MODELING NEWTONIAN FLUID FLOW IN AN OPEN RECTANGULAR CHANNEL WITH LATERAL INFLOW CHANNEL

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2014

DECLARATION

This thesis is my original work and has not been submitted to any other university for examination.

Signature:................................. Date:..

Karimi Samuel Macharia

This thesis report has been submitted for examination with our approval as University supervisors.

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DEDICATION

This thesis is dedicated to my late brother Boniface Ndung'u Karimi, his wife Katrina Ndung'u and his lovely daughter Ashley Njeri. Rest in peace my brother and friend.

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NOMENCLATURE

Roman symbols

Symbol	Meaning
A	Cross-sectional area of flow of the main channel,[m^2]
a	Cross-sectional area of the lateral inflow channel, [m^2]
b	Top width of the main channel,[m]
$b1$	Top width of the lateral inflow channel, [m]
C	Resistance coefficient of flow (Chezy coefficient)
y	Depth of flow in the main channel, [m]
$y1$	Depth of flow in the lateral inflow channel, [m]
K	Constant of proportionality in Chezy formula
L	Length of the lateral inflow channel, [m]
P	Wetted perimeter of the main channel,[m]
Q	Discharge in the main channel,[m^3 s^{-1}]
R	Hydraulic radius, [m]
Re	The Reynolds number
Fr	The Froude number
F_R	Resisting force, [N]
S_o	Slope of the channel bottom
S_f	Friction slope
T	Top width of the free surface, [m]
$\vec{T}$	Traction force,[N]
V	Mean velocity of flow in the main channel,[ms^{-1}]
u	Mean velocity of flow in the lateral inflow channel, [ms^{-1}]
g	Acceleration due to gravity, [ms^{-2}]

n	The manning roughness coefficient, [s $m^{-1/3}$]
q	Lateral discharge,[m^3s^{-1}]
t	Time, [s]
x	Distance along the main flow direction, [m]
$\vec{F}$	Body force, [N]
W	Weight of the fluid, [N]
$\vec{q}$	Velocity vector ($u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$)
Δ	Forward difference
∇	Divergence vector

Greek symbols

θ	Angle of lateral discharge channel (degrees)
τ	Shear stresses, [Nm^{-2}]
ν	Kinematic viscosity, [m^2s]
γ	Fluid specific weight, [N]
ρ	Density, [kg m^{-3}]
υ	Coefficient of viscosity, [Ns^{-1}]
α	Energy coefficient

ABSTRACT

The study aim is to propose a mathematical model to be used to investigate the flow in an open rectangular channel with a lateral inflow channel. An incompressible flow of a Newtonian fluid through a man-made open rectangular channel is considered. The study has considered the effects of angle as it varies from zero to ninety degrees and variation of the cross-sectional area, velocity and length of the lateral inflow channel on how they affect the flow velocity in the main open rectangular channel. Increase in the flow velocity leads to increase in the discharge. The equations governing the flow are the continuity and momentum equations. The resulting partial differential equations are coupled and nonlinear and cannot be solved analytically. Therefore, an approximate solution of these partial differential equations is determined numerically using the finite difference method. Matlab software has been used to write a code that has been used to compute and solve the problem. Solutions are represented by making use of graphs and tables. At zero degrees angle of the lateral inflow channel, the results compare to earlier research done by Kwanza *et al* (2007) and Thiong'o (2011). It is also found out that an increase in the cross-sectional area and the length of the lateral inflow channel leads to a reduction in the velocity while an increase in the velocity of this channel leads to an increase in the velocity of the main channel. Finally, an increase in the angle of the lateral inflow channel does not necessarily lead to an increase in the velocity in the main channel. The study shows that angles of between 30^o and 50^o exhibits higher optimum values of velocities in the main open channel compared to other angles of the lateral inflow channel.

CHAPTER ONE

INTRODUCTION

1.1 Background information

In the year 2002, Kenya experienced heavy rainfall, which resulted in bridges being swept away as rivers over flooded. During this heavy rainfall, water flowed from highlands where the soil became saturated with water. Moreover, some areas still suffer from floods during normal rain. Thus, designing channels that would control such an environmental disaster and more so divert the same water to agricultural land is very important. The fact that the flood problem still persists and the need to convey water for irrigation is still in demand, there is need to come up with a hydraulically efficient model of a channel with lateral inflow to convey the maximum discharge.

A channel may be closed or open. The channels that have an open top are referred to as open channels while those with a closed top are referred to as closed channels. Good examples of open channels are rivers and streams while examples of closed conduits are pipes and tunnels. Open channels made of earth and concrete have been designed which have been of different cross-sections such as trapezoidal, rectangular and circular.

This chapter begins with some definitions of open channel terminologies followed by literature review related to the study of open channels with lateral inflow channel. It ends with the model of the problem, null hypothesis, objectives, research questions and justification of the research are presented towards the end of the chapter.

1.2 Definitions

1.2.1 Fluid

A fluid is deformable if when subjected to stress the distance and the orientation between two neighbouring molecules change. A fluid has no definite shape but assumes the shape of the container. Fluids are conventionally classified as liquids or gases. Liquids do not change significantly in volume when subjected to change in pressure and temperature. For this reason they are treated as incompressible fluids. Gases show notable volume changes when subjected to change in pressure and temperature. This implies that they are compressible.

1.2.2 Newtonian fluid

Gutfinger and Pnueli (1992) defined that a fluid as Newtonian if it obeys the Newton's law of viscosity, which states that the shear stress is proportional to the velocity gradient, and the coefficient of viscosity is taken as a constant. This means the viscosity for a Newtonian fluid depends on pressure and temperature only and not on the forces acting upon it. In a simple definition, it means that the fluid will continue to flow along a channel regardless of the forces acting on it. Since the fluid is considered to be Newtonian, the traction forces in the momentum equation are a sum of the pressure gradient and viscous forces (which are assumed to be uniform throughout the flow). However, no natural free flowing fluid is Newtonian. This is because as the fluid flows in layers, the distance between the layers at different points gradually changes, which makes the coefficient of viscosity to change. In addition, as the fluid moves from one section of the channel due to either variation of the slope or width of the channel due to erosion, the velocity of the fluid is bound to change which will affect the viscosity of the fluid. So for the fluid to be considered as Newtonian is an assumption. The coefficient of viscosity is defined as the shear stress multiplied by the distance between the two adjacent layers of the fluid, and then divided by the relative change in velocity between the two layers.

1.2.3 Open channel

An open channel is a channel that is characterized by a free surface, whereas pipe flow has none. A free surface is defined as the surface of contact between the liquid and the overlying gaseous fluid. The flow of a liquid, for example, water in a conduit may either be in an open channel or pipe flow. The two kinds of flow are similar in many ways but differ in one important aspect. According to Chow (1973), in an open channel a fluid does not fill the conduit completely. Flow in open channels is due to the difference in the potential energy.

1.2.4 Lateral channel

A lateral inflow channel has discharge resulting from the addition of fluid or water along the direction of flow. A lateral channel that adds water to a stream, river, or lake is referred to as a lateral inflow channel while the one that draws water from the latter is referred to as a lateral outflow channel. Lateral inflow may also include groundwater flow and overland flow.

1.2.5 Steady flow

There are several types of flows classified according to changes in flow depth with respect to time and space. The flow is said to be steady if the depth and discharge of flow at a particular point does not change with time interval under consideration.

1.2.6 Unsteady flow

Flow in which depth and discharge changes with time and space is said to be unsteady. This is the most common type of flow and requires the solution of the energy, momentum and friction equations with time.

1.2.7 Uniform flow

Open channel flow is said to be uniform if the depth and velocity of flow are the same at every section of the channel. Hence it follows that uniform flow can only occur in prismatic channels.

1.2.8 Steady uniform flow

For steady, uniform flow, depth and velocity is constant as you move along the channel. This constitutes the fundamental type of flow in an open channel. It occurs when gravitational forces are in equilibrium with resistance forces.

1.2.9 Steady non-uniform flow

A flow in which depth varies with distance, but not with time is called steady non-uniform flow. The type of flow may either be gradually varied or rapidly varied. The former requires application of energy and frictional resistance equations.

1.2.10 Natural and artificial channels

There are two types of open channels, namely natural and artificial channels. Artificial channels are channels made by man. They include irrigation canals, navigation canals, spillways, sewers, culverts and drainage ditches. They are usually constructed in a regular cross-section shape throughout and are thus prismatic channels. In the field, they are commonly constructed of concrete and have the surface roughness reasonably well defined although this may change with age. Analysis of flow in such channels will give reasonably accurate results.

Natural channels are not regular or prismatic and their materials of construction can vary widely. The surface roughness will often change with time, distance and elevation. Consequently, it becomes more difficult to accurately analyze and obtain satisfactory results for natural channels than it is with man-made ones. This situation may be further complicated if the boundary is not fixed due to erosion and deposition of sediments occur. For analysis, various geometric properties of the channel cross-sections are required. For artificial channels, these can usually be defined using simple geometric equations given the depth of flow.

1.2.11 Laminar flow

The effects of viscosity in relation to the inertia forces of the flow govern the state or behavior of open channel flow. . In laminar flow the fluid particles appear to move in thin layers of fluid which seem to slide over adjacent layers with no disruptions between the layers.

1.2.12 Turbulent flow

The flow is turbulent if the inertial forces are strong relative to the viscous forces. In turbulent flow, the fluid particles move in irregular paths.

1.2.13 Transitional flow

A flow is termed transitional if it is neither laminar nor turbulent.

1.2.14 Reynolds's number

Reynolds's number is a non-dimensional parameter which represents the effect of viscosity relative to inertia. It is defined as $Re = VL / \nu$, where ν is the kinematic viscosity, V is the mean velocity of flow and L is the characteristic length.

1.2.15 Froude number

The dimensionless Froude number is important in analyzing the effect of gravity in fluid flow. It is defined as the ratio of inertial forces to gravity forces. It is defined as$Fr = V / \sqrt{Dg}$, where D is the hydraulic depth; V is the mean velocity and g is the acceleration due to gravity. If Fr is number one, it means that the inertial forces and gravity are equivalent and critical flow exists.

1.3 Statement of the problem

This research seeks to model mathematically an open rectangular channel by varying the angle, velocity, cross-sectional area and length of the lateral inflow channel to investigate how these four parameters affectthe flow velocity in the open rectangular channel.The fluid to be considered is Newtonian and the flow is uniform. This study, therefore, aims at coming up with a hydraulically efficient model of flow through an open rectangular channel with alateral inflow channel.

1.4 Geometry of the problem

In the present study, the lateral inflow channel is introduced into the open rectangular channel as illustrated in Figure 1. The discharge in the open rectangular channel and the lateral inflow channel are denoted by Q and q respectively. L and θ represent the length and the varying angle respectively, of the lateral inflow channel. The net volume of fluid that enters through the cell dx is considered at a time interval dt.

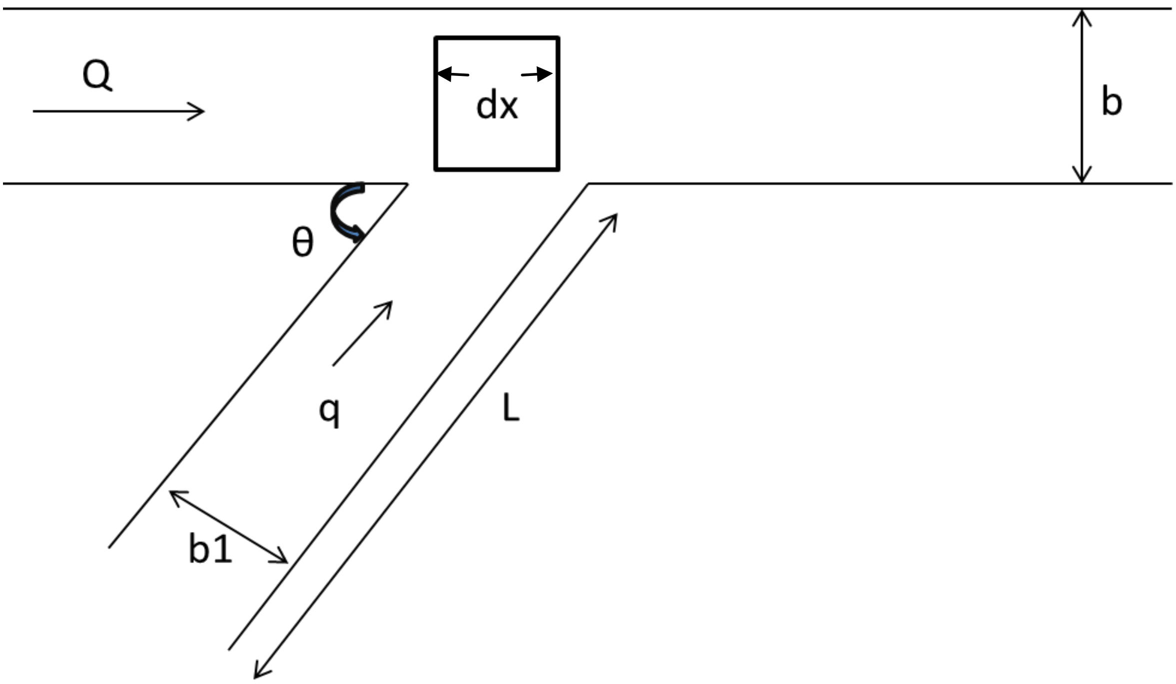

Figure 1: Model of the open rectangular channel with lateral inflow channel at an angle.

1.5 Null hypothesis

The variation of the cross-section area, length, velocity and angle of the lateral inflow channel do not affect the velocity in the main open channel.

1.6 Objectives of study

1.6.1 General objective

To analyze fluid flow in an open rectangular channel with a lateral inflow channel.

1.6.2 Specific objectives

i. To investigate how the variation of the angle of the lateral inflow channel affects velocity in the main flow channel.
ii. To investigate how the cross-sectional area of lateral inflow channel affects velocity in the main flow channel.
iii. To investigate how the length of the lateral inflow channel affects velocity in the main flow channel.
iv. To investigate how the velocity of the lateral inflow channel affects velocity in the main flowchannel.

1.7 Research questions

i. How does the variation of the angle of the lateral inflow channel affect the velocity in the main open channel?
ii. How does the cross-sectional area of lateral inflow channel affect velocity in the main open channel?
iii. How does the length of the lateral inflow channel affectvelocity in the main channel?
iv. How doesthe velocity of the lateral inflow channel affect velocity in the main channel?

1.8 Justification

For any civilization to exist and thrive, it needs water. Water is life, yet too much water can lead to death due to flooding. To direct water to lakes and rivers, man has constructed channels and canals. However,the problem of flood still persists, especially when there is heavy rain. Upto date, there is still a challenge to construct a channel that has a lateral inflow channel that will convey the maximum amount of water in an efficient way. Therefore, an efficient model of open channels with lateral inflow channels has to be formulatedto meet these needs. Moreover, mathematical modeling of an open channel with lateral inflow channel has received little attention.The mathematical model in this study is useful for constraction of lateral inflow channels when irrigating farms, draining water from flooded areas, in factories when producing flour or textiles and in water mills.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

The literature review in this chapter deals with three aspects of the current study. In the first section of this review, the equations that govern open channel flow are discussed, followed by review of lateral flow studies and equations. Then the literature on one of the numerical methods that is used to solve open channel flow is discussed and finally a summary why there is a gap in this literature to conduct the present study.

An open channel, whether in the form of an artificial or man-made channel constructed with a view to convey water to the required destination is common in many places. The forces at work in open channels are the inertia, gravity and viscosity forces. By neglecting forces that are generated in the junction area, the momentum equation is used to analyze lateral inflow in an open channel.

2.2 Review of literature

Chezy equation was one of the earliest equations developed for average computations of the velocity of a uniform flow, Henderson (1966). However, the formula that is mostly used in open channel problems is the Manning formula, Bilgil (1998). The Manning formula is very useful compared to the Chezy equation because it takes into account the degree of the channel irregularity, the bed materials, the relative effect of obstruction of the channel, vegetation growing in the channel, variation in shape and size of the channel and meandering in the channel, Chadwick and Morfet (1993). Many relationships, such as the velocity formula for open channel flow were studied by Chow (1959), which helped in analyzing open channels. His findings were that for all flow depth, the roughness coefficient *n* is assumed to be a constant if the bank and bed are equal in roughness and the slope is uniform.

The principles of conservation of momentum and mass are preserved by the Saint Venant equations and that is why they are used to govern unsteady open channel flow. The mathematical models of these unsteady open channel flows are applicable in flood defense and irrigation control design, Chaudhry (1993). Tuitoek and Hicks (2001), with the aim of controlling floods modeled unsteady flow in compound channels. They incorporated some terms to account for the momentum transfer phenomenon to incorporate unsteady flow in compound channels by developing a model based on the Saint Venant equations of flow.

Kwanza *et al* (2007) studied the effects of slope of the channel, the width of the channel, velocity and depth as they vary from one point to another in the channel and the lateral discharge on the fluid velocity and channel discharge for both rectangular and trapezoidal channels. They noted that to increase the discharge in the channels, the slope of the channel, width of the channel and the lateral discharge need to be increased. In addition, by minimizing the wetted perimeter, the velocity of the fluid flow increased. Thiong'o (2011) investigated fluid flow in open rectangular and triangular channels. Her findings on the study of rectangular channels agreed with those of Kwanza *et al* (2007). According to their findings, the flow velocity in the open rectangular channel increased when the slope, discharge and width increased. However, increasing the wetted perimeter of the channel resulted in a decrease in the flow velocity. To solve the continuity and momentum equations, they both used the finite difference method as a numerical tool.

The outflow and inflow discharge, the downstream and upstream and depth of water and the recirculation flow in the branch channel characterizes open channel dividing flow. Ramamurthy and Satish (1988), and Ingle and Mahankal (1990) established that the downstream to the upstream discharge ratio of the main channel is the most relevant parameter that analyzes open flow with a lateral channel at 90^o. Comparing these results with some experimental observations, it was observed that the results of the above analysis were satisfactory. The study of Neary and Odgaard (1993) concluded that the roughness of the bed as well as the branch-channel to main-channel velocity-ratio affects the structure of the flow. Barkdoll *et al* (1999) demonstrated in his research that the diversion flow ratio has the greatest effect on the sediment delivery ratio of the lateral intake, which is carried out in a straight path with 90^o intake angle.

Yang (2009) studied flow structures with diversion angles of 90°, 45° and 30°. A diversion angle of between 30° and 45° was recommended to get a better flow pattern of the fluid. Fan and Li (2005) formulated the diffusive wave equations for open channel flows with uniform and concentrated lateral inflow. In their formulation, they were able to present the continuity and momentum equations of an open channel with a lateral inflow channel that joins the main open channel at a varying angle.

Ramamurthy and Satish (1988) theoretically and experimentally investigated dividing flows with a submerged lateral branch when focusing on the sub-critical flow regime. The investigators developed a model theoretically by relating the discharge ratios and the downstream-to-upstream depth with the upstream Froude number. Their findings were that, a contraction in the channel section is caused by the re-circulatory zone downstream of the junction which makes the flow to change to super critical flow. The discharge in the branch of the lateral channel can be computed by the formula formulated by Mizumura *et al* (2003) for super-critical overflowing rivers, which compared well with results of Mizumura (2005).

Mohammed (2013) investigated how the discharge coefficient is affected by varying four different angles using an oblique weir with respect to the side of the channel wall in the flow direction. The four angles were 30^o, 60^o, 75^o and 90^o all of which were varied along the flow direction. The study found that maximum discharge was achieved at angle 30^o of the side weir. Masjedi and Taeedi (2011) studied in the laboratory the effect of intake angle on discharge ratio in lateral intakes in 180^o bend. The experiments were conducted with varying Froude number of various intake angles. The investigations showed that the discharge ratio increased at a lateral intake angle of 45^o in all locations of the 180^o flume bend.

Shamaa (2002) used the finite difference Preissmann implicit model to solve open channel operation-type problems which were based on the Saint Venant equations. Comparing with an explicit model, the implicit finite difference method model showed less oscillation and more accuracy. Akbari and Firoozi (2010) investigated the Preissmann and Lax diffusive schemes which are two different numerical methods for the numerical solution of the Saint Venant equations. These equations govern the propagation of flood wave in natural rivers with the objective of gaining better understanding of the propagation process. The study showed that the hydraulic parameters play an important role in these waves. Chagas and Souza (2005) provided the solution of Saint Venant equation through the study of flood in rivers. In their research they used a discretization for the Saint Venant equations with the objective of understanding better the propagation process. Their findings in the propagation of the flood wave showed that the hydraulic parameters play an important role in these waves.

From the forgoing, the Saint Venant equations are used to analyze fluid flow in both open channels and lateral inflow channel. The finite difference method is then used as a numerical tool to solve the equations. The literature above demonstrates that much research seems to have been done in open channels with no lateral inflow channel. However, the research that has been done on the lateral inflow channel has only been carried out in the laboratory. Therefore, little research has been done in open channels with lateral inflow channel by modeling the problem mathematically and solving the equations numerically. That is why in this research, the problem is modeled mathematically using the Saint Venant equations and the finite difference method is used to solve the equations.

CHAPTER THREE

GOVERNING EQUATIONS AND METHODOLOGY

3.1 Introduction

In this chapter, some uniform flow formulas like the Chezy and Manning formulae that have been developed over the years are reviewed. Next, the saint Venant equations for open channel flow are outlined and modified to incorporate an open channel with the lateral discharge at an angle the followed by the method of solution discussed. The governing equations are written in finite difference forms and their initial and boundary conditions are presented. Finally, investigations are carried out on how the variation of the angle, the cross-sectional area, velocity and length of the lateral inflow channel affect the velocity in the main rectangular channel.

3.2 Assumptions

i. The flow is one-dimensional.
ii. The forces causing the flow are due to gravity alone.
iii. The velocity of the fluid is too small compared to that of light.
iv. The fluid is considered incompressible.
v. The fluid is Newtonian.
vi. The flow is unsteady.
vii. Turbulent formation between the lateral inflow channel and the main open channel is negligible.

3.3 The Chezy and Manning formula

3.3.1 The Chezy formula

This formula is based on two assumptions, Cecen (1982). From the first assumption, the force resisting the flow is proportional to the square of the velocity, which implies it is equal toKV^2 where K is a constant of proportionality. Hence

$$F_R = KV^2 \tag{3.1}$$

For an open channel, the resisting force per unit of wetted area is proportional to the velocity squared,

$$F_R = KLPV^2 \tag{3.2}$$

Wetted area = Wetted perimeter (P) * Length (L) = PL, where$P = b + 2y$

From the second assumption, the force causing the flow is equal to the component of the weight of water in the direction of flow

$$F_m = \gamma ALsin\theta = \gamma ALS \tag{3.3}$$

where S is the slope of the channel and θ is the angle of inclination of the channel bottom with the horizontal force causing flow. Equating equations (3.2) and (3.3)

$$KLPV^2 = \gamma ALS \tag{3.4}$$

which implies that

$$V = \sqrt{\frac{\gamma}{K}}\sqrt{\frac{A}{P}}\sqrt{S} = C\sqrt{RS} \tag{3.5}$$

Where

$$R = \frac{A}{P} \tag{3.6}$$

and

$$C = \frac{\gamma}{K} \tag{3.7}$$

This coefficient is believed to be dependent upon the channel slope S, the hydraulic radius R and the coefficient of roughness n.

3.3.2 The Manning formula

The formula was developed by Bilgil (1998) through studies he performed. The Manning formula is given by

$$V = \frac{1}{n} R^{\frac{2}{3}} S_f^{\frac{1}{2}} \tag{3.8}$$

According to the above equation, the mean velocity of flow is a function of the hydraulic radius, channel roughness and the energy gradient slope where in uniform flow the energy gradient slope is assumed to be equal to the channel bottom slope.

Given the velocity, the discharge Q is defined as the product of cross-sectional area and velocity,

$$Q = AV \tag{3.9}$$

Substituting the Manning equation (3.8) into equation (3.9) yields

$$Q = KAR^{\frac{2}{3}} \tag{3.10}$$

where

$$K = \frac{1}{n} S_f^{\frac{1}{2}} \tag{3.11}$$

The equation (3.10) gives the flow rate through a channel of a given slope, radius and roughness coefficient.

From equation (3.6), R is inversely proportional to$P$. Furthermore, from equation (3.10), an increase in R will lead to increase in the discharge Q.Hence, to get maximum discharge from the channel, P needs to be minimized, which is the wetted perimeter.

For an open rectangular channel with depth y and width b, the wetted perimeter is given as

$$\mathrm{P} = \mathrm{b} + 2\mathrm{y} \tag{3.12}$$

While the cross-section area of the channel is given as

$$A = by \tag{3.13}$$

Making b the subject of the formula from the equation (3.13) yields

$$b = \frac{A}{y} \tag{3.14}$$

Substituting equation (3.14) into equation (3.12) yields

$$P = \frac{A}{y} + 2y \tag{3.15}$$

For a given channel of slope S, surface roughness n and area A, the wetted perimeter P, will be minimised when

$$\frac{dP}{dy} = 0 \tag{3.16}$$

Now,

$$\frac{dP}{dy} = -\frac{A}{y^2} + 2 \tag{3.17}$$

From the condition (3.16) and (3.17),

$$-\frac{A}{y^2} + 2 = 0 \tag{3.18}$$

$$\frac{A}{y^2} = 2 \tag{3.19}$$

$$A = 2\,y^2 \tag{3.20}$$

To confirm that P is minimized, $\frac{d^2P}{d^2y} > 0$ is checked.

Differentiating equation (3.17) with respect to y yields

$$\frac{d^2P}{d^2y} = \frac{2A}{y^3} \tag{3.21}$$

This means that since A and y is always positive, then from the equation (3.21),

$$\frac{d^2P}{d^2y} > 0$$

Substituting equation(3.13) in equation (3.20) yields

$$b = 2y \tag{3.22}$$

Therefore, to achieve maximum discharge for an open rectangular channel, the width b should be twice the depth y. This is because this will minimize the wetted perimeter P which is inversely proportional to the Hydraulic radius R. According to equation (3.10), this will in turn maximize the discharge Q.

3.4 Governing equations

The Saint Venant's equations basically describe the propagation of a wave in an open channel, predominantly one dimensional flow where the fluid is incompressible. These equations are the equation of continuity and equation of momentum, derived from Newton's second law of motion. Moreover, according to the number of elements considered in the model, waves can be classified as gravitational waves, diffusive waves, dynamic waves or cinematic waves.

3.4.1 Continuity equation

The principle of continuity is based on the law which states that mass can neither be created nor destroyed. Therefore, the equation of continuity is a type of differential equation that describes the transport of some kind of conserved quantity, for example mass.

According to Thiong'o (2011), for any arbitrary shape, the continuity equation governing unsteady flow in an open channel is

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} - q = 0 \tag{3.23}$$

According to Figure 1, the cell with lateral inflow distancedx, in a time interval dt is considered. The net volume of the fluid in this cell is $\frac{\partial Q}{\partial x}\ dxdt$. But since the lateral inflow channel is inclined at an angle θ, the lateral inflow is $\frac{q}{L}\sin\theta\ dxdt$. The increment of the fluid in this cell is $\frac{\partial A}{\partial t}\ dxdt$. Considering the density of the fluid is a constant and in line with the conservation law of the fluid,

$$\frac{\partial Q}{\partial x}dx\ dt + \frac{\partial A}{\partial t}dx\ dt = \frac{q}{L}\sin\theta\ \ dxdt \tag{3.24}$$

Diving each term of equation (3.24) by$dxdt$ yields

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = \frac{q}{L}\sin\theta \tag{3.25}$$

Substituting equation (3.9) into equation (3.25) and differentiating partially with respect to x yields

$$V\frac{\partial A}{\partial x} + A\frac{\partial V}{\partial x} + \frac{\partial A}{\partial t} = \frac{q}{L}\sin\theta \tag{3.26}$$

The flow area can be assumed to be a known function of the depth y and length x. Therefore, the derivatives of A can be expressed in terms of y as

$$\frac{\partial A}{\partial x} = \frac{dA}{dy}\frac{\partial y}{\partial x} = T\frac{\partial y}{\partial x} \tag{3.27}$$

$$\frac{\partial A}{\partial t} = \frac{dA}{dy}\frac{\partial y}{\partial t} = T\frac{\partial y}{\partial t} \tag{3.28}$$

Here T is the channel top width. Franz (1982) assumed that T is determined by

$$T = \frac{dA}{dy} \tag{3.29}$$

Substituting equation (3.27) and (3.28) into equation (3.26) yields

$$VT\frac{\partial y}{\partial x} + A\frac{\partial V}{\partial x} + T\frac{\partial y}{\partial t} = \frac{q}{L}\sin\theta \tag{3.30}$$

Dividing equation (3.30) throughout by T and rearranging yields

$$\frac{\partial y}{\partial t} + V\frac{\partial y}{\partial x} + \frac{A}{T}\frac{\partial V}{\partial x} = \frac{q}{T\,L}\sin\theta \tag{3.31}$$

The term q can be defined by

$$q = ua \tag{3.32}$$

Furthermore, the area of the lateral inflow channel is given by,

$$a = b1.y1 \tag{3.33}$$

Equation (3.31) is the general equation of continuity for open channel flow with lateral inflow channel at an angle.

3.4.2 Momentum equation

Since the equation of energy does not account for the dissipation of energy due to the turbulence of the fluid that is generated by the mixing of the open rectangular channel and the lateral inflow channel, the momentum equation becomes an appropriate equation for lateral inflow problems. The momentum equation describes the motion of fluid particles. This equation is derived from the Newton's second law of motion, together with the assumption that fluid stress is the sum of diffusing viscous term, plus a pressure term. This equation relates the sum of forces acting on an element of fluid to its acceleration or rate of change of momentum.

The law of conservation of momentum requires that the time rate of change of the momentum accumulated within the element is equal to the sum of the net rate of momentum transfer into the element and sum of the external forces in the flow direction. From Figure 1, in a time interval of dt, the net momentum for the cell dx is $\frac{\partial(QV)}{\partial x} dxdt$. The lateral inflow component of velocity in the flow direction is $u\cos\theta$. Thus, lateral inflow momentum into cell dx at a time interval dt becomes $\frac{q}{L}\sin\theta\ u\cos\theta\ dxdt$. The fluid pressure and fluid weight in the direction of flow are $g\frac{\partial(yA)}{\partial x}dx\,dt$ and

$gA(S_f - S_0)\, dxdt$ respectively. The increment in the momentum for the cell dx is $\frac{\partial Q}{\partial t}\, dxdt$.Therefore, according to the law of conservation of momentum equation,

$$\frac{\partial Q}{\partial t} dx\, dt + \frac{\partial (QV)}{\partial x}\, dx\, dt + g\frac{\partial (yA)}{\partial x} dxdt + gA(S_f - S_0)dxdt$$
$$= \frac{q}{L}\sin\theta\;\, u\cos\theta\;\, dx\, dt \qquad (3.34)$$

Dividing each term of equation (3.34) by $dxdt$simplifies to

$$\frac{\partial Q}{\partial t} + \frac{\partial (QV)}{\partial x} + g\frac{\partial (yA)}{\partial x} + gA(S_f - S_0) = \frac{q}{L}\sin\theta\;\, u\cos\theta \qquad (3.35)$$

Substituting equation (3.9) into equation (3.35) and differentiating partially with respect to *x* considering the area A is a constant yields

$$A\frac{\partial V}{\partial t} + V\frac{\partial A}{\partial t} + Q\frac{\partial V}{\partial x} + V\frac{\partial Q}{\partial x} + gA\frac{\partial y}{\partial x} + gA(S_f - S_0) = \frac{q}{L}\sin\theta\;\, u\cos\theta \qquad (3.36)$$

Rearranging equation (3.36) yields

$$V\left(\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x}\right) + A\frac{\partial V}{\partial t} + Q\frac{\partial V}{\partial x} + gA\frac{\partial y}{\partial x} + gA(S_f - S_0) = \frac{q}{L}\sin\theta\;\, u\cos\theta \qquad (3.37)$$

Substituting equation (3.25) into equation (3.37) yields

$$V\left(\frac{q}{L}\sin\theta\right) + A\frac{\partial V}{\partial t} + Q\frac{\partial V}{\partial x} + gA\frac{\partial y}{\partial x} + gA(S_f - S_0) = \frac{q}{L}\sin\theta\;\, u\cos\theta \qquad (3.38)$$

Substituting$Q = AV$ into equation (3.38) and dividing throughout by A yields

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial y}{\partial x} + g(S_f - S_0) + \frac{V}{A}\left(\frac{q}{L}\sin\theta\right) = \frac{q}{A\,L}\sin\theta\;\, u\cos\theta \qquad (3.39)$$

Rearranging the equation (3.39) yields

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial y}{\partial x} + g\left(S_f - S_0\right) = \frac{q}{A\,L}\sin\theta\ \left(u\cos\theta - V\right) \tag{3.40}$$

Equation (3.40) is the general momentum equation of an open channel with lateral inflow channel at varying angles.

3.5 Method of solution

The equations governing the flow considered in the problem are non-linear. The non-linearity is due to the term $V\frac{\partial V}{\partial x}$ in the momentum equation. These equations are non-linear first order partial differential equations. It is not possible to solve these equations analytically; thus, the finite difference method is used to obtain approximate solutions. In this method, the partial differential equations are estimated from a set of linear equations linking the values of the functions at each mesh point. Finally, these sets of algebraic equations are solved using Matlab code.

Accuracy is a measure of how well the discrete solution represents the exact solution of the problem. A technique is accurate if the truncation error is negligible. A technique is consistent if the truncation error decreases as the step size is reduced. A technique is stable if the errors in the solution will remain bounded. Finally, a numerical technique is convergent if the solution approaches the exact solution as the grid spacing is reduced to zero. All these four factors have been proven in the finite difference method, Nicholas (1996). Hence, that is why it is used in this thesis to solve the non-linear Saint Venant equations.

3.5.1 Finite difference method

The finite difference approximations of these partial differential equations are obtained from Taylor’s series expansion of the independent variables. This formula can be used as an approximation to the derivative of u at x taking Δx is very small. From Taylors series expansion

$$u(x + \Delta x) = u(x) + \Delta x \, u_x(x) + \frac{(\Delta x)^2}{2} u_{xx}(x) + \cdots \tag{3.41}$$

On dividing each term by Δx and rearranging

$$\frac{u(x + \Delta x) - u(x)}{\Delta x} = u_x(x) + \frac{\Delta x}{2} u_{xx}(x) + \cdots \tag{3.42}$$

Solving for u_x yields

$$u_x = \frac{u(x + \Delta x) - u(x)}{\Delta x} - \frac{R(x)}{\Delta x} \tag{3.43}$$

If the step Δx is considered to be very small, then the square, the cube and higher powers of this step will be very small and hence the product of the step size and their derivatives will be negligible. The term $R(x)$is the reminder term. From this fact, the approximation of u_x becomes

$$u_x \approx \frac{u(x + \Delta x) - u(x)}{\Delta x} \tag{3.44}$$

This equation is a first order finite approximation and it is helpful in solving the non-linear Saint Venant equations. The above analysis deals with continuous solution, however the objective is to calculate u at a set of discrete points on the mesh. The numerical solution of equations (3.31) and (3.40) will be approximated from a rectangular grid approximated at a discrete number of points. This rectangular grid is obtained by dividing the (x, t) plane into a network of rectangles of sides Δx and $\Delta t$by drawing the set of lines where h and k are the equal spacing in the x an t axis respectively.

$x = i\Delta x = ih, \quad i = 0,1,2,\ldots \qquad (3.45)$

$t = j\Delta t = jk, \quad j = 0,1,2,\ldots \qquad (3.46)$

The nodes or mesh points of the network occur at the intersections of the straight lines drawn parallel to the *x* and *t* axes. The index *i* refer to spatial points, whereas the index j refers to time. The lines parallel to the x axis represent time while those drawn parallel to they axis represent locations along the channel. The location lines are drawn with spacing Δx while the time lines are drawn with spacing Δt. Two indices identify each node in the network. The first designates spatial point (location) of the node in the time while the second designates the time.

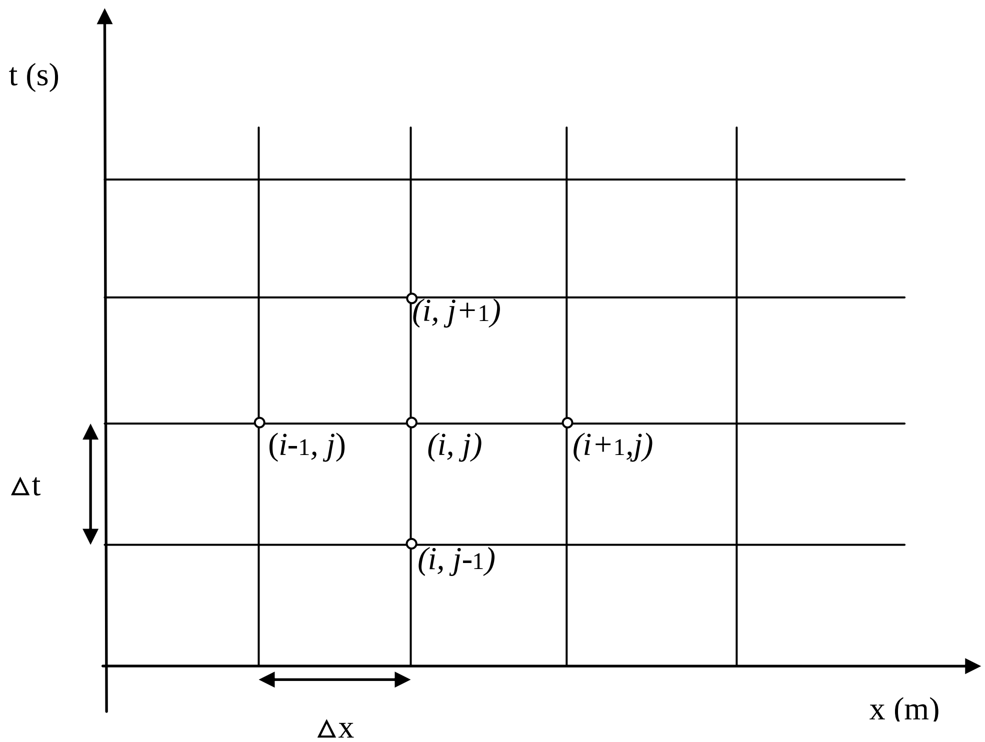

Figure 2: Finite difference mesh.

Letting $u_{i,j}$ be the numerical approximation for $u(x_i\ ,t_j)$,then the first order forward difference approximation of the derivatives of V and y with respect to time t respectively are given by,

$$\frac{\partial V}{\partial t} = \frac{V_{i,j+1} - V_{i,j}}{k} + 0(k) \tag{3.47}$$

$$\frac{\partial y}{\partial t} = \frac{y_{i,j+1} - y_{i,j}}{k} + \ 0(k) \tag{3.48}$$

Similarly the first order finite difference derivatives of V and y with respect to x respectively are given by,

$$\frac{\partial V}{\partial x} = \frac{V_{i+1,j} - V_{i,j}}{h} + 0(h) \tag{3.49}$$

$$\frac{\partial y}{\partial x} = \frac{y_{i+1,j} - y_{i,j}}{h} + 0(h) \tag{3.50}$$

Now the governing equations (3.31) and (3.40) are replaced by the finite difference analogies of the partial differential equations.

3.6 Governing equations in finite difference form

The equations (3.31) and (3.40) are coupled and non-linear hence cannot be solved analytically. That is why in this research, the finite difference method is used to solve them subject to the initial conditions

$$V(x,0) = \ 0, \quad y(x,0) = 0 \qquad for\ all\ x > 0 \tag{3.51}$$

and boundary conditions

$$V(0,t) = \ V_0, \quad y(0,t) = y_0 \qquad for\ all\ t > 0 \tag{3.52}$$

$$V(x_N, t) = V_0, \quad y(x_N, t) = y_0 \qquad for\ all\ t > 0 \tag{3.53}$$

The point x_N refers to the exit point of the section of the open rectangular channel. Thetotal length of the channel was considered to be 10m. Due to the numerical unstable solutions of the finite implicit difference method, Viessman *et al* (1972) noted that more stable solutions could be obtained by diffusing the finite difference approximations. Neglecting higher powers the derivative become,

$$\frac{\partial V}{\partial t} = \frac{V(i, j+1) - 0.5\big(V(i-1, j) + V(i+1, j)\big)}{\triangle t} \tag{3.54}$$

$$\frac{\partial y}{\partial t} = \frac{y(i, j+1) - 0.5\big(y(i-1, j) + y(i+1, j)\big)}{\triangle t} \tag{3.55}$$

$$S_f = \frac{S_f(i-1, j) + S_f(i+1, j)}{2} \tag{3.56}$$

$$\frac{\partial V}{\partial x} = \frac{V(i+1, j) - V(i-1, j)}{2 \triangle x} \tag{3.57}$$

$$\frac{\partial y}{\partial x} = \frac{y(i+1, j) - y(i-1, j)}{2 \triangle x} \tag{3.58}$$

Now, converting equation (3.31) into finite difference form yields

$$\frac{y(i, j+1) - 0.5\big(y(i-1, j) + y(i+1, j)\big)}{\triangle t} + V(i, j)\frac{y(i+1, j) - y(i-1, j)}{2 \triangle x} + \frac{A}{T}\frac{V(i+1, j) - V(i-1, j)}{2 \triangle x} = \frac{q}{T\,L}\sin\theta \tag{3.59}$$

$$y(i, j+1) = 0.5\big(y(i-1, j) + y(i+1, j)\big) - \triangle t\left\{V(i, j)\frac{y(i+1, j) - y(i-1, j)}{2 \triangle x} + \frac{A}{T}\frac{V(i+1, j) - V(i-1, j)}{2 \triangle x} - \frac{q}{T\,L}\sin\theta\right\} \tag{3.60}$$

Equation (3.60) is the finite difference form of the continuity equation for an open channel with lateral inflow channel at an angle. Converting the momentum equation (3.40) into the finite difference form yields

$$\frac{V(i,j+1) - 0.5\big(V(i-1,j) + V(i+1,j)\big)}{\Delta t} + V(i,j)\frac{V(i+1,j) - V(i-1,j)}{2\Delta x}$$

$$+g\frac{y(i+1,j) - y(i-1,j)}{2\Delta x} + g\left(\frac{S_f(i-1,j) + S_f(i+1,j)}{2} - S_0\right)$$

$$= \frac{q}{A\,L}\sin\theta\ \left(u\cos\theta - V(i,j)\right) \tag{3.61}$$

The friction slope S_f in unsteady flow can be estimated by either using the Chezy or the Manning resistance equations.From the Manning equation,

$$V = \frac{1}{n}R^{\frac{2}{3}}S_f^{\frac{1}{2}} \tag{3.62}$$

Now letting S_f to be our friction slope and making it the subject of the formula

$$S_f^{\frac{1}{2}} = \frac{V\,n}{R^{\frac{2}{3}}} \tag{3.63}$$

Now squaring both sides yields

$$S_f = \frac{V^2 n^2}{R^{\frac{4}{3}}} \tag{3.64}$$

Substituting equation (3.64) into equation (3.61) yields

$$V(i,j+1) = 0.5\big(V(i-1,j) + V(i+1,j)\big) - \vartriangle t\Bigg\{V(i,j)\frac{V(i+1,j)-V(i-1,j)}{2 \vartriangle x}$$

$$+g\frac{y(i+1,j)-y(i-1,j)}{2 \vartriangle x} + g\left[\frac{n^2}{2R^{\frac{4}{3}}}\big(V^2(i-1,j)+V^2(i+1,j)\big) - S_0\right]$$

$$-\frac{q}{A\,L}\sin\theta\ \ (u\cos\theta - V(i,j))\Bigg\} \qquad (3.65)$$

Equations (3.60) and (3.65) are the continuity and the momentum equations respectively in finite difference form. The terms $y(i,j+1)$and $V(i,j+1)$in equations (3.60) and (3.65) respectively, are computed subject to the initial and boundary conditions below.

Initial conditions of equation (3.60) and (3.65) are,

$$V(i,0) = 0 \qquad y(i,0) = 0 \qquad (3.66)$$

The boundary conditions of equation (3.60) and (3.65) are,

$$V(0,j) = 10 \qquad y(0,j) = 10 \qquad (3.67)$$

$$V(N,j) = 10 \qquad y(N,j) = 10 \qquad (3.68)$$

Now taking the velocity V_o =10 m/s and depth of the channel to be y_o = 0.5 m, the initial and boundary conditions in finite difference form become where the index i stands for x which is the distance along the channel while j stands for time.

The two equations (3.60) and (3.65) are solved using very small values of $\vartriangle t$. In this research$\vartriangle x = 0.1$ and $\vartriangle t = 0.0001$.According to Crank (1975), the finite difference method is known to be convergent and numerically stable whenever $\frac{\vartriangle t}{(\vartriangle x)^2} < \frac{1}{2}$ .The number of sub-divisions along the channel was taken to be 100 while along the time was taken to be 10000 sub-divisions.

CHAPTER FOUR

RESULTS AND DISCUSSION

The Matlab code in Appendix 1 is used to simulate the equations (3.60) and (3.65) which appear in Appendix 1 together with their initial and boundary condition. This was done by varying i and j at various nodal points. Then various graphs were plotted using the values of the velocity V against the depth y at a certain location along the channel. Various flow parameters of the angle, cross-sectional area, velocity, length of the lateral inflow channel and width, slope, roughness coefficient and energy coefficient were varied to investigate how they affect the flow velocity in the main flow channel.

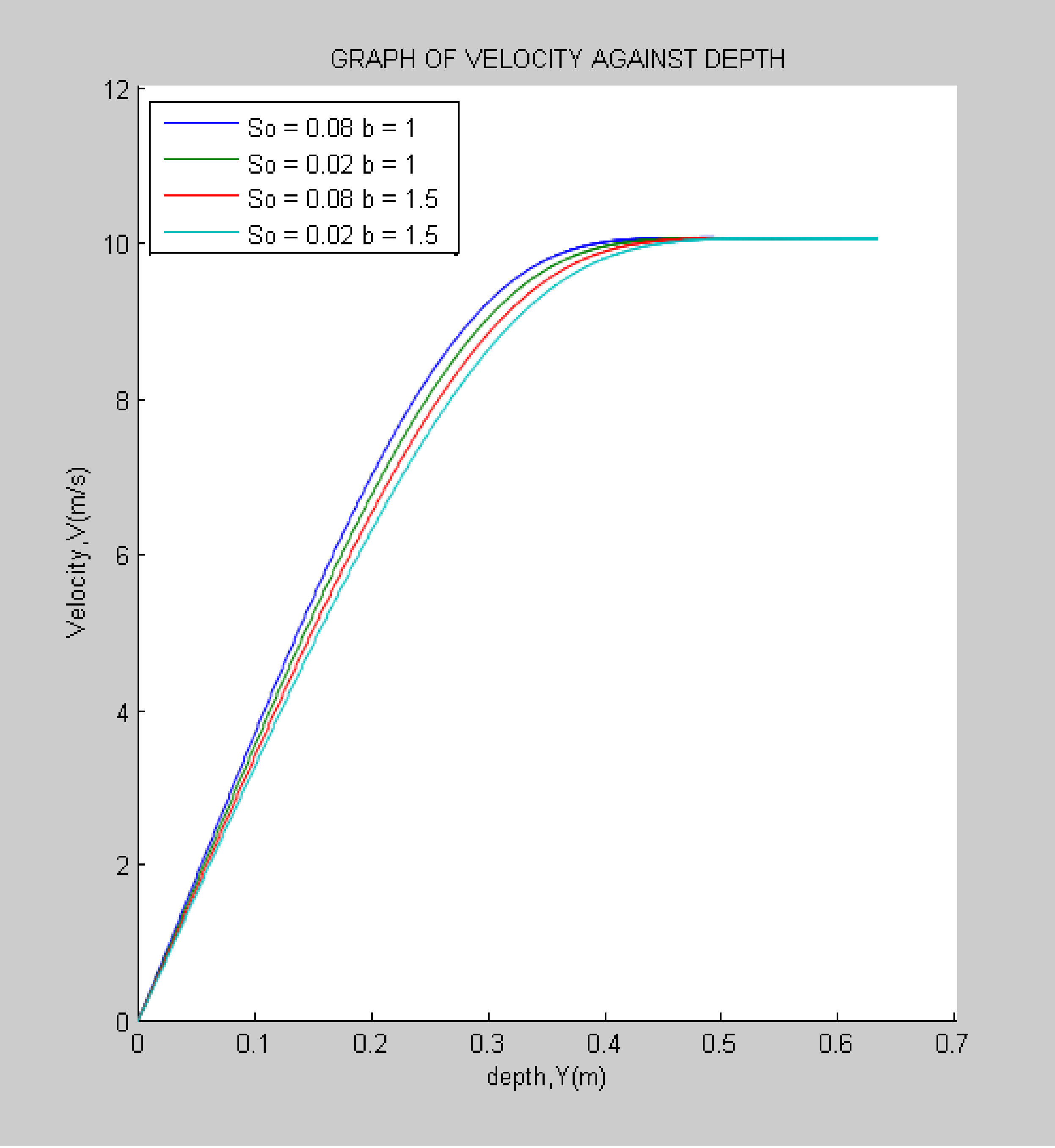

Figure 3: Velocity profiles versus depth at angle zero for varying width and slope of the channel.

From Figure 3, the velocity increases with increase in depth from the bottom of the channel to the free surface. At a depth of 0.5m that where the free surface occurs and the velocity at this depth is 10m/s. It is noted that maximum velocity occurs just below the free surface at a depth of 0.4m.The fluid flow velocity at the channel bottom is zero due to the non-slip condition of fluids. The no-slip condition states that a fluid in contact with a surface will achieve the same velocity at the surface. Since at the channel bottom the surface is not moving, the flow velocity at this section of the channel will be zero. However, as you move vertically upwards, the velocity increases since the frictional forces decrease and velocity becomes maximum slightly below the free surface. At the free surface the velocity is not maximum due to effects of surface tension.

An increase in the width of the main flow channel from 1m to 1.5m leads to a decrease in the velocity measured from the channel bottom to the free surface. However, the velocity becomes steady at a depth of 0.45m just below the free surface. This is because an increase in the width results in the increase in the wetted perimeter of the fluid under the conduit. This increase in the wetted perimeter will lead to an increase in the shear stress in the channel bottom, which will result in a reduction in the flow velocity. It is noted that an increase in the slope will lead to an increase in the fluid flow velocity. Thus, an increase in the slope from 0.002 to 0.08 will lead to the increase in the flow velocity. This can be seen in the Manning formula where an increase in the slope results in an increase in the flow velocity since the two are directly proportional to each other.

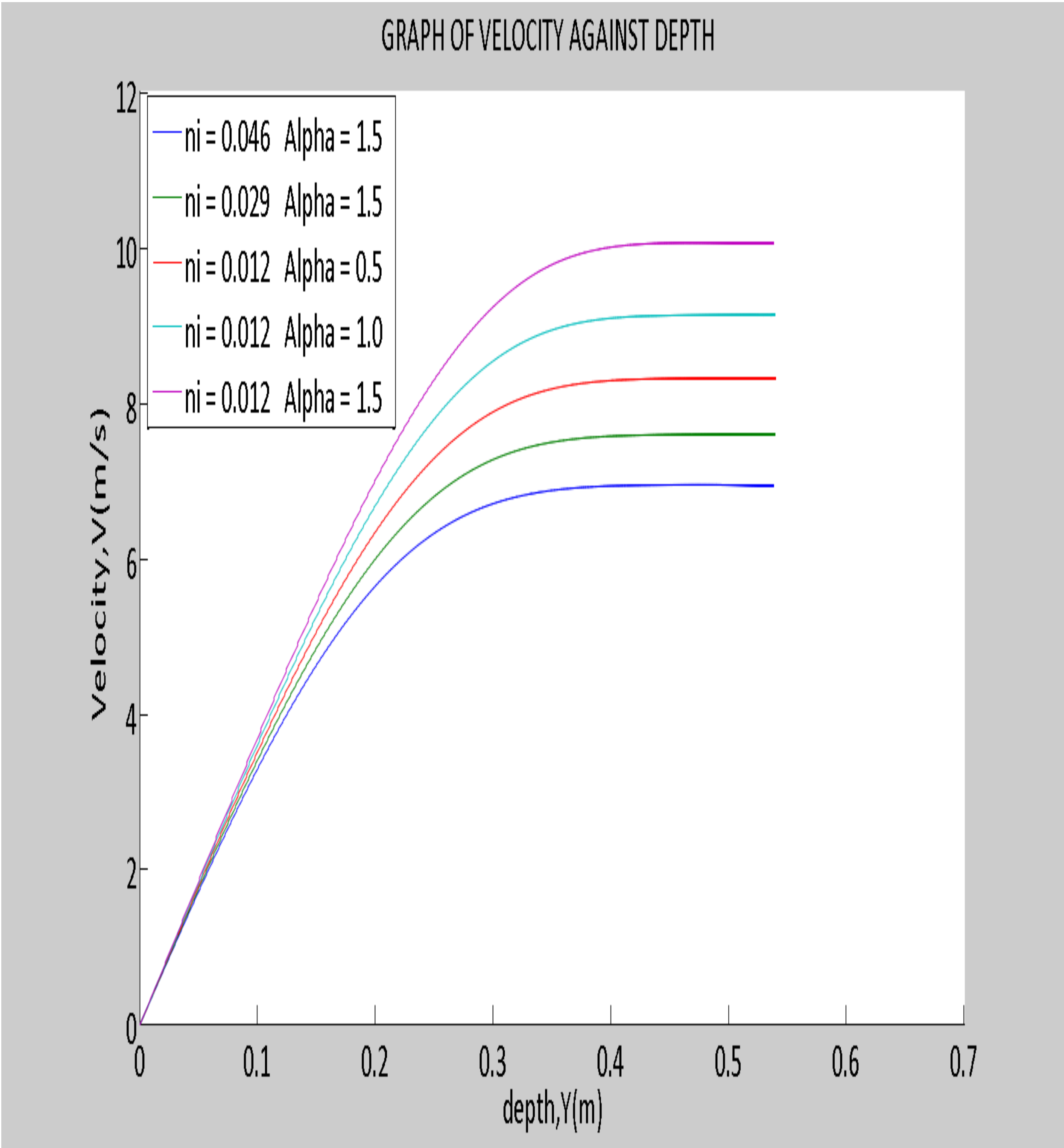

Figure 4: Velocity profiles versus depth at angle zero for varying roughness and energy coefficients.

From Figure 4, a decrease in the roughness coefficient from 0.029 to 0.012 will lead to an increase in the flow velocity. This is because an increase in the roughness coefficient leads to increase in the shear stresses at the sides of the channel which leads to a reduction in flow velocity. The speed of the particles which neighbor these slow moving molecules will be reduced which will result to the overall flow velocity to reduce. This will result in the overall velocity of the fluid being reduced. It is observed that an increase in the energy coefficient, Alpha, from 1 to 1.5 will lead to an increase in the flow velocity of the fluid. An increase in the energy coefficient will lead to an increase in the fluid particle's energy. This increase in the fluid particle's energy results to the particles attaining more random motion which causes constant bombardments with other fluid particles which leads to an increase in the flow velocity of the particles. This increase in the velocity of the particles will lead to an increase in the velocity of the fluid.

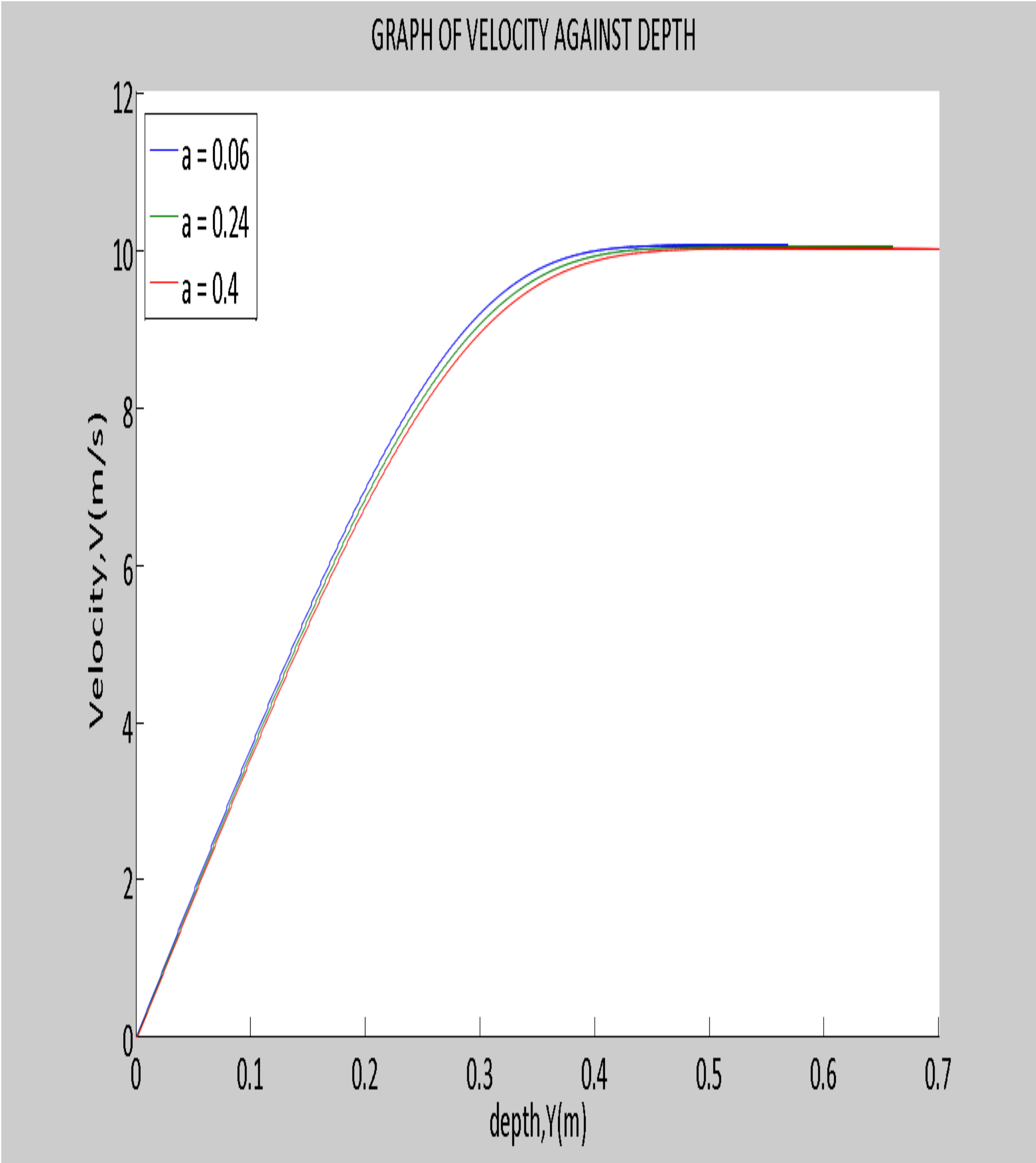

Figure 5: Velocity profiles versus depth at varying cross-sectional area of the lateral inflow channel at angle of 40^{o}.

From Figure 5, an increase in the cross-sectional area from 0.06 m^2 to 0.4 m^2 of the lateral inflow channel leads to a decrease in the flow velocity of the main channel. An increase in the area will lead to an increase in the wetted perimeter of the lateral inflow channel. A large wetted perimeter leads to large shear stress at the sides of the channel which results to the flow velocity being reduced.

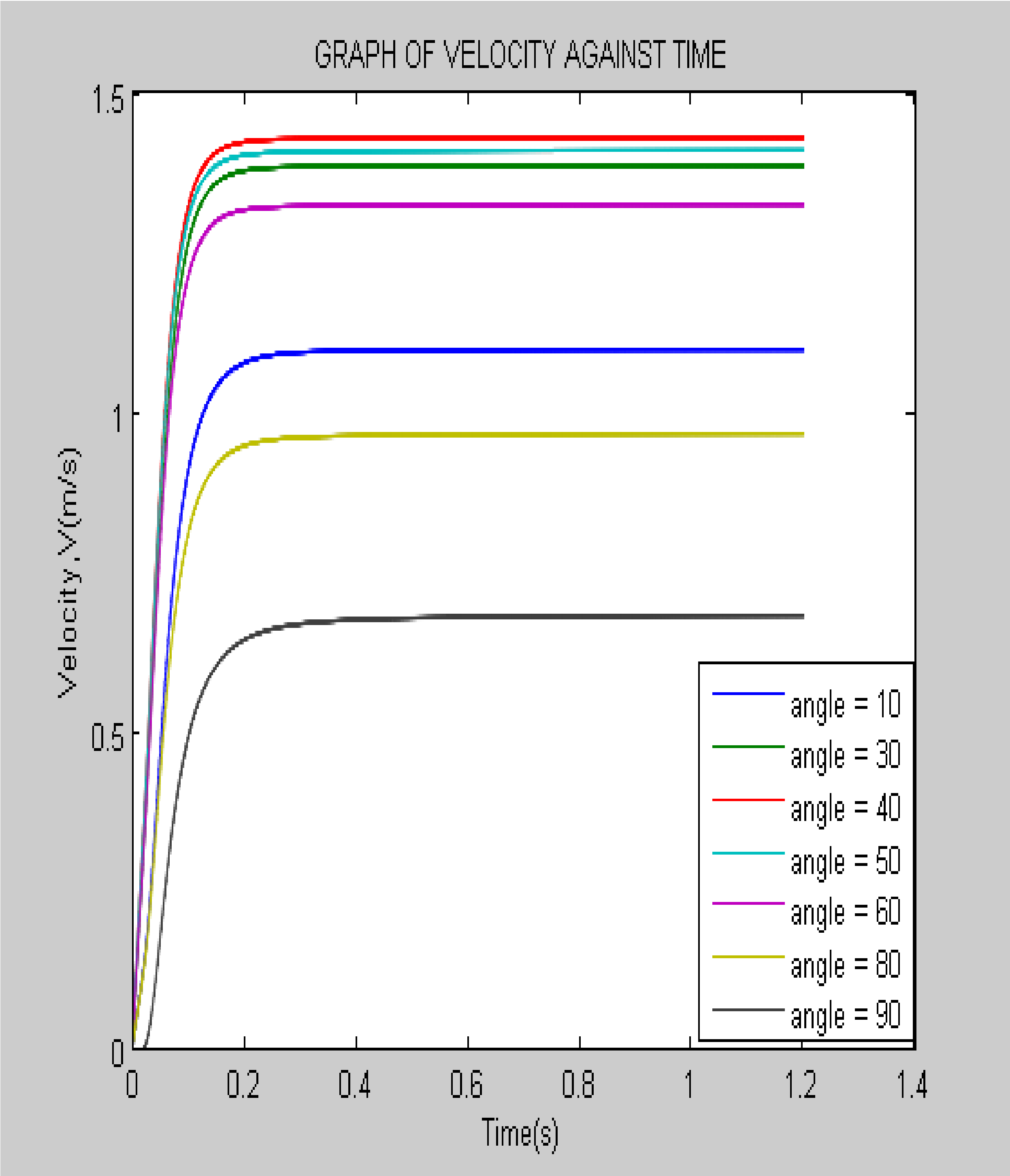

Figure 6: Velocity profiles versus time along the channel for varying angle of lateral inflow channel**.**

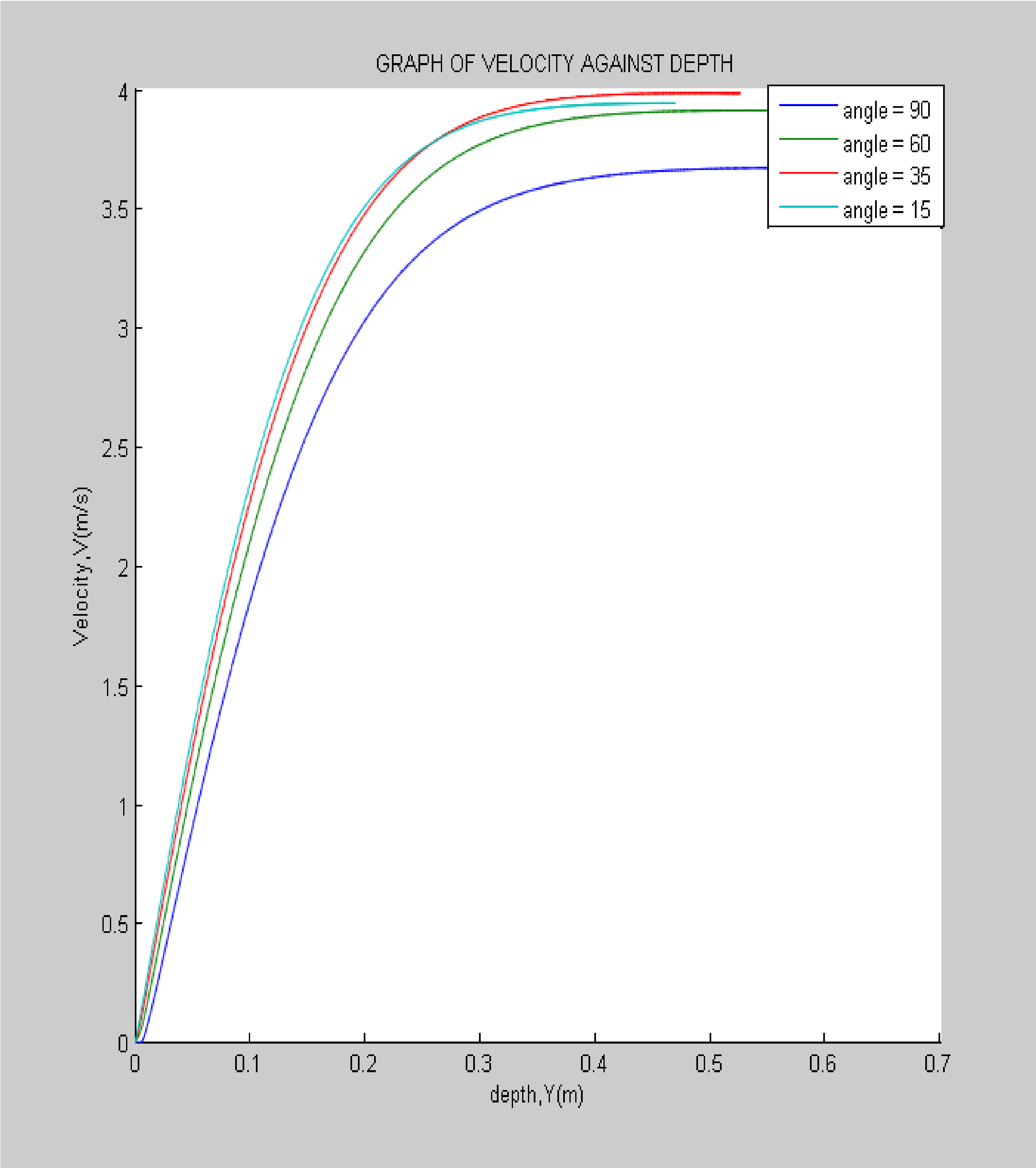

Figure 7: Velocity profiles versus depth for varying angle of the lateral inflow channel.

From Figure 6, an increase in the angle of the lateral inflow channel does not necessarily mean an increase in the fluid flow velocity of the open rectangular channel. It is observed that an angle of 40^o has a higher velocity value than an angle of 90^o.Angles of 10^o, 60^o, 80^o and 90^o have lower values of velocity compared to 30^o, 40^o and 50^o angles. Moreover, Figure 7shows that at 35^o of the lateral inflow channel has higher value of velocity compared to other angles. Therefore, to get maximum discharge from a lateral inflow channel, one has to construct it with an angle ranging from 30^o to 50^o. The reason why the flow velocity at an angle of 90^o of the lateral inflow channel is lower is because the velocity in this lateral inflow channel is maximized resulting to turbulence at the intersection of the two channels. This turbulence results in a reduction in the flow velocity in the main channel.

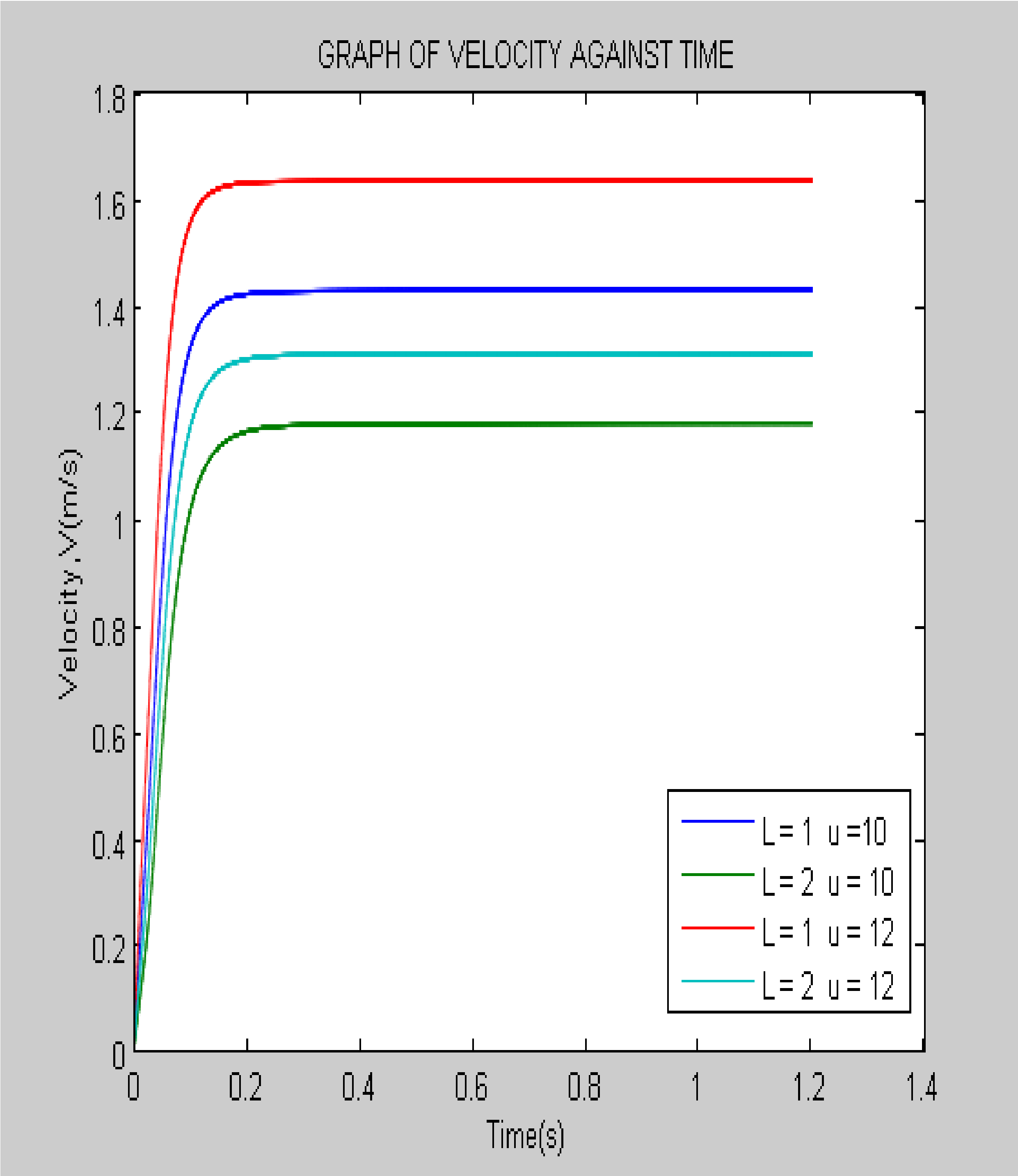

Figure 8: Velocity profiles versus time along the channel for varying length and velocity of the lateral inflow channel at angle 40^{o}.

From Figure 8, an increase in the velocity of the lateral inflow channel from 10m/s to 12 m/s will lead to an increase in the flow velocity of the main open channel. However, an increase in the length of the lateral inflow channel from 1m to 2m leads to a reduction in the flow velocity in the main open channel. An increase in the flow velocity of the lateral inflow channel means that more fluid particles at a given time will collide with the fluid particles in the main open channel resulting in more random motion of the particles. An increase in the length of the lateral inflow channel will lead to a decrease in the velocity. This is due to the increase in the shear stress on the walls and the channel bottom.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

The objectives of modeling fluid flow in an open channel with a lateral inflow channel have been achieved. The results also compare well with earlier research done when various similar flow parameters were varied. Here are the conclusion and recommendations on investigations of the effects of varying angle, the cross-sectional area, velocity and length of the lateral inflow channel on velocity in the main flow channel.

4.1 Conclusion

i. Increasing the angle of the lateral inflow channel does not necessarily mean an increase in the velocity in the main open channel. That angles of between 30^o and 50^0exhibits higher values of velocity in the main open channel than other angles.

ii. Increasing the cross-sectional area of the lateral inflow channel leads to decrease in the flow velocity in the main open channel.

iii. Increasing the velocity in the lateral inflow channel leads to an increase in the flow velocity in the main open channel.

iv. Increasing the length of the lateral inflow channel leads to a decrease in the velocity in the main open channel.

4.2 Recommendations

There is still a need to compare experimentally the theoretical results found in this research with laboratory results. Rectangular channels with lateral inflow channels can be developed in a laboratory and investigations on how the various flow parameters like length of the lateral inflow channel affects the discharge in the open channel. Finally, it is recommended that future research should be carried out on

i. The effect of lateral outflow channel on discharge.
ii. The effect of two or more lateral inflow channels at various locations on discharge in the main channel.
iii. Modeling fluid flow in a trapezoidal, triangular or circular open channel with lateral inflow or outflow.
iv. Studying the problem and solving using a different numerical technique like perturbation or finite element method.

4.3 Research paper published

Macharia Karimi, David Theuri and Mathew Kinyanjui. Modelling Fluid Flow in an Open Rectangular Channel with Lateral Inflow Channel. International Journal of Sciences: Basic and Applied Research (IJSBAR), Vol 17, No 1 (2014), 186-193.

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APPENDIX 1

Matlab code

The Matlab code that is used to generate the various graphs,

```
function lateralintakechannel()

%generate array x value

N=100;

x1=0;

xN=10;

dx =(xN-x1)/N;

x = zeros(1,N);

%generate array t values

K=10000;

t1=0;

tK=1;

dt=(tK-t1)/K;

t=zeros(1,K);

Y= zeros(N,K);

V= zeros(N,K);

%constants

g=9.81; So=0.002; yi=0.5; ni=0.012; b=1; Alpha=1;

b1=0.6; y2=0.3; u=10; L=1;

a=b1*y2;

A=b*yi;

P=b+(2*yi);
```

```
%variable angle

angle=40;

theta= (angle*pi/180);


for k=1:K+1

        V(1,k)=20;  Y(1,k)=0.5;            %ENTRY

        V(N,k)=20;  Y(N,k)=0.5;                        %EXIT

        t(k)=(k-1)*dt;

 end

for i=1:N+1

        V(i,1)=0;  Y(i,1)=0;


        x(i)=(i-1)*dx;

end

for k=1:K

for i=2:N

Y(i,k+1)= 0.5*(Y(i-1,k)+Y(i+1,k))-dt*((A/b)*(V(i+1,k)-V(i-1,k))/(2*dx)-V(i,k)*(Y(i+1,k)-
Y(i-1,k))/(2*dx)-(a*u*sin(theta))/(b*L));


V(i,k+1)= 0.5*(V(i-1,k)+V(i+1,k))dt*(Alpha*V(i,k)*(V(i+1,k)-V(i-
1,k))/(2*dx)+g*(Y(i+1,k)-Y(i-1,k))/(2*dx)-g*(So-((0.5*(ni^2)/(A/P)^(4/3))*(V(i-
1,k)^2)+(V(i+1,k)^2))))+ dt*a*u*sin(theta)*((u*cos(theta))-(V(i,k)))/(L*A);

end

end
```

```
figure(3)
 hold all;
     plot(Y(7,:),V(7,:),'LineSmoothing','on');
     title('GRAPH OF VELOCITY AGAINST DEPTH');
     xlabel('depth,Y(m)');
     ylabel('Velocity,V(m/s)');
     [~,~,~,current_entries] =legend;
     legend([current_entries {sprintf([texlabel('b'),' = %g'],b)}]);
 figure(4)
 hold all;
     plot(Y(7,:),V(7,:),'LineSmoothing','on');
     title('GRAPH OF VELOCITY AGAINST DEPTH');
     xlabel('depth,Y(m)');
     ylabel('Velocity,V(m/s)');
     [~,~,~,current_entries] =legend;
     legend([current_entries {sprintf([texlabel('Alpha'),' = %g'],Alpha)}]);
 figure(5)
 hold all;
     plot(Y(7,:),V(7,:),'LineSmoothing','on');
     title('GRAPH OF VELOCITY AGAINST DEPTH');
     xlabel('depth,Y(m)');
     ylabel('Velocity,V(m/s)');
     [~,~,~,current_entries] =legend;
     legend([current_entries {sprintf([texlabel('a'),' = %g'],a)}]);
```

```
 figure(6)
 hold all
        plot(t,V(50,:),'LineSmoothing','on');
        title('GRAPH OF VELOCITY AGAINST TIME');
        xlabel('Time(s)');
        ylabel('Velocity,V(m/s)');
        [~,~,~,current_entries] =legend;
        legend([current_entries {sprintf([texlabel('angle'),' = %g'],angle)}]);
figure (7)
hold all
       plot(Y(7,:),V(7,:),'LineSmoothing','on');
       title('GRAPH OF VELOCITY AGAINST DEPTH');
       xlabel('depth,Y(m)');
       ylabel('Velocity,V(m/s)');
       [~,~,~,current_entries] =legend;
       legend([current_entries {sprintf([texlabel('angle'),' = %g'],angle)}]);
  figure(8)
        plot(t,V(50,:),'LineSmoothing','on');
        title('GRAPH OF VELOCITY AGAINST TIME');
        xlabel('Time(s)');
        ylabel('Velocity,V(m/s)');
        [~,~,~,current_entries] =legend;
        legend([current_entries {sprintf([texlabel('L'),' = %g'],L)}]);
end
```