Modeling and Pricing Rainfall Derivatives to Hedge Weather Risk in Kenya

PHILLIP AKUMA OKEMWA

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DECLARATION

I, Phillip Okemwa, do declare that this is my original work, except to the extent acknowledged and cited herein and that to the best of my knowledge, it has not been submitted for any degree award in any University.

Signed____ Date ___________

Phillip Okemwa

Supervisors

This thesis is submitted for examination with our approval as the university appointed supervisors.

Signed____ Date ___________

Prof. Patrick G.O. Weke

University of Nairobi, Kenya

Signed____ Date ___________

Prof. John M. Kihoro

Co-operative University, Kenya

Signed____ Date ___________

Dr. Philip Ngare

University Of Nairobi, Kenya

DEDICATION

I dedicate this work to my parents Mr. Okemwa Mang'oa and Mrs. Fildha Mokeira, my siblings: the late Kelvin, Leonard, Sophie, Dennis, Elijah, Caleb and Malachi. To the later, may this be an inspiration in your academic endeavours. To my nephews and nieces, the sky is the limit, thank you for the joy you bring. To the love of my life Esther K, though far away, your love, support, prayers have made it possible.

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All errors and omissions in this document are entirely my own and should not be attributed to any person or entity herein mentioned.

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LIST OF ACRONYMS

BS- Black Scholes

MPR- Market Price of Risk

SDE- Stochastic Differential Equation

WD- Weather Derivative

ABSTRACT

This study set out to price rainfall derivatives based on rainfall at a particular location in Kenya over a given period. We employ a Markovian-Gamma model to model the rainfall process. In addition, its parameters are determined via maximum likelihood estimation. We assume existence of a tradable asset whose performance is rainfall dependent. To compute the prices of the rainfall derivatives, we rely on the Esscher transform, an actuarial tool. We then compare the Esscher prices with the standard Black-Scholes prices. The results suggest a certain pattern of movement of the prices in which the derivative price decreases as the strike price increases in the Black-Scholes whereas they increase on considering the Esscher. The study is conducted using rainfall and stock market data in Kenya.

CHAPTER ONE

INTRODUCTION

1.1Background

Weather conditions have pronounced influence on business revenues. They vary both seasonally and regionally. The weather influence presents both challenges that are adverse with huge losses and opportunities for the emergence, development and growth of the financial instruments like weather derivatives. Businesses use weather derivatives to hedge on their risks in order to make trading profits. Weather exposure can be hedged just the same way as currency exposure.

Weather conditions, that is, rainfall, temperature, frost, snow are always unpredictable. Moreover, by being so unpromising and their patterns being abnormal over the decades, many industries are becoming victims of the weather in profound ways (Geyser, Van derv enter, 2001).

Over the years, the agricultural based businesses have used futures contracts of agricultural commodities to hedge on weather related risks. However, traditional methods cannot cover a number of weather risks. This has given rise to the emergence of a more robust, flexible financial instrument called the weather derivatives (Geyser, 2002).

Weather derivatives are financial instruments with payoffs linked to specific weather events and are designed to provide protection against the financial losses that can occur due to unfavorable weather conditions. It is a contract that stipulates how payment will be settled between the parties involved based on the prevailing meteorological conditions during the contract period (Leobacher and Ngare, 2011). They are typically swaps, futures or options based on the underlying weather measures which can be temperature, humidity, rain or snowfall (Alaton, Djehiche and Stillberger, 2002).Weather derivatives, unlike traditional derivatives, have no underlying tradable instrument or stock. Therefore, they cannot be used to hedge price risk since the weather itself cannot be priced. Instead, they hedge against volumetric or quantity risk associated with weather conditions.

The weather derivatives market is well developed in the United States of America (USA) with the energy industry players being the leading participants. There is equally rapid growth in Asia (Japan, China and India among others) and larger Europe. The growth is rather unprecedented and significantly rapid (Douglas-Jones, 2002)

Closer home, in Africa, businesses in the agriculture and related industries have developed interest in the weather derivatives .Agriculture is largely unsubsidized and the energy industry is hugely regulated. Maize and wheat growers, silo owners, transport companies, sugar industry, fishing as well as insurance companies in South Africa have taken advantage of this new instrument (Bolin, 2002). Agriculture, clothing, construction, hospitality and outdoor entertainment industries which are highly weather sensitive and whose revenues and productivity are closely correlated with weather conditions are the ones heavily affected by harsh weather in the region.

The mentioned industries are differently affected by prevailing weather conditions. The hospitality, tourism and entertainment industries are mostly busy during summer, the same time that most of Africa receives its rainfall. Thus, the attendance figures dwindle. The construction industry is heavily hit in financial terms for projects that run beyond completion deadlines since operation of heavy machinery and working outdoor during rainy conditions are rather difficult. The clothing industry is equally dictated by weather since weather conditions dictate what people buy and wear for example, during winter, sweater and jacket products experience faster sales unlike during a milder than normal winter. In agriculture, weather conditions affect the quality and quantity of produce (Geyser, 2002).

The emergence of weather derivatives in the agriculture sector has particularly drawn considerable interest from international institutions like World Bank, International Finance Corporation (IFC) and International Monetary Fund (IMF).In the underdeveloped and developing world, farmers are never covered by government sponsored insurance programs yet the weather risk is most prevalent in devastating scales. The weather derivatives can therefore provide a sure way of protecting them against risk of drought and poor harvest or any weather related risk.

According to Cooper (2001), a large portion of South America's economy relates to growing commodities and selling them to the world market. In Brazil for instance, coffee harvest can be adversely affected by bad weather conditions, which will in turn have considerable effects on the economy. Weather derivatives can bring added stability.

1.2 Problem statement

The impact of weather conditions in regional and local markets plays a critical role in the overall economy. The energy industry, agriculture, retail, tourism, insurance and a host of many other weather dependent industries are either directly or indirectly exposed to weather risk.

This vulnerability explains the need for a strategy to manage weather risk in order to protect these businesses against uncontrolled weather risk exposures.

The development of the weather derivatives market is aimed towards managing the economic impact of weather events. Weather derivatives help businesses hedge their risk in much the same way as they can hedge foreign exchange or interest rate risk. There is therefore need to develop weather risk management in developing countries because they are worst hit when disaster strikes. They suffer heavier economic loss compared to their developed counterparts.

Kenya and Africa has a large potential in the weather derivative market. The climate radically varies with its vast lands spanning many latitude degrees with complicated terrain. Given that Africa is a large agriculturally productive continent and most of its economic activities are weather sensitive, it is urgent to adopt weather derivatives as a hedging mechanism to shield against weather risk. With telecommunication system networks and meteorological systems now becoming integrated and advanced, data for developing countries and pricing weather derivatives becomes more available. The ability to manage risk driven by weather events is important for most industries. There is need, therefore, for studies that will provide tools to manage the unpredictable financial risks associated with weather fluctuations. It is at this point that weather derivatives come in as an hedging mechanism against weather related risk.

1.3 OBJECTIVES OF THE STUDY

1.3.1 Main objective

The overall objective of this study was to develop a framework for modelling and pricing rainfall derivatives used to hedge weather risk in Kenya.

1.3.2 Specific objectives

1. To model rainfall process using Markovian –Gamma model
2. To construct a unique equivalent martingale measure Q via the Esscher transform.
3. To compute arbitrage-free prices of derivatives whose underlying is a rainfall process

1.4 Significance of the Study

This study sought to provide more information in the financial derivatives market on how to compute fair rainfall derivative prices, which can then be used to manage weather risk in Kenya. The study has greater significance to farmers in a developing economy whose livelihood is highly affected by fluctuations of the amount of rainfall received during the farming period. The weather derivative will compensate the farmers in case of above or below normal amount of rainfall required. In addition, the study is of significance to social event planners who will receive compensation in case of unexpected weather events.

1.5 Scope of the study

The study focuses on the rainfall at a single site thus we will not take into account the spatial correlations.

1.6 Limitations of the study

The study assumes rainfall as the main factor that affects crop yield, hydro-electric power generation.

1.7 Assumptions in the study

We assume an incomplete market, that is,

i) We assume a constant risk-free rate of interest.

ii) The market is frictionless and trading is continuous, that is, no taxes, no transaction cost and no restrictions on borrowing or short sales.

iii) All securities are perfectly and infinitely divisible.

Further, we assume that cultural practices, tradition plays no role in the study.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Weather is an essential production factor in agriculture and hydro-power generation. This factor of production is, however, uncontrollable. Therefore, it can pose risks that are the major sources of uncertainty in crop production and electricity reliability.

In the past decade research on weather derivatives has focused on temperature as the underlying weather measure because most traded weather derivatives are based on temperature indices. There are, however, several economic activities exposed to rainfall risk. A good example is that farmers and financial investors are affected by indirect losses as a result of abundant or scarce rainfall.

With the rainfall derivatives, firms are able to transfer rainfall risk to the capital market. The buyer of the derivative has a chance to reduce rainfall risk exposure, to profit from weather uncertainty, and stabilize cash flows and earnings. The buyer of the derivative receives a payout at a pre-determined settlement period if the weather event occurs regardless what the cause of the loss caused by the weather condition. Sellers of the derivative eliminate moral hazard and avoid the higher administrative and loss adjustment expenses of insurance contracts.

The Chicago Mercantile Exchange (CME) first introduced derivative contracts on weather indices in 1999.Both the over the counter and exchange traded derivatives written on weather variables range from temperature, hurricanes, frost and precipitation among others.

Unlike insurance that cover low probability extreme events, weather derivatives cover lower risk high probability events like colder than expected winter. Also the buyer of the derivative will receive the payoff at settlement period regardless of the loss caused by the weather events whereas insurance payoff depend on proof of damage. Further from the seller's perspective, weather derivatives eliminate moral hazard and avoid higher administrative costs and loss adjustment expenses of insurance contracts. However, a considerable risk may remain with the producer when using weather derivatives, because individual yield variations in general are not completely correlated with the relevant weather variable (Oliver et al, 2006).

The weather derivative market is a typical example of an incomplete market in the sense that the underlying weather variables are not tradable assets and cannot be replicated by other underlying instruments like in the equity market. Furthermore the market is relatively illiquid.

Campbell and Diebold (2005) suggest that the illiquidity is as a result of non-standardization of the weather. With this one may encounter inefficiencies in the weather derivative market. However, protection is achieved when the transaction counter parties: the hedger who wants to hedge the weather risk exposure and a speculator to whom the risk is transferred in anticipation for a reward meet.

The available academic literature on weather derivatives generally concentrates on two aspects. First, that the weather derivatives fundamentally differ from standard derivatives in that the underlying asset is not tradable. Secondly, for weather derivatives to be effective hedging instruments, good models of weather risks are needed. Filar et al (2008) argues that rather than studying derivative instruments with weather as the underlying asset, researchers and scholars ought to study pricing of derivatives where the underlying asset is sensitive to the weather. Like orange juice can be a classic example of a pure weather asset. In its production one requires extended development time and commitments of land and labor. Therefore, producers are exposed to price shocks predominantly created by adverse weather conditions like colder or warmer than normal temperatures.

Available literature increasingly deals with the question if weather derivatives can also play a role as risk management tools in agriculture, (Richards et al, 2004; Turvey ,2001). Cao et al (2004) proposed a pricing model for rainfall based on daily rainfall in which they calculated a fair premium while ignoring market price of rainfall risk. Carmona and Diko, (2005) proposed a Markov process model for the rainfall process for stochastic dynamics of the underlying rainfall. They assumed the existence of tradable rainfall assets and used utility indifference approach to price derivatives.

Leobacher and Ngare (2011) did construct a Markovian Gamma model for rainfall process with seasonal effects and gives utility indifference prices with exponential utility. Lee and Oren (2010), Hardle and Ospienko (2011) obtained equilibrium prices for weather derivatives on cumulative monthly rainfall by simulating market conditions of two types-farmers with profits exposed to weather and financial market investors aiming to diversify their financial portfolios.

Classical arbitrage theory assumes that stocks can accurately replicate options on tradable assets. However, for derivatives on weather conditions like rainfall, temperature indices cannot rely on hedging principles since the underlying cannot be traded. Therefore, with the market being incomplete there are many equivalent martingales to price rainfall derivatives. Moreover, these futures should be arbitrage free since they are indeed tradable.

As alluded to earlier we sought to find arbitrage free prices for rainfall derivatives via the Esscher transform with a constant Market Price of Risk. Esscher (1932) first introduced the transform for density approximations which was later developed as a general probabilistic model by Barndorff-Nielsen (1997). Gerber and Shiu (1994, 1996) used Esscher transforms in option pricing. Bühlmann et al. (1998), Bingham and Kiesel (1998), Chan (1999), also used the Esscher transform in their studies. We use Leobacher and Ngare (2011)'s model together with the Esscher transform. The only difference is that our model incorporates conditional risk adjusted expectation i.e. the price is given under the risk neutral valuation as opposed to indifference pricing method which ignores the market price of risk.

CHAPTER THREE

METHODOLOGY

3.0 Introduction

Standard pricing approaches for weather derivatives are based on historical weather data. Forward-looking information such as meteorological forecasts or the implied market price of risk is often not incorporated in usual pricing approaches.

In this study, we find arbitrage-free prices for rainfall derivatives by using an equivalent martingale measure via the Esscher transform with a constant market price of risk.

The proposed model captures the typical behaviour of daily rainfall and allows for daily pricing. Moreover, the resulting theoretical prices can be adjusted to market data by calibrating the market price of risk. At first, a standard model for daily rainfall is fitted to the available historical rainfall data. With this, the rainfall can be simulated for every day in the future especially in the period under consideration.

We calculate the prices under the risk neutral measure Q_θ. Since the market is incomplete, there will be many equivalent martingales Q_θ. By using an equivalent martingale $Q= Q_\theta$, we are able to find arbitrage free prices. This, however, requires an additional parameter θ, the market price of risk. Since our assumed distribution is non-normal, an Esscher transform of the distribution is performed with constant market price of risk. The mean of the transformed distribution is the expected price under the risk neutral measure, where θ is calibrated to the market data. Brenda et al (2013) shared this view.

3.1 The Rainfall Model

Carmona and Diko (2000) proposed a time homogenous jump markov process to model the rainfall process. To price the derivatives, they assumed the existence of tradable asset whose price depended on rainfall and relied on the utility indifference method to price the derivatives. This model was later to be improved by Leobacher and Ngare (2011) who constructed a Markovian- Gamma model for rainfall process which accounts for the seasonal effects of rainfall and calculates utility indifference prices with exponential utility.

We intend to exploit this model together with the Escher transform with constant market price rainfall risk to calculate the prices using an equivalent martingale measure.

To account for the seasonal effects of rainfall over a given period Leobacher and Ngare (2011) partitioned the period under consideration into equal sub-periods and separately modelled the total amount of rainfall within each sub-period.

By letting $Y_0, Y_1, Y_2, \ldots$ to be the sequence of total rainfall per sub-period, they assumed that in some sub-period k, the rainfall has a cumulative distribution function (CDF),$F_{\text{kmodm}}, k \geq 0$ where F_k is a continuous function and strictly increasing such that the inverse,$F_k{}^{-1}$ exists with similar properties ,that is, strictly increasing and continuous.

The assumptions above indicate that the sequence $(F_{\text{kmodm}}(Y_k)), k \geq 0$ constitutes of generally dependent random variables U_k uniform on (0, 1) which can generate a future rainfall sample path by setting $Y_k := F_{kmodm}{}^{-1}(U_k), k \geq 0,$ using the standard inverse transform method.

The sequence $F_{\text{kmodm}}(Y_k)$ is a discrete-time Markov process with state space (0, 1) and therefore rainfall amounts of two consecutive months or even the days are not independent.

It is assumed that the rainfall within sub-period k follows a gamma distribution with shape and scale parameters α and β respectively. Then, for some standard normal random variable Z,

$$F_k(z) = \int_{-\infty}^{z} f(y)dy, \qquad where$$

$$f_k(y) = \begin{cases} \frac{1}{\beta_k \Gamma \alpha_k} \left(\frac{y}{\beta_k}\right)^{\alpha_k - 1} e^{-y/\beta_k}, & y > 0, \alpha_k, \beta_k > 0 \\ 0, & y < 0 \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.1.1)$$

3.2 Parameter Estimation

This method is taken from Leobacher and Ngare (2011).

Suppose we are given data $y_0, y_1, \ldots, y_{n-1}$ of precipitation at a specific location with *m* observations per year. We want to fit our model to actual data, that is, we want to find a set of parameters $\alpha_0, \ldots, \alpha_{m-1}, \beta_0, \ldots, \beta_{m-1}$ such that if $F_k$is the CDF of a gamma distribution with parameters α, β for each k then $y_0, \ldots. y_{n-1}$ has the maximum likelihood. If the observations are monthly then *m*=12.

We are supposed to estimate (α_k, β_k) for every month using ordinary maximum likelihood estimation framework.

We note that consecutive observations are hardly independent; and since correlation between consecutive months cannot be ignored, no big error can be expected by assuming that consecutive Januaries are independent.

From this we would have estimated that the CDFs, $F_0, F_1, \ldots, F_{m-1}$ and thus can compute $\Phi^{-1}\big(F_{kmodm}(y_k)\big) \coloneqq z_k$.

3.3 Maximum Likelihood Estimation for Gamma Distribution Using Data Containing Zeros

Wilkis (1990), Leobacher and Ngare (2011) constructed their models using the method below.

Suppose the given data set contains M_0 censored data points (recorded as zeros) of censoring level $A = 0.1$ and M_v points with known values such that $M = M_0 + M_v$. Then the likelihood function for the distribution parameters is given by:

$$\text{Let} \quad Y(\alpha, \beta; y) = \prod_{j=1}^{M_0} G(A; \alpha, \beta) \prod_{i=1}^{M_v} g(y_i; \alpha, \beta)$$

$$\text{then} \qquad Y(\alpha, \beta; y) = [G(A; \alpha, \beta)]^{M_0} \prod_{i=1}^{M_v} \frac{1}{\beta \Gamma \alpha} \left(\frac{y_i}{\beta}\right)^{\alpha - 1} e^{\frac{-y_i}{\beta}}, where$$

$$G(A; \alpha, \beta) = \int_0^A g(y_i; \alpha, \beta) dy = \mathbb{P}[y_j \leq A]$$

If we assume that M_0=0 that is, all the data values are known, then the MLE of the parameters satisfy:

$$\log(\beta) + \varphi(\alpha) = \sum_{i=1}^{M_v} \frac{log y_i}{M_v}$$

$$\alpha - \frac{1}{\beta} \sum_{i=1}^{M_v} \frac{y_i}{M_v} = 0$$

where $\varphi(\alpha) = \frac{dlog[\Gamma\alpha]}{d\alpha}$ is the digamma function. Hence the MLE for $\beta\ and\ \alpha$ can be determined. In the case where$M_0 \neq 0$,

$$L(\alpha, \beta; y) = M_0 \log[G(A, \alpha, \beta)] - M_v[\alpha \log(\beta) + \log(\alpha)] + (\alpha - 1)\sum_{i=1}^{M_v} logy_i - \frac{1}{\beta}\sum_{i=1}^{M_v} y_i$$

…………………………………………………………………………………………… (3.3.2)

Can be evaluated numerically for the values of $\alpha\ and\ \beta$ using any of the available mathematical software like MATLAB.

3.4 The Market Model

The market model is given by stochastic differential equation:

$$dS_t = \mu(Y_t)S_t dt + \sigma(Y_t)S_t\, dZ_t \quad \text{……………………………} (3.4.3)$$

where $Z_t \sim iid\ N(0,1)$ and $\mu(Y_t)\ and\ \sigma(Y_t)$ are measurable functions whose concrete form we are yet to determine.

However we can take $\sigma(Y_t)$ as a constant and evaluate $\mu(Y_t)$ modelled as below:

$$\mu(Y_t) = alog(Y_t) + b, Y_t > 0 \text{………….. ……………………… } (3.4.4)$$

The parameters $a\ and\ b$ are estimated by MLE scheme by combining both the market and rainfall data. That is, given the rainfall records $y_0, y_1, \dots \dots \dots . y_{t-1}, y_t$ and asset prices $s_0, s_1, \dots \dots \dots . s_{t-1}, s_t$ of some hypothetical asset, we can set $\mathfrak{Y}_t = \log(Y_t)$ a.s and

$dS_t = S_t - S_{t-1}$ such that

$$dS_t = (a\mathfrak{Y}_t + b)S_t dt + \sigma S_t dZ_t$$

$$dS_t = \mu S_t dt + \sigma S_t dZ_t$$

$$\int \frac{dS_t}{S_t} = \mu t + \sigma Z_t, \quad Z_t \sim iidN(0,1)$$

Then by MLE the estimates for a, b, σ can be obtained.

$$\frac{InS_t - \mu t}{\sigma} = Z_t \sim N(0,1)$$

$$f(z,t) = \frac{1}{\sqrt{2\pi}}\ exp - \frac{1}{2}[Z_t]^2$$

$$\prod_{t=1}^{n} \frac{1}{\sqrt{2\pi}} \exp(\frac{1}{2\sigma^2}[{P_t}^2 - 2P_t \mu t + \mu^2 t^2]\ )$$

From which we maximize the log likelihood partials as follows

$$l = -\frac{n}{2} log(2\pi) - \frac{1}{2\sigma^2}\sum_{t=1}^{n}[{P_t}^2 - 2P_t \mu t + \mu^2 t^2]$$

$$\frac{\partial l}{\partial a} = \frac{-1}{2\sigma^2}\sum_{t=1}^{n}(-2tP_t\, \mathfrak{Y}_t + 2at^2{\mathfrak{Y}_t}^2 + 2bt^2{\mathfrak{Y}_t}^2) = 0$$

$$\frac{\partial l}{\partial b} = \frac{-1}{2\sigma^2}\sum_{t=1}^{n}(-2tP_t + 2at^2{\mathfrak{Y}_t}^2 + 2bt^2) = 0$$

On solving the two equations we obtain explicit expression for $a\ and\ b$ and even σ as below:

$$a = \frac{\beta\delta - \gamma\alpha}{\varepsilon\delta - \gamma^2} \quad and\ b = \frac{\alpha\varepsilon - \beta\gamma}{\varepsilon\delta - \gamma^2}$$

$$\sigma^2 = \tau + a^2\varepsilon + 2ab\gamma + b^2\delta - 2a\beta - 2b\alpha$$

Where $$n\alpha = \sum_{t=0}^{n-1} tP_t \quad ,$$

$$n\varepsilon = \sum_{t=0}^{n-1} t^2\, \mathfrak{Y}_t{}^2\ ,$$

$$n\beta = \sum_{t=0}^{n-1} tP_t\, \mathfrak{Y}_t\ \ ,$$

$$n\gamma = \sum_{t=0}^{n-1} t^2\, \mathfrak{Y}_t,$$

$$n\delta = \sum_{t=0}^{n-1} t^2$$

$$n\tau = \sum_{n=0}^{n-1} P_t{}^2,$$

$$P_t = InS_t = \log(\frac{S_t}{S_{t-1}})$$

3.6 THE ESSCHER TRANSFORM

3.6.1 Introduction

According to Buhlmann et al (1996), the standard fair pricing in finance uses the no arbitrage notion, that is, there is no such a thing as riskless gain. The mathematical formulation of this economic principle brings in the fundamental notion of risk neutral martingale measure. In practice, we mostly deal with incomplete markets. Consequently, risk cannot be fully hedged away and, in most cases, there will be infinitely many such equivalent martingale measures so that pricing is directly linked to an attitude towards risk. Whereas in classical insurance, the question would be ‘which premium principle to use’. Within the incomplete finance context it becomes “which equivalent martingale to use“. It is exactly at this point that the Esscher transform enters as a possible pricing candidate.

The Esscher transform is a tool in Actuarial Science. It is also an efficient technique for valuing derivative securities if the logarithms of the prices of the primitive securities are governed by certain stochastic processes with stationary and independent increments. This family of processes includes the Wierner process, the Poisson, the Gamma and the inverse Gaussian processes. These types of processes are called the Levy Processes (LP). They offer flexibility for accounting for basic features of financial series, that is, skewness, excess kurtosis, frequent small and large jumps. To be applicable for pricing derivatives, statistical distributions have to be adjusted for market price of risk and turned into martingale processes This is done by applying the Esscher transform to statistical processes.

The market price of risk is an equivalent property of the equivalent martingale measure. It is therefore required in order to derive an expression for future rainfall prices in this scheme. We first need to specify the risk neutral probability Q. We can say, safely, that Q~P (P is the physical measure) such that all tradable assets in the market are martingales after discounting.

The rainfall derivatives market is inherently incomplete since the weather is not a tradable asset.

It is not possible to find a unique risk neutral measure Q, the equivalent martingale. Therefore many martingales exist and according to (Jensen and Nielsen, 1996), (Benth 2004) only bounds for prices on contingent claims can be provided based on no -arbitrage principles.

We therefore, in this case, specify a class of probability measure using the Esscher transform which will provide us with market price of risk parameterized by a parameter θ. Buhlman 1980 derived the Esscher premium as a Pareto-optimal solution to a market situation where all the participants are characterized by an exponential utility function with all the risks being stochastically independent. The parameter θ represents risk aversion of the market participants.

The Esscher transform changes a probability density $f(x)$ of a random variable X to a new probability density$f(x,\theta)$) given as

$$f(x,\theta) = \frac{\exp(\theta x)\, f(x)}{\int_{-\infty}^{\infty} \exp(\theta x)\, f(x)\, dx}$$

Since the transformation is state dependent, the state price density $f(x)$ can be represented in an exponential form (Duffie and Kan ,1996).

3.6.2 Equivalent Martingale Measure Q Using the Esscher Transform

Modern financial derivatives theory is mainly based on martingale theory.

We consider a given filtration (flow of information) $\{\mathcal{F}_t\}_{t\geq 0}$. $\mathcal{F}_t$ can be thought of as the information generated by all the observed events up to time t. Therefore for any stochastic variable Y we let $E(Y/\mathcal{F}_t)$ denote the expected value of Y given the information available after at time t.

It is important to note that for a fixed t, then $E(Y/\mathcal{F}_t)$ is a stochastic variable. Indeed if the filtration is generated by a single observed process say X, then the information available at time t will depend on the behavior of X over the interval $[0, t]$.So the conditional expectation $E(Y/\mathcal{F}_t)$ will in this case be a function of all past values of X,that is $\{X_s : s \leq t\}$

Proposition 3.6.3

If $Y\ and\ Z$ are stochastic variables, and $Z\ is\ \mathcal{F}_t$ measurable, then $E[Z.Y|\mathcal{F}_t] = Z.E(Y/\mathcal{F}_t)$

Also $E[E[Y|\mathcal{F}_t]|\mathcal{F}_s] = E[Y|\mathcal{F}_s\ ]$

In the expected value $E[Z.Y|\mathcal{F}_t]$ we condition upon all information available at time t.Then if $Z\epsilon\mathcal{F}_t$, it means that given the information $\mathcal{F}_t$,we know the exact value of Z ,so we can treat its conditional expectation as a constant and thus taken outside the expectation. The second result is the law of iterated expectations which is basically the law of total probability. We can therefore proceed to define the martingale concept.

Definition

A stochastic process X is referred to as $(\mathcal{F}_t)$ martingale if the following hold:

i) X is adapted to the filtration $\{\mathcal{F}_t\}_{t\geq 0}$

ii) For all t, $E[|X_t|] < \infty$

iii) For all $s\ and\ t$ with $s \leq t$, then $E[X(t)|\mathcal{F}_s] \leq X(s)$

Definition: Equivalent probability measures

Let $P\ and\ Q$ be probability measures on $(\Omega, \mathcal{F})$

Lemma 3.6.4

For two probability measures $P\ and\ Q$, the relation $P{\sim}Q\ on\ \mathcal{F}$ holds if and only if:

$P(A) = 1 \leftrightarrow Q(A) = 1$, for all $A \in\ \mathcal{F}$

Proof

We note that, in the context of probability measures, although two equivalent measures $P\ and\ Q$ may assign totally different probabilities to a fixed event A, all events impossible under P,

that is ,$P(A) = 0$ are also impossible under Q. Equivalently all events possible under $P, that\ is, P(A) = 1$ are also possible under Q.From the definition it also follows that if an event has a strictly positive $P - probability$, then it also has a strictly positive $Q - probability$.

The study is on a risk neutral distribution, a martingale measure associated with a Markovian gamma process.

A perfect hedge cannot be obtained and there is always a residual risk that cannot be hedged. Indeed, there are many different equivalent martingale measures under which the discounted asset price process is a martingale. The existence of a martingale is related to the absence of arbitrage, while uniqueness of a martingale measure is related to market incompleteness., that is, perfect hedging.

Gerber and Shiu (1994) proposed one approach for finding an equivalent martingale measure using the Esscher transform. According to their findings, given a statistical model P, the Esscher transform induces an equivalent probability measure Q and a martingale process. The Esscher parameter is determined so that the discounted asset price is a martingale under the new probability measure Q**.**

Let

$$S_t = S_0 e^{X_t}, \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots .. (3.6.5)$$

where $\{X_t\}_{t\geq 0}$ is a process with stationary and independent increments and $X_0 = 0$ then for each t the random variable X_t has an infinitely divisible distribution with probability density given by:

$$f(x,t), t > 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.6.6)}$$

In addition, the moment-generating function, assumed to exist, is defined as-

$$M(u,t) = E[e^{uX_t}] = \int_{-\infty}^{\infty} e^{ux} f(x,t)dx \ldots\ldots\ldots\ldots\ldots \text{(3.6.7)}$$

Assuming that $M(u,t)$ is continuous at $t = 0$, then by infinite divisibility:-

$$M(u,t) = [M(u,1)]^t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.6.8)}$$

Let θ be a real number such that $M(\theta) = \int_{-\infty}^{\infty} e^{\theta x} f(x)dx$ exists, then the Esscher transform of $\{X_t\}_{t\geq 0}$ with parameter θ is defined as a Levy Process with stationary and independent increments where the new probability density of $X_t, t > 0$, is :

$$f(x,t;\theta) = \frac{e^{\theta x} f(x,t)}{\int_{-\infty}^{\infty} e^{\theta y} f(y,t)dy} = \frac{e^{\theta x} f(x,t)}{M(\theta,t)} \ldots\ldots\ldots\ldots\ldots\ldots \text{(3.6.9)}$$

The modified distribution of $X(t)$ is the Esscher transform of the original distribution whose moment-generating function given by :-

$M(u,t;\theta) = \int_{-\infty}^{\infty} e^{ux} f(x,t;\theta)dx = \frac{M(u+\theta,t)}{M(\theta,t)}$, and

$$M(u,t,\theta) = [M(u,1;\theta)]^t$$

Therefore, an equivalent Esscher measure is given by:

$$\frac{dQ/F_t}{dP/F_t} = \frac{e^{\theta X_t}}{E(e^{\theta X_t})} = \exp(\theta X_t - t\log(M(\theta)))$$

Proposition 3.6.5

The Esscher measure of a gamma process has a MGF at $t = 1$ given by:

$$\left\{\frac{1-\theta\beta}{1-(u+\theta)\beta}\right\}^{\alpha} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.6.5.1)}$$

Proof:

For $M(u+\theta) = \int_0^\infty e^{(u+\theta)y} \frac{1}{\beta^\alpha\Gamma\alpha} y^{\alpha-1} e^{\frac{-y}{\beta}} dy$

$$= \frac{1}{\beta^\alpha\Gamma\alpha}\int_0^\infty y^{\alpha-1} e^{-\left(\frac{1-\beta(u+\theta)}{\beta}\right)y} dy$$

$$= \frac{1}{\beta^\alpha\Gamma\alpha}\int_0^\infty y^{\alpha-1} e^{-\left(\frac{1-\beta(u+\theta)}{\beta}\right)y} dy * \left[\frac{\frac{1-\beta(u+\theta)}{\beta}}{\frac{1-\beta(u+\theta)}{\beta}}\right]^{\alpha-1}$$

$$=\left(\frac{\beta}{1-\beta(u+\theta)}\right)^{\alpha-1} \frac{1}{\beta^\alpha\Gamma\alpha}\int_0^\infty \left(\frac{(1-\beta(u+\theta)y}{\beta}\right)^{\alpha-1} e^{-\left(\frac{1-\beta(u+\theta)}{\beta}\right)y} dy \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.6.5.2)}$$

Now let $y_* = \left(\frac{1-\beta(u+\theta)}{\beta}\right) y$

Therefore $dy = \left(\frac{\beta}{1-\beta(u+\theta)}\right) dy_*$

Equation (11) becomes

$$\left(\frac{\beta}{1-\beta(u+\theta)}\right)^{\alpha-1} \frac{1}{\beta^\alpha\Gamma\alpha}\int_0^\infty (y_*)^{\alpha-1} e^{-y_*} \left(\frac{\beta}{1-\beta(u+\theta)}\right) dy_* \ldots\ldots\ldots \text{(3.6.5.3)}$$

Recall: $\Gamma\alpha = \int_0^\infty y^{\alpha-1}\, e^{-y}\, dy$

This leads (12) to $\left(\frac{\beta}{1-\beta(u+\theta)}\right)^{\alpha-1} \frac{1}{\beta^\alpha \Gamma\alpha} \left(\frac{\beta}{1-\beta(u+\theta)}\right) \Gamma\alpha$

$= \left(\frac{1}{1-\beta(u+\theta)}\right)^{\alpha}$ …………………………………………………………………… ……... (3.6.5.4)

And for $M(\theta,1) = \int_0^\infty e^{\theta y}\ \frac{1}{\beta^\alpha \Gamma\alpha} y^{\alpha-1} e^{\frac{-y}{\beta}} dy$

$= \frac{1}{\beta^\alpha \Gamma\alpha} \int_0^\infty y^{\alpha-1}\, e^{\frac{-(1-\beta\theta)y}{\beta}} dy$

$= \frac{1}{\beta^\alpha \Gamma\alpha} \int_0^\infty y^{\alpha-1}\, e^{\frac{-(1-\beta\theta)y}{\beta}} dy * \left(\frac{\frac{1-\beta\theta}{\beta}}{\frac{1-\beta\theta}{\beta}}\right)^{\alpha-1}$

$= \frac{1}{\beta^\alpha \Gamma\alpha} \left(\frac{\beta}{1-\beta\theta}\right)^{\alpha-1} \int_0^\infty \left(\left(\frac{1-\beta\theta}{\beta}\right)y\right)^{\alpha-1}\ e^{-\left(\frac{1-\beta\theta)y}{\beta}\right.} dy$

Let $y_* = \frac{y(1-\beta\theta)}{\beta}$, then $dy = \left(\frac{\beta}{1-\beta\theta}\right) dy_*$

Which now gives rise to, as above,

$$= \frac{1}{\beta^\alpha \Gamma\alpha}\left(\frac{\beta}{1-\beta\theta}\right)^{\alpha-1} \int_0^\infty (y_*)^{\alpha-1}\, e^{-y_*} \left(\frac{\beta}{1-\beta\theta}\right) dy_*$$

$$= \left(\frac{\beta}{1-\beta\theta}\right)^{\alpha-1} \frac{1}{\beta^\alpha \Gamma\alpha} \left(\frac{\beta}{1-\beta\theta}\right) \Gamma\alpha = \left(\frac{1}{1-\beta\theta}\right)^{\alpha} \text{ ... ... (3.6.5.4)}$$

Dividing (3.6.5.3) by (3.6.5.4) completes the proof.

The probability measure of the process has in fact changed and its exponential function is positive. Therefore, the modified probability measure is equivalent to the original probability measure, that is, they both have same null sets –sets of probability measure zero.

The parameter θ is determined so that the modified probability measure Q is an equivalent martingale measure to the original statistical probability measure P. The aim is to find $\theta = \theta^*$, so that the discounted stock price process $\{e^{-rt} S_t\}_{t\geq 0}$ is a martingale with respect to the probability measure corresponding to θ^*

With the martingale condition that,

$$S_0 = E^Q[e^{-rt}S_t] = e^{-rt}E^Q[S_t] ,$$

the parameter θ^* is a solution to:-

$$S_0 = E^Q[e^{-rt}S_t] = e^{-rt}E^Q[S_0e^{X_t}]$$

$$= e^{-rt}S_0\frac{E^P\left[e^{(\theta+1)X(t)}\right]}{E^P[e^{\theta X(t)}]} = e^{-rt}S_0\frac{M(\theta+1,t)}{M(\theta,t)}$$

where r is the constant risk free rate of interest.

This is equivalent to:

$$1 = e^{-rt}E^Q[e^{X_t}] \; or \; e^{rt} = M(1,t;\; \theta^*)$$

We note that the solution is independent of t and then by setting $t = 1$, we obtain

$e^r = M(1,1;\theta^*)$

And in logarithm form, the parameter is a solution to:-

$$r = \log[\mathrm{M}(1+\theta^*)] = \log[\mathrm{M}(1+\theta^*;1)] - \log[\mathrm{M}(\theta^*,1)]$$

That is,

$$r = \log\left[\left\{\frac{1-\theta\beta}{1-(u+\theta)\beta}\right\}^{\alpha}\right] = \alpha\left[\log(1-\beta\theta) - \log\left(1-\beta(\mu+\theta)\right)\right] \ldots\ldots\ldots\ldots (3.6.5.5)$$

3.6.6 The Escher Transform Parameter $\boldsymbol{\theta}$

From equation (3.6.5.5) we have:

$$e^{r} = \left\{\frac{1-\theta\beta}{1-\beta(\theta+1)}\right\}^{\alpha}$$

$$e^{\frac{r}{\alpha}} = \left\{\frac{1-\theta\beta}{1-\beta(\theta+1)}\right\}$$

It therefore follows from above that *theta* can be expressed in terms of the other parameters as:

$$\theta = \frac{e^{\frac{r}{\alpha}}(1-\beta)-1}{\beta(e^{\frac{r}{\alpha}}-1)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.6.5.6)$$

We need to show that θ^* is unique. We know that the parameter $\theta = \theta^*$ is chosen such that the process $\{e^{-rt}S_t\}_{t\geq 0}$ is a martingale with respect to the probability measure corresponding to θ^*.

Precisely, $S_0 = E[e^{-rt}S_t; \theta^*]$; hence $e^{rt} = E[e^{X_t}; \theta^*] = [M(1,1,\theta^*)]^t$

that is, $$r = \log[M(1,1;\theta^*)]$$

The Esscher measure corresponding to the parameter θ^* is the risk neutral Esscher measure.

Proposition 3.6.7

For a given time interval $[0,T], where\ T > 0$, the underlying asset price S_t is determined by the model $S_t = S_0 exp(X_t)$ with X_t is identically distributed with independent and stationary increments. The payoff function is given by :

$$V(S_T) = \max(S_T - K, 0) = f(x) = \begin{cases} S_T - K, for\ S_T > K \\ 0, for\ S_T\ \leq K \end{cases}$$

Is a European pay off function, where K is the option strike price.

Then the European call option price C_0 at $t = 0$ is given by:

$$C_0 = S_0[1 - F(g|T, \theta^* + 1)] - Ke^{rT}[1 - F(g|T, \theta^*)]$$

Where g ≡ log $\left({}^{K}\!/_{S_0}\right)$ and $F(g|T, \theta^* + 1)$ is the risk neutral probability measure.

Proof

The price of a derivative security, whose payments depend on $\{S_t\}$ is calculated as a discounted expected value where the expectation is taken with respect to the risk -neutral Esscher measure.

The value of a European option, at time $t = 0$, whose exercise price and date are $K\ and\ t$ respectively is given as :

$$E^Q[e^{-rt}(S_t - K)_+] = e^{-rt} \int_\tau^\infty [S_t e^x - K] f(x, t; \theta^*)\, dx$$

$$= e^{-rt} S_t \int_\tau^\infty e^x f(x, t; \theta) dx - e^{-rt} K[1 - F(\tau, t; \theta^*)] \text{ , for } \tau = \log \left[{}^{K}\!/_{S(0)}\right]$$

It therefore follows that:

$$e^x f(x,t;\theta) = \frac{e^{(\theta^*+1)} f(x,t)}{M(\theta^*,t)} = \frac{M(\theta^*+1,t)}{M(\theta^* t)} f(x,t;\theta^*+1)$$

$$= M(1,t;\theta) f(x,t;\theta^*+1) = e^{-rt} f(x,t;\theta^*+1)$$

Letting $I(.)$ denote the indicator function and as above $\tau = \log[\frac{K}{S(0)}]$,the price of the option at $t = 0$ is :-

$$e^{-rt} E[(S_t - K) I(S_t > K); \theta^*]$$

$$= e^{-rt} E[S_t I(S_t > K; \theta^*] - e^{-rt} K[I(S_t > K; \theta^*]$$

The expectation on the right hand side is equivalent to

$$\Pr[S_t > K; \theta^*] = 1 - F(\tau, t; \theta^*)$$

Thus, the price of a European call option with exercise price $K\ and\ date\ t$ can be given as:

$P_{EC} = S_t[1 - F(\tau, t; \theta^* + 1) - e^{-rt} K(1 - F(\tau, t; \theta]$ ……………………………… (3.6.5.7)

Accordingly (3.6.5.7) can be written as

$S_0 \Pr[S_\tau > K; \theta^* + 1] - e^{-rt} KPr[S_\tau > K; \theta^*]$………………………………… ….. (3.6.5.8)

We intend to use this expression (3.6.5.8) in our numerical implementation later.

3.7 The Black Scholes Model

The seminal paper of Black-Scholes (1977) provides an analytical framework for the pricing of contingent claims and in particular options. However the Black-Scholes formula relies on some fairly stringent assumptions. Most importantly it assumes that the underlying process is driven by geometric Brownian motion:

$$\frac{dX_t}{X_t} = \mu dt + \sigma dW_t$$

Then if $F(X_t) = \log(X_t)$, then by Ito's formulae :

$dF = \left(\mu - \frac{1}{2}\sigma^2\right) dt + \sigma dW_t$, which with the initial condition, X_0, has a solution given by:

$$F(X_t) = \log X_0 + \left(\mu - \frac{1}{2}\sigma^2\right)(t - t_0) + \sigma W_{t-t_0}$$

$$= X_t = X_0 e^{[\left(\mu - \frac{1}{2}\sigma^2\right)(t-t_0)+\sigma W_t]}$$

And by using the martingale approach it results into:

$V_t = E_q[X_0 e^{\left[\left(\mu - \frac{1}{2}\sigma^2\right)(t-t_0)+\sigma W_t\right]}|F_t]$, where Q represents the risk neutral martingale measure and V_t is the payoff of the option contract.

CHAPTER 4

DATA ANALYSIS AND RESULTS

4.1 Data Analysis and Numerics

We obtain daily rainfall data from the Kenya Meteorological Department for Dagoretti weather station in Kenya. The data spans a period of 11 years (2002-2012).We calculate the monthly averages and estimate the parameters, α and β from which we then proceed to plot density curves for all the months (Fig 4.1). From the curves, it is evident that the rainfall pattern closely (approximately) follows both the gamma and NIG distributions.

Also we obtained daily share prices for the Kenya Power Ltd from the Nairobi Securities Exchange for the same period (2002-2012). This was used to estimate the parameters for the claimed market model.cf equation (3.4.3).We adopted the KPLC share price because electricity in Kenya is mainly hydro generated and therefore rainfall dependent.

In the empirical analysis, we use the simulated paths of daily rainfall amounts under the historical measure P which we will then shift using the parameter as determined by the Esscher transformation.

Parameter Estimation

The rainfall data obtained from the Kenya Meteorological Department was analysed under the Gamma and Normal Inverse Gaussian distributions. The associated parameters were then estimated using the maximum likelihood estimation scheme given in section 3.3. The estimated parameters are given below in Table 4.1and 4.2 respectively correct to 4 decimal places.

Table 4.1: Gamma Distribution parameters

Month	Shape	Rate		Month	Shape	Rate
Jan	2.1293	0.8046		July	0.45450	0.0861
February	1.2291	0.3949		August	0.6908	0.1777
March	2.1856	0.9364		September	0.3887	0.2532
April	0.3644	0.0910		October	0.3675	0.2632
May	1.4837	0.3572		November	1.0196	0.2922
June	0.6193	0.3136		December	0.6226	0.2661

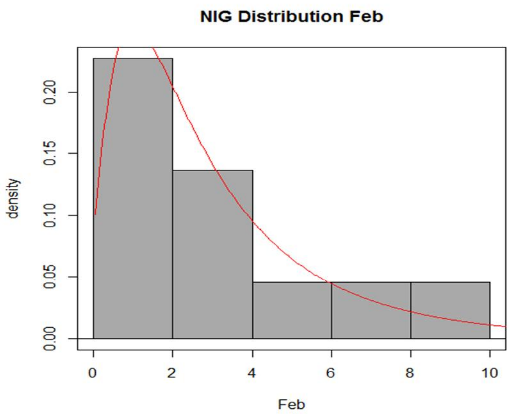
Table 4.2: Normal Inverse Gaussian distribution Parameters

	January	February	March	April	May	June
Alpha	115.8912	375.3693	169.2344	291.7944	12.1659	405.4666
Beta	3.0502	375.1475	-34.3779	291.6410	-10.2764	405.3924
Delta	280.5836	0.1324	232.8769	0.1677	12.6333	0.0396
Mu	- 4.7410	-0.7373	50.6416	-1.1672	24.1080	-0.0955

Table 4.2 cont': Normal Inverse Gaussian distribution Parameters

	July	August	September	October	November	December
Alpha	158.6296	237.3810	734.7128	270.6253	325.0203	341.8861
Beta	158.6173	237.3639	734.2090	-42.1334	324.8511	341.7867
Delta	0.0678	0.0452	0.0771	298.8518	0.1406	0.0615
Mu	-0.1477	0.1129	-0.5462	48.4987	-0.8656	-0.2111

To justify the claim that the rainfall per period follows the gamma distribution, the density plots and histograms for the determined parameters for the months of February are below. The other plots and histograms for the rest of the months are included in the appendix.



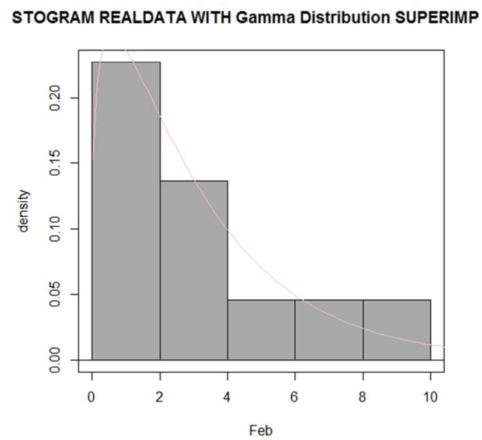


Fig 4.1 .Density Curve for Rainfall Process

4.2 Estimation of the Market Model Parameters.

From the model in equation (3) and equation (4) we sought to estimate the actual parameters based on the rainfall and share price data at hand. The parameters are estimated through a code written in R. Thus by using the monthly data, the estimated parameter values obtained are as in Table 4.3.

Table 4.3 Market Model Parameter Values

	a	b	μ	σ
Jan	-0.0241	0.6901	0.6708	1.9740
Feb	0.1576	0.5648	0.7216	1.7347
Mar	-0.1446	0.7680	0.6859	1.8896
Apr	0.0228	0.7075	0.6711	1.9227
May	0.0166	0.6458	0.6665	2.0031
Jun	-0.0704	0.6398	0.7045	1.9005
Jul	0.05980	0.7260	0.7065	1.8626
Aug	0.05069	0.6353	0.6571	2.0596
Sept	0.00573	0.6790	0.6678	2.1153
Oct	0.01682	0.6602	0.6682	2.1206
Nov	0.03116	0.6357	0.6682	2.1205
Dec	-0.0520	0.6802	0.6796	2.0936

We observe from table 4.3 that all months have their mean $\mu > 0$ and the deviation from the mean, that is the stock volatility is significantly positive.

Table 4.4 Esscher Parameter Values

Month	$\boldsymbol{\theta}$	$\mathbf{1-\beta\theta}$	$\mathbf{1-\beta(\theta+1)}$
January	-19.056978	24.68503	23.44218
February	-11.286193	29.57988	27.04759
March	-19.436885	21.75703	20.68911
April	-3.746845	42.17412	31.18511
May	-13.637160	39.17794	36.37839
June	-5.831194	19.59437	16.40559
July	-4.565867	54.02982	42.41541
August	-6.615564	38.22884	32.60137
September	-3.803988	16.02365	12.07420
October	-3.602615	14.68775	10.88836
November	-9.485880	33.46365	30.04134
December	-5.908616	23.20449	19.44651

Table 4.4 presents the values for theta and transformed scale parameter for each month based on the estimated parameters. From table 4.4 we realize that the value of theta varies roughly between -19 and -2, that is $-19 \leq \theta \leq -2$. With values of $\theta \neq 0$ will change the mean and variance of the transformed distribution

With the monthly Esscher parameter determined, which in this case is regarded as the market price of risk, it is possible to determine the risk neutral prices of the derivative under Q_θ using equation (3.6.5.8). The estimated prices as a result of the Esscher transformation and those due to the Black-Scholes implementation are in Table 4.5.

From the Table 4.5 we observe that the prices increase with the strike price when we consider the Black-Scholes pricing scheme. They however increase with increase in the strike price when we consider the esscher pricing method.

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

Weather risk and in particular rainfall risk is of great interest to researchers lately. Several methods on pricing and hedging derivatives have been proposed. For example, to price under the no-arbitrage condition, we need an equivalent martingale measure. In an incomplete market model, there are an infinite number of equivalent martingale measures. Also in an incomplete market model, any martingale measure which is equivalent to the physical measure, is a potential pricing measure. In this research we adopted the risk neutral conditional Esscher transform to determine an equivalent martingale measure and used the resulting measure to price the options.

In this research focus was on the European call options. The resulting pricing formulae was developed based on the equivalent martingale measure Q.Moreover we were able to conduct a comparison between the prices for the European call options obtained by our equivalent martingale measure method with those by the standard Black-Scholes method.

The model of a gamma distribution shifted by an Esscher transform parameter is used to obtain the risk neutral prices of options. The prices are hypothetical since they are not obtained from actual trading. This is so because there is no established derivative market in the region. The prices vary depending on the values of K, the strike price (see Table 4.5). And according to the shift parameter and by extension the actual rainfall distribution parameters and thus the rainfall process. We observe from table of prices that, except for some two months, the option prices under the Esscher scheme increase with increase in the strike price whereas the prices under the Black-Scholes scheme decrease with increase in K.

We observe from table 4.4 that the market price of rainfall risk changes in size with passing time.it does not necessarily increase and become positive during warmer months. This means that months with extreme amounts of rainfall like April have greater risk premium .A negative (positive) estimate of the MPR implies that the monthly rainfall under Q_θ coincides with the index written on the same underlying under P ; with the higher (lower) expected drift.

This is so because hedgers decide to enter contracts in presence of negative expected payoffs to eliminate their risk since this hedging instrument is less expensive than insurance contracts. To compensate in speculators from baring hedger's risk, there must be an expectation of increasing future prices.

From table 4.5, it can be realized that the derivative prices can be adjusted by adjusting the value of θ, the MPR . In actual trading, the MPR can be chosen such that the price that results under Q equals the market price as a result of actual trading. This appropriate choice of MPR is called the implied market price of rainfall risk since it's calculated from actual data.

Recommendations

A method on how to calculate risk-neutral prices for rainfall derivatives has been presented. In this method, a standard model for the rainfall process is used for simulation. In particular, we used a markovian gamma model represent the rainfall process. We then shift the rainfall process distribution by the Esscher transform to obtain the risk neutral prices. This procedure is flexible and can be applied to any rainfall derivative. The Esscher parameter θ, describes the market price of rainfall risk and can be calibrated from real market data.

Rainfall derivatives do not trade in the Kenyan market. In fact, they were recently introduced in the Chicago Mercantile Exchange. Therefore the reported prices are actually hypothetical prices since they are not from actual trading.

Hopefully, in the near future, when derivative trading gets established in Kenya or in the region, similar approaches can be used to investigate the behavior of rainfall derivatives and their use in managing risk associated with the weather and the nature of the market price of risk.

Our calculation can be used for daily trading to analyze temporal behavior of market price of risk and spatial behavior among different regions in the country.

It may be of great interest to investigate the more general form of univariate and multivariate esscher transforms, and the possibility of representing an ad hoc risk neutral measure through the esscher transform.

Further studies are recommended in directions of both hedging and pricing options under regime switching models. Of particular interest would be to explore application of Esscher transform to more complex derivatives, and dynamic hedging strategies in the incomplete markets. Further research can consider optimal hedging under regime switching lognormal models. We are remark that the hedging may not be simply obtained from pricing due to lack of a replicating process for regime switching. Thus we recommend use of a dynamic optimization process

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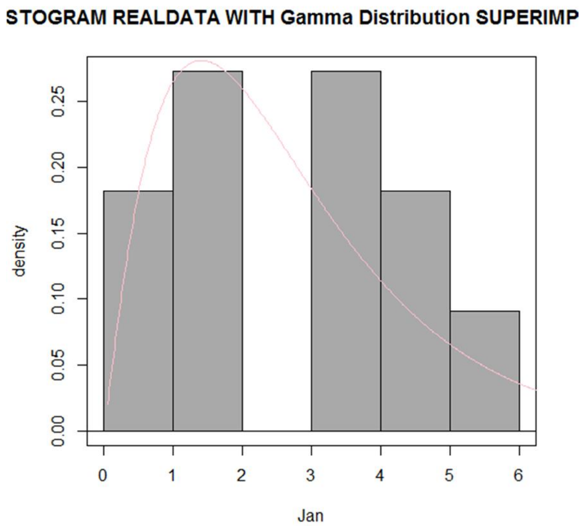
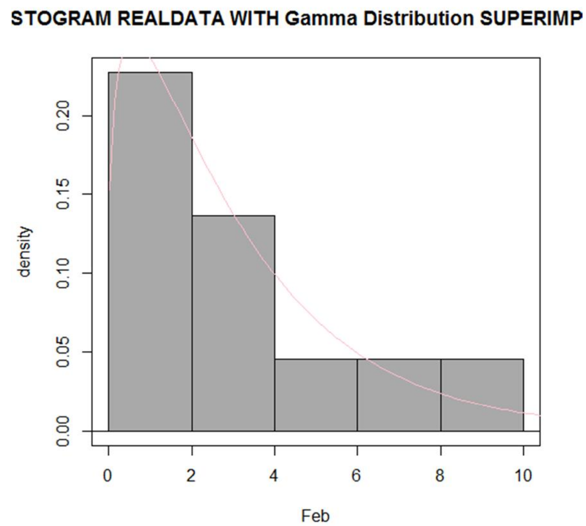
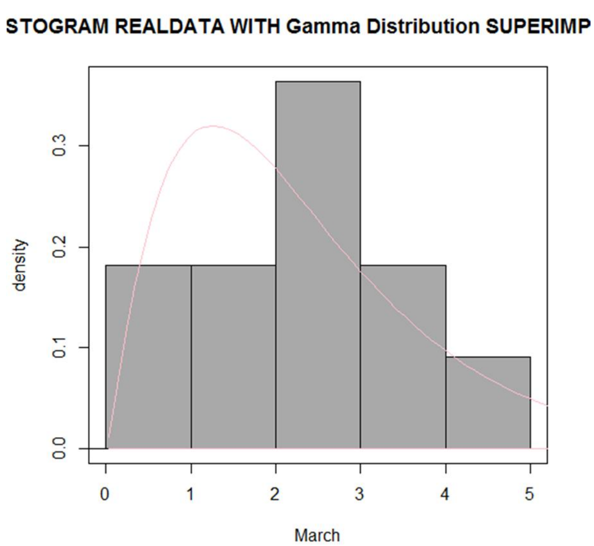
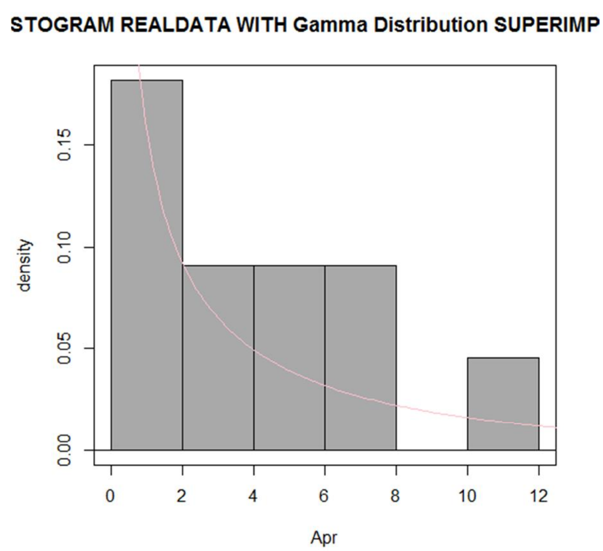
APPENDIX:

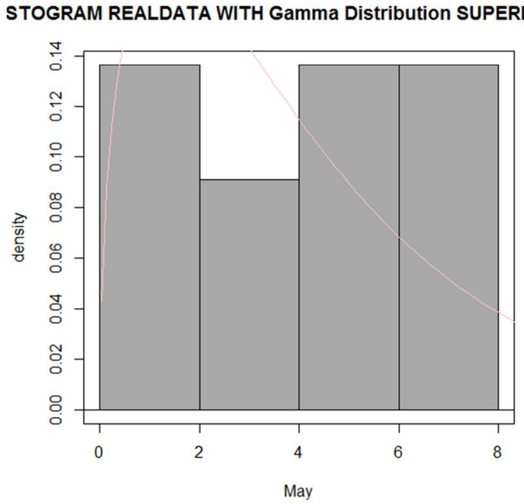
Density Curves for the rainfall process -Gamma Distribution

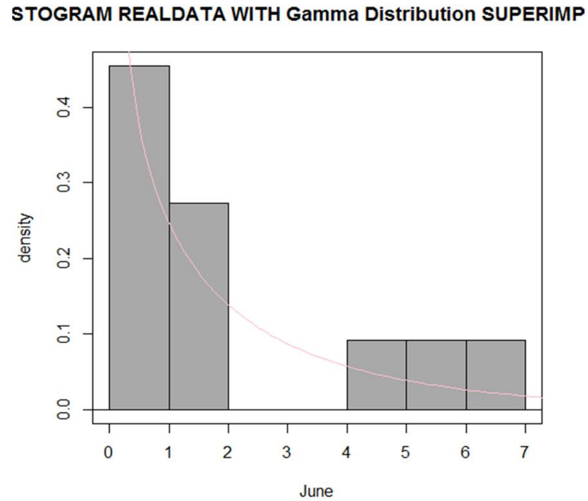


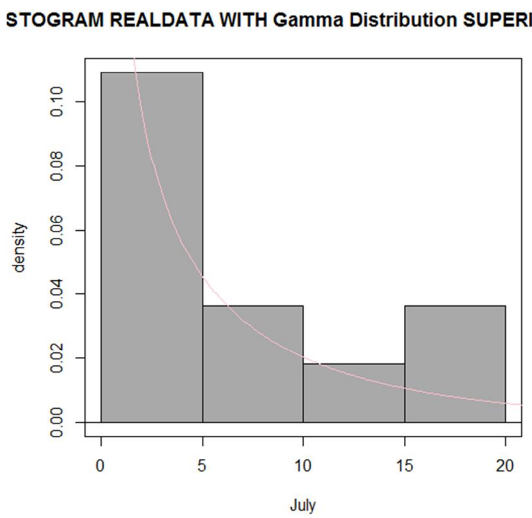


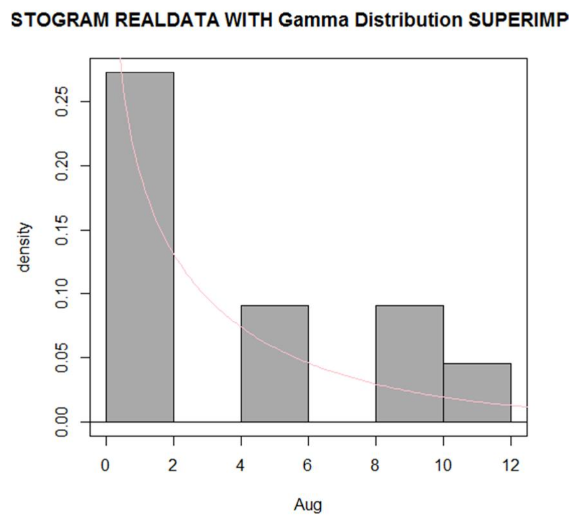


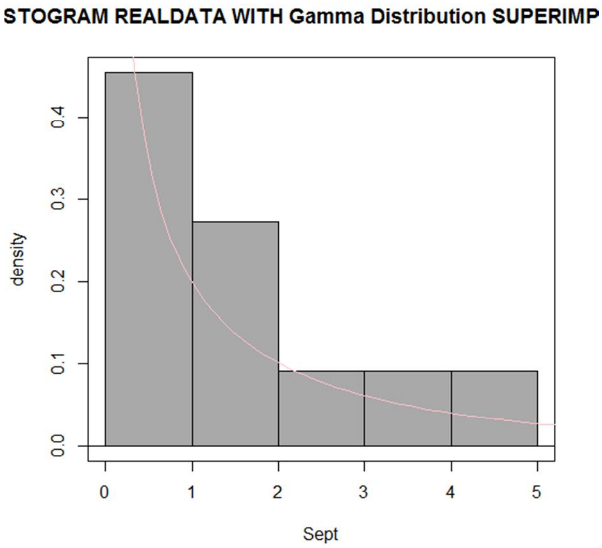


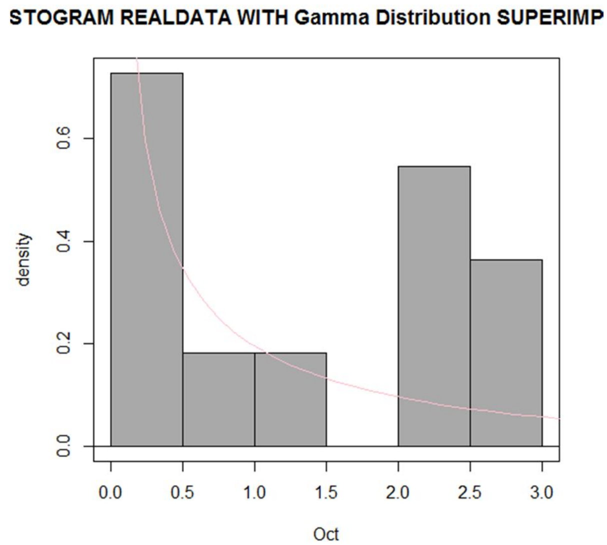

STOGRAM REALDATA WITH Gamma Distribution SUPERIMP
density
May

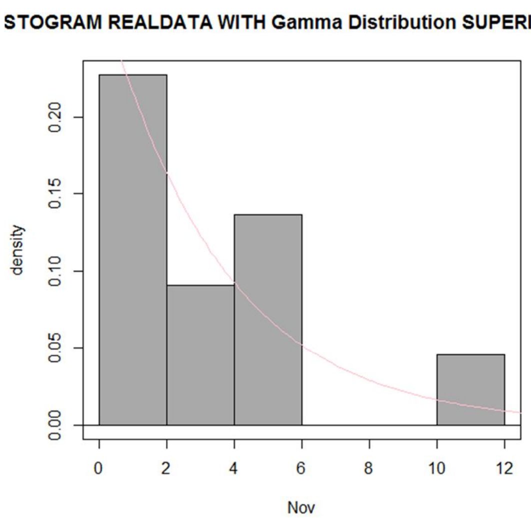

STOGRAM REALDATA WITH Gamma Distribution SUPERIMP
density
June

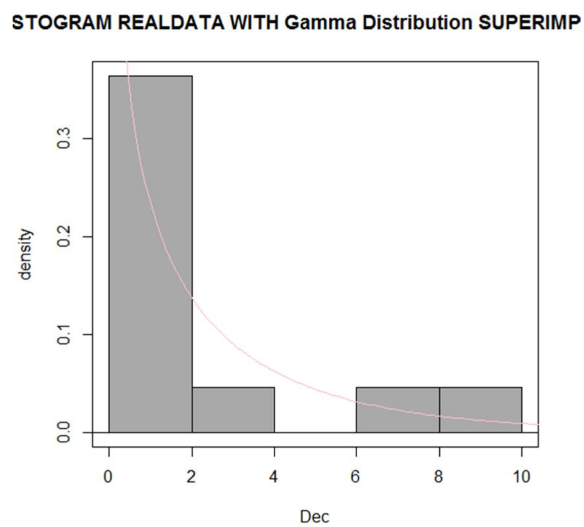

STOGRAM REALDATA WITH Gamma Distribution SUPERIMP
density
July


STOGRAM REALDATA WITH Gamma Distribution SUPERIMP
density
Aug

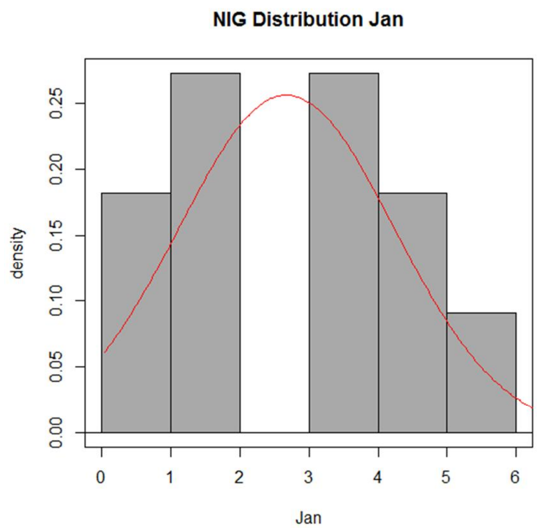

STOGRAM REALDATA WITH Gamma Distribution SUPERIMP
density
0.0
0.1
0.2
0.3
0.4
0
1
2
3
4
5
Sept

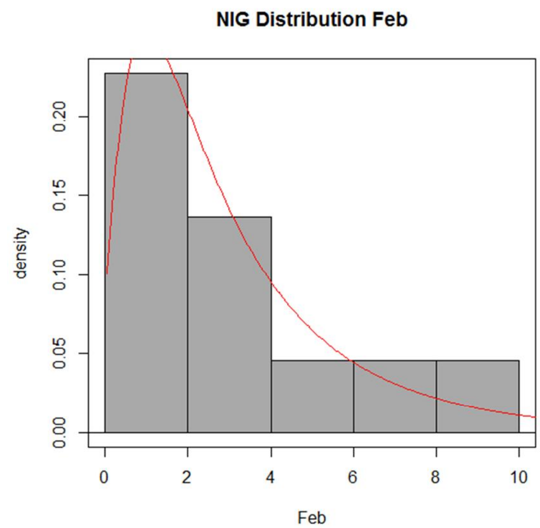

STOGRAM REALDATA WITH Gamma Distribution SUPERIMP
density
0.0
0.2
0.4
0.6
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Oct

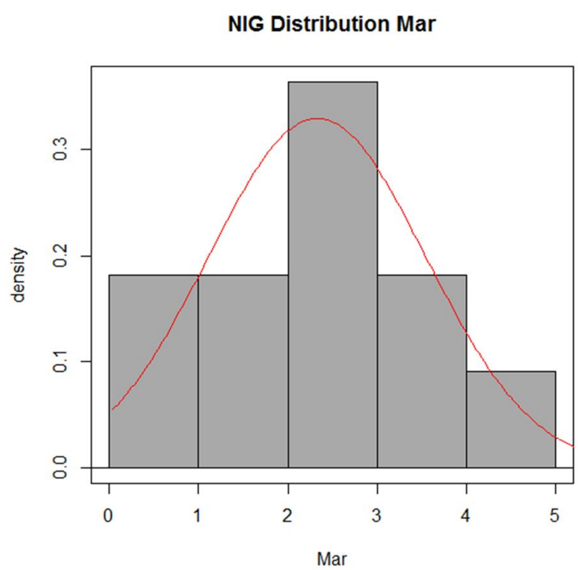

STOGRAM REALDATA WITH Gamma Distribution SUPERIMP
density
0.00
0.05
0.10
0.15
0.20
0
2
4
6
8
10
12
Nov

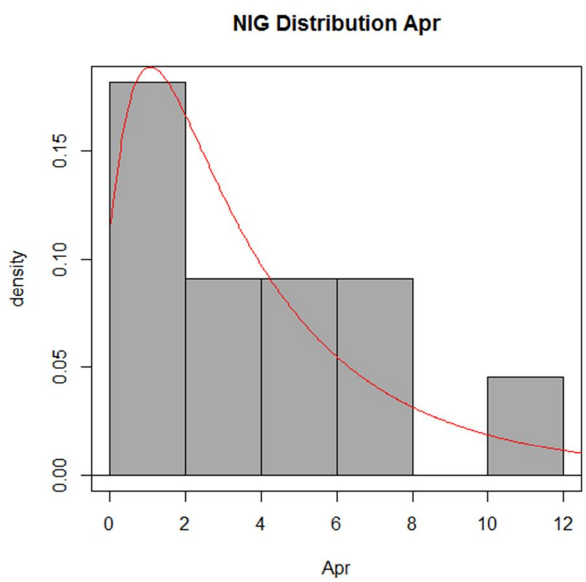

STOGRAM REALDATA WITH Gamma Distribution SUPERIMP
density
0.0
0.1
0.2
0.3
0
2
4
6
8
10
Dec

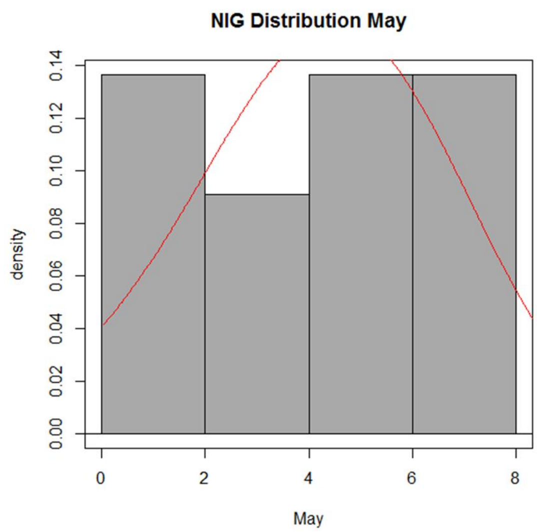
NIG Distribution Histograms and density curves

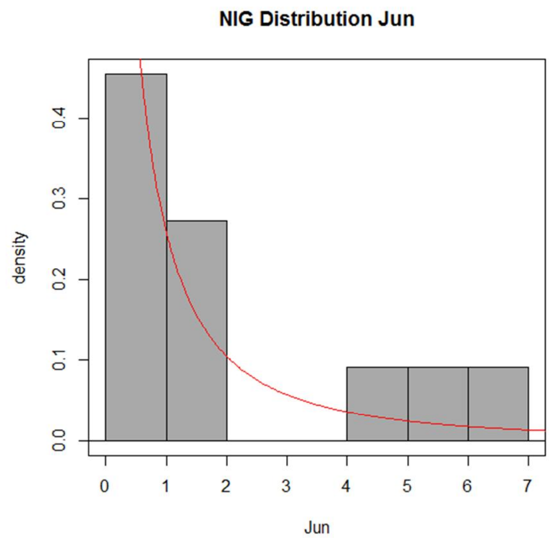


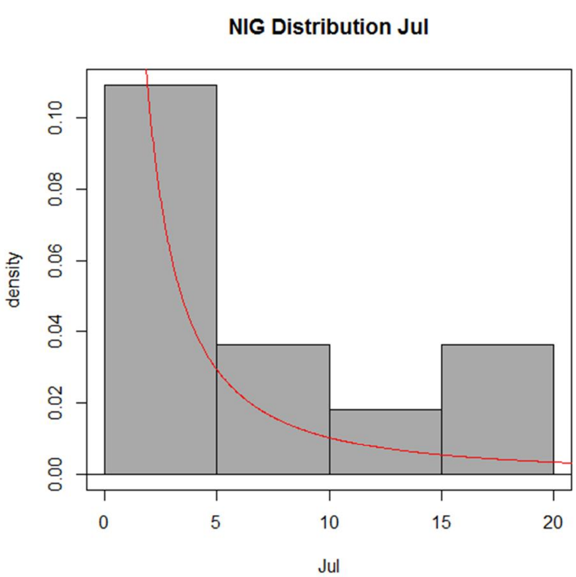


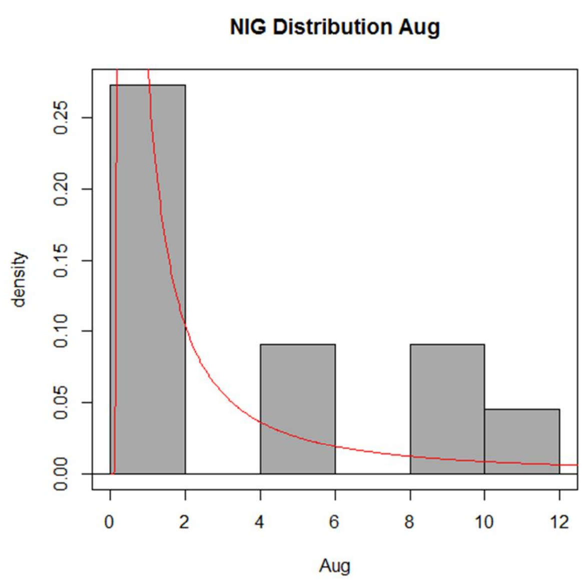


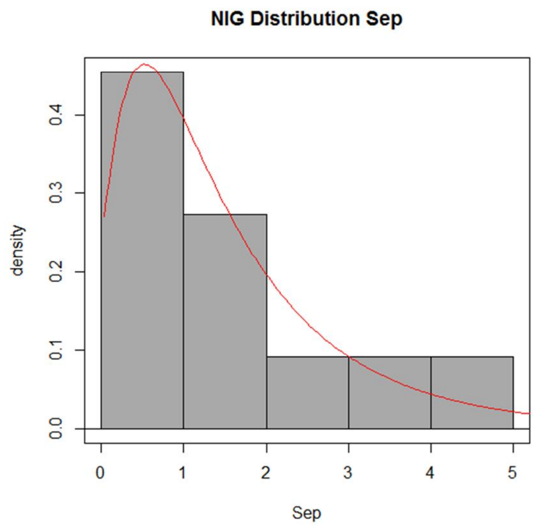


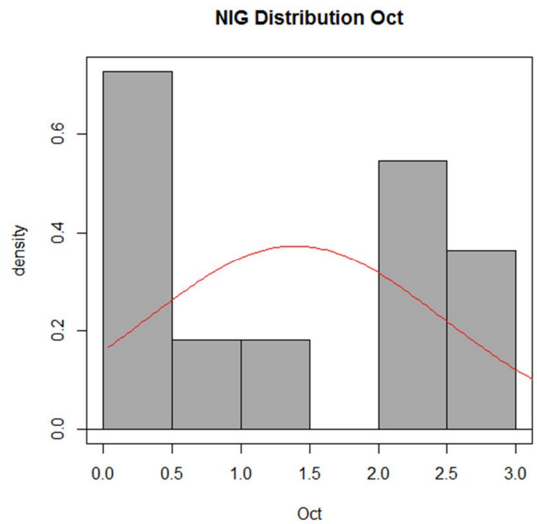

NIG Distribution May
density
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0
2
4
6
8
May

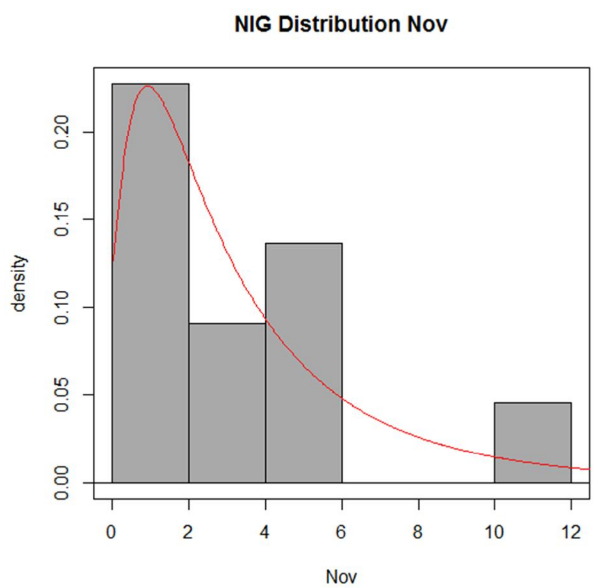

NIG Distribution Jun
density
0.0
0.1
0.2
0.3
0.4
0
1
2
3
4
5
6
7
Jun

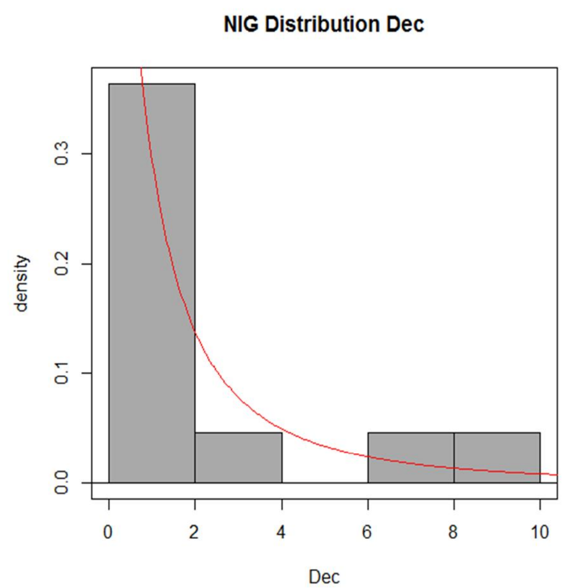

NIG Distribution Jul
density
0.00
0.02
0.04
0.06
0.08
0.10
0
5
10
15
20
Jul

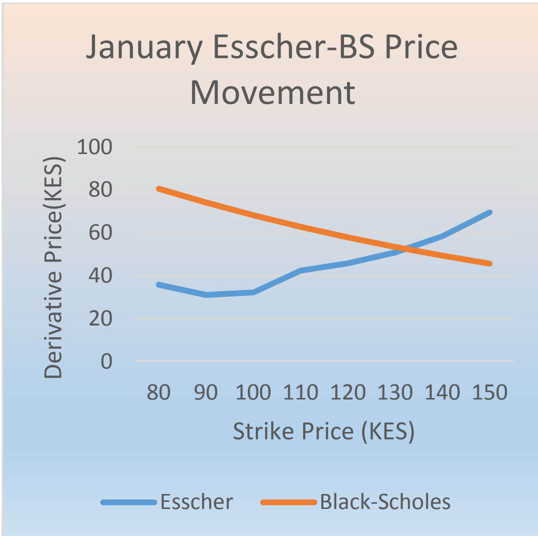

NIG Distribution Aug
density
0.00
0.05
0.10
0.15
0.20
0.25
0
2
4
6
8
10
12
Aug


NIG Distribution Sep
density
Sep
0.0
0.1
0.2
0.3
0.4
0
1
2
3
4
5

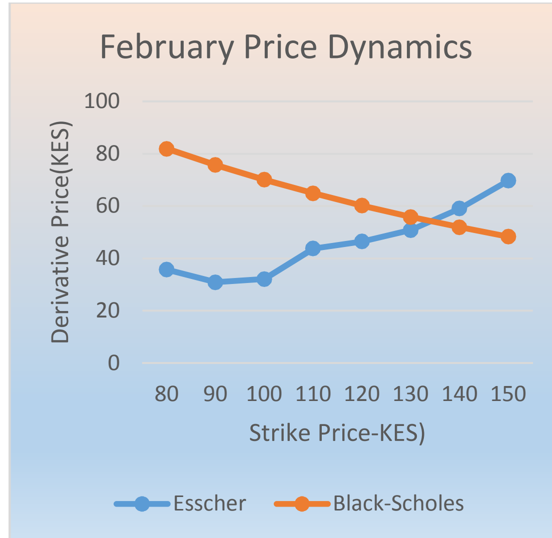

NIG Distribution Oct
density
Oct
0.0
0.2
0.4
0.6
0.0
0.5
1.0
1.5
2.0
2.5
3.0


NIG Distribution Nov
density
Nov
0.00
0.05
0.10
0.15
0.20
0
2
4
6
8
10
12

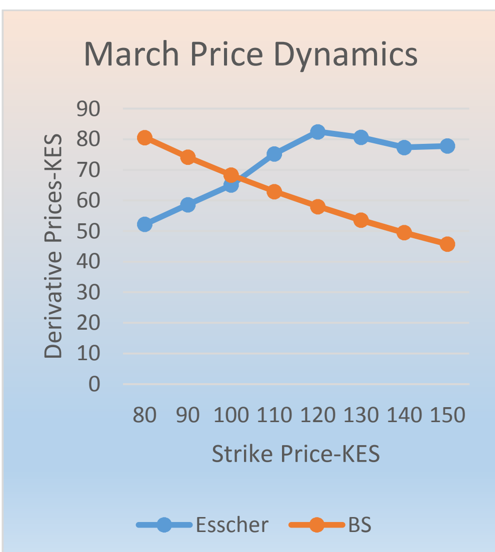

NIG Distribution Dec
density
Dec
0.0
0.1
0.2
0.3
0
2
4
6
8
10



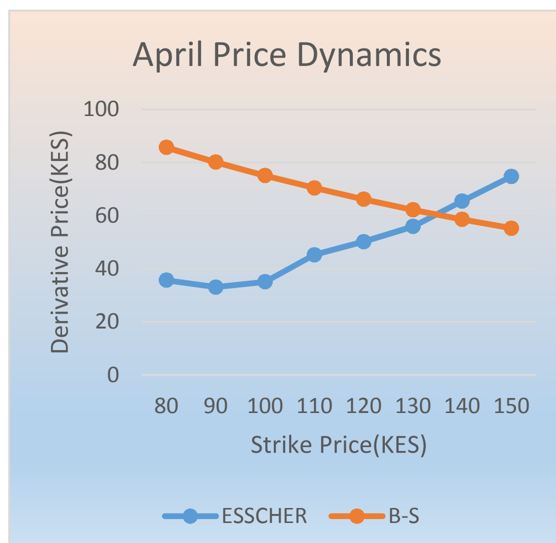
January Esscher-BS Price Movement
Derivative Price(KES)
100
80
60
40
20
0
80 90 100 110 120 130 140 150
Strike Price (KES)
Esscher
Black-Scholes



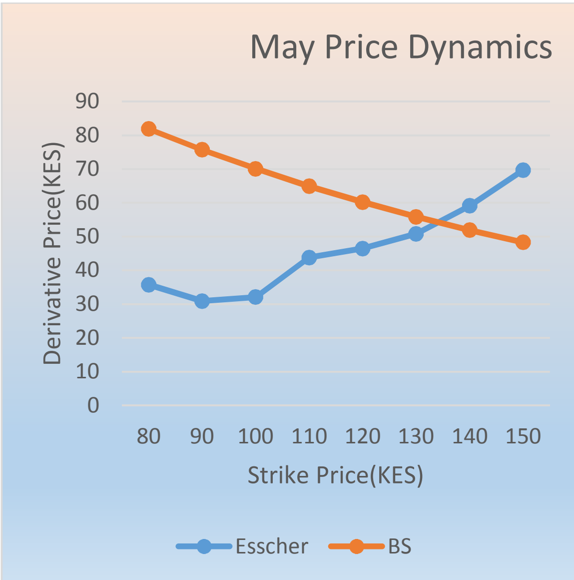
February Price Dynamics
Derivative Price(KES)
100
80
60
40
20
0
80 90 100 110 120 130 140 150
Strike Price-KES)
Esscher
Black-Scholes

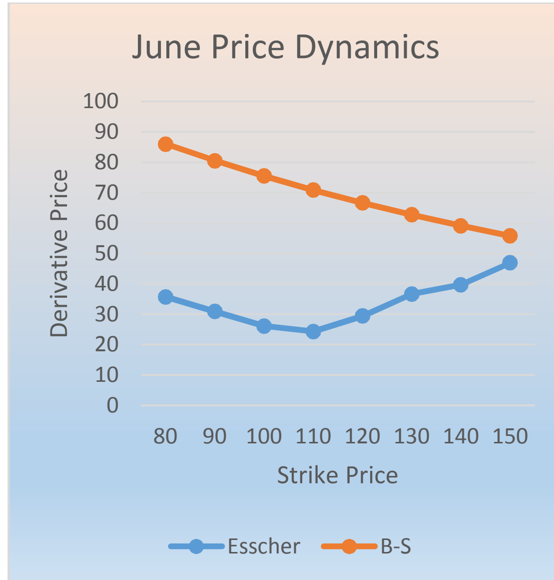


March Price Dynamics
Derivative Prices-KES
90
80
70
60
50
40
30
20
10
0
80 90 100 110 120 130 140 150
Strike Price-KES
Esscher
BS

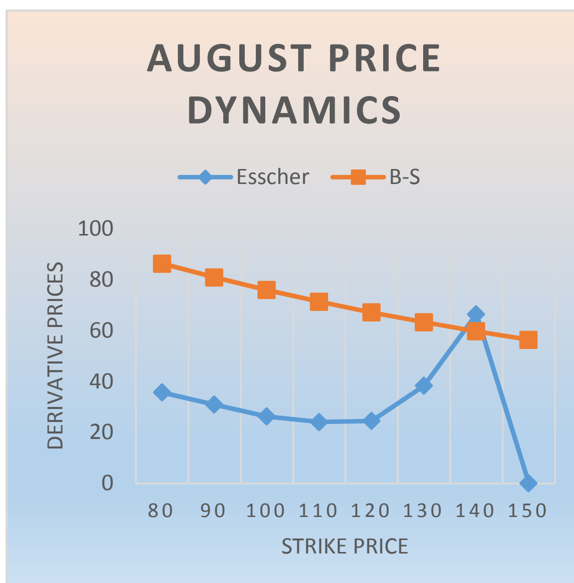


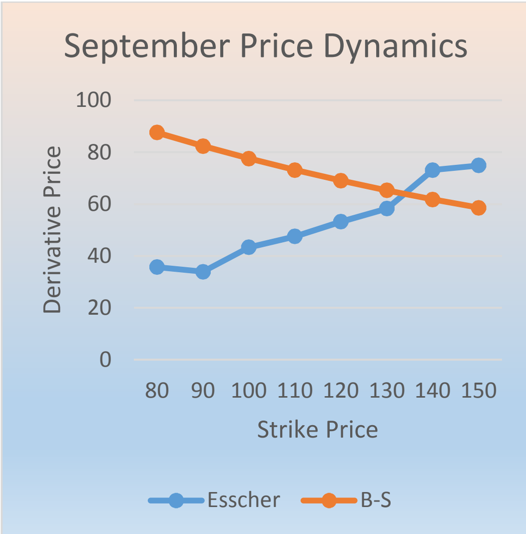
April Price Dynamics
Derivative Price(KES)
100
80
60
40
20
0
80 90 100 110 120 130 140 150
Strike Price(KES)
ESSCHER
B-S

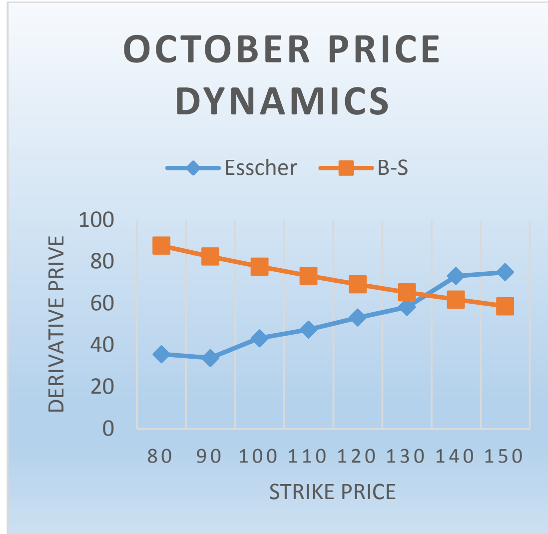

May Price Dynamics
90
80
70
60
50
40
30
20
10
0
Derivative Price(KES)
80 90 100 110 120 130 140 150
Strike Price(KES)
Esscher
BS

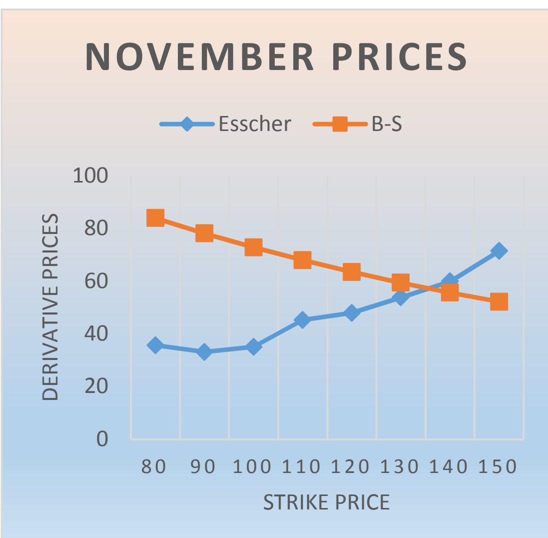

June Price Dynamics
100
90
80
70
60
50
40
30
20
10
0
Derivative Price
80 90 100 110 120 130 140 150
Strike Price
Esscher
B-S


JULY PRICE DYNAMICS
Esscher
B-S
100
80
60
40
20
0
DERIVATIVE PRICE
80 90 100 110 120 130 140 150
STRIKE PRICE


AUGUST PRICE DYNAMICS
Esscher
B-S
100
80
60
40
20
0
DERIVATIVE PRICES
80 90 100 110 120 130 140 150
STRIKE PRICE


September Price Dynamics
Derivative Price
100
80
60
40
20
0
80 90 100 110 120 130 140 150
Strike Price
Esscher
B-S


OCTOBER PRICE DYNAMICS
Esscher
B-S
DERIVATIVE PRIVE
100
80
60
40
20
0
80 90 100 110 120 130 140 150
STRIKE PRICE


NOVEMBER PRICES
Esscher
B-S
DERIVATIVE PRICES
100
80
60
40
20
0
80 90 100 110 120 130 140 150
STRIKE PRICE

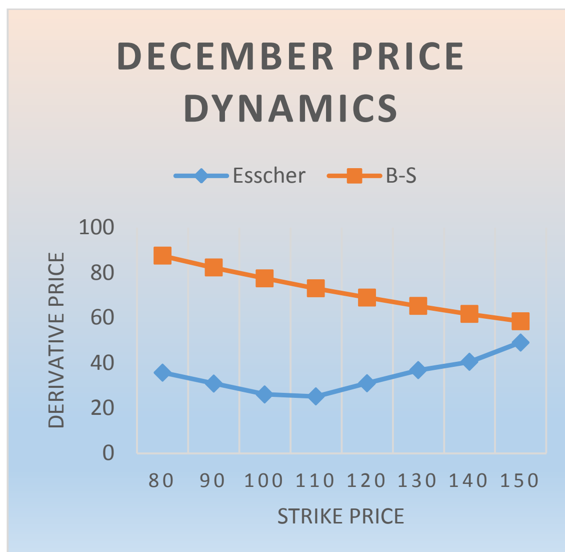

DECEMBER PRICE DYNAMICS
Esscher
B-S
DERIVATIVE PRICE
100
80
60
40
20
0
80 90 100 110 120 130 140 150
STRIKE PRICE

Table 4.5: Derivative prices

K/Month	Jan	Feb	Mar	Apr	May	Jun
80	35.724	35.724	52.224	35.724	35.724	35.724
90	30.908	30.908	58.658	33.158	30.908	30.908
100	32.093	32.093	65.093	35.093	32.093	26.093
110	42.277	43.777	75.277	45.277	43.777	24.350
120	45.717	46.467	82.461	50.217	46.467	29.450
130	50.775	50.803	80.646	55.974	50.804	36.635
140	58.247	59.094	77.330	65.557	59.094	39.733
150	69.495	69.717	77.811	74.837	69.638	46.918

K/Month	Jul	Aug	Sept	Oct	Nov	Dec
80	37.974	35.724	35.725	35.724	35.724	35.724
90	38.408	30.927	33.908	33.908	33.158	30.908
100	50.093	26.264	43.343	43.343	35.093	26.103
110	54.282	24.107	47.527	47.527	45.277	25.137
120	58.533	24.531	53.217	53.217	47.999	31.054
130	71.856	38.337	58.275	58.275	53.845	36.770
140	78.098	66.449	73.136	73.136	59.958	40.483
150	79.737	00.000	74.906	74.906	71.487	49.081