Modeling Inflation Rates in Liberia; SARIMA Approach

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DECLARATION

This thesis is my original work and has not been presented for a degree in any other University.

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DEDICATION

This work is dedicated to my parents Mr. and Mrs W. Fannoh , the Dean of college of Business and Public Administration University of Liberia Mr. Geegbae A. Geegbae and my beloved daughter, Rodell Rose Fannoh.

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TABLE OF CONTENTS

DECLARATION . . . . **ii**

DEDICATION . . . . **iii**

ACKNOWLEDGEMENT . . . . **iv**

TABLE OF CONTENTS . . . . **v**

LIST OF TABLES . . . . **vii**

LIST OF FIGURES . . . . **viii**

ABBREVIATIONS . . . . **ix**

ABSTRACT . . . . **xi**

CHAPTER ONE: INTRODUCTION **1**

1.1 Background Information 1

1.1.1 Some facts on Liberia and its inflationary experience 6

1.1.2 Structure and composition of Liberia consumer price index (CPI) 8

1.2 Statement of the problem 9

1.3 Objectives 10

1.3.1 Main Objective 10

1.3.2 Specific Objectives 10

1.4 Justification of the Study 10

1.5 Scope of the Study 11

1.6 Limitation of the Study 11

1.7 Organization of the thesis 12

CHAPTER TWO: LITERATURE REVIEW **13**

CHAPTER THREE: METHODOLOGY **18**

3.1 Introduction 18

3.2 Review of the SARIMA Model 18

3.2.1 Model Identification 20

3.2.2 Seasonal Unit Roots Test 26

3.2.3 Properties of the SARIMA Model 30

3.2.4 Parameter Estimation 30

3.2.5 Forecasting from Seasonal ARIMA Model 35

CHAPTER FOUR: RESULTS AND DISCUSSIONS **38**

4.1 Descriptive Statistics 38

4.2 SARIMA Modeling 41

4.2.1 Stationarity Test and Model Identification 41

4.2.2 Parameter Estimation and Evaluation 45

4.2.3 Forecasting 47

CHAPTER FIVE: CONCLUSION **49**

REFERENCES **50**

LIST OF TABLES

Table 3.1: Behavior of ACF and PACF for Non-seasonal $ARMA(p,q)$. 25

Table 3.2: Behavior of ACF and PACF for Pure Seasonal $ARMA(P,Q)s$ 25

Table 4.1: Descriptive Statistics of the inflation rates 41

Table 4.2: Augmented Dickey-Fuller test for the Inflation Rates 42

Table 4.3: Augmented Dickey-Fuller test for the First difference Inflation Rates 42

Table 4.4: HEGY Seasonal Unit Root Test for the inflation rates 43

Table 4.5: AIC and BIC for the Suggested SARIMA Model 44

Table 4.6: Estimates of the Parameters for ARIMA $(0,1,0)(2,0,0)_{12}$ . . 46

Table 4.7: Residuals Diagnostics Test for SARIMA model 46

Table 4.8: Forecasting Test 47

Table 4.9: Twelve Month Out - Sample forecast 48

LIST OF FIGURES

Figure 4.1: Monthly Inflation Rates and ACF of Liberia (2006:1-2013:12) 38

Figure 4.2: PACF of Liberia Monthly Inflation Rates (2006:1-2013:12) . 39

Figure 4.3: ACF and PACF of First difference series 44

Figure 4.4: Plot of the first difference inflation series 44

Figure 4.5: ACF and PACF plots of the Residuals of ARIMA $(0,1,0)(2,0,0)_{12}$ 46

Figure 4.6: Plot of the In-Sample forecast for inflation data 48

ABBREVIATIONS

ACF	Autocorrelation Function
ADF	Augmented Dickey-Fuller
AIC	Akaike Information Criterion
AICc	Corrected Akaike Iinformation Criterion
AR(p)	AutoRegressive of order p
ARCH	Autoregressive Conditional Heteroskedasticity
ARMA(p,q)	AutoRegressive Moving Average of order (p,q)
BIC	Bayesian Information Criterion
COICOP	Classification of Individual Consumption by Purpose
CPI	Consumer Price Index
GDP	Gross Domestic Product
HEGY	Hylleberg- Engle - Granger - Yoo test
IMF	International Monetary Fund
MA(q)	Moving Average of order q
MAE	Mean Absolute Error

MAPE	Mean Absolute Percentage Error
ME	Mean Error
MPE	Mean Percentage Error
MSE	Mean Square Error
PACF	Partial Autocorrelation Function
RMSE	RootMean Square Error
SARIMA	Seasonal AutoRegressive Integrated Moving Average

ABSTRACT

This thesis models Liberia's Inflation rates using the Box-Jenkins Methodology. It was modeled using seasonal autoregressive integrated moving average which extends the ARIMA model to capture seasonality. The monthly inflation data spanning the period January 2006, to December 2013 from the Research Department of the Central Bank of Liberia was used in this study. It is important to understand the pattern of inflation in a country in order to formulate better policies that will control inflation rates.The Hyndman-Khandakar algorithm selected ARIMA $(0,1,0)(2,0,0)_{12}$ as the best model for Liberia inflation series. Further residual analysis such as Autoregrssive Conditional Heteroscedsaticity Lagrange Multiplier test and Li-Jung Box test show no evidence of ARCH effect and serial correlation respectively. Lastly, a 12 months forecast for the year 2013 with the model revealed that Liberia is likely to experience single digit inflation values. In glow of the forecast result, it is recommend that vigorous monetary policies and appropriate economic measure be adopted by government and some policy makers to make certain that the single digit inflation values aim are met.

CHAPTER ONE

INTRODUCTION

1.1 Background Information

Blanchard (2000) defined inflation as the sustained increase in the general level of prices and services over time. An increase in the general price level causes a reduction in the purchasing power of money. Inflation reflects a reduction in the purchasing power per unit of money – a loss of real value in the medium of exchange and unit of account within the economy (Walgenbach *et al.*, 1973). A chief measure of price inflation is the inflation rate, the annualized percentage change in a general price index (normally the consumer price index) over time (Mankiw, 2002). In recent years, developments in many sub-Saharan Africa economies have caught the attention of international investors who are looking for higher returns in emerging markets. The democratic renaissance in South Africa sparked increase in interest in the region initially and, since then, many countries have undertaken reforms backed by the International Monetary fund (IMF) and World Bank promoting debt relief from the official community.

Inflation in sub-Saharan Africa had a downward trend in 2012. This partly reflects the reversal of the 2010 – 2011 spike in global food and fuel prices. But also reflects monetary tightening, especially in East Africa, where the rise in inflation had been more pronounced. Against this backdrop, inflation in sub Saharan Africa maintains its downward trend and reached eight percent in 2012. South Africa was an exception, the effect of the global crisis on sub Saharan Africa was relatively muted. South Africa, Nigeria, Kenya, Tanzania, Zambia, Ghana and Ivory Coast achieved an average growth rate of 4.7% in five years from 2007 to 2011 in which they benefited from the commodity boom and related increase in foreign direct investment, notably from China, the leading emerging market that is covered by the Global Association of Financial Institutions.

In contrast to the dis-inflationary trend in the global economy in recent years, many countries in sub-Sahara Africa have struggled to contain inflationary pressures. This, together with burgeoning budget deficits, means that central banks have tendancy to keep monetary policy fairly tight and a large interest rate differential has been maintained in respect of developed markets. High yields have resulted in portfolio surge flows, which together with increased inflows of foreign direct investment from resources rich countries have helped countries build reserves, despite having current account deficits. There has also been a renewed focus on gaining access to international capital markets. Several countries besides South Africa have a formal credit rating and have issued international bond over the past years, they include Ghana, Nigeria, Zambia, Gabon and Senegal. Kenya and Tanzania are actively considering tapping the market in the near future (Trevino, 2012).

Overtime, inflation has been considered a key indicator of a country's economic performance. No country can claim to have an economy free from inflation. Exploring the behavior of inflation is necessary because it allows for a better understanding of the role that monetary policymakers play in controlling inflation behavior. Moreover, in-depth knowledge of how inflation behaves is needed because policy efficiency is challenged by the fact that inflation effects are often lagged. African governments respond to inflation with a combination of measures to sustain domestic demand and support industrial production. Chief among these measures has been the easing of monetary policy through reduction of the policy rate, injection of liquidity in the system and other interventions in the foreign exchange markets to influence the value of the national currency.

Central banks play a key role in controlling inflation by easing the monetary policy stance, with an aim of supporting domestic economic activity. The mechanism has clearly demonstrated both the powerful role of counter-cyclical policy and the advantages of flexibility in monetary policy in responding to exogenous shocks to alleviate the impact of the inflation. Central banks' responses have been guided primarily by pragmatism rather than adherence to any prescribed policy regimes. Policy inconsistencies have often made it difficult for some policy makers to achieve the targeted rates of inflation in their countries, thus leading to poor living standards as a result of inflation (Kasekende *et al.*, 2010). Liberia is one of the countries whose economy suffered the devastating effects of civil unrest from 1989 to 2003. The First Liberian Civil War in 1989-1996 destroyed much of Liberia's economy. Many businessmen fled the country, taking away capital and expertise. Some returned during 1997 after which there was an elected government, but many have not. The democratically elected government, installed in 1997, inherited massive international debts and relied on revenues from its maritime registry to provide the bulk of its foreign exchange earnings. The restoration of the infrastructure and the raising of incomes in this war ravaged economy depend on the implementation of sound macro economic and micro-economic policies. This includes encouraging new foreign investment.

The primary objective of monetary policy in Liberia has been and remains to attain price and exchange rate stability. Despite the apparent continuity in this objective, Liberia's inflation experience since 1970 has been varying. Since the coup d'état of 1980, the country's economic growth rate has slowed down because of a decline in the demand for iron ore on the world market and the political upheavals. The average inflation in Liberia was reported at 7.43 percent change in 2009, according to the International Monetary Fund (IMF). In 2009, Liberia's economy share of world total gross domestic product (GDP), adjusted by Purchasing Power Parity, was 0.00 percent. Inflation rate in Liberia was recorded at 8.40 percent in March of 2013. Liberia's economic policy and statements revealed so many challenges in relation to 2012 estimates. The double digit food inflation of 10.4% shows a challenge of the Liberia economic indicator of 2012, declined in reserve money by 8.1%, worsening term of trade and increasing domestic debt to gross domestic product (GDP) of more than 50%. These require prompt policy coordination and effective monetary tightening that indicate hope for smooth economic drive. Globally rising price of fuel, has affected food supply, necessitated higher food imports and eventually exerting pressure on the inflation.

Given the external shocks (an unexpected change in an economic variable which takes place outside the economy),it was anticipated that despite the forecast of Africa's strong economic growth beyond 2013, Liberia's economy could face tougher economic times ahead, underpinned by pervasive structural constraints which includes (lack of electricity, shortage in water supply and poor road network). This is an indcator that the economy could present a daunting task for economic agents to cope and contend with critical macroeconomic pressures. Though the exchange rate shows relative stability for most parts of 2012, other exogenous factors seem to put the market on a risky growth path. Containing commodity inflation and bridging the fiscal deficit that average between 5% and 46.5million over the last three and five years respectively are an uphill challenge. In a small open economy like Liberia, it is anticipated that exchange rate depreciation could induces inflation. However, the exchange rate remains relatively stable, but with food, inflation is usually large. Food and commodity inflation's are critical macroeconomic policy concerns because they are the main absorber of liquidity by economic agents. This inflationary surge does not only lower the real money balance to constraint consumer spending, but it also undermines the monetary value of Liberia's tax revenues. The crust of the issue is that though inflation seems to be driven by some weak economic fundamentals and external shocks such as high fuel price, low demand for exports and supply side constraints remain the main drivers of inflation. These external shocks do have implication macroeconomic internal imbalance relative to exchange rate depreciation, higher inflation and slower growth of the economy.

Infusion of foreign exchange to contain the exchange rate pressure and extension of low cost credit to viable businesses in Liberia, especially small midsize neterprises (SMEs) are indication of effective utilization of monetary instrument. Aside from other external shocks, the widening trade deficit exposes economy to further macroeconomic risks. Trade deficit continues to depreciate over the last year with imports increasing at twice the level of exports. This translates into higher demand for foreign exchange within the domestic economy, characterized by rising number of import oriented styled of business. Combination of monetary intervention and capital inflow is helping to ease the exchange rate pressure (IMF, 2012).

In reference to the Central Bank of Liberia's annual bulletin (Vol.13, No.3, July-September, 2012), Liberia's exports have risen slightly, but not in excess of imports. This has led to the need for a structural shift towards import substitution growth model that will allow for improved domestic production of goods, not only for domestic consumption but also for exports. Liberia's relative currency stability achieved due to sound monetary prudence could be undermined by some form of fiscal dominance (the extent to which government deficits condition the growth of the money supply). From the fiscal front, Government planned spending has increased by more than 25% in reference to the last fiscal budget. At the same time, Government's spending is pressured by its rising recurrent expenditure against increased expenditures on infrastructure, even as revenues dwindle. The planned spending of almost 50% of gross domestic product (GDP) is a potential risk to economic growth if appraised development projects are not efficiently handled to ensure that the relevant budgetary allocation trickle to sectors with pervasive multiplier effects. The country's budget estimates of more than $650 million for 2012/2013 financial year is risked from being met, with predictable divergence between planned and actual revenues.

Since the inception of the Central Bank of Liberia in 1999 inflation has been high and rather volatile over the years. Volatile inflation can also be explained in relation to the exchange rate regime and fiscal deficit. To accomplish this goal section 1.1.1 explain some facts on Liberia and its inflationary experience. Section 1.1.2 look at the structure and composition of the consumer price index of Liberia.

1.1.1 Some facts on Liberia and its inflationary experience

The Liberian economy is sensitive to the external shocks especially adverse price movement under a flexible exchange rate regime like any other developing economies in Sub-Sahara Africa. Against this backdrop, one of the most serious problems facing the Liberian economy is that of inflation. The onset of this problem can be traced to the oil crisis of the mid-1970s in which the economy was severely hit and deteriorated, this was followed by a military coup d'états of April 12, 1980. The civil crisis of 1989 affected the length and breadth of the country causing the commodities and money markets to collapse where in the high level of foreign investments which provided jobs opportunities for Liberians and other foreign nationals were completely eroded.

There have been some substantial changes in the overall performance of the Liberian economy since the pre and post-war situation. The country is a small open economy situated in the West Africa with a population of more than 3.5 million people and well endowed with natural resources, a favorable geographical location and a vibrant informal and formal sectors. The informal sector in Liberia is mainly subsistence with small scale enterprise, such as cook shops, petty trading in any goods, used clothing and domestically Consumer agriculture products such as, okra, beans, sugar Cain, palm oil, vegetable etc.

The formal sector nearly collapse due to the massive movement of displaced people into Monrovia which is the capital and other urban centers. The civil war gives incentive for entry into the informal sector in the country. Thus, Liberia inflationary experience begins just before and after the fourteen years of civil conflicts and continued thereafter.

The rate of inflation before the civil war was very medicates as prices of basic goods and services were relatively low. During the civil crisis prices of goods and services continue to increased due to scarcity and therefore, inflation attained a double digits during the year 2005. In 2006 the average rate of inflation took a down ward trend which averaged to 7.4.

Thus, these fluctuations were occasioned by a combination of unfavorable conditions both man-made and natural conditions. In 2007, the rate of inflation went on an increase from 7.4 to 11.4 percents. Unlike in situation where inflation is increasing gradually, where the process of rising price is protracted and not generally noticeable except by studying post market price, inflation in Liberia seems to be a little bit moderate and continued to fluctuate due to increase in price and in the supply of money and the cost of goods and services. In 2008, the rate of inflation increased to 17.5 percent due to gradually increases in the price of commodity in the world market. Inflation is also deeply rooted in deterioration of the monetary base thus, the confidence that there is a store of value which the currency will be able to command later. From 2009 to 2012, inflation remained moderate on a single digit at the average rate between 6.9 and 8.5 percent respectively. The moderate rate of inflation during this period was punctuated by vigorous macroeconomic reforms.The rate of inflation is measured by an index such as the consumer price index (CPI) and/or by the implicit price deflector for some market basket of goods for the Liberian economy. The rapid experience of Liberia's economic growth however is that during the period of sleekly growth rate of the economy inflation largely remained persistent and high in most periods. Attempts to fame inflationary process over the year were unsuccessful. During the year, inflation remained in single digit despite the exchange rate pressure. At end-December 2013, average annual headline inflation slightly increased from 6.9% to 7.6% in 2012 due largely to the rise in the prices of imported food items **?**.

1.1.2 Structure and composition of Liberia consumer price index (CPI)

The Consumer Price Index (CPI) measures changes overtime in the general level of prices of goods and services that households acquire, (use or pay for) for the purpose of consumption.

In many countries, they are originally introduced to provide a measure of the changes in the cost of living faced by workers, so that wage increases could be related to the changing levels of prices. Over the years, CPI's have widened their scope, and currently are widely used as a micro economic indicator of inflation, as a tool for monitoring price stabilityas well as deflectors in the national accounts.

Currently, CPI's is one of the most important economic and social indicators produces by National Statistics Offices (NSOs) throughout the world. Against this background, the challenge is to identify users' needs, to conceptualize users' needs in terms of economic theory, etc.

In Liberia, the computation of the CPI is currently capital city based in that the data is collected only in the Capital city Monrovia. Its computation is based on a fixed weight basket of goods and services that totals 235 commodities and services. The CPI data is collected from four general markets and 39 shopping centers and supermarkets which also include barbing salon, beauty saloon, furniture shops, etc. The data is collected once a week for the first three weeks of the month, and is then edited, entered and processed using Laspeyres method. Average prices are also generated for each commodity monthly. The data is made available on the 16^{th} day of the current month for the previous month. In the calculation of CPI 2005 is made the base period.

Liberia follows the classification of individual consumption by purpose (COICOP) classification for calculating and presentation of CPI. The COICOP is the term for classifying consumer expenditures and prices on goods and services by purpose. The COICOP consists of 12 Divisions or Major GroupsLISGIS (2013).

1.2 Statement of the problem

As a major macroeconomic variable, inflation is a measure of a country's economic performance. Inflation presents challenges to policy makers especially in post war countries. Mainly, a country facing inflation has a government with lots of uncertainty in prices of commodities and even in delivery of services. Inflation increases costs of investment and accordingly, many investors may turn away from investing in such a country. Most cases in which inflation has been experienced have to device tools of managing the inflation so as to have a stable economy. Liberia is a country within the African continent which from the recent past is moving towards double inflation. It is in this light that the study aim to come up with a model that will help in controlling inflation in Liberia.

1.3 Objectives

1.3.1 Main Objective

The main objective of this study is to Model Inflation Rates in Liberia using the SARIMA Model.

1.3.2 Specific Objectives

The specific objectives are;

1. To determine a Seasonal Auto-regressive Integrated Moving Average (SARIMA) model for the inflation rates in Liberia.
2. To determine the properties of the SARIMA Model.
3. To forecast inflation in Liberia using SARIMA model.
4. To use the forecasts in (3) to advice the policy makers in Liberia.

1.4 Justification of the Study

Liberia's economic performance has been declining as a result of high inflation rates compounded by inconsistency over time. The base lending interest rate in Liberia as reported by Central Bank of Liberia was last recorded at 13.02 percent. Liberia Interest Rate averaged 14.96 percent between 2003 and 2013, reaching an all time high of 20.30 percent in September of 2004 and a record low of 13.02 Percent in March of 2013. The Central Bank of Liberia does not use the interest rate as a monetary policy tool. Many investors have continued to withdraw from the market and this continues to bite the country's economic stability. With decreased investment, job opportunities for Liberians have reduced. Prices of commodities have continued to fluctuate and this affects consumer's . This study will help model inflation. Modeling inflation rates using SARIMA model has not been tried for the case of Liberia and yet elsewhere in the world, SARIMA has offered good insights on matters of inflation. The Liberian case may provide new fronts for modeling inflation rates.

1.5 Scope of the Study

This study is to be conducted only for Liberia, but is predictive and therefore its findings can be adopted by other countries. The study analyzes the fluctuations in inflation in Liberia using Seasonal Autoregressive Integrated Moving Average (SARIMA) model,determine the properties of SARIMA model and make a forecast of Liberia inflation's series using the SARIMA model.

1.6 Limitation of the Study

This study is relevant for the period understudy. The study uses time series data and requires much caution in interpreting the data estimated result because better interpretation will yield to consistent policy measure. Due to the current knowledge and time constraint the study was unable to take into consideration testing of structural breaks in the inflation series.

1.7 Organization of the thesis

The remainder of this thesis is organized as follows: Chapter two presents literature review on SARIMA model. Chapter three present the theory about the model, the chosen tests, how the forecasting accuracy is evaluated and also presents the properties of the model. In Chapter four descriptive statistics of the data set of monthly inflation are presented and also in this chapter analyzes of our inflation data and illustration of how the theoretical methodology can be applied for modeling and forecasting is done. Chapter five concludes the thesis by summarizing the results giving conclusions and recommendations.

CHAPTER TWO

LITERATURE REVIEW

Inflation is one of the economic variables that have received much attention in time series modeling. Inflation reflects a reduction in the purchasing power per unit of money a loss of real value in the medium of exchange and unit of account within the economy (Walgenbach *et al.*, 1973). Price inflation is measure by the inflation rate (Mankiw, 2002).

Suleman and Sarpong (2012), carried out an empirical approach for modeling and forecasting inflation in Ghana. They used monthly data and it was modeled using Seasonal Autoregressive Integrated Moving Average (SARIMA) stochastic model. ARIMA $(3,1,3)(2,1,1)_{12}$ was identified as the best model for inflation rates. An eleven month forecast was made and concluded that the country is likely to experience single digit inflation for the year 2012.ODuro *et al.* (2012) conducted a study on application to microwave transmission of Yeji-Salaga (Ghana). The applied the Seasonal Autoregressive Integrated Moving Average (SARIMA) model to analyze the monthly data, the result shows that ARIMA $(1,1,1)(0,1,2)_{12}$ was the best fitted model. Inflation was found to be integrated of order one and follow the (6, 1, 6) order. Inflation was predicted highest for the months of March, April and May to be 8.95%, 10.07% and 10.24% respectively.

Globally, economists have differed in their analysis of the causes of inflation. As a result, prescribed solutions for inflation have also differed. Specifically, the debate about the causes of inflation is generally between the monetarists and the structuralists. The monetarists hold the view that sustained money growth in excess of the growth of output produces inflation (Meltzer and Monetarism, 2002). They argue that, secular inflation cannot persist without a corresponding increase in the money supply over and above the growth in real output. In this regard, they conclude that inflation can only be reduced by slowing down the growth of the money supply, and the monetarists are of the believe that, maintaining a stable price level through control of the money supply would take care of economic imbalances and rigidities that occur in developing countries.

The structuralists on the other hand assert that, inflationary pressures can exist independently of monetary conditions. In their view, inflation is due mainly to supply rigidities in key sectors of the economy and so money supply is the effect, rather than the cause of inflation (Pennant and Emmontt, 1990). The structuralists believe that, the direction of causation runs from identified challenges in different sectors of the economy to low output and then to rising prices and finally to increases in the money supply. For instance, challenges in the agricultural sector have an adverse effect on food production, which result in an increase in the prices of food. Cost of living then goes up and this in turn, leads to high money wages. Similarly, lack of foreign exchange in many developing countries leads to balance of payments deficits (An imbalance in which payments made by the country exceed payments received by the country), which often results in currency devaluation and import restrictions. Such measures lead to an increase in the prices of imported goods and their substitutes, and this has an adverse effect on domestic prices. An increase in the general price level then implies that, the money supply has to be further increased; since the inability to accommodate the price increase will have a negative impact on the recurrent and development expenditures. The structuralists suggest that, to be able to run a modern economy at low inflation, governments need an income policy.

Agenor and Montiel (1999) have argued that, wage indexation on past inflation rates directly or indirectly play a crucial role in inflation persistence by transmitting exchange rate movements to domestic price. In support of the structuralist view, Fisher and Vegh (2000) have indicated that, using a very broad cross-country panel and fixed effect estimates that, fiscal deficits have been a determinant of high inflation. In a similar work Fischer *et al.* (2002) found that, the relationship between fiscal deficit and inflation is only strong in high inflation countries but there is no obvious relationship between them for low inflation countries. The lack of relationship between fiscal deficit and inflation could be due to mainly the liability of the government to borrow money from domestic sources. In that respect, the transfer of money from the private to the public sector would not cause inflation. Again, using an econometric specification explicitly derived from an inter-temporal optimization model that relates long-run inflation to the permanent component of the fiscal deficit.Catoa and Terrones (2001) found that one percentage point reduction in the ratio of fiscal deficit to GDP typically lowers long run inflation by one and a half to six percentage points depending on the size of the inflation.

There are several factors that can account for the contradiction in empirical studies on the effects of inflation variability on growth. First, the measure of variability affecting growth is a key to the research results. Normally, inflation variability is measured by the variance or standard deviation of inflation. However, the variance of inflation is highly correlated to its level, making it difficult to distinguish the effects on growth of the level of inflation from the effects of the variability of inflation (Dornbusch and Fischer, 1993; Semali and Khan, 2000). Second, according to kuang Liang and Chi-Wei (2010), the empirical results may depend on the sample. In long-run macroeconomic time-series data, structure changes are common. Third, and most importantly, the effect of inflation variability on growth could vary with the inflation level, that is, at lower rates of inflation, the effect is not significant, but at higher rates it's very significant. In addition,Montiel (1989) carried out an empirical analysis of high inflation episodes in Argentina, Brazil and Israel and found that, there was rather little support for fiscal view. The study suggested instead that, exchange rate shocks have been the main cause of inflationary pressures.

Several methodologies have been previously employed by other researchers in modeling inflation in various countries. Dewan *et al.* (1999), using a mark-up model to describe inflation processes in Fiji, concluded that, inflation is driven by both foreign and domestic factors in a manner consistent with conventional theoretical models. The International Monetary Fund (IMF) for its part maintains that in the long-run, inflation in developed and developing countries is a monetary phenomenon. Inflation is said to be mainly caused by increase in money supply. The recent New Keynesian formulation of optimal policy has also raised the prominence of inflation forecasting in policy making (Woodford, 2003). Central banks aim to keep inflation stable, and perhaps also to keep output near an efficient level. With these objectives, the New Keynesian model makes explicit that optimal policy will depend on optimal forecasts (e.g., Svensson, 2005), and further that policy will be most effective when it is well understood by the general public.

The forecasting advantage of SARIMA model compared to other time series models have been investigated by many studies. For instance, Aidan *et al.* (1998) used SARIMA model to forecast Irish inflation, Junttila (2001) applied ARIMA model approach in other to forecast finnish inflation, and Pufnik and Kunorac (2006) applied SARIMA model to forecast short term inflation in croatia. Schulz and Prinz (2009) applied SARIMA model and Holt - Winters exponential Smoothing approach to forecast container transshipment in Germany, the results show that SARIMA approach yields slightly better values of modeling the container throughout than the exponential smoothing approach.

Numerous studies have investigated the relative accuracy of alternative inflation forecasting models. One approach has been to compare the accuracy of survey respondents' inflation forecasts relative to uni-variate time-series models. Another approach is the methodology associated with the work of Fama (1975) and recently extended by Fema and Gibbons (1984). This approach extracts from observed nominal interest rates the market's inherent expectation of inflation. Based on a uni-variate time-series modeling of the real interest rate,Fema and Gibbons (1984) found that the interest-rate model yields inflation forecasts with a lower error variance than uni-variate model, and that the interest-rate model's forecasts dominate those calculated from the Livingston survey of 1983 June. Simone (2000) estimated two time-varying parameter models of Chilean Inflation Box-Jenkins models outperform the two models for short-term out-of-sample forecasts; their superiority deteriorates in longer forecasts. Stockton and Glassman (1987) concuded that ARIMA model of inflation should turn in such a respectable forecast performance relative to the theoretically based specifications.

CHAPTER THREE

METHODOLOGY

3.1 Introduction

The main objective of this study was to model inflation rates in Liberia and that SARIMA model is used towards this end. This section starts by introducing the linear SARIMA model and the modeling cycle of the SARIMA model.

3.2 Review of the SARIMA Model

The seasonal ARIMA model incorporates both non-seasonal and seasonal factors in a multiplicative model which is the generalization of Box and Jenkins ARIMA model. An ARIMA model is a combination of Autoregressive (AR) which shows the relationship between present and past values, and a Moving average (MA) model which shows that the present values has something to do with the past residuals. The ARIMA model is applied in the case where the series has no seasonal features and also difference stationary. Which implies that there is a need for an initial differencing for the data to be stationary. A nonseasonal ARIMA model is classified as an "ARIMA (p,d,q)" model where the parameter p refer to the the number of auto-regressive lags, the parameter d refers to the order of integration that make the data stationary and the parameter q give the number of moving average lags (Pankratz, 1983; Hurvich and Tsai, 1989; Hamilton, 1994; Kirchgassner and Wolters, 2007; Klieber and Zeileis, 2008; Pfaff, 2008).

For time series with polynomial trend of degree d, the trend can be eliminated by differencing a process. The process $X_t = \Delta^d Y_t$ is an ARMA (p,q) satisfying stationary process. The original process Y_t is said to be ARIMA of order (p,d,q) is denoted as

$$\begin{aligned} \phi(L)(1-L)^d y_t &= \theta(L)\varepsilon_t; \\ &\sim WN(0,\sigma^2) \end{aligned} \tag{3.1}$$

where ε_t follows a white noise (WN) process, L is the lag operator , $\phi(L)$ and $\theta(L)$ are the auto-regressive operator and the moving average operator respectively and defined as follow:

$$\phi(L) = 1 - \phi_1 L - \phi_2 L^2 - ... - \phi_p L^p \tag{3.2}$$

$$\theta(L) = 1 + \theta_1 L + \theta_2 L^2 + ... + \theta_q L^q \tag{3.3}$$

$\phi(L) \neq 0$ for $\mid \phi \mid < 1$, the process $\{y_t\}$ is stationary if and only if $d = 0$, in which case it reduces to an ARMA (p,q) process.

The SARIMA model sometimes called the multiplicative seasonal auto-regressive integrated moving average model is denoted as ARIMA (p,d,q)×(P,D,Q)$_s$. This can be written in the lag form as (Halim and Bisono, 2008).

$$\phi(L)\Phi(L^2)(1-L)^d(1-L^s)^D y_t = \theta(L)\Theta(L^s)\varepsilon_t \tag{3.4}$$

$$\phi(L) = (1 - \phi_1 L - \phi_2 L^2 - ... - \phi_p L^p) \tag{3.5}$$

$$\Phi(L^s) = (1 - \Phi_1 L^s - \Phi_2 L^{2s} - ... - \Phi_p L^{ps}) \tag{3.6}$$

$$\theta(L) = (1 + \theta_1 L + \theta_2 L^2 + ... + \theta_q L^q) \tag{3.7}$$

$$\Theta(L^s) = (1 - \Theta_1 L^s - \Theta_2 L^{2s} - ... - \Theta_q (L)^{Qs}) \quad (3.8)$$

where,

p, d and q are the order of non-seasonal AR, differencing and MA respectively

P, D and Q are the order of the seasonal AR, differencing and MA respectively

y_t represent the observable time series data at time t

ε_t represent white noise or error (random shock) at period t

L represent the backward shift, where $L^m y_t = y_{t-m}$

S represent seasonal order, $s = 12$

$\phi(L)$and $\Phi(L^s)$ are the non seasonal and seasonal autoregressive operators respectively

$\theta(L)$ and $\Theta(L^s)$ are the non seasonal and seasonal moving average operators respectively

3.2.1 Model Identification

The possible SARIMA model is determined that best fit the data under consideration. SARIMA model is appropriate for stationary time series therefore, the data under consideration must satisfy the condition of stationarity that is the mean, variance and autocorrelation are constant over time.

3.2.1.1 Stationary Series

A stationary series is a variable with constant mean and constant variance across time. A time series $\{r_t\}$is said to be strictly stationary if the joint distribution of

$(r_{t_1+t}, \ldots r_{t_k+t})$ is identical to that of $(r_{t_1+t}, \ldots, r_{t_k+t})$ for all t, where k is an arbitrary positive integer and $(t_1, \ldots, t_k)$ is a collection of k positive integers. Strictly stationarity requires that the joint distribution of $(r_{t_1}, \ldots, r_{t_k})$ is invariant under time shift. Likewise, a time series $\{r_t\}$ is weakly stationary if both the mean of r_t and the covariance between r_t and r_{t-l} are time invariant, where l is an arbitrary integer or better still if

$$(a) \qquad E(r_t) = \mu$$

which is a constant and

$$(b) \qquad cov(r_t, r_{t-l}) = \gamma_l$$

3.2.1.2 Non Stationary Series

According to Box and Jenkins (1976) non-stationary series normally exhibit trend,seasonal and cyclic components. The series becomes stationary if the trend component is removed. By differencing the series with the intend of removing the polynomial that exhibited by the data and the logarithmic and square root transformation know as the Box-cox transformation which can be used to stimulates stationary are the two forms in which the series can be stationary.

3.2.1.3 Testing for Stationary Series

There are several formal and informal method that can be used to determine whether the series is stationary or non stationary.

INFORMAL METHOD: this is done by inspecting the time series plot or by looking at the plot of the series; such method maybe misleading in determining stationary and non stationary therefore,

FORMAL METHOD: is preferable in determining stationary and non stationary. For this case, the Augmented Dickey-Fuller (ADF) test which is one of the precise formal ways of testing stationarity and non stationarity is considered. The augmented Dickey-Fuller (ADF) test is a test for unit root in time series model and it is an extension of Dickey and W.A.Fuller (1979) test for large and complicated time series model.

To test for unit root, we assume that

$$\begin{aligned} \phi_p(L) &= (1-L)\varphi_{p-1}(L) && (3.9) \\ \varphi_{p-1}(L)(1-L)Y_t &= \theta_0 + \varepsilon_t && (3.10) \\ \varphi_{p-1}(L)\triangle Y_t &= \theta_0 + \varepsilon_t && (3.11) \\ \triangle Y_t - \sum_{j=1}^{p-1} \varphi_j \triangle Y_{t-j} &= \theta_0 + \varepsilon_t && (3.12) \end{aligned}$$

where $\varphi_{p-1}(L) = 1 - \varphi_1 L - ... - \varphi_{p-1} L^{p-1}$has root lying outside the unit cycle. So the Augmented Dickey-Fuller test equation is given as:

$$\begin{aligned} \triangle Y_t &= \phi Y_{t-1} + \sum_{j=1}^{p-1} \varphi_j \triangle Y_{t-j} + \theta_0 + \varepsilon_t && (3.13) \\ \triangle Y_t &= (\theta - 1)Y_{t-1} + \sum_{j=1}^{p-1} \varphi_j \triangle Y_{t-j} + \theta_0 + \varepsilon_t && (3.14) \\ with(\theta - 1) &= \delta \end{aligned}$$

Better still the ADF test equation is:

$$\triangle Y_t = \sigma Y_{t-1} + \sum_{j=1}^{p-1} \varphi_j \triangle Y_{t-j} + \theta_0 + \varepsilon_t \qquad (3.15)$$

with the null hypothesis given as $H_0 : \phi = 1$ and $H_0 : \delta = 0$.

3.2.1.4 Multiplicative SARIMA Model Using the ACF

The multiplicative SARMA $(p,q)\times(P,Q)_s$model for fitting the stationary w_t. The theoretical ACF of w_t is given by:

$$\begin{aligned} \rho_k &= \frac{\gamma_k}{\gamma_0} \\ k &= 0,1,2 \end{aligned} \tag{3.16}$$

where $\gamma_k = E[w_t, w_{t-k}]$. Using the result from Godolphine (1977), it follows that for s large enough

$$\begin{aligned} \rho_{i\pm js} &= \rho_i(u_t)\rho_j(U_t) \\ i &= 0,1,...,\frac{s}{2} \\ j &= 0,1.... \end{aligned} \tag{3.17}$$

where $\rho_{i\pm js}$ is the ACF of w_t at lags $i \pm js$ and $\rho_i(u_t)$ and $\rho_j(U_t)$ are the values of the ACF at lags i and j of u_t and U_t in the models $\phi(B)u_t = \theta(B)a_t$ and $\Phi(B^s)U_t = \Theta(B^s)a_t$, respectively.

Godolphine (1977) demonstrates using a vector representation that equation (3.17) holds exactly when $p = P = 0$ and $s > 2q$. In any case, the general approximation follows from the fact that a SARMA $(p,q)\times(P,Q)_s$ model can be approximated by a $(0,q)\times(P,Q)_s$model for suitable Q when s is large enough.

Theorem 3.1. *Generating Function of the Godolphin's (1977)Result*

If $s > 2q$, the autocovariance function γ_k of a $(0,q)\times(P,Q)_s$ model maybe expressed for negative lags as:

$$\gamma_{js\pm i} = \begin{cases} \gamma_i(u_t)\gamma_j(U_t) & for\, 0 \leq i \leq q\, and\, 0 \leq j \leq Q \\ 0, & otherwise \end{cases} \tag{3.18}$$

where $\gamma_i(u_t)$ and $\gamma_j(U_t)$ denote the auto-covariance functions of the processes $\phi(L)u_t = \theta(L)a_t$ and $\Phi(L)U_t = \Theta(L)a_t$.

Proof. For convenience, it may be assumed that $var(a_t) = 1$

$$\Gamma(L) = \frac{\theta(L)\theta(L^{-1})}{\Phi(L)\Phi(L^{-1})} \tag{3.19}$$

$$\Gamma(B) = \sum_{k=-\infty}^{\infty} \Gamma_k L^k \tag{3.20}$$

Then the auto-covariance generating function of the $(0,q) \times (P,Q)_s$model may be written as:

$$\gamma(L) = \sum_{k=-\infty}^{\infty} \gamma_k L^k \tag{3.21}$$

$$\gamma(L) = \theta(L)\theta(L^{-1})\Gamma(B^s) \tag{3.22}$$

$$\gamma(L) = \left[\left(\sum_{i=0}^{q}\sum_{i=0}^{q-l} \theta_i\theta_{i+l}\right)(L^l + L^{-l})\right]\Gamma(L^s) \tag{3.23}$$

where $\theta_0 = -1$. Thus, provides $s > 2q$, the coefficient of $B^{js+i}(i = 0,...,q)$ is

$$\gamma_{js\pm i} = \Gamma_j \sum_{i=0}^{q-i} \theta_l\theta_{l+i} \tag{3.24}$$

Similarly, the coefficients of B^{js+i} and B^{js-i} can be shown to be equal. □

The Sample PACF for SARIMA Model

If the sample PACF damps out at lags that are multiples of s, this suggests the incorporation of a seasonal MA component into the model. The failure of the sample PACF to truncate at other lags may imply that a nonseasonal MA term is required. The ACF and PACF suggest the model we should build. Checking the ACF and PACF plots, we should both look at the seasonal and non-seasonal lags. Usually the ACf and PACF has spikes at lag k and cuts off after lag k at the non seasonal level. Also the ACF and the PACF has spikes at lag ks and cuts off after lag ks at the seasonal level. The number of significant spikes suggests the order of the model. $Table$ 3.1 and $Table$ 3.2 describes the behavior of the ACF and PACF for both seasonal and non seasonal series (Shumway and Stoffer, 2006).

Table 3.1: Behavior of ACF and PACF for Non-seasonal $ARMA(p,q)$

	$AR(p)$	$MA(q)$	$ARMA(p,q)$
ACF	Tails off at lag k, $k = 1,2,3,...$	Cuts off after lag q	Tails off
PACF	Cuts off after lag p	Tails off at lags k,$k = 1,2,3,...$	Tails off

Table 3.2: Behavior of ACF and PACF for Pure Seasonal $ARMA(P,Q)s$

	$AR(P)s$	$MA(Q)s$	$ARMA(P,Q)s$
ACF	Tails off at lag ks, k =1,2,3,...	Cuts off after lag Qs	Tails off at lag ks
PACF	Cuts off after lag Ps	Tails off at lags ks, k =$1,2,3,...$	Tails off at lag ks

The ACF and PACF plot suggest the possible models that can be obtained for the data but it does not give the final model for the data. Fortunately, Hyndman and Khandakar (2008) developed an algorithm that can be used to speed up this selection process. The HK - algorithm has previously been used by Saz (2011) to identify SARIMA model for the Turkey monthly inflation.

3.2.2 Seasonal Unit Roots Test

The most common approach that is used to determine if the seasonal behavior in the data is deterministic, stochastic or stationary under the seasonal frequencies is the one of Hylleberg *et al.* (1990). Hylleberg - Engle - Granger - Yoo (HEGY) test was proposed by Hylleberg *et al.* (1990) to check for seasonal unit roots in quarterly time series. The approach was extended by Franses (1990) to monthly time series. Franses (1991) came up with the seasonal differencing operator $\triangle_{12}$with 12 roots on the unit circle which can be decomposed as follow:

$$1 - L^{12} = (1-L)(1+L)(1-iL)(1+iL) \tag{3.25}$$

$$x[1+(\sqrt{3}+i)L/2][1+(\sqrt{3}-i)L/2]$$

$$x[1-(\sqrt{3}+i)L/2][1-(\sqrt{3}-i)L/2]$$

$$x[1+(\sqrt{3}+i)L/2][1-(\sqrt{3}-i)L/2]$$

$$x[1-(\sqrt{3}+i)L/2][1+(\sqrt{3}-i)L/2]$$

where all the terms other than $(1-L)$ correspond to seasonal unit roots. Testing for the significance of the parameter in the auxiliary regression below is equivalent to testing for unit roots in monthly time series;

$$\begin{aligned}\beta*(L)y_t &= \pi_1 z_{1,t-1} + \pi_2 z_{2,t-1} + \pi_3 z_{3,t-1} + \pi_4 z_{3,t-2} + \pi_5 z_{4,t-1} + \pi_6 z_{4,t-2} + \pi_7 z_{5,t-1} + \pi_8 z_{5,t-2} \\ &\quad + \pi_9 z_{6,t-1} + \pi_{10} z_{6,t-2} + \pi_{11} z_{7,t-1} + \pi_{12} z_{7,t-2} + \mu_t + \varepsilon_t \end{aligned} \tag{3.26}$$

where μ_t represent the deterministic part in the regression model consisting of a constant and eleven seasonal dummy variables or a trend. $\beta^*(L)$ is a polynomial function of L for which the usual assumption applies and where

$$z_{1,t} = (1+L)(1+L^2)(1+L^4+L^8)y_t, \tag{3.27}$$

$$z_{2,t} = -(1-L)(1+L^2)(1+L^4+L^8)y_t, \tag{3.28}$$

$$z_{3,t} = -(1-L^2)(1+L^4+L^8)y_t, \tag{3.29}$$

$$z_{4,t} = -(1-L^4)(1-\sqrt{3}L+L^2)(1+L^2+L^4)y_t, \tag{3.30}$$

$$z_{6,t} = -(1-L^4)(1-L^2+L^4)(1-L+L^2)y_t, \tag{3.31}$$

$$z_{7,t} = -(1-L^4)(1-L^2+L^4)(1+L+L^2)y_t, \tag{3.32}$$

$$z_{8,t} = (1-L^{12})y_t, \tag{3.33}$$

The ordinary least square method is applied to obtain the estimates of the π_i. Testing for both seasonal and non seasonal unit roots is also implies that testing for the significant of π_i. The t-test is used to test the separates $\pi's$ of the null hypothesis. It is a one sided t-test that $\pi_1 = 0$ and $\pi_2 = 0$ of the null hypothesis respectively. The two sided t-test are use in testing for the null hypothesis of $\pi_i = 0$, $i = 3, ..., 12$. The F-test is used to test for the joint null hypothesis that π_2,$\pi_3$and $\pi_4$are all zero and that all four $\pi's$ are jointly zero ($\pi_1 = \pi_2 = \pi_3 = \pi_4 = 0$). The asymptotic distribution of the test statistics under the respective null hypothesis depend on the deterministic terms in the model.

There is no seasonal unit root if $\pi_2$through $\pi_{12}$are significantly different from zero. If $\pi_1 = 0$, then the presence of non seasonal unit root 1 cannot be rejected. According to Franses (1991), pairs of the complex unit roots are conjugates, so roots are only present when pairs of $\pi's$ are equal to zero simultaneously and also in the case of all $\pi_i, i = 1, 2, ..., 12$ are equal to zero. It is appropriate to apply the $\triangle_{12}$filter. The critical values for both t-tests and F-tests for the separates $\pi's$ as well as for the joint F-test of $\pi_3 = ... = \pi_{12}$ can be taking from Franses (1991).

In order to select the best model among the possible models, the penalty function statistics such as Akaike Information Criterion (AIC or AICc) or Bayesian Information Criterion (BIC) can be used, (Sakamoto *et al.*, 1986; Akaike, 1974; Schwart, 1978). The measure of goodness of fit of an estimated statistical model is the AIC, AICc and BIC. With the given data set, several competing models are ranked according to their AIC, AICc and BIC and the one having the lowest information criterion values is considered the best. How close the model fitted values tend to be the true values in terms of certain expected values are judge by the information criterion. The criterion attempts to find the model that best explains the data with a minimum of free parameters but also includes a penalty that is an increasing function of the number of estimated parameters. This penalty discourages over fitting. In general case, the AIC, AICc and BIC take the form as shown below:

$$
\begin{aligned}
AIC &= 2k - 2log(L) && (3.34)\\
AIC &= 2k + nlog(\frac{RSS}{n}) && (3.35)\\
AICc &= AIC + \frac{2k(k+1)}{n-k-1} && (3.36)\\
BIC &= -2log(L) + klog(n) && (3.37)\\
BIC &= log(\sigma_e^2) + \frac{k}{n}log(n) && (3.38)
\end{aligned}
$$

where

k = the number of parameters in the statistical model, $(p+q+P+Q+1)$

L = the maximized value of the likelihood function for the estimated model

RSS= the residual sum of squares of the estimated model

n= the number of observation

σ_e^2= the error variance

The AICc is a modification of the AIC by Hurvich and Tsai (1989) and it is AIC with a second order correction for small sample sizes. Burnham and Anderson (1998)found that since AICc converges to AIC as n gets large, AICc should be employed regardless of the simple size.

- When two or more different models have the same or similar AIC or BIC values then the principles of parsimony can also be applied in order to select a good model. This principle states that a model with fewer parameters is usually better as compared to a complex model. Also some forecast accuracy test between the competing models can also help in making a decision on which model is the best.

3.2.3 Properties of the SARIMA Model

This section discussed the property of stationarity and invertibility condition of the SARIMA model.

Stationarity and Invertibility Conditions

For an ARMA model to be stationary the roots of the characteristics equation $\phi(L) = 0$ must lie outside the unit cycle, Likewise for the invertibility the root of $\theta(L) = 0$ must fall outside the unit cycle.

For Seasonality stationarity the roots of the characteristics equation $\Phi(L^s) = 0$ must lie outside the unit cycle. Similarly, for seasonal invertibility, the roots of the characteristics equation $\Theta(L^s) = 0$ must fall outside the unit cycle(Hipel and McLeod, 1994).

3.2.4 Parameter Estimation

The identification process of SARIMA model having led to the formulation of the model, we then need to obtain efficient estimates of the parameters. As such we will concentrate on the maximum likelihood estimation method.

3.2.4.1 Maximum Likelihood Estimation for SARMA Model

McLeod and Salas (1983) provide an algorithm for calculating an approximation to the likelihood function of the multiplicative SARMA model given as:

$$\phi(L)\Phi(L^s)w_t = \theta(L)\Theta(L^s)a_t \tag{3.39}$$

where

$$\phi(L) = 1 - \phi_1 L - ... - \phi_p L^p \quad (3.40)$$

$$\theta(L) = 1 - \theta_1 L - ... - \theta_q L^q \quad (3.41)$$

$$\Theta(L^s) = 1 - \Theta_1 L^s - ... - \Theta_{qs} L \quad (3.42)$$

$$\Phi(L^s) = 1 - \Phi_1 L^s - ... - \Phi_{p_s} L^{sp_s} \quad (3.43)$$

L is the backshift operator, s the seasonal period and a_t a sequence of independent normal variables with mean 0 and variance σ^2. The $a_t's$, called the innovations represent the one step forecast errors when the vector of the model parameters,

$$\beta = (\phi_1, ..., \phi_1, \theta_1, ..., \theta_q, \Phi_1, ..., \Phi_{ps}\Theta_1, ..., \Theta_{qs}) \quad (3.44)$$

are known. The ARMA (p,q) model is obtained by taking $p_s = q_s = 0$. The stationarity and invertibiliy conditions stated above applied to the equation (3.39) respectively. Although the SARMA model maybe consider as a special case of the ARMA (p*,q*) model by taking $p* = p + sp_s$, $q* = q + sq_s$, $\phi^*(L) = \Phi(L^s)\phi(L)$ and $\theta^*(L) = \Theta(L^s)\theta(L)$, it will be shown how a more efficient estimation algorithm can be developed utilizing the multiplicative structure of the SARMA model.

Given the observations $z_t(t = 1, 2, ..., n)$ the exact log-likelihood function maximized over σ^2 may be written, apart from an arbitrary constant, as

$$logL(\beta) = -nlog\left(\frac{\frac{S_m}{n}}{2}\right) \quad (3.45)$$

where S_m, the modified sum of squares, is

$$S_m = S[M_n(p, q, p_s, q_s, s)]^{\frac{-1}{n}} \quad (3.46)$$

S represents the unconditional sum of squares of Box and Jenkins (1976) defined by

$$S = \sum_{t=-\infty}^{n} [a_t]^2 \tag{3.47}$$

where [.] denotes the expectation given $z_1, ..., z_n$

The evaluation of S by the iterative unconditional sum of squares method may involve two types of truncation error. First, the infinite sum in (3.47) is replaced by

$$S = \sum_{t=1-Q}^{n} [a_t]^2 \tag{3.48}$$

for suitably large Q. Theoretically, Q should be chosen so that

$$\frac{\gamma_0}{\sigma_a^2} - \sum_{i=0}^{Q} \psi_i^2 < e_{to1} \tag{3.49}$$

where $\gamma_0 = var(z_t)$, $\psi_i$is the coefficient of a_{t-i}in the infinite moving average representation of (equ. 3.47) and $e_{to1}$is an error tolerance. Thus if the model contains an autoregressive factor with roots near the unit cycle, a fairly large Q might be necessary. In practice,

$$Q = q + sq_s + 20(p + sp_s) \tag{3.50}$$

is often sufficient. The other truncation error involves terminating the iterative procedure used to calculate $[a_t]$. Several iterations maybe required to obtain convergence when the model contains a moving average factor with roots near the unit cycle. However, sufficient accuracy is usually obtained on the first evaluation.

McLeod (1977) suggested that the term $M_n(p, q, p_s, q_s, s)$ be replaced by $m(p, q, p_s, q_s, s)$, given by:

$$m(p,q,p_s,q_s,s) = M(p,q)[M(p_s,q_s]s \tag{3.51}$$

where $M(p,q)$is defined for any ARMA(p,q) model as

$$M(p,q) = M_p^2 M_q^2 / M_{p+q} \tag{3.52}$$

where the terms M_p, M_q and M_{p+q} are defined in terms of the auxiliary autoregressions, $\phi(L)v_t = a_t$ and $\theta(L)u_t = a_t$ and the left-adjoint autoregression $\phi(L)\theta(L)y_t = a_t$. For the autoregression, $\phi(B)v_t = a_t$, M_p is the determinant of the $p \times p$ matrix with (i, j) entry

$$\sum_{k=1}^{min(i,j)} \phi_{i-k}\phi_{j-k} - \phi_{p+k-i}\phi_{p+k-j} \tag{3.53}$$

and similarly for the autoregressions. The $p \times p$ matrix define by equation (3.53) is called the Schur matrix of $\phi(B)$. Pagano (1973) has shown that a necessary and sufficient condition for stationarity of an autoregression is that its Schur matrix be positive-definite. Thus calculation of $m(p,q,p_s,q_s,s)$ also provides a check on the stationary and invertibility conditions and so during estimation the parameters maybe constrained to the admissible region. Modified Cholesky decomposition is used to evaluate $M(p,q)$. To obtain the MLE for the model parameters, the modified sum of squares must be minimized using standard optimization algorithm.

3.2.4.2 Diagnostic Checking of the SARIMA Model

After estimating the parameters of our chosen model, the last step is model diagnostics. At this stage we determine the adequacy of the chosen model. These checks are usually based on the residuals of the model. One assumption of the SARIMA model is that, the residuals of the model should be white noise. The ACF of the residuals is approximately zero, when the residuals are white noise. If the assumption is not fulfilled then the different model must be searched to satisfy the assumption. Several statistical tools such as Ljung - Box Q statistic, ARCH - LM test and t-test can be used to test the hypothesis of independence, constant variance and zero mean of the residuals respectively.

Ljung-Box statistic proposed by Ljung and Box (1978) is used to check if a given observable series is linearly independent. The test usually check if there is higher order serial correlation in the residuals of a given model. The null hypothesis of linearly independence of the series is examined by the test. The Ljung - Box test statistic is given by:

$$Q(m) = T(T+2)\sum_{k=1}^{m}\frac{\hat{\rho}_k^2}{T-k} \tag{3.54}$$

where

$\hat{\rho}_k$ = the sample autocorrelation at lag k

T = the sample size

m = the number of time lags included in the test

$Q(m)$ is asymptotically χ^2 distributed with m degree of freedom, when the null hypothesis is satisfied. If the p - value associated with $Q(m)$ is small $(p - value < \alpha)$ or when the value of $Q(m)$ is greater than the selected critical value of the chi-square distribution with m degree of freedom, then the null hypothesis of linear dependence is rejected.

ARCH-LM test of Engle (1982) and SHAPIRO NORMALITY test can also be used to check for conditional homoscedasticity and normality among the residuals respectively. Let us consider the given linear regression:

$$
\begin{aligned}
b_t^2 &= \alpha_0 + \alpha_1 b_{t-1}^2 + \ldots + \alpha_m b_{t-m}^2 + e_t \\
t &= m+1, \ldots, T
\end{aligned} \tag{3.55}
$$

where e_t is the error term, m prespecified positive integer, and T is the sample size. The test for conditional heteroskedasticity which is also known as the Arch effect is the Lagrange Multiplier test which is also equivalent to the F statistic for testing $\alpha_i = 0\,(i = 1, \ldots, m)$in the above equation. The null hypothesis of no Arch effect in the square residuals is examine by the test. The F statistic as in Tsay (2005) is given by:

$$
F = \frac{(SSR_0 - SSR_1)/m}{SSR_1/(T-2m-1)} \tag{3.56}
$$

where

$SSR_0 = \sum_{t=m+1}^{T} (b_t^2 - \bar{w})^2$ where $\bar{w} = (\frac{1}{T}) \sum_{t=1}^{T} b_t^2$ is the sample mean of b_t^2, and

$SSR_1 = \sum_{t=m+1}^{T} \hat{e}_t^2$ where $\hat{e}_t^2$ is the least squares residual of equation (31).

F is asymptotically $\chi^2(m)$ distributed with m degree of freedom, when the null hypothesis is satisfied. If the p-value associated with F is small $(p - value < \alpha)$ or when the value of F is greater then the selected critical value of the chi-square (χ^2) distribution with m degree of freedom, then the null hypothesis of no Arch effect is rejected.

3.2.5 Forecasting from Seasonal ARIMA Model

Forecasting is the process of making a statements about events whose actual outcomes have not yet been observed. It is an important application of time series.

After the model has passed the entire diagnostic test, it becomes adequate for forecasting which is the last step in Box-Jenkins model building approach. For instance, let us consider the given Seasonal ARIMA $(0,1,1)(1,0,1)_{12}$ we can forecast the next step which is given by Cryer and Chan (2008).

$$\begin{aligned} z_t - z_{t-1} &= \Phi(z_{t-12} - z_{t-13}) + \varepsilon_t - \theta\varepsilon_{t-1} - \Theta\varepsilon_{t-12} + \theta\Theta\varepsilon_{t-13} && (3.57) \\ z_t &= z_{t-1} + \Phi z_{t-12} - \Phi z_{t-13} + \varepsilon_t - \theta\varepsilon_{t-1} - \Theta\varepsilon_{t-12} + \theta\Theta\varepsilon_{t-13} && (3.58) \end{aligned}$$

The one step ahead forecast from the origin t is given by

$$\hat{z}_{t+1} = z_t + \Phi z_{t-11} - \Phi z_{t-12} - \theta\varepsilon_t - \Theta\varepsilon_{t-11} + \theta\Theta\varepsilon_{t-12} \tag{3.59}$$

The next step is

$$\hat{z}_{t+2} = \hat{z}_{t-1} + \Phi z_{t-10} - \Phi z_{t-11} - \Theta\varepsilon_{t-10} + \theta\Theta\varepsilon_{t-11} \tag{3.60}$$

and so on. The noise term $\varepsilon_{13}, \varepsilon_{12}, \varepsilon_{11}, \varepsilon_{10}, ..., \varepsilon_1$(as residuals) will enter into the forecasts for lead times $l = 1, 2, ..., 13$, but for $l > 13$ the autoregressive part of the model takes over and we have;

$$\hat{z}_{t+l} = \hat{z}_{t=l-1} + \Phi z_{t+l-12} - \Phi z_{t+l-13} \text{ for l} > 13 \tag{3.61}$$

Forecasting Performance

The accuracy for each model can be checked to determine how the model performed in terms of in-sample forecast. In terms of out sample forecasting, some of the observations are left out during model building. The accuracy of the model can be compare using forecast measure or some statistic such as mean error (ME), root mean square error (RMSE), mean absolute error (MAE), mean percentage error (MPE), mean absolute percentage error (MAPE), mean square error (MSE) etc. The model with the minimum of MAE or RMSE is considered to be the best for forecasting. The mathematical expression are define as:

$$MAE = \frac{1}{T}\sum_{t=1}^{T} \mid \hat{y}_t - y_t \mid = \frac{1}{T}\sum_{t=1}^{T} \mid e_t \mid \tag{3.62}$$

$$MSE = \frac{1}{T}\sum_{t=1}^{T} (\hat{y}_t - y_t)^2 = \frac{1}{T}\sum_{t=1}^{T} (e_t)^2 \tag{3.63}$$

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T} (\hat{y}_t - y_t)^2} = \sqrt{\frac{1}{T}\sum_{t=1}^{T} (e_t)^2} \tag{3.64}$$

where y_t is the actual observation, $\hat{y}_t$is fitted or the forecast value and T is the sample size. If we have perfect forecast then $MAE = MSE = RMSE = 0$. The smaller the value the better the prediction and the big the value the poorer the predictive power of the model.

CHAPTER FOUR

RESULTS AND DISCUSSIONS

This section start by describing the source and properties of the inflation series and how the model under consideration was applied. This section also describes the forecast results.

4.1 Descriptive Statistics

In this research we analyze ninety six (96) monthly observations of inflation rates from January 2006 to December 2013.

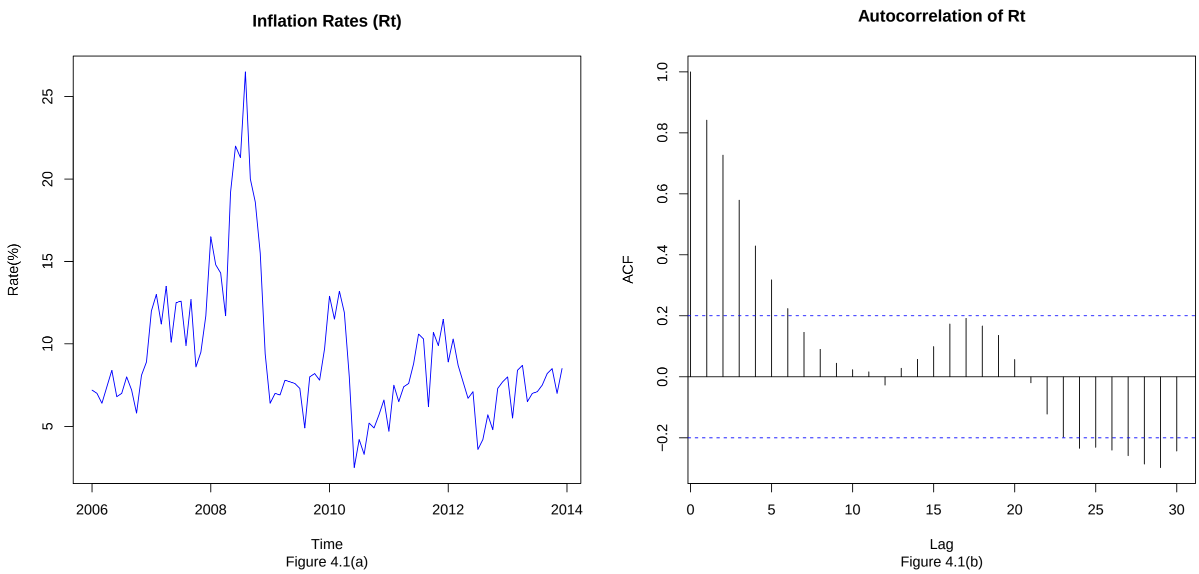

Figure 4.1: Monthly Inflation Rates and ACF of Liberia (2006:1-2013:12)

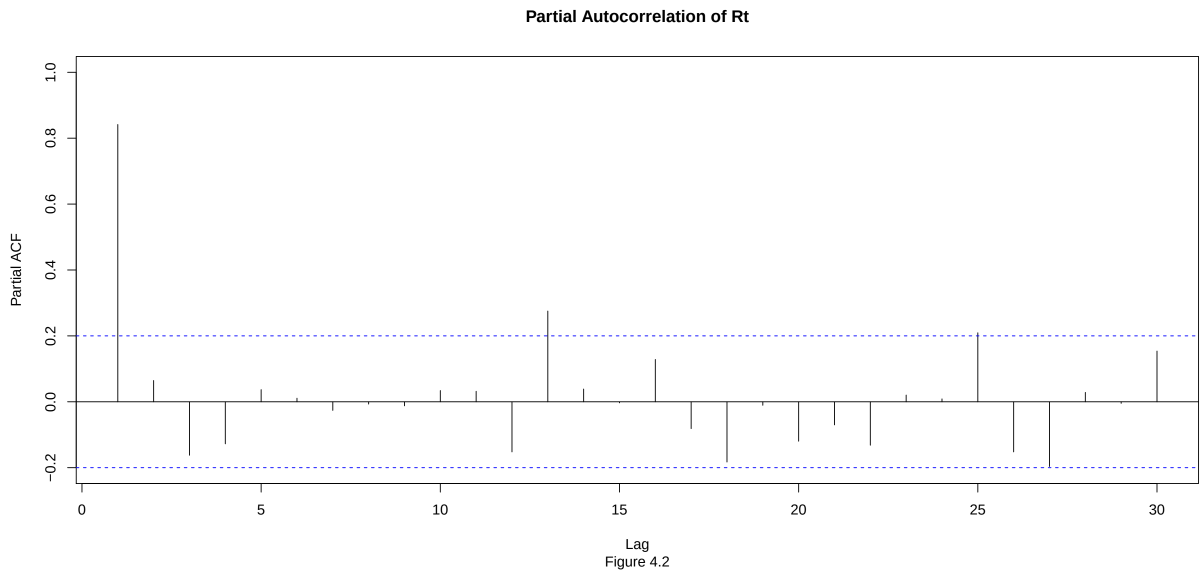

Figure 4.2: PACF of Liberia Monthly Inflation Rates (2006:1-2013:12)

The data was obtained from the Research Department of the Central Bank of Liberia (CBL) and the Statistic department of the Liberia Institute of Statistic and Geo - Information Services (LISGIS).

Figure (4.1)(a&b) display the plot of the original series and the autocorrelation function of the original series where inflation rates is represented by R_t, Figure (4.2) display the plot of the partial autocorrelation function of the original series where inflation is also represented as R_t. It can be seen that inflation exhibit volatility . The volatility in Liberia inflation series can be explained by so many factors including the supply of money, increase of prices on the world market such as petroleum and the poor agriculture sector of the country. The idea that Liberia is an import base economy contributed a lots to these fluctuations. Between 2008 and 2009 there exhibit a very high inflation, this was because prices of essential commodities had increased due to high importation costs and transportation costs. The main group that increase in the rate of inflation was the food and fuel groups. Because of the increment of Petroleum prices increased on the world market, this filter down to Liberian economy and consequently food prices also increased. Some food items that were responsible for the increase in inflation rates include: rice, oil palm, Irish potatoes, plantain and fish products(LISGIS, 2013). After 2008 single digits inflation rates were experienced all through up to 2013 at the average rate between 6.9 and 8.5 percent respectively. The vigorous macroeconomic reforms presented by the CBL was the immediate cause of the moderate rate of inflation. From figure (4.1)(b) it can also be seen that the original series is non stationary. From (4.1)(b) it can be seen that the original plot dies in a sine wave pattern which implies that there is a seasonal and non seasonal components of the series and as such a SARIMA model is required. Table (4.1) below presents the descriptive statistics of the the inflation rate in which the minimum and maximum values are $(2.5000, 26.5000)$ respectively. The mean and median is $(9.2688, 8.0000)$ respectively. Standard deviation measures the amount of variation or dispersion from the average. A low standard deviation indicates that the data points tend to be very close to the mean; a high standard deviation indicates that the data points are spread out over a large range of values. Hence, the standard deviation is 4.1905. Skewness measures the lack of symmetry of a probability distribution and also characterizes the shape of a distribution - that is, its value does not depend on an arbitrary change of the scale and location of the distribution. The skewness of our data is 1.6375 which skewed to the right or it has a long tail that extends to the right. Alternatively, as general rule, most of the time for data skewed to the right, the mean will be greater than the median. Hence, our table satisfies the general rule. Excess kurtosis is kurtosis relative to normal distribution. Kurtosis of normal distribution equals 3. Therefore excess kurtosis equals kurtosis less 3. The excess kurtosis of table (4.1) is 3.2921, this implies that our distribution is different from the normal distribution or better still our distribution is 0.2921 higher than the normal distribution and also our distribution fatter tails then the normal distribution since it is positive.

Table 4.1: Descriptive Statistics of the inflation rates

Statistic	Sub-Sample (N=96)
Minimum	2.5000
Maximum	26.5000
Mean	9.2688
Median	8.0000
Standard Deviation	4.1905
Skewness	1.6375
Ex. Kurtosis	3.2921
Jarque-Bera (p-value)	$1.86366e-019$

4.2 SARIMA Modeling

Seasonal Autoregressive Integrated Moving Average (SARIMA) model approach was used to model the Inflation rates in Liberia in this section. Box and Jenkins procedure was follow as describe in section 3.2 in the modeling cycle.

4.2.1 Stationarity Test and Model Identification

Since SARIMA modeling require that the series be stationary, therefore we have to test for stationarity by using the unit root test in inflation series as describe in section 3. We apply the method of Augmented Dickey- Fuller (ADF) test in testing for stationarity. This method is used to tackle the problem of autocorrelation. We test the null hypothesis that the inflation rate (R_t) is not stationary or has unit root. From (4.2) which present the result of the originl series of the unit root test, we accept the null hypothesis of the presence of a unit root at the 5% significance level because the absolute value of the test statistics is less then the absolute value of the 5% critical value. we consider first differencing the data to render it stationary and it is denoted by Y_t. We again apply the same method to check whether the series is stationary after the first differencing. From the result of the test shown in Table (4.3), we can now conclude that it is stationary at a 5% significance because the absolute value of the test statistics is greater then the absolute value of the 5% critical value.

Table 4.2: Augmented Dickey-Fuller test for the Inflation Rates

	Interpolated Dickey-Fuller			
	Test Statistics	1% critical value	5% critical value	10% critical value
z(t)	-2.245	-3.532	-2.903	-2.586

Table 4.3: Augmented Dickey-Fuller test for the First difference Inflation Rates

	Interpolated Dickey-Fuller			
	Test Statistics	1% critical value	5% critical value	10% critical value
z(t)	-4.164	-3.534	-2.904	-2.587

It is now necessary to test for the presence of seasonal unit roots in the inflation series. The seasonal unit root will enable us to know whether the data is stationary for modeling which is one of the requirements for modeling using SARIMA model. We use the HEGY test in section 3 to test for seasonal unit root in the series. Seasonal frequencies in monthly data are $\pi, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{6}\, and \frac{\pi}{6}$. These are equivalent to $6, 3, 4, 2, 5\, and\, 1$cycles per year respectively (Hylleberg *et al.*, 1990). The thumb rule for the test states that, for the null hypothesis, there exist seasonal unit root. Table (4.4) present the result of the HEGY test. From the test results, we reject the presence of seasonal roots at all frequencies and fail to reject the presence of unit root at the non-seasonal frequency at 5% level. In this regard, at seasonal level, we do not need to make differences for the data.

Table 4.4: HEGY Seasonal Unit Root Test for the inflation rates

Auxilliary Regression	Seasonal Frequency	Critical values	Constant Test Statistics
t-test:$\pi = 0$	0	-3.37	-0.2130
t-test: $\pi_2 = 0$	π	-1.94	-2.6723
F-test: $\pi_3 = \pi_4 = 0$	$\frac{\pi}{2}$	3.05	8.8215
F-test: $\pi_5 = \pi_6 = 0$	$\frac{2\pi}{3}$	3.05	11.1900
F-test: $\pi_7 = \pi_8 = 0$	$\frac{\pi}{3}$	3.08	6.4395
F-test: $\pi_9 = \pi_{10} = 0$	$\frac{5\pi}{6}$	3.08	13.3892
F-test:$\pi_{11} = \pi_{12} = 0$	$\frac{\pi}{6}$	3.09	6.3867
F-test:$\pi_1 = \pi_2, ..., \pi_{12} = 0$		1.88	11.7866
F-test: $\pi_2 = \pi_3, ..., \pi_{12} = 0$		2.30	10.3502

Note: the null hypothesis seasonal unit root is rejected at 5% significant

The next step is to determine the order of the AR and MA for both seasonal and non - seasonal components following the Box and Jenkins procedure. This can be determine by using the sample ACF and PACF plots of the series as suggested by Box and Jenkins as described above. Figure (4.3) displays the plots of the acf and pacf of the first order difference series and Figure (4.4) displayed the plot of the first difference series of the Monthly Inflation Rates represented as Y_t. It can clearly be seen that after the first difference the data becomes stationary and from figure (4.3) it can also be seen from both the acf and pacf that the first several spikes are insignificant, however there is a significant spike at seasonal lag 12. Hyndman-Khandakar (HK) was used to select the best model since identifying the orders from the ACF and PACF seems infeasible. The model is presented in (4.5).

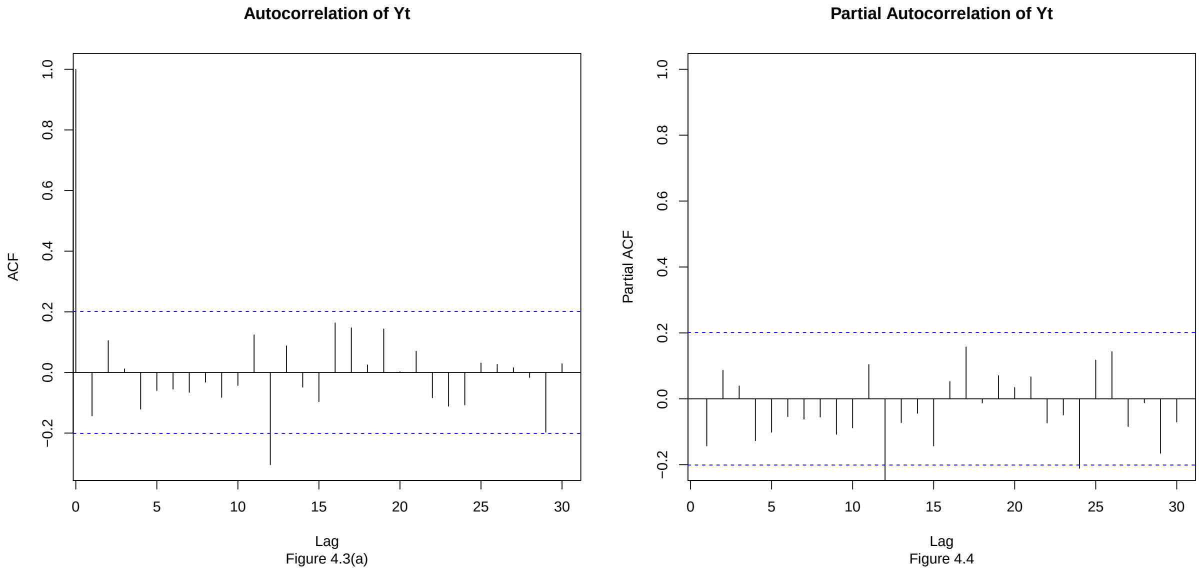

Figure 4.3: ACF and PACF of First difference series

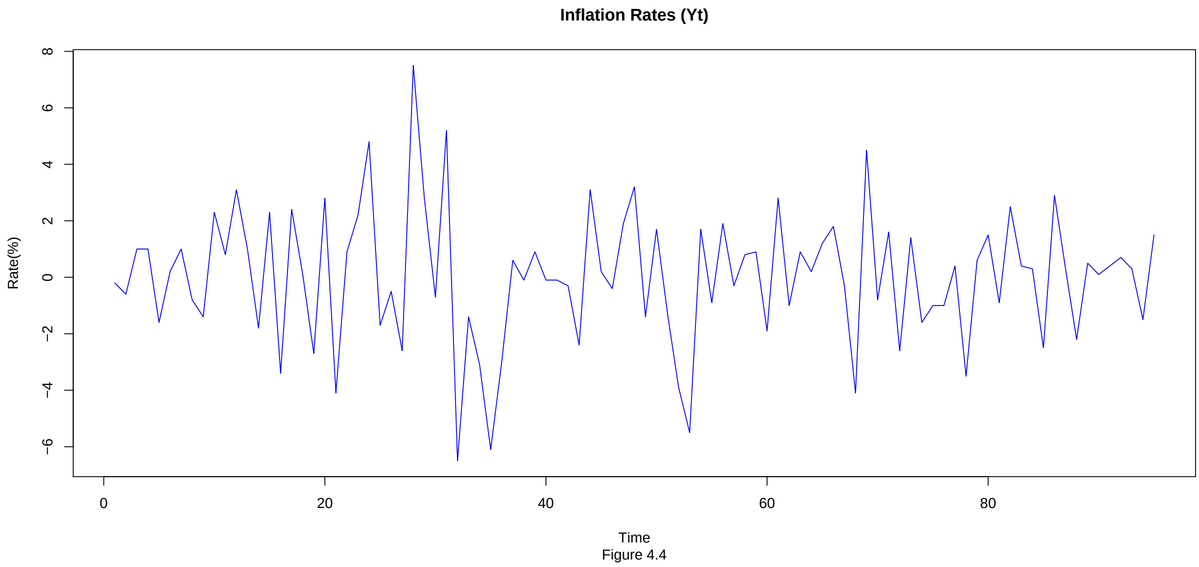

Figure 4.4: Plot of the first difference inflation series

Table 4.5: AIC and BIC for the Suggested SARIMA Model

Model	AIC	AICc	BIC
ARIMA $(0,1,0)(2,0,0)_{12}$	420.1	420.54	430.31

4.2.2 Parameter Estimation and Evaluation

The selected model using the Hyndman-Khandakar algorithm is ARIMA $(0,1,0)(2,0,0)_{12}$. After the model has been identified, we then estimate its parameters. There are several methods of maximum likelihood estimation namely: conditional, unconditional, exact maximum likelihood estimators etc. For our case we used the exact maximum likelihood estimator for the proposed model. We now checked the estimated model after the parameter of the model have been estimated as to whether it satisfies all the assumption of Seasonal ARIMA model which is, the residuals of the model must follow a white noise process meaning that the residual should have zero mean, constant variance and also uncorrelated. Figure (4.5) display the autocorrelation function of the residuals of the selected SARIMA model. From the plot it can be see that the autocorrelation of the residual from the model are all zero, therefore, we can conclude that the residual are uncorrelated. The ARCH-LM test and the Ljung - Box test results are provided in Table (4.7). We can test for constant variance and zero mean assumptions of the residual of the selected model by using the ARCH-LM test. From table (4.7), since the p-value of the ARCH-LM test is greater than 5% significant level, we fail to reject the null hypothesis of no ARCH effect (homoscedasticity) in the residuals of the selected model. Therefore, we conclude that there is a constant variance among the residuals of the selected model and the true mean of the residuals is approximately equal to zero. Also since the p-values for the Ljung-Box test exceed 5%, this indicates that there is no significant departure from white noise for the residuals. Thus, the model can provide an adequate representation of the data since it satisfy all of the necessary assumptions.

Table 4.6: Estimates of the Parameters for ARIMA $(0,1,0)(2,0,0)_{12}$

Parameters	Estimations	Standard Error	P-Values
Constant	-0.0211	0.1581	
$\hat{\phi}_1$	-0.4048	0.1099	$0.0002***$
$\hat{\phi}_2$	-0.2787	0.11435	$0.0148**$

*** Parameters are Statistically Significant

Table 4.7: Residuals Diagnostics Test for SARIMA model

	P - value	
Model	ARCH-LM test	Ljung - Box test
ARIMA $(0,1,0)(2,0,0)_{12}$	0.4618	0.7377

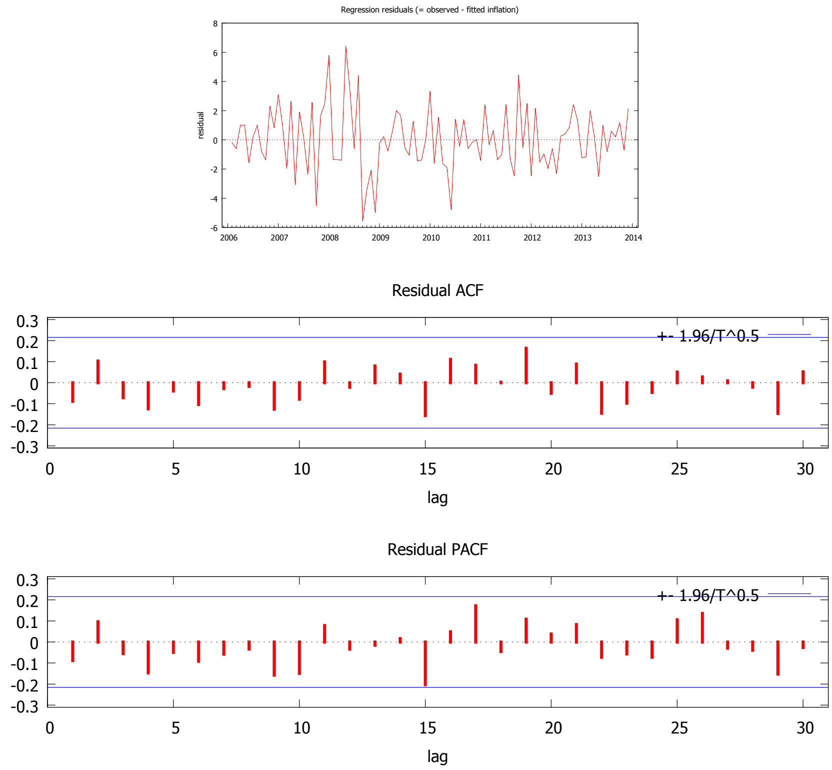

Figure 4.5: ACF and PACF plots of the Residuals of ARIMA $(0,1,0)(2,0,0)_{12}$

4.2.3 Forecasting

One of the most important objectives in the analysis of a time series is to forecast its future values. Forecasting as described by Box and Jenkins (1976) provide basis for economic and business planning, inventory and production control and optimization of industrial processes. Forecasting is a planning tool that helps management in its attempts to cope with the uncertainty of the future, relying mainly on the data from the past and present and analysis of trends. Forecasting is also the process of predicting some unknown quantities. Table (4.8) present the model from the algorithm of Hyadman and Khandakar and since all model assumptions have been satisfied we now conduct in-sample and out-of-sample forecasting of the Liberian inflation rates. Forecast errors will be measured using Mean Error (ME), Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE), and Mean Square Error (MSE).

Table 4.8: Forecasting Test

Model	ME	RMSE	MAE	MPE	MAPE	MSE
$ARIMA(0,1,0)(2,0,0)_{12}$	-1.237	1.626	1.413	-18.051	20.117	2.644

We conducted both in sample and out of sample forecast. Figure (4.6) is a plot of the in-sample forecast. From Figure (4.6), the forecast show that our model was able to ape the behavior of the actual observations although the values were not exactly the same. The 12 months out of sample forecast for the year 2013 was conducted. Table (4.9) summarizes the out-of-sample forecasting results of the inflation rates from the period January 2013 to December 2013 with a 95% confidence interval.

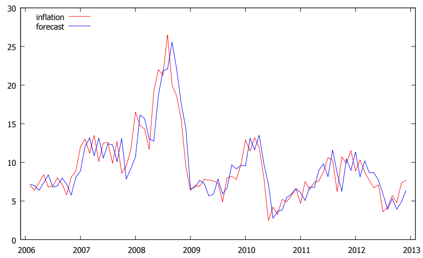

Figure 4.6: Plot of the In-Sample forecast for inflation data

Table 4.9: Twelve Month Out - Sample forecast

Month	Observed Values	Prediction	LCL	UCL
January	8.0	9.2	4.9	13.6
February	5.5	7.9	1.7	14.0
March	8.4	8.8	1.2	16.3
April	8.7	8.9	0.2	17.6
May	6.5	9.2	−0.5	18.9
June	7.0	8.7	−2.0	19.3
July	7.1	9.5	−2.0	21.0
August	7.5	9.3	−3.0	21.7
September	8.2	9.8	−3.2	22.9
October	8.5	8.9	−4.8	22.7
November	7.0	8.1	−6.3	22.5
December	8.5	7.4	−7.6	22.5

$UCL: 95\% \, upper \, confidence \, Limit; \, LCL: 95\% \, Lower \, confidence \, limit$

Comparing the observed values with the prediction value, we can see that there is a increase in pattern and decrease in pattern of the inflation series; as such Liberia is likely to experience single digit inflation for the year 2014.

CHAPTER FIVE

CONCLUSION

In this study, the inflation rates of Liberia was modeled using Seasonal Autoregressive Integrated Moving Average (SARIMA) model of Box and Jenkins (1976) approach to analyze monthly inflation rates from January 2006 to December 2013. The best model identified for the inflation rates was based on the algorithm developed by Hyadman and Khandakar in 2008, ARIMA $(0,1,0)(2,0,0)_{12}$ with maximum log-likelihood and minimum values of AIC, AICc and BIC. ARCH-LM test and Ljung-box test performed on the residuals showed no evidence of ARCH effect and serial correlation respectively. Having satisfied all the model assumptions, ARIMA $(0,1,0)(2,0,0)_{12}$ was selected to be the best model for forecasting. The out-of-sample forecasts for the year 2013 shows a fluctuation in inflation rates. From the out-of-sample forecast, it was deduced that Liberia is likely to experience single digit inflation for the year 2014. In glow of the forecast results, it is recommended that vigorous monetary policies and appropriate economic measures be adopted by the government and other policy makers to make certain that single digit inflation values are met.

RECOMMENDATIONS

The nucleus point for every forecaster is how well the value of the forecast is accurate. Reason is that, the quality of the policies implementation based is affected by the forecasted values. With this reason,

Comparing SARIMA model with other seasonal models for Liberia inflation rates and evaluating their forecasting performances can be considered for future research. Fractional integration and long memory approach may also be considered.

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