Multipurpose Reservoir Operating Policies: A Fully Fuzzy Linear Programming Approach

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ABSTRACT

A Fully Fuzzy Linear Programming (FFLP) formulation for the reservoir operation of a multipurpose reservoir in presented in the ongoing paper. In the real world, water resources systems usually have complexities among social, economic, natural resources and environmental aspects, which lead to multi-objective problems of significant uncertainties in system parameters, objectives and in their interactions. These uncertainties in FFLP reservoir operation model are considered by being treated as fuzzy sets. In the present study, an FFLP reservoir operation model is developed where all parameters and decision variables are fuzzy numbers. The developed model is demonstrated through a case study of Jayakwadi reservoir stage–II, Maharashtra, India with the objectives of maximization of annual releases for irrigation and hydropower generation. The FFLP reservoir operation model is solved to obtain a compromised solution by simultaneously optimizing the fuzzified objectives and the corresponding degree of truthfulness, using linear membership function. The degree of correspondence (Correspondence) obtained is equal to 0.78 and the corresponding annual releases for irrigation amount of 367 Mm^3 and while annual releases for hydropower generation being 216 Mm^3. the present study clearly demonstrates that, use of FFLP in multipurpose reservoir system optimization presents a potential alternative to attain an optimal operating policy.

Keywords: Fully fuzzy linear programming, Fuzzy decision variable, Reservoir operation, Triangular fuzzy numbers.

INTRODUCTION

Linear Programming (LP) is a popular method for optimization of a wide range of applications because of its simplicity and compatibility. However LP in its classical form is not well equipped in handling information of fuzzy uncertainty (Nazemi *et al.,* 2002). In many practical situations, it is not reasonable to require that the constraints or the objective function in linear programming problems be specified in precise, crisp terms. In such situations, it is desirable to use some type of FLP (Klir and Yuan, 2000). The solutions obtained through FLP are efficient. FLP models are not uniquely defined types of models. Many variations are possible, depending on the assumptions or features of the real situations to be modeled (Zimmermann, 1978; 1996). The problem of irrigation planning becomes more complex by considering an uncertainty. The uncertainties can be tackled by formulating the problem of irrigation planning as FLP. Fuzzy linear programming models can incorporate the scenario of a real world problem (Regulwar and Gurav, 2010). A key difficulty in optimization under uncertainty is in dealing with an uncertainty space that is huge, frequently leading to very large-scale optimization models. Decision making under uncertainty is often made further complicated by the presence of

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integer decision variables to model logical and other discrete decisions in a multi-period or multi-stage setting (Sahinidis, 2009). Various uncertainties are inherent in modeling any reservoir operation problem (Aktar and Simonovic, 2004). These uncertainties are defined here as the ones that result from imprecise knowledge of either current or future demands placed on the system (Carron _et al.,_ 2006). Fuzzy systems play essential roles in this fuzzy modeling which can formulate uncertainty in actual environments. Bellman and Zadeh (1970) have proposed the concept of decision making in a fuzzy environment. Several such kinds of FLP problems have been appeared in the literature as Li and Li (2006), Ganesan and Veeramani (2006), Stanciulescu _et al._ (2003), Arikan and Gungor (2007), Wang and Wang (1997), Rommelfanger (1996), Jimenez and Bilbao (2009). Modeling of the uncertain model parameters using fuzzy set theory have been utilized in many water resources' decision-making problems including reservoir optimization by Darell _et al._ (1997), Jairaj and Vedula (2009), Panigrahi and Mujumdar (2009), Regulwar and Anand Raj (2008; 2009).

Choudhari and Anand Raj (2009) demonstrated the operation of a complex system of multi-reservoir with multiple objectives. Uncertainties of inflow and demands are addressed by fuzzy set theory. A fuzzy stochastic based violation analysis approach is developed by Li and Huang (2009) for the planning of water resources management system of uncertain information. Uncertainty in objectives and various parameters of reservoir operation is addressed by Kamodkar and Regulwar (2010) through fuzzy set theory using linear membership function. Shrestha _et al._ (1996) proposed that the input to the reservoir operating principles (e.g. initial storage, inflows, and demands), as well as outputs (historical releases) could be described by means of fuzzy relations. These fuzzy inputs are combined to produce fuzzy output relations, which can be combined and

defuzzified to get crisp output. A variety of optimization models have been developed so far to facilitate the real time operation of the reservoir system, a summary being found in Yeh (1985). Azamathulla _et al._ (2008) developed a Genetic Algorithm (GA) and Linear Programming (LP) model for a real time reservoir operation. The performance of the models are analyzed by being applied to an existing Chiller reservoir system in Madhya Pradesh, India. Zahraie and Hosseini (2010) presented an Integrated Optimization-Simulation based Genetic Algorithm model (IOSGA) to develop the operational policies for a multi-purpose reservoir system. Rani and Moreira (2009) have presented a survey of simulation and optimization modeling approach utilized in a reservoir system operational problem.

Recent research in modeling uncertainty in water resources system has highlighted the use of fuzzy logic based approaches. A number of research contributions exist in the literature that deal with uncertainty in water resources system including fuzziness, subjectivity, imprecision and lack of adequate data (Mujumdar and Ghosh, 2009). However in all the above mentioned works, those cases of application of fuzzy set theory were studied in which not all parts of the problem were assumed to be fuzzy (e.g. either only right hand side or the objective function coefficient being taken as fuzzy). In this study, a problem is taken into consideration where all the variables and parameters are fuzzy numbers as described by Dehghan _et al._ (2006), Allahviranlo _et al_. (2008), Lotif (2009), Amit Kumar _et al._ (2011), and Liu (2010). An application of Fully Fuzzy Linear Programming (FFLP) problem to the single reservoir operation, where all the parameters and decision variables of the reservoir operation model are represented by triangular fuzzy numbers, is hereby demonstrated through a case study of Jayakwadi reservoir stage–II, Maharashtra state, India. The results obtained by solving the FFLP model are utilized to obtain the compromised solution for the intended objectives to obtain a

maximized degree of correspondence (truthfulness) (λ).

METHODOLOGY

LP is one of the most frequently applied operational research techniques. In the conventional approach, values of the parameters in the LP model must be well defined and precise. However, in the real world, this is not a realistic assumption. In the real life problems, there exists uncertainties regarding about the parameters. For example in case of the reservoir operation, reservoir storages are uncertain due to variation in inflows and sometimes vague due to poor operation. Crop water requirement can be stochastic contributing to uncertainty in irrigation demands. In such a situation the parameters of LP problem can be represented as fuzzy numbers. In the present FFLP reservoir operation model, the parameters and variables are treated as triangular fuzzy numbers.

The definitions of triangular fuzzy number, ranking function, formulation of FFLP problem (Allahviranloo *et al.,* 2008) and fuzzy compromised approach are hereby given.

Definition 1

A fuzzy set $\tilde{A}$, is called triangular fuzzy number with peak (or center) a, left width α and right width β if its membership function is of the following form:

$$\mu_{\tilde{A}}(x) = \begin{cases} 1-(a-x)/\alpha, & a-\alpha \le x \le a \\ 1-(x-a)/\beta, & a \le x \le a+\beta \\ 0, & otherwise \end{cases}$$

(1)

and the set of all triangular fuzzy numbers is denoted by $FT(\Re)$ where in parametric form is:

$$\tilde{A} = \left(\alpha(r-1)+a, \beta(1-r)+a\right)$$

Definition 2

A fuzzy number $\tilde{A}$ is said to be an L_R type of fuzzy number if:

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left((a-x)/\alpha\right), & x \le m, \alpha > 0 \\ R\left((x-a)/\beta\right), & x \ge m, \beta > 0 \\ 0, & otherwise \end{cases}$$

(2)

in which L denotes the left and R stands for right reference. a is the mean value of $\tilde{A}$, where α and β are called left and right spreads respectively. Symbolically $\tilde{A}$ is written as:

$$\tilde{A} = (a, \alpha, \beta)$$

If $L(x)$ and $R(x)$ be the linear functions, then the corresponding LR number is said to be a triangular fuzzy number.

In the present study the various parameters of FFLP model are treated as triangular fuzzy numbers. We use $\tilde{a} = (a, a', a'')$ for fuzzy numbers where a is the core, a' is the left margin and a'' is the right margin. The graphical representation of triangular fuzzy number is shown in Figure 1.

Ranking Function

An efficient approach to ordering the element is to define a ranking function D: $D : F(\Re) \to \Re$ which maps for each fuzzy number into the real line, where a natural order exits. We define these natural orders by:

$\tilde{A} \succeq \tilde{B}$ if and only if $D(\tilde{A}) \ge D(\tilde{B})$

$\tilde{A} \preceq \tilde{B}$ if and only if $D(\tilde{A}) \le D(\tilde{B})$

$\tilde{A} = \tilde{B}$ if and only if $D(\tilde{A}) = D(\tilde{B})$

Where $\tilde{A}, \tilde{B}$ are in $F(\Re)$. Also we write $\tilde{A} \succeq \tilde{B}$ if and only if $-\tilde{A} \preceq -\tilde{B}$. The following lemma is now immediate.
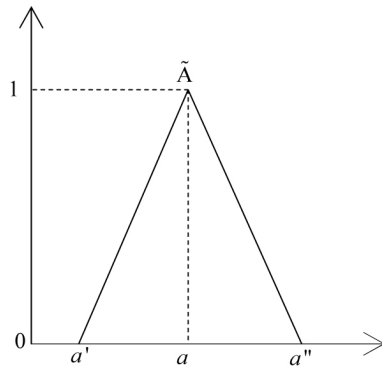
Figure 1. Triangular fuzzy number.

Let *D* be any linear ranking function then:

$$\tilde{A} \succeq \tilde{B} \ iff \ \tilde{A} - \tilde{B} \succeq 0 \ iff \ -\tilde{A} \succ -\tilde{B}$$

$$\tilde{A} \succeq \tilde{B} \ and \ \tilde{C} \succeq \tilde{D} \ then \ \tilde{A} \oplus \tilde{C} \succeq \tilde{B} \oplus \tilde{D}$$

Attention is restricted to linear ranking function, that is a ranking function *D* such that:

For any $\tilde{A}, \tilde{B}$ belonging to $F(\Re)$.

Here we introduce the linear ranking function. For any arbitrary fuzzy number $\tilde{A} = (\underline{A}(r), \overline{A}(r))$, a ranking function is used as follows:

$$D(\tilde{A}) = 1/2 \int_{[0,1]} (\underline{A}(r)) + \int_{[0,1]} (\overline{A}(r))$$

For a triangular fuzzy number this is reduced to:

$$D(\tilde{A}) = A + 1/4(A'' - A')$$

Then, for triangular fuzzy number $\tilde{A} \ and \ \tilde{B}$, we have:

$$\tilde{A} \succeq \tilde{B} \ \text{if} \quad \text{and} \quad \text{only}$$

if $A + 1/4(A'' - A') \geq B + 1/4(B'' - B')$

Fully Fuzzy Number Linear Programming Problems (FFLP)

Using definitions (1) and (2), triangular fuzzy numbers can be defined for all the parameters and variables while LP model

being written as Fully Fuzzy Linear Programming model as presentedbelow:

$$\begin{cases} \max \tilde{z} = \tilde{c}_1 \otimes \tilde{x}_1 \oplus .... \tilde{c}_n \otimes \tilde{x}_n \\ s.t. \\ \tilde{a}_{11} \otimes \tilde{x}_1 \oplus .... \tilde{a}_{1n} \oplus \tilde{x}_n \preceq \tilde{b}_1 \\ \vdots \\ \tilde{a}_{m1} \otimes \tilde{x}_1 \oplus .... \tilde{a}_{mn} \oplus \tilde{x}_n \preceq \tilde{b}_m \\ \tilde{x}_1 \succeq 0, \tilde{x}_2 \succeq 0, ..., \tilde{x}_n \succeq 0 \end{cases} \quad (3)$$

The matrix form of the above equation is:

$$\begin{cases} \max \tilde{z} = \tilde{c} \otimes \tilde{x} \\ s.t. \\ \tilde{A} \otimes \tilde{x} \preceq \tilde{b} \\ \tilde{x} \succeq 0 \end{cases}$$

$$(4)$$

The coefficient matrix $\tilde{A} = [\tilde{a}_{ij}]_{mxn}, 1 \leq i, j \leq n$ is $m \times n$ fuzzy matrix where $\forall i, j, \tilde{a}_{ij} \succ 0$ or $\tilde{a}_{ij} \prec 0$ and $\tilde{x}_i, b_j \in F(\Re)$.

If matrix $\tilde{A}$ be denoted by: $\tilde{A} = (A, A', A'')$

that $A = [a_{ij}], A' = [a'_{ij}], A'' = [a''_{ij}]$,

$\tilde{x} = (x, x', x''), \tilde{b} = (b, b', b'')$, then one has:

$$\begin{cases} \max \tilde{z} = (c, c', c'') \otimes (x, x', x'') \\ s.t. \\ (A, A', A'') \otimes (x, x', x'') \preceq (b, b', b'') \\ (x, x', x'') \succeq 0. \end{cases}$$

$$(5)$$

Fuzzy Compromised Approach

The fuzziness in both single and multiple objective problems of the fuzzy parameters of constraints and/or satisfaction levels attained with objective function(s) can be solved by using compromised approaches. In literature this is mostly applied to multiple objective decision making problems. To construct such a compromised model with fuzzy objectives, solve the model using Equation (5) by taking one objective at a time and find for each

objective (Z_K) respectively, the best (Z_K^+) values and worst ones (Z_k^-) being correspondent with the set (decision variables) of solutions (x_k^*).

Define a linear membership function $\mu_k(x)$ for each objective as equation 6.

An equivalent LP problem (crisp model) is then defined as:

Maximized λ

Subject to

$$(Z_k - Z_k^-)/(Z_k^+ - Z_k^-) \geq \lambda \ \ for \ k = 1, 2,..$$

(7)

and all the original constraints set as well as non negativity constraints for X and a degree of truthfulness (λ). The problem is then solved using Equation (7). The solution is the degree of truthfulness (λ) which is achieved for the solution X^*. The corresponding values of the objective function Z_k^* are obtained and this is the most suitably compromised solution. Methodology explained in the above section is used to develop the FFLP model for the

reservoir operation to obtain the optimum release policy from the reservoir. Ranking of the fuzzy numbers is achieved by using linear ranking function as explained in the methodology section.

Case Study

The methodology discussed in the previous section is used for modeling of operation of Jayakwadi reservoir stage –II, a multipurpose project, created by constructing a dam across the river Sindaphana, a tributary of river Godavari, in Aurangabad district, Maharashtra State, India in operation since 1987. The location map of the reservoir is depicted in Figure 2. The gross storage of the reservoir is 453.64 Mm^3 and live storage 313.30 Mm^3. This is partly supplied by water from the upstream part of Jayakwadi reservoir stage I. Main canal (the Majalgaon right bank canal) carries a discharge of 82.63 $m^3 \ s^{-1}$. The length of the canal is 165 km. The total installation capacity for power generation is

$$\mu_{zk} = \begin{cases} 0 & Z_k \leq Z_k^- \\ (Z_k - Z_k^-)/(Z_k^+ - Z_k^-) & Z_k^- \leq Z_k \leq Z_k^+ \ \ for \ k = 1, 2,.. \\ 1 & Z_k \geq Z_k^+ \end{cases}$$
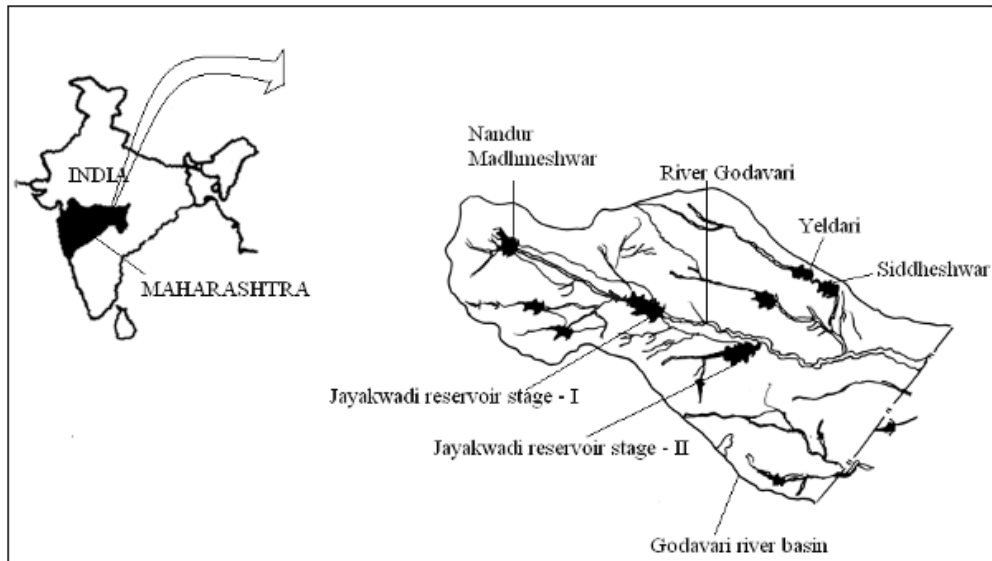
(6)

Figure 2. Location Map of Jayakwadi reservoir stage–II.

2.25MW. Irrigable command area is 938.85 km^2.

The 75% dependable monthly inflows into the reservoir are shown in Figure 3. Monthly irrigation demands were determined by the help of crop calendar, water requirements for various crops during different growth stages and the types of soils. Monthly irrigation demands in a water year also are shown in Figure 3.

Modeling Formulation

As mentioned in methodology, a wide range of models of water resources system planning and management are developed within a fuzzy environment. The first reason is a lack of adequate data, the second is the application of previous conditions to the future states, and third is an interaction of parameters that may not be so obvious. So application of FFLP as an optimization tool is both intuitive and plausible. Below is the generalized LP model developed for monthly operation of the reservoir assuming stationary inflows in a water year. As explained in methodology, FFLP formulations are incorporated in the following generalized reservoir operational model. The triangular fuzzy numbers are defined for each parameter and variable of

the model. The linear ranking function is used in defuzzifying the FFLP problem for the reservoir operation. Finally the model is solved for both maximized (λ) objectives by using FLP model. The reservoir operational model developed in the present study is explained as bellows:

Objective Function

The two objectives considered in the model are:
(1) Maximization of releases for irrigation (i.e., RI).
(2) Maximization of releases for hydropower production (i.e., RP).

$$Max\, Z_1 = Max\left(TOTRI\right)$$

$$Max\, Z_2 = Max\left(TOTRP\right)$$

(8)

Where, *TOTRI* is the Total Releases for Irrigation within all the time periods and *TOTRP* the Total Releases for Hydropower production. These objective functions can be written as:
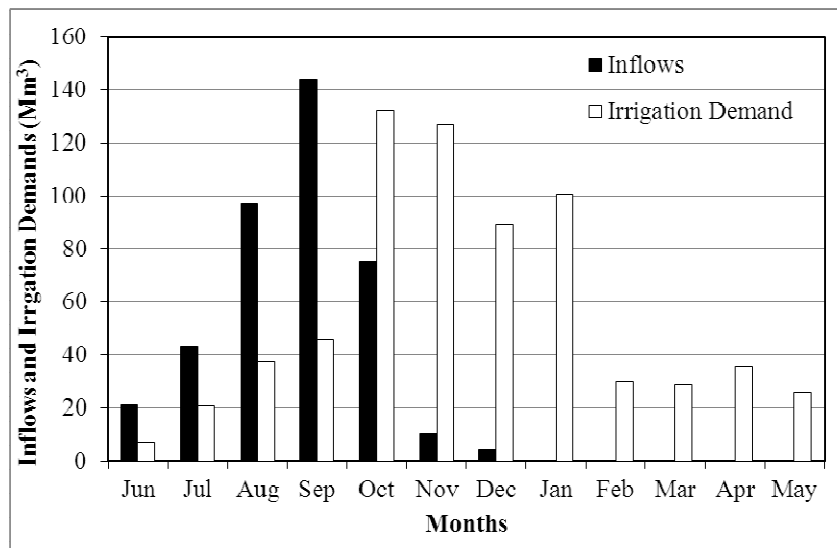

Figure 3. Inflows and irrigation demand for Jayakwadi reservoir stage–II**.**

$$Max \ Z_1 = \sum_{t=1}^{12} RI_t$$

$$Max \ Z_2 = \sum_{t=1}^{12} RP_t$$

$$(9)$$

Constraints

Turbine Release Constraint

Release for the turbine for hydropower production should be less than or equal to Turbine Capacity (TC) within each month (t), and it should be greater than or equal to the firm release committed for that month.

$$RP_t \leq TC \qquad \forall t = 1, 2, ..., 12$$

$$RP_t \geq FR_t \qquad \forall t = 1, 2, ..., 12$$

$$(10)$$

Irrigation Demand Constraint

Release into canals for irrigation (RI) should be less than or equal to maximum Irrigation Demand (ID). Release should also be greater than the minimum releases required for irrigation so that crop will not wilt. In the present case 30% of the maximum irrigation demand is considered as minimum irrigation demand for all the time periods.

$$RI_t \leq ID_t \qquad \forall t = 1, 2, ..., 12$$

$$RI_t \geq ID_{Mint} \qquad \forall t = 1, 2, ..., 12$$

$$(11)$$

Reservoir Storage Capacity Constraint

Live storage in the reservoir should be less than or equal to the maximum Storage Capacity (SC) and greater than or equal to minimum Storage Capacity (S$_{Min}$) for all the time periods.

$$S_t \leq SC \qquad \forall t = 1, 2, ..., 12$$

$$S_t \geq S_{Min} \qquad \forall t = 1, 2, ..., 12$$

$$(12)$$

Reservoir Storage Continuity Constraint

These constraints are related to the Releases for the Turbine (RP)$_t$, Releases for Irrigation (RI)$_t$, reservoir Storage (S)$_t$, Inflow (I)$_t$ into the reservoir, Feeder Canal Release (FCR)$_t$, Overflows (OVF)$_t$, Release for Water Supply (RWS)$_t$, and the Evaporation Losses (L)$_t$ for the whole time periods. Here evaporation losses are considered as a function of storage by assuming a linear relationship between reservoir water surface area and storage. The storage continuity constraints can be written as equation 13:

where,

$$a_t = A_a e_t / 2$$

In which A_a is the surface area of the reservoir per unit active storage volume; A_o is the surface area of the reservoir corresponding to the dead storage volume, e_t is evaporation rate for month t in depths unit.

The FFLP model formulated in this section is applied to the case study, and is solved using LINGO (Language for Interactive General Optimization). LINGO is a simple tool for utilizing the power of linear and nonlinear optimization to concisely formulate large problems, solve them, and have the solution analyzed.

RESULTS AND DISCUSSION

Throughout this study, the applicability of the reservoir operation model is improved

$$(1-a_t) S_t + I_t + FCR_t - RI_t - RP_t - A_0 e_t - OVF_t - RWS_t = (1+a_t) S_{t+1} \qquad (13)$$

by incorporating the FFLP model which involves uncertainties in model parameters and variables, representing them representing as fuzzy sets instead of crisp values

The model is applied to the case study of Jayakwadi reservoir stage – II, Maharashtra state, India. Two objectives namely maximization of annual releases for irrigation, and maximization of annual releases for hydropower production are considered in the model. Uncertainties in parameters and variables of the reservoir operation model are addressed by the triangular fuzzy number. The linear ranking function is used for defuzzification of the

FFLP problem. The model is initially solved for the individual objective function, the consequences of the two objectives being combined to determine an optimal compromised solution (Zimmermann, 1978). The results obtained from the individual optimization of the objectives are given in Tables 1 and 2. From the obtained results for irrigation presented in Table 1, it can be seen that for the months of July, August, September, October, April and May, the left and right spreads of the triangular fuzzy variable amount to zero. This means that in these months the irrigation demands are fully satisfied. The level of satisfaction of the objectives in these particular months is

Table 1. Release policy for the maximization of the releases for irrigation.

Month	Maximization of releases for irrigation (Mm^3)			Hydropower releases (Mm^3)		
	Left spread	Mean	Right spread	Left spread	Mean	Right spread
June	0	2.9	16.8	0	8.7	0
July	0	20.8	0	0	8.7	0
August	0	37.6	0	0	8.7	0
September	0	46.0	0	0	8.7	0
October	0	132.0	0	0	8.7	0
November	0	38.2	355.7	0	8.7	0
December	0	26.8	250.4	0	8.7	0
January	0	30.2	281.9	0	8.7	0
February	0	9.0	84.0	0	8.7	0
March	0	8.7	81.1	0	8.7	0
April	0	35.6	0	0	8.7	0
May	0	25.9	0	0	8.7	0
Total	0	413.7	1069.9	0	104.4	0

Table 2. Release policy for the maximization of the releases for hydropower.

Months	Maximization of power releases (Mm^3)			Releases for irrigation (Mm^3)		
	Left spread	Mean	Right spread	Left spread	Mean	Right spread
June	0	9.4	78.0	0	2.1	0
July	0	27.9	4.0	0	6.2	0
August	0	29	0	0	11.2	0
September	0	29	0	0	13.8	0
October	0	29	0	0	39.6	0
November	0	29	0	0	38.1	0
December	0	8.7	81	0	26.8	0
January	0	8.7	81	0	30.2	0
February	0	8.7	81	0	9.0	0
March	0	8.7	81	0	8.7	0
April	0	29	0	0	10.7	0
May	0	29	0	0	7.8	0
Total	0	246.1	406	0	204.2	0

the highest. However for the months of June, November, December, January, February and March, a minimum of irrigation requirement is satisfied through the model. Also for these months the fuzzification of the releases are possible as there is significant variation between the mean value and right spread of the fuzzy variable. For the month of June there is 83% of right spread observed, and for the remaining month the observed right spread is about 89%. It means that for these months the level of satisfaction for irrigation is lowest. When the objective of the maximization of annual releases for irrigation is in priority the corresponding releases obtained for hydropower amount to the firm releases required for hydropower generation. As a result of this, the left and right spreads are zero and minimum releases obtained for hydropower generation correspond to mean of the triangular fuzzy number. Similarly the optimal operating policy for hydropower releases is presented in Table 2. From the result it is observed that model has satisfied the maximum turbine capacity during the months of August, September, October, November, April and May. The level of satisfaction of the objective in these particular months is the highest. However for the remaining months, model has satisfied the releases requires for the firm power production of the turbine. In these months the fuzzification of the releases are possible, as there is 90% variation between the mean value and the right spread of the

triangular fuzzy variable. When the objective of the maximization of annual releases for hydropower is in priority the corresponding releases obtained for irrigation have satisfied the minimum irrigation requirement. As a result of this the left spread and right spread is zero and minimum releases obtained for irrigation corresponds to mean of the triangular fuzzy number.

Fuzzy Compromised Approach

Results obtained from the individual optimization are used to obtain the compromised solution for both objectives as explained in the methodology section. The best and worst values of either objective are obtained from the mean values of both of the objectives of triangular fuzzy number. When Z_1 is maximized, the corresponding value of Z_2 is considered to be the worst and vice-versa. These values are shown in Table 3. These values are used to develop the fuzzy compromised model using linear membership function to fuzzify both the objective functions and it the being solved for the maximization of degree of truthfulness (λ). The linear membership function developed for both objectives are given by Equations (14) and (15). The graphical representations of Equations (14) and (15) are shown by Figures 4 and 5.

$$\mu_{Z_1}(x) = \begin{cases} 0 & \text{if } Z_1 \leq 204.2 \\ (Z_1 - 204.2/(413.7-204.2) & \text{if } 204.2 \leq Z_1 \leq 413.7 \\ 1 & \text{if } Z_1 \geq 413.7 \end{cases} \tag{14}$$

Table 3: Best (Z_k^+) and worst (Z_k^-) values of the objective function.

Objective function	Bounds	
	Best value (Z_k^+) (Z_k^-)	Worst value
Releases for irrigation (Mm^3)	413.7	204.2
Releases for hydropower (Mm^3)	246.1	104.4

$$\mu_{Z_2}(x) = \begin{cases} 0 & if\ Z_2 \leq 104.40 \\ (Z_2 - 104.40/(246.10 - 104.40)) & if\ 104.40 \leq Z_2 \leq 246.10 \\ 1 & if\ Z_2 \geq 246.10 \end{cases}$$
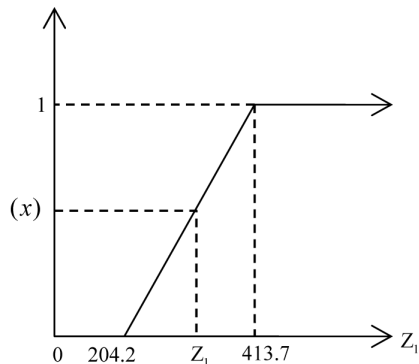
(15)
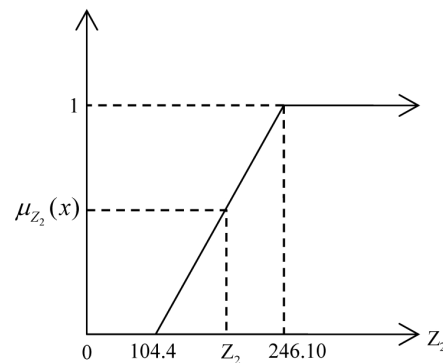

Figure 4. Membership function (Z_1).

Figure 5. Membership function (Z_2).

By using the above information, the following fuzzy compromised model is formulated using Equation (7) and it is the solved to obtain maximized degree of truthfulness (λ).

$MAX = \lambda$,

Subject to,

$(Z1 - 204.2)/(413.7.0 - 204.0) \geq \lambda$,

$(Z2 - 104.40)/(246.10 - 104.40) \geq \lambda$,

and along with all the original constraints given in the model and $\lambda \geq 0$. In this formulation λ is the degree of truthfulness obtained by simultaneously optimizing the fuzzified objectives Z_1 and Z_2.

Results obtained by the solving compromised model, using Equation (7) are presented in Table 4. From the obtained results it is observed that the FLP model has satisfied the minimum irrigation requirement for the month of June, July and as well from December to May. However for the month of August, September and October the FLP model has satisfied the maximum irrigation demand. For the month of November, the releases are 40% of maximum irrigation demand. The degree of truthfulness of both objectives is 0.78. Similarly from releases for hydropower production, it is observed that FLP model has satisfied the firm release requirement of the turbine for the month of

Table 4. Optimal operating policy.

Months	Releases for irrigation (Mm^3)	Release for power (Mm^3)
June	2.13	9.493
July	6.24	27.96052
August	37.64	29
September	46.02	29
October	132.01	28.75
November	50.29	29
December	26.87	19.33
January	30.2	8.7
February	9	8.7
March	8.69	8.7
April	10.67	8.7
May	7.764	8.7
Total	367.524	216.03352

January to May. However for the remaining months i.e. July, August, September, October and November the releases are equal to maximum turbine capacity. For the month of December the releases are only 33% less than the turbine capacity. For the month of June the releases are slightly more than the firm power releases. It means that the maximum power production can be achieved in the month of July to November. However in the months of January to May a minimum of power production can be maintained. The results are also compared with the operating policy obtained by considering the fuzzy coefficients and crisp variables of the reservoir operation model (Regulwar and Kamodkar, 2010) and it is observed that the degree of truthfulness obtained is only 0.53 as compared with the FFLP model; i.e. 0.78. The operating policy obtained through FFLP model is preferred to the policy obtained by considering the fuzzy coefficients and the crisp variables of the reservoir's operation model.

CONCLUSIONS

In literature, fuzzy linear systems of equations do not usually consider the fuzzy decision variables. In this article, the fully fuzzy linear systems i.e. fuzzy linear systems with fuzzy coefficients involving fuzzy variables are investigated and applied to the reservoir operation problem to decide the optimal release policy of the Jayakwadi reservoir stage – II, Maharashtra state, India. The uncertainty is inevitable in the reservoir operation modeling due to a lack of a perfect understanding of the phenomenon and of the process involved, in addition to random nature of the events. These uncertainties involved in the various parameters and variables are addressed here by fuzzy set theory. Releases for irrigation, releases for hydropower generation, irrigation demands, hydropower demands and storages in the reservoir during all the time periods are defined by triangular fuzzy numbers. Fully Fuzzy Linear Programming (FFLP) model

as explained in methodology for reservoir operation is developed by considering two objectives, i.e. maximization of annual releases for irrigation and maximization of annual releases for hydropower generation. Results obtained by solving FFLP model for individual optimization of objectives are used to formulate the fuzzy compromised model for both objectives while the model being solved for a maximization of the degree of truthfulness (λ). The value of (λ) obtained is equal to 0.78 and the corresponding values of the objectives i.e. optimum releases for irrigation, and for hydropower generation are respectively recorded as equal to 367.5 Mm^3, and 216.0 Mm^3.

From the methodology and the obtained results it can be concluded that when limited information is available on model parameters and on boundary conditions, fuzzy modeling can be employed to address the uncertainty from the various parameters of the reservoir operation to be used in the optimization of the model. The linear ranking function can then be efficiently used for the defuzzification of the triangular fuzzy numbers.

Notations

$\tilde{A}$: Triangular fuzzy number.
a : Peak of triangular fuzzy number.
α : Left spread of triangular fuzzy number.
β : Right spread of triangular fuzzy number.

$\mu_{\tilde{A}}(x)$: Grade of Membership (Degree of Belonging) of x in A.

$FT(\Re)$: Set of all triangular fuzzy numbers.

$\Re$: Set of all fuzzy numbers.

$F(\mathfrak{R})$: Set of all fuzzy subset of $\mathfrak{R}$.

λ : Degree of truthfulness

FP_t : Release for firm power during month t.

ID_t : Maximum irrigation demand during month t.

ID_{Mint} : Minimum irrigation demand during month t.

I_t : Inflows into the reservoir during month t.

RI_t : The releases into canals for irrigation during month t.

RP_t : The releases for hydropower production during month t.

RWS_t : Release for water supply during month t.

FCR_t : Feeder canal releases during month t.

SC_t : The storage capacity of the reservoir during month t.

S_t : The storage in the reservoir during month t.

S_{Mint} : Minimum storage capacity during month t.

TC : Turbine Capacity (flow through).

OVF_t : Overflow from the reservoir during month t.

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سیاست‌های بهره برداری از مخزن چند منظوره: رویکرد برنامه‌ریزی خطی تمام فازی

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چکیده

در این مقاله فرمول‌بندی برنامه ریزی خطی تمام فازی برای بهره برداری از مخازن چند منظوره ارائه شده است. در دنیای واقعی سامانه‌های منابع آب عموماً دارای پیچیدگی‌هایی از جنبه‌های اجتماعی، اقتصادی، منابع طبیعی و زیست محیطی هستند که به مسائل چند هدفه با عدم قطعیت‌های قابل توجه در پارامترهای

سیستم، اهداف و در برهم‌کنش‌های متقابل آن‌ها منجر می‌شوند. این عدم قطعیت‌ها در مدل برنامه ریزی خطی تمام فازی بهره برداری مخزن، در غالب مجموعه‌های فازی در نظر گرفته می‌شوند. در این تحقیق یک مدل برنامه ریزی خطی تمام فازی بهره برداری مخزن توسعه یافت که در آن تمام پارامترها و متغیرهای تصمیم به صورت اعداد فازی هستند. مدل توسعه یافته به وسیله مطالعه موردی فاز دو مخزن جایاکاوادی هند با هدف بیشینه کردن رها سازی سالانه آب برای آبیاری و تولید برقابی، تبیین شده است. مدل برنامه ریزی خطی تمام فازی بهره برداری مخزن، به منظور به‌دست آوردن یک پاسخ سازگار از طریق بهینه سازی همزمان اهداف فازی و درجه اعتماد پذیری متناظرشان با استفاده از تابع درجه عضویت خطی، حل شده است. درجه اعتماد پذیری به‌دس0.ت آمده برابر ۰/۷۵ و مقدار رهاسازی سالیانه مربوطه برای آبیاری برابر ۳۶۷ میلیون متر مکعب است، در حالی که رها سازی برای تولید برق‌آبی معادل ۲۱۶ میلیون متر مکعب می‌باشد. مطالعه حاضر به روشنی نشان داد که استفاده از مدل یاد شده در بهینه سازی سیستم‌های مخازن چند منظوره گزینه بالقوه ای است که برای به‌دست آوردن سیاست بهره برداری بهینه می‌توان از آن استفاده نمود.