ESTIMATING DEPENDENCE STRUCTURE AND RISK OF FINANCIAL MARKET CRASH

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DECLARATION

This thesis is my original work and has not been presented for a degree in any other University.

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DEDICATION

This research work is dedicated to God Almighty.

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CONTENTS

LIST OF FIGURES

LIST OF TABLES

ABBREVIATIONS

df	Degree of freedom
GARCH	Generalized Autoregressive Conditional Heteroskecity
GPD	Generalized Pareto Distribution
USA	United State of America
UK	United Kingdom
VaR	Value at Risk
T-Bill	Treasury Bills

ABSTRACT

Dependence structure of financial market is crucial in determining investment positions and strategies to reduce financial market risk. Linear correlation model is not suitable to capture asymmetries and dependence structure of financial market as the only capture the degree of correlation. In order to address the problem, the study estimates dependence structure between financial markets using the copula concept. Different relationships that exist in normal and extreme periods were estimated using copula. The Inference Functions for Margins method was used in estimating copula parameter thereby obtaining dependence estimates. The study show analytically how dependence estimates are imputed into Value-at-Risk. The Inference Function for Margin estimator was found to be consistent and asymptotically normal. From the empirical findings, to diversify market risk during the crisis period (2007-2009) the market pairs with the highest maximum possible loss is evident in the stock market and followed by the stock market-Tbills pairs. However, the less risk portfolio is the stock-bond. As bonds are financially considered as safer investments over time, implies that investment in a stock-bond portfolio is less risky during the crisis period.

CHAPTER 1

INTRODUCTION

1.1 Background Information

Dependence structure between random variables is crucial in multivariate analysis. In mathematical finance and economics, the dependence structure between financial markets are critical in order for investors, policymakers and researchers to make informed decision about investing their resources and making correct investment strategies (Ling, 2006). This is because in general, interest rates and equity prices move in opposite directions but in times of extreme stress, they both move in the same direction. Investors therefore, are concerned with the rate of return on their assets in order to diversify and get better returns from their portfolios.

The extent of interconnectedness and interdependence of the financial system was highlighted by 2007/09 financial crisis (Aloui et al., 2011). The studies of (Longin and Solnik, 2001) and (Dennis, 2013) suggest and further confirm that financial markets are likely to be more correlated in period of burst than periods of booms. When dependence structure of financial markets (stock, bond, exchange rates and money markets) are closely related, they tend to be faced with a possibility of a market crash.

Extreme events in financial system could occur in various forms such as default in credit, collapse in volatile asset prices and the recently experienced financial crisis of 2007/2009. In 1982 currencies of Mexican, Argentinean alongside other developing countries depreciated sharply, making stock prices in these economies tumble. As a result, several banks in these economies failed owing to the large loan losses (Kindleberger and Aliber, 2005). Similarly, the Asian currency crisis of 1997, began with the devaluation of the Thai baht which lead to wide spread contagion of financial market.

According to (Allen et al., 2009) the fall in house prices led to a fall in the prices of securitized loans and mortgages, which affected financial markets globally. During the fall of 2007, the prices of securitized mortgages continued to fall and many financial institutions started to come under strain.

Until recently, extreme events (financial crisis) were regarded as outliers and often excluded from statistical analysis of financial market (Wu et al., 2012). The financial turmoil has highlighted the importance of analyzing extreme events in investment decision, pricing of financial assets as well as risk management. Therefore, extreme events are likely to alter the dependence structure of financial market which could have implications for investment decisions and ability to estimate the risk of financial markets crashing.

1.2 Statement of the problem

Financial markets tend to be more associated in period of extreme events. However, a danger arises when correlation of financial markets are assumed to be linearly dependent.The literature documents that financial market data are leptokurtic and asymmetry in nature. Therefore, analyzing market correlation due to financial market cycles in a linear sense is not a diversifiable risk as the structure of dependence is ignored.

Market crashes are considered catastrophic events for instance when the values of equity market suddenly decline, exchange rates depreciates rapidly as well as when there is a credit default. When these rare events occur, they could lead to instability of the financial system and exposure to systemic risk. In recent times, extreme events in financial market are no longer considered as just an outlier having a negligible probability. Modeling of extreme events is now important in order to reduce financial market risk. In analyzing extreme events, assumption about normality is often made in multivariate analysis when dealing with financial series (Chen et al., 2004). However, restriction to elliptical distributions such as t – student and Gaussian distribution, implies dealing with measures which only captures dependence in the linear sense. Traditional linear correlation model such as Pearson and Spearman correlations have been used to analyze dependence (Britt and Napoli, 2005). Correlation does not imply dependence. Linear correlation models however, from empirical literature have been found not to be appropriate for measuring non normal distributions. Since financial market data exhibit heavy tails as a result, linear correlation models cannot capture the structure of dependence (Embrechts et al., 2002). Financial decision based on linear correlation models may be misleading as it is not robust in modeling of nonlinear dependence (Boyer et al., 1999). With the drawback of linear correlation models, it may lead to underestimation of the risks that could be associated with financial market crash. This was evident using Gaussian copula which was limited in capturing tail dependence and asymmetries of financial market variables during the financial crisis. Therefore, it is pertinent to use models which capture financial market non-linearity. This research shall fill the gap in the literature by modeling dependence structure and estimating the risk associated with a financial market crash.

1.3 Objectives of the Study

1.3.1 Main Objective

To estimate dependence structure and risk of financial market crash.

1.3.2 Specific Objectives

The specific objectives are to:

(i) Estimate dependence structure in periods of extreme events (financial crisis).

(ii) Determine changes on the dependence structure caused by extreme events.

(iii) Estimate the risk of a market crash.

1.4 Justification of the study

Extreme events such as financial crisis often occur and it is important to study them in other to make better strategies and investment decisions. However, during the financial crisis of 2007/2009 it was revealed that assuming financial distribution and using Gaussian copula model to measure dependence and inferring for investment might be misleading. This therefore makes it significant to use models that can capture the asymmetries of financial market data.

This study intends to fit different copula models to ascertain the structure of dependence and not assume a Gaussian distribution of the financial market data. Since there is a consensus that financial market data are non-linear, the structure of dependence between financial markets have implications for market participants. The study incorporates asymmetry of correlations into portfolio decisions which can add substantial economic value to investor in terms of investing in markets which could lead to greater returns on their investments.

Incorporating asymmetries in the analysis of dependence structure are equally beneficial to supervisory authorities such as the central banks such that understand the structure of dependence of financial markets have implications for the financial stability of an economy.

1.5 Scope of The Study

The study aimed at modeling changes in dependence structure of financial markets (Stock, T-bill and Bonds) in advanced market (USA, UK) and developing markets (Kenya, Nigeria, South Africa). For uniformity, consideration for data is given from January, 2000 to March 2016 on the selected countries. The choice of the sample period is informed to capture periods of market boom and burst. Similarly, the choice of the sample is based on the availability of market data and the interest of modeling dependence structure and estimating the risk of financial market crash using a copula approach, which gives insight to diversification of financial market risk.

CHAPTER 2

LITERATURE REVIEW

Several studies have analyzed changes in dependence structure of financial markets. Studies of (Costinot et al., 2000); (Patton, 2012); (Rodriguez, 2007) and (Okimoto, 2008) analyze changes of dependence structure of financial markets using different methods. In this section we examine some of the previous studies that found the copula theory handy in modeling dependence structure of financial markets. The review of literature is carried out in sections below as it relates to dependence structure of market.

2.1 Dependence Structure (linear and nonlinear)

Several studies have estimated linear and non linear dependence of financial markets. Guilheme et al. (2015) analyzed dependence structure between risk and returns in Latin financial markets. Using daily data from the Brazilian, Argentinean, Mexican and U.S markets, they estimated a copula based multivariate GARCH model. Furthermore, they employed Normal, Student's t, Frank, Gumbel and Galambos Copula to fit data. They found that the linear correlation between risk and returns in these countries are not significant, emphasizing the lack of linear dependence. However, this study differs from the assumption of a Gaussian distribution since there is a consensus that financial returns are not symmetric.

2.2 Dependence Structure (normal and extreme periods)

Empirical studies have evolved to analyze dependence structure of financial market, concentrating on normal and extremely volatile periods. To mention a few studies done in tranquil and turmoil periods include (Samitas et al., 2007) and (Chai, 2015).

Chai (2015) analyzed dependence structure and optimal hedge ratio of U.S. spot and futures markets in financial crisis. To estimate the optimal hedge ratio, a Gumbel copula-threshold-GARCH model was employed, simultaneously capturing asymmetric nonlinear behaviour in univariate returns of spot and futures markets and bivariate dependency.

2.3 Copula in Modeling Extreme Markets

Using the theory of Copulas, (Costinot et al., 2000) examined the dependence structure among financial markets during the crisis periods. Extreme events were better modelled through copulas rather than simple correlation analysis. Similarly, (Longin and Solnik, 2001) used a Gumbel copula to analyze extreme correlation in international equity markets. The Gumbel copula is a popular method and widely used in various applications. However, the Gumbel copula is restrictive in the sense that it failed to reflect various tail dependence in reality as financial market tend to exhibit various patterns. Furthermore, (Poon et al., 2004) used sub-models in estimating tail dependence and suggested several different tail independence structures to deal with the tail dependence underlying equity markets.

Tsui and Zhang (2010) investigates the relationships between five currencies in Asia around the period of Asian Financial Crisis in 1997, including the Singapore Dollar, Japanese Yen, South Korea Won, Thailand Baht and Indonesia Rupiah. Employing various time-varying copula models to examine the possible structural breaks. The results indicate significant changes at the dependence level, tail behavior and asymmetry structures between returns of all permuted pairs from the five currencies before and after the crisis. The results show that the copula approach seems to have more explanatory power than the existing ones in identifying structure breaks.

Ghorbel and Trabelsi (2013) using time varying copula model to investigate the impact of the global financial crisis on dependence between US and each of six major stock markets and on risk management strategies. The model is implemented with an AR-GARCH-t for the marginal distribution and the extreme value copula for the joint distribution, which allow taking into account non-linear dependence, tails behaviour and their development over time. The study investigate whether there are significant changes in the time-varying dependence structure of market and in VaR and ES measures especially during global financial crises period. Empirical results show that market dependence between US, European and Brazilian markets tend to increase considerably during crisis period and this increase started around the beginning of 2008. In the other hand, market volatility registered record levels around the end of 2008 due to the increase of the degree of uncertainty in this period. As a consequence, investors will allow more amounts to cover against negative evolution of portfolio value.

He and Zhao (2013) used copula approach to investigate the extreme dependence between crude oil and stock sectors in China. Empirical results show that oil-stock linkages have been changed by the occurrence of the recent financial crisis. Before the financial crisis, only two stock sectors have weakly negative tail dependence, while in the post-crisis period much more sectors become positively dependent with oil at extreme levels. Meanwhile, heterogeneity of sector dependence with crude oil is identified. The sector of Basic materials has the largest tail dependence (94.5%) with crude oil prices, which is followed by Financial and Construction & Materials. Asymmetric tail dependence is found in the Basic materials-crude oil pair, indicating that two returns exhibit greater correlation during market crashing than booming. Empirical findings in this paper have potentially important implications for financial market participants.

2.4 Copula as a Tool for Measuring Dependence Structure

Dependence structure of financial markets have been studied using regime switching copula. (Ang and Bekaert, 2002) in a regime switching setup finds that the cost of ignoring changes in regimes of high and low dependence increases in presence of a risk-free asset. Similarly, Rodriguez (2007) modeled dependence using switching parameter copula to determine financial contagion. Estimating daily returns from five East Asian stock indices in the Asian crisis period and four Latin American stock indices during the Mexican crisis, the study found changing dependence structure during periods of crisis. Furthermore, the study concludes that Asian countries were characterized by increased asymmetry and tail dependence. Latin American countries were symmetric and thus exhibit tail independence. Okimoto (2008) in a regime switching copula framework analyzed asymmetric dependence for various international stock indices.

Regarding the regime switching framework, the above authors have analyzed the importance of changing dependence structure. However, they did not explore the importance of identifying the change in dependence structure in relation to reducing financial market losses. This study will fill the gap in the literature by analyzing the implication of the change in dependence structure for investors portfolio diversification.

A consideration on time varying copula to analyze dependence structure was proposed by (Patton, 2006), extending Sklar's theorem for conditional distributions and proposed an observation which is driven by a conditional copula model. This model defined the time-varying dependence parameter of a copula which served as parametric function of transformations of lagged data and an autoregressive term. The study tested for dependence between the Deutsche Mark and the yen. The empirical result reveals that the mark-dollar and yen-dollar exchange rates are more correlated when the are depreciating against the dollar than when they are appreciating. Similarly, using a time varying copula (Aloui et al., 2011) investigated the oil-equity relationships for six economies. Their empirical results show that dependence structure of oil-equity market change especially during the financial crisis as the lower tail dependence is more stronger than the upper tail dependence indicating the possibility of financial contagion. Nguyen and Nguyen (2014) analyzed the dependence structure of bond market, equity money markets in the United States and Australia. Using a combination of empirical distribution and time- varying copula model, their empirical findings provide some important implications of investment in Australian and US financial markets. Hu (2010) used time varying conditional copula to model dependence structure of Chinese and US stock market returns. The study used Autoregressive Generalized Autoregressive Conditional Heteroscedastic-t (AR-GARCH – t) to model marginal distributions, while Normal and Generalized Joe-Clayton (GJC) copula models were employed to analyze the joint distributions. Using a two-step maximum likelihood method, the study found three results. First, the time varying dependence model doesn't always perform better than the constant model. Second, the upper tail dependence is much higher than lower tail dependence in some short periods. Third, the tail dependence with other financial markets is much lower in China than the US. Bartram et al. (2007) used a time-varying copula model to investigate the impact of the introduction of the Euro on the dependence between seventeen European stock markets during the period 1994-2003. They modeled marginal distribution by a GJR-GARCH-t and used the Gaussian copula for the joint distribution. Their results indicate that dependence across the market increased after the introduction of the common currency only for large equity markets, such as in France, Germany, Italy, the Netherlands and Spain. They also estimated time-varying dependence between the Euro and non-Euro European equity markets and found that the UK and Sweden, but not other countries outside the Euro area, exhibited an increase in equity market co-movement. Ni and Jiang (2011), used a Copula – GARCH model to investigate dependence structure and tail dependence between Shanghai Composite index and CSI 300 index (Chinese financial futures returns). The study found a high positive dependence and tail dependence between the variables of consideration. They conclude that the two return sequences have a joint thick tail, with a higher upper tail and lower tail dependence.

The authors shed light on the static and time varying parameters using a copula and documenting that dependence is not static. However, their findings were not comprehensive to capture the risk inherent in changing dependence structure of the financial market. This study goes beyond the static and time varying copula to identify the risk of a change in dependence structure of financial market.

Other authors have also used a mixed copula approach to analyze financial market dependence. Samitas et al. (2007) use copulas to measure the dependence between the stock exchanges in period of crises. The approach was based on multivariate copula functions and Markov switching parameters. They found that dependence increases for both the BRIC and the developed markets of the US and the UK when at least one of the countries is in a financial crisis. Further, they find that all crises drive higher market dependence when compared with stable periods. The results also indicate that not only co-variances are higher during a crisis time, but also that correlations and co variances increase during a crisis. Necula (2010) using a t- copula and a mixture and the mixture of Gumbel– Clayton to estimates the dependence structure between stock indices. The study finds that the t-copula and the Gumbel- Clayton mixture copula are appropriate copula functions to capture the dependence structure of two financial return series.

Severally studies have empirically investigated dependence structure of financial markets using a Copula- Garch model.Dajcman (2013)analyzed dependence structure between returns of Croatian and five European stock markets (Austrian, French, German, Italian, and the UK) using a copula GARCH approach. The return series were modeled as univariate GARCH processes and the dependence structure between the return series using a copula function. Furthermore, four different copulas were fitted (a constant and conditional normal and symmetric Joe-Clayton (SJC) copulas) using a semi-parametric method. The study reveals that time-varying normal copula yields the best fit for Croatian and other European stock market pairs while time-varying SJC copula is the best fit for (Croatain and Austrian) and (Croatian and UK FTSE100). The study further reveals that the probability of simultaneous extreme positive and negative returns in Croatian and other European stock markets can increase to 0.77 during turbulent times. Similarly, lower and upper tail dependence dynamics between Croatian and other European stock markets is similar in pattern, differing only in scale. The paper concludes that the dependence between the stock markets of Croatia and five major European stock markets is dynamic and can be properly captured by either a dynamic normal or symmetric Joe-Clayton copula GARCH models.

2.5 Dependence Structure And Financial market Loss

Another dimension to the literature is measuring the dependence structure of financial markets as it relates to estimating the risk of financial loss. Goorbergh et al. (2005) examined the behaviour of bivariate option prices in the presence of association between the underlying assets. Parametric families of copulas offering various alternatives to the Gaussian dependence structure were used to model this association, which is explicitly assumed to vary over time as a function of the volatilities of the assets. These dynamic copula models were applied to better-of-two-markets and worse-of-two-markets options on the Standard and Poor's 500 and Nasdaq indexes. Their results demonstrated that option prices implied by dynamic copula models can differ substantially from prices implied by models that fix the dependence between the underlyings, particularly in times of high volatilities. In the study, the Gaussian copula also produced option prices that differed significantly from those induced by non-Gaussian copulas, irrespective of initial volatility levels. Within the class of alternatives considered, option prices were robust with respect to the choice of copula.

Romano (2002) described some possible uses of copula functions in risk management applications and proved how some kind of copula functions are easy to implement in Monte Carlo simulation models to estimate risk measures. A practical application with a portfolio of ten Italian equities was performed and proved that the common hypothesis of multi-normal distribution for asset returns (or risk factor returns) underestimates the VaR and the Expected Shortfall of a market portfolio. A Monte Carlo simulation, modeling asset returns using fat tail marginal distributions and a copula function with tail dependence was also performed and obtained a more accurate estimate for the two risk measures. The study provide that a methodology using a multivariate Gaussian distribution of the latent variables does not capture the risk of many joint counterparty defaults. On the contrary, events of this kind are effectively captured using the Student-t copula to describe the dependence structure of the latent variables. Therefore, the Student-t copula can be very useful to model the extreme risk that worries risk managers and supervisors.

Shamisia et al. (2011) focused on estimating risk due to extreme events which goes beyond multivariate normal distribution of joint returns. Using a multivariate copula –EGRACH approach, they investigate the presence of conditional dependence between financial markets. Their finding reveals significant dependence for Singapore and Malaysia. Canela and Pedreora (2012) used continuous two-dimensional copulas to estimate dependence structures of daily returns in a pairwise sense. The study reveals that inter-market dependence between Latin America stock market showed higher probabilities of extreme losses. Therefore, the dependence structure of stock markets strengthens more in crisis periods than in tranquil ones.

Xiao and Dhesi (2011) investigates volatility spillover effect between FTSE100 and S&P500 stock indices. Strong lagged volatility of stock market itself and asymmetric spillover effect between UK and US stock markets are found out based on the multivariate GARCH-BEKK model. Furthermore, on a two step Copula-GARCH model to examine the correlation and tail dependence of returns. The study found evidence that a volatility spillover effect significantly exists between the UK and US stock markets. The spillover effect is asymmetric; the S&P500 dominates the volatility effect. The study concludes that time-varying copulas perform better amongst all the six copula candidates.

Mahfound (2012) studied the asymmetry and the non-linearity in the dependence structure between two vectors, the copula approach was applied. To deal with non-normality, heavy tails and heteroscedasticity in the returns series an ARMA-GHARCH with student – t distributed error terms was applied. Using Clayton, Gumbel and the Frank copula from the Archimedean copulas were fitted to a portfolio that consists from two major stock indices from the Eurozone, namely to the German DAX- 30 and to the French CAC-40 index. As a result, it was found that the copula that best fit the dependence structure between the two indices is characterized by a relatively strong upper tail dependence described by the Gumbel copula. The joint distribution of the DAX-30 and the CAC-40 stock indices exhibit higher upper tail dependence. This means that large gains from DAX-30 index and the CAC-40 index have more tendency to occur simultaneously then large losses.

2.6 Dependence Between Financial Asset Classes

The review also takes into consideration dependence structure of different financial assets. Kang (2007) modeled joint distribution of excess returns of four major assets (one year and ten-year Treasury bonds and S&P 500 and Nasdaq indices). Using a copula –GARCH approach and a two stage estimation procedure where marginal distributions are estimated separately in the first stage and then the copula is estimated in the second stage. Furthermore, consideration on three approaches in building multidimensional copula for the dependence structure of multiple variable: (1) n-dimensional normal copula and n-dimensional Student's t copula, (2) hierarchical Archimedean copula and (3) mixed copula. The study reveals that that n-dimensional Students' t copula yields the highest log likelihood while hierarchical Archimedean copula yields significantly low log-likelihoods. Similarly, plotting exceedance correlation and tail dependence between four series show that the dependence structure is asymmetric between two stock indices are not significant.

Wu and Lin (2010) proposes three classes of copula-based GARCH models to describe the time-varying dependence structure of stock-bond returns, examining the economic value of copula-based GARCH models in a mean-variance framework. We compare their out-of-sample performance with other models, including the passive, the constant conditional correlation (CCC) GARCH, and the dynamic conditional correlation (DCC) GARCH models. The empirical results, show that a dynamic strategy based on the GJR-GARCH model with Frank copula yields larger economic gains than passive and other dynamic strategies. Moreover, a more risk-averse investor will pay higher performance fees to switch from a passive strategy to a dynamic strategy based on GARCH-based copula models.

Adam et al. (2013) investigated dependence structure between Polish and foreign financial assets class (stocks, bonds and foreign exchange). Using several copula families, the result reveals that Polish equities, currency and to some extent long-term sovereign bonds exhibit economically significant tail dependence, while short-term bonds appear relatively unaffected. Symmetric tail behaviour characterizes the majority of asset pairs. The study finds significant asymmetries in a number of cases, with assets more likely to post large losses when global conditions significantly deteriorate, rather than to gain when they improve.

Benlagha (2014) investigates the dependence structure related to four French nominal and index-linked bonds with various maturities and reference indices. Employing various copulas approach to select the appropriate one for our data and compare results obtained using the copula method with multivariate dynamic conditional correlation GARCH (DCC-GARCH) modelling. The best copulas used to model the dependence among bond returns are the Plackett and Student models. The results reveal a dynamic correlation between bond returns. In particular, the relationship between nominal and indexed bonds is characterized by an asymmetric dependence.

Yang et al. (2015) explore the dependence structures among international stock markets, including developed, emerging, and frontier markets, using the hierarchical Archimedean copula model. Empirical results indicate that emerging markets show the strongest dependence with European markets. Frontier markets show the weakest dependence with other market. After the global financial crisis, the lower dependence structure among the international stock markets has changed. Negative news have a larger impact on the degree of dependence than positive news. Contagion effect is observed in both the global financial crisis and the EU debt crisis.

Jammazi et al. (2015) studied dependence pattern between stock and long-term government bond returns for several range of developed countries over the last two decades by using a DCC-GARCH-copula model. The empirical results show that the dependence structure between stock and 10-year government bond returns varies significantly over time for most countries. In particular, a positive stock–bond association is observed during the 1990s, while the relationship becomes negative from the early 2000s, supporting the presence of flight-to-quality effects. In addition, no evidence of asymmetric and tail dependence is found for the vast majority of countries.

Pastpipatkul et al. (2015) analyze co-movement and dependence of three stock markets, oil market, and gold market. These are gold prices as measured by gold future, crude oil prices as measured by Brent, and stock prices as measured by three developed stock markets comprising the U.S. Dow Jones Industrial Average, the London stock exchange, the Japanese Nikkei 225 index. In a bid to capture the correlation and dependence, an application of C-vine copula and D-vine copula were employed. The study demonstrate that the C-vine copula is a structure more appropriate than the D-vine copula. Furthermore, the study finds positive dependence between the London Stock Market and the other markets. Complicated results were found when the London stock exchange, the Dow Jones Industrial Average, and Brent were given as the conditions. Finally, the study found that gold might be a safe haven in this portfolios.

2.7 Market Crashes and Risk Analysis

Chow et al. (1999) splitted data into sub-samples for "stressful " and "normal" periods, in the context of the determination of mean-variance efficient porfolios. They define outliers to be those observations that fall outside of the centre of the distribution with entails a portfolio comparising 8 asset classes. Chow et al. (1999) observe the optimal portfolios to be quite different during stressful and normal conditions. They observe that the normal porfolio performs poorly during times of extreme market movements, and that equally, the stressful portfolio performs poorly under normal market conditions.

CHAPTER 3

METHODOLOGY

3.1 Introduction

This chapter gives an overview of theoretical framework, linear correlation models as well as contextualizing the copula theory to analyze financial market dependence and associated risk of a market crash.

3.2 Theoretical Framework

Assume a portfolio of two financial market index. The initial value of the portfolio is

$$P_0 = n_1 A_1 + A_2 n_2 \tag{3.1}$$

where $n_1$and n_2 are the number of units of the two financial markets index, which are valued at A_1 and A_2 at the beginning of a period. We denote $A_1^{'}$ and $A_2^{'}$ their values at the end of the period. The new value of this portfolio at the end of the new period is given by

$$P_1 = n_1 A_1^{'} + n_2 A_2^{'} \tag{3.2}$$

$$= n_1 A_1 e^x + n_2 A_2 e^y$$

to get log returns on the individual financial market index, we denote

$$X = log\left(\frac{A_t}{A_{t-1}}\right) \tag{3.3}$$

and

$$Y = log\left(\frac{A_{2t}}{A_{2t-1}}\right) \tag{3.4}$$

The log return of the portfolio is given by

$$R_t = log\left(\frac{A_t}{A_{t-1}}\right) = log\left(\frac{n_1A_1}{n_1A_1+n_2A_2}e^X + \frac{n_1A_1}{n_1A_1+n_2A_2}e^Y\right)$$

$$R_t = log\left(\lambda_1 e^x + \lambda_2 e^Y\right) \tag{3.5}$$

where $\lambda_1$and λ_2 are the individual market index.

The framework is based on copula modeling which originates from (Sklar, 1959). A copula is defined as a multivariate distribution function with uniform marginal distributions and its functional form joins all the margins to form a joint distribution of multiple random variables. It is a dependence function between random variables which links the univariate marginal together without losing any information from the initial multivariate distribution.

Mathematically, according to Nelsen (2006) a copula can be defined as a function C which is a mapping of the form $C : [0,1]^n \to [0,1]$ i.e a mapping of the n-dimensional unit code $[0,1]^n$ such that every marginal distribution is uniform on the interval $[0,1]$ the following three properties must hold for an n-dimensional copula: $C(u,0) = C(0,v) = 0$ for all $u, v\,\epsilon[0,1\}$

i. For all $u_i\epsilon\,[0,1], C(u_1,u_2...u_p) = 0$, if at least one coordinate of u_i equals 0.

ii. For all $u_i\epsilon\,[0,1], C(1,...,1,u_i1...1) = u_i$

iii. $C(u_1,u_2...,u_p)$ is grounded and p-increasing in each component u_i

Using a copula model has several advantages in modelling dependence structure of financial variables (Nguyen and Nguyen, 2014). Copula functions are very flexible in modelling dependence since it allows us to separately model marginal and the corresponding dependence structure. Second, copula functions enable us to directly model tail dependence; furthermore, they tell us not only the degree but also the structure of the dependence (Nguyen and Nguyen, 2014). Third, copula functions are invariant under transformations of the data while the linear correlation is not (Nguyen and Nguyen, 2014). Therefore, its flexibility in model specification allows us to use any specified marginal distributions to join with any available copulas, thus creating complex non-normal distributions which can fully capture dependency. Importantly, some copulas can help model tail dependence or co-movements, which show whether variables boom or crash together.

Modeling tail dependence is crucial to this study in two ways. Firstly, it is essential to be able to determine which markets co-move together. Second, inference from the analysis can guide decision making process for financial analyst and regulators on the need to diversify against market risk.

3.2.1 Sklar's Theorem

Theorem 3.1

Let H be an $n-dimensional$ distribution function with margins $F_1, ..., F_n$. Then there exists an $n-copula$ C such that for all x in $\Re^n$,

$$H\left(x_1, ..., x_n\right) = C\left(F_1\left(x_1\right), ..., F_n\left(x_n\right)\right) \tag{3.6}$$

If $F_1, ..., F_n$ are all continuous, then C is unique; otherwise C is uniquely determined on $RanF_1 * ... * RanF_n$. Conversely, if C is an $n-copula$ and $F_1, ..., F_n$ are distribution functions, then the function H defined above is an $n-dimensional$; distribution function with margins $F_1, ..., F_n$.

The Sklar's theorem shows that for continuous multivariate distribution functions, the univariate margins and the multivariate dependence structure can be separated which allows the dependence structure to be represented by a copula.

Corollary 1

Let H be an n-dimensional distribution function with continuous margins $F_1, ..., F_n$ and copula C. Then for any u in $[0,1]^n$,

$$C\left(u_1, ..., u_n\right) = H\left(F_1^{-1}\left(u_1\right), ..., F_n^{-1}\left(u_n\right)\right) \tag{3.7}$$

see Nelsen (2006)

3.2.2 Empirical Copula

Empirical copula's are obtained using the empirical cumulative density transform of the original data Eric and Jiahui (2006).

Let $\{x_i, y_i\}_{i=1}^{N}$ be a sample of size of N from a continuous bivariate distribution. The empirical copula is given by

$$C_\theta\left(u, v\right) = \frac{\{(x_i, y_i) : F_X\left(x_i\right)\} \leq u, F_Y\left(y_i\right) \leq v}{N} \tag{3.8}$$

and its probability density function is given by

$$C_\theta\left(u, v\right) = \frac{1}{N}\sum_{k=1}^{n}\delta\left(u - F_X\left(x_i\right),\ v - F_Y\left(y_i\right)\right) \tag{3.9}$$

3.3 Tail Dependence

An important property of copula is the tail dependence. It describes the behaviour of copulas when the value of the marginal cdf F_t reaches its bounds of zero (lower tail dependence) or one upper tail dependence which is defined as the limiting probability of a subset of a variable X has extreme values given that its compliment Y has extreme values (Joe and Xu, 1996).

A bivariate random vector, is expressed as upper tail if $\Lambda_u(1,1)$ exist and

$$\lambda_u = \Lambda_u(1,1) = v \overset{\lim}{\rightarrow} 1\ P((X > G^{-1}(V) \mid Y > H - 1(v)) > 0 \tag{3.10}$$

Consequently (x, y) is called upper tail independent when λ_U equals 0. λ_u is referred to as upper tail dependence coefficient. The lower tail dependence is defined by

$$\lambda_L = \Lambda_L(1,1) \tag{3.11}$$

A summary of various tail dependence coefficient for Elliptical and Archimedan is given by table 3.1 below .

Table 3.1: Tail dependence coefficients for Elliptical and Archimedan Copula

Family	λ_{UR}	λ_{LL}
Gaussian	0	0
Student t	$2t_{v+1}\left(-\sqrt{\frac{(v+1)(1-p)}{1+p}}\right)$	$2t_{v+1}\left(-\sqrt{\frac{(v+1)(1-p)}{1+p}}\right)$
Gumbel	$2 - 2^{\frac{1}{\theta}}$	0
Clayton	0	$\lambda_l = 2^{-\frac{1}{\theta}}$
Frank	0	0

3.3.1 Elliptical Copula

Elliptical copula has a class of elliptical distributions in which random variables are linearly correlated through a Pearson correlation coefficient. In a bivariate case,

$$\rho_\rho = \frac{Cov(Y_1, Y_2)}{\sqrt{Var(Y_1).Var(Y_2)}} \tag{3.12}$$

In empirical research, the most common elliptical copulas are the Gaussian and the student t.

3.3.1.1 Gaussian Copula

The Gaussian copula is mostly the widely used copula. It is implied by assuming a multivariate distribution (normal distribution). A multivariate Gaussian distribution entails a set of normally distributed marginal distributions that are linked with a Gaussian copula. The random vector $X = (X_1, ..., X_n)$ is multivariate normal iff:

1. the univariate margins $F_1, ..., F_n$ are Gaussian;
2. the dependence structure among the margins is described by a unique copula function C (the normal copula).

Traditionally, Gaussian copula has been used in modelling dependence with a cdf:

$$C_n^\phi(U; \Omega^\phi) = \phi_n(\phi^{-1}(u_1), ..., \phi^{-1}(U_n), \Omega^\phi) \tag{3.13}$$

where U is the vector of marginal probabilities, denotes the standard univariate Gaussian cumulative distribution function. ϕ_n is the CDF for the n-variate standard normal distribution with correlation matrix Ω^ϕ and ϕ^{-1} is the inverse of the CDF for the univariate standard normal distribution.

$$C(u, v; p) = \int_{-\infty}^{\phi-1(u)} \int_{-\infty}^{\phi-1(v)} \frac{1}{2\pi\sqrt{(1-p)^2}} \{ - \frac{-(r^2 - 2prs + s^2)}{2(1-p^2)} drds \tag{3.14}$$

where u and v are the distribution of the standard residuals for the marginal models. The properties of the Gaussian Copula are

Upper tail dependence: $\lambda_u = 0$

Lower tail dependence $\lambda_l = 0$

Kendall's ρ_τ=$\frac{2*arcsin(p)}{\pi}$

Gaussian copula are mainly employed in a linear sense. This implies that Gaussian copula can only be used for symmetric dependence. Empirical studies such as Mahfound (2012) have used Gaussian copula to analysis dependence between financial variables. However, the fundamental drawback of this copula is its inability to capture asymmetric dependence between variables. This implies that the Gaussian copula is not able to model properly the joint extreme.

The Gaussian copula lacks tail dependence since the tail dependence are zero, i.e $\lambda_u = \lambda_l = 0$. This implies that heavy tail events or dependency cannot be modeled by the Gaussian copula. Therefore, the Gaussian copula is not always suitable, because Gaussian copulas are unable to account for tail events like stock market crash.

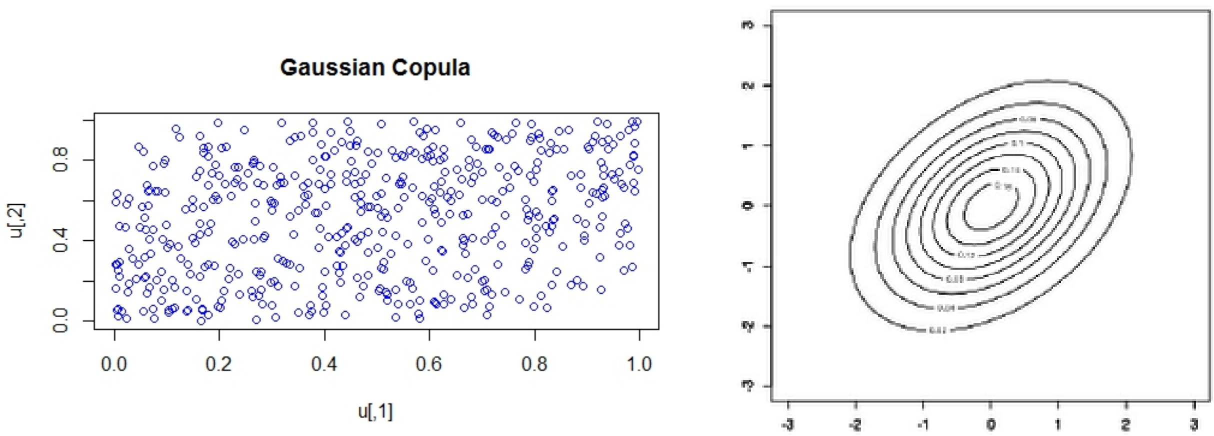


Figure 3.1: Gaussian and Contour Simulation of 500 Random Variable

3.3.1.2 Student t Copula

The Student t copula is a copula that is implied by a multivariate Student t distribution (Student t marginal distributions combined by a Student t copula). Student's t-copula allows for joint fat tails and an increased probability of joint extreme compared with Gaussian copula. This copula can be written as

$$C(u,v;p) = \int_{-\infty}^{t_v-1(u)} \int_{-\infty}^{t_v-1(v)} \frac{1}{2\pi\sqrt{(1-p^2)^{\frac{1}{2}}}} \{ -\frac{-(x^2-2pxy+y^2)}{v(1-p^2)} - \frac{v+2}{2} dsdt \tag{3.15}$$

where p and v are the parameters of the copula, and $t_v - 1$ is the inverse of the standard univariate students t distribution with v degrees of freedom, expectation 0 and variance $\frac{v}{v-2}$ (Kjersti, 2004). If the marginal distribution F_1 and F_2 are two Student-t distributions with (same) v degrees of freedom and C is a Student-t copula with parameters v and p^2, then the bivariate distribution function H defined by $H(x,y) = C(F_1(x), F_2(y))$ is the standardized bivariate t distribution, with $\mu = 0$, linear correlation coefficient p^2 and v degree of freedom. In this case, the t-Copula is the copula function which join the marginal t distributions with same degrees of freedom to the bivariate t distribution. The t Student copula generalizes the bivariate t distribution since we can adopt any marginal distribution.

Upper tail dependence: $\lambda_u = 2t_{v+1}\left(-\sqrt{\frac{(v+1)(1-p)}{1+p}}\right)$

Lower tail dependence $\lambda_l = 2t_{v+1}\left(-\sqrt{\frac{(v+1)(1-p)}{1+p}}\right)$

Kendall's $\rho_\tau = \frac{2*arcsin(p)}{\pi}$.

An important advantage of the t-copula which can not be overlooked is it ability to be extended to the multidimensional case, unlike some other copulas which are limited to two risks only. However, a limitation of the t-copula when modelling more than two risks is that aside from the pairwise correlation coefficient themselves there is only one variable, the *df* that controls the tail dependency structure which implies that all tail dependency have the same tail dependency structure which is not realistic. Studies such as Ling (2006) have used t-copula in analyzing dependence structure.

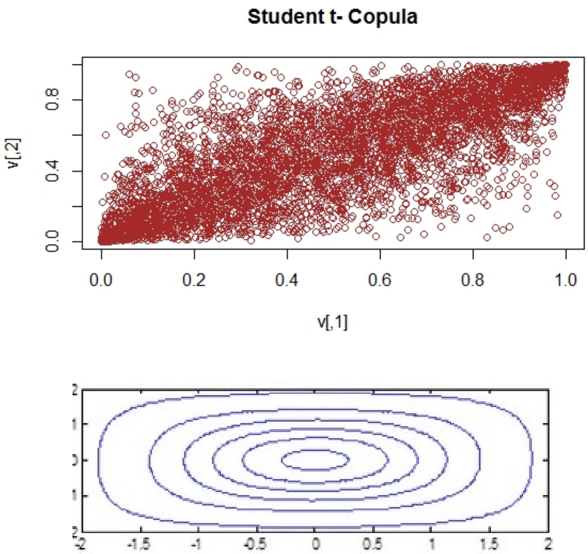


Figure 3.2: Student t Copula and Contour Simulation of 500 Random Variable

3.3.2 Archimedean Copula

The Archimedean Copula can capture various dependence structures such as tail events. This feature makes it possible to use them in modelling extreme events. A copula C is termed Archimedean if there exist a generator function ψ such that C has the form:

$$C\left(u_1, ...u_n\right) = \psi^{-1}\left(\psi\left(u_1\right) + ... + \psi\left(u_n\right)\right) \tag{3.16}$$

for all $0 \leq u_1, ..., u_n \leq 1$, where $\psi : [0,1] \longrightarrow [0, \infty]$ is continuous and strictly decreasing such that $\psi(1) = 0$ and $\psi(0) = \infty$ and ψ^{-1} is the inverse function of the generator. The generator satisfies the following conditions:

1. $\psi(1) = 0$

2. for all $t\epsilon\,[0,1]$, $\psi^{'}\,(t) < 0$, i.e $\psi$is decreasing.

3. for all $t\epsilon\,[0,1]$, $\psi^{'}\,(t) \geq 0$, i.e ψ is convex.

3.3.2.1 Gumbel

Gumbel copula exhibits greater dependence in the positive tail than in the negative. This copula is given by

$$C_{\theta}(u,v) = exp\,(-[(-\ln u)^{\theta} + (-\ln v)^{\theta}]^{\frac{1}{\theta}} \tag{3.17}$$

its generator is given by

$$\varphi_{\theta}(X) = (-\ln(x))^{\theta} \tag{3.18}$$

where $0 < \theta \leq 1$ is a parameter controlling the dependence. Perfect dependence is obtained if $\theta \to 0$,while $\theta = 1$ implies independence. $\theta\epsilon[1,\infty)$, (Kjersti 2004). The properties of the Gumbel Copula are

Upper tail dependence: $\lambda_u = 2 - 2\frac{1}{\theta})$

Lower tail dependence $\lambda_u = 0$

Kendall's ρ_{τ}=$1 - \frac{1}{\theta}$

The limitation of Gumbel copula is its in ability to model extreme dependence in the lower tails. This however, makes it not suitable to meet the objective of the study.

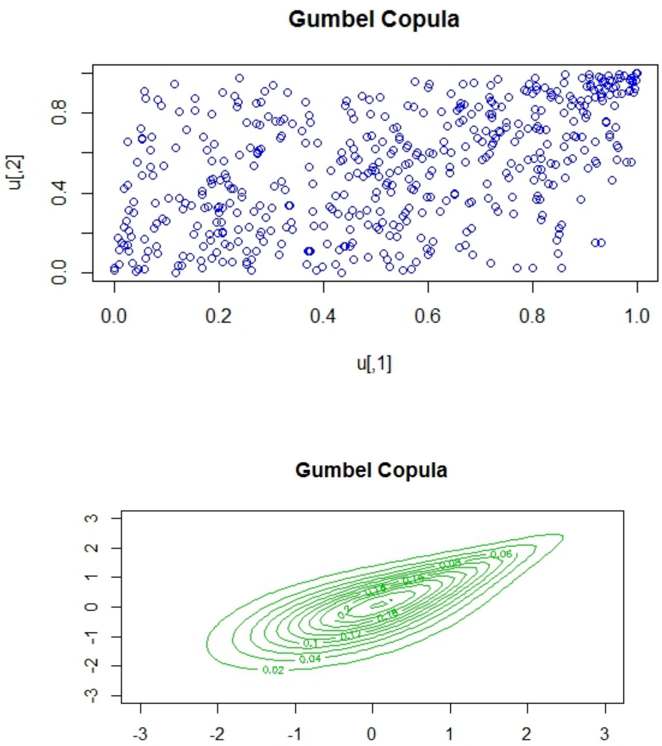


Figure 3.3: Gumbel Copula and Contour Simulation of 500 Random Variables

3.3.2.2 Clayton Copula

The Clayton copula is an asymmetric copula, exhibiting greater dependence in the negative tail than in the positive. Kjersti (2004) gives a mathematically expression as

$$C_\theta(u, v) = (u^{-\theta} + v^{-\theta} - 1)^{-\frac{1}{\theta}} \tag{3.19}$$

its generator is

$$\varphi_\theta(X) = \frac{1}{\theta}(x^{-\theta} - 1) \tag{3.20}$$

where $0 < \theta < \infty$ is a parameter controlling the dependence. Perfect dependence is obtained if $\theta \to \infty$,while $\theta \to 0$ implies independence $\theta \epsilon [-1, \infty]$.

θ has the following characteristics:

1. θ is continuous strictly decreasing function
2. $\theta\left(0\right)=\infty$
3. $\theta\left(1\right)=0$
4. θ^{-1} is completely monotonic on $[0,\infty]$i.e $\left(-1\right)^{k}\frac{d^{k}}{dt^{k}}\theta^{-1}\left(t\right)\geq 0$ for all t in $0,\infty$ and for all k

Upper tail dependence: $\lambda_{u}=0$

Lower tail dependence λ_{l}=$2^{-\frac{1}{\theta}}$

Kendall's ρ_{τ}= $\frac{\theta}{\theta+2}$

The Clayton copula is therefore characterised by lower tail dependence and upper tail independence. Therefore, markets crashing jointly can be modeled using the Clayton approach which is the main focus area of this research as it can tell about market risk in periods of extreme financial events. An empirical investigation using Clayton Copula haven be used by Nguyen and Nguyen (2014).

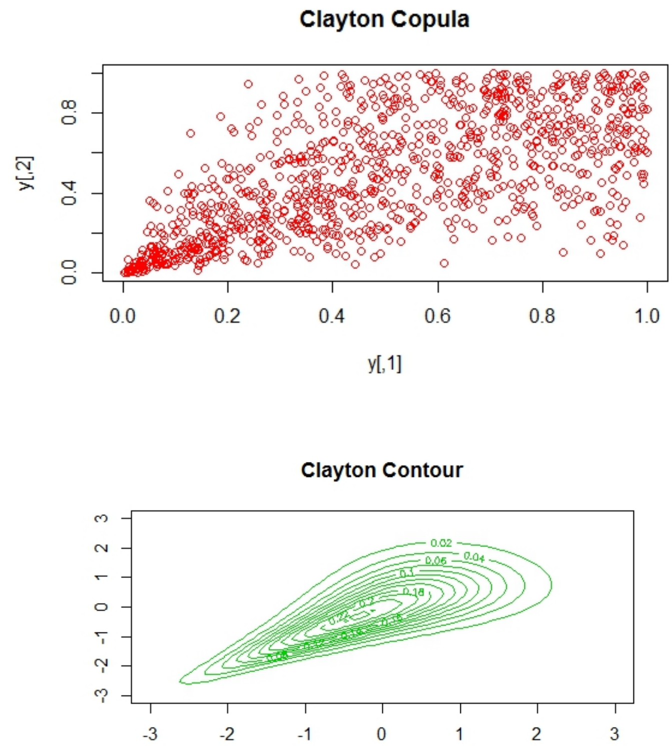


Figure 3.4: Clayton Copula and Contour Simulation of 500 Random Variables

3.3.2.3 Frank Copula

The frank copula is an Archimedean copula with a generator given by

$$C_\theta(u,v) = -\frac{1}{\theta}\ln\left\{1+\frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{e^{-\theta}-1}\right\} \tag{3.21}$$

The properties of the frank copula are:

Upper tail dependence: $\lambda_u = 0$

Lower tail dependence $\lambda_l = 0$

Kendall's $\rho_\tau = 1 - \frac{4}{\theta}\left(1 - \frac{1}{\theta}\int_0^\theta \frac{a}{e^{a-1}}da\right)$

The Frank Copula is symmetric and therefore differs from the Clayton and Gumbel copulas as it does not show tail dependence. Therefore will the objective of this study in mind, the frank copula is limited in capturing the tail dependence of financial market co-movements which is not preferred in the context of this study.

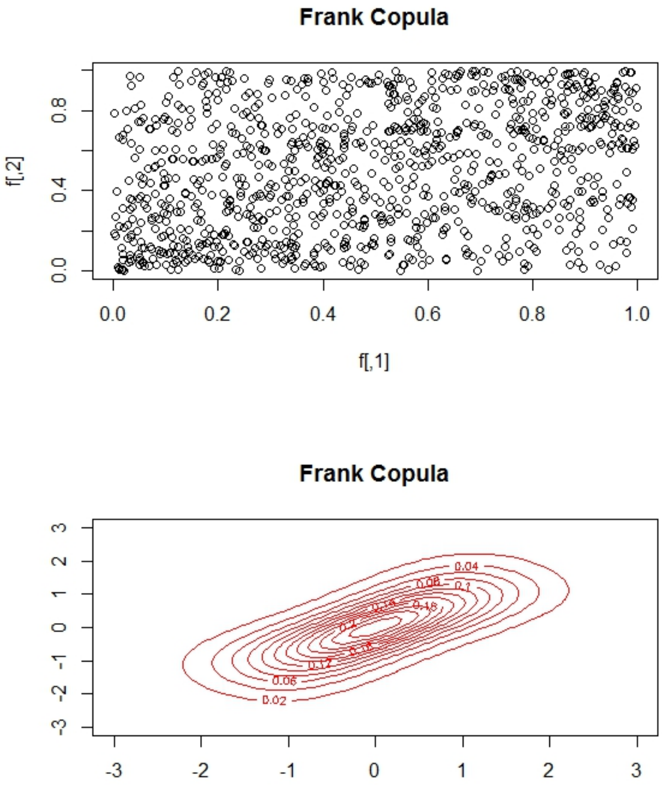


Figure 3.5: Frank Copula and Contour Simulation of 500 Random Variable

3.3.3 GARCH Model

Definition

Let $\{Z_t\}_{t\epsilon Z}$ be a series of i.i.d white noise variables, then the GARCH (p, q) process is defined by the equation

$$X_t = \sigma_t Z_t$$

where

$$\sigma_t^2 = \alpha_0 + \sum_{k=1}^{p} \alpha_k X_{t-k}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-1}^2 \tag{3.22}$$

such that $\alpha_0 > 0$,$\alpha_k \geq 0,\ k = 1, ..., p,\ \beta_j \geq 0,\ j = 1, ..., q$ are parameters. X_t is the return series while $\{Z_t\}_{t\epsilon Z}$ is the innovation series.

GARCH $(1, 1)$ model is defined as (see Engle 1982, Bollerslev 1986):

$$\sigma_t^2 = \alpha_0 + \left(\beta_1 + \alpha_1 Z_{t-1}^2\right) \sigma_{t-1}^2 \tag{3.23}$$

From equation 3.23, let $B_t = \alpha_0,\ A = \left(\beta_1 + \alpha_1 Z_{t-1}^2\right)\ Y_t = \sigma_t^2$, then we have a random variable recurrence $Y_t = A_t Y_{t-1} + B_t$ where (A_t, B_t) is i.i.d for $t\epsilon Z$. Thus $\{\sigma_t^2\}$is function of the innovation process $\{Z_t\}$.

Theorem 3.2

Assume $\{X_t\}_{t\epsilon Z}$ be a GARCH $(1, 1)$process and $\{\sigma_t^2\}, \{Z_t\}$as in 3.23. Then $\{X_t\}_{t\epsilon Z}$ and $\{\sigma_t^2\}$ have a strictly stationary solution if and only if $E\left(log\left(\alpha_1 Z_0^2 + \beta_1\right)\right) < 0$. The solution is unique and its squared volatility is

$$\sigma_t^2 = \alpha_0 \sum_{i=0}^{\infty} \left(\prod_{j=0}^{t-1} \left(\beta_1 + \alpha_1 Z_{t-j}^2\right) \right) \tag{3.24}$$

Proof can be found in Nelsen (1990).

3.3.3.1 Quasi Maximum Likelihood Estimation

Assume $X_t = \sigma_t Z_t$ be i.i.d innovations $Z_t \sim WN(0, 1)$. Given that the admissable GARCH parameter set is

$$\neg := \Big\{\gamma = (\alpha_0, \alpha_1, ..., \alpha_p, \beta_1, ..., \beta_p) : \alpha_i, \beta_j > 0, \sum_{j=1}^{q} \beta_j \leq \triangle_0^* < 1, 0\ \triangle_1^* \leq$$

$$min\ \{\gamma\} \leq max\ \ \{\gamma\} \leq \triangle_2^*, q\triangle_1^* < \triangle_0^* \tag{3.25}$$

with $\triangle_0^*, \triangle_1^*, \triangle_2^*$ being constants the logarithm of the quasi likelihood function is defined as

$$L(\gamma : \mathbf{x}) = -\frac{1}{2}\sum_{t=1}^{N}\left(log\ w_t(\gamma) + \frac{X_t^2}{w_t(\gamma)}\right) \tag{3.26}$$

where $w_t(\gamma) + \sum_{j=1}^{\infty} c_j(\gamma) X_{t-j}^2$is the ARCH ∞representation of GARCH (p,q) process. in this representation

$$c_0 = \frac{1}{1 - \sum_{j=1}^{q}\beta_j}, \sum_{j=1}^{\infty} c_j z^j = \frac{A(z)}{B(z)}, \qquad for|Z| \leq 1 \tag{3.27}$$

where$A(z) = \sum_{v=1}^{p}\alpha_v z^v,\ B(z) = 1 - \sum_{u=1}^{q}\beta_u z^u$

For identifiability of the GARCH (p,q) parameters, the following conditions hold: the polynomials $A(z)$ and $B(z)$ should have no common roots, should be a non-degenerate random variable. (Berkes et.al., 2003)

The qusai maximum likelihood estimator for $\gamma\epsilon\Gamma$ is defined as

$$\hat{\gamma} = argmax - \frac{1}{2}\sum_{t=1}^{N}\left(log\ w_t(\gamma) + \frac{X_t^2}{w_t(\gamma)}\right) \tag{3.28}$$

The consistency and asymptotic normality of the QMLE estimator is proofed in (Berkes et.al, 2003, pg. 215)

3.4 Threshold Selection

3.4.1 Extreme Value Theory

Extreme value theory studies the stochastic behavior of the extreme values in a process. For a single process, the behavior of the maxima can be described by the three extreme value distributions–Gumbel, Frechet and negative Weibull (Fisher and Tippett, 1928). The first application of extreme value distributions was probably made by Fuller in 1914. Thereafter, several researchers have provided useful applications of extreme value distributions.

3.4.2 Generalized Pareto Distribution (GPD)

An equivalent result form Extreme Value Theory describe the behaviour of large observations which exceed high thresholds. The distribution which come to fore in these results is the generalized Pareto distribution (GDP).

The GPD is usually expressed as a two parameter distribution with df

$$G_{\xi,\sigma}(x) = \begin{matrix} 1-\left(1+\frac{\xi x}{\sigma}\right)^{-\frac{1}{\xi}} & If\,\xi \neq 0. \\ 1-exp\left(-\frac{x}{\sigma}\right) & If\,\xi = 0 \end{matrix} \tag{3.29}$$

where $\sigma > 0$, and the support is $x \geq 0$ when $\xi \geq 0$ and $0 \leq x \leq \frac{-\sigma}{\xi}$ when $\xi < 0$. G_{ξ} is a standard generalized Pareto Distribution.

Introducing the scale parameter β,

$$G_{\xi,\sigma}(x) = 1-\left(1+\xi\frac{x}{\beta}\right)^{-\frac{1}{\xi}}, \qquad x\epsilon D(\xi,\beta) \tag{3.30}$$

where

$$x\epsilon D(\xi,\beta) = \begin{cases} [0,\infty) & if \quad \xi \geq 0 \\ [0,\frac{-\beta}{\xi}] & if \xi < 0 \end{cases}$$

The Generalized Pareto Distribution (GPD), $G_{\xi,\beta}$, $\xi\epsilon\mathbb{R}$, appears as the limit distribution of scaled excesses over high thresholds.

Definition: Mean Excess Function

Let x be a random variable with distribution function (df) F and right endpoint x_F. For a fixed $u > x_F$,

$$F_u(x) = P(X-u \leq x|X > u), \qquad x \geq 0$$

is the excess of the random variable X $(of\, the\, df\ F)$over the threshold u.

The function

$$e(u) = E(X - u|X > u)\,,\ \ 0 \leq u < x_F$$

is called the mean excess function of X.

3.4.3 The Pickands-Balkema-De-Haan Theorem

Theorem 3.3

Suppose that $X_1, X_2, ..., X_n$ are n independent realizations of a random variable X with a distribution function $F(x)$. Let x_F be the finite or infinite right endpoint if the distribution F. The distribution function of the excesses over certain (high) threshold u is given by

$$F_u(x) = Pr\{X - u \leq x/X > u\} = \frac{F(x+u) - F(u)}{1 - F(u)}$$

for $0 \leq x < x_F - u$.

The Pickands-Balkema-De Haa theorem (Balkema and Haan 1974; Pickands, 1975) states that if the distribution function $F \epsilon\ DA(H_\xi)$then $\exists$a positive measureable function $\beta(k)$such that;

$$\begin{matrix} lim & sup \\ u \to x_0 & \leq y < x_0 - u \end{matrix} |F_u(y) - G_{\xi,\sigma(u)}(y)| = 0 \tag{3.31}$$

where $G_{\xi,\beta(u)}(x)$denote the Generalized Pareto distribution. The theorem shows that as the threshold K becomes large, the distribution of the excesses over the threshold tends to the Generalized Pareto distribution, provided the underlying distribution F belongs to the domain of attraction of the Generalized Extreme Value Distribution.

3.5 Risk Measure

In estimating financial market risk, the literature offers several risk measures. However,Value at Risk is reviewed.

3.5.1 Value-at-Risk

Value-at-Risk (VaR) measures the maximum or worst loss we would expect over a given time horizon (Thomas et al., 2002). Carmona (2004) defined VaR_q as the 100qth percentile of the loss distribution given as:

$$q = Pr\{R \leq r\} = F_{R_t}(r) \tag{3.32}$$

where q is the probability R_t is the random variable and r is the particular value that R_t takes.

From our earlier setting of a portfolio with two financial index and log returns denoted by X and Y, we solve for r in equation 3.32 to get our VaR with our copula parameter inputted in the VAR (Copula –VAR) by computing the CDF of the log return R. The latter can be expressed analytically as

$$q = \int\int_{\{(x,y);\lambda_1 e^x + \lambda_2 e^y\} \leq e-r} f\,(X,Y)^{(x,y)dxdy} \tag{3.33}$$

where ; $\lambda_1 e^x + \lambda_2 e^y$ are log returns of the portfolio and $f\,(X,Y)$ is the CDF of the returns.

For a continuous case, The continuous distribution function (CDF) is given by:

$$F(X,Y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f\,(x,y)dxdy$$

Note: For double integral, we assume that one variable is constant and the other is varying. So here we assume X to be the only variable and Y as a constant.

The above generates the upper limit of X as follows.

$$log\left(\lambda_{1e}x+\lambda_{2e}y\right)\leq -r\Rightarrow\lambda_{1e}x+\lambda_{2e}y\leq e^{-r}$$

when Y is considered a constant, we then have

$$\lambda_{1e}X\leq=\lambda_1\leq\frac{e^{-r}}{e^X}\Rightarrow\lambda_1\leq e^{-r-X}\Rightarrow log\left(\lambda_1\right)\leq -r-X\leq -r-log\lambda_1$$

Hence the Upper limit for X is $-r-log\lambda_1$

Then for$Y$, we vary both x and y. This is because we cannot assume X to be constant as we have already assumed Y to be one but now we integrate with respect to X. So we have,

$$\lambda_1e^X+\lambda_2e^y\leq e^{-r}\Rightarrow\frac{\lambda_1e^X}{\lambda_2}+\frac{\lambda_2e^X}{\lambda_2}\leq\frac{e^{-r}}{\lambda_2}-\frac{\lambda_1e^X}{\lambda_2}\Rightarrow Y\leq log\left(\frac{e^{-r}}{\lambda_2}-\frac{\lambda_1}{\lambda_2}\right)$$

As the upper limit for Y.

Change of variables and Integration

Now, we do a change of variable so as so intergrate in a mathematically correct way. Recall that for copulas, $F\left(x_1,x_2,...,x_n\right)=C\left(F_1\left(x_1\right),F_2\left(x_x\right),...,F_n\left(x_n\right)\right)$from Sklar (1959).

Recall also that

$$F\left(X,Y\right)=\frac{f_{(X,Y)^{x,y}}}{f_X\left(x\right)f_Y\left(y\right)}=C\left(F_X\left(x\right),F_Y\left(y\right)\right)$$

give us equation (34)

$$q=\int_{-\infty}^{-r-log\lambda_1}dx\int^{log(e^{-r}/\lambda_2-\lambda_1/\lambda_2e^x)}c(F_X(x),F_Y(y))f_x(x)f_y(y)dy \qquad (3.34)$$

To bring in u, we do a transformation

$u = f(X)$ such that when $X = -\infty$, $u = 0$ and when $X = -r - log\lambda_1$, $u = F_X(-r - log\lambda_1)$

for v, $v = f(Y) = F_Y(y)$ such that when $Y = -\infty$, $v = 0$ and when

$$Y = \frac{e^{-r}}{\lambda_2} - \frac{\lambda_1 e^X}{\lambda_2}$$

$$v = F_Y\left(\frac{e^{-r}}{\lambda_2} - \frac{\lambda_1 e^X}{\lambda_2}\right)$$

$$q = \int_0^{F_X(-r-log\lambda_1)} du \int^{F_Y(log(e^{-r}/\lambda_2 - \lambda_1/\lambda_2 e_X^{F^{-1}(u)})} dv\ c(u,v) \tag{3.35}$$

By integrating, we have

$$q = \int_0^{F_X(-r-log\lambda_1)} du\ \frac{\partial}{\partial u} C(u,v)|_{v=F_Y(log(e^{-r}/\lambda_2 - \lambda_1/\lambda_2 e_X^{F^{-1}(u)})} \tag{3.36}$$

An expression for VaR is given as

$$Var_t(\theta) = F_{t+1}^{-1}(\theta)$$

where F^{-1} is the inverse of the distribution.

In order to estimate the VaR in the given framework, we estimate the copula dependence parameter from a sample pair of log returns. The estimated parameter is then used to compute the VaR at different confidence levels which gives estimates for VaR.

3.6 Model

From the above methodology, this study uses a Clayton copula approach with estimator to measure dependence in periods of extreme events and how this extreme events change the dependence structure of financial markets. Since market crashes occur mostly on the lower tail of the distribution, a copula which can capture the left dependence is required. The Clayton copula captures the left tail dependence which is suitable to model dependence structure of financial market at the lower left tail. Therefore, an asymmetric tail dependence which are in most case the expectations of financial markets as losses occur more often jointly than gains do (Ninga, 2004).

3.7 Estimating Copula Parameters

The section discusses a brief survey of estimation procedures for copula modelling. These procedures are parametric, semi parametric and non-parametric copula inference for random variables.

3.7.1 Maximum Likelihood Estimation

Let $f_\theta; R^2 \to (0, \infty)$ be the density function of a given random vector $\mathbf{X}$, $F_\theta : D \to [0, 1]$ the distribution function of the given random vector with $D \subseteq R^2$, and $\theta \epsilon \, \Theta \subseteq R^d$ be the parameter. Then the log-likelihood function is defined as $l : D \, x \, \Theta \to R$ where $l = log \; f \, (\mathbf{x} : \theta)$ and the joint likelihood function is

$$L : D^n x \; \Theta \to R, \; L \, (x_1, ..., \, x_n : \theta) = \sum_{K=1}^{N} l(x_k; \, \theta) \tag{3.37}$$

for a random sample $(x_1, ..., \, x_N)$ of i.i.d vector with distribution F_θ. The maximum likelihood estimator for parameter θ is given by

$$\hat{\theta} \, (X_1, ..., \, X_N) = argmaxL \; (X_1, ..., \, X_N; \theta) \tag{3.38}$$

Normality conditions of the MLE as proof in (Lehmann and Casella, 1998, Pg.449) $n \to \infty$,there exists $\hat{\theta}_n = \hat{\theta} \, (X_1, ..., \, X_n)$ such that $\frac{\partial}{\partial \theta} L \left(X_1, ..., \, X_n; \hat{\theta} \right) = 0$ and $\hat{\theta}_n \to \theta_0$as $n \to \infty$.

The weakly consistent condition is satisfied as proofed by Casella (1998).

$$\sqrt{n}\left(\hat{\theta}_n - \theta_0\right)^d \rightarrow N\left(0, (I(\theta))^{-1}\right)$$

3.7.2 Inference Function For Margins

We use the inference function for margins method (IFM) to fit copula and estimate the structure of dependence. The IFM is based on the pioneering work of Joe and Xu (1996). The estimation method of IFM is presented below:

Assume we observe n independent observations $X_t = (x_{t1}, x_{t2}, \ldots, x_{tp})$ from a multivariate distribution, which can be constructed with p marginal distributions and a copula function $C(F_1(x), \ldots, F_n(x); \theta)$ with parameter θ. The probability distribution function (PDF) of the marginal distributions is defined as$f_i(x; \theta_i)$with a cumulative density distribution (CDF) as $F_i(x; \theta_i)$, where θ_i is the parameter of marginal distributions. The IFM method estimates the parameters of the marginal distribution in the first step.

The log-likelihood function of the first step can be written as

$$Logl(\theta) = \sum_{i=1}^{n}\sum_{j=1}^{p} log\ f_1\left(x_{ij}; \theta_i\right) \tag{3.39}$$

The estimation of the parameter$\theta = (\theta_1, \ldots, \theta_n)$of marginal distribution can be made through maximizing the log-likelihood function.

$$\hat{\theta}_i = argmax \sum_{i=1}^{n}\sum_{j=1}^{p} log\ f_1\left(x_{ij}; \theta_i\right) \tag{3.40}$$

The parameter θ of the copula function is estimated in the second step of IFM, with the parameter $\hat{\theta}$ of the p marginal distributions.

$$\hat{\theta} = argmax \sum_{i=1}^{n} log\ C\left(F_1\left(x_{i1};\hat{\theta}_i\right), ...F_P\left(x_{ip};\hat{\theta}_p\right);\theta\right) \tag{3.41}$$

The IFM is given by a vector

$$\theta^{IFM} = \left(\hat{\theta}, \hat{\theta}_{IFM}\right)$$

where

$$\hat{\theta}_{IFM} = \left(\hat{\theta}_i, \hat{\theta}_p, \theta\right)$$

3.7.2.1 Asymptotic Properties of Inference Function for Margin Estimator

We use theorem 3.4 to show consistency and asymptotic normality of the IFM estimator $\hat{\theta}_{IFM}$.

Theorem 3.4

Let $X_1, \ldots, X_n$ be independent and identically distributed random vectors with density f_θ. Let $\theta \epsilon \Theta$ and $x \epsilon S := supp(f_0) \subseteq R^2$ where $supp(f_0)$ is the support of (f_0). Assuming the following conditions hold:

1. The parameter space $\Theta \subseteq R$ is an open interval.
2. The Support S is independent of θ
3. $f(x;\theta)$ is three times continuously differentiable with respect to θ
4. $El_\theta(X:\theta)^2 + El_{(\theta\theta)}(X:\theta) = 0and\int_s \frac{\partial}{\partial\theta} f\left(x:\theta\right)dx = \frac{\partial}{\partial\theta}\int_s \quad f\left(x:\theta\right)dx = 0)$
5. The Fisher information $I(\theta) = El_\theta\left(X:\theta\right)^2 = -El_{\theta\theta}\left(X:\theta\right)$is positive and finite

6. For all $\theta_0 \epsilon \Theta$ and $\theta \epsilon \Theta$ and $\theta \epsilon U_\delta (\theta_0)$ there exists a measurable function $M_{\theta_0}$with $E_{\theta_0} (M (X : \theta_0)) < \infty$such that $\mid l_{\theta\theta\theta} (y : \theta) \mid \leq M (x : \theta_0)$for all x.

Imposing the regularity conditions from White (1994) and Patton (2006b) to the marginal likelihood in equation 3.39, and the copula likelihood function, equation 3.41, a joint normality condition holds such that as$n \to \infty$,

$$\sqrt{n} \left(\left(\hat{\theta}_{IFM} \right) - \theta \right) \;\; \to \;\; N \left(0, \hat{G} \right)$$

where $\hat{G}$ is the estimator of the Godambe Information matrix (Joe, 1997).

G is defined as

$$G = \left(D_g^{-1} M D_g^{-1} \right)^t ,$$

where

$$D_g = E \left(\frac{\partial g^t (X, \eta)}{\partial \eta} \right)$$

and

$$M_g = E \left(g^t (X, \eta) \, g (X, \eta) \right)$$

3.7.3 Canonical Maximum Likelihood Method

The canonical maximum likelihood method (CML) does not assume an a priori assumption on the distributional form of the margins. It relies on the concept of the empirical marginal transformation, which tends to estimate the unknown parametric marginal $F_n (.)$, for $n = 1, ..., N$, with the empirical distribution functions $F_n (.)$ defined as follows

$F_n (.) = \frac{1}{T}$

$$F_n\left(.\right) = \frac{1}{T}\sum_{t=1}^{T} 1_{X_{\{nt\leq .\}}} \tag{3.42}$$

For $1_{X_{\{nt\leq .\}}}$ represents the indicator function. The CML method is them implemented via a two stage procedure:

1. Transformation of the initial data set $X = \{X_{1t}, X_{2t}...X_{Nt}\}_{t=1}^{T}$ into uniform variates, using the empirical marginal distribution.

2. Estimation of the Vector of the Copula parameters α.

The CML estimator is defined as the vector $\theta^{CML} = \hat{\alpha}_{CML}$.

3.7.4 Copula Selection

The Akaike Information Criterion (AIC) has been used by several authors such as Embrechts et al. (2002) for copula selection. The rule of thumb

$$AIC = 2l^{\#}_{n,max} - 2length\left(\theta\right) \tag{3.43}$$

where $l^{\#}_{n,max}$ is the maximized likelihood for the model.

CHAPTER 4

EMPIRICAL STUDY

4.1 Introduction

This chapter discusses data used and results from fitting copula (dependence structure in pre-crisis, crisis and post crisis periods) between financial market variables and VaR estimates.

4.1.1 Data Description

In the analysis of financial market dependence, monthly data were collected from January, 2000 to March, 2016 from Central Bank websites, Yahoo finance, Stock Exchanges of five countries namely, Kenya, USA, UK, Nigeria and South Africa. Dealing with three markets (Stock, Bond, Money Markets), consideration for proxy are all share index of different countries for stock markets, 10-year government bond for USA and UK; 30day T-bills money market for USA and UK.

4.1.2 Empirical Analysis

Figure 4.1 shows the evolution of monthly stock market (Kenya, USA, UK, South Africa and Nigeria). Generally, the stock market data from observation exhibit a downward trend across stock markets from 2000 to 2003. However, markets began to trending upward from 2004 to 2007. Stock markets exhibit a decline from 2007 to 2009 reflecting the financial turmoil. From a visual analysis, the graph reveals that there is a co-movement of stock market in a similar direction either in an upward or downward trend within the period under consideration.

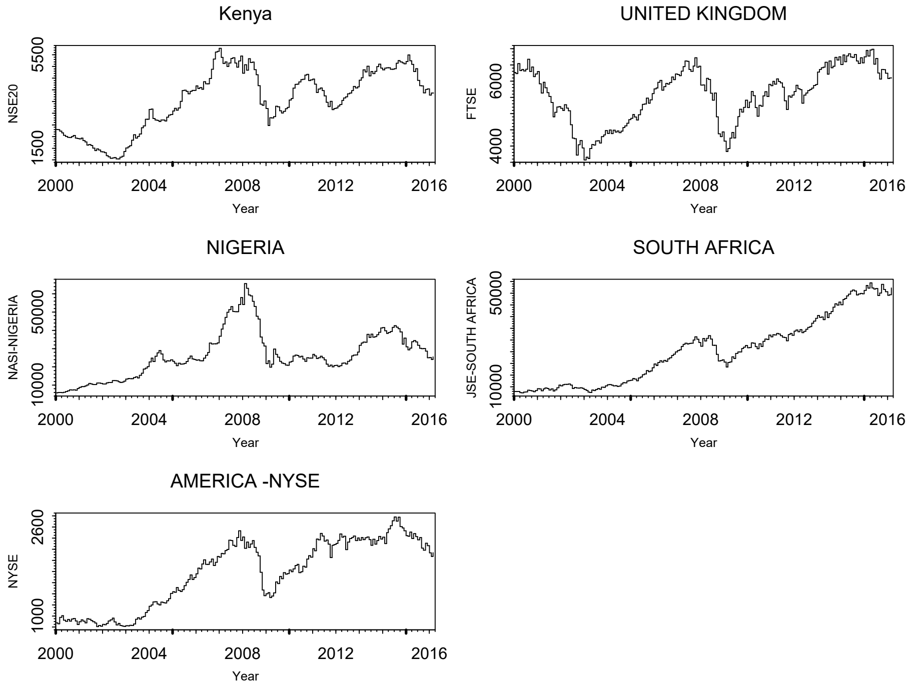


Figure 4.1: Stock Markets Evolution from January, 2000- March 2016

Log returns for respective financial market index were calculated using negative returns given as

$$R_t = -\ln\left(\frac{A_t}{A_{t-1}}\right)$$

where A_t is today's index and A_{t-1} is the previous day's index.

Monthly log returns on stock markets are presented in figure 4.2 below. The graphs of various stock market reveal the features of financial time series with volatility clusters and asymmetry been evident.

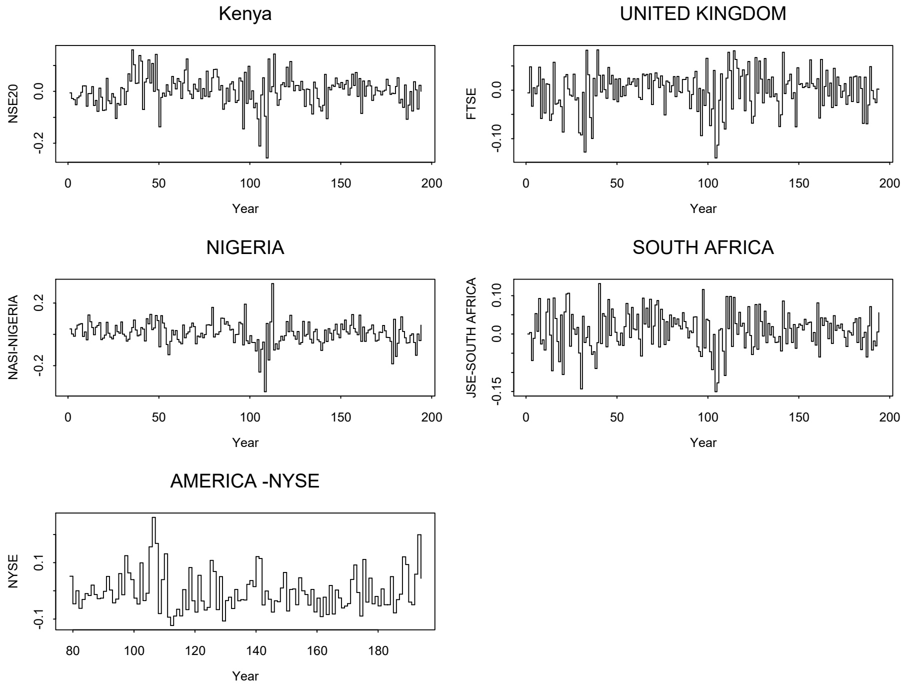


Figure 4.2: Log Returns on Monthly closing index for Stock Markets

Table 4.1: Descriptive Statistics for Monthly Data from January, 2000 to March 2016

Statistic	NSE20	FTSE100	NSE-ASI	JSE	USA
Min	-0.257	-0.1395	-0.1503	-0.2730	-0.1973
Max	0.1602	0.0829	0.1313	0.0003	0.1287
Mean	0.00265	-0.00013	0.00972	0.1930	0.0045
Standard dev	0.0300	0.0404	0.0490	0.0570	0.0464
Skewness	-0.474	-0.6716	-0.6716	-0.9514	-0.6070
Kurtosis	2.502	0.7711	0.771	0.5635	1.7886
Number of Observation	194	194	194	194	194

Table 4.1 presents summary statistics for the five stock index returns. These include the mean, variance, skewness, and excess kurtosis statistics. The Min represent the lowers value in the data series and the Max is the highest value in the series. The mean gives us the middle value in our respective series while the standard deviation show the spread of the data are from the mean. All five returns series have a negative skewness, which means that extreme negative

returns are a dominant feature for all five indices. The kurtosis is significantly greater than zero. This implies a distribution with kurtosis > 3 (excess kurtosis >0) for all series, implies that they exhibit heavy tails which are characters of financial market data.

Following Pickands, Balkema & De Hann Theorem, a choice of a high threshold gives our data a Generalized Pareto Distribution. A graphical test is established to check the behavior of the returns of the markets in consideration. This behaviour is estimated by the mean excess. The choice of a threshold from the returns is done by choosing the threshold with an approximation by the GPD, detecting a linear shape on the graph.

Figure 4.3 show the mean excess function for stock market returns. Using the criteria, the graph reveals the threshold of 0.032 for NSE20, 0.026 for FTSE100, 0.049 NSE-ASI; 0.041 and 0.030 for JSE respectively.

Figure 4.4 gives an overview of 3 months treasury bills for USA and UK. From 2007, T-bills rate in UK and USA trend downwards in both markets which shows a significant effect of the 2007-2009 crisis on the T-bills market in this countries. Similarly, log returns were computed for the various markets. Log returns plot show the stylized fact of volatility clustering where large (small) returns are followed by large (small) returns.

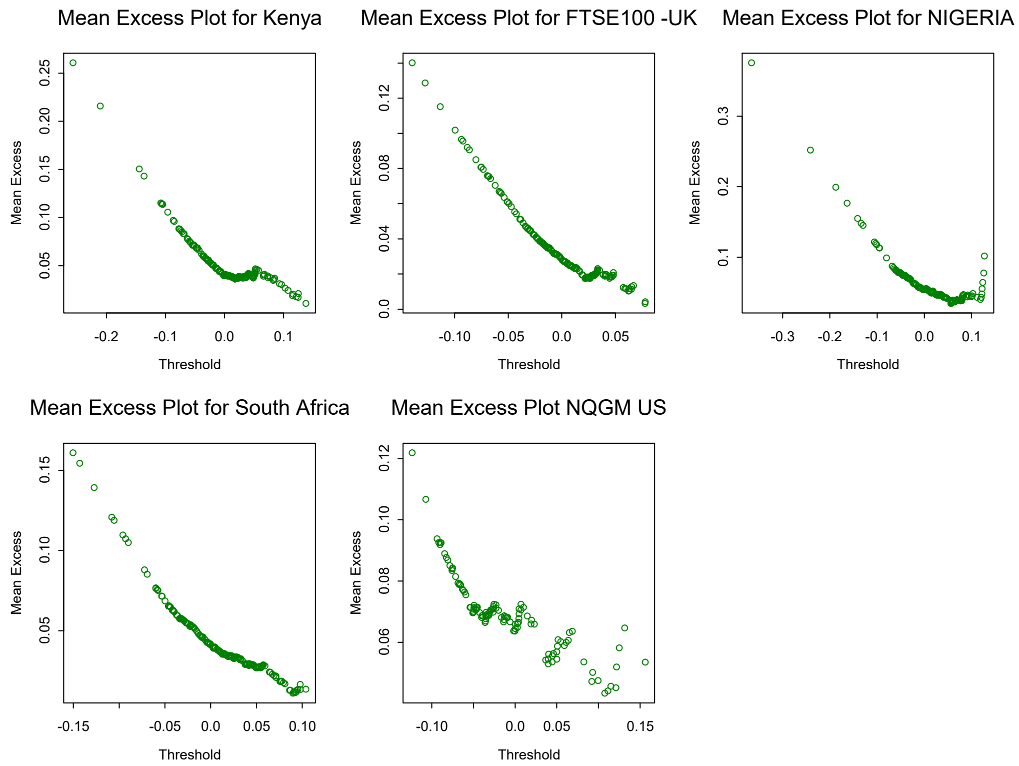


Figure 4.3: Mean Excess Function for Stock market

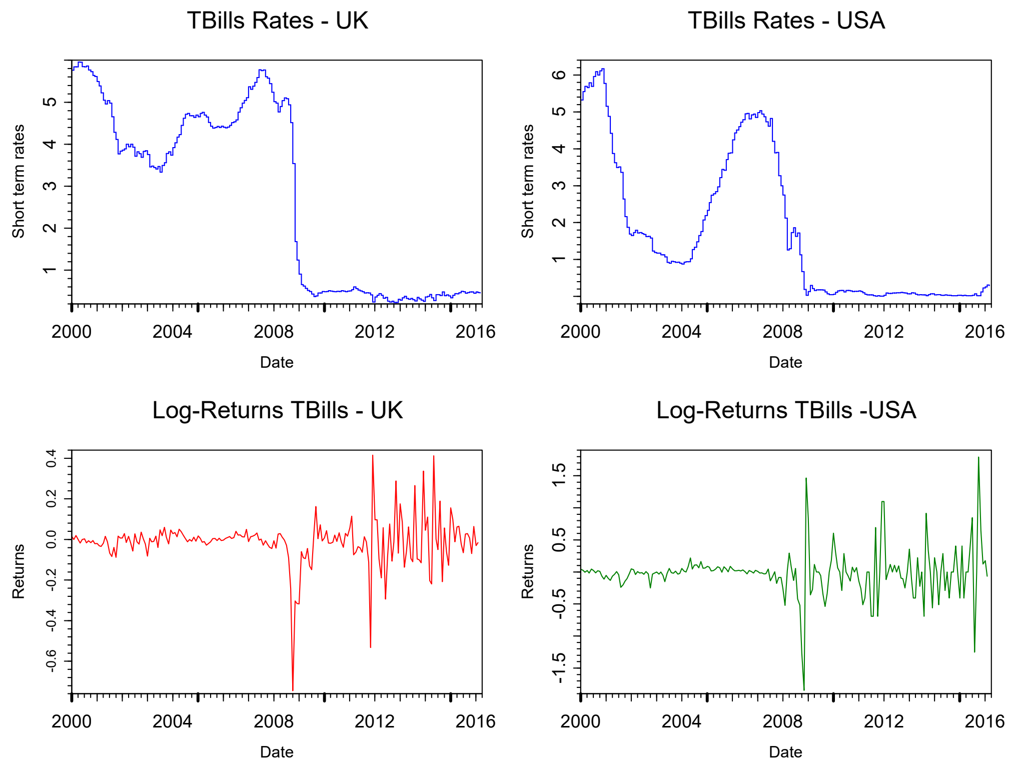
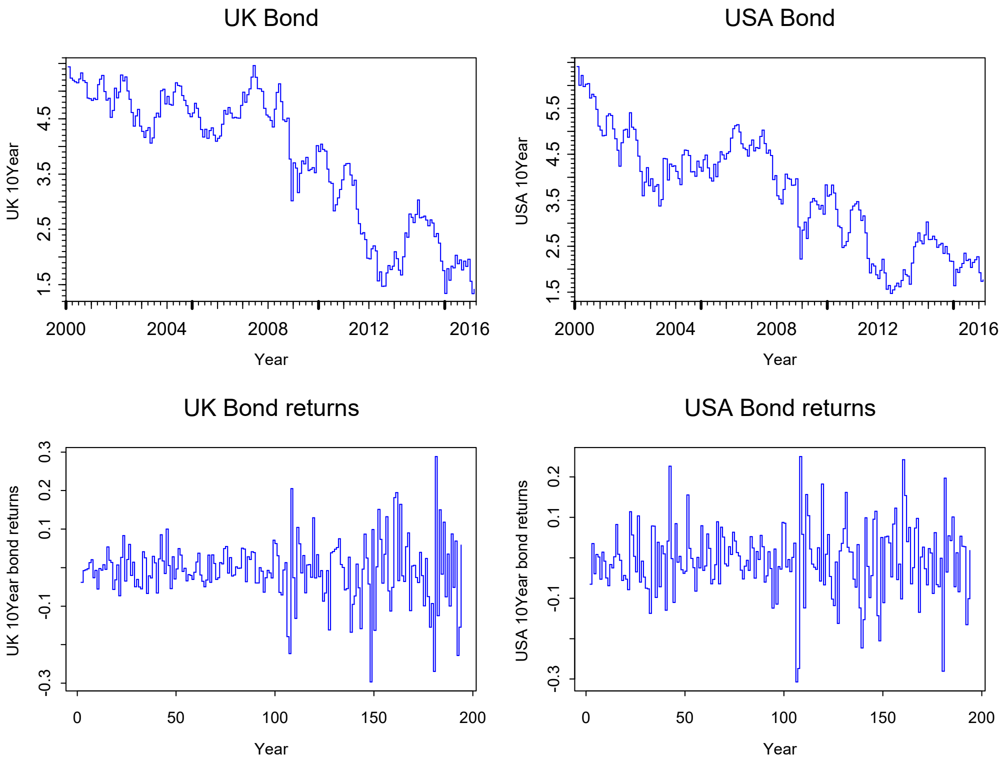


Figure 4.4: Evolution for Tbills market

Figure 4.5: Evolution of Bond Markets



Bond trends in figure 4.5 shows a consistent declined for 10 years UK and USA bond markets. The log returns series show a volatility clustering in the series and a downward movement from 2008 as presented in figure 4.5.

Threshold from mean excess plot for Tbills are reported in figure 4.6. The estimates reveal threshold levels of 0.038 for NSE20, 0.023 for FTSE100, NSE-ASI 0.002 and JSE 0.008.

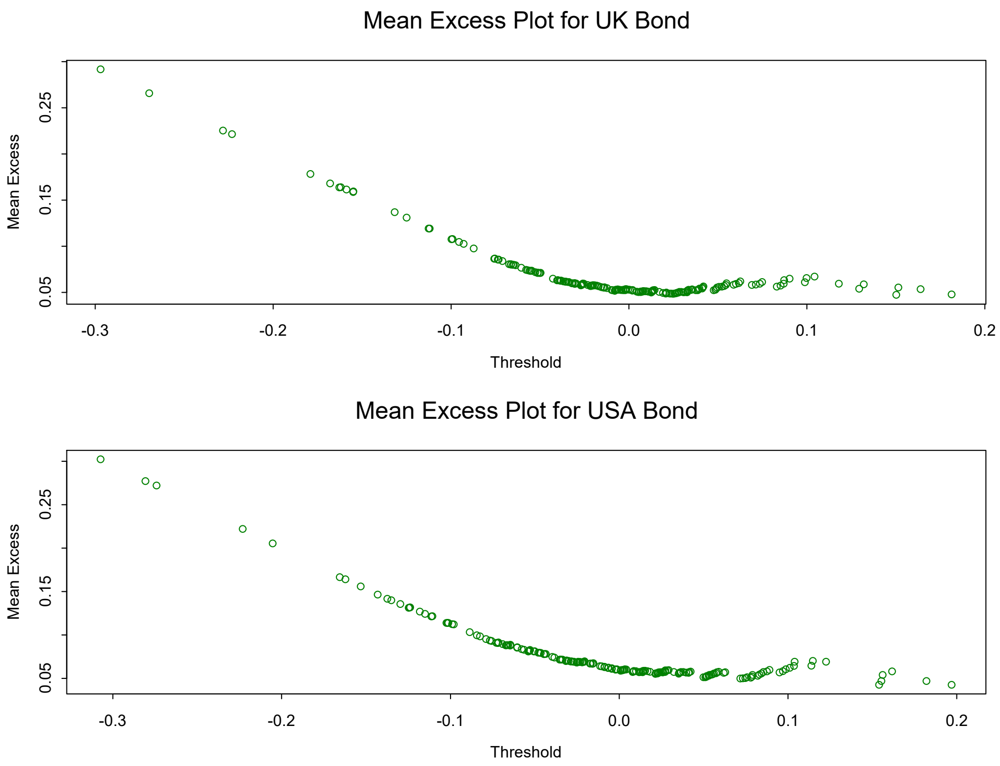


Figure 4.6: Mean Excess Plot for Bond Markets

4.2 Modelling Dependence Structure

In order to achieve objective 1 which was to estimate dependence structure of financial market in extreme periods, the study fits copula and selects the best copula that fit the market pairs using the AIC as described in the methodology section.

4.2.1 Copula Fit and Estimates for Dependence Structure

Table 4.2 gives estimates from fitting copula to stock market pair for a period considered as stability between 2000-2006.The best model is selected using the AIC criteria which serves as a rule of thumb in copula selection.

Table 4.2: Fitting of Copula for Stock Market Pair 2000-2006 (Pre Crisis Period)

Kenya - Nigeria							
Copula	Parameter						
	θ	LogLike	AIC	BIC	HQ	Kendall's	Spearman
Gaussian	0.175 (0104)	1.289	-0.587*	1.840	0.392	0.119	0.1673
Frank	0.699 (0.648)	0.581	0.8382	3.257	1.810	0.117	0.175
Gumbel	1.073 (0.073)	0.658	0.6589	3.101	1.653	0.111	0.167
Clayton	0.157 (0.137)	0.847	0.304	2.723	1.276	0.072	0.109
Kenya - USA							
Copula	Parameter						
	θ	LogLike	AIC	BIC	HQ	Kendall's	Spearman
Gaussian	0.0057 (0.1087)	0.00137	1.9972	4.4160	2.968	0.00363	0.0054
Frank	0.67931 (0.6449)	0.55317	0.8936*	3.3124	1.865	0.0751	0.1125
Gumbel	1.007 (0.0609)	0.0072	1.9854	4.4043	2.957	0.0073	0.0107
Clayton	0.0537 (0.1209)	0.1107	1.7785	4.1974	2.750	0.0261	0.0392
Kenya - UK							
Copula	Parameter						
	θ	LogLike	AIC	BIC	HQ	Kendall's	Spearman
Gaussian	0.1904 (0.192)	-15.88	33.76	35.32	34.30	0.054	0.080
Frank	1.069 (1.101)	0.469	1.061*	2.616	1.598	0.007	0.114
Gumbel	1.010 (0.041)	0.032	1.936	4.353	2.906	0.009	0.014
Clayton	0.072 (0.155)	0.115	1.768	4.187	2.740	0.034	0.052
Kenya - South Africa							
Copula	Parameter						
	θ	LogLike	AIC	BIC	HQ	Kendall's	Spearman
Gaussian	0.0612 (0.108)	0.1578	1.684	4.103	2.656	0.039	0.0585
Frank	0.2307 (0.662)	0.0606	1.878	4.297	2.850	0.0256	0.0384
Gumbel	1.0153 (0.059)	0.0360	1.927	4.346	2.899	0.0151	0.0226
Clayton	0.0994 (0.1259)	0.3628	1.274*	3.693	2.246	0.0473	0.0709

Note: The table gives a summary of empirical copula fit for Kenya stock market index (NSE20) pairs with stock index for Nigeria, USA, UK and South Africa. θ is the dependence parameter for respective copula (Gaussian, fit, Loglike is the Log-Likelihood, AIC is the Akaike Information Criterion and BIC is the Bayesian Information Criterion. * represent the selected copula in the given period.

The copula fit for Kenya-Nigeria stock market in table 4.2 reveals that the best fit copula is the Gaussian copula with an AIC of -0.5787 which implies that Kenya and Nigeria stock market with the characteristics of a Gaussian copula can boom and crash together. The Kendall's tau reports a correlation of 0.1119 and Spearman rho of 0.1673. For Kenya-USA pair and Kenya - UK pair, the Frank copula using the AIC reveals the best fits with an AIC of 0.89365 for Kenya-USA and 1.0612 for Kenya-UK. The correlation coefficient are 0.0751 and

01125 for Kendall's tau and Spearman rho respectively for Kenya-USA and 0.074 and 0.114 for Kendall's tau and Spearman rho respectively. However, the Kenya-South Africa pair reveals that the Clayton copula is the best fit copula implying that both market have the probability of crashing together in the period under review. Figure 4.7 gives a visual picture of the scatter plots for stock market pairs.

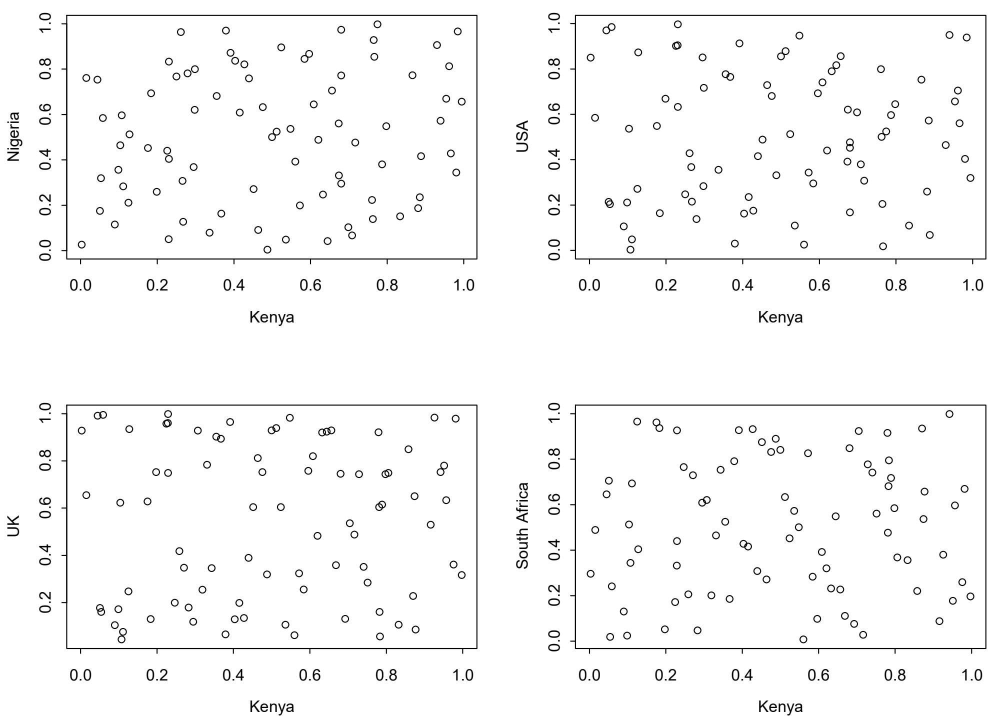


Figure 4.7: Scatter Plot of Stock Market Pairs

Table 4.3 gives an overview of stock-tbills pairs for USA and UK markets. The copula reveals that the best copula fit for the Kenya-USA market data is the Frank copula with the lowest AIC of 1.7065, the pair has a correlation of 0.047 and 0.0709 for Kendall's tau and Spearman rho respectively. On the other hand, the Kenya - UK pair reveals that the best copula fit is the Gumbel copula with an AIC of 1.1101. This implies that the pair has the probability of moving up together.

Table 4.3: Fitting of Copula Stock Market -TBills Pairs 2000-2006

KENYA -USA				
Parameters	**Gaussian**	**Frank**	**Gumbel**	**Clayton**
θ	0.0214 (0.1468)	0.4267 (0.2787)	1.0206 (0.0957)	0.05724 (0.2054)
LogLike	0.01063	0.1467	0.0244	0.0394
AIC	1.978742	1.7065*	1.9412	1.92116
BIC	4.39783	4.1253	4.3700	4.34000
HQ	2.9504	2.6783	2.9229	2.8929
Kendall's	0.01363	0.0473	0.02026	0.02782
Spearman	0.02045	0.0709	0.03035	0.04172
KENYA-UK				
Parameters	**Gaussian**	**Frank**	**Gumbel**	**Clayton**
θ	0.0898 (0.1078)	0.4911 (0.7007)	1.0568 (0.0698)	0.1077 (0.1366)
LogLike	0.3405	0.2450	0.4444	0.37914
AIC	1.3188	1.5098	1.1101*	1.2417
BIC	3.7376	3.9286	3.5289	3.6605
HQ	2.2905	2.4815	2.0818	2.2134
Kendall's	0.0572	0.0544	0.0537	0.0511
Spearman	0.0858	0.0815	0.0803	0.0765

Copula fit for Kenya stock market index (NSE20) with T-bills index for USA and UK. θ is the dependence parameter for respective copula * represents the selected copula.

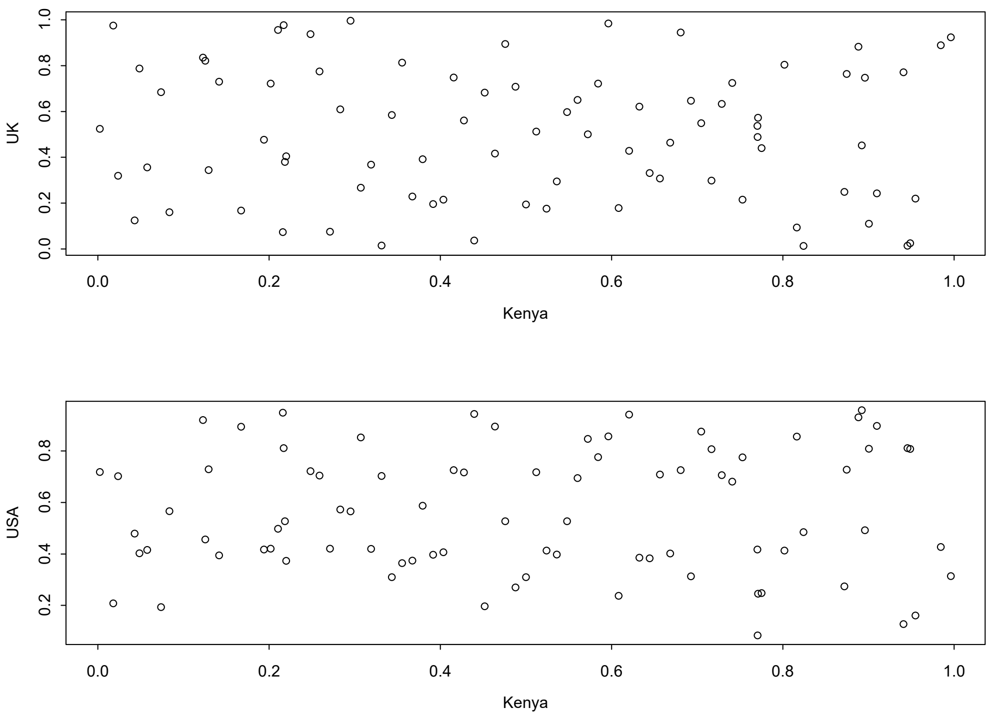


Figure 4.8: Scatter plot of copula stock market - Tbills Pairs 2000-2006

Table 4.4 gives results of fitting copula to stock market and bond index pair for USA and UK for 2000-2006 period. In this period, the dependence between Kenya stock market and USA Bond index is described by the Gumbel copula having the smallest AIC of 1.714. In the case of Kenya-UK stock-bond pair, the Gumbel copula has the lowest AIC of 1.335.

KENYA -USA				
Parameters	**Gaussian**	**Frank**	**Gumbel**	**Clayton**
θ	0.0200 (0.1086)	0.2445 (0.6643)	1.0296 (0.0610)	1.6614 (0.090)
LogLike	0.0170	0.0678	0.1426	-0.0000067
AIC	1.9659	1.8643	1.7147*	2.0001
BIC	4.3847	4.2831	4.1336	4.418854
HQ	2.9376	2.8361	2.6865	2.9176
Kendall's	0.0127	0.0271	0.02879	8.307e-007
Spearman	0.0191	0.0407	0.04310	1.246e-006
KENYA-UK				
Parameters	**Gaussian**	**Frank**	**Gumbel**	**Clayton**
θ	0.0836 (0.1077)	0.3257 (0.6892)	1.0521 (0.0704)	0.04501 (0.1116)
LogLike	0.2967	0.1115	0.3324	0.0929
AIC	1.4064	1.7768	1.3351*	1.8141
BIC	3.8253	4.1957	3.7540	4.2329
HQ	2.3782	2.7486	2.3069	2.7859
Kendall's	0.0533	0.0361	0.0496	0.0220
Spearman	0.0799	0.0542	0.0741	0.0330

Table 4.4: Fitting Stock Market-Bond Pairs 2000-2006

Copula fit for Kenya stock market index (NSE20) pairs with Bond index for USA and UK. θ is the dependence parameter for respective copula. * represent the selected copula

Table 4.5: Copula Fitting for Stock Market-Stock Pairs 2007-2009

KENYA -USA			
Parameters	**Gaussian**	**Frank**	**Clayton**
θ	0.4125 (0.1352)	2.2675 (1.0856)	0.7512 (0.2902)
LogLike	-5.2073	2.1217	5.2651
AIC	12.414	-2.2435	-8.5303*
BIC	13.970	-0.6882	-6.9750
HQ	12.951	-1.7066	-7.9934
Kendall's	0.2706	0.2400	0.2730
Spearman	0.3967	0.3543	0.3973

KENYA-UK			
Parameters	**Gaussian**	**Frank**	**Clayton**
θ	0.4124 (0.1352)	2.2675 (1.085)	0.7512 (0.29202)
LogLike	-5.207	2.121	5.265184
AIC	12.414	-2.2435	-8.5303*
BIC	13.970	-0.6882	-6.9750
HQ	12.951	-1.7066	-7.9934
Kendall's	0.27067	0.2400	0.2730
Spearman	0.3967	0.3543	0.3973

KENYA-SOUTH AFRICA			
Parameters	**Gaussian**	**Frank**	**Clayton**
θ	0.5115 (0.1101)	3.4175 (1.1306)	0.9201 (0.3189)
LogLike	5.4477	4.3773	6.6607
AIC	-8.8955	-6.7546	-11.3215*
BIC	-7.3401	-5.1993	-9.766156
HQ	-8.3586	-6.2177	-10.7845
Kendall's	0.3418	0.3425	0.3151
Spearman	0.4939	0.4968	0.4542

KENYA -NIGERIA			
θ	0.099 (0.1660)	1.0210 (1.1378)	0.2863 (0.2486)
LogLike	0.1765	0.4026	0.9114
AIC	1.6469	1.1946	0.1771*
BIC	3.2023	2.7500	1.7324
HQ	2.1839	1.7315	0.7140
Kendall's	0.0636	0.1123	0.1252
Spearman	0.0953	0.1678	0.1865

Copula fit during extreme financial period for Kenya stock market index (NSE20) with stock market index for Nigeria, South Africa, USA, UK and Kenya. θ is the dependence parameter for respective copula. * represent the selected copula.

Table 4.5 gives an overview of fitting copula to stock market pair in extreme period considering the 2007-2009 period. In this period, the dependence between

Kenya stock market and other stock markets are described by the Clayton copula since they exhibit the smallest AIC. In the case of the Kenya-USA stock market pair, the Clayton copula has the lowest AIC of -8.53. Similarly, for Kenya-South Africa, the lowest AIC of -11.32 was recorded. Kenya-Nigeria stock pair had the lowest AIC of 1.077. Figure 4.9 below confirms the result of the AIC from the respective scatter plots showing the graphical representation.

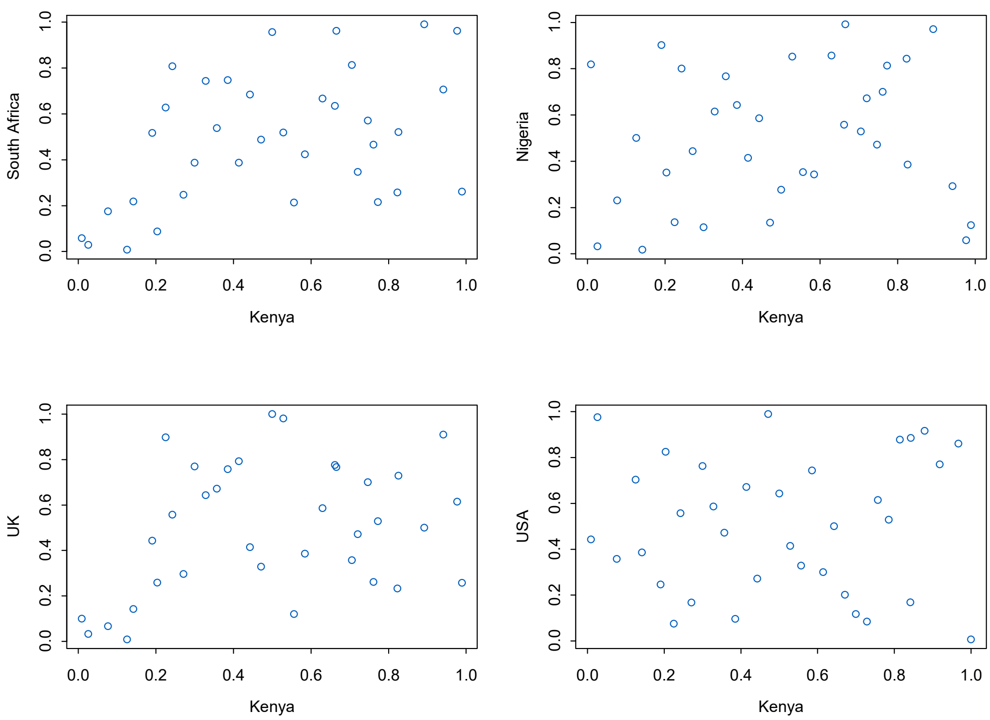


Figure 4.9: Scatter Plot Fit for Stock Market Pair 2007-2009

In analyzing dependence between stock market and Tbills in extreme period, Kenya stock market index was fitted with Tbills index of USA and UK markets. The best copula describing dependence between various market pairs is the Clayton. The Clayton Copula fit had the lowest values in AIC. This result signifies that there is a negative dependence in the lower tail of the market pairs. Figure 4.9 which shows the scatter plots from fitting stock market pairs during 2007 - 2009, reveals lower tail dependence which signifies stock markets jointly crashing together during the financial crisis period.

Table 4.6: Fitting Stock Market -TBills Pairs 2007-2009

KENYA -USA			
Parameter	Gaussian	Frank	Clayton
θ	0.2513 (0.1664)	0.8609 (1.0687)	0.8609 (1.0687)
LogLike	-7.2008	0.3231	1.234
AIC	16.4017	1.3536	-0.4695*
BIC	17.957	2.9090	1.08057
HQ	16.938	1.8905	0.0673
Kendall's	0.1617	0.0949	0.1355
Spearman	0.2480	0.1421	0.2016
KENYA-UK			
Parameter	Gaussain	Frank	Clayton
θ	0.3607 (0.1458)	1.7102 (1.0127)	0.5106 (0.2548)
LogLike	-6.0012	1.42710	2.7257
AIC	14.0024	-0.8542	-3.45141*
BIC	15.557	0.7011	-1.8960
HQ	14.539	-0.3173	-2.91451
Kendall's	0.2349	0.1847	0.2033
Spearman	0.3464	0.2745	0.2997

Copula fit for stock-Tbills market index for USA and UK. θ is the dependence parameter for respective copula. * represents the selected copula

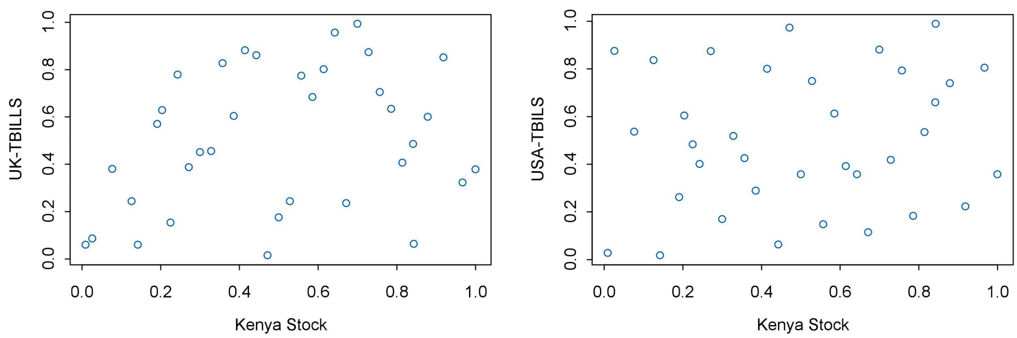


Figure 4.10: Scatter Plots for Stock Tbills 2007-2009

In the case of dependence between stock and bond markets, copula fit for Kenya, USA and UK pairs reveal that frank copula better capture the dependence between the market pairs. The Kenya-USA market has and AIC of 1.3215. Similarly, for the Kenya - UK market, the Frank Copula has the lowest AIC of 1.792. Figure 4.10 which shows the scatter plots from fitting stock market Tbill pairs during 2007-2009. It reveals no tail dependence during the period under study.

KENYA -USA			
Parameter	Gaussian	Frank	Clayton
θ	0.2195 (0.1662)	0.8536 (1.0335)	0.1237 (0.1949)
LogLike	-7.39257	0.3392	0.2354
AIC	16.783	1.3215*	1.5291
BIC	18.3404	2.8769	3.0844
HQ	17.3220	1.8584	2.06602
Kendall's	0.1409	0.0941	0.0582
Spearman	0.2101	0.1409	0.087
KENYA-UK			
Parameter	Gaussian	Frank	Clayton
θ	0.1340 (0.1748)	0.4571 (1.0019)	2.2792 (1.2320)
LogLike	-7.8880	1.0381	-3.853
AIC	17.760	1.792338*	2.0000
BIC	19.315	3.3476	3.5555
HQ	18.297	2.3294	2.5369
Kendall's	0.0855	0.0506	1.139
Spearman	0.12805	0.07597	1.707

Table 4.7: Fitting Stock Market-Bond Pairs 2007-2009

The scatter plot in figure 4.11 gives a visual display for both USA and UK bond fitting during the crisis period.

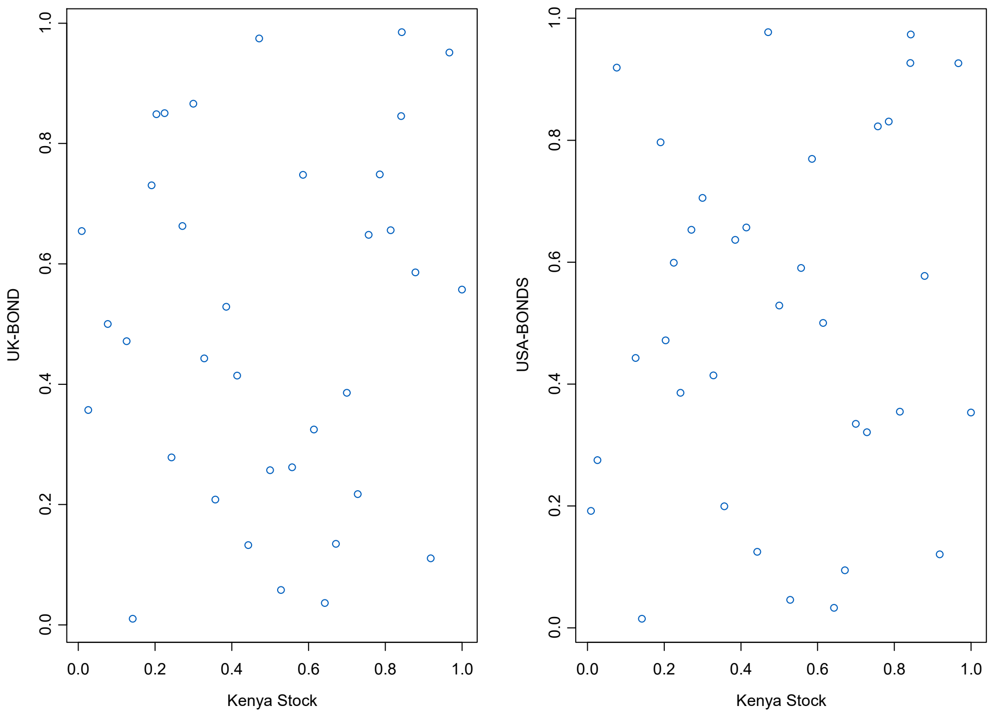


Figure 4.11: Scatter Plot For Stock Market- Bond Pairs 2007-2009

In periods after extreme events (2010-2016) the dependence between financial markets, the Frank copula explains the dependence between Kenya-Nigeria stock market. The Kenya-South Africa market pair, the Gaussian copula describes the dependence structure of the the market with an AIC of -9.328 which is the smallest of the fitted copulas. Figure 4.10 which shows the scatter plots from fitting stock and bond market, reveals no tail tail dependence during the period under study. The markets can boom or crash together.

Table 4.8: Fitting Stock Market Pairs 2010-2016

KENYA NIGERIA				
Parameter	**Normal**	**Frank**	**Gumbel**	**Clayton**
θ	0.3879 (0.094)	2.398 (0.7397)	2.057 (0.1324)	0.4307(0.1847)
LogLike	-3.288	5.304	3.465	3.328
AIC	8.576	-8.608	-4.564*	-4.657
BIC	10.88	-6.304	-3.4859	-2.3539
HQ	9.495	-7.689	-6.9903	-3.7388
Kendall's	0.1772	0.2525	0.2343	0.2536
Spearman	0.2622	0.3721	0.9093	0.3727
KENYA -USA				
Parameter	**Normal**	**Frank**	**Gumbel**	**Clayton**
θ	0.668(0.07)	1.732 (0.21)	5.664 (01.16)	1.047(0.30)
LogLike	11.36	8.40	12.27	8.221
AIC	7.901	5.81	3.789*	6.349
BIC	6.926	7.679	4.910	7.207
HQ	5.081	4.556	3.826	6.578
Kendall's	0.0426	0.0422	0.049	0.0343
Spearman	0.0651	0.0591	0.0690	0.0491

Copula fit for Kenya stock market index (NSE20) pairs with stock market index for Nigeria and USA. θ is the dependence parameter for respective copula. *represents the selected copula.

The Frank copula explains the dependence between Kenya-Nigeria stock market. The Kenya-South Africa market pair, the Gaussian copula describes the dependence structure of the the market with an AIC of -9.328 which is the smallest of the fitted copulas.

KENYA -UK				
Parameter	**Normal**	**Frank**	**Gumbel**	**Clayton**
θ	0.01522 (0.1151)	1.3800 (0.6715)	1.000 (0.0880)	5.4035 (0.0431)
LogLike	0.00874	0.00211	-1.11022e-015	-1.7059
AIC	1.9825	1.9577*	2.0000	2.0019
BIC	4.2865	4.2618	4.304065	4.304065
HQ	2.9016	2.8768	2.91920	2.91920
Kendall's	0.0096	0.01533	0.0034	2.70175
Spearman	0.0145	0.02295	0.0287	3.8901
KENYA -USA				
θ	0.1347 (0.1109)	0.1330 (0.1382)	0.56560(0.6815)	1.05156 (0.0809)
LogLike	1.34416	0.3441	0.2276	0.5611
AIC	0.5886*	1.3116	1.5447	0.8776
BIC	2.89272	3.6157	3.8488	3.1816
HQ	1.50778	2.2307	2.4638	1.7966
Kendall's	0.08599	0.0626	0.0490	0.06239
Spearman	0.12869	0.0938	0.0732	0.09342

Table 4.9: Fitting Stock Market - Tbills Pairs 2010-2016
Copula fit for Kenya stock market index (NSE20) pairs with Tbills market index for USA and UK. θ is the dependence parameter for respective copula. * represents the selected copula

In table 4.9, the paring of Kenya stock index and Tbills market index for USA and UK reveal that the Frank copula described the dependence between Kenya-UK pair while the Gaussian copula described the Kenya - USA market pair. Figure 4.12 gives a visual overview of the scatter plot of stock -Tbills pair index.

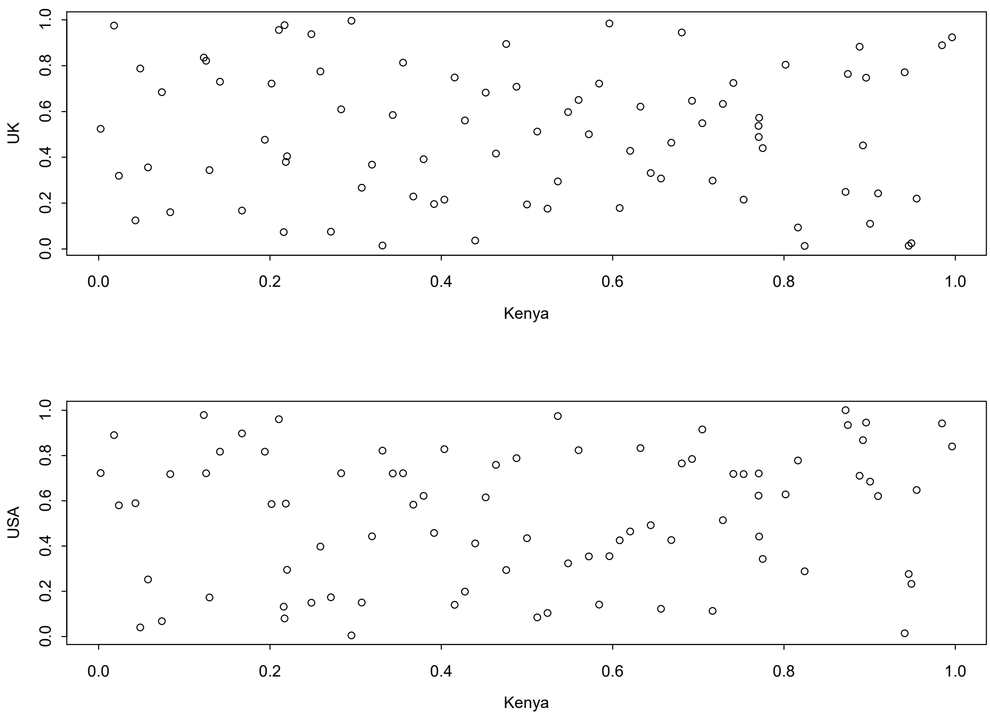


Figure 4.12: Scatter plot for copula Stock Market -Tbills Pairs 2010-2016

Table 4.10 gives an overview of fitting copula to stock market pairs between 2010-2016 period. In this period, the dependence between Kenya stock market and bond market is described by the Gumbel copula since they exhibit the smallest AIC. In the case of the Kenya-USA market pair, the Gumbel copula has the lowest AIC of -0.949. Similarly, for Kenya-USA, the lowest AIC of -1.173 was recorded. Figure 4.13 below, gives the respective copula showing the graphical representation. Figure 4.12 shows that the markets are likely to crash or boom together.

KENYA -UK				
Parameter	**Normal**	**Frank**	**Gumbel**	**Clayton**
θ	0.0889 (0.1142)	0.58120 (0.7110)	1.6504 (0.0789)	0.0543 (0.1401)
LogLike	0.2973	0.3350	0.41349	0.08116
AIC	1.4050	1.3298	1.17301*	1.83767
BIC	3.7094	3.6339	3.47707	4.1417
HQ	2.3245	2.2490	2.0921	2.75679
Kendall's	0.05668	0.06436	0.0611	0.02647
Spearman	0.08494	0.09643	0.0911	0.03969
KENYA -USA				
Parameter	**Normal**	**Frank**	**Gumbel**	**Clayton**
θ	0.1917 (0.1091)	1.01625 (0.7175)	1.1294 (0.0873)	0.1229 (0.1487)
LogLike	1.4098	1.0005	1.4746	0.3968
AIC	-0.8196	-0.00113	-0.9492*	1.2063
BIC	1.4843	2.3029	1.3548	3.5103
HQ	0.0994	0.91798	-0.03011	2.1254
Kendall's	0.1228	0.1117	0.06106	0.05789
Spearman	0.1833	0.1670	0.09113	0.08671

Table 4.10: Fitting Stock Market - Bond Pairs 2010-2016
Copula fit for Kenya stock market index (NSE20) pairs with bond market index for USA and UK. θ is the dependence parameter for respective copula. * represents the selected copula.

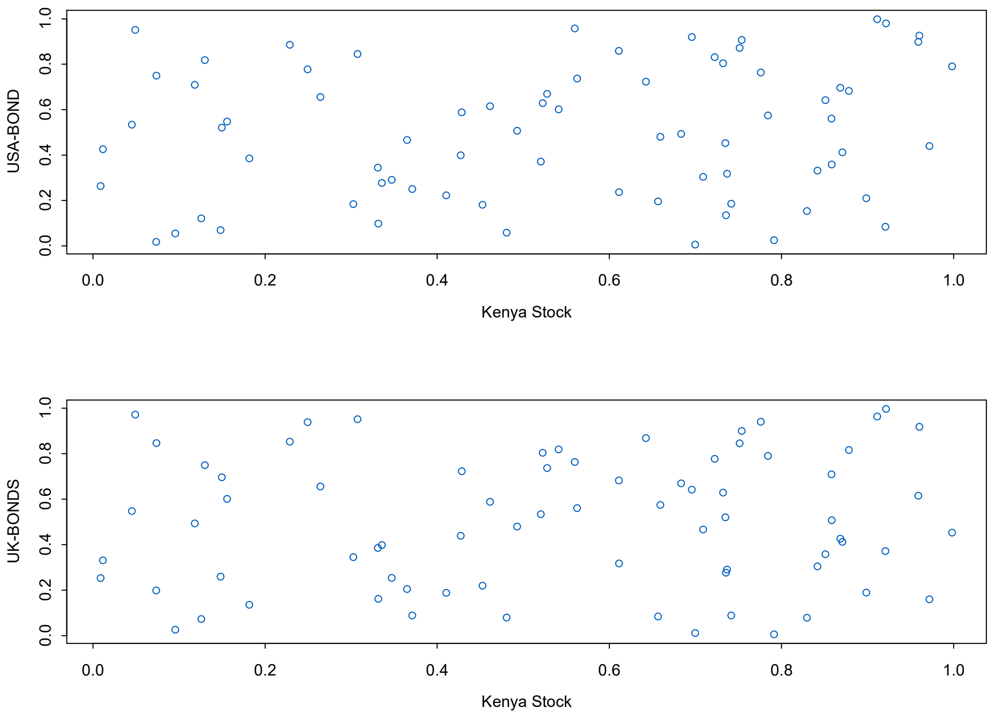


Figure 4.13: Scatter Plot for fitted Copula for Stock Bond Pairs 2010-2016

In view of objective 2 which is to determine changes on the dependence structure caused by extreme event, fitting of copula in the three periods (pre-crisis, crisis and post crisis) show that different copula characterized the pre-crisis period and post crisis period. However, the crisis period for financial market pair was captured mainly by the Clayton copula. This implies that extreme events change the dependence structure as different copula model the structure of dependence at times of extreme events and normal times.

4.3 Estimating Market Risk

After estimation of copula parameter θ, we substitute the dependence parameter into the VaR functions to estimate risk measures at 90%, 95% and 99% confident interval. Table 4.11 give estimates of the possible loss for stock market portfolio during the crisis period. Using a 99% confident interval, the Kenya-Nigeria pair records 0.0837, Kenya-UK reveals 0.0690 and the Kenya-USA estimate is 0.0735. This implies that the losses from holding this portfolios are 8.37% for Kenya-Nigeria stock market, 6.9% for Kenya-UK and 7.35% for Kenya-USA stock market pairs. Kenya South Africa stock market pair has the highest risk of 0.085 implying 8.5%.

Table 4.11: VaR Estimates for Stock Market Pairs

(a) Crisis Period : Clayton Copula			
NSE20-NASI	90% VaR	95% VaR	VaR 99%
	0.0759	0.0784	0.0837
NSE20-JSE	90% VaR	90% ES	VaR 95%
	0.0776	0.0795	0.0855
NSE20-UK	90% VaR	90% ES	VaR 95%
	0.0657	0.0680	0.0690
NSE20-USA	90% VaR	90% ES	VaR 95%
	0.0693	0.0721	0.0735

For a portfolio consisting of stocks and bonds, we considered the Kenya stock and the USA and UK bond index. The VaR estimates are presented in table 4.12. Using a 99% confident interval, the Kenya-USA pair records 0.0182, Kenya-UK reveals 0.0162 This implies that the losses from holding this portfolios are 1.82% for Kenya-USA and Kenya-UK and 1.62%.

Table 4.12: VaR Estimates for Stock-Bond Market Pairs

(a) Crisis Period : Clayton Copula			
NSE20-USA	90% VaR	95%VAR	VaR 99%
	0.0175	0.0178	0.0182
NSE20-UK	90% VaR	90% ES	VaR 95%
	0.0156	0.0159	0.0162

In the context of stocks and Tbills, we considered the Kenya stock and the USA and UK Tbills index. The VaR estimates are presented in table 4.13. Using a 99% confident interval, the Kenya-USA pair records 0.0321 and Kenya-UK reveals 0.049. This implies that the losses from holding this portfolios are 3.2% for Kenya-USA portfolio and 4.9% for Kenya-UK. This indicates that the Kenya-UK portfolio has a higher risk.

Table 4.13: VaR for Stock-Tbills Market Pairs

(a) Crisis Period : Clayton Copula			
NSE20-USA	90% VaR	95% VaR	VaR 99%
	0.0263	0.0289	0.0321
NSE20-UK	90% VaR	90% ES	VaR 95%
	0.038	0.042	0.049

From the empirical findings, to diversify market risk during the crisis period (2007-2009) the market pairs with the highest maximum possible loss is evident in the stock market with market pairs Kenya-Nigeria stock with a possible loss of 8.3%, 8.5% for South Africa, 6.9% for UK and 7.3% for USA. The Stock -Tbills market using a 99% VaR, the Kenya USA could have a maximum of 2.8% and for Kenya -UK, 4.9% of the portfolio holding. Since bonds are financial considered as saver investments over time, Using a 99% VaR, the Kenya-USA pair could loss a 1.8% and 1.6% for UK. This implies that investment in bond is less riskier during the crisis period.

CHAPTER 5

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

The research used the concept of copula and Value at Risk to estimate the possible loss from a market crash. The copula parameter was imputed into the Value at risk framework analytically. The Inference Function for Margin was used to estimated the copula parameter and the consistency and normally properties of the estimator were also derived. This was applied to real data to estimate market risk. The study focused on stock markets (Kenya, Nigeria South Africa, UK and USA), Bond market (US, UK) and Tbills Market (UK, US) using the concept of copula, which revealed the estimates of dependence structure of stock markets during pre-crisis, crisis and post-crisis periods. Estimation of clayton copula parameters using the Inference Function for Margins method was used to obtain the dependence parameter. To measure VaR, the dependence parameter was used in estimating the VaR. The results revealed that during the crisis period, the maximum possible loss of market value is 75.9% and 77.6% with a confident interval of 90% for the Kenya-Nigeria and Kenya-South Africa portfolios respectively. This implies that the Kenya-South Africa portfolio has the highest risk. A further implication is that dependence during crisis period imply that opportunities for portfolio diversification are reduced than at periods of booms.

5.2 Recommendation

Since financial market are prone to recessions and recovery, it is important to analyse and estimate the structure of dependence and risk associated with investments so as to diversify to minimize losses. Therefore, risk managements and monetary authorities should constantly monitor the structure of dependence in this wise.

Further studies on estimating financial market risk using copula can be extended beyond Value-at-Risk measure but also analytically using the Expected Shortfall. Since Expected Short fall accounts for coherent risk and the VaR does not. This will give a robust result in estimating for financial market dependence and risk during periods of extreme events. Further studies which could capture factors that can lead to market crash and the exploration of other classes of copulas such as the vine copula models to capture high dimensional dependence and used to estimate financial market risk could be research on in future.

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