

**MODELLING AND FORECASTING STOCK RETURNS
VOLATILITY ON UGANDA SECURITIES EXCHANGE USING
UNIVARIATE GARCH MODELS**

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Declaration

I, Jalira Namugaya (Reg. No: MF300-0005/12) hereby declare that this thesis is my original work and has not been presented for a degree in any other university.

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Dedication

To my Mum Namulesa Madinah, my Dad Ndhaye Abdu, my loving husband Habumugisha Isaac and my beloved Sons H. Abdulatif, H. AbdulWahab and H. AbdulHamid.

Acknowledgement

In the journey of an academician, there is need for commitment, hard work, endurance, focus, patience and teamwork in order to reach the destination, SUCCESS. These however need to be coupled with the works of our giants on which we base on to attain the ultimate goal. At this moment in time, I would like to humbly acknowledge the great works of my giants that have made me reach this point in my academic journey.

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Abstract

Stock returns volatility of daily closing prices of the Uganda Securities Exchange (USE) all share index over a period of 04/01/2005 to 18/12/2013 is Modelled. We employ different univariate Generalised Autoregressive Conditional Heteroscedastic(GARCH) models; both symmetric and asymmetric. The models include; GARCH(1,1), GARCH-M, EGARCH(1,1) and TGARCH(1,1). Quasi Maximum Likelihood(QML) method was used to estimate the models and then the best performing model obtained using two model selection criteria; Akaike Information criterion(AIC) and Bayesian Information criterion(BIC). Their forecasting abilities was determined using two loss functions; Mean square error (MSE) and Mean absolute error (MAE). The GARCH(1,1) model outperformed the other competing models in modelling volatility while EGARCH(1,1) performed best in forecasting volatility of USE returns.

Contents

Declaration	i
Dedication	ii
Acknowledgement	iii
Abstract	v
Table of Contents	vi
List of Figures	x
List of Tables	x
List of definitions	xii
1 INTRODUCTION	1
1.1 Background of the study	1
1.1.1 Overview of Uganda Securities Exchange (USE)	3
1.2 Statement of the Problem	4
1.3 Objectives of the study	5
1.3.1 General objective	5
1.3.2 Specific objectives	5

1.4	Research Questions	5
1.5	Justification of the study	6
1.6	Study scope	7
2	LITERATURE REVIEW	8
2.1	Introduction	8
2.2	Volatility Modelling and Forecasting	8
2.3	Studies carried out on African Markets	10
2.4	Research gap	12
2.5	Conclusion	12
3	UNIVARIATE GARCH MODELS	13
3.1	Introduction	13
3.2	ARCH(q) Model	13
3.3	GARCH(p, q) Model	14
3.4	Symmetric GARCH Models	15
	3.4.1 GARCH (1,1) Model	16
	3.4.2 GARCH-in-Mean (GARCH-M(1,1)) Model	18
3.5	Asymmetric GARCH models	19
	3.5.1 The Exponential GARCH (EGARCH(1,1)) Model	19
	3.5.2 The Threshold GARCH (TGARCH(1,1)) Model	20
3.6	Model Estimation	20

3.6.1	Estimation of the GARCH Models using QML method . . .	21
3.7	Model selection criteria	23
3.7.1	The Akaike Information Criterion (AIC)	24
3.7.2	The Bayesian Information Criterion (BIC)	24
3.8	Forecasting volatility	25
3.8.1	Forecasting ability of the Models	25
4	DATA ANALYSIS AND RESULTS	27
4.1	Introduction	27
4.2	Data	27
4.2.1	Asset returns	27
4.2.2	Basic statistics of USE returns series	28
4.2.3	Quantile-Quantile (Q-Q) and ACF plots	29
4.2.4	Testing for stationarity	31
4.2.5	Testing for Heteroscedasticity	32
4.3	Empirical results	33
4.3.1	Selection of GARCH(p, q) Model	33
4.3.2	Simulation	34
4.3.3	Model estimation	35
4.4	Forecasting performance of the Models	37
5	CONCLUSIONS AND RECOMMENDATIONS	38

List of Figures

4.1	USE Daily prices and returns distributions(Jan.2005-Dec.2013) . .	30
4.2	Q-Q plot and ACF of USE returns	31
4.3	QMLE parameter distribution	35

List of Tables

4.1	Descriptive statistics of USE returns Series	29
4.2	ADF unit root test for the USE All Share index and returns series	31
4.3	ARCH-LM Test for residuals of the USE returns Series	33
4.4	Comparison of different GARCH(p, q)Models	33
4.5	Simulation results of GARCH(1,1)process	34
4.6	Estimation results of different GARCH Models for USE	36
4.7	Forecast results of different GARCH Models for USE	37

Definitions

Definition 0.0.1 : *Time series (TS)*

A TS model for a single risk factor is a stochastic process $\{\varepsilon_t\}_{t \in \mathbb{Z}}$, ie a family of random variables indexed by the integers and defined on some probability space (Ω, \mathcal{F}, P) .

Definition 0.0.2 : *Strict stationary*

A TS $\{\varepsilon_t\}_{t \in \mathbb{Z}}$ is strictly stationary if $(\varepsilon_{t_1}, \dots, \varepsilon_{t_n}) \stackrel{d}{=} \varepsilon_{t_1+k}, \dots, \varepsilon_{t_n+k}, \forall t_1, \dots, t_n, k \in \mathbb{Z}$ and $\forall n \in \mathbb{N}$

Definition 0.0.3 : *Covariance stationary (CS)*

A TS $\{\varepsilon_t\}_{t \in \mathbb{Z}}$ is CS if the first two moments exist and satisfy;

$$\mu(t) = \mu, t \in \mathbb{Z},$$

$$\gamma(t, s) = \gamma(t + k, s + k), t, s, k \in \mathbb{Z}.$$

Definition 0.0.4 : *ACF*

ACF of a CS process $\{\varepsilon_t\}_{t \in \mathbb{Z}}$ is defined as, $\rho(h) = \rho(\varepsilon_h, \varepsilon_0) = \gamma(h)/\gamma(0), \forall h \in \mathbb{Z}$

Definition 0.0.5 : *White noise*

A TS $\{\varepsilon_t\}_{t \in \mathbb{Z}}$ is a white noise process if it is CS with ACF

$$\rho(h) = 1 \text{ if } h = 0 \text{ and } \rho(h) = 0 \text{ if } h \neq 0.$$

Definition 0.0.6 : *Martingale Difference*

$\{\varepsilon_t\}_{t \in \mathbb{Z}}$ is known as a martingale difference sequence with respect to the filtration

$\{\mathcal{F}_t\}_{t \in \mathbb{Z}}$ if $E|\varepsilon_t| < \infty, \varepsilon_t$ is \mathcal{F}_t and $E(\varepsilon_t | \mathcal{F}_{t-1}) = 0 \forall t \in \mathbb{Z}$.

Definition 0.0.7 : *Compact space*

A subset A of a topological space X, τ is said to be compact if every open covering

of A has a finite subcovering. If the compact subset equals X , then (X, τ) is said to be a compact space.

Definition 0.0.8 : *Sigma algebra.*

A collection of subsets of A is called a sigma algebra or Borel field, denoted by \mathcal{B} , if it satisfies the following three properties.

1. $\emptyset \in \mathcal{B}$
2. If $A \in \mathcal{B}$, then $A^c \in \mathcal{B}$
3. If $A_1, A_2, \dots \in \mathcal{B}$, then $\bigcup_{i=1}^{\infty} A_i \in \mathcal{B}$.

Definition 0.0.9 : *Jensen's inequality*

Let X be a random variable, g a convex function, and suppose that X and $g(X)$ are integrable. Then $g(EX) \leq Eg(X)$.

Definition 0.0.10 : *Strong law of large numbers*

Let $\{X_n, n = 1, 2, \dots\}$ be a sequence of i.i.d random variables with $E |X_n|^4 = M < \infty$. Then $\frac{S_n}{n} \xrightarrow{a.s.} \mu$

Chapter 1

INTRODUCTION

This chapter gives the background of the study, problem statement, research objectives and questions, justification and scope of the study undertaken.

1.1 Background of the study

The financial sector all over the world is working tirelessly to see to it that success is recorded in any financial activity engaged in. To this regard, one of the important aspects that need special attention by any investor or policy maker is the volatility of stock returns. Volatility can be defined as a statistical measure of the dispersion of returns for a given security or market index and it can either be measured using the standard deviation or variance between returns from that same security or market index, (Suliman and Winker, 2012).

Hongyu and Zhichao (2006), assert that modelling volatility in financial markets is important because it sheds further light on the data generating process of the returns. Volatility forecasting is an important area of research to financial markets and a lot of effort has been expended in improving volatility models since better forecasts translates into better pricing of options and better risk management.

There is observed considerable uncertainty and volatility both in the emerging and mature stock markets. Great concern is about the fluctuating returns of their investments due to the market risk and variation in the market price speculation as well as the unstable business performance, (Alexander, 1999). In the real world of financial markets, investors and financial analysts are generally more interested in the profit or loss of the stock over a period of time that is; the

increase or decrease in the price, than in the price self.

Trilochan and Luis (2010), clearly state that volatility has been used as a proxy for riskiness associated with the asset. The conditional variance of financial time series is important for pricing of derivatives, calculating measures of risk, and hedging. It is for this reason that enormous volatility models have been developed by many renown researchers and scholars since Engle's seminal paper of 1982.

ARCH model proposed by (Engle, 1982) and its extension; GARCH model by (Bollerslev, 1986), were the first models to be introduced into the literature. These were found to be useful and have become very popular in that they enable analysts to estimate the variance of a series at a particular point in time, (Enders, 2004). Since then, there have been a great number of empirical applications of modelling and forecasting the volatility of financial time series by employing these models and their many specifications, (Suliman and Winker, 2012).

The assumption that variance is constant through time is statistically inefficient and inconsistent, (Campbell et al., 1997). In real life, financial data for instance stock market returns data, variance changes with time (a phenomenon termed as heteroscedasticity), hence there is need for studying models which accommodate this possible variation in variance. Many studies have suggested that volatility of returns in stock markets world over can be modelled and forecasted using the GARCH type models.

Financial time series usually exhibit stylized characteristics. Firstly, it was observed by (Mandelbrot, 1993), that financial returns displayed volatility clustering meaning that large changes in the price of an asset are often followed by other large changes, and small changes are often followed by other small changes. Secondly, (Fama, 1965) demonstrated that financial data exhibit leptokurtosis meaning that the distribution of the returns is fat-tailed. Finally, (Black, 1976) introduced the leverage effect meaning that volatility is higher after negative shocks

than after positive shocks of the same magnitude. A good volatility model, then, must be able to capture and reflect these stylized facts,(Engle and Patton, 2001).

Frimpong and Oteng-Abayie (2006), assert that many stock markets over the last two decades have been established in Africa due to financial sector development and reforms in many Sub-Saharan Africa (SSA) countries aimed at shifting their financial systems from one of bank-based to security market-based. Most of the emerging stock markets in Africa are at the early stages of taking off striving to register success. Knowledge on volatility of stock returns is a crucial area of concern that needs special attention to compete favorably with developed stock markets.

1.1.1 Overview of Uganda Securities Exchange (USE)

USE was licensed to operate as an approved Stock Exchange in June 1997 by the CMA of Uganda with a mission of developing and managing the most efficient, transparent securities market that matches international standards, and promotes a partnership with the general public, foreign and institutional investors, employees, the government and other stake holders in the development of Ugandas capital markets industry.

The USE began formal trading operations in January 1998 following the listing of its maiden instrument, EADB Bond. Currently the products listed on the Exchange include bonds and 9 equities (3 of the equities are cross listings).

The current state of Uganda's financial markets can be described as an emerging market, which is at an early stage of development. It has 16 companies listed. The market generally consists of:- The regulator, CMA which is an autonomous body that was set up following the enactment of the CMA Statute 1996 to regulate, promote and develop capital markets in the country.

- (i) The market - USE, which is the only stock exchange in the country. It is a self regulated organisation meaning that it creates, amends and implements its own rules and regulations.
- (ii) The market Players who include broker/dealers (have a license to trade on the USE floor), investment advisors, collective investment schemes (who pool the funds of their clients for investment purposes), registrars and the investing public.

The exchange is open five days a week from Monday to Friday. Official working hours are 8:30a.m. to 5:30p.m. and stays closed on all national public holidays. Trading of equities is done five days a week (Monday to Friday) starting from 10.00 a.m. to 12.00 noon. Trading of fixed income securities is done anytime through licensed broker/dealers. However, the trades must be reported to the exchange during work hours.

USE, NSE and DSE have established a working relationship among them in the spirit of integrating and developing capital markets in the EAC. The exchanges operate under the umbrella of the EASEA which is a member of the Capital Markets Development Committee of the EAC, <http://www.use.or.ug/>

1.2 Statement of the Problem

As far as stock markets are concerned, volatility of returns is one of the important aspects. There is, however limited literature on modeling and forecasting volatility on stock markets in Africa and to the best of my knowledge, there is no such empirical studies on the USE in particular. There is therefore need for the financial specialists, economic practitioners and policy makers to be aware of the important aspects that need to be addressed for efficiency and success. Thus, one of the contributions of this study is to provide empirical evidence on the fit

of volatility models for the USE.

1.3 Objectives of the study

These include the general objective and specific objectives of the study to be undertaken.

1.3.1 General objective

To model and forecast volatility of stock returns for USE using univariate GARCH models.

1.3.2 Specific objectives

- (i) To formulate the parametric estimators for the GARCH(p, q) model and obtain their consistency and asymptotic normality properties.
- (ii) To fit the univariate GARCH models to the USE data and measure volatility.
- (iii) To establish which model is best at forecasting volatility of stock returns for USE.

1.4 Research Questions

- (i) Are estimators consistent and asymptotically normal?
- (ii) Which GARCH model is best in modelling volatility of market returns?
- (iii) Which GARCH model is effective in forecasting volatility of market returns?

1.5 Justification of the study

Stock market volatility is one of the most important aspects of financial market developments, providing an important input for option pricing and market regulation, (Poon and Granger.C., 2003). As a matter of fact, there is a relationship between volatility and risk, in a sense that having knowledge on volatility directly means you have knowledge on risk. This study will thus add to the literature in three ways. Firstly, the study will examine whether the estimators of the GARCH model are consistent and asymptotically normal. Secondly, it empirically investigates and tests whether the USE time series exhibit the stylized characteristics. Finally, this work attempts to determine the modelling and forecasting abilities of these GARCH-type models. The results of the study will be of great significance to academicians and researchers, policy makers and investors as follows:

- (i) To Academicians and Researchers;

This research study is expected to add to existing knowledge and basis for further studies in the same area or other related fields of study.

- (ii) To policy makers;

The study is intended to contribute to policy making through developing models which can be used to forecast volatility thereby guiding policy makers in formulating policies.

- (iii) To traders or investors;

This study is intended to inform investors about which decision is to make on a particular asset; for instance in portfolio selection since volatility has been used as a proxy for riskiness associated with the asset and sheds further light on the data generating process of the returns.

1.6 Study scope

The study was carried out in Kampala, the capital city of Uganda where the USE offices are located. Under this study, different univariate GARCH models were used. Two symmetric and two asymmetric models; GARCH(1,1), GARCH-M(1,1) models and EGARCH(1,1), TGARCH(1,1) models respectively were used to capture the main characteristics of financial time series such as volatility clustering, fat-tails, and the leverage effect.

Chapter 2

LITERATURE REVIEW

2.1 Introduction

This chapter presents the rationale of conducting research on modelling and forecasting volatility of stock returns. It describes the different models that have been used to model and forecast volatility by different authors both theoretically and empirically, and identifying the gap to be alleviated by the study.

2.2 Volatility Modelling and Forecasting

Engle (1982), studied on ARCH and GARCH models, and revealed that, these models were designed to deal with the assumption of non-stationarity found in real life financial data. He further pointed out that these models have become widespread tools for dealing with time series heteroscedasticity. The ARCH and GARCH models treat heteroscedasticity as a variance to be modelled. The goal of such models is to provide a volatility measure like a standard deviation that can be used in financial decisions concerning risk analysis, portfolio selection and derivative pricing.

Parvaresh and Bavaghar (2012), studied on forecasting volatility in Tehran Stock Market with GARCH Models for a period 9/28/1997 to 8/20/2010. The performance of GARCH, TARARCH, EGARCH and component ARCH (CARARCH) fitted to daily Tehran Stock Market was compared. They found that the estimation and test results for models suggest that the leverage effect term, was significant in EGARCH model(even with a one-sided test) indicating that asymmetric effect in

Tehran stock market was present. In addition evaluation forecasting with Mean Square Error criteria indicated that GARCH models had the same forecasting power, but when Log-Likelihood is evaluation criteria, CGARCH had the best forecasting power.

Brook and Burke (2003), studied stochastic volatility models and found that most time series models such as GARCH, will have forecasts that tend towards the unconditional variance of the series as the prediction horizon increases. This is a good property for a volatility forecasting model to have, since it is well known that volatility series are mean-reverting. This implies that if they are at a low level relative to their historic average they will have a tendency to rise back towards the average. This feature is accounted for in GARCH volatility forecasting models.

Banerjee and Sarkar (2006), studied on modelling volatility in the daily return of the National Stock Exchange, India using data which was collected over a five-minute interval. It was found that GARCH models predict the market volatility better than the other models such as historical EWMA model. Also, among the GARCH models they found that the asymmetric models provide a better fit than the symmetric models. They also conclude that the change in volume of trade positively affects market volatility.

Ajay (2005), modelled and forecasted volatility in Indian capital markets. He compared the performance of various unconditional estimators and conditional volatility models (GARCH and EGARCH) using time series data of S&PCNX Nifty, a value-weighted index of 50 stocks traded on the National Stock Exchange, Mumbai for 3 years (1999-2001). As far as forecasting ability of models and estimators is concerned, it was found that the conditional volatility models fare extremely poorly in forecasting five-day (weekly) or monthly realized volatility. In contrast, extreme value estimators, except the Parkinson estimator, perform

relatively well in forecasting volatility over these horizons.

Franses and Dijk (1996), studied on the performance of the GARCH model and two of its non-linear modifications to forecast weekly stock market volatility. The models were the Quadratic GARCH and the Glosten *et al*(1993), Glosten et al. (1993) or (GJR) models which had been proposed to describe the negative skewness in stock market indices. Their forecasting results for five weekly stock market indices showed that the QGARCH model could significantly be improved on the linear GARCH model so that it could be good at calibrating data including extreme events. Based on the results they concluded that the forecasting of GARCH type models appeared sensitive to extreme within sample observations. The GJR model on the other hand was not recommended for forecasting.

Hamilton (1994), carried out a study on the importance of forecasting the conditional variance and pointed out that sometimes we might be interested in forecasting not only the level of the series, r_t , but also its changing variance. He further described that changes in the variance are quite important for understanding financial markets, since investors require higher expected returns as compensation for holding riskier assets. A variance that changes over time also has implications for the validity and efficiency of statistical inference about the parameters that describe dynamics of the level of the series, r_t .

2.3 Studies carried out on African Markets

Suliman and Winker (2012), studied modelling stock volatility using univariate GARCH Models from two African markets; Sudan and Egypt. Five models were used; GARCH(1,1), GARCH-M(1,1) models and EGARCH(1,1),TGARCH(1,1) and PGARCH (1,1) models. They found out that volatility of the Khartoum Stock Exchange was an explosive process whereas for the case of Cairo and Alexandria Stock Exchange, it was quite persistent. The results also provided

evidence on the existence of a positive risk premium in both markets, which supports the hypothesis of a positive correlation between volatility and the expected stock returns. Furthermore, the asymmetric GARCH models found a significant evidence for asymmetry in stock returns in the two markets, confirming the presence of leverage effect in the returns series.

Onwuke et al. (2011), investigated the time series behaviour of daily stock returns of four firms listed in the Nigerian Stock Market from 2nd January, 2002 to 31st December, 2006, using three different models of heteroscedastic processes, namely: GARCH(1,1), EGARCH(1,1) and GJR-GARCH models respectively. The four firms whose share prices were used in this analysis were UBA, Unilever, Guinness and Mobil. All the return series exhibited leverage effect, leptokurtosis, volatility clustering and negative skewness, which are common to most economic financial time series. Except for Guinness, other series display significant level of second-order autocorrelation, satisfying covariance-stationary condition. These models were estimated assuming a Gaussian distribution using Brendt-Hall-Hall-Hausman (BHHH) algorithms program in Eviews software platform. The estimation results revealed that the GJR-GARCH(1, 1) gives better fit to the data and were found to be superior both in-sample and out-sample forecasts evaluation.

Frimpong and Oteng-Abayie (2006), applied GARCH models to the Ghana Stock Exchange in order to examine the behavior of stock returns as well as the market efficiency and volatility effects in the Ghana stock exchange using GARCH models. They concluded that GARCH (1,1) model outperformed the other models under the assumption that the innovations follow a normal distribution.

Ogum et al. (2005), applied the EGARCH model to investigate the market volatility of the Kenyan and Nigerian Stock market returns series. Results of the EGARCH model indicate that asymmetric volatility found in the U.S. and other developed markets is also present in Nigerian stock exchange, but Kenya

shows evidence of significant and positive asymmetric volatility. Also, they show that while the NSE return series indicate negative and insignificant risk-premium parameters, the Nigerian Stock Exchange return series exhibit a significant and positive time-varying risk premium.

2.4 Research gap

From the available literature, it is evidenced that the empirical study on the USE is lacking and yet knowledge on the behavior of the stock market is paramount to the development of Ugandas financial and economic sectors.

2.5 Conclusion

It is evidenced from available literature that volatility of stock market returns can be modelled and forecasted using different GARCH models. Most of the studies have employed more than one model since it is noted that not a particular model works best for all stock markets.

Chapter 3

UNIVARIATE GARCH MODELS

3.1 Introduction

The ARCH model by (Engle, 1982) and its generalization, GARCH by (Bollerslev, 1986) are the major and widely used methodologies in modelling and forecasting volatility of financial time series.

In this chapter, different univariate GARCH models under study are discussed. QML estimation, model selection and volatility forecasting are also discussed.

3.2 ARCH(q) Model

This model was developed by (Engle, 1982) and below is its specification. Let $\{Z_t\}_{t \in \mathbb{Z}}$ be a sequence of *iid.* random variables such that $Z_t \sim N(0, 1)$. $\{\varepsilon_t\}_{t \in \mathbb{Z}}$ is called the ARCH(q) process if

$$\begin{aligned}\varepsilon_t &= \sigma_t Z_t; \\ \sigma_t^2 &= \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2,\end{aligned}\tag{3.1}$$

where $\omega > 0, \alpha_i \geq 0$ (for $i = 1, \dots, q$), to ensure non-negativity of the conditional variance Engle (1982).

Let $\{\mathcal{F}_t\}_{t \in \mathbb{Z}}$ be the information set given at a time t denoted by $\mathfrak{F}_t = \{\varepsilon_t, \dots, \varepsilon_{t-1}\}$. Equation 3.1 ensures that σ_t is measurable with respect to \mathcal{F}_{t-1} . Provided that $E(|\varepsilon_t|) < \infty$,

$$E(\varepsilon_t | \mathcal{F}_{t-1}) = E(\sigma_t Z_t | \mathcal{F}_{t-1}) = \sigma_t E(Z_t | \mathcal{F}_{t-1}) = \sigma_t E(Z_t) = 0 \tag{3.2}$$

This means that the ARCH process has a martingale difference property with respect to $\{\mathcal{F}_t\}_{t \in \mathbb{Z}}$. The conditional standard deviation is a continually changing function of the previous squared values of the process. Despite the simplicity of the ARCH model, one of its draw backs is that it often requires many parameters to estimate to adequately describe the volatility process of an asset return, (Tsay, 2010). This violates the principle of parsimony and can present difficulties when using the model to adequately describe the data. To solve this problem , Bollerslev (1986) introduced the GARCH model which is discussed in the following section.

3.3 GARCH(p, q) Model

This is an extension of the ARCH model and was developed by (Bollerslev, 1986). Let $\{Z_t\}_{t \in \mathbb{Z}}$ be a sequence of *i.i.d.* random variables such that $Z_t \sim N(0, 1)$. $\{\varepsilon_t\}_{t \in \mathbb{Z}}$ is called the GARCH(p, q) process if

$$\begin{aligned} \varepsilon_t &= \sigma_t Z_t; \\ \sigma_t^2 &= \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2, \end{aligned} \tag{3.3}$$

where ω, α_i and β_j are the parameters to be estimated. In order for the variance to be positive the necessary condition is that $\omega > 0, \alpha_i \geq 0$ (for $i = 1, \dots, q$) and $\beta_j \geq 0$ (for $j = 1, \dots, p$).

For $p = 0$, the process reduces to an ARCH(q) process and, for $p = q = 0, \varepsilon_t$ is simply white noise. The sizes of the parameters α_i and β_j determine the short run dynamics of the resulting volatility process. Large ARCH coefficients, α_i imply that volatility reacts significantly to market movements while large GARCH coefficients, β_j indicate that shocks are persistent, (McNeil et al., 1967).

If we write the variance equation of Equation (3.3) in terms of the lag-operator,

L where ($L\varepsilon_t = \varepsilon_{t-1}$), we obtain

$$\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2, \quad (3.4)$$

where

$$\begin{aligned} \alpha(L) &= \alpha_1 L + \alpha_2 L^2 + \dots + \alpha_q L^q; \\ \beta(L) &= \beta_1 L + \beta_2 L^2 + \dots + \beta_p L^p. \end{aligned} \quad (3.5)$$

If the roots of the characteristic equation, i.e, $1 - \alpha_1 x + \alpha_2 x^2 + \dots + \alpha_q x^q = 0$ lie outside the unit circle and the process $\{\varepsilon_t\}$ is stationary, then the variance equation of Equation(3.3) can be written as

$$\sigma_t^2 = \frac{\omega}{1 - \beta(1)} + \frac{\alpha(L)}{1 - \beta(L)} \varepsilon_t^2. \quad (3.6)$$

If we let $\omega^* = \frac{\omega}{1 - \beta(1)}$, and δ_i the coefficients of L^i in the expansion of $\frac{\alpha(L)}{1 - \beta(L)}$, Equation (3.6) becomes

$$\sigma_t^2 = \omega^* + \sum_{i=1}^{\infty} \delta_i \varepsilon_{t-i}^2. \quad (3.7)$$

Equation (3.7) clearly indicates that the GARCH(p, q) process is an ARCH(∞) process with a fractional structure of the coefficients. This means that $\{\varepsilon_t\}$ is also a martingale difference. The conditional variance of ε_t is given by;

$$\sigma_t^2 = \frac{\omega}{1 - \sum_{i=1}^q \alpha_i - \sum_{j=1}^p \beta_j}, \quad (3.8)$$

and we have that $\sum_{i=1}^q \alpha_i + \sum_{j=1}^p \beta_j < 1$ for the conditional variance to be positive. There are many extensions of this model which will be discussed in the subsequent sections.

3.4 Symmetric GARCH Models

For the symmetric models, the conditional variance only depends on the magnitude, and not the sign, of the underlying asset. Under this study, there are two symmetric GARCH models that were used; the standard GARCH(1, 1) and GARCH-M(1, 1). The models are discussed below:

3.4.1 GARCH (1, 1) Model

In this model, the conditional variance is represented as a linear function of its own lags. It is a particular case of GARCH(p, q) with $p = 1 = q$. The basic and most widespread model GARCH (1, 1) is given by;

$$\text{Mean equation} \quad r_t = \mu + \varepsilon_t \quad (3.9)$$

$$\text{Variance equation} \quad \sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \quad (3.10)$$

where $\omega > 0, \alpha_1 \geq 0$ and $\beta_1 \geq 0$ and r_t is the return of the asset at time t, μ is the average return. σ_t^2 is the conditional variance, $\varepsilon_t =$ residual returns, defined in Equation (3.3). For GARCH(1, 1), the constraints $\alpha_1 \geq 0$ and $\beta_1 \geq 0$ are needed to ensure that conditional variance is positive, (Poon, 2005).

Equation (3.5) can be written as a stochastic recurrence equation (SRE). Using $\varepsilon_t = \sigma_t Z_t$ from Equation (3.3) and substituting it in Equation(3.5), yeilds

$$\sigma_t^2 = \omega + \sigma_{t-1}^2(\alpha_1 Z_{t-1}^2 + \beta_1) \quad (3.11)$$

which is a SRE of the form

$$X_t = A_t X_{t-1} + B_t \quad (3.12)$$

$\{A_t\}$ and $\{B_t\}$ are sequences of *i.i.d* random variables defined $\forall t \in Z$. Comparing Equations (3.11) and (3.12), it follows that $A_t = \alpha_1 Z_{t-1}^2 + \beta_1$ and $B_t = \omega$. Sufficient conditions for a solution are that

$$E(\ln^+ | B_t |) < \infty \quad \text{and} \quad E(\ln | A_t |) < 0, \quad (3.13)$$

where $\ln^+ | B_t | = \max\{0, \ln | B_t |\}$. Iterating Equation (3.12) k times yields,

$$\begin{aligned} X_t &= A_t(A_{t-1}X_{t-2} + B_{t-1}) + B_t \\ &= B_t + \sum_{i=1}^k B_{t-i} \prod_{j=0}^{i-1} A_{t-j} + X_{t-k-1} \prod_{i=0}^k A_{t-i} \end{aligned}$$

The conditions (3.13) ensure that the middle term on the right hand side converges absolutely and the last term disappears as indicated below;

$$\frac{1}{k+1} \sum_{i=0}^k \ln |A_{t-i}| \xrightarrow{a.s.} E(\ln |A_t|) < 0$$

by the strong law of large numbers. So

$$\prod_{i=0}^k |A_{t-i}| = \exp \left(\sum_{i=0}^k \ln |A_{t-i}| \right) \xrightarrow{a.s.} 0.$$

The unique solution of Equation (3.12) is given by

$$X_t = B_t + \sum_{i=1}^{\infty} B_{t-i} \prod_{j=0}^{i-1} A_{t-j}, \quad (3.14)$$

where the sum converges absolutely, almost surely. The general solution of Equation(3.11) becomes

$$\sigma_t^2 = \omega \left(1 + \sum_{i=1}^{\infty} \prod_{j=1}^i \alpha_1 Z_{t-1}^2 + \beta_1 \right) \quad (3.15)$$

The solution of the GARCH(1, 1) defining equations is then

$$\varepsilon_t = Z_t \sqrt{\omega \left(1 + \sum_{i=1}^{\infty} \prod_{j=1}^i \alpha_1 Z_{t-1}^2 + \beta_1 \right)} \quad (3.16)$$

Proposition 3.4.1 *GARCH(1, 1) process is covariance- stationary white noise process if and only if $\alpha_1 + \beta_1 < 1$. The variance of the covariance-stationary process is given by $\frac{\omega}{1 - \alpha_1 - \beta_1}$.*

Proof 3.4.2 *From equation (3.10),*

$$\varepsilon_t^2 = \sigma_t^2 Z_t^2 = \omega Z_t^2 + \alpha_1 Z_t^2 \varepsilon_{t-1}^2 + \beta_1 Z_t^2 \sigma_{t-1}^2.$$

Assuming covariance-stationarity, it follows from the equation of ε_t^2 and $E(Z_t^2) = 1$ that

$$E(\varepsilon_t^2) = \omega + \alpha_1 E(\varepsilon_{t-1}^2) + \beta_1 E(\sigma_{t-1}^2).$$

$$\text{But } E(\varepsilon_t^2) = E(\varepsilon_{t-1}^2) = E(\sigma_{t-1}^2) = \sigma^2$$

Clearly, $\sigma^2 = \frac{\omega}{1 - \alpha_1 - \beta_1}$ and we must have that, $\alpha_1 + \beta_1 < 1$.

Conversely, if $\alpha_1 + \beta_1 < 1$, then, by Jensen's Inequality,

$$E[\ln(\alpha_1 Z_{t-1}^2 + \beta_1)] \leq \ln[E(\alpha_1 Z_{t-1}^2 + \beta_1)] = \ln(\alpha_1 + \beta_1) < 0.$$

Using Equation (3.16), it is seen that $\sigma^2 = \frac{\omega}{1 - \alpha_1 - \beta_1}$. □

3.4.2 GARCH-in-Mean (GARCH-M(1, 1)) Model

The return of a security may depend on its volatility, (Ahmed and Suliman, 2011). Thus, one may consider the GARCH-M Model of (Engle et al., 1987) in order to model such a phenomenon, where "M" stands for GARCH in the mean, (Tsay, 2010). It is an extension of the basic GARCH model and allows the conditional mean of a sequence to depend on its conditional variance or standard deviation. The simple GARCH-M(1, 1) model is given by;

$$\text{Mean equation} \quad r_t = \mu + \xi \sigma_t^2 + \varepsilon_t \quad (3.17)$$

The variance equation is similar to that defined for the GARCH(1, 1) process in Equation (3.10). The parameter ξ is called the risk premium parameter. A positive ξ indicates that the return is positively related to its volatility, that is; a rise in mean return is caused by an increase in conditional variance as a proxy of increased risk, (Suliman and Winker, 2012). Engle et al. (1987), assume that the risk premium is an increasing function of the conditional variance of ε_t . Enders (2004), also contends that, the greater the conditional variance of returns, the greater the compensation necessary to induce the agent to hold the long term asset.

3.5 Asymmetric GARCH models

One of the shortcomings of symmetric GARCH models is that they are unable to capture the asymmetry or leverage effects and yet such effects are believed to be very important in studying the behavior of stock returns. This has led to the introduction of a number of models; asymmetric models, to deal with this phenomenon. For asymmetric models, the shocks of the same magnitude, positive or negative, have different effects on future volatility. Under this study, EGARCH by (Nelson, 1991) and TGARCH by (Glosten et al., 1993) models are employed to capture the asymmetric phenomenon in the USE returns.

3.5.1 The Exponential GARCH (EGARCH(1,1)) Model

This model captures the asymmetric responses of the time varying variance and at the same time, it ensures that the conditional variance is always positive even if the parameter values are negative, (Piejje, 2009). This means that there is no need for parameter restrictions to impose non negativity. It was developed by (Nelson, 1991) with the following specification:

$$\ln(\sigma_t^2) = \omega + \beta_1 \ln(\sigma_{t-1}^2) + \alpha_1 \left\{ \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| - \sqrt{\frac{2}{\pi}} \right\} - \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}}, \quad (3.18)$$

where $\pi = \frac{2}{\sqrt{2}}$, γ is the asymmetric response parameter or leverage parameter. In most empirical cases, γ is expected to be positive so that a negative shock increases future volatility or uncertainty while a positive shock eases the effect on future uncertainty, Suliman and Winker (2012).

3.5.2 The Threshold GARCH (TGARCH(1,1)) Model

This was developed by Zakoian(1994), (Zakoian, 1994). It a special case of APARCH by (Ding et al., 1993) and below is its model specification

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \gamma d_{t-1} \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2, \quad (3.19)$$

where d_{t-1} is a dummy variable, *i.e.*,

$$d_{t-1} = \begin{cases} 1 & \text{if } \varepsilon_{t-1} < 0, \quad \text{bad news} \\ 0 & \text{if } \varepsilon_{t-1} \geq 0, \quad \text{good news.} \end{cases}$$

Again γ is the asymmetric response parameter or leverage parameter. The model reduces to the standard GARCH form when $\gamma = 0$. Otherwise, when the shock is positive (*i.e.*, good news) the effect on volatility is α_1 , but when the shock is negative (*i.e.*, bad news) the effect on volatility is $\alpha_1 + \gamma$. Carter et al. (2007), asserts that when γ is significant and positive, negative shocks have a larger effect on σ_t^2 than positive shocks.

3.6 Model Estimation

Another area heavily researched in the GARCH world is the method of estimation. GARCH models can be estimated using Maximum Likelihood (ML) and Quasi Maximum Likelihood (QML) approaches. ML assumes and maximizes a density function for the parameters that are conditional on a set of sample outcomes. Bollerslev and Wooldridge (1992), propose a QML technique that adjusts for small deviations from normality. Under this study, the models were estimated using QML.

3.6.1 Estimation of the GARCH Models using QML method

The QMLE is studied for the process $\{\varepsilon_t\}_{t \in \mathbb{Z}}$ defined as follows: For a positive constant τ , let

$$\begin{aligned} \varepsilon_t &= \sigma_t Z_t \\ \sigma_t^\tau &= \omega_0 + \sum_{i=1}^p \alpha_{0i+} (\varepsilon_{t-i}^+)^{\tau} + \alpha_{0i-} (-\varepsilon_{t-i}^-)^{\tau} + \sum_{j=1}^q \beta_{0j} \sigma_{t-j}^\tau \end{aligned} \quad (3.20)$$

Where $\omega_0 > 0, \alpha_{0i+} \geq 0, \alpha_{0i-} \geq 0 (i = 1, \dots, p), \beta_{0j} \geq 0 (j = 1, \dots, q)$. Using the notation $x^+ = \max(x, 0), x^- = \min(x, 0)$ it is assumed that

A0 : $\{Z_t\}$ is a sequence of *i.i.d* random variables with finite mean, $E|Z_t|^s < \infty$ for some $s > 0$.

All the above discussed GARCH models can be written in the form of Equation (3.20). The vector of parameters

$$\begin{aligned} \theta &= \left(\theta_1, \dots, \theta_{2p+q+1} \right)' \\ &:= \left(\omega, \alpha_{1+}, \alpha_{1-}, \dots, \alpha_{p+}, \alpha_{p-}, \beta_1, \dots, \beta_q \right)', \end{aligned}$$

is assumed to belong to a parameter space $\Theta \subset (0, \infty) \times [0, \infty)^{2p+q}$. The unknown true parameter value is given by $\Theta_0 = \left(\omega_0, \alpha_{01+}, \alpha_{0-}, \dots, \alpha_{0p+}, \alpha_{0p-}, \beta_{01}, \dots, \beta_{0q} \right)'$. Let $(\varepsilon_1, \dots, \varepsilon_n)$ be the realization of length n of the unique non-anticipative stationary solution $\{\varepsilon_t\}$ to Equation (3.20). Conditional on the initial values $\varepsilon_0, \dots, \varepsilon_{1-q}, \tilde{\sigma}_0 \geq 0, \dots, \tilde{\sigma}_{1-p} \geq 0$, the Gaussian quasi-likelihood is given by, (Hamadeh and Zakoian, 2011)

$$L_n(\theta) = L_n(\theta, \varepsilon_1, \dots, \varepsilon_n) = \prod_{t=1}^n \frac{1}{\sqrt{2\Pi\tilde{\sigma}_t^2}} \exp\left(-\frac{\varepsilon_t^2}{2\tilde{\sigma}_t^2}\right), \quad (3.21)$$

where the $\tilde{\sigma}_t^2$ are defined recursively, for $t \geq 1$ by

$$\tilde{\sigma}_t^\tau = \tilde{\sigma}_t^\tau(\theta) = \omega + \sum_{i=1}^q \alpha_{i+} (\varepsilon_{t-i}^+)^{\tau} + \alpha_{i-} (-\varepsilon_{t-i}^-)^{\tau} + \sum_{j=1}^p \beta_j \tilde{\sigma}_{t-j}^\tau \quad (3.22)$$

A QMLE of θ is defined as any measurable solution $\hat{\theta}_n$ of

$$\hat{\theta}_n = \arg \max_{\forall \theta \in \Theta} L_n(\theta) = \arg \max_{\forall \theta \in \Theta} \tilde{I}_n(\theta), \quad (3.23)$$

where

$$\tilde{I}_n(\theta) = \frac{1}{n} \sum_{t=1}^n \tilde{\ell}_t, \quad \text{and} \quad \tilde{\ell}_t = \tilde{\ell}_t(\theta) = \frac{\varepsilon_t^2}{\tilde{\sigma}_t^2} + \ln \tilde{\sigma}_t^2. \quad (3.24)$$

Let

$$\mathcal{A}_{\theta_+}(z) = \sum_{i=1}^p \alpha_{i+} z^i, \mathcal{A}_{\theta_-}(z) = \sum_{i=1}^p \alpha_{i-} z^i, \mathcal{B}_\theta(z) = 1 - \sum_{j=1}^q \beta_j z^j$$

By convention, if $p = 0$, $\mathcal{A}_{\theta_+}(z) = \mathcal{A}_{\theta_-}(z) = 0$ and if $q = 0$, $\mathcal{B} = 1$. Before stating the strong consistency of the QMLE, the following assumptions are made.

A1 : $\theta_0 \in \Theta$ and Θ is compact.

A2 : $E(Z_t^2) = 1$ and $P(Z_t > 0) \in (0, 1)$. Moreover if $P(Z_t \in \Gamma) = 1$ for a set Γ , then Γ has a cardinal $|\Gamma| > 2$

A3 : $\gamma(C_0) < 0$ and $\forall \theta \in \Theta, \sum_{j=1}^q \beta_j < 1$

A4 : If $q > 0$, $\mathcal{B}_{\theta_0}(z)$ has no common root with $\mathcal{A}_{\theta_0+}(z)$ and $\mathcal{A}_{\theta_0-}(z)$. Moreover $\mathcal{A}_{\theta_0+}(1) + \mathcal{A}_{\theta_0-}(1) \neq 0$ and $\alpha_{0p+} + \alpha_{0p-} + \beta_{0q} \neq 0$.

Hamadeh and Zakoian (2011) assert that there is no need to assume that $E(Z_t) = 0$ and that assumption **A2** is made for identifiability reasons. In particular $P(Z_t > 0) \in (0, 1)$ ensures that the process $\{\varepsilon_t\}$ takes positive and negative values with a positive probability (if, for instance, the $\{\varepsilon_t\}$ were almost surely positive, the parameters α_{0i-} could not be identified). Assumption **A3** implies that for the true value θ_0 , the model admits a strictly stationary solution but is less restrictive concerning the other values θ belonging to the parameter space. The second part of **A3** implies that the roots of $\mathcal{B}_\theta(z)$ lie outside the unit disk. **A4** is a standard identifiability assumption.

According to (Hamadeh and Zakoian, 2011), it is convenient to approximate the sequence $\{\tilde{\ell}_t(\theta)\}$ by an ergodic stationary sequence. Denote by $\{\sigma_t^\tau\} = \sigma_t^\tau(\theta)$ the

strictly stationary, ergodic and non-anticipative solution of

$$\sigma_t^\tau = \omega + \sum_{i=1}^q \alpha_{i+} (\varepsilon_{t-i}^+)^{\tau} + \alpha_{i-} (-\varepsilon_{t-i}^-)^{\tau} + \sum_{j=1}^p \beta_j \tilde{\sigma}_{t-j}^{\tau}, \quad \forall t \in \mathbb{Z} \quad (3.25)$$

Let $I_n(\theta) = \frac{1}{n} \sum_{t=1}^n \ell_t$, and $\ell_t = \ell_t(\theta) = \frac{\varepsilon_t^2}{\sigma_t^2} + \ln \sigma_t^2$.

Theorem 3.6.1 *Let $\hat{\theta}_n$ be a sequence of QML estimators satisfying Equation (3.23). Then under **A0-A4**, $\hat{\theta}_n \xrightarrow{a.s.} \theta_0$ as $n \rightarrow \infty$,*

The following two conditions are needed to show asymptotic normality.

A5 : $\theta_0 \in \overset{\circ}{\Theta}$, where $\overset{\circ}{\Theta}$ denotes the interior of Θ .

A6 : $k_Z := E(Z_t^4) < \infty$ where k is the kurtosis.

The theorem for asymptotic normality property of the parameter vector $\hat{\theta}_n$ is now stated.

Theorem 3.6.2 *Let $\hat{\theta}_n$ be a sequence of QML estimators satisfying Equation (3.23). Then under **A0-A6**, $\sqrt{n}(\hat{\theta}_n - \theta_0) \implies N(0, (k_Z - 1)J^{-1})$,*

where

$$J = E_{\theta_0} \left(\frac{\partial^2 \ell_t(\theta_0)}{\partial \theta \partial \theta'} \right) = \frac{4}{\tau^2} E_{\theta_0} \left(\frac{1}{\sigma_t^{2\tau}(\theta_0)} \frac{\partial \sigma_t^\tau(\theta_0)}{\partial \theta} \frac{\partial \sigma_t^\tau(\theta_0)}{\partial \theta'} \right) \quad (3.26)$$

The proof of the above theorems can be found in Francq and Zakoian (2004), where strong consistency and asymptotic normality of the QMLE of GARCH processes is given.

3.7 Model selection criteria

In financial modelling, one of the main challenges is to select a suitable model from a candidate family to characterize the underlying data. The choice of a

good model in the application of time series analysis is crucial; the total process cannot be automated since the context is all important and there is never a perfect or unique model. Model selection criteria provide useful tools in this regard and assesses whether a fitted model offers an optimal balance between goodness—of—fit and parsimony. Ideally, a criteria will identify candidate models that are either too simplistic to accommodate the data or unnecessarily complex. The most common model selection criteria are the AIC and the BIC. A desirable model is one that minimizes the AIC or the BIC.

3.7.1 The Akaike Information Criterion (AIC)

AIC was introduced in 1973 by Hirotogu Akaike as an extension to the maximum likelihood principle and was the first model selection criterion to gain widespread acceptance. Conventionally, maximum likelihood is applied to estimate the parameters of a model once the structure of the model has been specified.

$$AIC = -2(\text{Loglikelihood}) + 2k, \quad (3.27)$$

where k is the number of parameters.

3.7.2 The Bayesian Information Criterion (BIC)

The Bayesian information criterion (BIC) is related to the Bayes factor and is useful for model comparison in its own right. The BIC is defined by

$$BIC = -2 \ln(\text{likelihood}) + k \ln N, \quad (3.28)$$

where N is the number of observations or equivalently, the sample size and k denotes the number of parameters. BIC penalizes more complex models (those with many parameters) relative to simpler models. This definition permits multiple models to be compared at once; the model with the highest posterior probability is the one that minimizes BIC.

3.8 Forecasting volatility

One of the important aspects of time series modeling is volatility forecasting as evidenced by the enormous studies that have been carried out. Consider the basic GARCH(1, 1)-model from Equation (3.10) which is fitted to a data set for the time period $t = 1, \dots, T$. The 1-step forecast of the variance, given the information at time T is given by

$$E(\sigma_{T+1}^2 | \sigma_T^2) = \omega + \alpha_1 \varepsilon_T^2 + \beta_1 \sigma_T^2, \quad (3.29)$$

where σ_T^2 and ε_T^2 are the fitted values from the estimation process. The above derivation can be iterated to get the k -step forecast ($k \geq 2$)

$$E(\sigma_{T+k}^2 | \sigma_T^2) = \omega \sum_{i=1}^{k-2} (\alpha_1 + \beta_1)^{i+} (\alpha_1 + \beta_1)^{k-1} (\omega + \alpha_1 \varepsilon_T^2 + \beta_1 \sigma_T^2) \quad (3.30)$$

As $k \rightarrow \infty$, the variance forecast approaches the stationary variance $\frac{\omega}{1 - \alpha_1 - \beta_1}$ (if the GARCH process is stationary). The smaller this quantity $\eta = \alpha_1 + \beta_1$, the more rapid is the convergence to the long-term volatility estimate.

3.8.1 Forecasting ability of the Models

Engle and Patton (2001), state that establishing the effectiveness of a volatility forecast is not straightforward since volatility itself is not observed. There are several measures for determining the predictive accuracy of an ARCH-GARCH model. The forecasting performance of the competing models under study was evaluated and compared using two different statistical error functions, MSE and MAE defined below:

Let h be the number of lead steps, S the sample size, $\hat{\sigma}_t^2$ is the forecasted variance and σ_t^2 is the actual variance.

$$MSE = \frac{1}{h+1} \sum_{t=s}^{s+h} (\hat{\sigma}_t^2 - \sigma_t^2)^2 \quad (3.31)$$

Among the alternative measures are the mean absolute error (MAE) by Lopez (1999) Lopez (1999) defined by

$$MAE = \frac{1}{h+1} \sum_{t=s}^{s+h} | \hat{\sigma}_t^2 - \sigma_t^2 |^2 \quad (3.32)$$

On the over all, the best model is one that minimizes the error functions.

Chapter 4

DATA ANALYSIS AND RESULTS

4.1 Introduction

Having explored the general theory of the above GARCH models under study in the preceding chapter, this chapter is dedicated to fitting the GARCH family of models to the USE data. A description of the data is given in Section 4.2 where the general statistical features of the USE data are investigated. The rest of the sections discuss the application of the four Univariate GARCH models in real life data is given.

4.2 Data

Secondary data from the USE was used. Daily closing prices of USE All share index data over a period of 9 years extending from 04/01/2005 to 18/12/2013(1426 observations) was used. The USE All Share Index is the major stock market index on the USE. It tracks the daily performance of the 16 most capitalized companies listed on the USE. In order to make forecasts, the full sample was divided into two parts; in-sample and out-of-sample observations.

4.2.1 Asset returns

Most financial studies involve returns, instead of prices, of assets. This is because the return of an asset is a complete and scale-free summary of the investment opportunity for average investors and return series are easier to handle than price series because the former have more attractive statistical properties, (Campbell

et al., 1997). Let P_t and P_{t-1} denote the closing market index of USE at the current (t) and previous day ($t-1$), respectively. The USE All Share returns (log returns or continuously compounded returns) at any time are given by:

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right) \quad (4.1)$$

4.2.2 Basic statistics of USE returns series

Descriptive statistics for the returns was carried out to describe the behavior of USE return series. The skewness, kurtosis and Jarque-Berra test for normality were used as the diagnostic tools under this study. They are;

	Test statistic	Distribution
Skewness, $S(r) = \frac{1}{T} \sum_{(t=1)}^T \left(\frac{r_t - \mu}{\sigma}\right)^3$		$N(0, \frac{6}{T})$
Kurtosis, $K(r) = \frac{1}{T} \sum_{(t=1)}^T \left(\frac{r_t - \mu}{\sigma}\right)^4$		$N(3, \frac{24}{T})$
Jarque-Berra, $JB = \frac{T}{6}[S(r)]^2 + \frac{T}{24}[K(r) - 3]^2$		χ_2^2

(4.2)

This is implemented by using the estimated mean, μ and standard deviation, σ . The distributions stated on the right hand side of Equation(4.2) are under the null hypothesis that r_t is independently and identically distributed, (*iid*) $N(\mu, \sigma^2)$. The quantity $K(r) - 3$ is called the excess kurtosis. If excess kurtosis is zero (0), then the return series are said to be normally distributed. For excess kurtosis greater than zero, the return series will be heavy tailed or leptokurtic and when excess kurtosis is less than zero, then the return series will be short tailed or platykurtic. The null hypothesis of normality is rejected if the p -value of the JB statistic is less than the level significance.

Descriptive statistics for the USE return series are shown in Table (4.1). As is expected for a time series of returns, the mean is close to zero. The return series

Table 4.1: Descriptive statistics of USE returns Series

Mean	0.00 09705
median	0.0002228
Maximum	0.4766000
minimum	-0.4844000
Std.Dev	0.03649952
Skewness	0.3190972
Kurtosis	102.9358
Jarque-Bera	630969.7487
JB probability	<2.2e-16
No. of observations	1425

Sample: Jan 04, 2005 to Dec 18, 2013

are positively skewed an indication that the USE has non-symmetric returns. The kurtosis is greater than three for the normal distribution. This indicates that the underlying distribution of the returns are leptokurtic or heavy tailed. The series is non-normal according to the JB test which rejects normality at the 1% level.

Figure (4.1) shows the distribution of the USE All share index Figure (4.1a) and the return series 4.1b. From the graphs, the stock prices are non stationary while the return series are stationary with a mean return of zero. There is also evidenced volatility clustering in the return series. This is analogous to other studied stock exchanges.

4.2.3 Quantile-Quantile (Q-Q) and ACF plots

The Q-Q graphical examination was employed to further check whether the USE index return series is normally distributed. According to (Alexander, 1999), a Q-Q plot is a scatter plot of the empirical quantiles against the theoretical quantiles

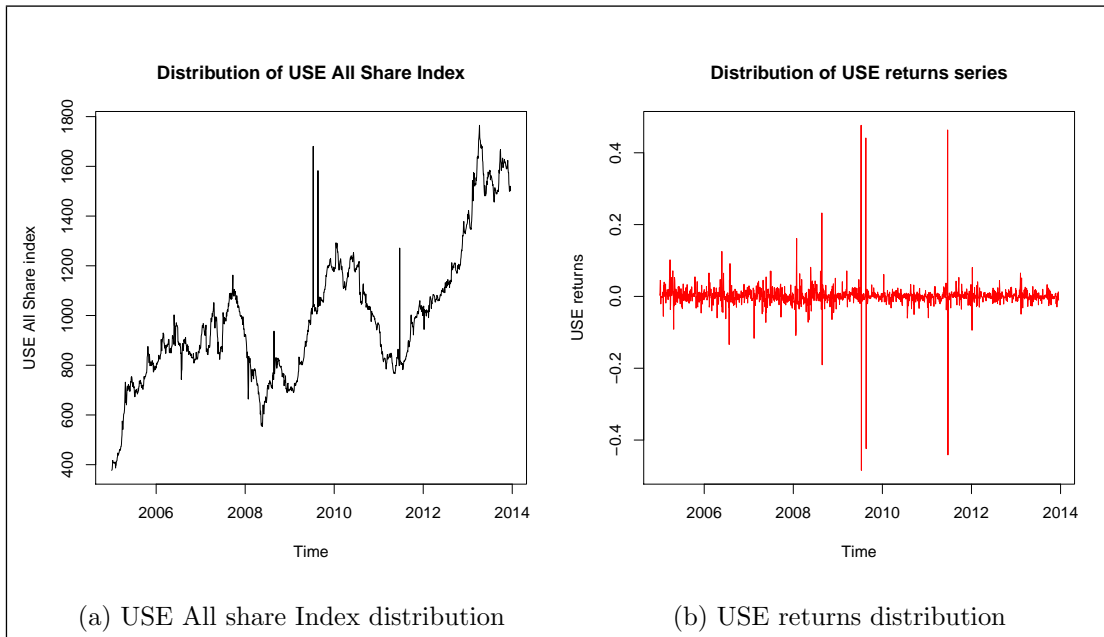


Figure 4.1: USE Daily prices and returns distributions(Jan.2005-Dec.2013)

of a given distribution. Suliman and Winker (2012), assert that if the sample observations follow approximately a normal distribution with mean equal to the empirical mean and standard deviation equal to the empirical standard deviation, then the resulting plot should be roughly scattered around the 45-degree line with a positive slope, the greater the departure from this line, the greater the evidence for the conclusion that the series is not normally distributed. The behaviour of the USE return series can also be deduced using a correlogram. This helps in establishing whether there is serial correlation in the series.

The Q-Q plot in Figure (4.2a) shows that the return distribution of USE exhibit fat tails confirming the results in Table(4.1) that the USE returns data do not follow the normal distribution. Figure (4.2b) above, there is little evidence of serial correlation in the USE return series. The absolute returns series show that there is serial correlation or dependency which seems to confirm volatility clustering. This result is similar to that of Ding *et al*,(1993) Ding et al. (1993).

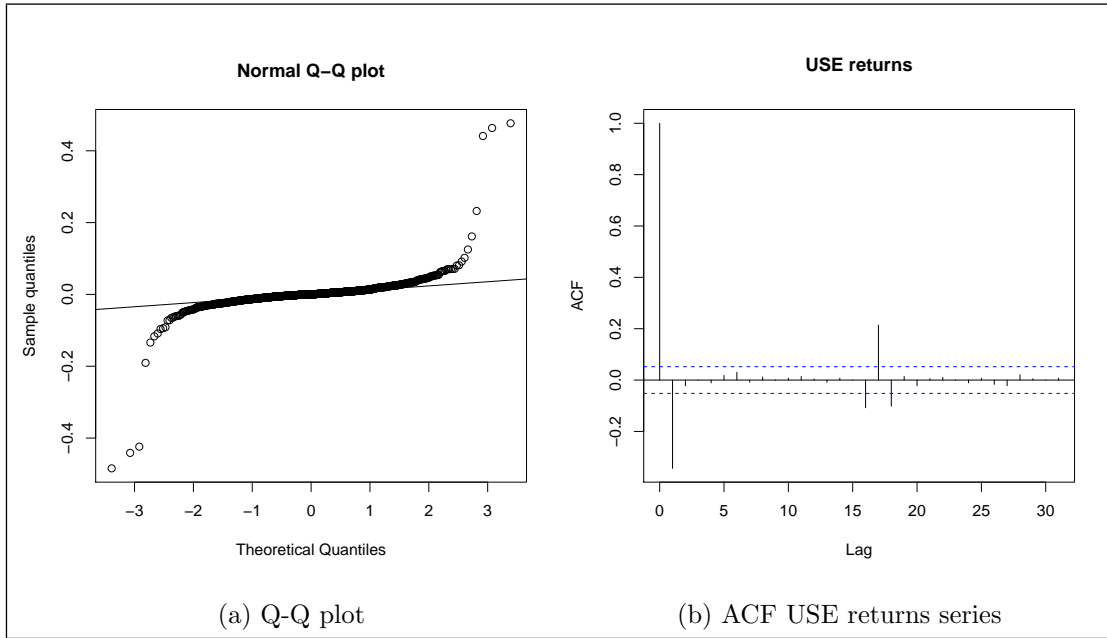


Figure 4.2: Q-Q plot and ACF of USE returns

4.2.4 Testing for stationarity

Before estimating the parameters of the models under consideration, it is required that one checks whether the series are stationary. Under this study, ADF test, (Dickey and Fuller, 1981) was used to investigate whether the daily price index and its returns are stationary series. The ADF test includes a constant term without trend.

Table 4.2: ADF unit root test for the USE All Share index and returns series

Series	ADF Statistic	Critical values		
		1%	5%	10%
Price Index	-2.2484(11)	-3.43	-2.86	-2.57
Return series	-10.1458(11)	-3.43	-2.86	-2.57

Note: Critical values are taken from MacKinnon(1996)

Table 4.2 shows the ADF test results for both the USE All share index and the return series. The ADF test for the price index indicate that they have to be considered as non-stationary. On the other hand, the null hypothesis of a unit root is rejected for the return series at all levels of significance. This means that the return series might be considered as stationary over the specified period.

4.2.5 Testing for Heteroscedasticity

It is always sensible to pre-test the data if ARCH effects are suspected in a series. Engle (1982), proposes a Lagrange Multiplier (LM) test for ARCH. Therefore the LM test was used to test for ARCH effects. Below is the procedure for the LM test: Let $\varepsilon_t = r_t - \mu$ be the residuals of the mean equation. The squared series ε_t^2 is then used to check for conditional heteroscedasticity, which is also known as the ARCH effects. This test is equivalent to the usual F statistic for testing $\alpha_i = 0 (i = 1, \dots, q)$ in the linear regression

$$\varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \dots + \alpha_q \varepsilon_{t-q}^2 + e_t, \quad t = q + 1, \dots, T \quad (4.3)$$

e_t is the error term, q is the pre-specified integer and T is the sample size. The null hypothesis that there are no ARCH effects up to order q can be formulated as: $H_0 : \alpha_1 = \alpha_2 = \alpha_3 = \dots = \alpha_q = 0$ against the alternative $H_1 : \alpha_i > 0$, for at least one, $i = 1, 2, \dots, q$. The test statistic for the joint significance of the q -lagged squared residuals is given by TR^2 where T is the number of observations and is evaluated against the $\chi^2(q)$ distribution. In order to test for ARCH effects, ARMA(1,1) model for the conditional mean in the return series was employed as an initial regression. Then the null hypothesis that there are no ARCH effects in the residual series up to lag 12 was tested. The results are summarised in Table 4.3.

The null hypothesis is rejected basing on the ARCH-LM test results in Table 4.3. This means that there are ARCH effects in the residual series of the mean

Table 4.3: ARCH-LM Test for residuals of the USE returns Series

ARCH-LM test statistic	Prob.chi-square(12)
128.9103	< 2.2e-16

H_0 : There are no ARCH effects

equation, an indication that the variance of the USE return series is non-constant.

4.3 Empirical results

After analysing the characteristics of the USE data, and most importantly the ARCH-LM test providing strong evidence for heteroscedasticity, the GARCH models under study can now be applied.

4.3.1 Selection of GARCH(p, q) Model

The choice of the GARCH(p, q) model to be used can be made basing on AIC, BIC, standard error(SE) and on the significance tests. R software was used to determine the best fitting model. The idea is to have a parsimonious model that best describes the data. To this effect, GARCH(1, 1), GARCH(1, 2) and GARCH(1, 3) have been chosen as the competing models. The model with the smallest value of AIC, BIC and standard error will be taken to be the best.

Table 4.4: Comparison of different GARCH(p, q)Models

Model	AIC	BIC	SE	Log-Likelihood
GARCH(1, 1)	-4.337832	-4.323062	0.09014864	3094.705
GARCH(1, 2)	-4.336178	-4.317715	6.226534	3094.527
GARCH(1, 3)	-4.334607	-4.312452	0.2955442	3094.408

The models were estimated using QMLE and from Table 4.4 above, GARCH(1, 1) model out performed the other models using QMLE. This means that the lag order (1, 1) is enough to describe the data generating process. Brook and Burke (2003), assert that the lag order (1, 1) is sufficient to capture all the volatility clustering that is present in the data.

4.3.2 Simulation

This subsection examines the performance of the QML estimators. Data are generated through the GARCH(1, 1) model

$$\begin{aligned}\varepsilon_t &= \sigma_t Z_t \\ \sigma_t^2 &= 0.1 + 0.3\varepsilon_t^2 + 0.4\sigma_{t-1}^2,\end{aligned}\tag{4.4}$$

for Gaussian innovations *i.e* $Z_t \sim N(0, 1)$. The properties of the QML estimators are investigated for different observations, $n = 500$ and $n = 1000$. The MSE and MAE for each n are compared.

Table 4.5: Simulation results of GARCH(1,1)process

Θ_0		MSE	MAE
$\omega_0 = 0.1$	n=500	0.002090954	0.04572695
	n=1000	2.538395e-05	0.005038249
$\alpha_0 = 0.3$	n=500	0.00183678	0.04285767
	n=1000	0.0006093317	0.02468465
$\beta_0 = 0.4$	n=500	0.05915487	0.2432177
	n=1000	0.003885085	0.06233045

Note: Number of simulations is 1000, it is assumed that $Z_t \sim N(0, 1)$.

From Table 4.5, it is observed that the QML parameters are consistent as $n \rightarrow \infty$. Figure 4.3 shows that the distribution of the QMLE tend to the normal distribution as $n \rightarrow \infty$.

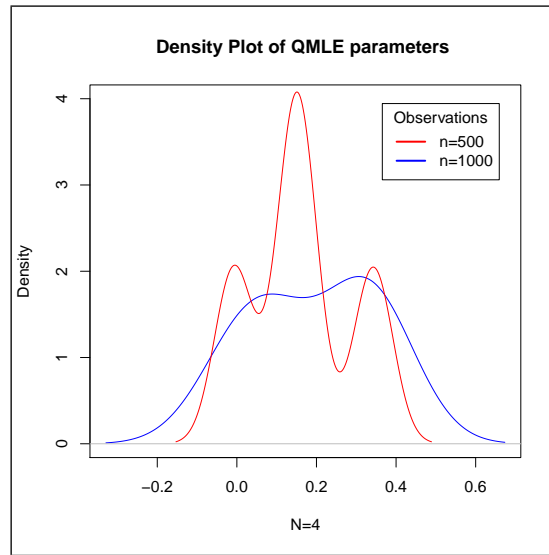


Figure 4.3: QMLE parameter distribution

4.3.3 Model estimation

The parameter estimates of the different models for the USE returns for the study period are showed in the table below. The diagnostics test results of the models, including the AIC and BIC are also provided.

From Table 4.6 , the coefficients ω, α and β in the variance equation of the GARCH(1, 1) are statistically significant and have the expected sign. The significance of α and β indicates that news about volatility from the previous period have an impact on the current volatility. Also the persistence, $\alpha + \beta > 1$ meaning that the volatility process is explosive suggestive of an integrated process.

In the GARCH-M(1, 1) model, the risk premium, λ is significant and positive, indicating that volatility used as a proxy for risk of returns is positively related to the returns. These results are consistent with the asset price theorem, which states that the returns of an asset depend on the level of risk as the asset takes.

The asymmetric EGARCH(1, 1) model estimated for the USE returns indicates that all the estimated coefficients are statistically significant at the 1% confidence

Table 4.6: Estimation results of different GARCH Models for USE

Models	GARCH	GARCH-M	EGARCH	TGARCH
Mean Equation				
μ	-0.0005361	0.01402	-0.000239	0.001001
λ	-	0.2438	-	-
Variance equation				
ω	7.725e-06	1e-06	-0.1404	7.0e-05
α	0.6135	0.1882	0.02353	0.2420
β	0.7629	0.7643	0.96	0.8002
γ	-	-	0.510324	-0.08638
$\alpha + \beta$	1.3764	0.9525	0.9835	1.0422
ARCH-LM test				
Statistic	0.4075927	0.14297	0.11248	0.1535
Probability	0.9999999	0.9996	0.9998	0.9995
Model Performance				
LL	3094.705 ¹	2396.472 ⁴	3087.225 ²	3053.652 ³
AIC	-4.338 ¹	-3.3565 ⁴	-4.326 ²	-4.2788 ³
BIC	-4.323 ¹	-3.3380 ⁴	-4.308 ²	-4.2603 ³

Note: * denotes significance at 1% level, and ** at 5% level, superscripts denote rank of the model

level. The leverage parameter, γ is also statistically significant with a positive sign, suggesting the presence of leverage effects in the the returns of USE during the study period.

The TGARCH(1, 1) model; an alternative asymmetric model estimated also indicates that all the coefficients are significant and have the expected signs. The persistence is very long and explosive suggestive of an integrated process. The leverage parameter is negative and significant, also confirming the presence of

leverage effects. The significance means that positive shocks(good news) have a larger effect on volatility than negative shocks(bad news) of the same magnitude.

The ARCH-LM statistic for all the GARCH models under study also reported in Table 4.6 indicate that there are no additional ARCH effects remaining in the residuals of the models meaning that the models were well specified.

Overall, using the maximum LL, minimum AIC and BIC as the model selection criteria, GARCH(1, 1) outperformed the other models in modelling volatility of USE returns for the study period.

4.4 Forecasting performance of the Models

The forecasting ability of volatility of the four models under study were also evaluated. In order to make forecasts, the full sample was divided into two parts; 1325 in-sample observations from 04/01/2005 to 12/06/2013 and 100 out-of-sample observations from 13/06/2013 to 18/12/2013. The forecast performance of the models is shown in the table below.

Table 4.7: Forecast results of different GARCH Models for USE

Models	GARCH	GARCH-M	EGARCH	TGARCH
MSE	$5.418536e - 3^4$	$3.101637e - 4^3$	$1.407942e - 4^1$	$1.443793e - 4^2$
MAE	$7.721101e - 03^1$	$1.491336e - 2^4$	$8.759654e - 3^2$	$8.956583e - 3^3$
Overall rank	2	4	1	2

Note: Forecast sample: 13/06/2013 to 18/12/2013. Superscripts denote rank of the model

The results in Table 4.7 indicate that EGARCH(1, 1) model outperformed the other models under study in forecasting volatility of USE returns while GARCH-M(1, 1) performed worst.

Chapter 5

CONCLUSIONS AND RECOMMENDATIONS

The volatility of USE returns has been modelled and forecasted for a period of 04/01/2005 to 18/12/2013 using both symmetric and asymmetric univariate GARCH models that capture the the most common stylised characteristics of index returns. The models are GARCH(1, 1), GARCH-M(1, 1) ,EGARCH(1, 1) and TGARCH(1, 1) with the first two models being symmetric and the last two models being asymmetric.

Basing on the empirical results obtained, the following can be concluded: Firstly, it was found that the USE returns are non-normal and heteroscedasticity was found to be present in the residual return series and therefore the support for the use of the above models. Secondly, the USE return series also exhibit volatility clustering and leptokurtosis as evidenced from the high kurtosis values. Thirdly, the QMLE for the GARCH(1, 1) parameters were found to be consistent and normally distributed as $n \rightarrow \infty$. Forthly, the parameter estimates for the GARCH(1, 1) model $\alpha + \beta > 1$ indicating that the volatility of USE stock returns is an explosive process. Fifthly, the risk premium parameter, λ in the GARCH-M(1, 1) model is statistically significant and positive implying that an increase in volatility is affected by an increase in returns. This result is in agreement with the asset price theory. sixth, there is evidence of leverage effects based on the estimation results of the leverage parameter γ of the EGARCH(1, 1) and TGARCH(1, 1) models. The GARCH(1, 1) model outperformed the other competing models in modelling while EGARCH(1, 1) performed best in forecasting volatility of USE returns.

It is recommended that IGARCH(1, 1) is used in place of GARCH(1, 1) to adequately describe the USE returns. Further research can be done to determine the

effects of macroeconomic factors like inflation, exchange rate on the stock prices.

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Appendix

R commands used

```
>data=P
> length(P)
># declaring P as a time series
> P=ts(P , start=2004, frequency=159)
> #Returns generation
> rt=log(Data1[2:1426]/Data1[1:1425])
> rt
># declaring rt as time series
>rt=ts(rt, start=2004, frequency=159)
>par(mfrow=c(2,1))
>plot(P,type="l",ylab="",)
>plot(rt,type="l",col="red")
> summary(rt)
>sd(rt)
> par(mfrow=c(2,2))
> acf(rt)
> acf(abs(rt))
>qqnorm(rt,main="Normal Q-Q plot",xlab="Theoretical Quantiles",
ylab="Sample quantiles",plot.it=TRUE)
> qqline(rt)
> skewness(rt)
> kurtosis(rt)
>jarqueberaTest(rt)
> #load package: tseries
> adf.test(P)
```

```

> adf.test(rt)
> #load package: finTS
> #fit ARMA(1,1)
>arm=arma(rt, order = c(1, 1), lag = NULL, coef = NULL,
include.intercept = TRUE, series = NULL, qr.tol = 1e-07)
> arm
> ArchTest(residuals(arm))
> #GARCH fitting(fgarch package)
> g11=garchFit(formula = ~garch(1, 1), data = rt, cond.dist = "QMLE")
> g12=garchFit(formula = ~garch(1, 2), data = rt, cond.dist = "QMLE")
> g13=garchFit(formula = ~garch(1, 3), data = rt, cond.dist = "QMLE")
> summary(g11)
># GARCH forecasting
> g11@sigma.t
> vol=g11@sigma.t
> vol=ts(vol,start=2004,frequency=159)
> plot(vol,type="l",colour="red")
> vf=forecast(vol[1:1325], h=100, level=95, fan=FALSE)
>summary(vf)
> plot(vf,col="red",main="Volatility forecast of USE return series"
,ylim=c(0,0.5),ylab="volatility of USE return series")
>lines(vol,col="red")
>#Gjr GARCH(rgarchpackage)
>tspec = ugarchspec(mean.model = list(armaOrder = c(0,0),
include.mean = TRUE),variance.model = list(model = "gjrGARCH",
garchOrder=c(1,1)), distribution.model = "norm")
>tfit = ugarchfit(data = Rt, spec = tspec)
>#TGARCH forecasting
>tfit = ugarchfit(data = Rt, spec = rtspec, out.sample=100)

```

```

>tforc = ugarchforecast(tfit, n.ahead=100)
>tforc
>fpm(tforc)
>plot(tforc)
>#GARCH-M
>gmspec = ugarchspec(mean.model = list(armaOrder = c(0,0),
include.mean = TRUE, archm = TRUE, archpow = 1),
variance.model = list(model = "sGARCH",garchOrder=c(1,1)),
distribution.model = "norm")
>gmfit = ugarchfit(data = Rt, spec =gmspec)
>gmfit
># GARCH-M forecasting
>gmfit = ugarchfit(data = Rt, spec =gmspec,out.sample=100)
>eforc = ugarchforecast(gmfit, n.ahead=100)
>fpm(gmforc)
>plot(gmforc)
>#EGARCH
> espec = ugarchspec(mean.model = list(armaOrder = c(0,0),
include.mean = TRUE), variance.model = list(model = "eGARCH",
garchOrder=c(1,1)), distribution.model = "norm")
>e fit = ugarchfit(data = Rt, spec = espec)
> efit
>#EGARCH forecasting
>efit = ugarchfit(data = Rt, spec = espec,out.sample=100)
>eforc = ugarchforecast(efit, n.ahead=100)
>fpm(eforc)
>plot(eforc)
>#simulation
>spec=garchSpec(model = list(omega = 0.1 , alpha = 0.3, beta = 0.4),

```

```

presample = NULL,cond.dist = "snorm", rseed = 123)
>y1 = garchSim(spec, n = 500,n.start=1000)
fit1 = garchFit(~garch(1,1), cond.dist="QMLE",data = y1)
coef(fit1)
>d1 <- density(coef(fit1))
># returns the density data
>plot(d1,main="Density Plot of parameters for n=500")
# plots the results
>y2 = garchSim(spec, n = 1000,n.start=1000)
>fit2 = garchFit(~garch(1,1), cond.dist="QMLE",data = y2)
>coef(fit2)
>d2 <- density(coef(fit2))
># returns the density data
>plot(d2,xlab="N=4",main="Density Plot of QMLE parameters ",
ylim=c(0,4),col="blue")# plots the results
>lines(d1,col="red")
>labels <- c("n=500", "n=1000")
>legend("topright", inset=.05, title="Observations",
labels, lwd=2, lty=c(1, 1), col=colors)

```