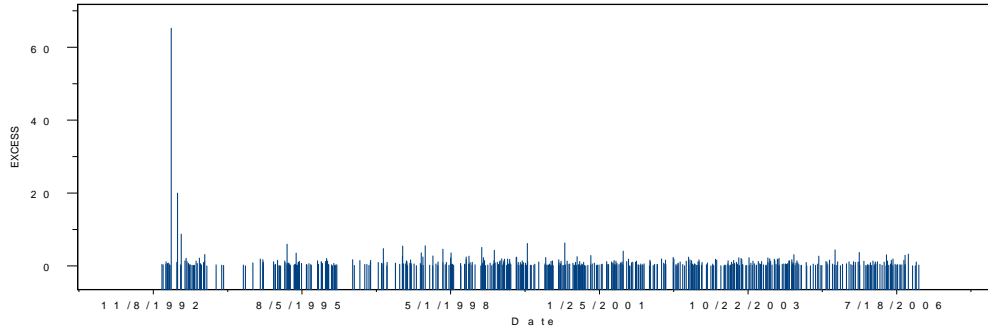
Appendix 5: Standardised residuals and estimated volatility from GARCH (1, 1) models for the exchange rate returns



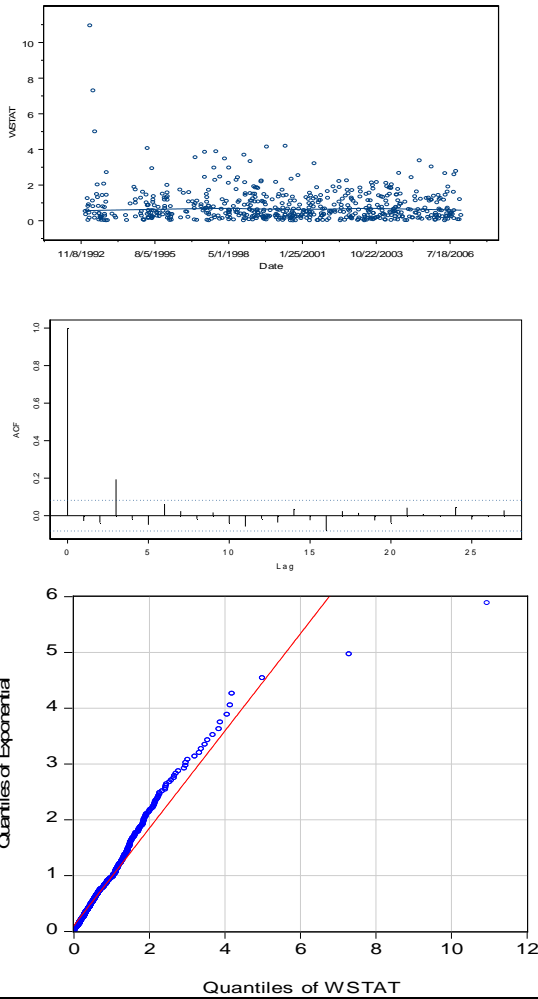
Appendix 6: Standardised Residuals and Estimated Volatility from EGARCH (1, 1) Models of Exchange rate returns

Appendix 7: Excesses and Diagnostic Plots for the POT models for Exchange rate returns

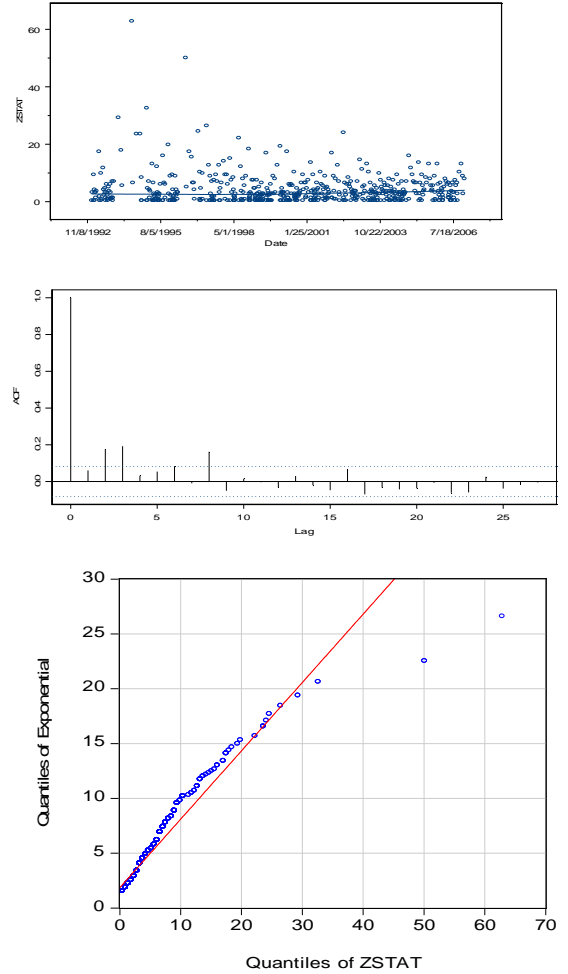
a) Excesses for the Residuals of the Ksh/USD



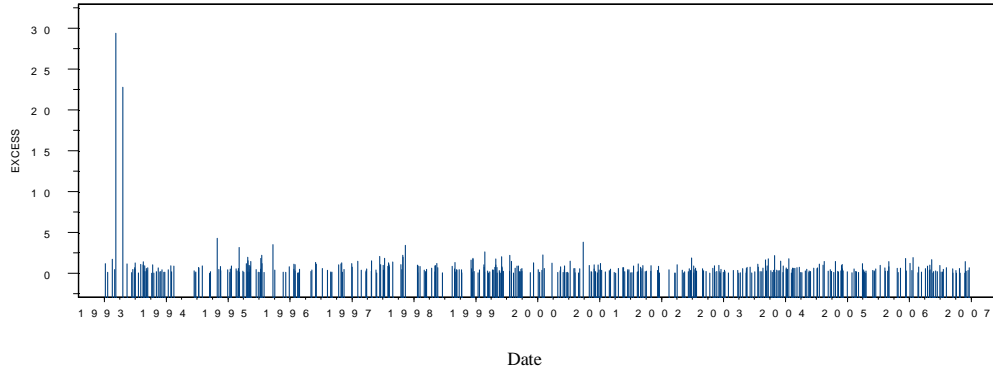
Diagnostics for the W-Statistics



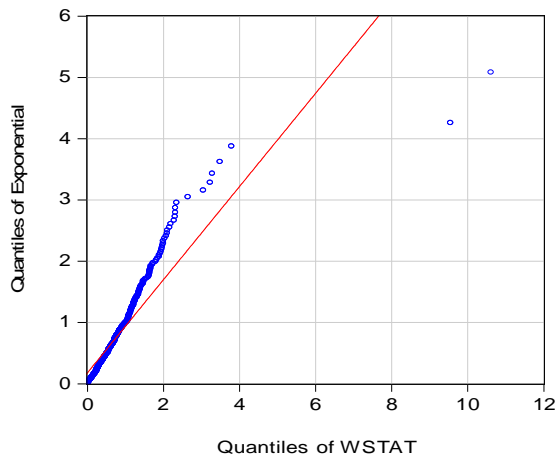
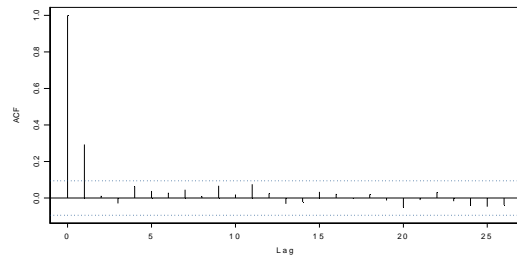
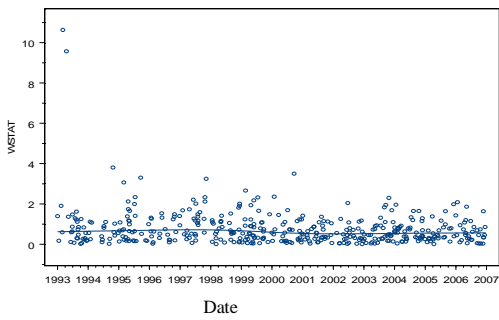
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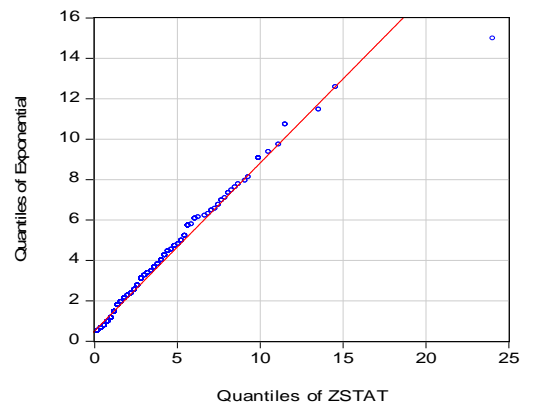
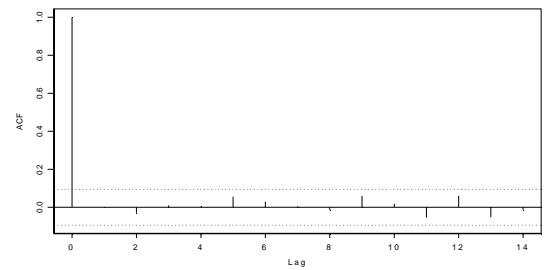
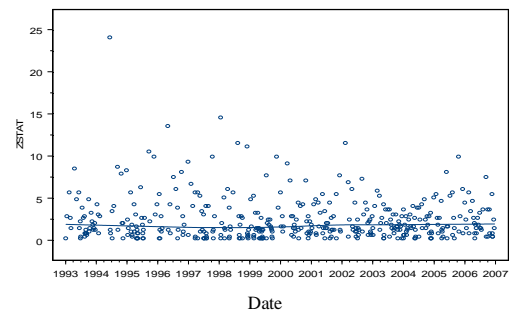
b) Excesses for the residuals of the Ksh/Pound



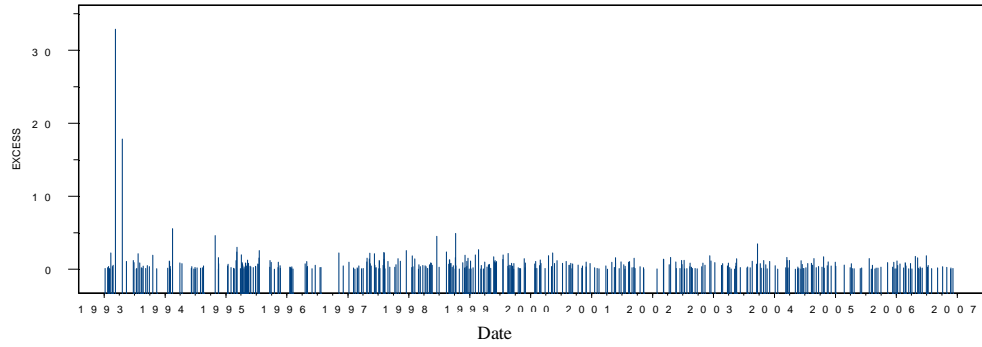
Diagnostics for the W-Statistics



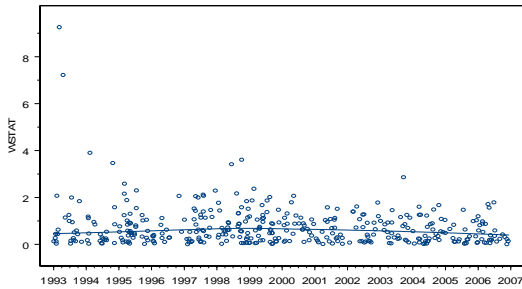
Diagnostics for the Z-Statistics



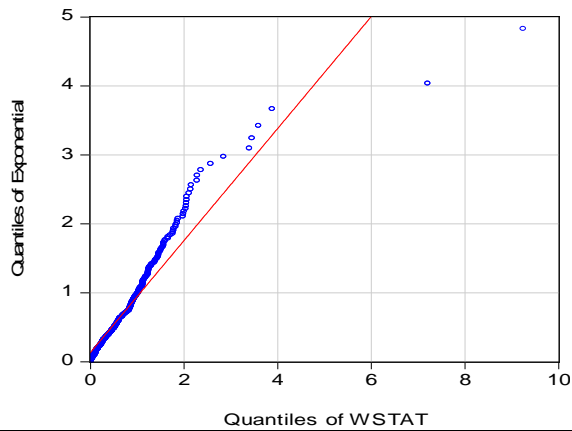
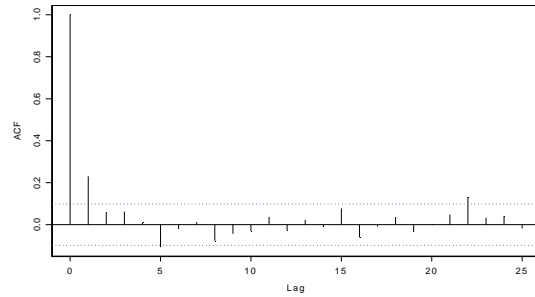
c) Excesses for the Residuals of the Ksh/Yen



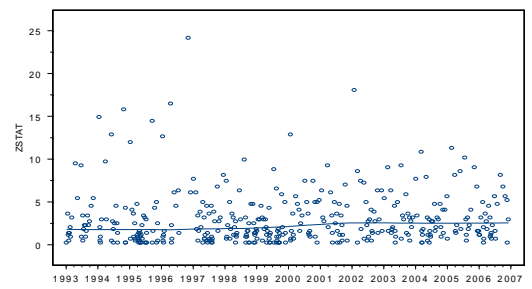
Diagnostics for the W-Statistics



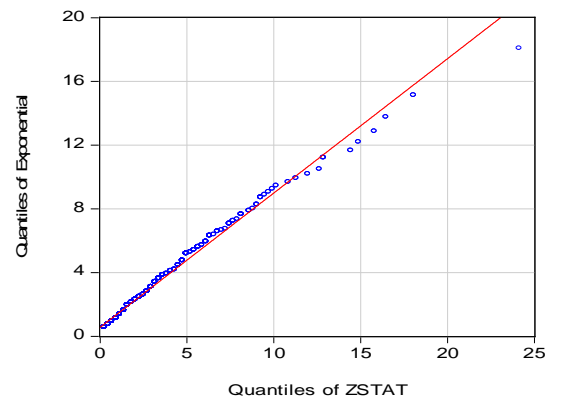
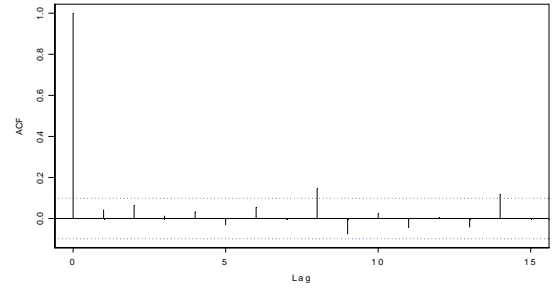
Date



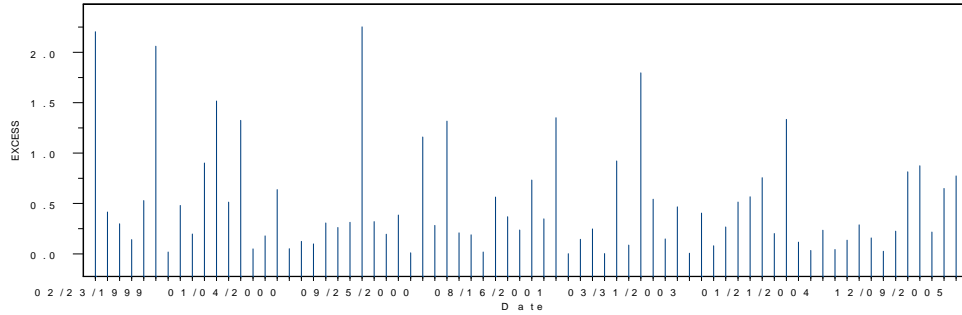
Diagnostics for the Z-Statistics



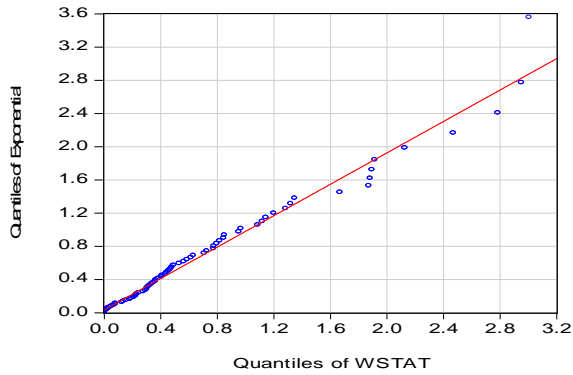
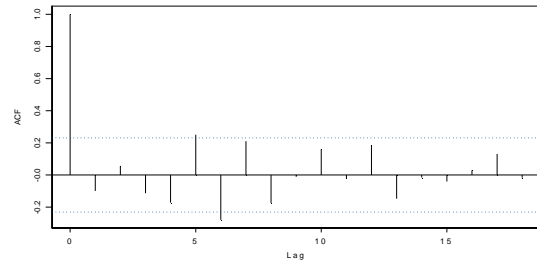
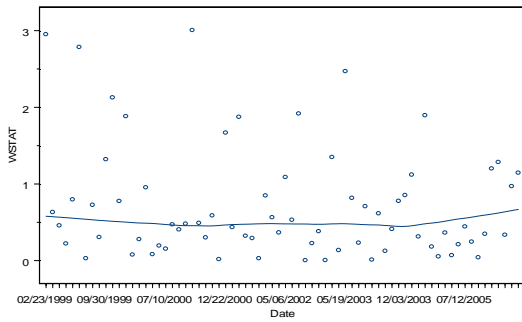
Date



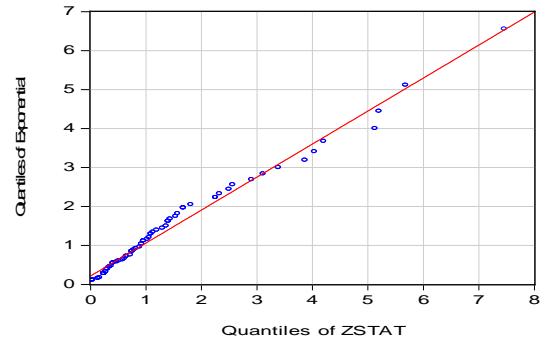
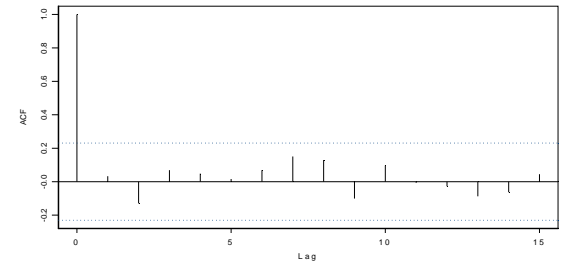
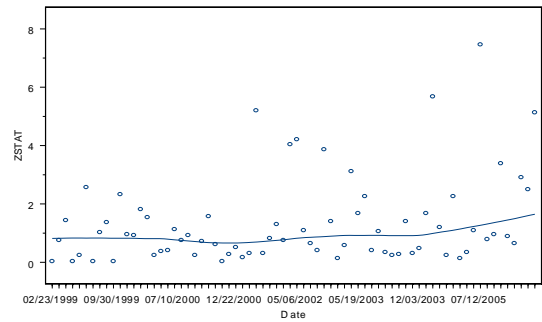
d) Excesses for the Residuals of the Ksh/Euro



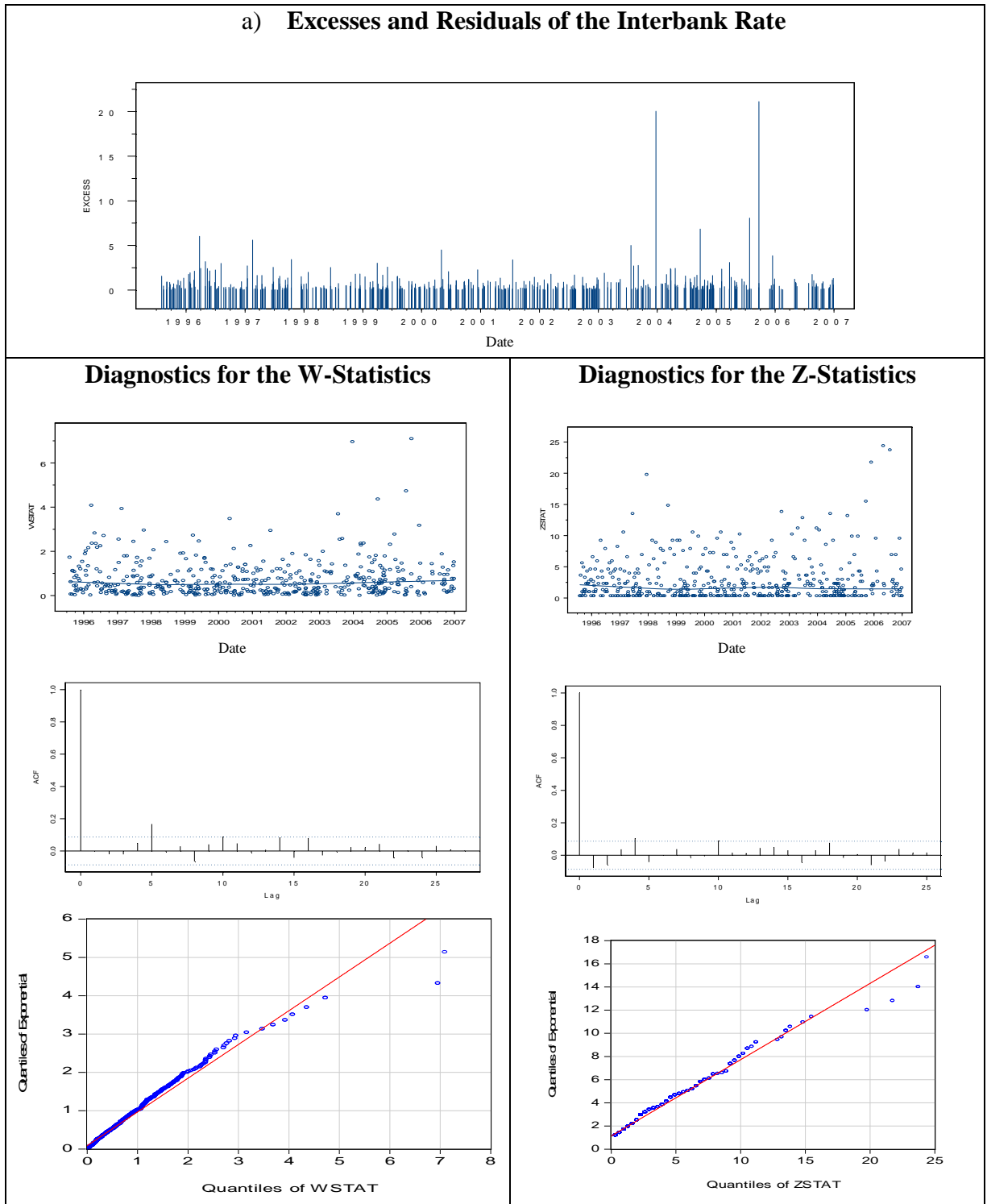
Diagnostics for the W-Statistics



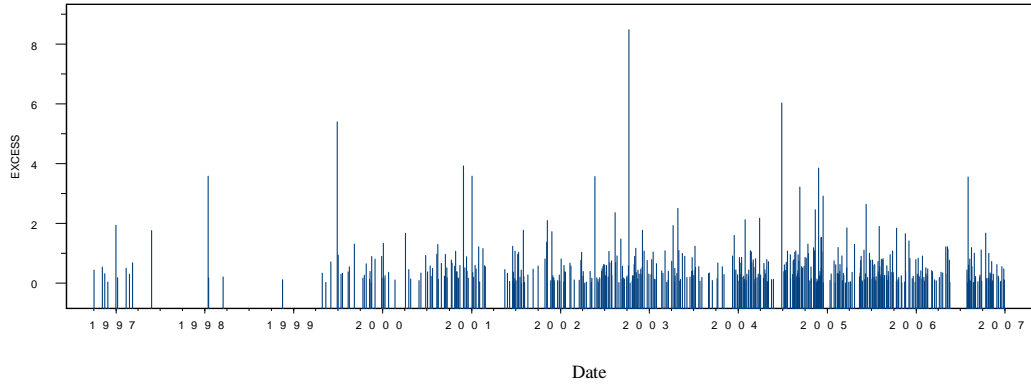
Diagnostics for the Z-Statistics



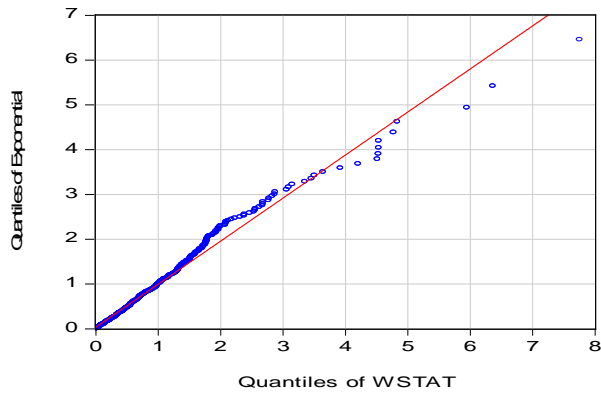
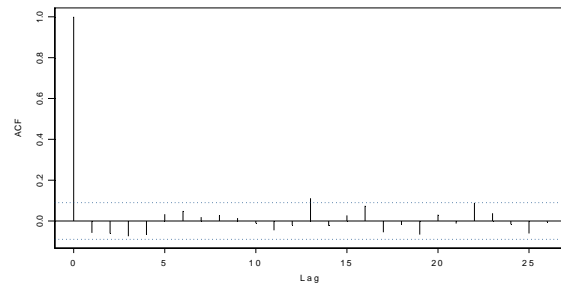
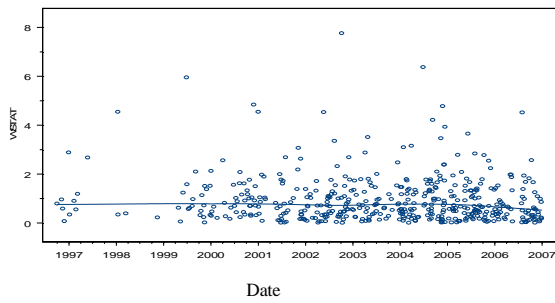
Appendix 8: Excesses and Diagnostic Plots for the POT models for Changes in Interest Rates



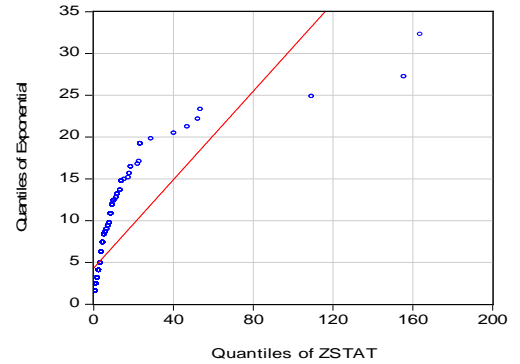
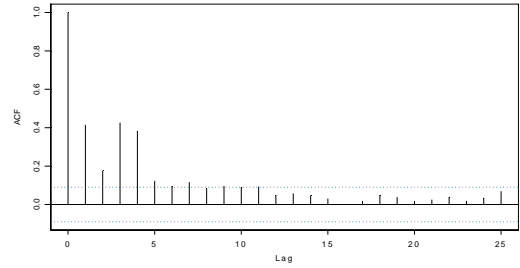
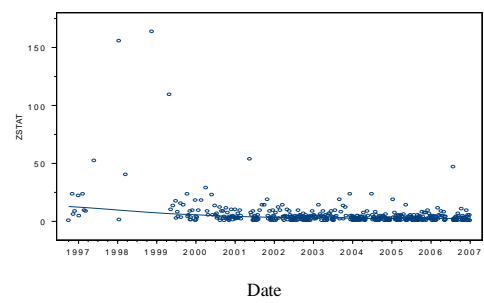
b) Excesses and Residuals of the Repo Rate



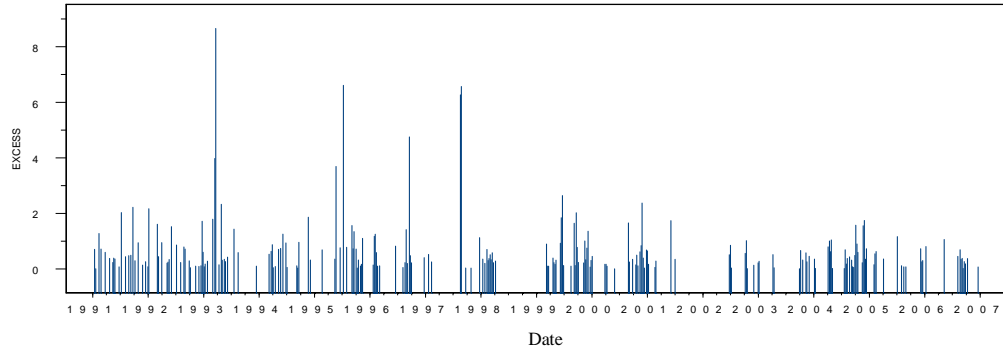
Diagnostics for the W-Statistics



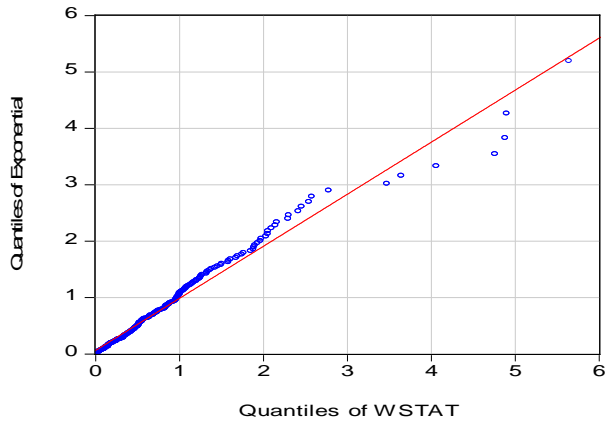
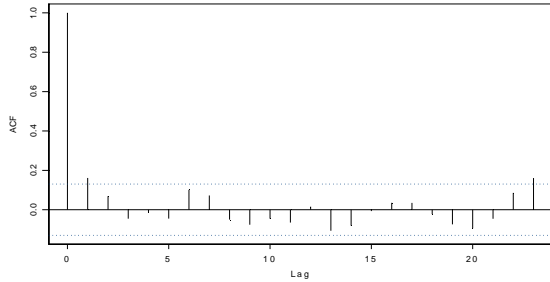
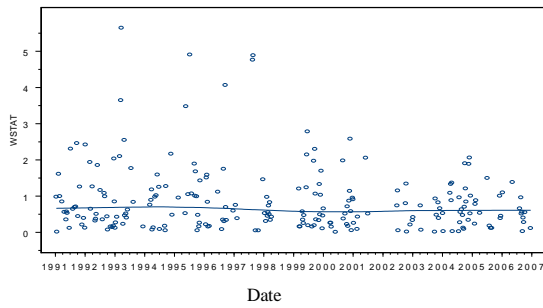
Diagnostics for the Z-Statistics



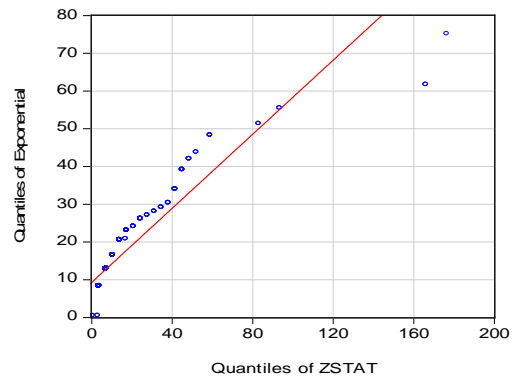
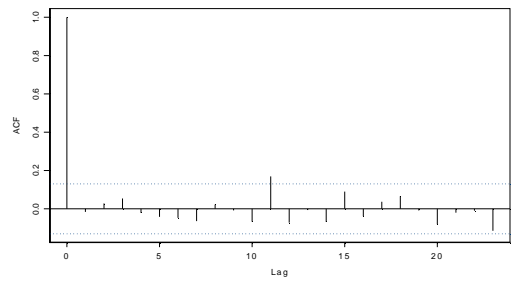
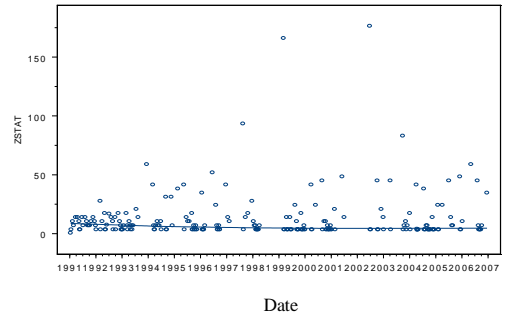
c) Excesses and Residuals of the Treasury Bill Rate



Diagnostics for the W-Statistics



Diagnostics for the Z-Statistics



Appendix 9: SPLUS Functions Used In Fitting the GPD Models

```
#####
findthreshold <- function(data, ne)
{
  data <- rev(sort(as.numeric(data)))
  thresholds <- unique(data)
  indices <- match(data[ne], thresholds)
  indices <- pmin(indices + 1., length(thresholds))
  thresholds[indices]
}
#####
fit.GPD <- function(data, threshold = NA, nextremes = NA, method = "ml", information = "observed")
{
  data <- as.numeric(data)
  n <- length(data)
  if(is.na(nextremes) & is.na(threshold))
    stop("Enter either a threshold or the number of upper extremes"
         )
  if(!is.na(nextremes) & !is.na(threshold))
    stop("Enter EITHER a threshold or the number of upper extremes"
         )
  if(!is.na(nextremes))
    threshold <- findthreshold(data, nextremes)
  exceedances <- data[data > threshold]
  excess <- exceedances - threshold
  Nu <- length(excess)
  if(method == "ml") {
    xbar <- mean(excess)
    a0 <- xbar
    gamma <- -0.35
    delta <- 0.
    pvec <- ((1.:Nu) + delta)/(Nu + delta)
    a1 <- mean(sort(excess) * (1. - pvec))
    xi <- 2. - a0/(a0 - 2. * a1)
    beta <- (2. * a0 * a1)/(a0 - 2. * a1)
    par.ests <- c(xi,beta)
    assign("excesses.nl", excess, frame = 1.)
    negloglik <- function(theta)
    {
      -sum(dGPD(excesses.nl,theta[1],abs(theta[2]),logvalue=T))
    }
    fit <- nlmin(negloglik, par.ests, max.fcal = 1000, max.iter = 200)
    par.ests <- fit$x
    par.ests[2] <- abs(par.ests[2])
    converged <- fit$converged
    ll.max <- -negloglik(fit$x)
    if(information == "observed") {
      fisher <- hessb(negloglik, fit$x)
      varcov <- solve(fisher)
    }
  }
  if(information == "expected") {
```

```

one <- (1. + par.ests[1.]^2./Nu
two <- (2. * (1. + par.ests[1.] * par.ests[2.]^2.)/Nu
cov <- -((1. + par.ests[1.] * par.ests[2.])/Nu
varcov <- matrix(c(one, cov, cov, two), 2.)
}
}
if(method == "pwm") {
  xbar <- mean(excess)
a0 <- xbar
gamma <- -0.35
delta <- 0.
pvec <- ((1.:Nu) + delta)/(Nu + delta)
a1 <- mean(sort(excess) * (1. - pvec))
xi <- 2. - a0/(a0 - 2. * a1)
beta <- (2. * a0 * a1)/(a0 - 2. * a1)
par.ests <- c(xi, beta)
denom <- Nu * (1. - 2. * xi) * (3. - 2. * xi)
if(xi > 0.5) {
  denom <- NA
  warning("Asymptotic standard errors not available for PWM Method when xi > 0.5")
}
one <- (1. - xi) * (1. - xi + 2. * xi^2.) * (2. - xi)^2.
two <- (7. - 18. * xi + 11. * xi^2. - 2. * xi^3.) * beta^2.
cov <- beta * (2. - xi) * (2. - 6. * xi + 7. * xi^2. - 2. * xi^3.)
varcov <- matrix(c(one, cov, cov, two), 2.)/denom
information <- "expected"
converged <- NA
  ll.max <- NA
}
par.ses <- sqrt(diag(varcov))
p.less.thresh <- 1. - Nu/n
out <- list(n = length(data), data = exceedances, threshold = threshold,
  p.less.thresh = p.less.thresh, n.exceed = Nu, method = method,
  par.ests = par.ests, par.ses = par.ses, varcov = varcov,
  information = information, converged = converged, ll.max =
  ll.max)
names(out$par.ests) <- c("xi", "beta")
names(out$par.ses) <- c("xi", "beta")
out
}
#####
plotTail <- function(object, extend=2,fineness=1000,...)
{
  data <- as.numeric(object$data)
  threshold <- object$threshold
  xi <- object$par.ests[names(object$par.ests) == "xi"]
  beta <- object$par.ests[names(object$par.ests) == "beta"]
xpoints <- sort(data)
ypoints <- ppoints(sort(data))
xmax <- max(xpoints)*extend
  prob <- object$p.less.thresh
  ypoints <- (1- prob) * (1 - ypoints)

```

```

    x <- threshold+qGPD((0:(finess-1))/finess, xi, beta)
  x <- pmin(x,xmax)
  y <- pGPD(x-threshold, xi,beta)
  y <- (1- prob) * (1 - y)
  plot(xpoints, ypoints, xlim=range(threshold,xmax), ylim = range(ypoints, y), xlab = "x (on log
scale)", ylab = "1-F(x) (on log scale)", log="xy",...)
  lines(x,y)
  NULL
}
#####
showRM <- function(object, alpha, RM="VaR", extend=2, ci.p = 0.95, like.num = 50.)
{
  threshold <- object$threshold
  par.ests <- object$par.ests
  xihat <- par.ests[names(par.ests) == "xi"]
  betahat <- par.ests[names(par.ests) == "beta"]
  p.less.thresh <- object$p.less.thresh
  a <- (1-alpha)/(1 - p.less.thresh)
  quant <- threshold+betahat*(a^(-xihat)-1)/xihat
  es <- quant/(1-xihat) + (betahat-xihat*threshold)/(1-xihat)
  point.est <- switch(RM, VaR=quant, ES=es)
  plotTail(object,extend=2)
  abline(v=point.est)
  xmax <- max(object$data)*extend
  assign("excesses.nl", object$data - threshold,frame = 1.)
  assign("a.nl", a, frame = 1.)
  assign("u.nl", threshold, frame = 1.)
  assign("RM.nl",RM,frame=1)
  parloglik <- function(theta)
  {
    xi <- theta
    if (RM.nl=="VaR")
      beta <- xi*(xpi-u.nl)/(a.nl^(-xi)-1)
    if (RM.nl=="ES")
      beta <- ((1 - xi) * (xpi - u.nl))/(((a.nl^( - xi) - 1)/xi) + 1)
    if (beta<0) out <- NA
    else
      out <- -sum(dGPD(excesses.nl,xi,beta,logvalue=T))
    out
  }
  parmax <- NULL
  start <- switch(RM, VaR=threshold,ES=quant)
  xp <- exp(seq(from = log(start), to = log(xmax), length = like.num))
  for(i in 1:length(xp)) {
    assign("xpi", xp[i], frame = 1.)
    fit2 <- nlmn(parloglik, xihat, max.fcal = 1000, max.iter = 200)
    parmax <- rbind(parmax, -parloglik(fit2$x))
  }
  overallmax <- object$I.max
  crit <- overallmax - qchisq(0.999, 1)/2.
  cond <- parmax > crit
  xp <- xp[cond]

```

```

parmax <- parmax[cond]
par(new = T)
plot(xp, parmax, type = "n", xlab = "", ylab = "", axes = F,
      ylim = range(overallmax, crit), xlim=range(threshold,xmax), log = "x")
axis(4., at = overallmax - qchisq(c(0.95, 0.99), 1.)/2., labels
      = c("95", "99"), ticks = T)
aalpha <- qchisq(ci.p, 1.)
abline(h = overallmax - aalpha/2, lty = 2, col = 2)
cond <- !is.na(xp) & !is.na(parmax)
smth <- spline(xp[cond], parmax[cond], n = 200.)
lines(smth, lty = 2., col = 2.)
ci <- smth$x[smth$y > overallmax - aalpha/2.]
out <- c(min(ci), point.est, max(ci))
names(out) <- c("Lower CI", "Estimate", "Upper CI")
  out
}
#####
MEplot <- function(data, omit = 3., labels = T, ...)
{
  data <- as.numeric(data)
  n <- length(data)
  myrank <- function(x, na.last = T)
  {
    ranks <- sort.list(sort.list(x, na.last = na.last))
    if(is.na(na.last))
      x <- x[is.orderable(x)]
    for(i in unique(x[duplicated(x)])) {
      which <- x == i & !is.na(x)
      ranks[which] <- max(ranks[which])
    }
    ranks
  }
  data <- sort(data)
  n.excess <- unique(floor(length(data) - myrank(data)))
  points <- unique(data)
  nl <- length(points)
  n.excess <- n.excess[ - nl]
  points <- points[ - nl]
  excess <- cumsum(rev(data))[n.excess] - n.excess * points
  y <- excess/n.excess
  plot(points[1:(nl - omit)], y[1:(nl - omit)], xlab = "", ylab = "",
        ...)
  if(labels)
    title(xlab = "Threshold", ylab = "Mean Excess")
}
#####
RiskMeasures <- function(out, p)
{
  u <- out$threshold
  par.ests <- out$par.ests
  xihat <- par.ests[names(par.ests) == "xi"]
  betahat <- par.ests[names(par.ests) == "beta"]
}

```

```

p.less.thresh <- out$p.less.thresh
lambda <- 1./(1. - p.less.thresh)
quant <- function(pp, xi, beta, u, lambda)
{
  a <- lambda * (1. - pp)
  u + (beta * (a^(- xi) - 1.))/xi
}
short <- function(pp, xi, beta, u, lambda)
{
  a <- lambda * (1. - pp)
  q <- u + (beta * (a^(- xi) - 1.))/xi
  (q * (1. + (beta - xi * u)/q))/(1. - xi)
}
q <- quant(p, xihat, betahat, u, lambda)
es <- short(p, xihat, betahat, u, lambda)
cbind(p, quantile = q, sfall = es)
}
#####
xiplot <- function(data, models = 30., start = 15., end = 500., reverse = T, ci =
0.95, auto.scale = T, labels = T, table = F, ...)
{
  data <- as.numeric(data)
  qq <- 0.
  if(ci)
    qq <- qnorm(1. - (1. - ci)/2.)
  x <- trunc(seq(from = min(end, length(data)), to = start, length =
models))
  gpd.dummy <- function(nex, data)
  {
    out <- fit.GPD(data = data, nex = nex, information = "expected")
    c(out$threshold, out$par.ests[1.], out$par.ses[1.])
  }
  mat <- apply(as.matrix(x), 1., gpd.dummy, data = data)
  mat <- rbind(mat, x)
  dimnames(mat) <- list(c("threshold", "shape", "se", "exceedances"),
NULL)
  thresh <- mat[1., ]
  y <- mat[2., ]
  yrange <- range(y)
  if(ci) {
    u <- y + mat[3., ] * qq
    l <- y - mat[3., ] * qq
    yrange <- range(y, u, l)
  }
  index <- x
  if(reverse)
    index <- - x
  if(auto.scale)
    plot(index, y, ylim = yrange, type = "l", xlab = "", ylab = "",
axes = F, ...)
  else plot(index, y, type = "l", xlab = "", ylab = "", axes = F, ...)
  axis(1., at = index, lab = paste(x), ticks = F)
}

```

```

axis(2.)
axis(3., at = index, lab = paste(format(signif(thresh, 3.))), ticks = F
)
box()
if(ci) {
  lines(index, u, lty = 2., col = 2.)
  lines(index, l, lty = 2., col = 2.)
}
if(labels) {
  labely <- "Shape (xi)"
  if(ci)
    labely <- paste(labely, " (CI, p = ", ci, ")", sep = ""
)
  title(xlab = "Exceedances", ylab = labely)
  mtext("Threshold", side = 3., line = 3.)
}
if(table)
  print(mat)
NULL
}
#####
interarrival <- function(datathreshold)
{
  n <- length(datathreshold)
  m <- matrix(0., ncol=1, nrow=n)
  for(i in 1:n) {
    for(j in 1:1) {
      m[1,1] <- 0
      m[i,j] <- datathreshold[i] - datathreshold[i-1]
      out <- m
    }
  }
  m
}
}
#####

```