MODELING VOLATILITY IN THE GAMBIAN EXCHANGE RATE RETURNS USING VARIANTS OF ARMA–GARCH MODELS

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Modeling Volatility in the Gambian Exchange Rate Returns Using Variants of ARMA–GARCH Models

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DECLARATION

This thesis is my original work and has not been presented for a degree in any other University.

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DEDICATION

To my late parents Ebrima Marreh and Mama Suwa. May the Almighty God continue to have mercy on them.

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ABBREVIATIONS

APARCH	Assymetric Power Autoregressive Conditional Heteroscedasticity
FIGARCH	Fractionally Integrated Generalized Autogressive Conditional Heteroscedasticity
GARCH	Generalized Autoregressive Conditional Heteroscedasticity
GED	Generalized Error Distribution
GJR-ARCH	Glostan,Jaganathan and Runkle Autogressive Conditional Heteroscedasticity
GMD	Gambian Dalasis
IGARCH	Integrated Generalized Autoregressive Conditional Heteroscedasticity
iid	Independent and identically distributed
MISO	Midwest independent System Operator
TARCH	Threshold Autoregressive Conditional Heteroscedasticity
TS-GARCH	Threshold Seasonal Generalized Autoregressive Conditional Heteroscedasticity
USD	United States Dollars

ABSTRACT

This thesis aimed at modeling exchange rate volatility in the Gambian foreign exchange rate returns. Financial time series models that combined autoregressive moving average and generalized conditional heteroscedasticity were explored theoretically and applied to the exchange rate returns of the Gambian Dalasi (GMD) against the Euro and United States dollars (USD). The data covers the period from January 2003 through January 2013 and represents daily spot exchange rates.. The properties of the daily Gambian exchange rate and returns data were examined and the best fitting autoregressive moving average and generalized conditional heteroscedasticity was selected after various model building stages namely, identification, estimation and how well the model captures the variation in the data have been critically evaluated. The autoregressive moving average process is used to model the mean equation and the residuals were fitted with a generalized conditional heteroscedasticity model. The autoregressive moving average process as the mean equation serves as a filter in order to remove serial dependence in the returns and to produce independent and identically distributed residuals.. The goodness of fit of the models were assessed by the Aikaike Information Criteria . Based on the Aikaike Information Criteria , the autoregressive moving average of order (1,1) with generalized conditional heteroscedasticity of order (1,1) and the autoregressive moving average of order (2,1) with the generalized conditional heteroscedasticity of order (1,1) were judged to be the best to model the mean equation and residuals of the GMD/Euro and GMD/USD return series. To check for leverage effects in the Gambian exchange rate market, the autoregressive moving average of order (1,1) with assymetric power autoregressive conditional heteroscedasticity of order (1,1) and the autoregressive moving average of order (2,1) with assymetric power autoregressive conditional heteroscedasticity of order (1,1) were included and fitted to the GMD/Euro and GMD/USD return series respectively. The empirical results revealed that the distribution of the return series was heavy-tailed and volatility was highly persistent in the Gambian foreign exchange market.

Using the two models for each exchange rate returns, 150 out-of-sample forecast of volatility– measured as the conditional variance– were generated. The mean absolute error and the root mean square error were used to assess the forecast accuracy. Based on these metrics in assessing the out-of-sample forecast, the autoregressive moving average of order (1,1) with generalized conditional heteroscedasticity of order (1,1) slightly perform better than the the autoregressive moving average of order (1,1) with assymetric power autoregressive conditional heteroscedasticity of order (1,1) for the GMD/Euro whilst the autoregressive moving average of order (2,1) with the generalized conditional heteroscedasticity of order (1,1) forecasted the volatility better than theautoregressive moving average of order (2,1) with assymetric power autoregressive conditional heteroscedasticity of order (1,1) in the GMD/USD returns. The Diebold-Mariano test of forecast accuracy was performed on the two models applied to each currency to establish which model is superior in forecasting volatility . However, the results shows that the two models applied to each currency have the forecasting accuracy.

CHAPTER ONE

INTRODUCTION

1.1 Background Information

Since the Bretton Woods system of fixed exchange rates was abolished in 1973, exchange rates have experienced a great deal of volatility(Antonakakis, 2007). Exchange rate is one of the salient policy tools for many transitional economies. At the macroeconomic level, exchange rate fluctuations can have significant impact on trade volume. At the microeconomic level it can affect firms and individuals involved in international business. Trade, investments, finance, tourism and several other macroeconomic variables are all greatly influenced by foreign exchange rates. Governments especially in developing countries are continuing to search for mechanisms to cope with the uncertainty that can prevail in foreign exchange markets. Therefore, foreign exchange rates monitoring and evaluation has become one of the most important tool of interest to policymakers in governments as it widely explains to a certain degree a country's relative economic stability. This among others present why modeling exchange rates volatility has drawn much attention from researchers for the last two decades (Antonakakis and Darby, 2012).

The autoregressive conditional heteroscedasticity (ARCH) models, with its extension, the generalized autoregressive conditional heteroscedasticity (GARCH) models introduced by Engel (1982) and Bollerslev (1986) respectively, accommodate the dynamics of conditional heteroscedasticity. That is the changing variance nature over time present in a data. Developing countries are increasingly being regarded as alternative destinations for foreign investment flow and this has been accompanied by a huge increase in international transfers, and in many cases by unexpected changes in exchange rate volatility (Antonakakis and Darby, 2012). Such changes can be very costly for investors if they are unforeseen or inefficiently managed.

Volatility of an exchange rate can be termed as the variation of the price at which two different countries currencies are traded.Volatility models are important since they can observe the effect of economic factors on foreign exchange rates and also, to assist policymakers and governments in formulating policies related to money supply in the economy and those associated with the government expenditures and incomes (Alam and Rahman, 2012).

In the corporate world, GARCH models and their extensions are used as a tool in drawing portfolios, risk management and as an input for derivative asset pricing. As economies become globalized, more firm investors and workers find their fortunes linked to exchange rates and thus its impact on trade and financial flows . Therefore, exchange rate volatility in terms of international trade deserves special attention because it affect monetary policies specifically in countries where export growth provides a large stimulus to their domestic economy's growth.

Various exchange rate systems have been adopted by countries and the three main kinds are discussed below;

- Fixed exchange rate regime is a system whereby the rate of exchange between a home country's currency is fixed against an anchor currency. This does not allow for fluctuations in the rates. This is usually the case with economies having currency boards or with no separate national currency of their own. A separate national currency might not exist for a country, either when they have formally dollarized, or when the country is a member of a currency union, for example Euro, CFA Zone etc.

- Floating exchange rate is a system whereby a country's currency value is allowed to fluctuate according to market forces in the foreign exchange market. In this system monetary policy is allowed to pursue other goals such as stabilization of employment and prices.

- Managed floating regime is a floating exchange rate system whereby the market takes its own course but the monetary authorities intervene in the market to "manage" the exchange rate, if needed, to prevent high volatilities and to stimulate growth, without committing to a particular exchange rate level.

Given the small open and import dependent nature of the Gambian economy, the exchange rates is one of the most important macroeconomic variable. This is manifested as government reserves are kept in foreign currency, most imports and exports are paid in foreign currency and moreover, the remittances received by many Gambians from abroad shows that exchange rate is an important component of the monetary transmission process in The Gambia. The volatility in this price has significant effects on people as well as on prices. Consequently, modeling and forecasting volatility in exchange rates is vital for monetary policy and in the short term enables policymakers to make informed decisions on intervention monetary policies (Longmore and Robinson, 2004).

In this study, the aim is to model volatility in the Gambian Exchange rate returns data. The properties of the Gambian exchange rates data are explored and the theoretical properties of GARCH models that are suitable for modeling were examined. Specifically, an autoregressive moving average was used to model the returns while the residuals were fitted with a symmetric and asymmetric GARCH process. The forecasting accuracy of these models were also investigated and presented. This thesis contributes to knowledge in two ways: This is the first study on modeling volatility in the return series of the Gambian exchange rate data and to assess out-of-sample forecast performance using a symmetric and asymmetric ARMA-GARCH/APARCH

. Previous research employed only symmetric GARCH models and assumes that the returns follow a pure GARCH process. This assumption might not be plausible as it is restrictive that the returns series is generated by a white noise process (Francq and Zakoian, 2010).

1.1.1 Exchange Rate Regime and Stylized Facts about The Gambian Economy

The Gambian economy is small economy in West Africa particularly in terms of basic macroeconomic indicators. In terms of official exchange rate GDP measure, the economy is a total of 896 million US Dollars (WDI, 2013). Agriculture and fisheries are dominant activities and contributed about 19.7% percent of GDP in 2013, while Industry though small accounts for 12.6% and the main sector of the economy being services (mainly distributive trade, tourism, transportation and telecommunication) accounted for 67.7 percent of GDP in the same year.

Prior to 1986, it was the fixed exchange rate regime that was adopted by The Gambia supported by exchange control regulations.The foreign exchange market in the fixed exchange rate period was characterized by high demand for foreign exchange which cannot be adequately met with the supply of foreign exchange by the Central Bank of Gambia. As a result, in 1986 the managed floating exchange rate system was introduced as part of the economic and restructuring package program from the IMF. This allows the exchange rate against International currencies such as the US Dollar to be determine by the forces of demand and supply in the currency market. The Central Bank often intervenes only to maintain the required level of reserves and to smooth out volatility. In a bid to increase competition for foreign exchange and improve the allocation efficiency, the Central Bank of The Gambia allowed the licensing of Foreign Exchange Bureaus in 1990 to participate in the currency market. Foreign Exchange Bureaus are business entities different from banks that are authorized by the Central

Bank to exchange currencies. Currently, The Gambian operates a floating exchange rate regime. Currently, the The Gambia is adopting the floating exchange rate system. The major trading currency in the inter-bank foreign exchange market is the US dollar, followed by the Euro and the British Pound.

1.2 Problem Statement

In this period of globalization and financial liberalization, exchange rates play an important role in international trade and finance for countries especially in developing and transitional economies. Research have shown that it has a significant impact on fundamental macroeconomic variables like interest rates, inflation, wages, unemployment and the level of output. For examples see Longmore and Robinson (2004); Ramzan *et al.* (2012). The assumption of constant variance in modeling many financial time series returns can be misleading since the price fluctuates from time to time unpredictably and may also depend on lot of factors. Therefore there is need to accommodate such variance which changes over time when modeling and forecasting volatility in returns especially in high frequency exchange rate returns.

There is no work that has been done on modeling exchange rate volatility using GARCH models in The Gambia. This study attempts to fill this gap. In this thesis, a manifestation of the characteristics of the daily Gambian exchange rate series and returns is shown; an ARMA model is used to model the mean process and a symmetric and asymmetric GARCH models are proposed to model the residuals accordingly.

The suitability of the models in forecasting volatilities are also evaluated.

1.3 Objectives

1.3.1 Main Objective

The general objective of this study is to model volatility in The Gambian exchange rate data between 2003 to 2013.

1.3.2 Specific Objectives

1. To examine the movement and pattern in the Gambian exchange rates data.
2. To identify an appropriate ARMA–GARCH model for The Gambian exchange rate return series.
3. To estimate the proposed model in (2) by Quasi-Maximum Likelihood Method.
4. To give properties of the estimators in (3).
5. To use the estimated models to forecast volatility.

1.4 Limitations of the Research

In this study, the focus is solely on univariate time series modeling of exchange rate volatility. In other words, the return is not related to any other series. The emphasis is to explain its historical movement and model it accordingly. The mean equation is modeled with an ARMA process instead of a constant. This is inconsistent with the efficient market theory, but is deemed necessary as return series exhibits serial dependence and the assumption is that the market is not perfect which is most often the case in foreign exchange markets. We are dealing with daily data and therefore are interested only in the one-step ahead forecast.

1.5 Justification

Foreign exchange rate volatility is a crucial instrument for policymakers, financial institutions, investors as well as ordinary individuals. Accurate modeling and forecasting of exchange rate volatility could be a key indicator in giving warning of an upcoming economic crises within a country (Antonakakis, 2007). Also, a good knowledge of the existence and the extent of a soaring foreign exchange rate could help policymakers in developing and implementing suitable exchange rate policies.

After the Asian financial crises between 1997 to 1998, it became evident that exchange rate volatility deserve much more attention. In The Gambia, exchange rate movements affect people across all works of life. It is also very influential in the monetary mechanism process in the country. The Gambia being a small open economy that trades with rest of the world, deems it imperative for a more precise modeling and forecasting of its exchange rate volatility in order to make better informed decisions not only for international traders' imports and exports, but for the government as well. Therefore, the end-product of this thesis will benefit the government in managing exchange rates volatility and formulate appropriate policy. It will help end-users of exchange rate such as importers, exporters, currency traders, banks and foreign exchange bureau to optimize their future investment decisions.

CHAPTER TWO

LITERATURE REVIEW

2.1 Background Information

Since the seminal works in Engel (1982) and Bollerslev (1986), generalized autoregressive conditional heteroscedasticity (GARCH) processes have received considerable attention in the analysis of financial time series. Engle describe the conditional variance by a simple quadratic function of its lagged values, while Bollerslev modeled the conditional variance to be determined by its own lagged values and the square of the lagged values of the innovations or shocks. These time series models are known to capture several essential features of financial series such as leptokurticity and volatility clustering–volatility responding differently to different economic situations. Empirical studies have shown that such processes are successful in modeling time series. For example in the context of foreign exchange rate markets see earlier works by (Bailie and Bollerslev, 1989; Hsieh, 1989). Many drivers of the dynamics in exchange rate returns and volatility can best be identified in high frequency data. For more details see (Andersen and Bollerslev, 1998a,b). According to (Choy, 2002), knowledge of volatility and its estimation can ensure mitigation of long term risk of any investment which assists in promoting economic growth since investment is the main channel of increasing real output and employment.

The GARCH–in–mean was used by Ryan and Worthington (2004) to investigate the sensitivity of the Australian Bank stock returns to market interest rate and foreign exchange rate risks. Their results suggest that bank stock returns is mostly determined by market risks, together with short and medium term interest and foreign exchange rates. This suggest that the volatility in exchange rates has a strong effect on stock market returns.

In Ghana, Adjasi *et al.* (2008) investigated the influence of exchange rate volatility on stock market returns by using the exponential GARCH model. They established that there exists a negative relationship between exchange rate volatility and stock market returns. They argue that a depreciation of the local currency results to an increase in stock market returns in the long run. Olowe (2009) examines the volatility of Naira/ US Dollar exchange rates in Nigeria using monthly data over the period 1970 to 2007. Six different univariate GARCH models were applied to the data. The paper concluded that all the models show that volatility is persistent for both fixed exchange rate period and the managed floating regime and the best performing models are the APARCH and TS-GARCH.

Kamal *et al.* (2012)modeled volatility in the exchange rates of the Pakistani Rupee and the US Dollar using three ARCH type models namely: GARCH in mean model, EGARCH and TARCH Models. According to the results of their study, it was concluded that EGARCH model was the best model in explaining the volatility behavior of the exchange rate of Pakistani Rupee against the US Dollar. Vee *et al.* (2011) evaluated the volatility forecasts for US Dollar/Mauritius Rupee exchange rate using GARCH(1,1) under two distributional assumptions namely: Generalized Error Distribution (GED) and Student t-distribution. The forecasting performance under these distributional assumptions were evaluated by Mean Absolute Error (MAE) and Root Mean Square Error. It was found that both models performed well for the in-sample forecast, but the model with GED error assumption performs better for forecasting out-of-sample volatility.

A comparative study to established whether the univariate volatility models used widely in modeling and forecasting exchange rate volatility in developed countries were equally applied to data from developing countries by (Antonakakis and Darby, 2012). Three developing countries were selected and four developed countries. All exchange rates were against the US Dollar. It was concluded that for the case of developed countries the Fractionally Integrated GARCH models was superior to other models whereas in the case of developing countries the IGARCH models fitted the data better.

All these studies reviewed above assumes that the series follows a GARCH process. This implies the mean equation in their GARCH models is considered to be a constant. Since no study on modeling volatility in exchange rates is done on using ARMA-GARCH models, this thesis attempts to fill this Gap. These models have been successfully applied to the energy markets notably the oil and electricity markets. For example, a study in modeling and forecasting the conditional mean and volatility of weekly crude oil spot prices in eleven international markets from 2007 to 2009 was conducted by (Mohammadi and Su, 2010). To investigate the out-of-sample forecasting performance, they found that the conditional mean follows an MA(1) process and the conditional variance were fitted with four time-varying volatility models namely GARCH, EGARCH, APARCH and FIGARCH. They concluded that the APARCH model outperforms the others and that conditional standard deviation captures the volatility in oil returns better than the traditional the traditional conditional variance. Hickey *et al.* (2012) evaluate the forecasting performance of autoregressive moving average model with exogenous inputs (ARMAX) to model the conditional mean process and a GARCH models for the residuals on five MISO electricity pricing hubs (Cinergy, First Energy, Illinois, Michigan and Minnesota) using hourly data from June 1, 2006 to October , 2007. The conditional mean accommodates price-spikes, Seasonality, as well as cyclical and secular movement of electricity prices. Considering out-of-sample forecasting performance using MAE, RMSE and the Diebold-Mariano test, it was found that APARCH(1,1) performs well in most hubs, but was not always to produce results that were statistically better than GARCH, Exponential GARCH or Co-integrated GARCH over a longer forecasting horizon. Derek (2013) examined the dynamics of the World Trade Index (WIT) crude oil data from 1993 to 2013. The data was modeled with univariate ARIMA model and the residuals fitted with GARCH and APARCH. A competing regression model was built using eight explanatory variables (consumption, production, ending stock, net import, energy utilization rate, U.S. interest rate, New York MEX oil futures, contract 4 and Standard and Poor 500 index). It was found that on the basis of forecasting accuracy, GARCH and APARCH model performed well, with APARCH performing best in a turbulent market. Therefore, the target is to investigate whether such models can adequately describe exchange rate price behavior in the Gambian foreign exchange market.

2.2 ARCH and GARCH Models

The class of autoregressive conditional heteroscedastic (ARCH) and the generalized autoregressive conditional heteroscedastic (GARCH) models known as conditional heteroscedasticity models are particularly valuable in modeling time series that exhibit stylized properties of financial time series such as fat-tailless, volatility clustering etc. Since the introduction of ARCH several extensions have been made which proved to be useful in modeling and analyzing financial time series. For example, Nelson (1991) proposes the EGARCH (exponential GARCH) specification, modeling the leverage effect, which refers to the increase in volatility following a previous drop in stock returns. Glostan *et al.* (1989) extends the GARCH model with leverage effect in another way, called the GJR-ARCH model and the power ARCH of Ding *et al.* (1993) which embodied the features of asymmetries and long memory in financial returns.

Empirical Research on return distribution has been a subject of discussion among researchers since the 1960s. Badrinath and Chatterjee. (1988) and Rachev *et al.* (2005) have found that the distribution of returns in financial time series data is not characterized by normality but by the stylized facts of fat-tails, high peakedness (excess

Kurtosis) and skewness. Fat-tailness in returns implies that the probability density function is more peaked at the center with a heavy-tail than the normal distribution. The heavy-tailness and high peakedness is measured by its Kurtosis defined as

$$K(r_t) = E\left[\frac{(r_t - \mu)^4}{\sigma^4}\right] \tag{2.1}$$

where μ and σ are the mean and standard deviation of the return series, r_t. $K(r_t)$ greater than 3 implies that the distribution has a heavy-tail property.

Skewness gives a measure of whether a distribution is symmetric or not. Empirically, many financial time series has returns that are asymmetrical which usually contradicts the assumption of normality of returns. The Skewness of a normally distributed variable is defined as:

$$S(r_t) = E\left[\frac{(r_t - \mu)^3}{\sigma^3}\right] \tag{2.2}$$

where Negative values of skewness of returns indicate that data are skewed to the left which in terms of exchange rate refers to an appreciation of the currency, whereas positive values of skewness indicate that data are skewed to the right referring to a depreciation of the currency.

Return series shows significant values of Skewness and kurtosis, this makes the normality assumptions not being met. Therefore in modeling and forecasting with return series, the use of distributions that take into account the leptokurtic and skewness properties in the probability density function can be considered. Some of these distributions include the Levy distribution, the Student-t and the Generalized Error Distribution (Vee *et al.*, 2011).

2.2.1 Mean and Variance Equation

The mean equation of the return series ,r_t, is described by the process

$$r_t = E(r_t|F_{t-1}) + \varepsilon_t \tag{2.3}$$

where $E(r_t|F_{t-1})$ denotes the conditional expectation operator on r_t given F_{t-1}which represent the information set available up to time $t-1$. ε_t denotes the disturbances that are uncorrelated with mean zero and plays an important role in the analysis of the unpredictability of the time series. There are various techniques to model the mean equation. The most commonly used technique in modeling the mean equation in GARCH modeling is a constant, a pure AR or MA. In this study, the mean equation is modeled with an ARMA process. The residuals are then fitted with a GARCH and APARCH model.

2.2.1.1 ARMA Mean Equation

The autoregressive moving average, ARMA(P,Q) process of order P and Q can be described as

$$\begin{aligned} r_t - \mu &= \sum_{i=1}^{P} a_i (r_{t-i} - \mu) - \sum_{j=1}^{Q} b_j \varepsilon_{t-j} + \varepsilon_t \\ &= \phi(\mathrm{B})(r_t - \mu) + \tau(\mathrm{B})\varepsilon_t \end{aligned} \tag{2.4}$$

where

- a and b are constants
- $(r_{t-i} - \mu)$ represents the centered data

- μ is the mean of the return series.

The model can be expressed in a simple comprehensive form using the backshift operator, B, defined by $\mathrm{B}r_t = r_{t-1}$. The functions $\phi(\mathrm{B})$and $\tau(\mathrm{B})$are polynomials of degree P and Q respectively in the backward shift operator B. If $P = 0$ in equation (2.4), we have pure moving average process and if on the other hand $Q = 0$, we have a pure autoregressive process.The ε_t defined above is a white noise process, meaning that it is a sequence independent and identically distributed random variable which has a mean of zero, variance σ^2, and zero correlation across time, i.e. E($\varepsilon_t\varepsilon_{t-j}$) = 0 if $j \neq 0$.

In modeling ARMA process, the time series data needs to be stationary i.e. it has approximately a constant mean and variance over time. This is done by differencing the series or using the logarithmic difference of the series. The Augmented Dickey and Fuller (1979) ,and the Phillips and Perron (1988) tests are conducted to verify that the return series are stationary or not after observing the plotted series.

2.2.1.2 GARCH Variance Equation

In practical terms, the mean equation cannot take into account the heteroscedasticity effect time series processes typically observed in the form of fat-tails, volatility clustering and the leverage effects. A GARCH model incorporates such features. The general specification for the innovations ε_t can be described as

$$\begin{aligned} \varepsilon_t &= \sigma_t z_t \\ z_t &\sim D_\vartheta\left(0,\sigma^2\right) \end{aligned} \tag{2.5}$$

where z_t is an independent and identically distributed process and $D_\vartheta\left(0,\sigma^2\right)$ is the probability density function of the innovations or residuals with zero mean and variance σ^2. If it is assume the innovation to follow a standard normal distribution then $z_t \sim N(0,1)$. Otherwise ϑ are additional distributional parameters to describe the skew and the shape of the distributions. This is the case under the assumption that the innovations follow a Student's t-distribution or generalized error distribution.

The ARMA model which represent the mean equation is used as a filter in order to remove serial dependence in the return series and to produce independent and identically distributed residuals. The McLeod-Li test and ARCH test were conducted to check for independence and heteroscedasticity in the residuals. The probability quantile (QQ) plots are then used to check for normality of the innovations The theoretical background of some conditional heteroscedastic models are now discussed.

2.2.1.3 GARCH(p,q) Model

The Generalized ARCH (GARCH), as developed by Bollerslev (1986), is an extension of the ARCH model. It allows for both a longer memory and a more flexible lag structure.

Definition 2.1. The process ε_t, is called a GARCH(p,q) process if its first two conditional moments exists and satisfy:

$E(\varepsilon_t|\varepsilon_u, u<t) = 0, \ t\varepsilon\mathbb{Z}$

There exist constants, ω, $\alpha_i \geq 0, \ i=1,...,q$ and $\beta_j \geq 0, \ j=1,...,p$ such that

$$\begin{aligned} \sigma_t^2 &= Var(\varepsilon_t|\varepsilon_u, u<t) \\ &= \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2, \ t\varepsilon\mathbb{Z} \end{aligned} \tag{2.6}$$

where

$$p \geq 0, \quad q > 0, \quad \omega > 0$$

We can write equation (2.6) in a more compact for a

s

$$\begin{aligned} \sigma_t^2 &= Var\left(\varepsilon_t|\varepsilon_u, u < t\right) \\ &= \omega + \alpha\left(B\right)\varepsilon_t^2 + \beta\left(B\right)\sigma_t^2 \end{aligned} \tag{2.7}$$

where B is backshift operator, for example $B^i \varepsilon_t^2 = \varepsilon_{t-1}^2$and α and β are polynomials of degrees$q$ and p respectively:

$$\begin{aligned} \alpha\left(B\right) = \sum_{i=1}^{q} \alpha_i B^i \quad \beta\left(B\right) \\ = \sum_{j=1}^{p} \beta_j B^j \end{aligned} \tag{2.8}$$

The vector of parameters to be estimated is given by $\theta = \left(\omega, \alpha_1, ..., \alpha_q, \beta_1, ..., \beta_p\right)$ assuming that p and q are known. This parameter vector belongs to a parameter space

$$\Theta \subset (0, +\infty) \times [0.\infty)^{p+q},$$

where Θ is a compacts set.

However, setting $Z_t = \varepsilon_t/\sigma_t$ in the GARCH model we have

$$\begin{aligned} E(z_t) &= E(\varepsilon_t/\sigma_t) = E(E(\varepsilon_t/\sigma_t|F_{t-1})) \\ &= E(E(\varepsilon_t|F_{t-1}/\sigma_t) = 0, \end{aligned} \tag{2.9}$$

and

$$
\begin{aligned}
Var(z_t) &= E(z_t^2) - E^2(z_t) = E(z_t^2) = E(\varepsilon_t^2/\sigma_t^2) \qquad (2.10)\\
&= E(E(\varepsilon_t^2/\sigma_t^2|F_{t-1})) = E(E(\varepsilon_t^2|F_{t-1}/\sigma_t^2)) = 1.
\end{aligned}
$$

Under the assumption that z_t is normally distributed, the difference between strong and semi-strong GARCH models disappears. A GARCH process is said to be strong if $Var(\varepsilon_t|F_{t-1}) = \sigma_t^2$ *and* $Z_t = \varepsilon_t/\sigma_t$ and semi-strong when$Var(\varepsilon_t|F_{t-1}) = \sigma_t^2$.

Assuming without loss of generality that s > 0, we notice

$$
\begin{aligned}
Cov(z_t, z_{t+s}) &= E(z_t z_{t+s}) = E(E(\varepsilon_t \varepsilon_{t+s}/\sigma_t \sigma_{t+s}|F_{t+s-1}))\\
&= E(E(\varepsilon_t \varepsilon_{t+s}|F_{t+s-1}/\sigma_t \sigma_{t+s})) = 0, \qquad (2.11)
\end{aligned}
$$

and this, together with equation (2.11) and (2.13) shows that

$$z_t \sim N(0,1)$$

implying that

$$z_t \sim i.i.d.\ N(0,1).$$

The unconditional variance of our return series is:

$$Var(r_t) = \frac{\omega}{1 - (\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j)} \qquad (2.12)$$

The necessary and sufficient condition for existence of a stationary solution of ε_t to the GARCH model was established by (Bollerslev, 1986). The definition of the stationarity is:

Definition 2.2. The process $\varepsilon_t, t \in \mathbb{Z}$, is stationary if

$E(\varepsilon_t)$ is independent of t, and

$Cov(\varepsilon_{t+h}, \varepsilon_t)$ is independent of t for each h.

Theorem 2.1. *The GARCH(p,q) process as defined in (2.1) is stationary with* $E(\varepsilon_t) = 0$, $Var(\varepsilon_t) = \omega(1 - (\sum \alpha_i - \sum_{j=1}^{p} \beta_j)^{-1}$ *and* $Cov(\varepsilon_t, \varepsilon_s) = 0$ *for* $t \neq s$ *if and only if*

$$\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1 \tag{2.13}$$

The proof of this theorem can be found in (Bollerslev, 1986).

The simplest, but often very useful GARCH process is the GARCH(1,1) process given by

$$\begin{aligned} \sigma_t^2 &= \alpha_o + \alpha_1 r_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \qquad (2.14) \\ \alpha_0 > 0 \quad \alpha \geq 0 \quad \beta \geq 0 \end{aligned}$$

where $\alpha_1 + \beta_1 < 1$ being sufficient for wide sense stationarity. Many studies have found that GARCH(1,1) specification to exchange rate return series performs better than other alternative models. For Instance see Taylor (1986), McCurdy and Morgan (1988) among others.

2.2.2 Integrated GARCH Model

The GARCH models performs better in terms of capturing the short run volatility dependencies in financial returns. Many financial return series such as exchange rates exhibit persistent volatility. The IGARCH model shows that long-run dependencies in variances do exists. The IGARCH model is given by equation (2.6) together with the condition $\sum_{i=1}^{q}\alpha_i+\sum_{j=1}^{p}\beta_j=1$, indicating that the unconditional variance σ^2 is not definite anymore. According to Bourgerol and Picard (1992) the IGARCH model can be applied to financial returns even if they are not covariance stationary since they remain strictly stationary. This is because the unconditional density of the error does not change over time.

The IGARCH(1,1) model can be written as:

$$\begin{aligned}\sigma_t^2 &= \omega+(1-\beta_1)r_{t-1}^2+\beta_1\sigma_{t-1}^2 \\ 0< \beta_1 &\leq 1\end{aligned} \tag{2.15}$$

Several studies have shown the suitability of the IGARCH model in modeling and forecasting daily exchange rate volatility since the volatility seems to be highly persistent (see e.g. McCurdy and Morgan (1988), and Hsieh (1989)).

2.2.3 Asymmetric Power ARCH Model

The Asymmetric Power ARCH (APARCH) model has the structure

$$\begin{aligned}\varepsilon_t &= \sigma_t z_t \\ \sigma_t^\delta &= \omega+\sum_{i=1}^{q}\alpha_i(|\varepsilon_{t-i}|-\gamma_i\varepsilon_{t-i})^\delta+\sum_{j=1}^{p}\beta_j\sigma_{t-j}^\delta\end{aligned} \tag{2.16}$$

where

$$\alpha_0>0,\delta\geq 0,\alpha_i\geq 0 i=1,...,q,\beta_j\geq 0,j = 1,...,p,-1<\gamma_i<1$$

and $\omega, \gamma_i, \alpha_i, \beta_j$and δ are the parameters to be estimated. γ_i is the leverage effect and a positive value implies negative information has stronger impact than positive information on price volatility. This model imposes a Box-Cox transformation on the conditional standard deviation process and the asymmetric absolute residuals(Ding *et al.*, 1993). This model includes seven models (namely ARCH, GARCH, GJR-GARCH, Taylor/Schwert's GARCH, Nonlinear ARCH, Log-ARCH and Threshold ARCH) as special cases by assigning different values to the δ and an interval constraint on the γ_i parameters in equation (2.17). Assuming that the distribution of ε_t is conditionally normal, then the condition for existence of $E\sigma_t^\delta$and $E|\varepsilon_t|^\delta$ is outlined as

$$\sum_{i=1}^{q} \alpha_i E(|z_{t-i}| - \gamma_i z_{t-i})^\delta + \sum_{j=1}^{p} \beta_j < 1,$$

where

$$\begin{aligned} E(|z_{t-i}| - \gamma_i z_{t-i})^\delta &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} (|x| - \gamma_i x)^\delta exp\left\{\frac{x^2}{2}\right\} dx \qquad (2.17) \\ &= \frac{1}{\sqrt{2\pi}} \left\{(1+\gamma_i)^\delta + (1-\gamma_i)^\delta\right\} 2^{\frac{\delta-1}{2}} \Gamma\left(\frac{\delta+1}{2}\right) \end{aligned}$$

Therefore the condition becomes

$$\frac{1}{\sqrt{2\pi}} \sum_{i=1}^{q} \alpha_i \left\{1+\gamma_i)^\delta + (1-\gamma_i)^\delta\right\} 2^{\frac{\delta-1}{2}} \Gamma\left(\frac{\delta+1}{2}\right) \quad + \sum_{j=1}^{p} \beta_j < 1. \qquad (2.18)$$

If this condition is satisfied, $\delta \geq 0$ implies covariance stationarity and $\delta \geq 2$ is a sufficient condition for ε_t to be covariance stationary.

Since

$$\begin{aligned} E|\varepsilon_t|^\delta &= E|z_t|^\delta E\sigma_t^\delta \qquad (2.19) \\ &= \frac{1}{\sqrt{\pi}} 2^{\frac{\delta}{2}} \Gamma\left(\frac{\delta+1}{2}\right) E\sigma_t^\delta \end{aligned}$$

implying that the condition for existence of $E|\varepsilon_t|^\delta$ is the same as that of $E\sigma_t^\delta$.

When condition (2.19) is satisfied, then the conditional expectation of σ_t^δ follows as

$$E\sigma_t^\delta = \frac{\alpha_0}{\left(1 - \sum_{i=1}^{q} \alpha_i E(|z_{t-i}| - \gamma_i z_{t-i})^\delta\right) - \sum_{j=1}^{p} \beta_j} \tag{2.20}$$

and letting $E\sigma_t^\delta = \phi^\delta$

$$\begin{aligned} E|\varepsilon_t|^\delta &= \frac{1}{\sqrt{\pi}} 2^{\frac{\delta}{2}} \Gamma\left(\frac{\delta+1}{2}\right) E\sigma_t^\delta \\ &= \frac{1}{\sqrt{\pi}} 2^{\frac{\delta}{2}} \Gamma\left(\frac{\delta+1}{2}\right) \phi^\delta \end{aligned} \tag{2.21}$$

The GARCH (p,q) covariance stationarity condition is obtained when $\delta = 2$ and $\gamma_i = 0$ in the inequality (2.19) as

$$\begin{aligned} \frac{1}{\sqrt{2\pi}} \sum_{i=1}^{q} \alpha_i 2^{\frac{2-1}{2}} \Gamma\left(\frac{2+1}{2}\right) + \sum_{j=1}^{p} \beta_j &= \frac{1}{\sqrt{\pi}} \sum_{i=1}^{q} \alpha_i 2 \left(\frac{1}{2}\right) \Gamma\left(\frac{1}{2}\right) + \sum_{j=1}^{p} \beta_j \\ &= \sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1 \end{aligned} \tag{2.22}$$

which is the same condition as that of (2.13).

The other models from the APARCH model can be obtained as special cases as:

1. ARCH (p) when $\delta = 2$and $\gamma_i = 0, i = 1, ..., p,\ \beta_j = 0, j = 1, ..., q$
2. Taylor/Schwert's GARCH in standard deviation model when $\delta = 1$ and $\gamma_i = 0,\ i = 1, ..., p$.
3. GJR GARCH model[see Glostan *et al.* (1989)] when$\delta = 2$
4. Zakoian's Threshold ARCH model [see Zakoian (1991)]when $\delta = 1$ and $\beta_j = 0, j = 1, ..., q$

5. Higgins. and Bera (1990) nonlinear ARCH when $\gamma_i = 0, i = 1, ..., p,\ \beta_j = 0, j = 1, ..., q$

6. Log-ARCH when $\delta \to 0$.

CHAPTER THREE

METHODOLOGY

3.1 Returns

Since most high frequency financial time series data such as daily exchange rate prices are usually non-stationary, the logarithmic transformation is needed to convert the prices to returns which are notably known to be stationary over time. Let the daily exchange rate series be denoted by y_t, then its logarithmic difference series, r_t, is defined as

$$\begin{aligned} r_t &= \ln\left(\frac{y_t}{y_{t-1}}\right) \\ &= \ln(y_t) - \ln(y_{t-1}) \end{aligned}$$

where

- r_t is the return at time t
- y_t is the exchange rate price at time t
- y_{t-1} is the exchange rate at time $t-1$.

The underlying theory of econometric financial time series models is stationarity which is achieved with the returns.The returns defined above are used in modeling the mean equation by an ARMA process. This is necessary because it produces independent and identically distributed residuals which are then modeled with a GARCH process.

3.2 Model Identification

It is important to subject ε_t to residual analysis test before we proceed to model in order to determine whether there exists dependence in the returns. There are various residual analysis tests which include; sample autocorrelation function test, portmanteau test, ARCH test, Mcleod-Li test, turning point test, difference-sign test and normality checking. In this study, the test considered is on using the sample ACF, Mcleod-Li test and the ARCH test. In modeling time series data, choosing a correct or adequate model is essential since it aids in making accurate future predictions. It is important that the order of the model should be known before estimation and making predictions. The number of parameters in the model plays an important role in both analysis and forecasting. This is because addition of unnecessary lags reduces the sum of squares of estimated residuals and the forecasting performance as well. The Akaike Information Criterion (AIC) is used and is given as

$$AIC = -2\log(L) + 2k$$

where

- $\log(L)$ is the maximized likelihood of the parameters for the estimated model
- k is the number of parameters
- The term $2k$ is a penalty as an increasing function of the number of estimated parameters.

  This AIC function is used in determining the orders P,Q for the ARMA process as well as the orders p and q for the GARCH process. Several ARMA and GARCH orders are considered and the model with the lowest AIC value is preferred.

3.3 Model Estimation

The Quasi-maximum likelihood estimation of the ARMA(P,Q)-GARCH(p,q) model is adopted in this research. In the absence of normality, Weiss (1986) and, Bollerslev and Wooldridge (1992) have shown that in GARCH models, maximizing the Gaussian likelihood produces a Quasi-Maximum Likelihood (QML) estimator that is consistent and asymptotically normally distributed provided that the conditional mean and variance equation are correctly specified. The ARMA-GARCH process under mild conditions of Francq and Zakoian (2004) is used and the QML estimator is shown together with its properties.

To compare the forecasting performance and to check for leverage effects in the Gambian foreign exchange data, an ARMA-APARCH is included.

3.4 Estimation of ARMA(P,Q)-GARCH(p,q) Model

The situation discussed here is the case whereby the GARCH process constitutes the the innovations of an ARMA process. This is important as in financial time series, it is restrictive to assume that the observed series is a realization of a noise. Assuming that the $r_1, ..., r_n$are generated by a a strictly stationary non-anticipative solution of the ARMA(P,Q)-GARCH(p,q) order defined as

$$\begin{aligned} r_t - \mu &= \sum_{i=1}^{P} a_i (r_{t-i} - \mu) - \sum_{j=1}^{Q} b_j \varepsilon_{t-j} + \varepsilon_t \\ \varepsilon_t &= \sigma_t z_t \\ \sigma_t^2 &= \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \end{aligned} \tag{3.1}$$

where z_t is an independent and identically distributed variable with a mean of zero and variance σ^2. Assuming that the orders P, Q, p and q are positive and known, the parameter vector is denoted by

$$\begin{aligned}\varphi &= \left(\vartheta', \theta'\right)' \\ &= \left(a_1, ..., a_P, b_1, ..., b_Q, \theta'\right).\end{aligned} \tag{3.2}$$

where $\theta = \left(\omega, \alpha_1, ..., \alpha_q, \beta_1, ..., \beta_p\right)$ .The parameter space is given by

$$\Phi \subset \mathbb{R}^{P+Q+1} \times (0, +\infty) \times [0, \infty)^{p+q}. \tag{3.3}$$

The true value of the parameter is given by

$$\begin{aligned}\varphi_0 &= \left(\vartheta_0', \theta_0'\right)' \\ &= \left(a_{01}, ..., a_{0P}, b_{01}, ..., b_{0Q}, \theta_0'\right).\end{aligned} \tag{3.4}$$

With the Gaussian quasi-maximum likelihood conditional on initial values when $q \geq Q$, we have the initial values as

$$r_1, ..., r_{1-(q-Q)-P}, \tilde{\varepsilon}_{-q+Q}, ..., \tilde{\varepsilon}_{1-q}, \tilde{\sigma}_0^2, ..., \tilde{\sigma}_{1-p}^2,$$

the last p of these values are positive and may depend on the parameter or on the observations. For any ϑ, the values $\tilde{\varepsilon}_t(\vartheta), t = -q+Q+1, ..., n$, and then for any θ, the values of $\tilde{\sigma}_t^2(\theta)$, for $t = 1, ...n$ is computed from

$$\begin{aligned}\tilde{\varepsilon}_t &= \tilde{\varepsilon}_t(\vartheta) = r_t - \mu - \sum_{i=1}^{P} a_i (r_{t-i} - \mu) + \sum_{j=1}^{Q} b_j \tilde{\varepsilon}_{t-j} \\ \tilde{\sigma}_t^2 &= \tilde{\sigma}_t^2(\varphi) = \omega + \sum_{i=1}^{q} \alpha_i \tilde{\varepsilon}_{t-i}^2 + \sum_{j=1}^{p} \beta_j \tilde{\sigma}_{t-j}^2.\end{aligned} \tag{3.5}$$

However, when $q < Q$, the fixed initial values are

$$r_1, ..., r_{1-(q-Q)-P}, \varepsilon_0, ..., \varepsilon_{1-Q}, \tilde{\sigma}_0^2, ..., \tilde{\sigma}_{1-p}^2. \tag{3.6}$$

Conditional on these initial values, the Gaussian Log-likelihood is obtained as

$$\begin{aligned} \tilde{I}_n(\varphi) &= n^{-1} \sum_{t=1}^{n} L_t,\ L_t \\ &= L_t(\phi) \\ &= \frac{\tilde{\varepsilon}_t^2(\vartheta)}{\tilde{\sigma}_t^2(\varphi)} + \log\left(\tilde{\sigma}_t^2(\varphi)\right). \end{aligned} \tag{3.7}$$

A QML estimator of the parameter vector is defined as any measurable solution of the equation

$$\begin{aligned} \tilde{\varphi}_n &= argmin \tilde{I}_n(\varphi),\ \varphi \\ &\in \Phi. \end{aligned} \tag{3.8}$$

3.5 Properties of the ARMA–GARCH Quasi-maximum Likelihood Estimator

3.5.1 Consistency of the Quasi-Maximum Likelihood

The consistency property of the quasi-maximum likelihood estimator is now discussed. To show this consistency, let $a_\vartheta(z) = 1 - \sum_{i=1}^{P} a_i z^i$ and $b_\vartheta(z) = 1 - \sum_{j=1}^{Q} b_j z^j$ and utilizing the following assumptions:

A1. $\gamma(A_0) < 0$ and for all $\theta \in \Theta, \sum_{j=1}^{p} \beta_j < 1$ where

$$A_0 = (A_{0t})$$
$$= \begin{pmatrix} \alpha_{01}z_t^2 & & \cdots & & \alpha_{0q}z_t^2 & \beta_{01}z_t^2 & & \cdots & & \beta_{0p}z_t^2 \\ 1 & 0 & \cdots & & 0 & 0 & & \cdots & & 0 \\ 0 & 1 & \cdots & & 0 & 0 & & \cdots & & 0 \\ \vdots & \ddots & \ddots & & \vdots & \vdots & \ddots & \ddots & & \vdots \\ 0 & & \cdots & 1 & 0 & 0 & & \cdots & 0 & 0 \\ \alpha_{01} & & \cdots & & \alpha_{0q} & \beta_{01} & & \cdots & & \beta 0_p \\ 0 & & \cdots & & 0 & 1 & 0 & \cdots & & 0 \\ 0 & & \cdots & & 0 & 0 & 1 & \cdots & & 0 \\ \vdots & \ddots & \ddots & & \vdots & \vdots & \ddots & \ddots & & \vdots \\ 0 & & \cdots & 0 & 0 & 0 & & \cdots & 1 & 0 \end{pmatrix}$$

is a top negative Lyapunov exponent, $\gamma(A_0) < 0$, where

$$\begin{aligned} \gamma(A_0) &= \inf_{t \in \mathbb{N}} \frac{1}{t} E\left(\log \|A_{0t}A_{0t-1}...A_{o1}\|\right) \\ &= \lim_{n \to \infty} \frac{1}{t} \log \|A_{0t}A_{0t-1}...A_{o1}\|, \end{aligned}$$

and admits a stationary solution of the GARCH (p,q) model as in Definition2.2.

A2. z_t^2 has a non-generate distribution and $Ez_t^2 = 1$

A3. $p > 0,_{\theta_0}(z)$ and $B_{\theta_0}(z)$ have no common roots, $A_{\theta_0}(1) \neq 0$ and $\alpha_{oq} + \beta_{0p} \neq 0$.

A4. $\varphi_0 \varepsilon \Phi$ and Φ is compact

A5. For all $\varphi \in \Phi, a_{\vartheta}(z) b_{\vartheta}(z) = 0$ implies $|z| > 1$

A6. $a_{\vartheta_0}(z)$ and $b_{\vartheta_0}(z)$ have no common roots and $a_{oP} \neq 0$ and $b_{oQ} \neq 0$.

Under assumption **A1-A5**, gives the unique strictly stationary solution of equation (3.1). By letting $\varepsilon_t = \varepsilon_t(\vartheta) = a_{\vartheta_0}(B) b_{\vartheta}^{-1}(B)(r_t - \mu)$ and $L_t(\phi) = \frac{\varepsilon_t^2(\vartheta)}{\sigma_t^2(\varphi)} + \log\left(\sigma_t^2(\varphi)\right)$ where $\sigma_t^2 = \sigma_t^2(\varphi)$ gives the non-anticipative and ergodic strictly non-stationary solution of (equation of GARCH) and lead to the following theorem.

Theorem 3.1. *Let $\tilde{\varphi}_n$ be a sequence of QML estimators satisfying Equation 3.10. Assume $Ez_t = 0$, then under the six assumptions, almost surely*

$$\tilde{\varphi}_n \to \varphi_0, \qquad as\ n \to \infty$$

To proof this theorem the following four conditions must be shown

(a) $\lim_{n\to\infty} \sup_{\varphi \varepsilon \Phi} \left(I_n(\varphi) - \tilde{I}_n(\varphi)\right) = 0$ *almost surely*

(b) $(\exists t \varepsilon \mathbb{Z}$ *such that* $\varepsilon_t(\vartheta) = \varepsilon_t(\vartheta_0)$ *and* $\sigma_t^2 = \sigma_t^2(\varphi) P_{\varphi_0}$*almost surely)*$\Rightarrow \varphi = \varphi_0$ *and P is the best linear projection.*

(c) *If* $\varphi \neq \varphi_0, E_{\varphi_0}\mathscr{L}_t(\varphi) > E_{\varphi_0}\mathscr{L}_t(\varphi_0)$ *where* $\mathscr{L}_t$*is defined in* 3.7

(d) *For any* $\varphi \neq \varphi_0$*, there exists a neighborhood such that* $V(\varphi)$ *such that*

$\lim_{n\to\infty} \inf \inf_{\varphi \varepsilon V(\varphi)} \tilde{I}_n(\varphi^*) > E_{\varphi_0} L_1(\varphi_0)$, *almost surely.*

The following corollary and lemma will help us achieve the results above.

Corollary 3.1. *: Let γ denote the lyapunov exponent of the sequence (A_t) defined in **(A1)**. Then*

$$\gamma < 0 \Rightarrow \exists s > 0, \quad E\sigma_t^{2s} < \infty, \quad E\varepsilon_t^{2s} < \infty$$

Lemma 3.1. *Let $\{A_t\}$be a sequence of independent and identical sequence of a posi-*

tive matrix with top lyapunov exponent γ. Then

$$\gamma < 0 \Longleftrightarrow \exists s > 0, \quad \exists k_0 \geq 1, \quad \delta := E\left(||A_{k_0}A_{k_0-1}...A_1||^s\right)\right) < 1$$

where $\varepsilon_t = \sigma_t z_t$ is the strictly stationary solution of the GARCH(p,q) model in (2.5) and definition (2.1).

The proof of the Lemma and Corollary can be found in (Francq and Zakoian, 2010).

We now the show the proof of the four conditions (***a to d***) which will establish the consistency of the Quasi-Maximum Likelihood estimator.The condition in **(a)** refers to the nullity of the asymmetric impact of the initial values. In the following proofs, K and $\rho$are generic constant whose values varies from argument to argument. Letting

$$\tilde{\sigma}_t^2 = \tilde{c}_t + B\tilde{c}_{t-1} + ... + B^{t-1}\tilde{c}_1 + B^t\tilde{\sigma}_0^t \tag{3.9}$$

where $\tilde{c}_t = \left(\omega + \sum_{i=1}^{q} \alpha_i \varepsilon_t^2, 0, ..., 0\right)'$ are the initializing variables. Assumption **A1** implies that $\sum_{j=1}^{p} \beta_j < 1$ implies that $\rho(B) < 1$. The compactness of Φ implies that

$$\sup_{\theta\varepsilon\Phi} \rho(B) < 1 \tag{3.10}$$

Assumptions **A4** and **A5** implies that for any $k \leq 1$ and $1 \leq i \leq q$

$$\sup_{\varphi\varepsilon\Phi} |\tilde{\varepsilon}_{k-i} - \varepsilon_{k-i}| \leq K\rho^K,$$

It follows almost surely

$$\begin{aligned} \|c_k - \tilde{c}_k\| &\leq \sum_{i=1}^{q} |\alpha_i| \left|\tilde{\varepsilon}_{k-i}^2 - \varepsilon_{k-i}^2\right| \qquad (3.11) \\ &\leq \sum_{i=1}^{q} |\alpha_i| \left|\tilde{\varepsilon}_{k-i}^2 - \varepsilon_{k-i}^2\right| (|2\varepsilon_{k-i}| + |\tilde{\varepsilon}_{k-i} - \varepsilon_{k-i}|) \\ &\leq K\rho^K \left(\sum_{i=1}^{q} |\varepsilon_{t-i}| + 1\right) \end{aligned}$$

By equations

$$
\begin{aligned}
\left\|\sigma_t^2-\tilde{\sigma}_t^2\right\| &= \left\|\sum_{k=0}^{t} B^{t-k}\left(c_k-\tilde{c}_k\right)+B^t\left(\sigma_0^2-\tilde{\sigma}_0^2\right)\right\| \\
&\leq K\sum_{k=0}^{t}\rho^{t-k}\rho^k\left(\sum_{i=1}^{q}\left|\varepsilon_{t-i}\right|+1\right)+K\rho^t \qquad (3.12)\\
&\leq K\rho^t\sum_{k=-q}^{t}\left(\left|\varepsilon_k\right|+1\right).
\end{aligned}
$$

By using the same arguments, we have almost surely $\left|\tilde{\varepsilon}_t^2-\varepsilon_t^2\right|\leq K\rho^t\sum_{k=-q}^{t}\left(\left|\varepsilon_k\right|+1\right).$ The difference between the theoretical Log-likelihoods with and without initial values is bounded as

$$
\begin{aligned}
\sup_{\varphi\varepsilon\Phi}\left|I_n(\varphi)-\tilde{I}_n(\varphi)\right| &\leq n^{-1}\sum_{t=1}^{n}\sup_{\varphi\varepsilon\Phi}\left\{\left|\frac{\tilde{\sigma}_t^2-\sigma_t^2}{\tilde{\sigma}_t^2\sigma_t^2}\right|\varepsilon_t^2+\left|\log\left(1+\frac{\sigma_t^2-\tilde{\sigma}_t^2}{\tilde{\sigma}_t^2}\right)\right|+\frac{\left|\varepsilon_t^2-\tilde{\varepsilon}_t^2\right|}{\tilde{\sigma}_t^2}\right\}\\
&\leq \left\{\sup_{\varphi\varepsilon\Phi} max\left(\frac{1}{w^2},\frac{1}{w}\right)\right\}Kn^{-1}\sum_{t=1}^{n}\rho^t\left(\varepsilon_t^2+1\right)\sum_{k=-q}^{t}\left(\left|\varepsilon_k\right|+1\right). \qquad (3.13)
\end{aligned}
$$

Letting $\zeta_t=\left(\varepsilon_t^2+1\right)\sum_{k=-q}^{t}\left(\left|\varepsilon_k\right|+1\right)$ .It suffices to show that for all $r>0$, $E\left(\rho^t\zeta_t\right)^r$is the general term of a finite series. That is

$$
\begin{aligned}
E\left(\rho^t\zeta_t\right)^{\frac{s}{2}} &\leq \rho^{t\frac{s}{2}}\sum_{k=-q}^{t}E\left(\varepsilon_t^2\left|\varepsilon_k\right|+\varepsilon_t^2+\left|\varepsilon_k\right|+1\right)^{s/2}\\
&\leq \rho^{t\frac{s}{2}}\sum_{k=-q}^{t}\left[\left\{E\left(\varepsilon_t^{2s}\right)E\left|\varepsilon_k\right|^s\right\}^{1/2}+E\left|\varepsilon_k\right|^s+E\left|\varepsilon_k\right|^{\frac{s}{2}}+1\right]\\
&=O\left(t\rho^{t\frac{s}{2}}\right). \qquad (3.14)
\end{aligned}
$$

Condition ***(a)*** follows from corollary (3.1), since $E\left(\varepsilon_t^{2s}\right)<\infty$.

The condition in ***(b)*** refers to the identifiability of the parameters. If $\varepsilon_t(\vartheta)=\varepsilon_t(\vartheta_0)$ almost surely, assumption **A5** and **A6** implies that there exists a constant linear combination of the variables, $r_{t-j}, j\geq 0$. The linear innovation of (r_t), equal to$r_t-E(r_t|r_u,u<$

$t) = z_t\sigma_t^2(\varphi_0)$ is almost surely if $z_t = 0$ almost surely since $\sigma_t^2(\varphi_0) \geq w_0 > 0$. This holds true, since $E\left(z_t^2\right) = 1$. Therefore, it follows that $\vartheta = \vartheta_0$and therefore, $\theta = \theta_0$.

The condition in ***(c)*** implies that the limit criterion is minimized at the true value. To show this, we use the fact that $E_{\varphi_0}l_n(\varphi) = E_{\varphi_0}\mathscr{L}_t(\varphi)$is define in $\mathbb{R}\cup\{+\infty\}$, and in $\mathbb{R}$ at $\varphi = \varphi_0$, we have that

$$\begin{aligned} E_{\varphi_0}L_t(\varphi) - E_{\varphi_0}L_t(\varphi) &= E_{\varphi_0}\log\frac{\sigma_t^2(\varphi)}{\sigma_t^2(\varphi_0)} + E_{\varphi_0}\left[\frac{\varepsilon_t^2(\vartheta)}{\sigma_t^2(\varphi)} - \frac{\varepsilon_t^2(\vartheta_0)}{\sigma_t^2(\varphi_0)}\right] \\ &= E_{\varphi_0}\left\{\log\frac{\sigma_t^2(\varphi)}{\sigma_t^2(\varphi_0)} + \frac{\sigma_t^2(\varphi_0)}{\sigma_t^2(\varphi)} - 1\right\} + E_{\varphi_0}\frac{\left\{\varepsilon_t^2(\vartheta) - \varepsilon_t^2(\vartheta_0)\right\}^2}{\sigma_t^2(\varphi)} \\ &+ E_{\varphi_0}\frac{2z_t\sigma_t(\varphi_0)\{\varepsilon_t(\vartheta) - \varepsilon_t(\vartheta_0)\}}{\sigma_t^2(\varphi)} \geq 0, \end{aligned} \tag{3.15}$$

because the last expectation is equal to zero (noting that $\varepsilon_t(\vartheta) - \varepsilon_t(\vartheta_0)$ belongs to the past as well as $\sigma_t(\varphi_0)$ and $\sigma_t(\varphi)$), the other expectations being positive or null by the arguments already used. This inequality is strictly stationary only if $\varepsilon_t(\vartheta) = \varepsilon(\vartheta_0)$ and if $\sigma_t^2(\varphi) = \sigma_t^2(\varphi_0)P_{\varphi_0}$almost surely which, by ***(b)*** implies$\varphi = \varphi_0$ and completes the proof of condition ***(c).***

To show condition ***(d)*** the compactness of Φ and the ergodity of $L_t(\varphi)$is used. For more details see (Francq and Zakoian, 2010).

3.5.2 Asymptotic Normality of Quasi-Maximum Likelihood Estimator

To establish the asymptotic normality of the QML estimator, further assumptions are required. These are:

A7: $\rho\{E(A_{0t}\otimes A_{0t})\} < 1$ and, for all $\theta\varepsilon\Theta$, $\sum_{j=1}^{p}\beta_j < 1$

A8: $\varphi_0 \in \Phi^o$, where Φ^o denotes the interior of Φ

A9: There exists a set $\Lambda$of cardinality of real numbers such that $P(z_t \in \Lambda) = 1$, where P refers to probability.

Assumption **A8** implies that $k_z = E\left(z_t^4\right) < \infty$ and makes assumption **A1** superfluous. Now the asymptotic normality of the QML estimator is established by the following theorem.

Theorem 3.2. *Assume that $Ez_t = 0$ and that assumptions **A2**, **A3** and **A5-A9** hold true. Then*

$$\sqrt{n}\left(\tilde{\varphi}_n - \varphi_0\right) \to N\left(0, \Sigma\right)$$

where $\Sigma = J^{-1}IJ^{-1}$

$$\begin{aligned} I &= E_{\varphi_0}\left(\frac{\partial L_t\left(\varphi_0\right)}{\partial \varphi}\frac{\partial L_t\left(\varphi_0\right)}{\partial \varphi'}\right) \\ J &= E_{\varphi_0}\left(\frac{\partial^2 L_t\left(\varphi_0\right)}{\partial \varphi \partial \varphi'}\right) \end{aligned}$$

If the distribution of z_t is symmetric, we have

$$I = \begin{pmatrix} I_1 & 0 \\ 0 & I_2 \end{pmatrix}$$

$$J = \begin{pmatrix} J_1 & 0 \\ 0 & J_2 \end{pmatrix}$$

with

$$
\begin{aligned}
I_1 &= (k_z-1)\,E_{\varphi_0}\left(\frac{1}{\sigma_t^4}\frac{\partial\sigma_t^2}{\partial\vartheta}\frac{\partial\sigma_t^2}{\partial\vartheta'}(\varphi_0)\right)+4E_{\varphi_0}\left(\frac{1}{\sigma_t^4}\frac{\partial\varepsilon_t}{\partial\vartheta}\frac{\partial\varepsilon_t}{\partial\vartheta'}(\varphi_0)\right),\\
I_2 &= (k_z-1)\,E_{\varphi_0}\left(\frac{1}{\sigma_t^4}\frac{\partial\sigma_t^2}{\partial\theta}\frac{\partial\sigma_t^2}{\partial\theta'}(\varphi_0)\right),\\
J_1 &= E_{\varphi_0}\left(\frac{1}{\sigma_t^4}\frac{\partial\sigma_t^2}{\partial\vartheta}\frac{\partial\sigma_t^2}{\partial\vartheta'}(\varphi_0)\right)+4E_{\varphi_0}\left(\frac{1}{\sigma_t^4}\frac{\partial\varepsilon_t}{\partial\vartheta}\frac{\partial\varepsilon_t}{\partial\vartheta'}(\varphi_0)\right),\\
J_2 &= E_{\varphi_0}\left(\frac{1}{\sigma_t^4}\frac{\partial\sigma_t^2}{\partial\theta}\frac{\partial\sigma_t^2}{\partial\theta'}(\varphi_0)\right).
\end{aligned}
$$

The proof of this theorem can be found in (Francq and Zakoian, 2010).

3.5.3 The ARMA-APARCH Model

The ARMA(P,Q)-APARCH(p,q) model has the form

$$
\begin{aligned}
r_t-\mu &= \sum_{i=1}^{P} a_i\,(r_{t-i}-\mu)-\sum_{j=1}^{Q} b_j\varepsilon_{t-j}+\varepsilon_t\\
\varepsilon_t &= \sigma_t z_t\\
\sigma_t^{\delta} &= \omega+\sum_{i=1}^{q}\alpha_i(|\varepsilon_{t-i}|-\gamma_i\varepsilon_{t-i})^{\delta}+\sum_{j=1}^{p}\beta_j\sigma_{t-j}^{\delta}.
\end{aligned}
\tag{3.16}
$$

The parameter vector to be estimated is $\varphi=\left(\vartheta',\theta'\right)'=\left(a_1,...,a_P,b_1,...,b_Q,\theta'\right)$ where $\theta'\left(\omega,\gamma_i,\delta,\alpha_1,...,\alpha_q,\beta_1,...,\beta_p\right)$. The constraints on the parameters remain the same as in equation (2.16). Since the ARMA(P,Q)-APARCH(p,q) does not show convergence when estimating the parameters using the Quasi-maximum likelihood technique, we use the traditional maximum likelihood approach under the assumption that the distribution of the residuals are student's t distributed. This is a good distribution as Bollerslev (1987) showed that this distribution performs better in capturing the volatility clustering and leptokurtic nature of speculative prices and exchange rates.

3.6 Statistical Tests

In this section, the model diagnostics test applied to the daily and return series of the exchange rates and residuals from the mean equation to assess whether a univariate time series GARCH process will be appropriate for modeling are explained below.

3.6.1 Augmented Dickey-Fuller Test

The augmented Dickey-Fuller(1979) is used to test if a time series is stationary or not. The test is applied to both the daily and return series. The test statistic is defined as

$$\triangle r_t = c_t + \beta r_{t-1} + \sum_{i=1}^{p-1} \phi_i \triangle r_{t-i},$$

where c_t is a deterministic function of the time index t and $\triangle$ is the differenced operator. The null hypothesis of the Augmented Dickey-Fuller is $\beta = 0$ (i.e. the returns has unit root) versus the alternative hypothesis that $\beta < 0$ (i.e. the returns exhibit no unit root).

3.6.2 Phillips-Perron Test

The Phillips-Perron(1988) test involves a regression of the form

$$r_i = \alpha + \rho r_{i-1} + u_i, \tag{3.17}$$

on the return time series data. The intercept term may be excluded or include a trend term. The test statistics calculated as

$$
\begin{aligned}
Z_p &= n(\hat{\rho}_n - 1) - \frac{1}{2}\frac{n^2\hat{\sigma}_t^2}{s_n^2}\left(\hat{\lambda}_n^2 - \hat{\gamma}_{0,n}\right)
\end{aligned}
$$

$$
\begin{aligned}
\hat{\gamma}_{j,n} &= \frac{1}{n}\sum_{i=j+1}^{n} \hat{u}_i\hat{u}_j \\
\hat{\lambda}_n^2 &= \hat{\gamma}_{0,n} + 2\sum_{j=1}^{q}\left(1 - \frac{j}{q+1}\right)\hat{\gamma}_{j,n} \\
s_n^2 &= \frac{1}{n-k}\sum_{i=1}^{n} u_i
\end{aligned}
$$

where $u_i$is the OLS residuals from (3.17), k is the number of covariates in the regression, q is the number of Newey-West lags to use in calculating $\hat{\lambda}_n^2$ and $\hat{\sigma}$ is the OLS standard error of $\hat{\rho}$. The null hypothesis is that there is unit root against the null hypothesis that the variable was generated by a stationary process. This test can be viewed as the Dickey-Fuller test adjusted for serial correlation by using heteroscedasticity and autocorrelation covariance matrix of (Newey and West, 1987).

3.6.3 ARCH test for Heteroscedasticity

This is the conditional heteroscedasticity test introduced by Engel (1982) and is also know as the Lagrange Multipier test. The test is similar to the F test for $\alpha_i = 0 (i = 1, ..., m)$in the linear regression

$$
\begin{aligned}
\varepsilon_t^2 = \alpha_o + \sum_{i=1}^{m} \alpha_i \varepsilon_{t-i} + a_t,\ t \\
= i + 1, ..., n,
\end{aligned}
$$

where a_t denote an error term and m is a pre-specified integer. Under the null hypothesis of no ARCH effect, the test is distributed $\chi^2_{1-\alpha}$ with m degrees of freedom and $\alpha$is the significance level. The null hypothesis is rejected when the reported p-value of the test is less than the significance level α.

3.6.4 Jarque-Bera Test for Normality

This test is used to check whether a specific time series is normally distributed or not. The residuals from the the fitted ARMA model in the mean equation is subject to this test. The statistics is

$$JB = \frac{n-k}{6}\left(S^2 + \frac{(K-3)^2}{4}\right)$$

- n is the number of observations
- k is the number of estimated parameters
- K is the Kurtosis
- S is the sample skewness

The test is χ^2-distributed with 2 degrees of freedom under the null hypothesis that the series is normally distributed. The null hypothesis is rejected when the p-value of the test is less than the significance level of 5%.

3.6.5 Ljung-Box Test

Ljung and Box (1978) test is used to test that several autocorrelations of a time series are zero. It has the test statistics

$$Q(m) = n(n+2)\sum_{l=1}^{m} \frac{\hat{\rho}_l^2}{n-l},$$

where

$$\hat{\rho}_l = \frac{\sum_{t=l+1}^{n} (\varepsilon_t - \bar{\varepsilon})(\varepsilon_{t-l} - \bar{\varepsilon})}{\sum_{t=1}^{n} (\varepsilon_t - \bar{\varepsilon})^2}$$

and m is the number of lag autocorrelations to be considered. Under the null hypothesis of no serial dependence in the first m autocorrelations, the decision rule is to reject the null hypothesis if the p-value is less than or equal to α, the significance level.

3.6.6 Mcleod-Li Test

McLeod and Li (1983) developed a test for sample autocorrelations in the squared residuals. This checks for independence as well as ARCH effects in the residuals. The test statistic is defined as

$$L = \left(n^2 + 2n\right) \sum_{k=1}^{K} \frac{\hat{\rho}_k^2}{n-k}$$

$$\begin{aligned} \hat{\rho}_k^2 &= \frac{\sum_{t=k+1}^{n} \left(\varepsilon_t^2 - \hat{\sigma}_n^2\right)\left(\varepsilon_{t-k}^2 - \hat{\sigma}_n^2\right)}{\sum_{t=1}^{n} \left(\varepsilon_t^2 - \hat{\sigma}_n^2\right)}. \\ \sigma_n^2 &= \frac{\sum_{t=1}^{n} \varepsilon_t^2}{n} \end{aligned}$$

If the residuals, $\hat{\varepsilon}_t$, is a sequence of iid variable, then L follows a χ_k^2asymptotically. The null hypothesis assumes independence in the series. If the p-values of the test lies below the 0.05 threshold, then we can conclude that the residuals are not independent.

3.7 Forecasting

One of the most important rationale for modelling time series is the ability to generate forecast that can be used for making better future decisions. There are three kinds of forecast which are usually of interest in forecasting. They are short, medium and long term forecast. However, many time series models are suitable for forecasting in the short term.

3.7.1 Forecasting Volatilities with ARMA-GARCH Model

Forecasting volatilities with GARCH models are obtained recursively. In this study, the volatility measured as the conditional standard deviation independent from the conditional mean. For a GARCH(p,q), the $h-$step ahead forecast of the conditional standard deviation $\hat{\sigma}_{t+h|t}$ is computed recursively from

$$\begin{aligned}\hat{\sigma}_{t+h|t} &= \sqrt{\hat{\sigma}^2_{t+h|t}} \\ \hat{\sigma}^2_{t+h|t} &= \hat{\omega} + \sum_{i=1}^{q} \hat{\alpha}_i \hat{\varepsilon}^2_{t+h-i|t} + \sum_{j=1}^{p} \beta_j \hat{\sigma}^2_{t+h-j|t}\end{aligned} \tag{3.18}$$

where $\varepsilon^2_{t+i|t} = \sigma^2_{t+i|t}$ for $i > 0$ and $\sigma^2_{t+i|t} = \sigma^2_{t+i}$ for $i \leq 0$, and $\hat{\sigma}^2_{t+h|t}$ is the estimate from the GARCH. It is obtained by modeling the residuals from the ARMA mean equation with a GARCH rprocess.

In an APARCH(p,q), the distribution of the innovations had an effect on the forecast. The optimal $h-$step ahead forecast of volatility is obtained from

$$\begin{aligned}\hat{\sigma}_{t+h|t} &= \sqrt{\hat{\sigma}^{\delta}_{t+h|t}} \\ \hat{\sigma}^{\delta}_{t+h|t} &= E\left(\hat{\sigma}^{\delta}_{t+h}|\theta_t\right) \\ &= \hat{\omega} + \sum_{i=1}^{q} \hat{\alpha}_i E\left[\left(|\varepsilon_{t+h-i}| - \hat{\gamma}_i \varepsilon_{t+h-i}\right)^{\hat{\delta}} |\theta_t\right] + \sum_{j=1}^{p} \beta_j \sigma^{\hat{\delta}}_{t+h-j|t},\end{aligned} \tag{3.19}$$

where $E\left[\left(|\varepsilon_{t+h-i}| - \hat{\gamma}_i \varepsilon_{t+h-i}\right)^{\hat{\delta}} |\theta_t\right] = k_i \sigma^{\hat{\delta}}_{t+k|t}$ for $k > 0$ and $k_i = E\left(|z| - \gamma_i z\right)^{\delta}$. The conditional variance specification estimates $\hat{\sigma}^{\delta}_{t+h|t}$ are obtained by modeling the residuals from the ARMA mean equation with APARCH model.

3.7.2 Forecast Performance measures based on loss function

A very good performance measure can be hard to find since the volatility is not observable directly. Therefore, it makes sense not to rely on one specific measure but rather on several of them. In this research work, two different forecasting performance measures are considered for evaluating the performance of volatility forecasts from the different ARMA-GARCH models. These are:

3.7.2.1 Mean Absolute Error (MAE)

The MAE is the average absolute forecast errors. It is defined as:

$$MAE = \frac{1}{n}\sum_{t=1}^{n}|\varepsilon_t| = \frac{1}{n}\sum_{t=1}^{n}|\hat{\sigma}_t - \sigma_t| \tag{3.20}$$

where σ_t is the square root of the estimated variance equation of the residuals fitted with a GARCH/APARCH process.

$$\sigma_t =$$

This method does not allow the offsetting effects of over and under predictions. The minimum MAE amongs the competing models will be preferred. It is also more robust to the presence of outliers than other forecasting performance measures.

3.7.2.2 Root Mean Square Error (RMSE)

The RMSE is defined as:

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(\hat{\sigma}_t - \sigma_t)^2} \tag{3.21}$$

The RMSE for a sample of size n is the square root of a squared loss function. The RMSE also attached more weight to large forecast errors. Again, the model with the smallest RMSE will be preferred.

For the two forecast criteria considered in this research study, we will use the ε_t^2 as a substitute for the actual variance, σ_t^2 as being the forecasted variance. This is done because the actual variance is never realized or observed. Therefore, given a set of competing models, smaller forecasting statistic values indicate the superior forecasting ability of a corresponding model.

3.7.2.3 Diebold-Mariano test

The forecast measures presented above are useful to compare a number of models. However, the main shortcoming of these methods is that they contain no information whether the forecasting accuracy of competing models are statistically different.. The test suggested by Diebold and Mariano (1995) involves using statistical test that examines forecast accuracy for two sets of forecast. If we have two forecast from different models: $e_{1,t}$ and $e_{2,t}$ with $t = 1,...,m$ and m is the number of forecast lags to be considered. Considering g as a function of forecast errors, the difference between the errors of two models is written as $d_t = g(e_{it}) - g(e_{jt})$. Under this function the hypothesis are:

1. $H_0 : d_t = 0$ i.e two models provide equal forecast accuracy.

2. $H_0 : d_t \neq 0$ i.e. model i provide more accurate forecast than model j different forecast accuracy.

The Diebold-Mariano test statistic is

$$DM = \left[V\left(\bar{d}\right)\right]^{\frac{1}{2}} \bar{d},$$

where

- $V\left(\bar{d}\right)$ is the asymptotic variance of the mean of the difference between the forecasting errors given as $V\left(\bar{d}\right) \approx m^{-1}\left[\gamma_0 + 2\sum \gamma_k\right]$, and γ_k is the kth autocorrelation of d_t.

- $\bar{d}$ is the sample mean loss differential defined as $\bar{d} = \frac{1}{m}\sum_{t=1}^{m}\left[g\left(e_{it}\right) - g\left(e_{jt}\right)\right]$.

Under the null hypothesis of equal forecast accuracy, the DM test statistic has an asymptotic standard normal distribution. Low negative value of the DM statistic suggests that Model i is statistically more superior than Model j in forecasting.

CHAPTER FOUR

DATA ANALYSIS AND DISCUSSION

4.1 Introduction

Financial time series such as, exchange rates, stock prices are known to exhibit features which are crucial for correct model specification, estimation and forecasting. The characteristics of financial return series include excess kurtosis (leptokurtic property), skewness, volatility clustering and at times long memory etc. Volatility clustering implies that the volatility is high for certain periods followed by periods in which is low. Long memory in financial returns implies that the serial dependency of volatilities does not decay out quickly. Practically, it could be interpreted that current return is affected by historical returns over a long time horizon. Since the basis of time series analysis is stationarity, this is the reason for using returns (first differences or logarithmic difference) instead of actual prices. Returns series are usually found to be stationary but exhibits serial dependence, thus the resason for modeling the mean equation with an ARMA process.

4.2 Movement and Patterns in the Gambian exchange rates data

4.2.1 The data

The data used in this thesis work consists of daily exchange rates of the Euro and US dollar against the Gambian currency (Dalasis) for ten years. The data cover the period from May 2003 to May 2013. It consists of 3653 observations. The data represents the average daily spot price exchange rates at which banks buy and sell these foreign currencies. The data were obtained from the Central Bank of the Gambia courtesy of the OANDA corporation (www.oanda.com/currency/historical–rates) historical exchange rate website.

In Figure 4.1, we noticed that the variation in the daily series of the Gambian Dalasi against Euro and USD currencies are not constant over time. This is termed as non-stationarity and it is widely observed in many applied time series data. The movement is an upward trend and indicates that the Dalasi on these international currencies have been depreciating over the last decade. This could be attributed to many factors. One of which include the diminishing nature of the country's re-export trade due to harmonization of external tariffs in the region and efficiency improvements in competing port facilities, notably in Senegal (WAMZ, 2013).

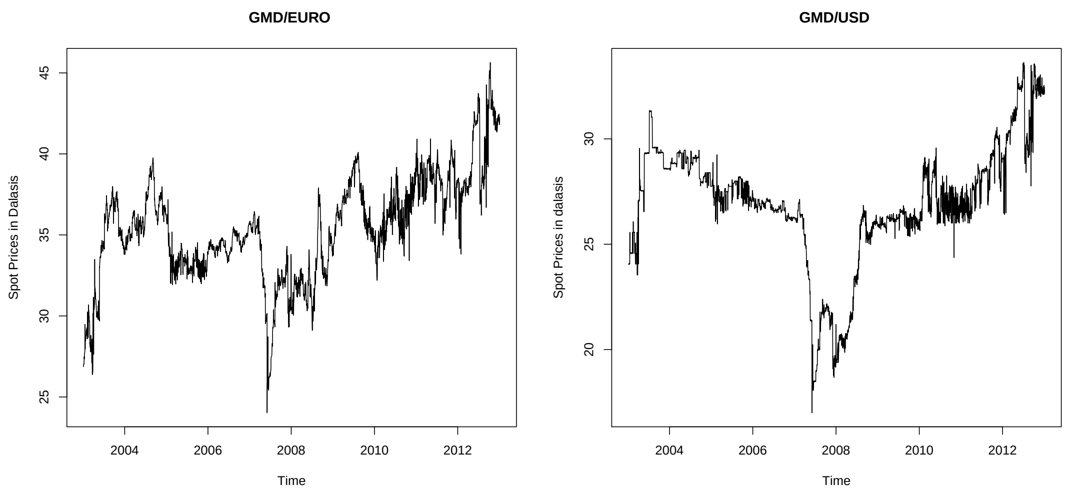

Figure 4.1: Time Series plot of daily Gambian exchange rate on Euro and USD

Since our daily exchange rate series in this study is non-stationary , we need to transform the original series to render it stationary. This will enable the application of time series models without violating the underlying theory.

Figure 4.2 illustrate the logarithmic returns plots obtain from the original daily series.

The return series appear stationary over time and fluctuating to an extend around mean zero. The volatility clustering can also be observed in the plots. This is evident as high return values tend to be followed by other high values and low values are followed by low values.

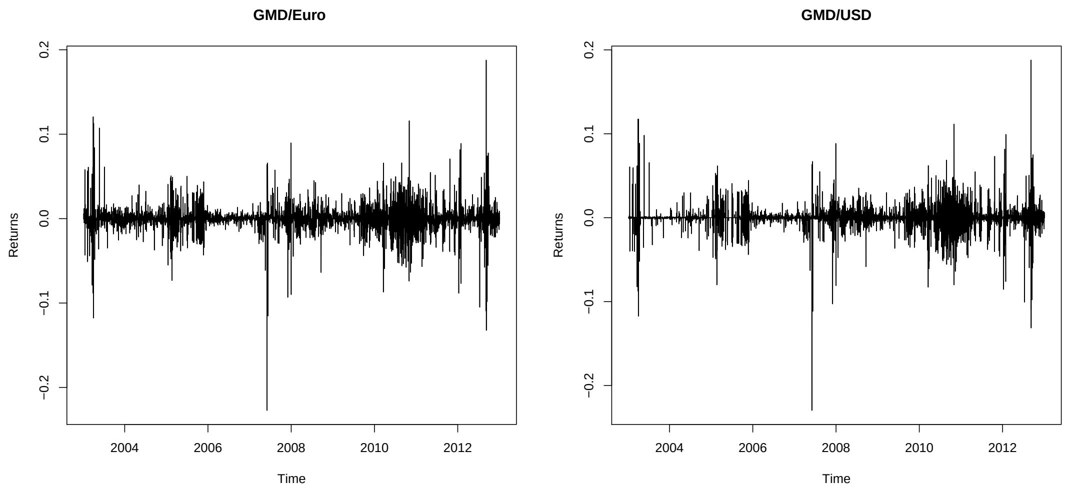

Figure 4.2: Logarithmic returns plots

The absolute returns plots in Figure 4.3 show the volatility clustering feature better than the return plots in figure 4.2. The suggestion by Mandelbrot (1963) and Fama (1965), that large absolute returns are more likely than small absolute returns to be followed by another large absolute returns can also be observed. It is very clear that the market or exchange rate volatility is changing over time which further suggest that a suitable model for the data should incorporate a time varying volatility structure as illustrated in the seminal on ARCH model [see Engel (1982)]. The volatility period observe around 2008 is much higher than any other period. This could be attributed to the world financial crises experienced particularly in Europe and America which undoubtedly trickle down to developing world coupled with low agricultural productivity the Country observed from the years 2008 through 2011.

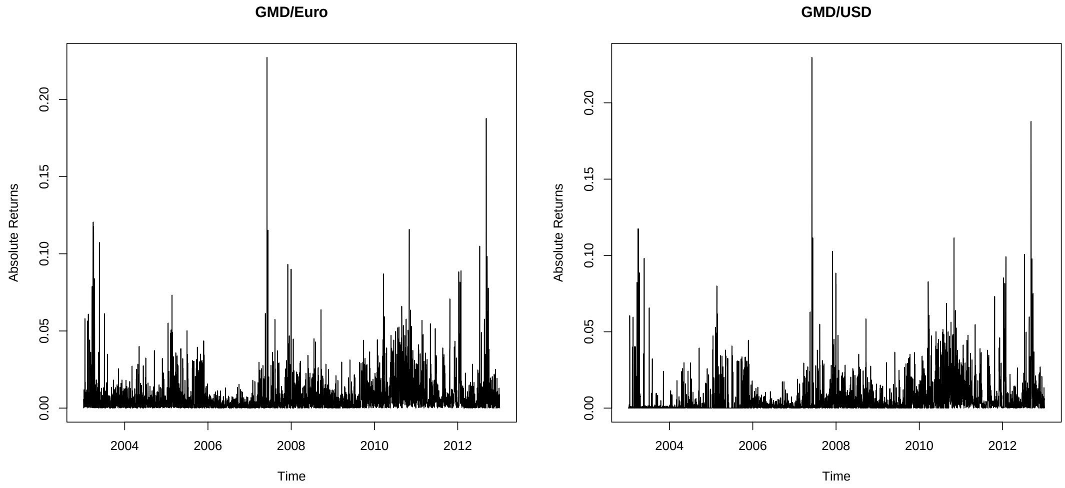

Figure 4.3: Absolute return plots

4.2.2 Descriptive Statistics

In this section, descriptive statistics carried out on the original and return series are discussed. The Table 4.1 gives the descriptive statistics. The average exchange of the Dalasi against the Euro and USD from 2003 to 2013 is 39.391 and 26.947 respectively. The excess kurtosis of the daily exchange rates are 0.419 and 1.0645 for the GMD/Euro and the GMD/USD series. This implies the distribution of the Euro and USD to the Dalasi has approximately the same kurtosis as that of a normal distribution which known to be 3.The excess kurtosis of returns indicate that they are heavy-tailed with values of 28.973 and 33.319 respectively. The excess kurtosis tell us by how much the kurtosis of a variable differs from that of a normally distributed variable. Therefore, the exact kurtosis of the daily and returns is the value shown in Table 4.1 plus 3. The mean of both return series is close to 0.0001. The standard deviation of the returns is close to 0.02 indicating that the spread of the returns from their mean value is not very small. Skewness is a measure of symmetry and is defined in (2.2) .The skewness of the returns are greater than zero and positive which implies that the distribution of the returns are slightly rightly skewed. In terms of exchange rates, the positive skewness indicates that the Gambian dalasi depreciates more often than appreciating (Maana *et al.*, 2010).

Table 4.1: Summary statistics of the Return Series

	Daily		Returns	
	GMD/Euro	GMD/USD	GMD/Euro	GMD/USD
Mean	39.391	26.947	0.00012	0.00008
Median	35.330	27.047	0.00004	0.0000
Maximum	45.647	33.631	-0.18778	0.18781
Minimum	24.024	16.999	-0.22725	-0.22975
Standard Deviation	3.213	2.854	0.01589	0.01532
Excess Kurtosis	0.491	1.065	28.973	33.319
Skewness	-0.066	-0.728	0.57349	0.657
Jarque-Bera Statistics	39.3786	495.2008	122007	169187.4
Jarque-Bera P-value	<0.0001	<0.0001	<0.001	<0.0001
Ljung -Box statistics	17155.91	17621.01	233.959	307.8463
Ljung-Box P-value	<0.0001	<0.0001	<0.0001	<0.0001
Number of Observations	3653	3653	3652	3652

The Jarque-Bera test at 1%,5% and 10% significance rejects the null hypothesis, confirming the departure from normality of the daily and return series for each currency (critical values are 9.21, 5.99 and 4.61 respectively). The Ljung-Box statistics up to lags 5 allows us to conclude the lack of randomness in the data, that is high presence of serial correlation since the p-values are less than 1%, 5% and 10% significance levels.

Table 4.2 gives the stationarity test results for the daily and return series. Both the Augmented Dickey-Fuller and the Phillips-Perron test confirms the presence of unit root in both daily series at the 1% significance level since their p-values are greater than or equal to it. For the returns, both test suggests stationarity at the 1%, 5% and 10% because the p-values associated with the test are all smaller than the respective significance levels. This implies that we do not need to difference the return series to achieve stationarity. The number of lags used in the Augmented-Dickey Fuller test was obtained by using the formula $k = (n-1)^{\frac{1}{3}}$ which corresponds to the suggested upper bound on the rate at which the number of lags, k, should be made to grow with the sample size for the general ARMA(p,q) setup.

Table 4.2: Augmented Dickey-Fuller and Philips-Perron tests for unit root

	Daily Series		Returns	
	Euro/GMD	USD/GMD	Euro/GMD	USD/GMD
Test Statistics	-3.7078	-1.818	-15.9198	-15.4906
P-value	0.02358	0.6554	<0.01	<0.01
	Phillips Perron Test			
Test Statistic	-40.2401	0.01	-3733.33	<0.001
P-value	-12.7666	0.3978	-3714.407	<0.001

From the graphs of the autocorrelation function of the returns in Figure 4.4, it is seen that there exists only two significant spikes at around lags 2 and 7 for both series. The PACFs exhibits several significant lags at 1 and 6. For the squared ACFs and PACFs in Figure 4.5 in both series it clear indicates an exponential declining of the spikes. This suggests a mixed ARMA process for the mean equation. From the graphs, it is observed there is no pattern of seasonal lags being present. Thus, the assumption of no seasonality in the returns is plausible to assume.

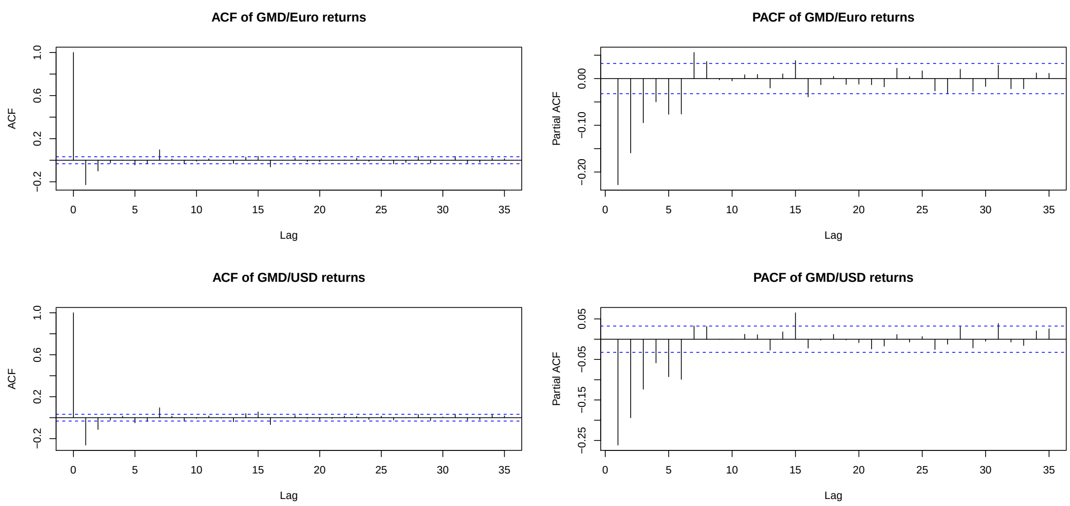

Figure 4.4: ACF and PACF of Returns

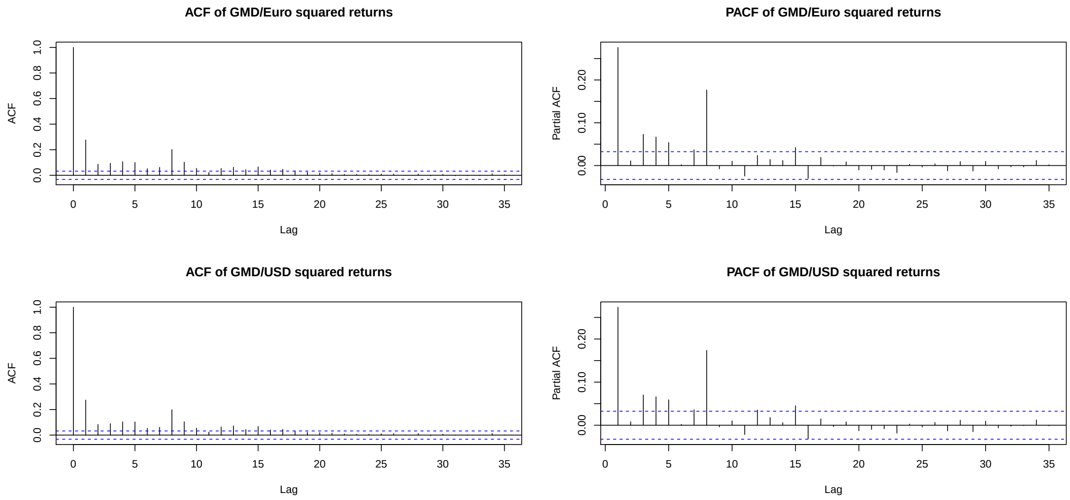

Figure 4.5: ACF and PACF of Squared Returns

4.3 Selection of ARMA–GARCH Model

In this section, the selection of the best ARMA–GARCH model to model the mean and variance equation for the GMD/Euro and GMD/USD returns is presented.

4.3.1 Selection of ARMA(P,Q) model

The selection of a suitable ARMA model to fit the returns as the mean equation is based on the Aikaike Information Criteria (AIC). Several ARMA models were fitted and evaluated based on this criteria. The AIC is a measure of the goodness-of-fit of an estimated statistical model. Given any two estimated models, the model with the lower value of AIC is the one to be preferred.

In Table (4.3), 9 ARMA models were fitted for each of our returns. In fitting the various ARMA model, we use the centered data. This means subtracting the mean from each of the returns respectively. The ARMA$(1,1)$ and ARMA$(2,1)$ appears to be the best candidates to model the mean equation for daily series of the GMD/Euro and GMD/USD respectively since they have the lowest values. These ARMA Models serves as a filter to remove serial dependence and to produce independent and identically distributed errors.

Table 4.3: AIC Criteria for ARMA mean equation selection

ARMA Models	GMD/Euro	GMD/USD
$(0,1)$	-19548.08	-19937.71
$(0,2)$	-19609.06	-20021.65
$(1,0)$	-19468.17	-19794.08
$(1,1)$	**-19614.31**	-20023.81
$(1,2)$	-19613.21	-20024.09
$(2,0)$	-19556.3	-19926.8
$(2,1)$	-19613.24	**-20024.23**
$(2,2)$	-19610.93	-20022.11

4.3.2 Residuals Analysis

After having explored and fitted the returns data with an ARMA mean equation, we modeled the obtained residuals with a GARCH/APARCH model. Although, the true residuals are unobserved, they are estimated with the ARMA residuals. Several statistical test have been performed on the residuals before comfirming that the GARCH/APARCH models are ideal in modeling them. These test are the Engel's ARCH test , McLeod-Li test, Ljung-Box test and Jarque-Bera discussed in section 3.6.

'The ACF plots in Figure 4.6 of the squared residuals in both models shows that autocorrelations does not die out very fast. The decay of the autocorrelations dies out at around lag 15. For Partial autocorrelation plots, we noticed that there exist very significant spikes around lag 8 after which it decays out very fast. This suggest a somewhat phenomenon of long memory process for the residuals of both models as stated by (Ding *et al.*, 1993).

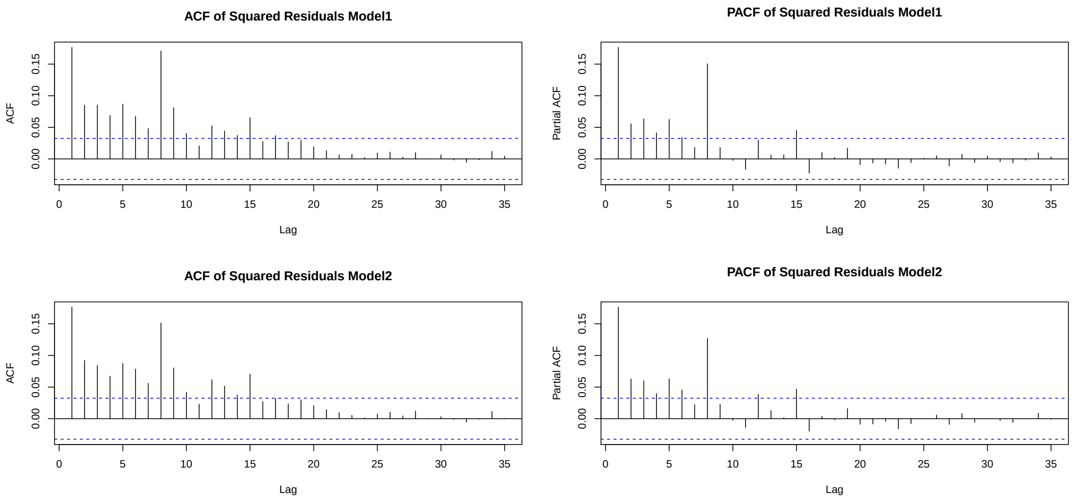

Figure 4.6: ACF and PACF of ARMA(1,1) and ARMA(2,1) squared residuals

In Figure 4.7, the QQ-plots does not suggest evidence of normality for the residuals. However, the Jarque-Bera (1987) test suggest a p-values less than 0.0001 respectively. This implies that we reject the null hypothesis that the residulas are normally distributed.

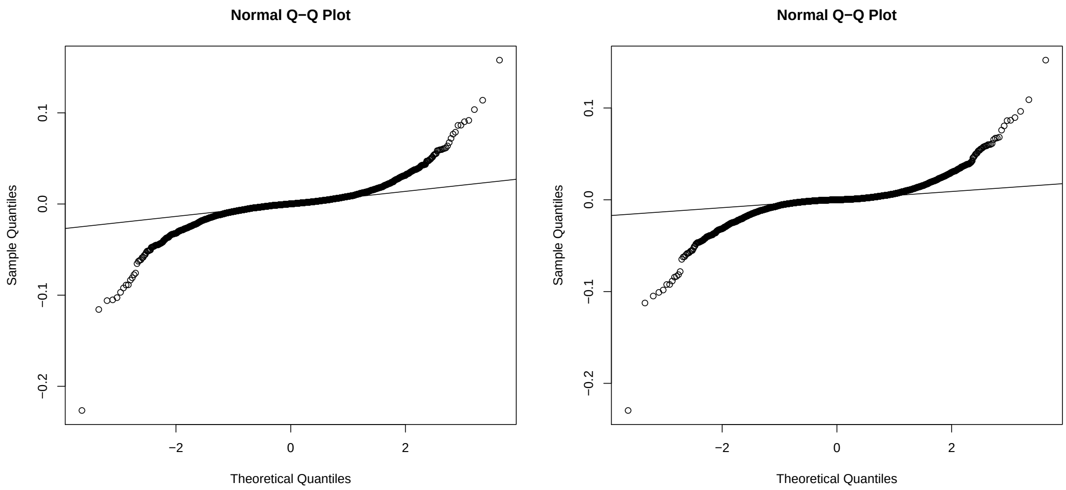

Figure 4.7: QQ Plots of the residuals

4.3.2.1 Heteroscedasticity Test

Having obtained the residuals, they are subject to a heteroscedasticty which is the main characteristic of conditional heteroscedasticity models (ARCH and GARCH type models).

In Table 4.4, it is found that the residuals from the fitted ARMA$(1, 1)$ and ARMA$(2, 1)$ to the GMD/Euro and GMD/USD returns at the various lags rejects the null hypothesis of no ARCH effects. This is evident as the p-values obtained are all less than the significant levels at 1%, 5% and 10% respectively. The same conclusion is reached by using the test statistics, since they are all greater than the theoritical values from a Chi-squared distribution with 4, 8 and 12 degrees of freedom respectively.

Table 4.4: Engel Heteroscedasticity Test at 5% significance level for the residuals

	Lag	Test Statistics	P-value	Critical value
Euro	4	144.51	<0.001	9.487729
	8	241.96	<0.001	15.50731
	12	246.64	<0.001	21.02606
USD	4	145.52	<0.001	9.487729
	8	224.04	<0.001	15.50731
	12	231.20	<0.001	21.026069

The McLeod-Li test which is performed on the squared residuals suggests strong evidence that they are highly autocorrelated and therefore lacks randomness. This is evident from Figure 4.8 in which the p-values of the squared residuals in the plots against different lags all lie below the 0.05 threshold. Accordingly, the results from the Engel's ARCH and McLeod-Li test indicates that a GARCH/APARCH are suitable candidate models to fit our residuals.

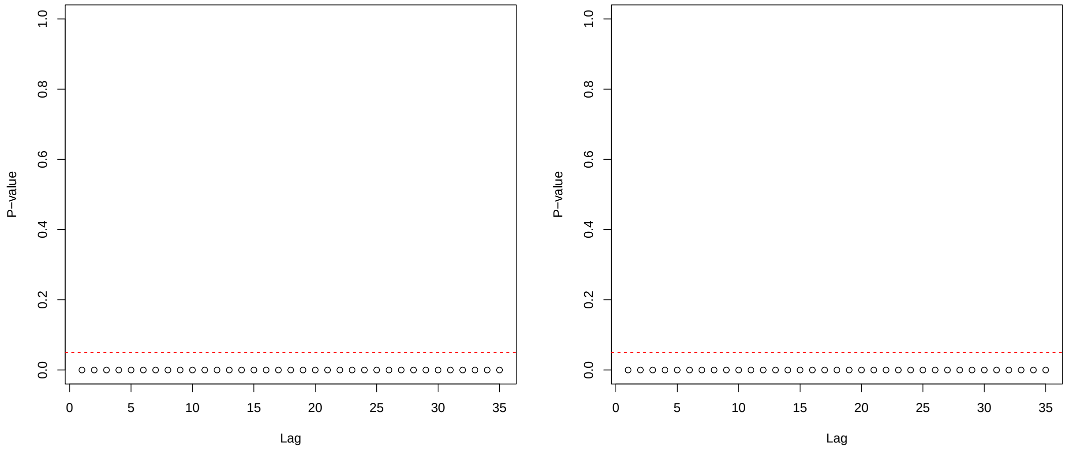

Figure 4.8: McLeod-Li test on squared residuals

4.3.3 Selection of GARCH(p,q) Model

The technique used in selecting the appropriate GARCH among competing models is based on the Aikaike Information Criteria (AIC). We evaluate the AIC at various combinations of the p and q of a GARCH model. In empirical applications, only small lag for p and q are often used. Typically, GARCH$(1,1)$, GARCH$(1,2)$ and GARCH$(2,1)$ are adequate in modeling volatilities in financial time series over long sample periods (Bollerslev and Wooldridge, 1992).

In Table 4.5, several GARCH models are included to check if they could be favourable models in modeling the heteroscedasticity in our data. Again as in the selection of the ARMA model in subsection 4.3.1, the samller the AIC the better a corresponding model.

Table 4.5: Various AIC of GARCH models

	GARCH Models					
	$(1,0)$	$(1,1)$	$(1,2)$	$(2,0)$	$(2,1)$	$(2,2)$
Residuals 1	-5.19612	**–5.21140**	-5.21018	-5.19603	-5.21090	-5.21139
Residuals 2	-5.20023	**-6.22128**	-5.21931	-6.20015	-5.22092	-5.22124

The table 4.5, shows the various AICs obtained by fitting various GARCH models to the residuals from our ARMA$(1,1)$ and ARMA$(2,1)$ mean equations respectively. The model given in bold is considered to be most appropriate according to our criteria. From the table, we deduce that the most appropriate GARCH model for our residuals is the GARCH$(1,1)$.

We include the APARCH$(1,1)$ to model the residuals in order to capture the long memory process observed from the ACF and PACF of the squared residuals in figures 4.6 This will give us a better platform to compare the in-sample forecasting performance between the two models and also, to check if there exist any leverage effect in the Gambian exchange rate data.

4.4 Estimation Results and Analysis

The estimated ARMA-GARCH models using the Quasi-maximum likelihood estimation method and the ARMA-APARCH models estimated with the classical maximum likelihood method under the assumption that the innovations are student t-distributed is presented below. Table 4.6 contain results of estimates of the ARMA-GARCH/APARCH applied to GMD/Euro and GMD/USD returns respectively. Values in parentheses are the p-values of the corresponding parameter estimates.

From Table 4.6, the sum of the GARCH parameters is approximately equal to one for all the models i.e. $\alpha_1 + \beta_1 \approx 1$. This shows that the volatility is persistent in our exchange rate data which is consistent with the findings of Beg and Anwar (2012) for the U.K. pound/ U.S dollar daily exchange rates. This implies that external economic shock have a long standing effect on exchange rates in Gambia. The coefficient α_1 captures the influence of new shocks on volatility. Estimates of this parameter

Table 4.6: Estimated results

	$ar1$	$ar2$	$ma1$	ω	α_1	β_1	γ_1	δ
Euro/GMD Returns								
ARMA(1,1)-GARCH(1,1)	0.0579		-0.7476	0.0000	0.0913	0.9261		
	(<0.001)		(<0.001)	(0.152)	(<0.001)	(<0.001)		
ARMA(1,1)-APARCH(1,1)	0.6057		-0.7090	0.0000	0.0811	0.9251	0.3237	2.0000
	(<0.001)		(<0.001)	(0.001)	(<0.001)	(<0.001)	(<0.001)	(0.001)
USD/GMD Returns								
ARMA(2,1)-GARCH(1,1)	0.4815	-0.0764	-0.7542	0.0000	0.0871	0.9262		
	(<0.001)	(0.0199)	(<0.001)	(0.0030)	(<0.001)	(<0.001)		
ARMA(2,1)-APARCH(1,1)	0.4765	-0.0875	-0.7252	0.0000	0.0756	0.9293	0.2261	1.9995
	(<0.001)	(0.006)	(<0.001)	(<0.001)	(<0.001)	(0.001)	(<0.001)	(<0.001)

are statistically significant for all four models and positive. The estimate, α_1, from the ARMA-GARCH fitted model is close to 0.085 for both returns. In the ARMA-APARCH models, the estimated α_1 is approximately 0.075 which is closer to the theoretical benchmark used in GARCH modeling (in general, we expect $\alpha \approx 0.05$). The parameter β_1, that measures persistence of volatility shocks, is positive and statistically significant. For all models considered, value of β_1 is close to 1 (around 0.93), indicating that old shocks to exhange rate prices tend to persist, instead of dying out quickly. This implies that economic shocks especially those of external have a permanent effect on exchange rate in the Gambia.

The APARCH estimates prove to be a good condidate as well, as all the model conditions are satisfied accordingly. The parameter δ is estimated and approximately equal to 2 in both APARCH models. It is also significant at the 1%, 5% and 10% levels since the associated p-value is less than the significant levels. We can also observe that the leverage effect parameter γ_1 is between 0 and1 in both models. These estimates of the δ and γ_1 suggest that our APARCH models exactly follows the GJR model by (**?**). The leverage effect parameter, γ_1, is significant and positive at 1%, 5% and 10% levels for both the Euro/GMD and USD/GMD returns. This indicates that shocks have asymmetric effects on the volatility of exchange rate prices and the positive sign suggests that positive shocks reduce volatility more than negative shocks.

4.4.1 Diagnostic Checking of ARMA-GARCH/APARCH models

Table 4.7 shows the ARCH test results together with AIC and BIC for each of the estimated models. Model 1 represents ARMA(2,1)-GARCH(1,1) while Model 2 indicates the ARMA(1,1)-APARCH(1,1) and ARMA(2,1)-APARCH(1,1) models accordingly. The ARMA-APARCH models have the smallest AIC and BIC for our Euro/GMD and USD/GMD returns.

Table 4.7: Engel's ARCH Test on squared residuals and Information Criteria

	GMD/Euro		GMD/USD	
ARCH Test	Model 1	Model 2	Model 1	Model 2
Test Statistics	2.3441	5.325	4.1340	6.0288
P-value	0.9987	0.946	0.9809	0.9146
Information Criteria				
AIC	-5.964	-5.9959	-6.2291	-6.244
BIC	-5.9561	-5.985	-6.2189	-6.2306

The ARCH test for heteroscedasticity accepts the null hypothesis of no ARCH effects in the residuals because the p-values are all greater than than 1%, 5% and 10% respectively. Based on the information criteria in Table 4.7, ARMA-APARCH models show more adequacy than the ARMA-GARCH models because the AIC and BIC values are smaller for them.

Moreover, if the model is successful in modeling the return series well, then there should be minimal or no autocorrelation left in the standardized residuals and squared standardized residuals.

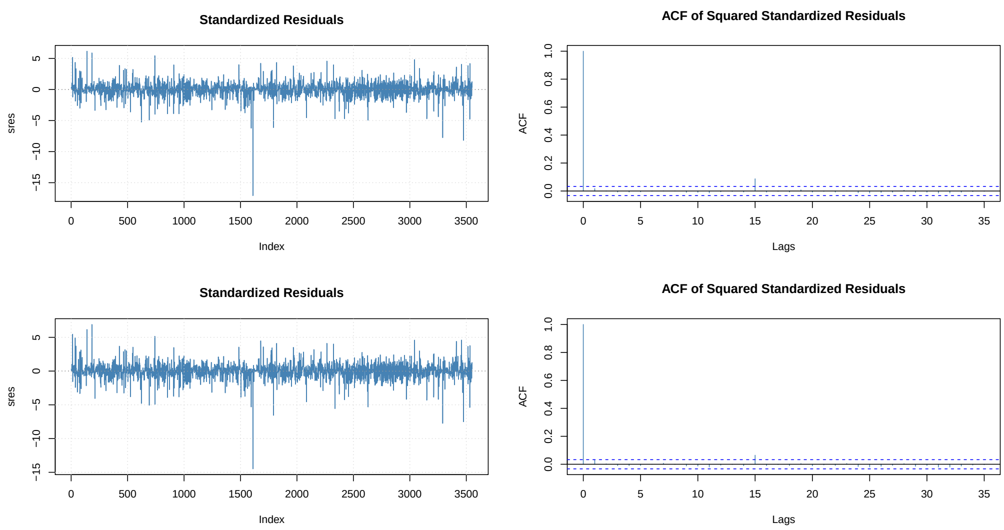

Figure 4.9: Plots of ARMA-GARCH/APARCH model residuals for Euro/GMD returns

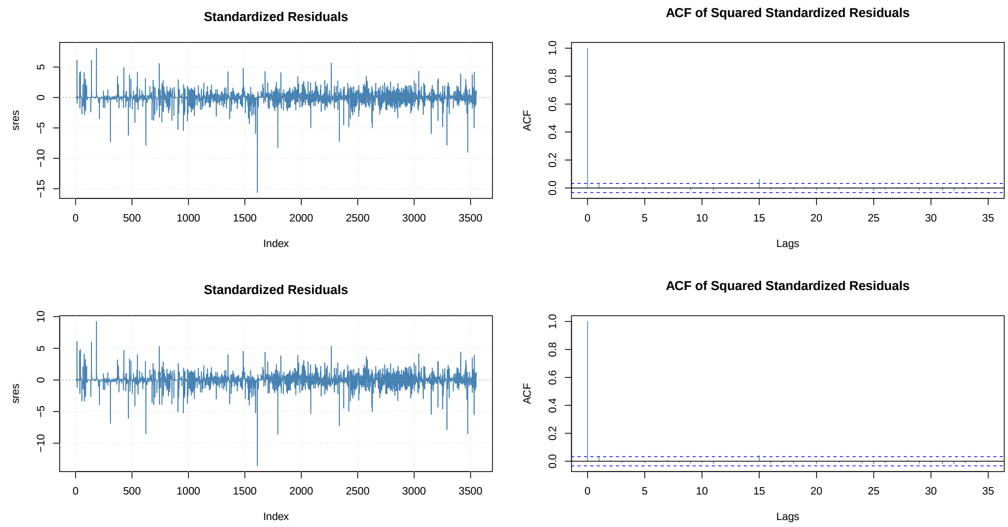

Figure 4.10: Plots of ARMA-GARCH/APARCH model residuals for USD/GMD returns

Figures 4.9 and 4.10 gives the standardized residuals and ACF plots of the ARMA-GARCH and ARMA-APARCH applied to the GMD/Euro and GMD/USD returns respectively. The standardized residuals plots show that they are stable and revolving around a constant mean equal to zero. By looking at the ACF plots we realized that the GARCH(1,1) and APARCH$(1,1)$ standardized residuals in both the Euro/GMD and USD/GMD returns shows no significant serial correlation in them whereas for the GARCH$(1,1)$ there exist a bit of autocorrelation around lag 15. This further suggest that the APARCH$(1,1)$ which suggest that the models fits our data well.

The volatilities measured as the conditional standard deviation from each of the estimated models are plotted and shown in figure (4.11). In all the plots the volatility pattern does not exhibit constant increase or decrease but instead a mixture of periods of high volatility followed by periods of low volatility. This suggest that the Gambian exchange rate during the last ten years have witnessed significant instability.

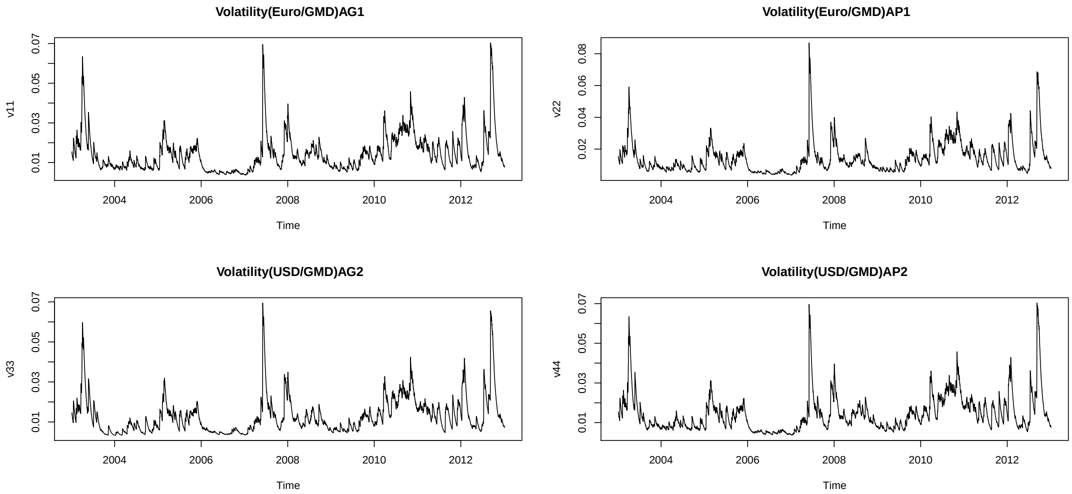

Figure 4.11: Plots of Volatility measured as the conditional standard deviation

4.5 Forecasting Volatilities

In this section, the forecasting performance of the models fitted to the Euro/GMD and USD/GMD returns are examined and analyzed. The out-of-sample forecast values are computed with one step ahead forecasting method (not re-estimating the coefficients). To get a better picture of how well the models forecast volatility, the RMSE and MAE are generated. Since two models were applied to each exchange rate data, the lowest value is preferred. The better model in term of forecasting for each returns data is highlighted. In Table 4.8, it can been that based on the metric used to forecast performance, the ARMA(1,1)-GARCH(1,1) is favored for the GMD/Euro returns while for GMD/USD the ARMA(2,1)-GARCH(1,1) is preferred.

Table 4.8: Comparison of forecast accuracy

	Model	RMSE	MAE
GMD/Euro	ARMA(1,1)-GARCH(1,1)	**0.001409**	**0.000613**
	ARMA(1,1)-APARCH(1,1)	0.00164	0.000651
GMD/USD	ARMA(2,1)-GARCH(1,1)	**0.001337**	**0.000559**
	ARMA(2,1)-APARCH(1,1)	0.001410	0.000633

To compare out-of-sample predictive ability for the models a statistical test is used. The Diebold and Mariano (1995) test is applied to each to the two models for each exchange rate returns. The null hypothesis of this test is that the two models have the same forecast accuracy against the alternative hypothesis that the second model is less accurate than the first model. The results are shown below in table (4.9). In both cases, we fail to reject the null hypothesis since the p-values are large approximately 50%, thus suggesting that the forecast accuracy of the ARMA-GARCH and ARMA-APARCH model in predicting out-of-sample volatilities in the Euro/GMD and Euro/GMD exchange rates does not differ significantly. This signifies that the test does not prefer any model over the other.

Table 4.9: Test for forecast accuracy

	Test Statistic	P-value
GMD/Euro	-0.0071	0.4972
GMD/USD	-0.002	0.4992

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

The time series of the Gambian exchange rate returns have been explored and a suitable ARMA-GARCH/APARCH model was formulated and applied to the data. Based on AIC criteria, the ARMA(1,1)-GARCH(1,1) and ARMA(1,1)-APARCH(1,1) were applied to the GMD/Euro returns whereas for the GMD/USD returns, the ARMA(2,1)-GARCH(1,1) and ARMA(2,1)-APARCH(1,1) were fitted to it. The theory on Quasi-maximum likelihood estimation of ARMA-GARCH were extensively evaluated and understood before applying the method to model the data. The ARMA process for the mean equation is used as filter for the returns in to produce indepenent and identically distributed residuals. The autoregressive parameter for the both the GMD/Euro and GMD/USD returns was found to be significant. The sum of the estimates of the GARCH parameters α_1 and β_1 were close to one, suggesting that the volatility in the exchange rates is persistent. The leverage effect parameter γ_1 in the APARCH model for both exchange rate returns was found to be significant. 150 out-of-sample volatility– measured as the conditional standard deviation– forecast were made for each of the two models applied to the returns. On the basis of forecast performance using RMSE and MAE, the ARMA(1,1)-GARCH(1,1) was deemed the best model in forecasting volatility in the GMD/Euro while the ARMA(2,1)-GARCH(1,1) was favored for the USD/GMD returns. The Diebold-Mariano test for the superiority of the two models in forecasting volatility for each returns were found to be indifferent. This indicates that the two models for each currency have the forecast accuracy.

The ARMA-GARCH models are successfully applied to the energy sector such as modeling oil and electricity prices and it was found that it has modeled the volatility in the Gambian exchange rates well.The volatility in the Gambian exchange rates witnessed significant instability during the last decade in the form of depreciation of the currency. This suggests that exchange rate risk in the Gambian market is high. This risk is important to understand as it affect transactional account exposure related to receivables (export contracts), payables such as import contracts and repartriation of dividends. It also impact revenues on domestic sales and input and also, on operating cost. Therefore, this thesis creates an avenue for understanding the volatility (risk) associated with the Gambian foreign exchange market which provides a good platform to relevant authorities and other parties in managing currency risk.

5.2 Recommendations

Suggestions for future researcher to improve on this study which focuses exclusively on exchange rate data, can be done through;

1. Modelling using MULTIVARIATE GARCH which may include fundamental macroeconomic variables such as interest rates, inflation rates etc. This will give a broader insight and through understanding to the Gambian financial market.

2. Since the sum of the GARCH parameters α_1 and β_1 were close to 1.This suggests persistent of volatility which is better modeled with an IGARCH process. Therefore, an ARMA-IGARCH could be a suitable candidate to model volatility in the Gambian forex market and also, to explore the concept of regime switching to increase the overall fit of the models.

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