PRICING OF A EUROPEAN CALL OPTION UNDER A LOCAL VOLATILITY INTERBANK OFFERED RATE MODEL

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DECLARATION

This research thesis is my own work and has not been presented elsewhere for a degree award.

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ABSTRACT

Financial derivatives offer a great investment opportunity when accurately priced. Developing countries such as Kenya are yet to establish the mechanisms of trading in derivatives. This research seeks to demonstrate how advances in developed money markets can be reflected towards the establishment of derivatives markets in developing countries. To achieve this, the dynamics of the inter bank offered interest rates in developing markets and developed markets are compared. The two interbank offered rates are found to be similar under an appropriate martingale measure. A European caplet for the developed money market is priced using the local volatility interbank offered rate model. The accuracy of the local volatility interbank offered rate is found to be better when benchmarked against the industry accepted Black's model. The local volatility model is used as it captures the volatility smiles more efficiently in one sweep.

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Chapter 1

INTRODUCTION

The Interbank Offered Rate (IBOR) is the average inter bank interest rate at which banks in a money market are prepared to lend money to each other. This interest rate is important to both professional and private individuals as it is the benchmark rate for several business loans, adjustable rate mortgages and financial instruments traded on the financial markets. Examples of the IBORS include the London IBOR, Kenyan IBOR, European IBOR.

An option is a financial derivative that gives the holder the right to buy or sell an underlying security at a specified price, at a specified future date. The underlying asset could be interest rates, stocks, exchange rates or any financial variable of interest to a researcher. Papapantoleon (2010) observes that an option's price depends on the volatility of the underlying asset price, which in this research is the IBOR interest rate. This volatility affects the distribution of the assets at expiry and hence the expected return from the option. Investors invest in options for the purposes of hedging, speculating or an existing arbitrage opportunity.

The derivatives market in Kenya is under developed. The market is inactive due to factors such as lack of awareness among the potential local and foreign investors, inadequate risk management techniques and poor legislation with regards to derivatives trading and taxation.This research seeks to price an interest rate derivative for the Kenyan money market thereby introducing a new investment security for the local and foreign investors.

Under continuous time modelling,traditionally, the Black-Scholes option pricing formula introduced by Black & Scholes (1973) has been used as the benchmark to price the European vanilla options. Previous researches on the relative merits of competing option pricing models have been conducted. Researchers have compared the Black-Scholes(BS) model and other stochastic volatility models and observe that the BS formula exhibits strong pricing biases across both maturity and moneyness.

One of the biases of Black- Scholes formula is the assumption that the distribution of the underlying stock is log- normal with known mean and variance. This is not true since the underlying stock has a high kurtosis and the assumption on constant variance is somewhat dubious, (Bakshi et al., 1997). The constant volatility assumption made in the Black-Scholes model cannot explain the long-term observed features of the implied volatility such as volatility smiles and skews, (Hull & White, 1987).

It is important to consider a model that takes into consideration the varying volatility of the underlying security under consideration. There are a number of models that take into account the volatility skews and smiles. These models include the stochastic volatility models, jump models, variance gamma models and local volatility models.

Some of the stochastic volatility models introduced were by Hull & White (1987), Stein & Stein (1991) and Heston (1993). Merton (1976) introduces the likelihood of jumps in the stochastic process for the underlying asset price that allows for the existence of volatility skew. The variance gamma process for the stock price was introduced by Madan et al. (1998). This process generalizes the Brownian Motion and allows for the existence of skewness and kurtosis in the return distribution.

Stochastic volatility models are computationally complex and they pose an extreme difficulty of fitting parameters to the current prices of interest rate options. Researchers found a simpler way of pricing exotic options consistently with the volatility skew. Dupire et al. (1994) introduced the concept of local volatility model whereby they noticed that under risk neutrality, there existed a unique diffusion process consistent with this distributions.

A local volatility model, is one that treats volatility as a deterministic function of both the current asset level and time. This is to say that, local volatilities represent the given averages over all possible instantaneous volatilities in a stochastic volatility framework, (Berestycki et al., 2002). In other words, a local volatility model is some generalisation of the Black-Scholes model. The local volatility model allows for the simplification of assumptions that allows practitioners to price exotic options consistently with the known prices of vanilla options.

Marabel (2012) find that the local volatility model can be a correct approach to price the European quanto options in the presence of volatility smile. Further, the local volatility type modelling captures the surface of the implied volatilities more precisely than other approaches, (Henry-Labordere, 2009). Of importance to note is that the local volatility framework is an arbitrage free and risk neutral valuation framework. The local volatility framework is adopted to determine the European call option price with the underlying asset as the Kenyan IBOR interest rate.

1.1 Statement of the Problem

Derivatives markets are greatly underdeveloped in developing countries as opposed to developed financial markets where derivatives trading is established. This research seeks to determine an accurate option pricing model in which the underlying security is interest rates for developing financial markets. The dynamics of the Kenyan IBOR rates and those of the London IBOR rates will be compared to ascertain if they are similar. This is so as to determine if the advances in derivatives markets in developed countries can be used to establish a derivatives market in Kenya.

If the option price is given by the market, an inversion of the relationship gives the implied volatility. Identical options with different strike prices have different implied volatilities. This shows the existence of volatility skews/ smiles over the long term. This study uses the local volatility framework as it takes into consideration the volatility skew of the underlying asset. The local volatility type modelling captures the surface of implied volatilities more precisely than other approaches. Moreover, the local volatility model is complete and thus ensures the uniqueness of option prices. The model will then be validated by calibrating it to market data to ascertain if it is accurate to price options in developing financial markets.

1.2 Justification of the Study

Interest rate derivatives have become increasingly popular among investors with 60% of derivatives traded on the global exchanges being interest rate derivatives, Fleming et al. (2012). However, derivative instruments if not accurately priced have been cited to cause economic crises. Options markets have been characterized by a persistent negative dependence of the implied volatility with respect to the strike price, which is known as the implied volatility skew. With the introduction of derivatives market at the Nairobi Securities Exchange, an accurate pricing model for the call options on the IBOR rates will offer the best investment option for both foreign and local investors. This is because the interest rate caplet can be used by investors to hedge against interest rates fluctuations.

1.3 Objectives of the study

1.3.1 General objective

To price a European caplet under a local volatility interbank offered rate model.

1.3.2 Specific objectives

1. To investigate whether the dynamics of the Kenyan IBOR and London IBOR are similar.

2. To formulate the local volatility interbank offered rate model.

3. To price the European call option using the local volatility model.

1.4 Research Questions

1. Are the dynamics underlying the Kenyan IBOR rates similar to the dynamics underlying London IBOR rates?

2. What is the caplet pricing formula for the local volatility IBOR rate model?

3. Is the caplet pricing formula obtained accurate in the pricing of a caplet?

This investigation seeks to give insight on how the advances in developed money markets can be reflected towards the establishment of derivatives markets in developing markets. The above research questions aid in achieving this.

1.5 Scope of the Study

The scope that this research focuses on is developing and developed financial markets. The underlying financial instrument is the London Interbank offered rates(LIBOR) and Kenyan Interbank offered rates. The derivative instrument of interest is the interest rate caplet.

1.6 Limitations

The major limitation of this research is the availability of the option prices market data for the developing financial markets. Developing financial markets with Kenya being the case study do not trade in derivatives case in point interest rate derivatives. As a result, there is no market data available to mark the model to developing markets.

Another limitation of the study is that historical data on interest rate derivatives from developed markets is very expensive to purchase. This limits the historic period through which the study can be conducted.

Chapter 2

LITERATURE REVIEW

2.1 Introduction

This section discusses pertinent literature reviewed regarding the dynamics of IBOR rates, the local volatility interbank offered rate model and the calibration and validation of the local volatility model.

2.1.1 Dynamics of the forward IBOR rates.

An important advancement in the pricing of interest rate derivatives was the emergence of market models that are consistent with market conventions. These market models have common suitable features that make them appropriate for term structure derivatives pricing. The features are that they are arbitrage free among bonds, they keep the rates positive and they price the caps in a Black-Scholes framework allowing automatic calibration to market data,Brace et al. (1997).

One of the market models was introduced by Heath, Jarrow, and Morton (HJM) in 1992. The HJM model incorporates log normal volatilities for forward rates. These forward rates are continuously compounded. Since these forward rates are continuously compounded, a weakness of the HJM model is that the log normal volatilities lead to forward rates that become infinite in finite time with positive probability, (Jamshidian, 1997).

In contrast to HJM, Miltersen et al. (1997) and Jamshidian (1997) overcome this difficulty by developing discretely compounded rates that admit a deterministic diffusion coefficient for the log normal volatilities. The rates are log normal under an appropriate change of measure but by themselves the rates are not simultaneously log normal. Similarly, Brace et al. (1997) model LIBOR rates in a log normal HJM framework by parametrizing it. That is, they assume that each LIBOR forward rate process has a log-normal volatility structure.

A similarity in the findings of Miltersen et al. (1997) ,Jamshidian (1997) and Brace et al. (1997) is that they observe that the corresponding market forward rates do not explode, and are positive and mean reverting. The Pricing of caps and floors is consistent with the Black formulas that is used in the market. The discretely compounded forward rates are employed in this research. The downside is that their methodologies yield volatility that is flat which contrasts the findings of Jarrow et al. (2007) who observes that interest caps exhibit smiles and skews .

Jarrow et al. (2007) allow the LIBOR rates to follow an affine jump diffusion process. From this process, they are able to obtain closed form solutions of pricing the caps. They are among the first to provide comprehensive evidence of volatility smiles in the caps market. This evidence of smiles in the interest rate caps necessitates the use of a model that captures this smile. In this research, the local volatility model is the model employed to capture the smile.

Papapantoleon (2010) critically reviews the construction and the properties of the different models of the LIBOR rates. Some of the models he discusses are the classical LIBOR market models, forward price models, Markov-functional models and the affine LIBOR models. He further discusses the requirements that these model should satisfy. This requirements are non-negativity, arbitrage- free,analytically tractable and able to provide good calibration to market data. The local volatility interbank offered rate model should satisfy these requirements.

2.1.2 Local volatility interbank offered rate model.

The development of the local volatility model by Dupire et al. (1994) was a major advancement in handling of volatility smiles. The local volatility models are arbitrage-free, self consistent and can easily be calibrated to match the observed market smiles, (Henry-Labordere, 2009). For this reasons, it is the model employed in the interest rate derivative pricing in this research.

Further, Sepp (2002) studies the local volatility model and prices the barrier options under the local volatility model. He observes that the local volatility solver produces call prices that are compatible with the market prices across all strikes and maturities. This is evidence that the local volatility framework can provide accurate prices.

The Dupire type local volatility model requires one to obtain differentiation against maturity. The traditional continuous stochastic process that has been used in interest rate vanilla pricing cannot allow for differentiation against maturity. As a result, this research shall employ the method of Zhu & Qu (2016). Zhu & Qu (2016) creates a spot process that allows one to differentiate against maturity and hence strip local volatility to price caps.

Another development in the capturing of interest rate derivatives smile in a local volatility framework was by Andersen & Andreasen (2000). They extend the Libor market model to markets with volatility skews in observable option prices. They expanded the family of forward rate in order to develop a special case of the local volatility models in order to capture the skews and smiles. This is an alternative methodology to pricing of interest rate derivatives in the local volatility framework.

On the downside, the local volatility model has a very general formulation and thus does not have an analytical solution for European options. Benhamou et al. (2010) present a new methodology to derive closed form solutions to price any European options. The formula results from the asymptotic expansion of the terms of the Black-Scholes model and related Greeks. The accuracy of the formula developed depends on the smoothness of the payoff and it converges with very few terms.

Marabel (2012) compares the standard quanto adjustment and the local volatility model in the pricing of quanto derivatives. She presents quanto adjustment that corresponds to the local volatility model that allows for the pricing of quanto derivatives consistently with the observed market equity skew and exchange rate smile. The results she obtains shows that the standard quanto adjustment can be subject to significant pricing errors compared to the local volatility model.Thus, the local volatility model provides less pricing errors for exotic derivatives.

Benhamou et al. (2012) present new approximation formulas for pricing European options in a local volatility model with stochastic interest rates. In their approach, they model the local volatility of the discounted spot rate in order to obtain accurate approximations with tight estimates of the error terms. An illustration with real market data shows accuracy.

2.1.3 Calibration of the local volatility IBOR model

There are different ways to calibrate the local volatility model to market data. One of them is by Berestycki et al. (2002) who propose a new formulation of the calibration problem for the local volatility model. They use the quasilinear degenerate parabolic partial differential equation to establish closed-form asymptotic formulae for the implied volatility near expiry and for deep in-the-money and out-of-the-money options. They show that this formulation is well posed.

Among the interest rate models that have been recently introduced is one by Chibane & Law (2013).They provide a quicker way to modelling of both stocks smiles and stochastic interest rates at long-dated maturities.They calibrate the parametric local volatility using numerical iterations under the Cheyette model.

Hok et al. (2014) derived dynamics of the system of foreign LIBOR rates to price quanto options. They used the local volatility function and expansion formulas introduced by Benhamou et al. (2010) to obtain the closed form solution to the local volatility model. On calibrating the model, they find that the model they developed has excellent accuracy.

Zhu & Qu (2016) present a simpler and practical model that handles the interest rate smile. The model allows direct Dupire-type local volatility stripping in the asset class of interest rates. The model also formulates a backward-pricing partial differential equation using a numeraire deflated value, which can be used to price suitable path-dependent interest rate derivatives with a smile. This self-contained smile model possesses all the good features of a Dupire-type local volatility model, including numerical simplicity and efficiency. This is the approach that is employed in this research due to it's simplicity and efficiency.

Chapter 3

METHODOLOGY

This section discusses the local volatility Libor model and the European call option model

3.1 Local Volatility IBOR model

3.1.1 Dynamics of the forward IBOR rate

The requirements that a IBOR model should satisfy are as follows:

1. The IBOR rates should be non-negative, that is, $L(t,T) \geqslant 0 \quad \forall \ t \in [0,T]$, where $L(t,T)$ is the time t forward IBOR rate settled at time T and received at time $T+\delta$.

2. The IBOR rates model should be free of arbitrage

3. The IBOR rate model should be analytically tractable,that is, it should be easy to implement and calibrate to market data of liquid derivatives.

Consider a probability space $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ where the filtration $\mathbb{F} = (\mathcal{F}_t)_{t\in[0,T]}$ satisfies the usual conditions of a filtration and T denotes a finite time horizon with a discrete tenor structure. Let $W = (W_t)_{t\in[0,T]}$ denotes a standard Brownian motion.
Denote the forward martingale measure by $(\mathbb{P}_t)_{t\in T}$ and where the corresponding zero

coupon bond $B(.)$ acts as a numeraire for each forward measure. All the forward measures are assumed to be equivalent to the measure $\mathbb{P}$

The forward IBOR rates over the future accrual period$[t, T+\delta]$ observed at time t, is set to satisfy the equation

$$1+\delta L(t,T) = \frac{B(t,T)}{B(t,T+\delta)} = F(t,T,T+\delta)$$

which is also expressed as

$$L(t,T) = \frac{B(t,T) - B(t,T+\delta)}{B(t,T+\delta)} \tag{3.1}$$

The dynamics of the system of IBOR rates under the forward martingale measure is given by:

$$dL(t) = \lambda_{(T,T+\delta)}(t).dW_{T,T+\delta}(t) \tag{3.2}$$

In relation to equation(3.2) when using the local volatility type model, the differentiation against maturity cannot be obtained thus a spot process introduced by Zhu & Qu (2016) that allows differentiation against maturity and local volatility stripping is employed.

3.1.2 Dynamics of fixed-tenor rolling IBOR

Having established that the dynamics of the IBOR rates are similar to those of the LIBOR rate, research follows the approach of Zhu & Qu (2016) in determining the dynamics of the rolling IBOR rates. Denoting the time- t rolling IBOR as $L_{t,t+\delta}(t)$ and the zero coupon bond price as $B(t, t+\delta)$, the spot process of the rolling IBOR is assumed to follow the normal process:

$$dL_{t,t+\delta}(t) = \mu_{t,t+\delta}(t)dt + \sigma_{(t,t+\delta)}(t).dW_{t,t+\delta}(t) \tag{3.3}$$

where $\mu_{t,t+\delta}(t)$ is the drift, $\sigma_{(t,t+\delta)}(t)$ is the diffusion coefficient term and $W_{t,t+\delta}(t)$ is the standard brownian motion under the spot measure Q^{spot} associated with the local numeraire $B(t, t+\delta)$. Zhu & Qu (2016) apply the measure- change technique to define the local numeraire $B(t, t+\delta)$ such that a deterministic function of IBOR $[f(L_{t,t+\delta}(t))]$ satisfies:

$$E^a f(L_{T,T+\delta}(T))|t = E^{spot}\left\{\frac{a_T B(t, t+\delta)}{a_t B(T, T+\delta} f(L_{T,T+\delta}(T))|t\right\}$$

where $E^a(.)$ is an expectation under the measure associated with a standard numeraire in this case the zero coupon bond. The use of a local numeraire in $E^{spot}[.]$ is allowed in this case provided that the drift $\mu_{t,t+\delta}(t)$ and the diffusion $\lambda_{(t,t+\delta)}(t)$ are calibrated to the market forward and option prices. They use the local numeraire for $f(L_{t,t+\delta}(t))$ in the measure-change technique to obtain the expectation under different measures.

The rolling IBOR is not a martingale under Q^{spot} but the forward IBOR $L_{T,T+\delta}(t)$ is a martingale under the forward measure Q^{fwd} thus we have that:

$$E^{fwd}\{L_{T,T+\delta}(T)|t\} = L_{T,T+\delta}(t) \tag{3.4}$$

Zhu & Qu (2016) show that by changing the measure technique, the expectations under Q^{fwd} and $Q^{spot}$are related as shown:

$$\begin{aligned} E^{fwd} L_{T,T+\delta}(T)|t &= E^{spot}\left\{\frac{B(T, T+\delta)}{B(t, T+\delta)}\frac{B(t, t+\delta)}{B(T, T+\delta)} L_{T,T+\delta}(T)|t\right\} \\ &= \frac{B(t, t+\delta)}{B(t, T+\delta)} E^{spot}\{L_{T,T+\delta}(T)|t\} \end{aligned} \tag{3.5}$$

Thus:

$$E^{spot}\left\{L_{T,T+\delta}(T)|t\right\} = \frac{B(t,T+\delta)}{B(t,t+\delta)}L_{T,T+\delta}(t)$$

From the prior definition of the spot process:

$$E^{spot}\left\{L_{T,T+\delta}(T)|t\right\} = L_{t,t+\delta}(t) + \int_t^T \mu_{s,s+\delta}(s)ds$$

Thus:

$$\int_t^T \mu_{s,s+\delta}(s)ds = \frac{B(t,T+\delta)}{B(t,t+\delta)}L_{T,T+\delta}(t) - L_{t,t+\delta}(t)$$

which gives the drift term $\mu_{t,t+\delta}(t)$ as:

$$\mu_{t,t+\delta}(t) = -\frac{\partial(\frac{B(t,T+\delta)}{B(t,t+\delta)}L_{T,T+\delta}(t) - L_{t,t+\delta}(t))}{\partial t} \tag{3.6}$$

The drift equation enables the derivation of the pricing formulae for interest rate vanillas under the rolling IBOR spot process.

3.2 European Caplet pricing formula under the spot process

The payoff of a European call caplet with expiry date T pays at $t \geqslant T$ the amount :

$$max(L_{T,T+\delta} - K, 0)^+$$

where the strike price $K \geqslant 0$.

The price of a caplet whose strike price is K starting at T and maturing at time $T+\delta$ under the forward measure Q^{fwd} is given by:

$$C(t;T,T+\delta,K) = \delta B(t,T+\delta)E^{fwd}\left\{[L_{T,T+\delta}(T)-K]^+|t\right\}$$

Zhu & Qu (2016)show that using the measure-change technique:

$$\begin{aligned}
E^{fwd}\left\{[L_{T,T+\delta}(T)-K]^+|t\right\} &= E^{spot}\left\{\frac{B(T,T+\delta)}{B(t,T+\delta)}\frac{B(t,t+\delta)}{B(T,T+\delta)}[L_{T,T+\delta}(T)-K]^+|t\right\} \\
&= \frac{B(t,t+\delta)}{B(t,T+\delta)}E^{spot}\left\{[L_{T,T+\delta}(T)-K]^+|t\right\}
\end{aligned}$$

Thus the market caplet price is:

$$C(t;T,T+\delta,K) = \delta B(t,t+\delta)E^{spot}\left\{[L_{T,T+\delta}(T)-K]^+|t\right\}$$

Assuming that the rolling IBOR follows a normal process defined in eqn(3.3, a Bachelier pricing formula is obtained as:

$$C(t;T,T+\delta,K) = \delta Z(t,t+\delta)[(F(T)-K)N(d)+\sigma_{t,t+\delta}(t)\sqrt{T-t}\phi(d)] \quad (3.7)$$

where

$$d = \frac{F(T)-K}{\sigma_{t,t+\delta\sqrt{T-t}}} \qquad F(T) = L_{t,t+\delta}(t) + \int_t^T \mu_{s,s+\delta}(s)ds$$

$F(T) = L_{0,t+\delta}(0) + \int_0^T \mu(s)ds$ is the forward IBOR. $N(.)$ is the Gaussian cumulative distribution function and $\phi(.)$is the Gaussian probability density function $\phi(x) = \frac{1}{\sqrt{2\pi}}\exp(-\frac{x^2}{2})$ and $\sigma_{t,t+\delta}(t)$ is the implied volatility of the rolling IBOR specific to the caplet under the spot process.

3.3 Caplet local volatility stripping formula under the spot process

The cap pricing formula earlier derived under the spot IBOR process can be used for local volatility stripping as it permits differentiation against the option maturity allowing the application of the Dupire-type local volatility.

The spot rolling IBOR process with the local volatility can be expressed as:

$$dL_{t,t+\delta}(t) = \mu_{t,t+\delta}(t)dt + \sigma_{(t,t+\delta)}(t,L).dW_{t,t+\delta}(t) \tag{3.8}$$

where $\sigma_{(t,t+\delta)}$ is the local volatility. For notational convenience, $\mu(t) = \mu_{t,t+\delta}(t)$ and $\sigma(t) = \sigma_{(t,t+\delta)}(t,L)$. For $t < T$ and denoting the expectation of the un discounted payoff as $\hat{V} = E^{spot}\left\{[L_{T,T+\delta}(T) - K]^{+}|t\right\}$ and applying the Fokker-Planck equation to the probability density function, a Dupire-type local volatility PDE for the rolling IBOR can be derived as:

$$\frac{\sigma^2(T,K)}{2}\frac{\partial^2\hat{V}}{\partial K^2} = \frac{\partial\hat{V}}{\partial T} + \mu(T)\frac{\partial\hat{V}}{\partial K}$$

From the above PDE the local volatility stripping formula is given by:

$$\sigma(T,K) = \sqrt{2\frac{\frac{\partial\hat{V}}{\partial T} + \mu(T)\frac{\partial\hat{V}}{\partial K}}{\frac{\partial^2\hat{V}}{\partial K^2}}}$$

The normal local volatility can be stripped directly from the implied volatility. Using the Bachelier call, the normal local volatility stripping formula is given by:

$$\sigma(T,K) = \sqrt{\frac{2\frac{\partial\sigma_1}{\partial T} + \frac{\sigma_1}{T} + 2\mu(T)\frac{\partial\sigma_1}{\partial K}}{\frac{1}{\sigma_1 T}(1 + \frac{(F(T)-K)}{\sigma_1}\frac{\partial\sigma_1}{\partial K})^2 + \frac{\partial^2\sigma_1}{\partial K^2}}} \tag{3.9}$$

3.4 Backward pricing PDE under the spot process

Zhu & Qu (2016) complete the local volatility model by deriving the appropriate backward PDE for pricing the contigent claims. To bypass the problem of having to handle stochastic discount or annuity on PDE nodes, Zhu & Qu (2016) derive the backward pricing PDE for the discounted value of the contigent claim.

For a contigent claim $C[L_{T,T+\delta}(T)]$, its zero-bond numeraire discounted value is defined as:

$$\hat{C}[L_{T,T+\delta}(T)] = \frac{C[L_{T,T+\delta}(T)]}{B(T, T+\delta)}$$

The expectation of the discounted value under the spot measure is denoted as:

$$\hat{P} = \hat{P}(L_{t,t+\delta}(t), t) = E^{spot}\left\{\hat{V}[L_{T,T+\delta}(T)]|t\right\}$$

Applying the Feyman-Kac theorem and the fact that $L_{t,t+\delta}(t)$ follows the local volatility normal process, Zhu & Qu (2016) arrive at the following Kolmogorov backward PDE:

$$\frac{\partial P}{\partial t} + \mu(t)\frac{\partial P}{\partial L} + \frac{1}{2}\sigma^2(t, L)\frac{\partial^2 P}{\partial L^2} = 0 \tag{3.10}$$

The above backward PDE provides an efficient way to price the interest rate call option in the presence of a volatility smile.

Chapter 4

RESULTS

These section discusses the results obtained in relation to each of the objectives of the research. The dataset used is London Interbank Offered Rate(LIBOR) for developing markets and Kenyan Interbank Offered Rate for developed markets. The period of study is 2013-2015.

4.1 Comparison of dynamics of developing markets IBOR and developed markets LIBOR

One of the objectives of the research was to determine if the dynamics of the developing markets and of the developed markets IBOR rates are similar. If the dynamics are similar, then the local volatility IBOR model is marked to market using the developed markets option prices to determine the accuracy. If the accuracy of the model is high then the model shall be used to price the call option for developing markets.
From Figure 4.1, it is observed that the Kenyan IBOR rate is more volatile and changes more dramatically than the London IBOR . This shows an option where the underlying security is the Kenyan IBOR is a riskier investment than one which the underlying security is the LIBOR. The LIBOR rates is more stable than the

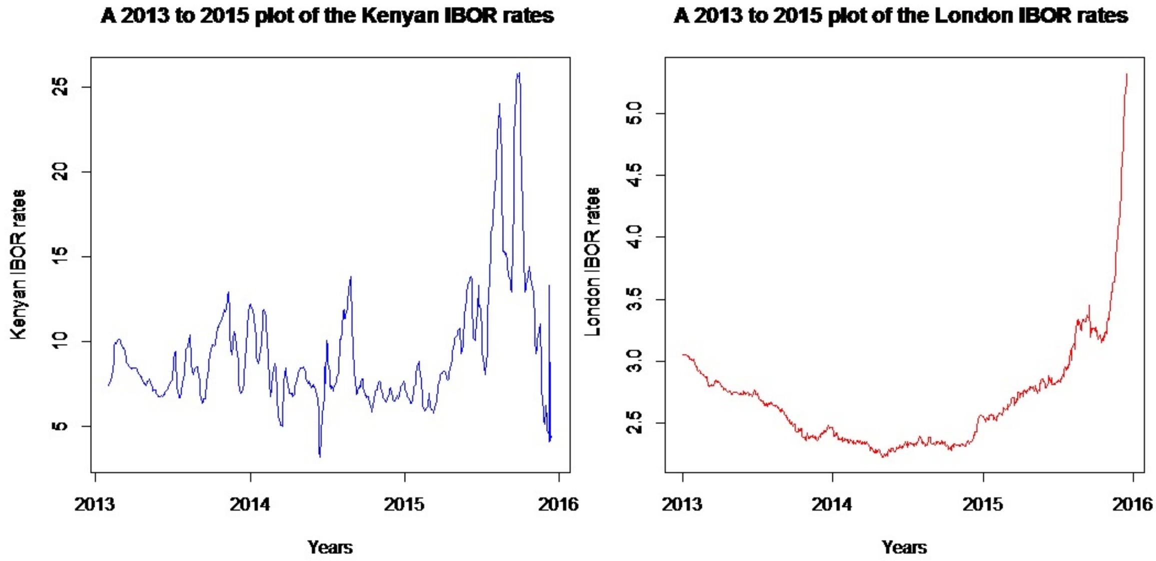

Figure 4.1: Kenyan IBOR and London IBOR
A comparison of the patterns and behaviors of the LIBOR and Kenyan IBOR over time .

Kenyan IBOR. This explains why for interest rate derivatives in developed financial markets the underlying asset is mostly the LIBOR.
The characteristic of volatility clustering is more evident on the Kenyan IBOR than the LIBOR. This is because the large changes in the Kenyan IBOR are followed by large changes and the small changes are followed by small changes. Moreover, in terms of the scale of the datasets, the Kenyan IBOR varies over a larger scale of 0-25 while the LIBOR varies over the scale of 0-5. This shows that the the Kenyan IBOR is more volatile.

All these show that an interest rate derivative on Kenyan IBOR is a very risky asset for investors. This explains why there are no interest rate derivatives trading in developing markets and the need to have a model that can accurately price these derivatives for a developing market.

Table 4.1: Descriptive Statistics of the IBORS

Descriptive Statistics	Kenyan IBOR	London IBOR
Mean	9.293	2.683
Median	8.229	2.607
Stdev	3.633	0.441
Skewness	2.225	2.482
Kurtosis	6.161	9.300

These descriptive statistics show the summaries difference between the developing markets IBOR rates and developed markets IBOR rates

The descriptive statistics indicate that both IBOR rates are not normally distributed. This is shown by the kurtosis of the two datasets which is greater than 3 indicating heavy tail. Moreover, both datasets are asymmetrical since they both exhibit a positive skew. This shows that the two data sets are from a similar distribution which is a heavy tailed one. This indicates similarity between the Kenyan IBOR and London IBOR rates.

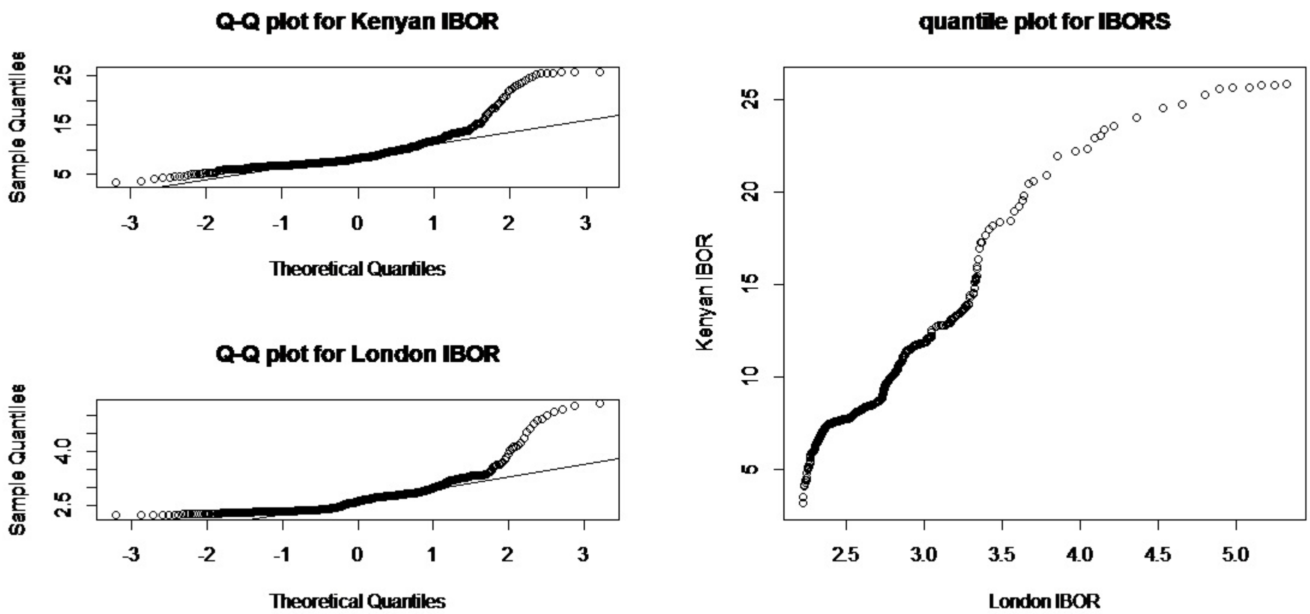

Figure 4.2: Quantile-Quantile plot for the IBORS

The quantile quantile plots also show that the Kenyan and London IBORS are not normally distributed but the underlying distribution is a heavy tailed distribution and one that has a positive skew . The quantile quantile plot of the Kenyan IBOR against the London IBOR shows that the two data sets are from a similar distribution with the same tail and skew.

4.2 Pricing of the European call option

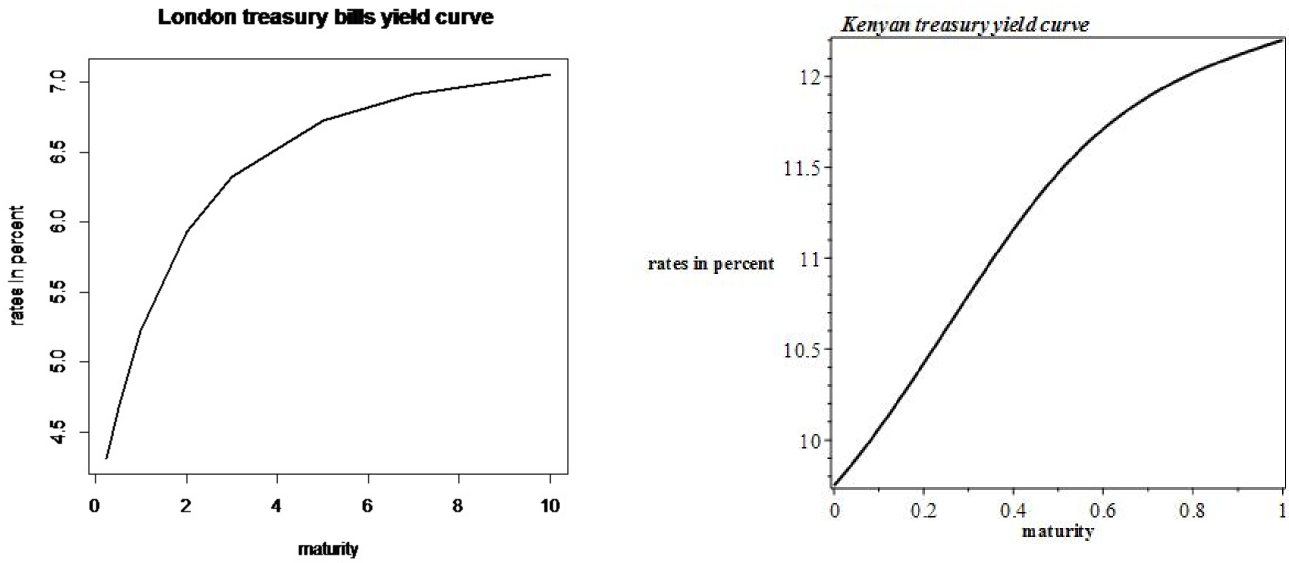

Figure 4.3: Today's yield curve(zero rate)

This shows the comparison of the developed market yield curve and the developing market yield curve.

Today's yield curve is of importance because from it the quantities of the drift term of the local volatility interbank offered rate model can be obtained thus the spot process equation can be accurately calculated.

The Local volatility Interbank offered rate model is benchmarked using the Black's model for pricing interest rate derivatives. Before pricing the caplet for the developing markets, the model is used to price for the developed markets and its prices compared to the Black's model.

This shows the comparison of the developed market yield curve and the developing market yield curve.

Table 4.2: Comparison of Black's model and IBOR model

ATM	Black's model	IBOR model
Relative error	89.97%	98.22 %

The findings are that the model prices interest rate options with an accuracy of 98% in the developed markets when marked to market compared to the Black's model. A comparison is made between the industry accepted black's model for pricing the European caplet and the local volatility IBOR model to ascertain which of the two models marks to market data more efficiently. The local volatility IBOR model marks to market more efficiently.

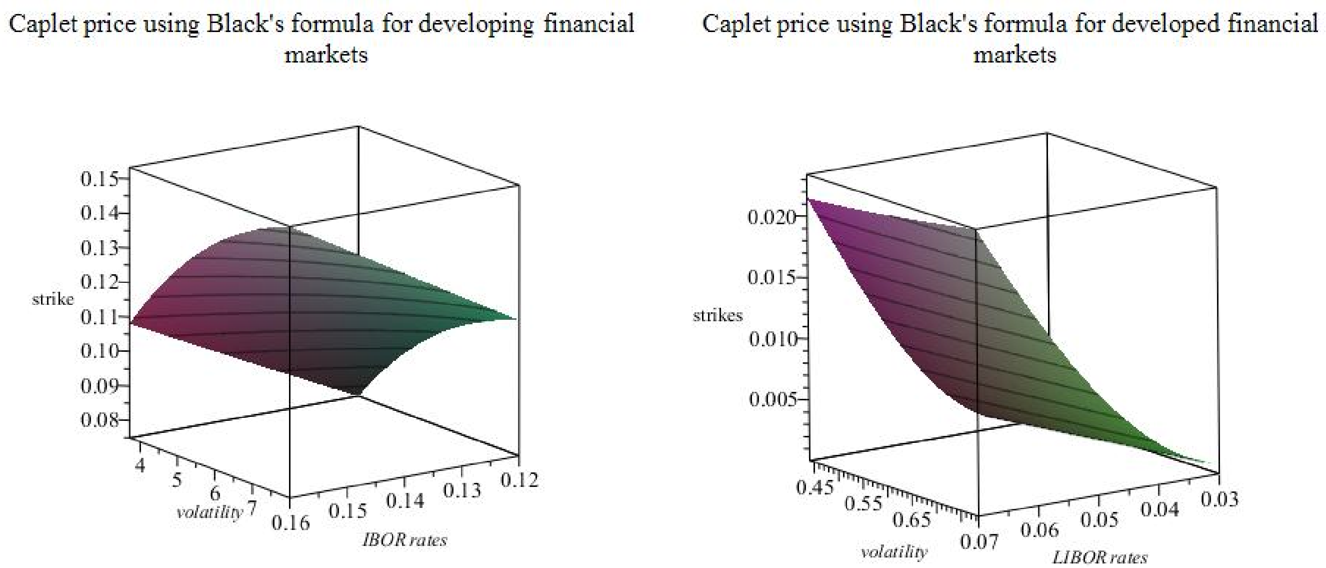

Figure 4.4: price evolution under black's model
This figure shows the difference in the price evolution for a developed market and developing market under the Black's formula.

The figure 4.5 show the price evolution of the European caplet under the local volatility interbank offered rate model for developing markets. The local volatility IBOR model captures the smile more efficiently.

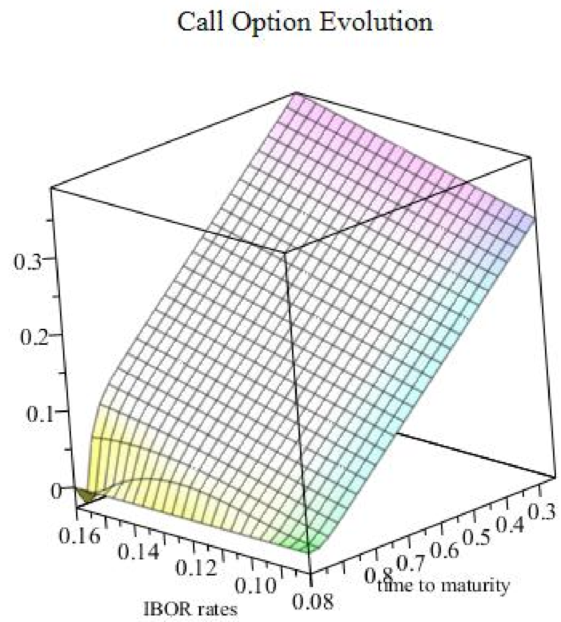

Figure 4.5: Caplet price evolution under the local volatility LIBOR model
For this Call option the price of the option where the underlying security is interest rate is 17.22.

Chapter 5

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

The dynamics of developing markets IBOR rates and those of the developed markets IBOR rates are found to be similar though the developing markets are more volatile. The reason for high volatility in the Kenyan IBOR rates is due to high fluctuation in determinants of interest rates such as inflation during the period under study. Similarity in the underlying dynamics of the two interest rates shows that a model used to price derivatives in a developed market can still be used to price derivatives for developing markets. The local volatility IBOR model is found to be efficient to price the interest rate caplet for developed markets. The model matches option prices in developing markets very well hence its use to price for developed markets. When benchmarked with the black's model, the local volatility model produces less errors. Thus, the obtained kolmogorov backward PDE can be used to price interest rate derivatives in markets that exhibit the volatility smile in the long run in both the developed and developing money markets.

5.2 Recommendation

A recommendation for further studies is to price these interest rate derivatives for developing markets with a model that has a closed form solution and still captures the local volatility surface. Moreover, researchers would be interested in constructing a model that is suitable for pricing exotic options consisting of a series of consecutive forward start options.

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