

## POINCAR'E CONJECTURE SOLVED!

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The Clay Mathematics Institute (CMI) announced on 18<sup>th</sup> March, 2010 the award of its Millennium prize to Dr Grigoriy "Grisha" Perelman of St.Petersburg, Russia for resolution of the 106 years old Poincar'e conjecture . The Poincar'e conjecture posed by the French mathematician Henri Poincar'e in 1904 was regarded as such an important problem in mathematics that it was identified as one of the seven Millennium prize problems by the CMI in 2000. The CMI carries the prize of one million dollars for the resolution of each problem.

According to the CMI, the Millennium prizes ' were recorded as some of the most difficult problems with which mathematicians were grappling at the turn of the second millennium increase consciousness of the general public the fact that in mathematics, the frontier is still open and abounds in important unsolved problems. The CMI was also convinced that the recognition of the problems would emphasize the importance of working towards a solution of the deepest most difficult problems and to recognize achievement in Mathematics of historical magnitude.

In 2006, Dr. Perelman, an eccentric genius had famously rejected the Fields Medal, the highest award in mathematics, generally regarded as the Mathematics equivalent of the Nobel Prize. The important difference being that the Fields Medal is awarded to mathematicians not over the age of 40 years.

Dr. Perelman had been chosen for the Fields Medal in May 2006 by a committee of nine mathematicians. The medal citation said "for his contributions to geometry and his revolutionary insights into the analytical structure of the Ricci flow". His results had provided had provided a way of resolving two outstanding problems in topology: the Poincar'e conjecture and Thurston geometrization conjecture. The latter in fact contains, the Poincar'e conjecture as a special case. The Fields Medal is customarily given at the International Congress of Mathematicians (ICM). In June 2006, Sir John Ball, president of IMU, approached Dr. Perelman to persuade him to accept the medal. His two days of persuasion were to no avail. On 22<sup>nd</sup> August 2006, he was publicly offered the medal during the 2006 congress at Madrid Spain. However he did not present himself at the ceremony and declined to accept the Medal. Topology is the branch of Mathematics that deals with fundamental properties of shapes that remain unchanged when the shapes were deformed, that is bent , stretched, shrunk , moulded, warped or twisted but not torn.

The conjecture was posed as a question in a paper that French mathematician Henri Poincar'e wrote in 1904: If a three dimensional shape is simply connected, is it homeomorphic to the three dimensional sphere? The conjecture is fundamental to achieve an understanding of all three dimensional shapes. To understand the statement, consider the analogous two dimensional situation. Think of an apple. It is easily seen that it can be shrunk to a point by moving it slowly, without tearing it or allowing it to leave the surface. On the other hand, if the same rubber band was stretched around a

doughnut, a torus, there would be no way of shrinking it to a point without tearing either the rubber band or doughnut. In the language of mathematics the apple is connected while the doughnut is not.

Now, how do we characterise the spherical apple surface mathematically? Think of a disk lying in a two dimensional plane. A sphere is nothing but the disk with its boundary lifted up and tied to a single point. Mathematically, we say that that all the boundary points are identified to a single point. Now one can do this if the two dimensional disk is lying in a 3-dimensional space. Now what we have is a 2 dimensional disk lying in a 3 dimensional space. Stating that a 2-dimensional sphere is characterised by the property of simple connectivity, Poincar'e question corresponds to asking whether similar characterisation is valid for all closed 3 dimensional objects (embedded in a 4 dimensional space) that are sufficiently like a 3 dimensional sphere.

The question has turned out to be extraordinarily difficult and mathematicians have been struggling to prove this ever since. The analogous result has been known to be true in higher dimensions for some time but the case of the three dimensional sphere has proved be the hardest of all. It definitely required a maverick Perelman to crack it and that is what happened.

Dr. Perelman's works have so far not been published in a peer reviewed journal as the rules for the Millennium prize require; he had only posted them in electronic form in the public research preprint archive of the Argonne National Laboratory (ANL). However, the mathematics community felt that the verification of his proof in his e prints was so intensive and thorough that it was a great deal more than the normal process of peer review. The CMI governing board had accordingly changed the formal prerequisites for the award.

The special advisory committee comprising of famous mathematicians – Drs. Simon Donaldson, David Gabai, Mikhail Gromov, Terence Tao (the Australian who had shared the 2006 Fields medal with Dr.Perelman) and Andrew Wiles (who proved the famous Fermat's last Theorem )- had recommended Dr. Perelman for the CMI Millennium Award . As per the governing rules of the institute, this was approved by its six members Scientific Advisory Committee and then finally decided by its three members Board of Directors." The resolution of the Poincar'e Conjecture brings to a close a century-long quest for the solution", said Dr. James Carlson, President of the CMI. "It's a major advance in the development of mathematics that will be long remembered", he added.

The CMI and the institut Henri Poincar'e(IHP) held a conference to celebrate Poincar'e Conjecture and it's resolution on 8<sup>th</sup>,9<sup>th</sup> June 2010 in Paris France. This was preceded by a public lecture by Dr. Etienne Ghys on 7<sup>th</sup> June 2010. The 2010 ICM was held in Hyderabad, India from 19<sup>th</sup> August 2010 to 27<sup>th</sup> August 2010. The next round of field medals winners were announced and given their awards there. A felicitation ceremony during the congress to mark Dr. Perelman achievements was held. Not surprisingly, Dr. Perelman did not turn up to receive the Clay Award either. Disillusioned with Mathematics especially with ethical standards, he is believed to have quit mathematics altogether and become a recluse, living with his mother at St.Petersburg on her modest pension. "It is not people who break ethical standards who are regarded as aliens, it is people like me who are isolated", he said after declining the Fields Medal.