ERROR DETECTION AND CORRECTION OF THE MODIFIED INTERNATIONAL STANDARD SERIAL NUMBER

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Error Detection and Correction of the Modified International Standard Serial Number

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DECLARATION

This thesis is my original work and has not been presented for a degree in any other University.

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This thesis has been submitted for examination with our approval as University

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DEDICATION

Dedicated

To

1.My wife

God wanted me to be happy that is why He made you my wife. You bring out the best in me.

2. My sons

I wouldn't be the man I am without you. You made me to believe in hard work and to trust in God

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SYMBOLS AND ABBREVIATIONS

a/b	a divides b
$a \nmid b$	a does not divide b
C	Code C
$modn$	modulo n
$\sum$	Summation
$V(n,q)$	Set of all ordered n-tuples over $GF(q)$
iff	if and only if
${}_nC_r$	n combination r
${}^nP_r$	n permutation r
(k,m)	gcd of k and m
$(\mathbb{Z}_n,\times,+)$	A ring of integers modulo n under the operations $\times$ and $+$

LIST OF ALGORITHMS

ABSTRACT

The International Standard Serial Number (ISSN) code is internationally used for identifying the title of serial publications. This research aims at determining the error detection and correction capabilities of the International Standard Serial Number code against the dictionary size. ISSN code has a relatively small dictionary size and is not effective in error detection and correction capabilities. The study develops a modified International Serial Number code and determines its error detection and correction capabilities against its dictionary size. The study utilizes weight checksum technique to detect and correct error(s). Moreover, the study designs an algorithm for conversion of ISSN code words to the modified ISSN code words. The total number of code words is established by how many digits permute. Modified ISSN code is effective in error detection and correction capabilities for it has dual mechanism for detection and correction of errors, if the weight checksum equation does not hold and if the conditions for the generating equation does not hold.

CHAPTER ONE

INTRODUCTION

1.1 Background information

A serial is an indefinitely continuous print or non-print publication. Serials include periodicals such as magazines, newspapers, annuals, journals, books in the series and monographs in series. Serials are subjected to numerous and constant changes. Moreover, with wide growth in the world's publishing researches caused the development of a standard identification of serials. International Standard Serial Number (ISSN) was formulated as an international organization for standardization (ISO) international standard in 1971 later published as ISO 3297 in 1975. The international standard serial number code words are used internationally for identifying the title of serial publications. Unlike International Standard Book Number (ISBN), ISSN uniquely identifies a title of a serial publication regardless of language or country in which the serial is published, ISSN International Centre (2010). International Standard Book Number (ISBN) is a 13-digit code which identifies a specific edition of a book from a specific publisher. An ISBN identifies a single volume such as a monograph, a novel or a particular issue of an annual. Moreover, ISBN assists in revealing the publisher and publication.

A prime number p is a positive integer with only two positive divisors, 1 and itself. A composite number n is an integer n with $n > 1$ and n is not prime. Thus a composite number n is given by $n = ab$, where a and b are integers, each strictly between 1 and n.

A code is a system of letters,words or signs used to represent messages in more convenient shortened form or in secret form for communication or storage.

A digit is an element of a code word and a code word is an element of a code. Total number of digits in a code word is referred as the length of a code word.

The Hamming distance $d(X, Y)$ between two code words $X, Y \in (F_q)^n$ is the number of digits in which they differ, for example in $(F_2)^6$, $d(001111, 110101) = 4$.

Let X, Y and Z be code words, then the Hamming distance, d, satisfies the usual conditions for a metric:

i. $d(X,Y) \geq 0$ and $d(X,Y) = 0$ iff $X = Y$

ii. $d(X,Y) = d(Y,X)$

iii. $d(X,Z) \leq d(X,Y) + d(Y,Z)$ for any $X,Y,Z \in (F_q)^n$, Huffman and Pless (2010).

Nearest neighbor. Given a code $C \subset (F_q)^n$ and a code word $Y \in (F_q)^n$ then code word $X \in C$ is a nearest neighbor to code word Y if

$d(X,Y) = \min(d(Z,Y) \mid Z \in C)$.

Let $X \in (F_q)^n$, then the **weight of** X**, w(x)** is the sum of non-zero digits in X.

Lemma 1.1.1. *For $X,Y \in (F_q)^n$, then $d(X,Y) = w(X-Y)$.*

The minimum distance of a code C, denoted $d(C)$, is $d(C)$= min $(d(X,Y) \mid X,Y \in C,\ X \neq Y)$. For code C, $|C| > 1$, where $|C|$ is the number of code words and $d(C) \geq 1$, Huffman and Pless (2010).

Theorem 1.1.2. *(a) If for a code C, $d(C) \geq s+1$ then code C can detect up to s errors.*

(b) If $d(C) \geq 2t+1$ then the code C can correct up to t errors, Huffman and Pless (2010)

Assumptions made in the study

i. Errors in different positions in a code word are independent; the existence of an error in one position in the code word does not affect the probability of an error in another position.

ii. Each digit in a code word has the same probability r of being inaccurately transmitted. The assumption is made that the probability of error is small, $r \ll \frac{1}{2}$.

iii. If a digit in a code word is received in error, then each of the $q-1$ possible errors is equally likely to be received. Such channels are known as $q-$ary symmetric channels.

The term dictionary, Kamaku (2013) is the total number of code words that a code can generate.

In a code C, symbols or digits (redundancy) may be added in order to achieve some degree of uniqueness. These symbols, referred to as the check digits help to ensure that the code satisfies a certain condition, Jacobus (1973).

Suppose a sender intends to send a code word to a receiver. If u and v are the sent and received vectors respectively, situations may occur such that $u \neq v$, this simply means that error(s) occurred. The ability of a code to detect the availability of an error(s) in a code word is referred to error detection whereas the ability of a code to correct existing error(s) in a code word or to reconstruct the sent code word is referred to error correction, Hill (1986).

A code C is a linear code in $(F_q)^n$ if whenever $x_0 x_1 ... x_n \in C$, $y_0 y_1 ... y_n \in C$ and $\lambda \in (F_q)$ then $[(x_0 x_1 ... x_n) + (y_0 y_1 ... y_n)] \in C$ and $\lambda (x_0 x_1 ... x_n) \in C$. A linear code C is a cyclic code if a right or left cyclic shift of a code word in C is also a code word, Van (1998). That is, a code word $a_1 a_2 a_3 ... a_{n-2} a_{n-1} a_n$ of a cyclic code C when cyclically shifted, code word $a_2 a_3 ... a_{n-2} a_{n-1} a_n a_1$ is obtained.

Suppose n is a positive integer. If a, b are integers such that $n/a-b$, then a is congruent to b modulo n and denoted as $a \equiv b\,(mod\,n)$. If n does not divide $a-b$ then a and b are incongruent modulo n . If $a \equiv b\,(mod\,n)$, then b is said to be a residue of a modulo n. For all $a \in \mathbb{Z}$ $\{b \in \mathbb{Z}\ |a \equiv b\,(mod\,n)\}$ is known as the residue class for a modulo n denoted by $[a]$, Fine and Rosenberger (2016).

Theorem 1.1.3. *Given n* > 0*, then congruence modulo n is an equivalence relation on the sets ofintegers. Therefore, the residue classes partition the integers, Fine and Rosenberger (2016).*

Theorem 1.1.4. $\mathbb{Z}_n$ *is field if and only if n is a prime, Fine and Rosenberger(2016).*

Corollary 1.1.5. *a field is an integral domain, Fine and Rosenberger(2016).*

Definition 1.1.6. An algorithm is a finite sequence of instructions that can be followed to perform a specific task, such as a sequence of instructions in a computer program, which must terminate on any valid input, Stein (2008).

Theorem 1.1.7. *(The Division Algorithm) Suppose a and b are integers with* $b \neq 0$*,then there exist unique integers q and r such that* $0 \leq r < |b|$ *and* $b = aq + r$ *, Stein (2008).*

Theorem 1.1.8. *(Dirichlet) Let a and b be integers with* $gcd(a,b) = 1$*. Then there are infinitely many primes of the form* $ax + b$*, Stein (2008).*

Theorem 1.1.9. *(The Euclidean Algorithm) Given integers b and* $a > 0$ *with* $a \nmid b$ *form the repeated divisions*

$$
\begin{aligned}
b &= q_1 a + r_1,\ 0 < r_1 < a \\
a &= q_2 r_1 + r_2,\ 0 < r_2 < r_1 \\
&\dots \\
r_{n-2} &= q_n r_{n-1} + r_n,\ 0 < r_n < r_{n-1} \\
r_{n-1} &= q_{n+1} r_n.
\end{aligned}
$$

The last non-zero remainder r_n is the GCD of a , b. Further r_n can be expressed as a linear combination of a and b by recursively eliminating the $r_i^{\prime}s$ in the intermediate equations, Fine and Rosenberger(2016).

Theorem 1.1.10. *(Inverse modulo n algorithm) Suppose a and n are integers and*

$gcd(a,n) = 1$, then the multiplicative inverse of a is an integer x such that $ax \equiv 1(mod\, n)$,Stein (2008).

A permutation is an arrangement of elements of a set in a specified order.

A function is a rule that maps the elements of a set A (domain) to the elements of another set B (range). Each element in set A maps to just a single element in set B.

The identity function on a set A is the set $\{(a,a) : a \in A\}$.

A function f from set A to set B is called injective or one to one if for every two elements, a and b in X with $f(a) = f(b)$ then $a = b$.

A function f from set A to set B is called surjective or onto if for every element $b \in B$ there exists an element $a \in A$ such that $f(a) = b$.

A function $f : A \to B$ is bijective iff there exists a function $g : B \to A$ such that the composite function of f and g is the identity function.

A permutation is a bijective function from set A to set A.

A group is a set which is closed under an operation that satisfies inverse, identity and associativity properties, Hill (1986).

A ring is a set which is closed under two operations usually addition and multiplication, satisfying the following axioms: abelian group under addition, associativity of multiplication and distributive property.

A field is a ring in which every non-zero element has a multiplicative inverse, Hill (1986).

1.2 Error correcting coding

An error is unintentional deviation from a correct information caused by ignorance or fault in a system. Error correcting coding is an effective technique of detecting and correcting errors which may occur due to environmental interference or physical defects such as human errors in the communication channels. Error correcting coding makes a communication channel reliable by ensuring that the information the receiver gets is the correct information the sender intended the receiver to get . Different communication channels have different error correcting coding schemes depending on the types of errors expected in a particular communication channel. An effective error coding requires an effective scheme which is selected based on the characteristics of the specific communication channel. A good communication channel should have a good error coding scheme which is excellent in error detection and correction capabilities. The primary principle of error correcting coding is to achieve error detection by adding some redundancy to the information which the receiver can apply to determine the consistency of the received information (Singh and Singh, 2012). There are two types of error correcting coding; systematic error correcting coding and non-systematic error coding. In systematic error correcting coding, the source of the information transmits the original information attached together with a fixed number of check digits. Systematic error correcting coding uses deterministic algorithms to detect the existence of errors. In non-systematic error correcting coding, the original information is transmitted as encoded information with at least the same length as the original information.

There two main error detection schemes; Repetition scheme and redundancy check scheme.

In repetition scheme the message is sent as a stream of bits broken up into blocks of bits. In sending each block is sent in some predetermined number of times.

The main principle used in error detection is redundancy. Redundancy is the addition of extra bits to a code word to facilitate in detecting and correcting errors. Redundant digits are usually added by the sender and removed by the receiver. In redundancy, the extra digit added to the data (code word) are discarded when the receiver receives the correct data.

Types of redundancy check

i. Parity check

ii. Cyclic redundancy

iii. Checksum

Parity check is based on the addition of extra digits in the code word called the parity. Parity digits are added in order to make the total numbers of 1's even (or odd) in a code word. Cyclic redundancy check is based on division, where the cyclic redundancy remainder is attached to the end of a code word so that the results of the code word is divisible by the second established binary number.When receiver receives the data which is divisible by the given binary number then the data is accepted. If the data is not divisible by the given binary number, there exists a remainder and the data is rejected. In the checksum method, a checksum is created on the data to be transmitted using some algorithms. When the receiver receives the data, the checksum is calculated to test the accuracy of the data.

There are two main techniques used in error correction; Automatic Repeat Request (ARQ) and Forward Error Correction (FEC). In Forward Error Correction (FEC), the sender encodes the message with an Error Correcting Code (ECC) and transmits the coded message. The receiver decodes the message without sending any feedback to the sender. The Error Correcting Code (ECC) is used to correct any error that might have occurred during the transmission of the message received. In Automatic Repeat Request (ARQ), the sender sends a message together with an error detection code. The receiver uses the error detection code to detect any error in the received message. If there are no errors detected, the receiver sends back a positive acknowledgement to the sender and in case of errors, the receiver requests for re-transmission of the message.

1.3 Types of errors in a code

i. Single error: A single error occurs when one digit is incorrectly typed in a code word. For example, a common typing single error of writing 2 instead of 8. In case of a single error in a code word the weight checksum equation does not hold.

ii. Silent error: Silent error occurs when there is an error in a code word but the weight checksum equation holds. A silent error may be a single silent error or a multiple silent errors.

iii. Double error: A double error occurs when two digits in a code word are incorrectly typed. In case of a double error, the weight checksum equation does not hold.

iv. Transpose error: A transpose error may occur due to interchanging of digits in a code word, Hill (1986).

v. Jump transpose error: A jump transpose error may occur when three digits are reversely interchanged. In case of a jump transpose error in a code word, the weight checksum equation does not hold. For example $523 \longrightarrow 325$.

vi. Jump twin error: A jump twin error may occur when two equal digits are interchanged for another. In case of a a jump twin error in a code word, the weight checksum equation does not hold. For example $535 \rightarrow 636$, Wachira, et al (2015).

vii. Phonetic error: A phonetic error may occur when a digit is replaced with another that sounds almost the same. For example 17 $\to$ 70.

viii. Twin error: A twin error may occur when a pair of similar digits are replaced with another pair. For example 66 $\to$ 33.

ix. Omission or insertion of a digit(s) error: These are errors that occur when a digit(s) are omitted or extra digit(s) are added.

1.4 Statement of the problem

ISSN code is only associated with the title of a publication. Therefore, in case a publication is modified appreciably, a new ISSN has to be assigned. This means that if a publication is modified ten times the same publication has to be assigned ten new ISSN code words. Despite of ISSN used internationally for identifying the title of serial publications, the code has a relatively small dictionary of about 10,000,000 code words. The above mentioned weaknesses of the ISSN code do not affect the ISSN assignment therefore little study has been done on these weaknesses. There is need to determine the error detection and correction capabilities of ISSN code against the dictionary size. This study is set to determine the error detection and correction capabilities of the International Standard Serial Number code. Moreover, the study develops a modified international serial number code which is improved in error detection and correction capabilities against the dictionary size.

1.5 Justification of the study

The International Standard Serial Number code words are used internationally for identifying the title of serial publications. The code has a length of 8 digits which uniquely identifies periodic publications. The code is assigned to the title of a periodical publication. ISSN code words are used for monographs-in- series, newspapers, magazines, annual publications and journals, National Library of New Zealand (1989). ISSN code words are widely used by:

i. ISSN makes trade distribution system more efficient and fast by providing an economical communication channel between suppliers and publishers, ISSN International Centre (2010).

ii. Publishers in identifying their serial publications and incorporating bar-codes on the magazines, newspapers and journal, Lowery (2014).

iii. Libraries in identifying identical serial titles and facilitating checking and ordering procedures. Moreover, libraries use ISSN code words for facilitating collection management, inter library loans and legal deposit.

iv. Catalogue databases use the ISSN code word as a record control number.

v. Documentation centers and databases through bibliographic references use ISSN code words for efficient and accurate citation, indexing services and abstracting (National Information Standards Organization, (U.S.), & American National Standards Institute, 1995).

vi. Subscription agencies use ISSN code words for accurate serial publication ordering, Lowery (2014).

vii. Academics use ISSN code words for accurate cite in full details of publications in researches, National Information Standards Organization (U.S.), & American National Standards Institute (1995).

viii. Retailers/wholesalers use ISSN code words for efficient assessing and control magazine/ newspapers circulation, Lowery (2014).

Error correcting coding is essential in day to day life because any communication system has the transmission of information, which may be in the form of analog or digital from the source to a certain destination. There exists no communication channel which is immune to communication channel noise and impairment. Therefore, different types of errors may occur during the transmission of information from the source to its destination. When errors are introduced in a communication channel, they can completely change the meaning of the information causing a lot of economic implications. Error detection and correction is very crucial to accurate information transmission, storage and redeem. Error coding has wide applications in imperfection tolerant computing in computer memory, optical and magnetic data, deep space communications, satellite, network communications and in all digital communications.

Serial numbers are widely used in the world today. In Kenya, the central bank of Kenya uses serial numbers to code Kenyan currencies. The government of Kenya applies serial numbers to code registered vehicles. Moreover, serial numbers are used in the registration of birth certificates and national identity cards. With the growth of digitization in the current world, there is need for a strong security program that can be an inherent component of a digitization strategy. Utilization of high secured international serial numbers is one way of securing digitization.

1.6 Objectives of the study

1.6.1 General objective

The main objective of this research is to determine the error detection and correction capabilities of the current ISSN code against its dictionary size and design a new ISSN code that is improved in error detection and correction capabilities against the dictionary size.

1.6.2 Specific objectives

The specific objectives are to:

i. Determine the error detection capabilities of the ISSN code.

ii. Determine the error correction capabilities of the ISSN code against the dictionary size.

iii. Develop a modified ISSN code and determine its error detection and correction capabilities against the dictionary size.

iv. Design an algorithm for conversion from the ISSN code to the newly modified ISSN code.

CHAPTER TWO

LITERATURE REVIEW

For many years error- correcting coding has been a key area in research with the aim of either creating new codes or enhancing the existing ones. Tilborg (1993), described a channel as a medium over which information is sent together with its characteristics which consist of an input A, an output B, and a transition probability function. He observed that unless stated otherwise, successive transmissions have the same transition probability function that is independent of each other.

Hill (1986), wrote on International Standard Book Number code of length 10 digits and showed that it was designed to detect a single error and a double error which can be caused by the transposition of two digits. Moreover, he gave some conditions that must be satisfied for error correction. He noted that International Standard Book Number code of length 10 digits can correct an error if only the digit in error is known. The basis of detection of an error in International Standard Book Number code of length 10 digits is through the weight check equation.

Kenneth (1993), studied on the International Standard Book Number code of length 10 digits and showed the check digit was a remainder upon division by 11 of the weighted sum of the first nine digits. Moreover, he proved that a single error or a transposition of two digits could be detected using the check digit and that it is possible to detect and correct the errors.

Tervo (1998), researched the secrets of International Standard Book Number code of length 10 digits and found that error control codes are often designed for known applications where certain types of errors are expected to occur. Most common errors expected are those which humans would typically make when writing a book order. These errors would normally be either a single digit incorrectly written or switch two adjacent digits. Moreover, he found that the ability to detect errors allows identification of invalid numbers which might be encountered . He also noted that a systematic approach to error detection can lead to better codes.

Egghe (1999), studied the detection and correction of multiple errors in general block codes and found the necessary condition for systems to be able to detect all $1,2,...,(k-1)$ errors $(k \in \mathbb{N})$ and showed that the results have applications in International Standard Book Number. He also showed that if the system detects all $1,2,...,k$ errors $(k \in \mathbb{N})$, then it corrects all of them.

Nyaga and Mwathi (2008), studied the cyclic International Standard Book Number code of length 10 digits to improve the efficient of conventional International Standard Book Number code of length 10 digits. They designed a code that was able to detect and correct multiple errors. The code was able to correct as many errors as it could detect without many conditions attached for error correction. However, the method used was trial and error calculation and therefore, it needed further study in order to be improved.

Kamaku (2013), studied on error detection and correction in the International Standard Book Number. He showed how the code dictionary of International Standard Book Number code of length 10 digits and the International Standard Book Number code of length 13 digits affects the error detection and correction capabilities by showing that both codes have major weaknesses in regard to the properties mentioned. Kamaku proposed code, the International Standard Book Number code of length 16 digits, which has a larger dictionary and better error detection and correction capability than the International Standard Book Number code of length 13 digits. An algorithm was designed to convert ISBN-13 code and ISBN-10 code to ISBN -16 code.

Sivan et al (2013), studied on the FPGA implementation of low Bandwidth ECC code. They devised a method of detecting two simultaneous bit error in the communication channel. Moreover, they devised a method of correcting a single bit error based on Hamming code error detection and correction technique. They found that the use of $8-$bit information transmission and the utilization of Hamming code along with low bandwidth data transmission improves the speed of data transmission.

Young (1988), researched on International Standard Serial Numbers. He explained the general background information on International Standard Serial Number code its and their uses.

According to Shelton (1999), the main reason for error coding scheme is to achieve dependable data storage where there are data corruption to provide high precision communication channel. The coding techniques depend on types of errors expected on a communication channel. Reliable and good communication channel which use complex coding schemes are better in error detection and correction capabilities.

According to Doumen (2003), the aims of cryptography is to provide secure transmission message in the sense that two or more persons can communicate in a way that guarantees confidentiality, data integrity and authentication.

Brown (2003), studied on the assignment of International Standard Serial Numbers (ISSNs) to online resources. It was found that there will always be problems in trying to ensure consistency with an international standard due to the differing social, cultural, political and economic environments within which standard agencies operate internationally. Moreover, she noted that for the International Standard Serial Number network these different problems are further exacerbated by the emergence and growth of electronic publications, and the associated problems these have brought with them.

Xambó-Descamps (2003), researched on the block error correcting codes. It was showed that the minimum distance decoder maximizes the likelihood of correcting errors if all the transmission symbols have the same probability of being altered by the channel noise. If in the transmission there is p probability of a symbol in a code vector been altered. The probability that j errors in a block of length n is given by $\binom{n}{j} p^j(1-p)^{n-j}$and therefore the probability that $t+1$ or more errors occur in a code vector and the probability that undetectable errors occurs is given by $p_e(n,t,p) = \sum_{j=t+1}^{n} \binom{n}{j} p^j(1-p)^{n-j} = 1 - \sum_{j=0}^{t} \binom{n}{j} p^j(1-p)^{n-j}$.

Egghe (2005), studied the coding of the International Standard Book Number code. He concluded that the minimum requirement for a useful code was that all single errors, as well as all permutations of two symbols, must be detectable. Moreover, he discussed the strength of alternative codes, in particular with respect to the detection of double errors. He gave a complete description of the method based on division by 11, described the power of the method with respect to the detection of three or four errors.

Egghe and Ronald (2005), studied the detection of double errors in International Standard Book Number code and International Standard Serial Number code and found that using division by 11, all coding methods detect the same percentage of errors. They discovered that using numerical experiments with larger prime numbers as divisors, different divisors have different detection capabilities for double errors. Moreover, they discovered that a minimum required for the used code is that all single errors, as well as all permutations of two symbols, must be detectable. They then left an open unsolved problem challenging the readers to formulate a new algorithm, which one check digit (or letter), such that all single and double errors are detected (or prove that this is not possible).

Singh and Singh (2012), studied on a comparative study of error detection and correction coding techniques. They found that the repetition codes are actually inefficient and are prone to problems especially when the error occurs in the same position for all blocks of bits. Single weight check method is inefficient to check the accuracy of a message if two bits are flipped. Checksum schemes such as the Verhoeff algorithm and Luhn algorithm work only for the common human errors. Cyclic redundancy check codes are the best to use in digital networks and storage devices.

Mukesh et al (2013), studied on a review on error correcting codes deployed in mobility prone digital communication. They showed that the Shannon Hartley capacity theorem can be mathematically expressed as $C = Blog_2(1+\frac{S}{N})\, bits$ per secs(bps) , where S is the average signal energy in each signaling interval of duration $T = \frac{1}{B}$, $\frac{N}{2}$ is the two-sided noise power spectral density. The proof of the theorem shows that for any transmission Rate $R \leq C$, (channel capacity) there exists a coding scheme that produces an arbitrary small probability of error and if $R \geq C$ then no coding scheme can achieve reliable results. Moreover, they found that any error control application should have a field of a definite size q. The efficiency and performance of error correcting codes depend on the implementation of Galois field arithmetic. Galois field arithmetic has a lot of applications especially on the high speed error control coding systems based on the field $GF(p^m)$ where $m = 2 or 3$ and p is a prime of sufficient value to produce the required field size.

Achalare et al (2014), researched on the significance of International Standard Serial Number and International Standard Book Number codes in publication. They concluded that International Standard Serial Number code uniquely internationally identifies serial publications. Publications that are intended to be continued indefinitely on either a regular or irregular basis require an International Standard Serial Number code, for example, Annual reports, Quarterly reports, Biannual reports, Bulletins, Newsletters, Journals or proceedings. The International Standard Book Number code is assigned for a single published book, and International Standard Serial Number code is assigned for a series of books.

Skrzypczak (2014), studied International Standard Serial Number (ISSN). He found that the use of the International Standard Serial Number code is not obligatory, but recommended for the purpose of easier identification of the title and ensuring certainty about turnover. He stressed that each ISSN code word is permanently assigned to key title of a publication at the time of its registration. The court that registers newspapers and periodicals is the one responsible to issue the permit to publish a periodical rather than the ISSN being assigned. It is impossible for ISSN code to be assigned *ex officio* for the code is assigned at the publisher's request. Each title is assigned a unique, unrepeatable identification number. The identity of the periodical depends on its title, however, rather than the International Standard Serial Number code which is only an auxiliary tool to identify the publication. He emphasized that the code is not conclusive when deciding about the right to the title and the more so about whether the publication of the periodical is legal.

Oury (2016), studied on Revision of the International Standard Serial Number involving stakeholders to adopt a bibliographic standard to its ever-changing environment. He discovered that International Standard Serial Number standard over time has evolved in order to meet the needs of its users. It has undergone four revisions since its first release. Each ISO standard regularly undergoes systematic revisions. In April 2016, a vote on the opportunity of a systematic revision was issued by TC46/SC9 to all ISO member bodies.

John et al (2016), researched om error detection and correction using Hamming and cyclic codes in a communication channel. It was found that in Hamming decoding, if $s(r) = 0$, then is assumed there is no error. If $s(r) \neq 0$ and it contains odd number of $1's$ then it is assumed there is a single error and if $s(r) \neq 0$ and it contains even number of $1's$ then it is assumed there is an uncorrectable error pattern.

Wachira (2017), studied the error detection and correction on the credit card number using the Luhn algorithm. It was found that the Luhn algorithm is not effective in error detection and correction. Usage of a composite modulo, that is modulo 10 and modulo 9 in error detection allows errors associated with zero divisors to be undetected. The study came up with a new algorithm, the modulo 13 algorithm, that surpasses the Luhn algorithm in error detection and captures all errors associated with zero divisors.

According to Moon (2020), if a communication channel introduces fewer than the minimum distance errors, $d(C)$, then these can be detected and that if $d(C) - 1$ errors are introduced, then error detection is guaranteed. The probability of error detection depends only on the error introduced by the communication channel and that the decoder will make an error if more than half of the received bit strings are in error.

According to the ISSN international centre website ISSN code is internationally used for identifying the title publications. Table 2.1 below shows the total number of ISSN code words used per type in the period of 2010 to 2018.

Table 2.1: **Total number of ISSN used as per type as from 2010 to 2018**

Type	2010	2011	2012	2013	2014	2015	2016	2017	2018
periodicals	1,066,973	1,121,393	1,171,612	1,218,137	1,263,939	1,367,744	1,367,791	1,415,805	1,460,005
annuals	284,035	289,834	295,467	300,696	307,501	313,874	318,749	323,794	328,773
monographic series	147,155	154,764	161,106	167,117	172,391	177,491	181,527	186,828	192,634
newspaper	41,091	43,019	44,741	47,484	50,098	53,913	56,201	58,185	60,455
updating websites	2,608	3,088	3,824	4,777	5,203	5,981	7,038	8,027	9,015
unspecified	10,893	8,7009	8,563	8,446	8,277	8,094	7,973	7,921	7,826
updating loose-leaf	1,996	2,107	2,185	2,282	2,543	2,609	2,664	2,736	2,787
updating database	556	652	777	1,032	1,158	1,284	1,629	1,794	1,895

It is evident that ISSN code is widely used. ISSN code is highly utilized in periodical publications and lowest used in updating databases. Moreover Table 2.2 shows the total number of ISSN code words utilized since 2010 to 2018.

Table 2.2: **Total number of ISSN records from 2010 to 2018**

	2010	2011	2012	2013	2014	2015	2016	2017	2018
Total number of records	1,555,307	1,623,566	1,688,275	1,749,971	1,811,110	1,884,990	1,943,572	2,005,090	2,063,390
Number of new records	65,534	68,259	64,709	61,696	61,139	73,880	58,582	61,518	58,300

The number of new records per each year since 2010 to 2018 can be represented graphically as follows;

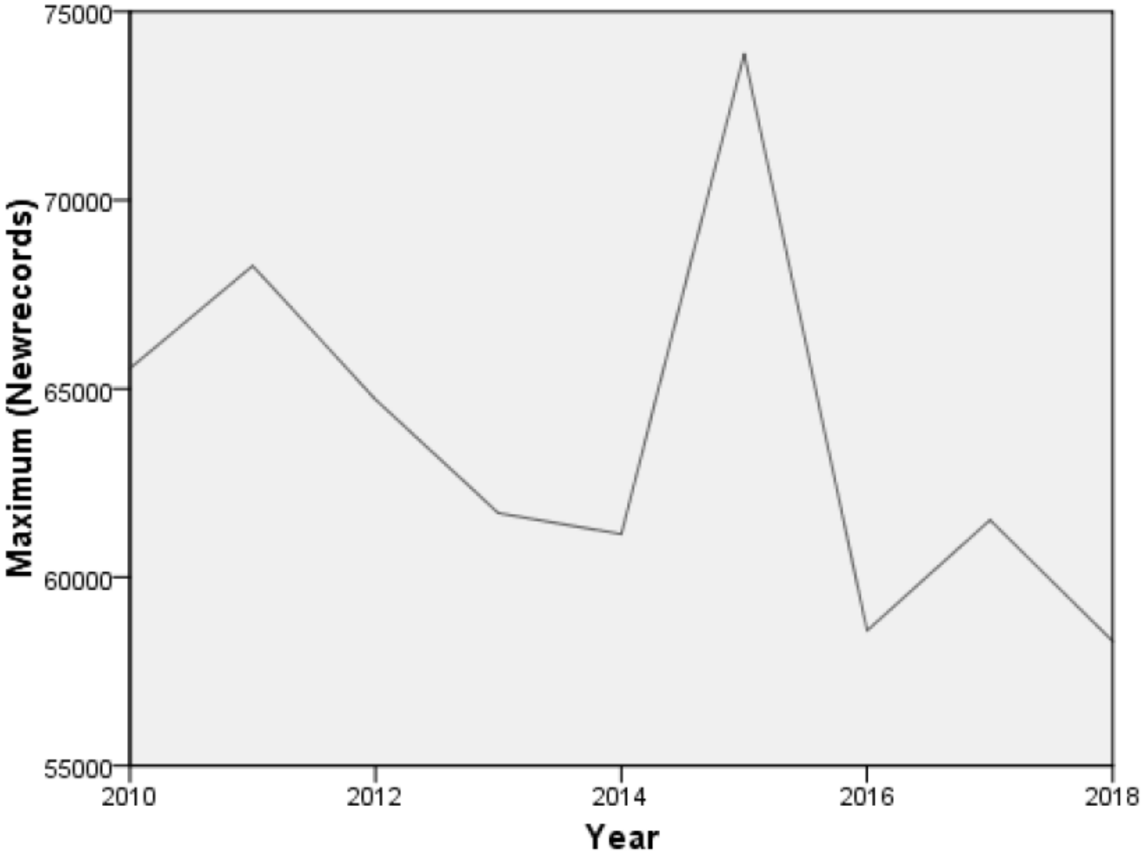

Figure 2.0.1: **The number of new ISSNs record since 2010 to 2018**

In 2016, there were lowest ISSN newly issued records. This was because publishers requested few ISSN code words. In 2016, there were many challenges between ISSN international centres and the editorial staff of professional publications. However, there were changes in policy regarding the time duration of contracts for the use of ISSN data files (ISSN International Centre, 2016). With the increase of publications the number of ISSN used per year is expected to increase.

From the studies done above, it is evident that many researchers have studied on

International Standard Book Number codes with little done on International Standard Serial Number codes. It is evident that, there is no research done on determination of error detection and correction capabilities the international standard serial number code against its dictionary size, despite, the code been used internationally for identification of serial publications. The aim of this research is to determine the error detection and correction capabilities of the International Standard Serial Number code against its dictionary size. Moreover, the study develops a modified International Serial Number code and determines its error detection and correction capabilities against its dictionary size.

CHAPTER THREE

METHODOLOGY

This chapter discusses the methodologies and methods to be used in order to realize the objectives of the study. This study employs mathematical formulas to detect the exact location of the corrupted digit/element of a code word. Once the corrupted digit of the code word is located, the study employs more mathematical algorithms to correct the corrupted digit(s) of the code word.

To detect and correct error(s) in a code word the study uses the checksum method where the weight checksum technique is employed. An ISSN code word has a length of eight digits, the eighth digit is called the check digit. ISSN code uses the check digit to detect the existence of error(s) in the code word. Let $X = (x_1, x_2, ..., x_7)$ be ISSN code without the check digit, then the check digit x_8 is computed by calculating

$$\xi = \sum_{i=1\ j=9-i}^{7} jx_i,$$

where the check digit $x_8 \equiv -\xi (mod\ p)$ where p is a prime number. This study uses the checksum method where the weight checksum technique is employed. The weight checksum equation is computed as

$$\sum_{i=1\ j=9-i}^{8} jx_i \equiv 0\ (mod\ 11).$$

If $\sum_{i=1\ j=9-i}^{8} jx_i \neq 0 (mod\, 11)$ then the code has detected error(s) in the ISSN cord word.

As seen earlier in section 1.2 the effectiveness of any error coding depends on the effectiveness of the error coding scheme chosen based on the characteristics of the specific communication channel. The effectiveness of ISSN code will be determined by the capability of the code to detect and correct errors. Moreover, the effectiveness of ISSN code will be determined by total number of code words which ISSN code can generate. The capability of error detection and correction will be determined by the number of errors in section 1.3 which the code can detect and correct. Furthermore, to determine the error detection and correction capabilities of the ISSN code against its dictionary size, the study will analyze the properties of the International Standard Serial Number code in terms of error detection, correction and the size of the code dictionary.

The modified ISSN code will be developed and generated by the use of functions, permutations and the properties of the ISSN code. The code will be developed on the basis of a good communication channel and an effective error coding scheme. The techniques used in determining the effectiveness of the ISSN code in error detection and correction will be utilized to determine the effectiveness of the modified ISSN code in error detection and correction capabilities. The study will design an algorithm for conversion from the already existing ISSN code to the modified ISSN code. The total number of code words will be established by how many digits permute. The code dictionary in the study is determined by permutation methods.

CHAPTER FOUR

PROPERTIES OF THE ISSN CODE

4.1 Introduction

The study in this chapter analyzes the properties of the International Standard Serial Number in terms of error detection, correction and the size of the code dictionary. The properties of the International Standard Serial Number in terms of error detection, correction and the size of the code dictionary determines the effectiveness of the ISSN code in terms of error correcting scheme.

4.2 Calculation of the check digit in an ISSN code word

According to the ISSN international centre website library the check digit of ISSN code word is calculated as follows;

i. Multiply the first digit of a ISSN code word by 8, the second digit by 7, the third digit by 6 that way until the seventh digit, that is multiplied by 2.

ii. Add the sum for (i).

iii. Divide this sum by the modulo 11.

iv. Subtract the remainder from 11. The answer is the check digit.

Mathematically the check digit is calculated as follows;

Let the ISSN code word be $X = x_1, x_2, \ldots, x_7$ without the check digit. Since the code word digits $x_1, x_2, \ldots, x_7$ are known we calculate $\sum_{i=1}^{7} \sum_{j=9-i} jx_i$. Let $\xi \equiv \sum_{i=1}^{7} \sum_{j=9-i} jx_i \ (mod\ 11)$ then $x_8 + \xi \equiv 0 (mod\ 11)$. Therefore $x_8 \equiv -\xi (mod\ 11)$. Alternatively,

$$x_8 \equiv \sum_{i=1}^{7} (2+i) x_i (mod\ 11).$$

Table 4.1 shows the additive inverses $(mod\ 11)$

Table 4.1: **Additive inverses** *modulo* 11

ξ	1	2	3	4	5	6	7	8	9	10
x_8	10=A	9	8	7	6	5	4	3	2	1

Table 4.2 shows the multiplicative inverses $(mod\ 11)$

Table 4.2: **Multiplicative inverse** *modulo* 11

x	1	2	3	4	5	6	7	8	9	10
x^{-1}	1	6	4	3	9	2	8	7	5	10

When the check digit is 10, replace an upper case A in the check digit position.

Example 4.2.1. Suppose that ISSN code word without the check digit is $0\,3\,1\,7\,8\,4\,7x_8$. Now to compute x_8 , calculate

$$\xi = \sum_{i=1}^{7} \sum_{j=9-i} jx_i = 10\ (mod\ 11)$$

Since $x_8 \equiv -\xi\,(mod\ 11)$ therefore

$$x_8 = -10\ (mod\ 11)$$
$$x_8 = 1.$$

or

$$x_8 \equiv \sum_{i=1}^{7} (2+i)x_i\ (mod\ 11)$$
$$x_8 \equiv 210\ (mod\ 11)$$
$$x_8 = 1$$

4.3 Error detection in an ISSN code word

Let the ISSN code word be $X = x_1, x_2, \ldots, x_8$, then the weight checksum equation is computed as $\sum_{i=1}^{n} \sum_{j=n-(i-1)} jx_i \equiv 0\,(mod\,11)$. Since $j = n-(i-1)$ this equation can be further simplified to $\sum_{i=1}^{n} (10-n+i)x_i \equiv 0\,(mod\,11)$ but in this case $n = 8$ therefore the weight checksum equation is computed as $\sum_{i=1}^{8} \sum_{j=9-i} jx_i \equiv 0\,(mod\,11)$. If $\sum_{i=1}^{8} \sum_{j=9-i} jx_i \neq 0\,(mod\,11)$ then the code has detected error(s). Alternatively, let the ISSN code word be $X = x_1, x_2, \ldots, x_8$, then the weight check is computed as $\sum_{i=1}^{8} \sum_{j=8-(i-1)} (11-j)x_i \equiv 0\,(mod\,11)$ or $\sum_{i=1}^{8} (2+i)x_i \equiv 0\,(mod\,11)$. If

$\sum_{i=1}^{8}(2+i)x_i \neq 0\,(mod\,11)$ then the code has detected error(s).

Omission and insertion of a digit(s) error in an ISSN code word can be detected by determining the length of an ISSN code word. An ISSN code word has a length of only eight digits if the code has a length of more or less than eight then there is an error.

4.3.1 Single error detection in an ISSN code word

Proposition 4.3.1. *ISSN code detects all single error in a code word.*

Proof. Let $X = x_1, x_2, ..., x_8$ be the ISSN code word. Suppose that a single error has occurred at digit x_τ then $Y = x_1, x_2, ..., x_{\tau-1}, y_\tau, x_{\tau+1}, ..., x_8$ with $y_\tau = x_\tau + \alpha, \quad \alpha \neq 0$. Since a single error has occurred at digit x_τ therefore, the weight checksum equation for the received vector with an error is

$$\begin{aligned}&(\sum_{i=1}^{8}(2+i)y_i)\\ =&(\sum_{i=1}^{8}(2+i)x_i)+(2+\tau)\,\alpha\\ =&(2+\tau)\,\alpha \neq 0\,(mod\ 11)\end{aligned}$$

where $\sum_{i=1}^{8}(2+i)x_i$ is the weight sum check equation of the ISSN code word with no error. Since the weight check equation for the received vector does not hold hence the single error is detected. □

Alternately, by use of $\sum_{i=1\ j=9-i}^{8} jx_i$ as the weight checksum equation.

Proof. Let $X = x_1, x_2, ..., x_8$ be the ISSN code word. Suppose that a single error has occurred at digit x_τ, then let the received vector be $Y = x_1, x_2, ..., x_{\tau-1}, y_\tau, x_{\tau+1}, ..., x_8$ with $y_\tau = x_\tau + \alpha, \quad \alpha \neq 0$. Then the weight checksum equation for the received vector with an error is

$$\begin{aligned}&(\sum_{i=1\ j=9-i}^{8} jy_i)\\ =&(\sum_{i=1\ j=9-i}^{8} jx_i)+(9-\tau)\,\alpha\\ =&(9-\tau)\,\alpha \neq 0\,(mod\ 11).\end{aligned}$$

Therefore the single error is detected. □

Remark 4.3.2. ISSN code can be only be used to correct single errors if the position of the error is known.

Corollary 4.3.3. *ISSN code cannot detect a silent error in a code word.*

Proof. Suppose a single error has occurred in an ISSN code word but the weight checksum equation $\sum_{i=1\ j=9-i}^{8} jx_i \equiv 0\,(mod\,11)$ holds.Then $Y = x_1, x_2, ..., x_{\tau-1}, y_\tau, x_{\tau+1}, ..., x_8$ with $y_\tau = x_\tau + \alpha,\ \ \alpha \neq 0$. A single error that has occurred in digit x_τ so the weight checksum equation is

$$\begin{aligned}
&(\sum_{i=1\ j=9-i}^{8} jy_i)\\
=&(\sum_{i=1\ j=9-i}^{8} jx_i) + (9-\tau)\,\alpha\\
=&\,(9-\tau)\,\alpha \equiv 0\,(mod\ 11).
\end{aligned}$$

This implies that either $9-\tau$ or α is a multiple of 11 or 0. Since $9-\tau$ cannot be a multiple of 11 or 0 hence $\alpha = 0$ and therefore no error has occurred. This is a contradiction. Moreover, if $\sum_{i=1\ j=9-i}^{8} jx_i \equiv 0\,(mod\,11)$ holds, this implies there is no any error in the code word but this is not true for there exists a silent error in a code word, hence ISSN code cannot detect a silent error in a code word. □

4.3.2 Transposition error detection in an ISSN code word

Proposition 4.3.4. *ISSN code detects any transposition error in a code word.*

Proof. Suppose that a transposition error has occurred in the ISSN code word $X = x_1, x_2, ..., x_8$, then the received vector $Y = x_1, ..., x_\beta, .., x_\tau, ..., x_8$ with x_τ and x_β the transposed digits. Then

$$\begin{aligned}
&(\sum_{i=1\ j=9-i}^{8} jy_i)\\
=&(\sum_{i=1\ j=9-i}^{8} jx_i) + (\tau-\beta)\,x_\tau + (\beta-\tau)x_\beta\\
=&(\tau-\beta)(x_\tau - x_\beta) \neq 0\,(mod\ 11)
\end{aligned}$$

provided $\tau \neq \beta$ and $x_\tau \neq x_\beta$. Therefore transposition error of digits x_τ and x_β detected. □

Example 4.3.5. Suppose that 03178471 is the ISSN code word. Now if x_4 and x_6 are transposed yielding $x_4 = 4$ and $x_6 = 7$. The weight checksum equation $\sum_{i=1}^{8} \sum_{j=9-i} jx_i \equiv 5\,(mod\,11)$ therefore $\sum_{i=1}^{8} \sum_{j=9-i} jx_i \neq 0\,(mod\,11)$ but $\sum_{i=1}^{8} \sum_{j=9-i} jx_i + (\tau-\beta)x_\tau + (\beta-\tau)x_\beta \equiv 0\,(mod\,11)$ since $5+14-8 \equiv 0\,(mod\,11)$. Now, because of transposition error β comes first before τ hence $\beta = 4$ and $\tau = 6$ $\Rightarrow (\tau-\beta)(x_\tau - x_\beta) = (6-4)(7-4) = (2)(3) = 6 \neq 0\,(mod\ 11)$ hence the transposition error detected.

Alternately, Proposition 4.3.4 can be proved by

Proof. Suppose $X = x_1, x_2, ..., x_8$ is the ISSN code word and the received vector $Y = x_1, x_2, ..., x_\beta, .., x_\tau, ..., x_8$ with x_τ and x_β interchanged digits. Then

$$
\begin{aligned}
&(\sum_{i=1}^{8} (2+i)y_i) \\
=&(\sum_{i=1}^{8} \sum_{j=9-i} (11-j)x_i) + (\beta-\tau)x_\tau + (\tau-\beta)x_\beta \neq 0\,(mod\ 11) \\
=&(\beta-\tau)(x_\tau - x_\beta) \neq 0\,(mod\ 11)
\end{aligned}
$$

provided $\tau \neq \beta$ and $x_\tau \neq x_\beta$.Therefore transposition error of digits x_τ and x_β detected. □

Corollary 4.3.6. *If a transposition error has occurred in an ISSN code word but the weight checksum equation holds then the transposed digits are equal.*

Example 4.3.7. Suppose that 03177475 is the ISSN code word. Now if x_4 and x_5 are adjacently transposed then yields the same ISSN code word 03177475. Therefore, the weight checksum equation $\sum_{i=1}^{8} \sum_{j=9-i} jx_i \equiv 0\,(mod\,11)$ holds.

Corollary 4.3.8. *ISSN code detects any jump transposition error in a code word.*

4.3.3 Double, twin, jump twin and phonetic error detection in an ISSN code word

ISSN code detects an error in a code word iff the weight checksum equation does not. However, the code cannot identify following errors in a code word:

i. Double error.

ii. Twin error.

iii. Jump twin error.

iv. Phonetic error.

v. Multiple transposition errors.

4.3.4 Omission and insertion errors detection in an ISSN code word

Omission and insertion errors in a ISSN code word can only be detected by counting the number of elements to confirm if they are more or less than eight.

4.3.5 Effectiveness of ISSN code in error detection capability

Proposition 4.3.9. *ISSN code is not effective in error detection.*

Proof. ISSN code only detects error(s) in a code word if and only if the weight checksum equation does not hold. This implies that ISSN cannot detect silent error(s) in a code word. Moreover, by use of the weight checksum equation the ISSN coding scheme only shows there is an error in the code word but cannot detect the exact location of the corrupted digit(s) or detect the type of error(s) in a code word. □

4.4 Error correction in an ISSN code word

4.4.1 Correction of single error in an ISSN code word

Proposition 4.4.1. *ISSN code corrects a single error only if the position of the error in a code word is known.*

Proof. Suppose $X = x_1, x_2, ..., x_8$ is an ISSN code word. If a single error has occurred at digit x_τ then the received vector $Y = x_1, x_2, ..., x_{\tau-1}, y_\tau, x_{\tau+1}, ..., x_8$ with $y_\tau = x_\tau + \alpha \quad \alpha \neq 0$. If the error has been detected and the position of the error is known. Then the error can be corrected by

$$\sum_{i=1\ i\neq\tau\ j=9-i}^{8} jx_i + (9-\tau)y_\tau \equiv 0\,(mod\,11)$$

$$(9-\tau)y_\tau \equiv -\left[(\sum_{i=1\ i\neq\tau\ j=9-i}^{8} jx_i)(mod\ 11)\right]$$

$$y_\tau = (9-\tau)^{-1} \times -\left[(\sum_{i=1\ i\neq\tau\ j=9-i}^{8} jx_i)(mod\ 11)\right].$$

Since the error had occurred in digit x_τ, this digit is replaced by y_τ then the weight checksum equation checked. □

Example 4.4.2. Let the ISSN code word be 2 0 4 9 3 6 3 0. Assume that there is an error in digit x_5. If the code word with the error is 2 0 4 9 7 6 3 0. Then

$$\sum_{i=1\ j=9-i}^{8} jx_i \neq 0\,(mod\,11)$$

$$137 \neq 0\,(mod\,11)$$

$$5 \neq 0\,(mod\,11)$$

therefore the error is corrected by solving for y_5 in the ISSN code word 2 0 4 9 y_5 6 3 0.

$$\sum_{i=1\ i\neq\tau\ j=9-i}^{8} jx_i + (9-\tau)y_\tau \equiv 0\,(mod\,11)$$

$$4y_5 \equiv -109\,(mod\,11)\ \ since\,\tau = 5$$

$$y_5 = 4^{-1} \times 1\ (mod\ 11)$$

$$y_5 = 3 \times 1\ (mod\ 11)$$

$$y_5 = 3.$$

Hence the error is corrected.

Corollary 4.4.3. *If a single error has occurred in an ISSN code word and the position of the error is unknown then the error cannot be corrected, however, by use of the weight checksum equation compute a new check digit for the received vector. In this case the error is corrected by generating a new code word rather than the original code word.*

4.4.2 Correction of transposition error in an ISSN code word

Proposition 4.4.4. *ISSN code corrects only a single adjacent transposition error in a code word.*

Proof. Suppose $X = x_1, x_2, ..., x_8$ is the ISSN code word and $Y = x_1, x_2, ..., x_\beta, x_\tau, ..., x_8$ is the received vector with x_τ and x_β adjacently transposed digits then

$$
\begin{aligned}
&(\sum_{i=1}^{8} \sum_{j=9-i} jy_i) \\
=&(\sum_{i=1}^{8} \sum_{j=9-i} jx_i) + (\tau - \beta)x_\tau + (\beta - \tau)x_\beta \\
=&(\tau - \beta)(x_\tau - x_\beta) \neq 0 \,(mod\, 11)
\end{aligned}
$$

Since x_τ and x_β are adjacently transposed digits then $\tau - \beta = 1$. The received vector Y has an error therefore the weighted checksum equation does not hold, therefore

$$
\begin{aligned}
(\sum_{i=1}^{8} \sum_{j=9-i} jy_i) &\neq 0 \,(mod\, 11) \\
(\sum_{i=1}^{8} \sum_{j=9-i} jy_i) &\equiv H(mod\, 11)
\end{aligned}
$$

where H is the error that has occurred due to the adjacently transposition of digits x_τ and x_β, therefore $-9 \leq H \leq 9, H \neq 0$.

If $H > 0$ then this implies that $x_\beta > x_\tau$ hence $x_\beta - x_\tau = H$. To find the position of x_β and x_τ, consider the received vector $Y = x_1, x_2, ..., , x_\beta, x_\tau, ..., x_8$ then find the position where there is $x_i - x_{i+1} = H$. Since there is only a single adjacent transposition error in the received vector Y, in case there are more than one positions with$x_i - x_{i+1} = H$,all cases are considered to get the correct position. Finally after finding the position of the transposed digits, the transposition error is corrected by just simply interchanging the transposed digits, then the weight checksum equation $\sum_{i=1}^{8} \sum_{j=9-i} jx_i \equiv 0 \,(mod\, 11)$ is checked. If $H < 0$ then this implies that $x_\beta < x_\tau$ hence $x_\beta - x_\tau = -H$. To find the position of x_β and x_τ, consider the received vector $Y = x_1, x_2, ..., , x_\beta, x_\tau, ..., x_8$ then find the position where there is $x_i - x_{i+1} = -H$. Finally after finding the position of the transposed digits, the transposition error is corrected by just simply interchanging the transposed digits, then the weight check equation $\sum_{i=1}^{8}\sum_{j=9-i} jx_i \equiv 0\,(mod\,11)$ is checked. $\square$

Remark 4.4.5. *If after the transposition error is corrected when $H > 0$ but $\sum_{i=1}^{8}\sum_{j=9-i} jx_i \neq 0\,(mod\,11)$ then consider $H < 0$. This can be shown by use of two examples, where the first example is when H>0 and the other example when H<0.*

Example 4.4.6. When $H > 0$

Suppose X =03178471 is an ISSN code word. Now if x_4 and x_5 are adjacently transposed then the received vector is $Y = 03187471$,yielding $x_4 = 8$ and $x_5 = 7$. To correct this transpose error:

$$(\sum_{i=1}^{8}\sum_{j=9-i} jy_i) \neq 0(mod\ 11) = 1\ (mod\,11)$$

$$H = 1$$

since $H > 0$ this implies $x_\beta > x_\tau$ therefore consider $x_i - x_{i+1} = 1$ in the code word $Y = 03187471$. Here there is only one case where $x_i - x_{i+1} = 1$ that is $8 - 7$ in the position x_4 and x_5. To correct the error interchange 8 and 7 yielding 03178471 then weight checksum equation $\sum_{i=1}^{8}\sum_{j=9-i} jx_i \equiv 0\,(mod\,11)$ holds, therefore transposition error corrected.

When $H < 0$

Suppose X =03178471 is an ISSN code word. Now if x_2 and x_3 are adjacently transposed then the received vector is $Y = 01378471$,yielding $x_2 = 1$ and $x_3 = 3$. To correct this transpose error:

$$(\sum_{i=1}^{8}\sum_{j=9-i} jy_i) \neq 0(mod\ 11) = 9\ mod\ 11\ or = -2\,(mod\,11)$$

$$\Rightarrow H = 9\ or\ -2$$

For $H = 9$ does not work since $Y = 01378471$ for there is no digits in Y such that $x_i - x_{i+1} = 9$. Consider $H = -2$ it implies that $x_\beta < x_\tau$ therefore $x_i - x_{i+1} = -2$. Here there is only one case where $x_i - x_{i+1} = -2$ that is $1-3$ in the position x_2 and x_3. To correct the error interchange 3 and 1 yielding 01378471 then weight check equation $\sum_{i=1}^{8}\sum_{j=9-i} jx_i \equiv 0\,(mod\,11)$ holds, therefore transposition error corrected.

Alternately, proposition 4.4.4 can be proved by

Proof. Suppose $X = x_1, x_2, ..., x_8$ is an ISSN code word and the received vector $Y = x_1, x_2, ..., , x_\beta, x_\tau, ..., x_8$ with x_τ and x_β the adjacent transposed digits, then

$$
\begin{aligned}
&(\sum_{i=1}^{8}\sum_{j=9-i} (11-j)y_i) \\
=&(\sum_{i=1}^{8}\sum_{j=9-i} (11-j)x_i) + (\beta-\tau)x_\tau + (\tau-\beta)x_\beta \\
=&(\sum_{i=1}^{8} (2+i)y_i) \\
=&(\sum_{i=1,}^{8} (2+i)x_i) + (\beta-\tau)x_\tau + (\tau-\beta)x_\beta \\
=&(\beta-\tau)(x_\tau - x_\beta) \neq 0\,(mod\ 11)
\end{aligned}
$$

Since $\tau - \beta = 1 \Rightarrow \beta - \tau = -1$. The received vector Y has an error therefore the weighted checksum equation does not hold, therefore

$$
\begin{aligned}
\sum_{i=1}^{8} (2+i)\, y_i \neq 0\,(mod\,11) \\
(\sum_{i=1}^{8} jy_i) = H\,(mod\,11)
\end{aligned}
$$

where H is the error that has occurred due to the adjacently transposition of digits x_τ and x_β, therefore $-9 \leq H \leq 9, H \neq 0$.

If $H > 0$ then this implies that $x_\beta > x_\tau$ hence $x_\beta - x_\tau = H$. To find the position of x_β and x_τ, consider the ISSN code word $Y = x_1, x_2, ..., x_\beta, x_\tau, ..., x_8$ then find the position where there is $x_i - x_{i+1} = H$. Finally after finding the position of the transposed digits, the transposition error is corrected by just simply interchanging the transposed digits, then the weight check equation $\sum_{i=1}^{8}(2+i)x_i \equiv 0\,(mod\,11)$ is checked. If $H<0$ then this implies that $x_\beta < x_\tau$ hence $x_\beta - x_\tau = -H$. To find the position of x_β and x_τ, consider the received vector $Y = x_1, x_2, ..., x_\beta, x_\tau, ..., x_8$ then find the position where there is $x_i - x_{i+1} = -H$. Finally after finding the position of the transposed digits, the transposition error is corrected by just simply interchanging the transposed digits, then the weight check equation $\sum_{i=1}^{8}(2+i)x_i \equiv 0\,(mod\,11)$ is checked. □

Remark 4.4.7. If after the transposition error is corrected when $H>0$ but $\sum_{i=1}^{8}(2+i)x_i \neq 0\,(mod\,11)$ then consider $H<0$.

Example 4.4.8. When $H>0$

Suppose X =$0\,3\,1\,7\,8\,4\,7\,1$ is an ISSN code word. Now if x_4 and x_5 are adjacently transposed then the received vector is $Y = 0\,3\,1\,8\,7\,4\,7\,1$,yielding $x_4 = 8$ and $x_5 = 7$. To correct this transpose error:

$$\sum_{i=1}^{8}(2+i)x_i \neq 0\,(mod\;11) = 1\,(mod\,11)$$
$$\Rightarrow H = 1$$

since $H>0$ this implies $x_\beta > x_\tau$ therefore consider $x_i - x_{i+1} = 1$ in the code word $Y = 0\,3\,1\,8\,7\,4\,7\,1$. Here there is only one case where $x_i - x_{i+1} = 1$ that is $8-7$ in the position x_4 and x_5. To correct the error interchange 8 and 7 yielding $0\,3\,1\,7\,8\,4\,7\,1$ then weight check equation $\sum_{i=1}^{8}(2+i)x_i \equiv 0\,(mod\,11)$ holds, therefore transposition error corrected.

When $H<0$

Suppose X =$0\,3\,1\,7\,8\,4\,7\,1$ is an ISSN code word. Now if x_2 and x_3 are adjacently transposed then the received vector is $Y = 0\,1\,3\,7\,8\,4\,7\,1$, yielding $x_2 = 1$ and $x_3 = 3$. To correct this transpose error:

$$\sum_{i=1}^{8}(2+i)x_i \neq 0\,(mod\ 11)$$

$$9\ (mod\,11)\ or$$

$$-2\ (mod\,11)$$

$$H = 9\ or$$

$$H = -2$$

For $H = 9$ does not work since $Y = 01378471$ for there is no digits in Y such that $x_i - x_{i+1} = 9$. Consider $H = -2$ it implies that $x_\beta < x_\tau$ therefore $x_i - x_{i+1} = -2$. Here there is only one case where $x_i - x_{i+1} = -2$ that is $1-3$ in the position x_2 and x_3. To correct the error interchange 3 and 1 yielding 0 1 3 7 8 4 7 1 then weight checksum equation $\sum_{i=1}^{8}(2+i)x_i \equiv 0\,(mod\,11)$ holds, therefore transposition error corrected.

Corollary 4.4.9. *ISSN code cannot correct a transposition error if it is not an adjacent transposition error. Moreover, the code cannot correct multiple transposition errors in a code word.*

4.5 Number of code words in ISSN code

Proposition 4.5.1. *For a $q-$ary code the total number of the code words generated depends on the following factors:*

i. The length n of each code word.

ii. The order of the field $GF(q)$.

iii. Whether repetition of digits is allowed.

iv. The defined permutation.

Proof. Suppose $X = x_1, x_2, ..., x_n$ is an arbitrary code word of a $q-$ary code. Each element of the code word$X$ is chosen from the field $GF(q)$, therefore there are q ways of choosing the first element of the code word X. Choosing the second digit to the last digit of the code word X depends on the defined permutation (the defined order of arranging the elements of the code word) and whether the repetition of elements in the code word X is allowed. Considering the four factors then the total number of code words which can be generated by a $q-$ary code is given by total number of choosing n elements of a code word X. □

Proposition 4.5.2. *ISSN code has a dictionary of maximum* 10^7 *number of code words*

Proof. An ISSN code word has a length of 8. Each elements of ISSN code word is chosen from $\{0,1,2,...,9\}$ except for the check digit number where $A = 10$ is used. The eighth digit(check digit) number does not permute for it is not chosen but calculated on the basis of weight check equation. There is no any defined permutation of elements of an ISSN code word. Repetition of the digits is allowed therefore there are 10^7 permutations yielding the dictionary of ISSN be $10^7 = 10,000,000$. □

4.6 Effectiveness of the ISSN code as an error correcting scheme

The effectiveness of the International Standard Serial Number (ISSN)in error detection and correction capabilities is determined by the following;

4.6.1 Durability of the ISSN code

From ISSN network statistics by January 2019, more than 2.5 million ISSNs had been issued. The number increases by approximately $60,000$ to $70,000$ ISSNs annually. However, about $130,000$ ISSNs are changed and corrected annually. ISSN is only associated with the title of the publication. Therefore in case a publication is modified appreciably, a new ISSN has to be assigned. This means that if a publication title changes ten times the same publication has to be assigned ten new ISSN. It is clearly evident that the number of ISSNs changed and corrected is about double the number of ISSN increase per year.

Additionally, the entire ISSN coding scheme has a relatively small dictionary as shown in proposition 4.5.2. Now, with the dictionary of $10,000,000$ code words and currently more than 2.5 million ISSNs have been issued by January 2019. This implies that with about 70,000 code words issued per year and $130,000$ ISSNs changes and correction annually, the remaining *ISSN* code words can last for maximum of 35 years. With high number of publications increase, currently ISSN coding scheme is not durable.

4.6.2 Precision of the ISSN code words

ISSN code is computed modulo 11, therefore each element of the code word should be chosen from the set $F_{11} = \{0,1,2,...,9,10\}$ but this is not the case. The element 10 has two digits 1and 0 , therefore, not applicable as an element of the code word. However, in the case when the check digit is 10, an alphabetic letter is used. An alphabetic letter is used to ensure that the length of the ISSN codes remains eight. The usage of the alphabetic letter only in the check digit but not is an element of the entire code word makes the ISSN code to loss its brevity and precision. ISSN uses only numeric digits except in check digit where alphabetic digit is used. Therefore there is a lot of similarities of the elements from one code word to another.

4.6.3 Reliability in the utilization of the ISSN code words

From the properties of ISSN in error detection and correction capabilities results, it is clear that the following deductions holds;

i. ISSN code can detect any error in a code word if and only if the weight check sum equation does not hold.

ii. ISSN code can detect and correct all single error in a code word if and if the position of the error is known.

iii. ISSN code can detect and correct adjacent transposition error in a code word even if the position of errors is unknown.

iv. ISSN code can detect all transposition errors in a code word but only corrects a single adjacent transposition error in a code word.

v. ISSN code can neither detect nor correct multiple transposition errors in a code word.

vi. ISSN code can neither detect nor correct silent error(s) in a code word.

vii. ISSN code can neither detect nor correct multiple errors in a code word.

viii. ISSN code cannot correct omission and insertion errors in a code word.

ix. ISSN code can neither identify nor correct the following errors in a code word;

a. Twin error.

b. Jump twin error.

c. Phonetic error.

d. double error.

Corollary 4.6.1. *ISSN code is not an effective error coding scheme*

Proof. From section 4.6, effectiveness of ISSN code as an error correcting coding scheme, ISSN code does not satisfy durability and precision conditions for a good error coding scheme. Moreover, it is clear that ISSN is not reliable in error correction for it only corrects a single error and a transposition error in a code word. Therefore, ISSN code is not effective in error correction capabilities. □

CHAPTER FIVE

MODIFIED ISSN CODE

5.1 Introduction

The Modified ISSN code is developed and generated by the use of functions, permutations and the properties of the existing ISSN code. The code is developed on the basis of a good communication channel and an effective error coding scheme. A good communication channel should have a good error coding scheme which is excellent in error detection and correction capabilities. Moreover, a good error coding scheme should be durable, precise and reliable.

5.2 Development and generation of modified ISSN

i. The Modified ISSN code is an alphanumerical code, that is, it is made of alphabetical digits and numerical digits.

ii. Each digit of the Modified ISSN code is chosen from the set F_{31}. The set F_{31} is the set $\mathbb{Z}_{31} = \{0,1,2,...,30\}$. Each code word of the modified ISSN code has a length of 8 digits, where each digit chosen from the set F_{31}. Choosing the elements of Modified ISSN alphabetical digits O and I are omitted for O resembles numerical digit 0 and I resembles 1 hence to avoid confusion the two digits are omitted. Thus the set F_{31}= $\{0,1,2,...,8,9,A,B,C,...,U,V,W\}$ without alphabetical digits I and O. F_{31} is the largest finite field which can be generated by alphabetical digits and numerical digits. The main reason of using alphabetical digits is to replace two digit numerical digits hence avoiding confusing and enhancing preciseness.

iii. The digits of modified ISSN code word are generated by the generating equation defined by $x_{i+1} = 3x_i + 5\,(mod\,31)$, $i = 1,2,...,6$ except the last digit of the code word. The last digit of code word is the check digit hence is computed. The first term(digit) of the generating equation is chosen randomly from the $F_{31} \cong \mathbb{Z}_{31}$.

iv. It is not a must for the generating equation defined by $x_{i+1} = 3x_i + 5\,(mod\,31)$, $i = 1,2,...,6$ to start from $i = 1$ to $i = 6$. The generating equation can be terminated within a code word. In the case where the generating equation is terminated, digit 0 is inserted between the last digit of the terminated sequence and the first digit of the other generating sequence . The first term(digit) of the other generating equation is also chosen randomly from the $F_{31} \cong \mathbb{Z}_{31}$.

v. The digit 0 does not have to necessarily be used when a sequence generated by the generating equation has been terminated, it may be used as an element of the code word itself. That is, digit 0 may be generated by the generating equation or may be at the beginning of the generating equation. In this case, digit 0 acts as a neutral digit. This implies that the generated sequence does not necessarily terminate once there is a digit 0.

vi. It is not a must for the generating equation defined by $x_{i+1} = 3x_i + 5\,(mod\,31)$, $i = 1,2,...,6$ to start at the beginning of a code word. In the case where the generating equation does not start at the beginning of a code word, the first digit of sequence is repeated from the beginning of a code word until where the start digit of the generating equation starts. Moreover, it is not a must for the generating equation to start at the beginning immediately after digit 0 in case the generated sequence had been terminated. In the case where the generating equation $x_{i+1} = 3x_i + 5(mod\,31)$ $i = 1,2,...,6$ does not start after digit 0, the first digit of the new sequence is repeated after digit 0 until where the start digit of generating equation starts.

vii. ISSN codes are only associated with the title of a publication, in case changes and corrections are made to the title of a publication, the code word will have an extra digit called a blind digit. A blind digit is separated from the check digit by a hyphen. A blind digit does not affect the weight check sum of the entire code word. It is given depending on the number of changes made in the title of a publication. Importance of the blind digit is to help to know the number of times the changes and corrections have been made to a title of a publication.

viii. The eight digits of the modified ISSN code must satisfy the weight checksum equation $\sum_{i=1}^{8} \sum_{j=9-i} jx_i \equiv 0\,(mod\,31)$ or $\sum_{i=1}^{8} (22+i)x_i \equiv 0\,(mod\,31)$.

Examples of modified ISSN code;

$1\,8\,V\,W\,2\,B\,7\,N$ $\qquad L\,3\,0\,R\,J\,U\,0\,3$

$B\,B\,B\,B\,B\,7\,R\,2$ $\qquad 3\,E\,G\,N\,9\,1\,8\,L-2$

$S\,M\,0\,V\,W\,2\,B\,3$ $\qquad V\,W\,2\,B\,0\,T\,Q\,D-3$

Table 5.1: **Additive inverse** $(mod\,31)$

ξ	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
x_8	W	V	U	T	S	R	Q	P	N	M	L	K	J	H	G

ξ	G	H	J	K	L	M	N	P	Q	R	S	T	U	V	W
x_8	F	E	D	C	B	A	9	8	7	6	5	4	3	2	1

Table 5.2: **Multiplicative inverse** $(mod\,31)$

x	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
x^{-1}	1	G	M	8	R	S	9	4	7	U	H	D	C	L	V

x	G	H	J	K	L	M	N	P	Q	R	S	T	U	V	W
x^{-1}	2	B	K	J	E	3	Q	T	N	5	6	P	A	F	W

Where

Table 5.3: **Elements of modified ISSN code**

0	1	2	3	4	5	6	7	8	9	A=10
B=11	C=12	D=13	E=14	F=15	G=16	H=17	J=18	K=19	L=20	M=21
N=22	P=23	Q=24	R=25	S=26	T=27	U=28	V=29	W=30		

5.2.1 Calculation of the check digit in a modified ISSN code word

Let the code word for modified ISSN be $X = x_1, x_2, ..., x_7$ without the check digit. To compute the check digit, x_8, Since the code word digits $x_1, x_2, ..., x_7$ are known, calculate $\sum_{i=1}^{7}\sum_{j=9-i} jx_i$. Let $\xi \equiv \sum_{i=1}^{7}\sum_{j=9-i} jx_i\ (mod\,31)$ then $x_8 + \xi \equiv 0\,(mod\ 31)$. Now since 0 is additive identity of $\mathbb{Z}_n$, therefore x_8 is the additive inverse of $\xi\,(mod\ 31)$. Thus, the check digit $x_8 \equiv -\xi\,(mod\ 31)$.

Example 5.2.1. Calculate the check digit for the modified ISSN code word $1\ 8\ V\ W\ \ 2\ B\ 7\ x_8$. To compute x_8, calculate $\xi \equiv \sum_{i=1}^{7}\sum_{j=9-i} jx_i\ (mod\,31) = 443\,(mod\,31) = 9\,(mod\,31)$. Find $-9\ (mod\,31)$,

$$x_8 = 31 - \xi$$
$$x_8 = 31 - 9 = 22 = N.$$

Alternatively, the check digit for a modified ISSN code word can be calculated as follows;

Let the code word for modified ISSN be $X = x_1, x_2, ..., x_7$ without the check digit. To compute the check digit, x_8, Since the code word digits $x_1, x_2, ..., x_7$ are known, calculate $\sum_{i=1}^{7} (22+i)x_i$. Let $\xi \equiv \sum_{i=1}^{7} (22+i)x_i \,(mod\,31)$ then $x_8 = \xi$.

Example 5.2.2. Calculate the check digit for the modified ISSN code word 1 8 V W 2 B 7 x_8. To compute x_8, calculate $\xi \equiv \sum_{i=1}^{7} (22+i)x_i \,(mod\,31) = 2285\,(mod\,31) = 22\,(mod\,31)$. Hence, $x_8 = 22 = N$.

Algorithm 5.1 ***How to generate a modified ISSN code word***

---

i.Choose randomly the first digit X(1) of the modified ISSN code word from the set $\mathbb{Z}_{31}$.
ii. To generate the second to seventh digit of the modified ISSN code word use the generating equation $x_{i+1} = 3x_i + 5\,(mod\,31), i = 1, 2, ..., 6$.
iii. If the generating equation does not start at X(1), the digit at X(1) is repeated until where the generating equation starts.
iv. If the sequence generated by the generating equation terminates, digit zero is inserted between the last digit of the generated sequence and the first of the next sequence. Step i and step ii is repeated to generate the new sequence by the generating equation.
v. After generating seven digits of the modified ISSN code word compute the check digit. Check digit is computed by calculating the summation $\sum_{i=1}^{7} (22+i)x_i$, where x_i are the seven generated digits of the modified ISSN code word. Convert the summation $\sum_{i=1}^{7} (22+i)x_i$ into $(mod\,31)$ and equate it to ξ . The check digit is ξ .
vi. Check whether the weight checksum equation $\sum_{i=1}^{8} (22+i)x_i \equiv 0\,(mod\,31)$. holds.
vii. Convert all the modified ISSN digits in the code word from $\mathbb{Z}_{31}$ into F_{31}.

5.2.2 Number of code words in modified ISSN code

Proposition 5.2.3. *Modified ISSN code has a dictionary of maximum* 28, 644, 217 *number of code words.*

Proof. The digits of the Modified ISSN code are chosen from the set F_{31}. The set F_{31} is the set $\mathbb{Z}_{31} = \{0, 1, 2, ..., 30\}$. Each code word of the modified ISSN code has a length of 8 symbols,where each symbol is chosen from the set F_{31}. Moreover, the eighth digit of the modified ISSN code is the check digit which is computed on the basis of weight checksum equation. The elements in each code word is generated by the generating equation defined by $x_{i+1} = 3x_i + 5\,(mod\,31)\; i = 1, 2, 3, ..., 6$. It is not a must for the sequence formula for the generation of the elements of a code word to start at the beginning of the code word. In the case where the generating equation does not start in the beginning of a code word, the first digit of generating equation is repeated from the first digit of the code word until where the start digit of generating equation starts. Total number of the code words depends on the generating equation and digit 0 for the termination of the generated sequence . Digit 0 for the termination of the generating equation can occur from the second digit of a code word to the seventh digit of a code word. There are 31 ways of choosing the first digit of a code word, two ways of choosing the second digit, that is, through generating equation or repetition if generating equation does not start in the beginning of a code word. From the third digit to the seventh digit there are three ways of choosing each digit. There is only one way of choosing the eighth digit for it is computed to satisfy the weight checksum equation. Moreover, if there is a digit 0 in second digit, there are 31 ways of choosing the third digit, similar to the fourth digit until the seventh digit. Therefore there are $31 \times 2 \times 3^5$ or 31^5 ways of choosing a code word, hence the dictionary of the modified ISSN code is $15066 + 28,629,151 = 28,644,217$. □

5.3 Properties of modified ISSN code

In this section we analyze the properties of the modified International Standard Serial Number in terms of error detection, correction and the size of the code dictionary. The effectiveness of the modified ISSN in error detection and correction will be determined by its properties in terms of error detection, correction and the size of the code dictionary.

5.3.1 Error detection in modified ISSN code

Modified ISSN code is an improved error coding scheme in terms of error detection and correction capabilities. The code has dual mechanism in terms of error detection and correction capabilities. Let the code word for modified ISSN be $X = x_1, x_2, ..., x_8$, then the weight checksum is computed $\sum_{i=1}^{8} (22 + i)x_i \equiv 0 \,(mod\, 31)$ or $\sum_{i=1}^{8} \sum_{j=9-i} jx_i \equiv 0 \,(mod\, 31)$. If $\sum_{i=1}^{8} (22+i)x_i \neq 0 \,(mod\, 31)$ then the code has detected error(s). Secondly, Let the code word for modified ISSN be $X = x_1, x_2, ..., x_8$, then $x_2 = (3x_1 + 5)\,(mod\, 31), x_3 = (3x_2 + 5)\,(mod\, 31), \quad x_4 = (3x_3 + 5)\,(mod\, 31), ..., x_7 = (3x_6 + 5)\,(mod\, 31)$. If $x_i \neq (3x_{i-1} + 5)\,(mod\, 31)$, $i = 2, 3, ..., 7$ then the code has detected error(s). An ISSN code word has a length of only eight digits unless there is change of the title of the publication where the code word has nine digits, therefore, if the code has a length of more or less than eight then there is an error.

Remark 5.3.1. $x_i \neq (3x_{i-1} + 5)\,(mod\, 31)$, $i = 2, 3, ..., 7$ does not necessarily indicate an error in the code word for there are two occasions where $x_i \neq (3x_{i-1} + 5)\,(mod\, 31)$, $i = 2, 3, .., 7$ will occur but no error has occurred in the code word:

a. In case there is a digit 0 indicating the termination of the generating equation $x_{i+1} = 3x_i + 5(mod\,31), i = 1,2,...,6$.

b. In case there is repetition of a digit indicating that the generating equation $x_{i+1} = 3x_i + 5(mod\,31), i = 1,2,...,6$ has not begun in the beginning of the code word. The code word has to satisfy the checksum weight equations $\sum_{i=1}^{8} \sum_{j=9-i} jx_i \equiv 0\,(mod\,31)$ and $\sum_{i=1}^{8} (22+i)x_i \equiv 0\,(mod\,31)$.

5.3.2 Single error detection in a modified ISSN code word

Proposition 5.3.2. *The modified ISSN code detects all single error in a code word.*

Proof. Suppose $X = x_1, x_2, ..., x_8$ is the modified ISSN code word and

$Y = x_1, x_2, ..., x_{\tau-1}, y_\tau, x_{\tau+1}, ..., x_8$ with $y_\tau = x_\tau + \alpha, \quad \alpha \neq 0$ is the received vector with a single error that has occurred in digit x_τ. Then

$$
\begin{aligned}
&\Big(\sum_{i=1}^{8} \sum_{j=9-i} jy_i\Big) \\
=&\Big(\sum_{i=1}^{8} \sum_{j=9-i} jx_i\Big) + (9-\tau)\,\alpha \\
=&\,(9-\tau)\,\alpha \neq 0\,(mod\ 31).
\end{aligned}
$$

Moreover, $y_\tau \neq (3x_{\tau-1}+5)\,(mod\,31)$.Therefore the single error is detected. □

Proposition 5.3.3. *The Modified ISSN code detects all silents error in the code word.*

Proof. Suppose a single error has occurred in a modified ISSN code word but the weight checksum equation holds, $\sum_{i=1}^{8} \sum_{j=9-i} jx_i \neq 0\,(mod\,31)$. Then if

$Y = x_1, x_2, ..., x_{\tau-1}, y_\tau, x_{\tau+1}, ..., x_8$ with $y_\tau = x_\tau + \alpha \quad \alpha \neq 0$ is the received vector with a single error that has occurred in the position x_τ Then

$$\begin{aligned}
&(\sum_{i=1}\sum_{j=9-i}^{8} jy_i)\\
=&(\sum_{i=1}\sum_{j=9-i}^{8} jx_i)+(9-\tau)\,\alpha\\
=&(9-\tau)\,\alpha\,(mod\,31)\equiv 0\,(mod\,31).
\end{aligned}$$

This implies that either $9-\tau$ or α is a multiple of 31 or 0. Since $9-\tau$ cannot be a multiple of 31 or 0 hence $\alpha = 0$ and no error. However, since there is an error in the code word, the generating equation $x_{i+1} \neq (3x_i+5)\,(mod\,31), i = 1,2,..,6$. Therefore $x_\tau \neq (3x_{\tau-1}+5)\,(mod\,31)$ and hence the silent error detected. Conversely, Suppose a single error has occurred in modified ISSN code but $x_i = (3x_{i-1}+5)\,(mod\,31)$ for $i = 2,3,..,7$, then $\sum_{i=1}\sum_{j=9-i}^{8} jx_i \neq 0\,(mod\,31)$ then the silent error is detected. □

Corollary 5.3.4. *The Modified ISSN code corrects a single error in a code word even if the position of the error in the code word is unknown.*

Proof. Suppose $X = x_1, x_2, ..., x_8$ is the modified and

$Y = x_1, x_2, ..., x_{\tau-1}, y_\tau, x_{\tau+1}, ..., x_8$ with $y_\tau = x_\tau + \alpha, \quad \alpha \neq 0$ is the received vector with a single error that has occurred in digit y_τ. The most important thing is the code to detect the position of the digit with error. From second digit to the second last digit of the code word is generated by the generating equation $x_{i+1} = 3x_i + 5\,(mod\,31), i = 1,2,..,6$ unless there is a repetition of digits or digit 0 indicating termination of the generating equation. By use of the generating equation $x_{\tau+1} = 3x_\tau + 5\,(mod\,31)$ the digit $y_\tau = x_\tau + \alpha \quad \alpha \neq 0$ is detected by $y_\tau \neq (3x_{\tau-1}+5)\,(mod\,31)$. After the position of y_τ is detected, the error is corrected by the computing $y_\tau = (3x_{\tau-1}+5)\,(mod\,31)$ and $y_\tau = (\frac{x_{\tau+1}-5}{3})\,(mod\,31)$. In case $x_{\tau+1} < 5$ or $(x_{\tau+1}-5)$ not divisible by 3 then

$$\begin{aligned}
&3y_\tau \equiv x_{\tau+1} - 5\ (mod\,31)\\
&3^{-1}y_\tau = 3^{-1} \times ((x_{\tau+1}-5)\ (mod\,31))\\
&y_\tau = 21 \times ((x_{\tau+1}-5)\ (mod\,31))
\end{aligned}$$

when $x_{\tau+1} < 5$

$$y_\tau = 21 \times ((31 + (x_{\tau+1}-5))\ (mod\,31))$$

Then the weight checksum equation $\sum_{i=1}^{8}\sum_{j=9-i} jx_i \equiv 0\,(mod\,31)$ is tested whether it holds. Additionally, when the error has been detected and the position of the error is known. Then the error can be corrected by

$$\sum_{i=1}^{8}\sum_{i\neq\tau}\sum_{j=9-i} jx_i + (9-\tau)y_\tau \equiv 0\,(mod\,31)$$

$$(9-\tau)y_\tau \equiv -\left[\Big(\sum_{i=1}^{8}\sum_{i\neq\tau}\Big(\sum_{j=9-i} jx_i\Big)\,(mod\,31)\right]$$

$$y_\tau = (9-\tau)^{-1} \times -\left[\Big(\sum_{i=1}^{8}\sum_{i\neq\tau}\Big(\sum_{j=9-i} jx_i\Big)\,(mod\,31)\right]$$

Since $y_\tau = x_\tau + \alpha, \alpha = 0$ hence $y_\tau = x_\tau$ yielding the original modified ISSN code word $X = x_1, x_2, ..., x_8$. □

Example 5.3.5. Let $1\,8\,V\,W\,2\,B\,7\,N$ be the modified ISSN code word and $1\,8\,V\,W\,C\,B\,7\,N$ be the received vector with a single error at x_5. By

$$\sum_{i=1}^{8}\sum_{j=9-i} jx_i \neq 0\,(mod\,31)$$

$$505 \neq 0\,(mod\,31)$$

$$9 \neq 0\,(mod\,31)$$

therefore the single error is detected. By use of the generating equation $x_{i+1} = (3x_i + 5)\,(mod\,31)$ the position of the error is found. The error is at x_5 for

$$C \neq (3W+5)\,(mod\,31)$$

$$12 \neq (3\times 30+5)\,(mod\,31)$$

$$12 \neq 95\,(mod\,31)$$

the error is corrected by the computing $y_\tau = (3x_{\tau-1} + 5)\,(mod\,31)$ and $y_\tau = (\frac{x_{\tau+1}-5}{3})\,(mod\,31)$

$$
\begin{aligned}
x_5 &= (3x_4+5) \Rightarrow x_5 = (3W+5)\,(mod\,31)\\
x_5 &= (3\times 30+5)\,(mod\,31)\\
x_5 &= 2.\\
Additionally \quad x_5 &= (\frac{x_6-5}{3})\ (mod\,31)\\
x_5 &= (\frac{B-5}{3})\ (mod\,31)\\
x_5 &= 2.
\end{aligned}
$$

Remark 5.3.6. In case $x_{\tau+1} < 5$ or $(x_{\tau+1}-5)$ not divisible by 3 . This can be demonstrated using an example

Example 5.3.7. Let 1 8 V W 2 B 7 N be the modified ISSN code word and 1 8 V 5 2 B 7 N be the received vector with a single error. By

$$
\begin{aligned}
&\sum_{i=1}^{8}\sum_{j=9-i} jx_i \neq 0\,(mod\,31)\\
&340 \neq 0\,(mod\,31)\\
&30 \neq 0\,(mod\,31)
\end{aligned}
$$

therefore the single error is detected. By use of the generating equation

$x_{i+1} = (3x_i+5)\ (mod\,31)$, the position of the error can be found. The error is at x_4 for

$$
\begin{aligned}
&5 \neq (3V+5)\,(mod\,31)\\
&5 \neq (3\times 29+5)\,(mod\,31)\\
&5 \neq 92\ (mod\,31)
\end{aligned}
$$

the error is corrected by the computing $y_\tau = (3x_{\tau-1} + 5)\,(mod\,31)$ and $y_\tau = (\frac{x_{\tau+1}-5}{3})\ (mod\,31)$

$$
\begin{aligned}
x_4 &= (3x_3+5)\,(mod\,31)\\
&= (3V+5)\,(mod\,31)\\
&= (3\times 29+5)\,(mod\,31\\
&= 92\,(mod\,31)
\end{aligned}
$$

and

$$
\begin{aligned}
&x_4 = (\frac{x_5-5}{3})\ (mod\,31)\\
&= (\frac{2-5}{3})\ (mod\,31)\,in\,this\,case\,(2<5)\\
&= 3^{-1}\times(31-3)\ (mod\,31)\\
&= 21\times 28\ (mod\,31)\\
&= 30\\
&= W.
\end{aligned}
$$

Hence the error is corrected.

Remark 5.3.8. Modified ISSN is more effective in single error detection and correction than ISSN code.

Algorithm 5.2 ***How to correct a single error in a modified ISSN code word***

i. Convert the modified ISSN code word digits into $\mathbb{Z}_{31}$.
ii. Check whether the weight checksum equation $\sum_{i=1\ j=9-i}^{8} jx_i \equiv 0\,(mod\,31)$ holds. If the checksum equation does not hold, error(s) are detected in the modified ISSN code word.
iii. Compute $x_{i+1} = (3x_i+5)\,(mod\,31)\,i=2,3,...,7$ if $x_{i+1} \neq (3x_i+5)\,(mod\,31)$ and $x_{i+2} \neq 9x_i+20\,(mod\,31)$. The position of the corrupted digit $X(i)$ is detected.
iv. Correct the corrupted digit $X(i)$ by computing $x_i = (3x_{i-1}+5)\,(mod\,31)$ and $x_i = (\frac{x_{i+1}-5}{3})(mod\,31)$.
v. Repeat step ii. If weight checksum holds, the single error in corrected.
vi. Convert the modified ISSN code word digits into F_{31}

5.3.3 Transposition error detection and correction in a modified ISSN code word

Proposition 5.3.9. *The Modified ISSN code detects all transposition errors in a code word.*

Proof. Suppose $X = x_1, x_2, ..., x_8$ is the modified ISSN code word and

$Y = x_1, x_2, ...,, x_\beta, ..., x_\tau, ..., x_8$ with x_τ and x_β the interchanged digits of the received vector. Then

$$
\begin{aligned}
&(\sum_{i=1}^{8} \sum_{j=9-i} jy_i) \\
=&(\sum_{i=1}^{8} \sum_{j=9-i} jx_i) + (\tau - \beta)x_\tau + (\beta - \tau)x_\tau \\
=&(\tau - \beta)(x_\tau - x_\beta) \neq 0 \,(mod\, 31)
\end{aligned}
$$

provided $\tau \neq \beta$ and $x_\tau \neq x_\beta$. Therefore transposition error of digits x_τ and x_β is detected. Alternatively, suppose $X = x_1, x_2, ..., x_8$ is the modified ISSN code word and $Y = x_1, x_2, ...,, x_\beta, ..., x_\tau, ..., x_8$ with x_τ and x_β the interchanged digits of the received vector. Transposition error is detected as errors at digit x_τ and digit x_β . Then

$$
\begin{aligned}
&x_i = (3x_{i-1} + 5)\,(mod\,31)\, i = 2, 3, ..., 7 \\
&x_\tau \neq (3x_{\tau-1} + 5)\,(mod\,31) \\
&x_{\tau+1} \neq (9x_\tau + 20)\,(mod\,31) \\
&and \\
&x_\beta \neq (3x_{\beta-1} + 5)\,(mod\,31) \\
&x_{\beta+1} \neq (9x_\beta + 20)\,(mod\,31).
\end{aligned}
$$

Therefore error detected at digits x_τ and x_β. □

Corollary 5.3.10. *The modified ISSN code detects an adjacent transposition error as follows:*

suppose $X = x_1, x_2, ..., x_8$ *is the modified ISSN code word and* $Y = x_1, x_2, ...,, x_\beta, x_\tau, ..., x_8$ with x_τ and x_β the adjacent interchanged digits of the received vector. Then

$$
\begin{aligned}
&x_i = (3x_{i-1} + 5)\,(mod\,31)\, i = 2, 3, ..., 7 \\
&x_\tau \neq (3x_\beta + 5)\,(mod\,31) \\
&and \\
&x_\beta \neq (\frac{x_\tau - 5}{3})\,(mod\,31)
\end{aligned}
$$

but

$$
\begin{aligned}
x_\beta &= (3x_\tau + 5)\,(mod\,31)\\
&and\\
x_\tau &= (\frac{x_\beta - 5}{3})\,(mod\,31).
\end{aligned}
$$

Therefore adjacent transposition error of digits x_τ and x_β detected.

Example 5.3.11. Let $1\,8\,V\,W\,2\,B\,7\,N$ be the modified ISSN code word and $1\,8\,V\,W\,B\,2\,7\,N$ be the received vector with adjacent transposed error at x_5 and x_6. Consider the received vector $1\,8\,V\,W\,B\,2\,7\,N$, the weight checksum equation $(\sum_{i=1}^{8}\sum_{j=9-i} jx_i \neq 0\,(mod\,31))$.

$$
\begin{aligned}
&\beta = 5, \tau = 6, x_\beta = 11, x_\tau = 2\\
&(\tau - \beta)(x_\tau - x_\beta) \neq 0\,(mod\ 31)\\
&(6-5)(2-11) = -9 \neq 0\,(mod\,31)
\end{aligned}
$$

Moreover, for

$$
\begin{aligned}
2 &\neq (3 \times 11 + 5)\,(mod\,31)\\
38\,(mod\,31) &= 7
\end{aligned}
$$

and $11 \neq (\frac{2-5}{3})\,(mod\,31)$ but $11 = (2 \times 3 + 5)(mod\,31)$ and $2 = (\frac{11-5}{3})$. Hence the transposed error detected.

Proposition 5.3.12. *The Modified ISSN code corrects all transposition errors in a code word.*

Proof. From the above proof(detection of transpose error) it is clear that

$$
\begin{aligned}
&x_i = (3x_{i-1} + 5)\,(mod\,31)\,i = 2,3,...,7\\
&x_\tau \neq (3x_{\tau-1} + 5)\,(mod\,31)\\
&x_{\tau+1} \neq (9x_\tau + 20)\,(mod\,31)\\
&and\\
&x_\beta \neq (3x_{\beta-1} + 5)\,(mod\,31)\\
&x_{\beta+1} \neq (9x_\beta + 20)\,(mod\,31).
\end{aligned}
$$

Thus implying errors at digits x_τ and x_β can be corrected by

$$
\begin{aligned}
&x_i = (3x_{i-1} + 5)\,(mod\,31)\,i = 2,3,...,7\\
&x_\tau = (3x_{\tau-1} + 5)\,(mod\,31)\\
&x_\tau = (\frac{x_{\tau+1} - 5}{3})\,(mod\,31)\\
&and\\
&x_\beta = (3x_{\beta-1} + 5)\,(mod\,31)\\
&x_\beta = (\frac{x_{\beta+1} - 5}{3})\,(mod\,31)
\end{aligned}
$$

□

Corollary 5.3.13. *The modified ISSN code corrects an adjacent transposition error as follows:*

$$
\begin{aligned}
&x_i = (3x_{i-1} + 5)\,(mod\,31)\,i = 2,3,...,7\\
&x_\tau \neq (3x_\beta + 5)\,(mod\,31)\\
&x_\beta \neq (\frac{x_\tau - 5}{3})\,(mod\,31)
\end{aligned}
$$

but

$$
\begin{aligned}
x_\beta &= (3x_\tau + 5)\,(mod\,31)\\
x_\tau &= (\frac{x_\beta - 5}{3})\,(mod\,31).
\end{aligned}
$$

Thus implying there is a transpose error which can be corrected by interchanging the digits x_τ and x_β.

Example 5.3.14. Consider the received vector 1 8 V W B 2 7 N. It is clear that $2 \neq (3 \times 11+5)\,(mod\,31)$ for $(3 \times 11+5)\,(mod\,31) = 38\,(mod\,31) = 7$ and $11 \neq (\frac{2-5}{3})\,(mod\,31)$ but $11 = (2 \times 3+5)\,(mod\,31)$ and $2 = (\frac{11-5}{3})$. Since the transposed error is detected the code can be corrected by transposing the digits yielding 1 8 V W 2 B 7 N by which the weight checksum equation holds ie $\sum_{i=1}^{8}\sum_{j=9-i} jx_i \equiv 0\,(mod\,31)$. Hence the error corrected.

Remark 5.3.15. Modified ISSN is more effective in transpose error detection and correction than ISSN code.

Algorithm 5.3 ***How to correct an adjacent transpose error in a modified ISSN cord word***

---

i. Convert the modified ISSN code word digits into $\mathbb{Z}_{31}$.
ii. Check whether the weight checksum equation $\sum_{i=1}^{8}\sum_{j=9-i} jx_i \equiv 0\,(mod\,31)$ holds. If the checksum equation does not hold, error(s) are detected in the modified ISSN code word.
iii . Compute $x_i = (3x_{i-1}+5)\,(mod\,31)$, $i = 2,3,...,7$ if $x_{i+1} \neq (3x_i+5)\,(mod\,31)$ and $x_i \neq (\frac{x_{i+1}-5}{3})(mod\,31)$ but $x_i = (3x_{i+1}+5)\,(mod\,31)$ and $x_{i+1} = (\frac{x_i-5}{3})(mod\,31)$.The transposition error in digits x_i and x_{i+1} are detected.
iv. Correct the adjacent transpose error digit x_i and x_{i+1} by interchanging the two digits in the received vector.
v. Repeat step ii. If weight checksum holds, the jump transposition error in corrected.
vi. Convert the modified ISSN code word digits into F_{31}

Corollary 5.3.16. *The Modified ISSN code detects jump transposition error in a code word.*

Proof. Suppose that $X = x_1,x_2,...,x_8$ is a modified ISSN code word and

$Y = x_1,..,x_{\tau+2},x_{\tau+1},x_\tau,...,x_8$ be the received vector with jump transposition error. The error can be detected by $x_{i+1} \neq (3x_{i+2}+5)\,(mod\,31), and\ x_{i+1} \neq (\frac{x_i-5}{3})(mod\,31)$

but $x_{i+1} = (3x_i+5)\,(mod\,31)$and $x_{i+1} = (\frac{x_{i+2}-5}{3})\,(mod\,31)$. Hence jump transposition error detected. □

Corollary 5.3.17. *The Modified ISSN code corrects all jump transposition errors in a code word.*

Proof. Suppose that $X = x_1,x_2,...,x_8$ is a modified ISSN code word and

$Y = x_1,..,x_{\tau+2},x_{\tau+1},x_\tau,..,x_8$ be the received vector with jump transposition error. Consider $Y = x_1,..,x_{\tau+2},x_{\tau+1},x_\tau,..,x_8$ once the error is detected by

$x_{i+1} \neq (3x_{i+2}+5)\,(mod\,31)$ and $x_{i+1} \neq (\frac{x_i-5}{3})\,(mod\,31)$ but $x_{i+1} = (3x_i+5)\,(mod\,31)$ and $x_{i+1} = (\frac{x_{i+2}-5}{3})\,(mod\,31)$. This implies that the error can be corrected by transposing x_i

and x_{i+2} and then test whether the weight checksum equation holds and the generating equation $x_i = (3x_{i-1}+5)\,(mod\,31)\; i = 2,3,...,7$ holds. □

Example 5.3.18. Let $1\,8\,V\,W\,2\,B\,7\,N$ be the modified ISSN and $1\,8\,2\,W\,V\,2\,7\,N$ be the received vector with jump transposed error at x_3 and x_5.The error is detected by the fact that $x_4 \neq (3x_5+5)\,(mod\,31), and\; x_4 \neq (\frac{x_3-5}{3})\,(mod\,31)$ but $x_4 = (3x_3+5)\,(mod\,31)$ and $x_4 = (\frac{x_5-5}{3})\,(mod\,31)$ therefore the jump transposition error can be corrected by interchanging x_3 and x_5 yielding $1\,8\,V\,W\,2\,B\,7\,N$ in which both $x_i = (3x_{i-1}+5)\,(mod\,31)\; i = 2,3,...,7$ and $\sum_{i=1\; j=9-i}^{8} jx_i \equiv 0\,(mod\,31)$ hold.

Algorithm 5.4 ***How to correct jump transposition error in a modified ISSN code word***

---

i.Convert the modified ISSN code word digits into $\mathbb{Z}_{31}$.
ii.Check whether the weight checksum equation $\sum_{i=1\; j=9-i}^{8} jx_i \equiv 0\,(mod\,31)$ holds. If the checksum equation does not hold, error(s) are detected in the modified ISSN code word.
iii.Compute $x_i = (3x_{i-1}+5)\,(mod\,31)\, i = 2,3,...,7$. If $x_{i+1} \neq (3x_i+5)\,(mod\,31)$ and $x_{i+1} \neq (\frac{x_{i+2}-5}{3})(mod\,31)$ but $x_{i+1} = (3x_{i+2}+5)\,(mod\,31)$ and $x_{i+1} = (\frac{x_i-5}{3})(mod\,31)$.The jump transposition error in digits x_i and x_{i+2} is detected.
iv Correct the jump transposition error interchange digits x_i and x_{i+2} in the modified ISSN code word.
v. Repeat step ii. If weight checksum holds, the jump transposition error is corrected.
vi. Convert the modified ISSN code word digits into F_{31}

5.3.4 Double error detection and correction in modified ISSN code

Proposition 5.3.19. *The Modified ISSN code detects and corrects all double errors in a code word.*

Proof. Suppose $X = x_1, x_2, ..., x_8$ is the modified ISSN code word and

$Y = x_1, ..., x_\tau, ..., x_\beta, ..., x_8$ be the received vector with double error in digit x_τ and x_β. Without loss of generality $Y = \sum_{i=1\; j=9-i}^{8} jx_i \neq 0\,(mod\,31)$ and error detected in the received vector Y. Suppose there is no error in Y, then $\sum_{i=1\; j=9-i}^{8} jx_i \equiv 0\,(mod\,31)$ and $x_i = (3x_{i-1}+5)(mod\,31)$ $, i = 2,3,...,7$, but this not the case. Since there is an error in Y, the double errors are detected by

$$
\begin{aligned}
&x_i = (3x_{i-1}+5)\,(mod\,31)\; i = 2,3,...,7\\
&x_\tau \neq (3x_{\tau-1}+5)\,(mod\,31)\\
&x_{\tau+1} \neq (9x_\tau+20)\,(mod\,31)\\
&and\\
&x_\beta \neq (3x_{\beta-1}+5)\,(mod\,31)\\
&x_{\beta+1} \neq (9x_\beta+20)\,(mod\,31).
\end{aligned}
$$

but for the rest $x_i = (3x_{i-1}+5)\,(mod\,31)$ holds. Since $x_{\tau-1} = (3x_{\tau-2}+5)\,(mod\,31)$ and $x_{\beta+1} = (\frac{x_{\beta+2}-5}{3})\,(mod\,31)$, the double errors in Y can be corrected by computing $x_\tau = (3x_{\tau-1}+5)\,(mod\,31)$ and $x_\beta = (\frac{x_{\beta+1}-5}{3})\,(mod\;31)$, then test whether the weight checksum equation holds. □

Example 5.3.20. Let $1\;8\;V\;W\;2\;B\;7\;N$ be the modified ISSN code word and $1\;8\;V\;5\;C\;B\;7\;N$ be the received vector with double error at x_4 and x_5. Since the weight checksum equation does not hold, the error is detected in the code word. The double error is detected by

$$
\begin{aligned}
C \neq& (3\times 5+5)\,(mod\,31)\\
C =& 12 \neq 20\,(mod\,31)\\
5 \neq& (\frac{C-5}{3})\,(mod\,31)\\
5 \neq& (3\times V+5)\,(mod\,31)\\
5 \neq& 30\,(mod\,31)
\end{aligned}
$$

and

$$
\begin{aligned}
C \neq& (\frac{B-5}{3})\,(mod\,31)\\
12 \neq& (\frac{11-5}{3})\,(mod\,31)
\end{aligned}
$$

Since

$$
\begin{aligned}
V =& (3\times 8+5)\,(mod\,31)\\
29 =& (3\times 8+5)\,(mod\,31)
\end{aligned}
$$

and

$$
\begin{aligned}
B &= (\frac{7-5}{3})\,(mod\,31)\\
11 =&3^{-1}\times 2\;(mod\,31)\\
11 =&21\;\times 2\;(mod\,31)\\
11 =&42\;(mod\,31)
\end{aligned}
$$

then

$$
\begin{aligned}
x_\tau &= (3\times V+5)\,(mod\,31)\\
x_\tau &= 30\\
x_\tau &= W
\end{aligned}
$$

and

$$
\begin{aligned}
x_\beta &= (\frac{B-5}{3})\,(mod\,31)\\
x_\beta &=2
\end{aligned}
$$

Therefore the code has detected and corrected double errors in the code word.

Algorithm 5.5 ***How to correct double errors in a modified ISSN code word***

i. Convert the modified ISSN code word digits into $\mathbb{Z}_{31}$.
ii. Check whether the weight checksum equation $\sum_{i=1\;j=9-i}^{8} jx_i \equiv 0\,(mod\,31)$ holds. If the checksum equation does not hold, error(s) are detected in the modified ISSN code word.
iii. Compute $x_i = (3x_{i-1}+5)\,(mod\,31)\,, i = 2,3,...,7$. If

$$
\begin{aligned}
&x_\tau \neq (3x_{\tau-1}+5)\,(mod\,31)\\
&x_{\tau+1} \neq (9x_\tau+20)\,(mod\,31)\\
&and\\
&x_\beta \neq (3x_{\beta-1}+5)\,(mod\,31)\\
&x_{\beta+1} \neq (9x_\beta+20)\,(mod\,31).
\end{aligned}
$$

but the rest $x_i = (3x_{i-1}+5)\,(mod\,31)$ holds. Double/twin errors at digit x_τ and x_β is detected.
iv. Since $x_{\tau-1} = (3x_{\tau-2}+5)\,(mod\,31)$ and $x_{\beta+1} = (\frac{x_{\beta+2}-5}{3})\,(mod\,31)$, the twin/ double errors are corrected by computing $x_\tau = (3x_{i-1}+5)\,(mod\,31)$ and $x_\beta = (\frac{x_{\beta+1}-5}{3})\,(mod\,31)$.
v. Repeat step ii. If weight checksum holds, the jump transposition error is corrected.
vi. Convert the modified ISSN code word digits into F_{31}

Remark 5.3.21. The relationship between error detection and error correction of Modified ISSN code is shown in the figure below

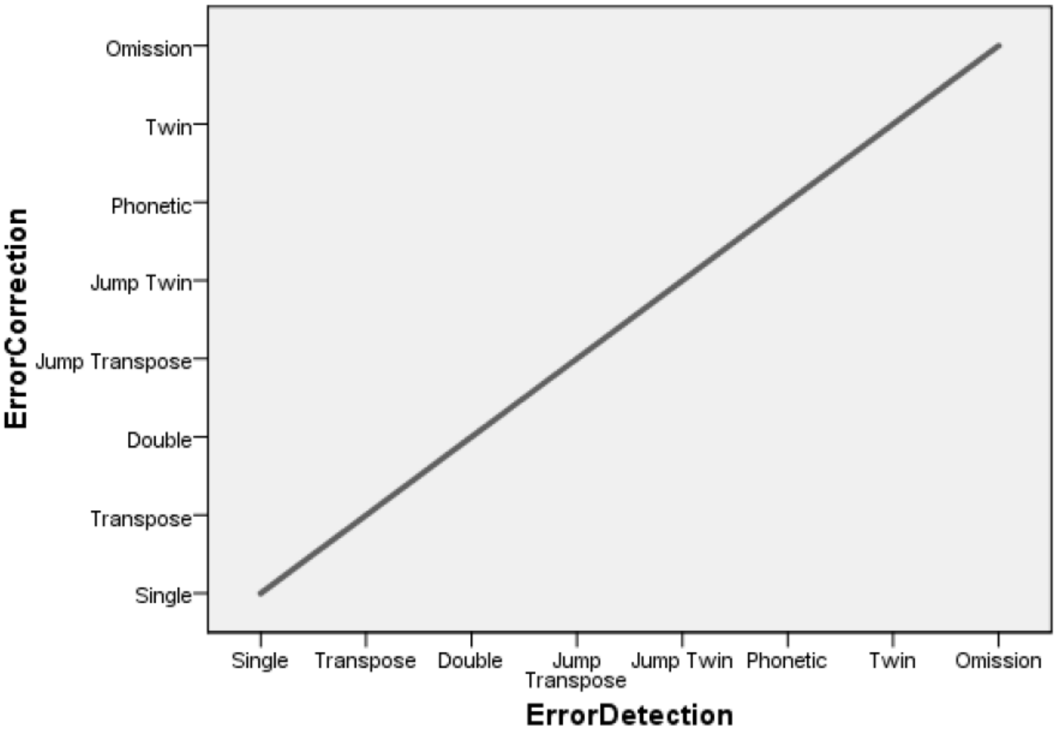

Figure 5.3.1: **Relationship between error correction and error detection of modified ISSN code**

5.4 Effectiveness of the modified ISSN in error detection and correction

The effectiveness of the modified international standard serial number in error detection and correction capabilities is determined by the properties of the international standard serial number in error detection and correction capabilities against the dictionary size.

Proposition 5.4.1. *Modified ISSN code is effective in error detection.*

Proof. It is clear that modified ISSN is effective in error detection for the code detects presence of any error if and only if the weight check equation and the generating equation do not hold. Conversely, if the weight check equation and the generating equation do not hold implies that the code has detected error(s) in the code word. □

Proposition 5.4.2. *Modified ISSN code is effective in error correction.*

Proof. It is clear that modified ISSN is effective in error correction for the code corrects any error that may occur by use of the weight checksum equation and the generating equation. Moreover, modified ISSN code can correct any error that may occur between the first digit and the seventh digit of the code word by use of generating equation. □

Proposition 5.4.3. *Modified ISSN code is an effective error coding scheme.*

Proof. Without loss of generality, modified ISSN code is effective in error detection and correction. Moreover, modified ISSN has a blind digit which shows the number of corrections and modification done of the title of publication. ISSN coding scheme is only associated with the title of the publication. Therefore in case a publication is modified appreciably, only the blind digit changes. This means that if a publication is modified ten times the modified ISSN code shows that the title of the publication has being changed ten times. □

5.5 Conversion tool from ISSN to modified ISSN

Modified ISSN code is more effective than ISSN code in error detection and correction capabilities, therefore the conversion tool from ISSN code to the Modified ISSN code is vital. In the conversion, the following steps are considered.

i.The generating equation $x_{i+1} = 3x_i + 5 \ (mod\ 31), i = 2,3,...,6$.

ii.The digit 0 for the termination of the generating equation.

iii. Since the Modified ISSN code has the same length as ISSN code and Modified ISSN code is generated by the generating equation $x_{i+1} = 3x_i + 5 \ (mod\ 31), i = 2,3,...,6$, the conversion tool from ISSN code to the Modified ISSN code is a bit challenging. ISSN code words have a length of eight digits in which the eighth is the check digit. In the conversion, only the seven digits of ISSN are considered. If the first digit of the ISSN code word is zero, only six digits of the code word are considered since the digit 0 does not have significance in this case. If the first two digits of the ISSN code word are zeros, only five digits of code word are considered and so on.

iv. The conversion tool from ISSN code word to the Modified ISSN code word is based on *mod* 31. Consider the number of the digits in a code word. In case, the first digit of a code word is not zero then, the code word has a considerable seven digits. The number is divided by 31 and its remainder considered then, its quotient is divided by 31 and remainder considered and so on until the quotient is less than 31. Consider the largest possible seven digit number and its quotient when divided by 31 and the smallest possible seven digit number and its remainder when its quotients are divided by 31.

Table 5.4: **Recursive division of the largest seven- digit number and its quotients by 31**

number	$x mod 31$
$9,999,999$	19
322580	25
10405	20
335	25
10	

Table 5.4 has five rows, when $9,999,999$ is divided by 31 and the remainders of the quotients in the second column of the table. It is evident from Table 5.4 that when $9,999,999$ is divided by 31 the answer is 322580 and the remainder is 19, when 322580 is divided by 31 the answer is 10405 and the remainder is 25. 10405 is divided by 31 the answer is 335 and the remainder is 20, When 335 is divided by 31 the answer is 10 and the remainder is 25. This indicates that the largest number in the last row (last quotient) is 10 for a seven- digit number. In the second column of Table 5.4, from the second row (second remainder) to the fourth row (last remainder), the number can be between 0 and 30. The first largest remainder (in the first row) is 19, while the other remainders and the last quotient remain the same.

Table 5.5: **Recursive division of the smallest seven- digit number and it quotients by 31**

number	$x mod 31$
$1,000,000$	2
32258	18
1040	17
33	2
1	

Table 5.5 indicates that the number has five rows of *modulo* 31. The last quotient that can exist in the last row is 1. In the second column of Table 5.5, the first remainder to the fourth remainder can be a number between 0 and 30.

Table 5.6: **Recursive division of the largest six- digit number and its quotients by** 31

number	$x mod 31$
$999,999$	1
32258	18
1040	17
33	2
1	

It is evident that if the number increases with one, the first remainder increases with one while the rest of the remainders and last quotient remain the same. If the first remainder is 30 and the number increases with one then the first remainder will be 0 and the second remainder will increase with one. The second remainder increases up to 30 then the third remainder increases with one and so on until the last quotient increases with one. The increment in remainders and the last quotient can be best likened to how weeks, days, hours, minutes and seconds of time change. It is clear that when there are five rows (four remainders) when a number and its quotients are divided by 31 the last quotient is a number between 1 and 10 while the remainders are numbers between 0 and 30.

Table 5.7: **Recursive division of the smallest six- digit number and its quotients by** 31

number	$x mod 31$
$100,000$	25
3225	1
104	11
3	

Table 5.7 has four rows and last row(last quotient) can have a number between 0 and 30. Moreover, the rest of remainders are numbers between 0 and 30.

Table 5.8: **Recursive division of the largest five- digit number and its quotients by** 31

number	$x mod 31$
$99,999$	24
3225	1
104	11
3	

Table 5.8 has four rows and last row(last quotient) can have a number between 0 and 30. Moreover, the rest of remainders are numbers between 0 and 30.

Table 5.9: **Recursive division of the smallest five- digit number and its quotients by 31**

number	$x\,mod\,31$
$10,000$	18
322	12
10	

It clear from tables above that the smallest possible η digits number and the possible $\eta-1$ digits number have the same number of rows when the number and its quotients are divided by 31 . The conversion tool from ISSN code to the modified ISSN code depends on the number of rows (remainders) that are there when a number and its quotients are divided recursively by 31.

If an ISSN code word to be converted has a number which forms only one row, that is, there is no remainder when the number is divided by 31, then the number is the first digit of the generating equation $x_{i+1} = 3x_i + 5\ (mod\ 31)$. The remaining digits of modified ISSN code word are generated by the generating equation and the last digit computed as $x_8 \equiv -\xi\,(mod\ 31)$, where $\xi = \sum_{i=1}^{7} \sum_{j=9-i} jx_i$ or $x_8 \equiv \sum_{i=1}^{7} (22+i)x_i\,(mod\,31)$

In case there are two rows, that is, a quotient and a remainder, then the number formed by the ISSN code word can be converted by having the digit at the quotient as the first digit of the modified ISSN code word and having a digit zero at the third digit of the modified ISSN code word followed by the digit which is at the remainder. The remaining digits are generated by the generating equation $x_{i+1} = 3x_i + 5\ (mod\ 31)$ and the last digit computed as $x_8 \equiv \sum_{i=1}^{7} (22+i)x_i\,(mod\,31)$ or $x_8 \equiv -\xi\,(mod\ 31)$, where $\xi = \sum_{i=1}^{7} \sum_{j=9-i} jx_i$.

In case there are three rows, that is, two remainders and the last quotient, then the number formed by the ISSN code word is converted by having a digit 0 at third digit and seventh digit of the modified ISSN code word. The digit at the last quotient is the first digit of the modified ISSN code word then the second digit at the $x\,mod\,31$ column (second remainder) after the digit 0 at the third digit. The digit at first remainder is omitted for the eighth digit is computed. The rest digits are generated by the generating equation $x_{i+1} = 3x_i + 5 \ (mod\ 31)$ and finally, the last digit computed as $x_8 \equiv \sum_{i=1}^{7} (22+i)x_i \, (mod\, 31)$ or $x_8 \equiv -\xi \, (mod\ 31)$, where $\xi = \sum_{i=1}^{7} \sum_{j=9-i} jx_i$.

When there are four rows, that is, three remainders and the last quotient, when a number and its quotients are divided recursively by 31, then the number formed by the ISSN code word is converted as shown in Table 5.10.

Table 5.10: **Conversion tool when a number has four rows when the number and its quotients are divided recursively by** 31

number	$x\, mod\, 31$	modified ISSN code word
$X_{1,1}$	$Y_{2,1}$ (first remainder)	$Y_{2,1} \rightarrow$ after digit 0 at 4^{th}digit,when the digit 0 is at 7^{th} digit the 8^{th} is computed
$X_{1,2}$	$Y_{2,2}$ (second remainder)	$Y_{2,2} \rightarrow$ after digit 0 at 2^{nd} digit
$X_{1,3}$	$Y_{2,3}$ (third remainder)	if $0 \leq Y_{2,3} \leq 15$ the digit 0 is at 4^{th}digit and if $16 \leq Y_{2,3} \leq 30$ the digit 0 is at 7^{th}digit
$X_{1,4}$(last quotient)		$X_{1,4} \rightarrow$first digit

The remaining digits are generated by the generating equation $x_{i+1} = 3x_i + 5 \ (mod\ 31)$ and the last digit is computed as $x_8 \equiv \sum_{i=1}^{7} (22+i)x_i \, (mod\, 31)$ or $x_8 \equiv -\xi \, (mod\ 31)$, where $\xi = \sum_{i=1}^{7} \sum_{j=9-i} jx_i$.

In case there are five rows when a number and its quotients divided recursively by 31, then the number formed by the ISSN code word is converted as shown in Table 5.11.

Table 5.11: **Conversion tool when a number has five rows when the number and its quotients are divided recursively by** 31

number	$x\, mod\, 31$	modified ISSN code word
$X_{1,1}$	$Y_{2,1}$	$Y_{2,1} \rightarrow$ either after the second digit 0 at 5^{th}or 6^{th} digit,
$X_{1,2}$	$Y_{2,2}$	$Y_{2,2} \rightarrow$either after first digit 0 at 2^{nd} or 3^{rd} digit
$X_{1,3}$	$Y_{2,3}$	if $0 \leq Y_{2,3} \leq 15$ the second digit 0 is at 5^{th}digit and if $16 \leq Y_{2,3} \leq 30$ the digit 0 is at 6^{th} digit
$X_{1,4}$	$Y_{2,4}$	if $0 \leq Y_{2,4} \leq 15$ the first digit 0 is at 2^{nd}digit and if $16 \leq Y_{2,4} \leq 30$ the first digit 0 is at 3^{rd} digit
$X_{1,5}$		$X_{1,5} \rightarrow$first digit

The remaining digits are generated by the generating equation $x_{i+1} = 3x_i + 5 \ (mod\ 31)$ and the last digit is computed as $x_8 \equiv \sum_{i=1}^{7} (22+i)x_i \, (mod\, 31)$ or $x_8 \equiv -\xi \, (mod\ 31)$,where $\xi = \sum_{i=1}^{7} \sum_{j=9-i} jx_i \ (mod\ 31)$.

This conversion tool is effective for it can convert all the ISSN code words.

Example 5.5.1. The ISSN code word 0 3 1 7 8 4 7 1 can be converted as follows. Consider the first seven digits and since the first digit is zero consider the six digits.

Table 5.12: **Conversion of ISSN** 0 3 1 7 8 4 7 1

number	$x\,mod\,31$
317847	4
10253	23
330	20
10	

In this case, the table has four rows, the last quotient is 10, therefore, the first digit for the new code word is $10 = A$. Since 20 is between 15 and 30, therefore, the digit 0 is at 2^{rd} digit and 7^{th} digit. Digit $23 = P$ is at 3^{rd} digit then it generates 4^{th}, 5^{th} and 6^{th} digit. Digit 4, in this case, is omitted because digit 0 is at 7^{th} digit then the eighth digit is computed yielding $A\,0\,P\,C\,A\,4\,0\,B$.

Example 5.5.2. The ISSN code word 1 0 4 1 5 6 5 3 can be converted as follows. Consider the first seven digits, then

Table 5.13: **Conversion of ISSN** 1 0 4 1 5 6 5 3

number	$x\,mod\,31$
1041565	27
33598	25
1083	29
34	3
1	

In this case, Table 5.14 has five rows, the first digit for the modified ISSN code word is 1 for the last quotient is 1. Since 3 is between 0 and 15 the first digit 0 is at 2^{rd} digit of the new code word and since 29 is between 16 and 30 the second digit 0 is at 6^{th} digit of the new code word. Digits 25 and 27 are at 3^{rd} and 6^{th} digits respectively. The rest of the digits are generated by the generating equation $x_{i+1} = 3x_i + 5\ (mod\ 31)$ and the last digit is computed as $x_8 \equiv -\xi\,(mod\ 31)$ where $\xi = \sum_{i=1}^{7} \sum_{j=9-i} jx_i$ yielding 1 0 25 21 6 0 27 0 which is equivalent to $1\,0\,R\,M\ 6\,0\,T\,0$

More examples of converted ISSN into modified ISSN are shown in Table 5.14 and Algorithm 5.6 below.

Table 5.14: **Conversion of ISSN** 10415653

ISSN codeword	converted modified ISSN codeword
23074531	$102B0W2H$
23269790	$2B0C0SM6$
23269812	$2B0C0UTK$

Algorithm 5.6 ***How to convert ISSN code word into modified ISSN code word***

i. Consider the first seven digits of the ISSN code word.
ii. Recursively divide the number formed in step i and its quotient by 31 until the last quotient is less than 31.
iii. Consider all remainders and the last quotient in step ii.
iv. Count the numbers of remainders in step iii.
v. (a) If the number of remainders in step iv is zero, then the number is the first digit of the generating equation $x_{i+1} = 3x_i + 5 \ (mod\ 31)$.
(b) Generate the remaining digits of the modified ISSN code word by generating equation and the last digit computed as $x_8 \equiv -\xi (mod\ 31)$ where $\xi = \sum_{i=1\ j=9-i}^{7} jx_i$.
(c) If the number of remainders in step iv is one, then the quotient is the first digit of the modified ISSN code word and the third digit is digit zero followed by the digit which is at the remainder. The remaining digits are generated as in step v(b).
(d) If the number of remainders in step iv is two, then digit 0 is at third digit and seventh digit of the modified ISSN code word. The last quotient is the first digit of the modified ISSN code word then the second remainder is after the digit 0 at the third digit. The first remainder is omitted for the eighth digit is computed. The remaining digits are generated as in step v(b).
(e) If the number of remainders in step iv is three, then the number formed by the ISSN code word is converted as follows;

Quotients	remainders	Modified ISSN code word
Q_1	R_1	$R_1 \rightarrow$ after digit 0 at 4^{th}digit,when the digit 0 is at 7^{th} digit the 8^{th} is computed
Q_2	R_2	$R_2 \rightarrow$ after digit 0 at 2^{nd} digit
Q_3	R_3	if $0 \leq R_3 \leq 15$ the digit 0 is at 4^{th}digit and if $16 \leq R_3 \leq 30$ the digit 0 is at 7^{th}digit
last quotient		Last quotient$\rightarrow$first digit

The remaining digits are generated as in step v(b).
(f) If the number of reminders in step iv is four, then the number formed by the ISSN code word is as follows;

Quotients	Remainders	Modified ISSN code word
Q_1	R_1	$R_1 \rightarrow$ either after the second digit 0 at 5^{th}or 6^{th} digit,
Q_2	R_2	$R_2 \rightarrow$either after first digit 0 at 2^{nd} or 3^{rd} digit
Q_3	R_3	if $0 \leq R_3 \leq 15$ the second digit 0 is at 5^{th}digit and if $16 \leq R_3 \leq 30$ the digit 0 is at 6^{th} digit
Q_4	R_4	if $0 \leq R_4 \leq 15$ the first digit 0 is at 2^{nd}digit and if $16 \leq R_4 \leq 30$ the first digit 0 is at 3^{rd} digit
Q_{last}		$Q_{last} \rightarrow$first digit

The remaining digits are generated as in step v(b).

5.6 Coding

Following the guidelines of development and generation of the modified ISSN code, the modified ISSN code word generator is developed using MATLAB program. Moreover, algorithms for error detection and correction properties to the modified ISSN code are coded. Finally, an algorithm of the conversion from ISSN code word to modified ISSN code word is also coded using MATLAB, see Appendix 2.

CHAPTER SIX

6.1 Conclusion

The study has determined the error detection and correction capabilities of the current ISSN code against its dictionary size. A new ISSN code that is improved in error detection and correction capabilities against the dictionary size has been designed. A conversion tool from ISSN code to the modified ISSN code has also been designed.

ISSN code is not effective in error detection. ISSN code detects an error in a code word iff the weight checksum equation does not hold. However, the code cannot identify; double error, twin error, jump twin error, phonetic error and multiple transposition errors in a code word.

ISSN code detects single errors and all transposition error in a code word. However, the code does not detect silent errors. ISSN code corrects a single error if the position of the error in a code word is known. The code corrects only a single adjacent transposition error in a code word. ISSN is not an effective error coding scheme. Modified ISSN code is effective in error detection and correction capabilities. The code has dual mechanism of detection of errors in a code word. First, if the weight checksum equation does not hold and secondly, if the generating equation does not hold. Modified ISSN code can detect and correct silent errors in a code word. Modified ISSN code is an effective error coding scheme for it is effective in error detection and correction capabilities. Moreover, the code has a relatively big dictionary.

6.2 Recommendations for future research

One can extend this research by

i. Determining the effectiveness of the International Standard Serial Number in error detection and correction capabilities by use of cyclic redundancy check.

ii. Determining the effectiveness in error detection and correction capabilities between checksum and cyclic redundancy check.

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MATLAB CODES

Listing 1: Generation of modified ISSN code words

```
%% This MATLAB code generates the modified ISSN code words
%
function generate_modified_ISSN()
clear all;    % clears the memory
clc;          % clears the working space
k(1)=randi(31); % generates a random integer between 1 and 31
for i = 1 : 6;
k(i+1) = 3*k(i)+5;
if i > randi(7)
  k(i+1) = 0;
  break;
    end
```

```
end
%
% Conversion into (mod 31)
%
k =mod(k, 31);
n(1)= randi(31);
 for j= 1: 6
n(j+1) = 3*n(j)+5;
end
n = mod (n, 31);
p = [k, n];
X = p( 17);
%
% Calculation of check digit
%
a = 8 : -1 : 2;
B = dot (a, X);
B = mod (B, 31);
check digit = (31 - B);
X =[X, check_digit]
str={'0','0','0','0','0','0','0','0'}; % initialization
for i=1:8;
if X(i)==0;
```

```
str{i}='0';
elseif X(i)==1;
str{i}= '1';
elseif X(i)==2;
str{i}= '2';
elseif X(i)==3;
str{i}= '3';
elseif X(i)==4;
str{i}= '4';
elseif X(i)==5;
str{i}= '5';
elseif X(i)==6;
str{i}= '6';
elseif X(i)==7;
str{i}= '7';
elseif X(i)==8;
str{i}= '8';
elseif X(i)==9;
str{i}= '9';
elseif X(i)==10;
str{i}= 'A';
elseif X(i)==11;
str{i}= 'B';
```

```
elseif X(i)==12;
str{i}= 'C';
elseif X(i)==13;
str{i}= 'D';
elseif X(i)==14;
str{i}= 'E';
elseif X(i)==15;
str{i}= 'F';
elseif X(i)==16;
str{i}= 'G';
elseif X(i)==17;
str{i}= 'H';
elseif X(i)==18;
str{i}= 'J';
elseif X(i)==19;
str{i}= 'K';
elseif X(i)==20;
str{i}= 'L';
elseif X(i)==21;
str{i}= 'M';
elseif X(i)==22;
str{i}= 'N';
elseif X(i)==23;
```

```
str{i}= 'P';
elseif X(i)==24;
str{i}= 'Q';
elseif X(i)==25;
str{i}= 'R';
elseif X(i)==26;
str{i}= 'S';
elseif X(i)==27;
str{i}= 'T';
elseif X(i)==28;
str{i}= 'U';
elseif X(i)==29;
str{i}= 'V';
elseif X(i)==30;
str{i}= 'W';
else
disp( 'INVALID' )
end
end
M_ISSN_CODE WORD=cell2mat(str);
fprintf('\n>>The modified ISSN code-word is: %,M_ISSN_code);
```

Listing 2:Error detection and correction of modified ISSN code

```
%% This MATLAB code detects and corrects errors in the
%% modified ISSN code words
%
function detect_correct_errors()
clears all;        % clears the memory
clc;        %clears the working space
%
%% Case 1: Single error detection and correction when there is a digit zero
%% for the termination of the generating formula
 %
 % Conversion of the modified code word digits into numerical digits
 % numerical digits for calculations
 function modified ISSN in numerical digits()
 % X given modified ISSN code word . For example,
 the one given below
 modified ISSN= {'1' , '0', 'S' , 'M' , '6' ,'0' ,'J' ,'C' };
 X=zeros(1,8);
 for i=1:8;
  elseif modified ISSN {i}=='0',
 X(i)=0;
 elseif modified ISSN {i}=='1',
 X(i)=1;
```

```
elseif modified ISSN {i}=='2',
X(i)=2;
elseif modified ISSN {i}=='3',
X(i)=3;
elseif modified ISSN {i}=='4',
X(i)=4;
elseif modified ISSN {i}=='5',
X(i)=5;
elseif modified ISSN {i}=='6',
X(i)=6;
elseif modified ISSN {i}=='7',
X(i)=7;
elseif modified ISSN {i}=='8',
X(i)=8;
elseif modified ISSN {i}=='9',
X(i)=9;
elseif modified ISSN {i}=='A',
X(i)=10;
elseif modified ISSN {i}=='B',
X(i)=11;
elseif modified ISSN {i}=='C',
X(i)=12;
elseif modified ISSN {i}=='D',
```

```
X(i)=13;
elseif modified ISSN {i}==’E’,
X(i)=14;
elseif modified ISSN {i}==’F’,
X(i)=15;
elseif modified ISSN {i}==’G’,
X(i)=16;
elseif modified ISSN {i}==’H’,
X(i)=17;
elseif modified ISSN {i}==’J’,
X(i)=18;
elseif modified ISSN {i}==’K’,
X(i)=19;
elseif modified ISSN {i}==’L’,
X(i)=20;
elseif modified ISSN {i}==’M’,
X(i)=21;
elseif modified ISSN {i}==’N’,
X(i)=22;
elseif modified ISSN {i}==’P’,
X(i)=23;
elseif modified ISSN {i}==’Q’,
X(i)=24;
```

```
elseif modified ISSN {i}=='R',
 X(i)=25;
 elseif modified ISSN {i}=='S',
 X(i)=26;
 elseif modified ISSN {i}=='T',
 X(i)=27;
 elseif modified ISSN {i}=='U',
 X(i)=28;
 elseif modified ISSN {i}=='V',
 X(i)=29;
 elseif modified ISSN {i}=='W',
 X(i)=30;
 else
 disp('INVALID')
 end
 end
 X;
   for i = 1:6
 if (X(i +1) ~= (mod(3*X(i)+5,31)) &
 (X (i +2) ~ = (mod(9*X(i)+20,31))))
 W(i)=X(i);
   end
 end
```

```
%
% in case of the presence of digit zero for the termination formula,
% the error is indicated then followed by zero. This error digit is
% omitted for there is no error
  %
count=0;
for i= 1 : length (W);
if W(i)~=0
 count=count +1;
 end
 end
 W
 % count
        if count<=2
 disp( 'A SINGLE ERROR DETECTED' )
 end
 %
 % Correction of a single error in a modified ISSN code word
 %
 for j=1:length (W);
 if ((W(j) == X(j))(X (j +1)=0));
 X(j)=(mod(3*X(j-1)+5,31)
 else if ((W(j) == X(j))(X (j +1)=0);
```

```
(X(j+1)=X(j+1)+31);
end
end
X=mod(X,31)
k = 8: -1: 1;
A= dot(X,k);
A=mod(A,31);
if A==0;
disp('NO_ERROR')
else
disp('ERROR')
end
%
% Case2: Single error detection and correction when there is repetition
%
% X     given modified ISSN code in numerical digits
% for example,the one given below
X=[30 30 2 11 7 26 18];
for i=1:6
if (X(i +1)~=(mod(3*X(i)+5,31)) & (X(i+2)~=(mod(9*X(i)+20,31))))
W(i)=X(i);
end
end
```

```
%
% in case of the presence of digit zero for the termination of generating
% equation, the error is indicated then followed by zero. This error
% is omitted for there is no error
%
count=0;
for i=1:length(W);
if W(i)~=0
count=count+1;
    end
end
W
% count
if count<=2
     disp('A SINGLE ERROR DETECTED')
end
%
% correction of a single error when there is repetition
%
for j=1:length(W);
if ((W(j)==X(j)) &X(j)~=X(j+1));
X(j)=(mod(3*X(j-1)+5,31));
else if    ((W(j))==X(j)) &X(j)~=X(j+1));
```

```
X(j)=X(j+1);
end
end
K=8:-11;
A=dot(X,k);
mod(A,31);
 if A==0;
disp('NO ERROR')
else
disp('ERROR')
end
%% Case 3: Transposition error detection and correction
%
% N/B: if there is no transposition error, this code won't execute since the
% variable W will be undefined.
%
 % X      given modified ISSN code word in numerical digits.
% For example, the one given below
%
X = [203141692205]
for i=1:6
if (X(i+1)==(mod(21*[X(i)-5],31))) &
(X(i+1)~=(mod(3*X(i),31))))
```

```
W(i)=X(i);
disp('NO TRANSPOSITION ERROR')
end
end
count=0;
for i=1:length(W);
if W(i)~=0
count=count+1;
K=X(i+1);
X(i)=k;
X(i+1)=W(i);
end
end
% count
if count==1;
disp('A TRANSPOSITION ERROR DETECTED')
end
disp(X)
```

Listing 3: Conversion of the existing ISSN to Modified ISSN code word

```
%% This MATLAB code converts the existing ISSN code word to
%% Modified ISSN code word
```

```
function converts_ISSN_to_ modified ISSN()
clear all;    % clears the memory
clc,       % clears the working space
%
% X     the first seven digits of the ISSN code word ( prompt user to user)
%
flag =0;       % loop check
 while flag==0
 flag=1;
 X(1)=str2num(input(>>>Enter first seven digits of the ISSN code word
   (e.g.,1041587):,’s’));
 % X(1)=1041587; % Example of user input
 numDigits=num1(num2str(X)); % count number of digits in an integer
 if numDigits~=7
 flag=0;
 clc;
  display (’>>>INVALID ENTRY, TRY AGAIN <<<’);
      end
 end
 for i=2:numDigits
 r(i-1)=mod(X(i-1),31);
    X(i)=((X(i-1)-r(i-1))/31);
 if X(i)<31
```

```
  break;
end
end
n=length(X); % compute the number of rows
str={’0’,’0’,’0’,’0’,’0’,’0’,’0’}; % initialization
initStr=str;
if n==3
str{1}= X(end);
str{2}= 3*X(end+5);
str{3}= 0;
str{4}= r(end);
str{5}= 3*str{4}+5;
str{6}= 3*str{5}+5;
str{7}= 0;
elseif n==4
str{1}= X(end);
str{2}= 0;
str{3}= r(end-1);
if r(end)<16
str{4}= 0;
str{5}= r(end-2);
str{7}= 3*str{6}+5;
str{6}= 3*str{5};
```

```
elseif r(end)>15
str{4}= 3*str{3}+5;
str{5}= 3*str{4}+5;
str{6}= 3*str{5}+5;
str{7}= 0;
str{4}= 3*str{3}+5;
end
elseif n==5
str{1}= X{end};
if r(end)<16
str{2}= 0;
str{3}=r{end-2};
str{4}= 3*str{3}+5;
elseif r(end)<15
str{2}= 3*str{1}+5;
str{3}= 0;
str{4}=r(end-2);
end
if r(end-1)<16
str{5}= 0;
str{6}=r(end-3);
str{7}= 3*str{6}+5;
elseif r{end-1}>15
```

```
str{5}= 3*str{ 4}+5;
str{6}=0;
str{7}= r{end-3};
end
end
Y=cell2mat(str);
clc;
if Y==cell2mat(initStr)
fprintf('\n Error: number of digits in X (1) should exceed
%d\n',numDigits);
else
display('The output is:')
Y=mod(Y,31)
%
% Calculation of check digit
%
a=8:-1:2;
B=dot(a,Y);
B=mod(B,31);
check_digit=31-B;
Y=[Y,check_digit];
Y=[char(Y)];
str={'0','0','0','0','0','0','0'};
```

```
for i=1:8;
if Y(i)==0;
str{i}= ’0’;
elseif Y(i)==1;
str{i}= ’1’;
elseif Y(i)==2;
str{i}= ’2’;
elseif Y(i)==3;
str{i}= ’3’;
elseif Y(i)==4;
str{i}=’4’;
elseif Y(i)==5;
str{i}=’5’;
elseif Y(i)==6;
str{i}=’6’;
elseif Y(i)==7;
str{i}= ’7’;
elseif Y(i)==8;
str{i}= ’8’;
elseif Y(i)==9;
str{i}=’9’;
elseif Y(i)==10;
str{i}=’A’;
```

```
elseif Y(i)==11;
str{i}='B';
elseif Y(i)==12;
str{i}='C';
elseif Y(i)==13;
str{i}= 'D';
elseif Y(i)==14;
str{i}='E';
elseif Y(i)==15;
str{i}= 'F';
elseif Y(i)==16;
str{i}= 'G';
elseif Y(i)==17;
str{i}= 'H';
elseif Y(i)==18;
str{i}= 'J';
elseif Y(i)==19;
str{i}= 'K';
elseif Y(i)==20;
str{i}= 'L';
elseif Y(i)==21;
str{i}= 'M';
elseif Y(i)==22;
```

```
str{i}= 'N';
elseif Y(i)==23;
str{i}= 'P';
elseif Y(i)==24;
str{i}= 'Q';
elseif Y(i)==25;
str{i}= 'R';
elseif Y(i)==26;
str{i}= 'S';
elseif Y(i)==27;
str{i}= 'T';
elseif Y(i)==28;
str{i}= 'U';
elseif Y(i)==29;
str{i}= 'V';
elseif Y(i)==30;
str{i}= 'W';
else
disp( 'INVALID')
end
end
M_ISSN_code=cell2mat(str);
fprintf('>>The existing ISSN code word is:%d\n>>
```

```
The modified ISSN code word is:%s\n',X(1),m_ISSN_code word);
end
```