PERFORMANCE EVALUATION OF DIRECTION OF ARRIVAL ESTIMATION USING UNIFORM AND NON-UNIFORM LINEAR ARRAYS FOR SIGNAL SOURCE LOCALIZATION

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DECLARATION

I hereby declare that this thesis is my original work and has not been presented for award of MSc degree in any University.

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DEDICATION

To my husband Murinzi Theogene, my mother, my father, and my siblings.

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ACRONYMS AND ABBREVIATIONS

AOA	Angle of Arrival
BER	Bit Error Rate
CH	Channel
DECOM	DOA Estimation with Combined MUSIC
DIT	Displacement Invariance Technique
DF	Direction Finding
DFT	Discrete Fourier Transform
DOA	Direction of Arrival
DOAE	Direction of Arrival Estimation
DTFT	Discrete Time Fourier Transform
ESPRIT	Estimation of Signal Parameters via Rotational Invariance Technique
FFT	Fast Fourier Transform
MSE	Mean Square Error
MUSIC	Multiple Signal Classification
NLA	Non-uniform Linear Array
OTHR	Over-The-Horizon-Radar
RDF	Radio Direction Finding
SNR	Signal to Noise Ratio
ULA	Uniform Linear Array
Wi-Fi	Wireless Fidelity

ABSTRACT

Due to the flexibility and convenience of wireless communication, many applications began to adapt to it. Hence, to comply with the requirements, a high speed data rate is necessary. With the advancement of technologies and plethora of applications, wireless communication tend to be overloaded which has resulted in the utilization of the higher frequencies in the spectrum owing to the larger bandwidth attributed to the higher band. The higher data rate, and consequently multipath fading and interference result in the limitation of data rate transmission. Therefore, the objective of this research project is to determine the desired signal location, minimize the multipath and interference by analyzing subspace techniques for direction of arrival (DOA) estimation using uniform linear array (ULA) and non-uniform linear array (NLA). Especially, this thesis focuses on enhancing the wireless communication capacity by estimating the DOA using Multiple Signal Classification (MUSIC) and Root-MUSIC algorithms. To apply these algorithms on the ULA and the NLA helps to analyze the accuracy and efficiency DOA estimation.

The first phase of this thesis is an extensive study of various high resolution directions of arrival estimation algorithms such as MUSIC and Root-MUSIC algorithms. Then apply them on ULA. The second phase is to estimate DOA using NLA. Thereafter, decompose it into two ULAs for the corresponding co-prime array before combining the two ULAs with MUSIC algorithm. The simulation of the DOA estimation with variation of snapshots, signal to noise ratio, array elements, signal source as a set of input parameters is carried out in MATLAB platform. Their performance under ULA and NLA are tested and the evaluation analysis of their accuracy and resolution towards the direction of arrival (DOA) estimation is checked.

The analysis is based on the evaluation of the DOA estimation performance using ULA and NLA in the presence of additive white Gaussian noise and the calculation of the estimation subspace methods through MATLAB platform. The performance of the algorithms has been analyzed by considering Mean Square Error (MSE) for eight trials as a function of array elements, of signal to noise ratio and as function of snapshots. Through extensive simulations NLA has shown to be more accurate and efficient in DOA estimation.

ABSTRAIT

En raison de la flexibilité et de la commodité de la communication sans fil, de nombreuses applications ont commencé à s'y adapter. Par conséquent, pour satisfaire aux exigences, un débit de données à haute vitesse est nécessaire. Avec l'avancement des technologies et la pléthore d'applications, la communication sans fil ont tendance à être surchargés ce qui a conduit à l'utilisation des fréquences plus élevées dans le spectre en raison de la plus grande bande passante attribuée à la bande supérieure. Le débit de données plus élevé et, par conséquent, la décoloration par trajets multiples et l'interférence entraînent une limitation de la transmission des débits de données. L'objectif de ce projet de recherche est de déterminer l'emplacement souhaité du signal, de minimiser le multi-trajet et l'interférence en analysant les techniques de sous-espace pour l'estimation de la direction d'arrivée (DOA) en utilisant une matrice linéaire uniforme (ULA) et une matrice linéaire non-uniforme. En particulier, cette thèse porte sur l'amélioration de la capacité de communication sans fil en estimant le DOA en utilisant la classification de signaux multiples (MUSIC) et les algorithmes Root-MUSIC. Appliquer ces algorithmes sur l'ULA et la NLA permet d'analyser l'estimation de DOA de précision et d'efficacité. La première phase de cette thèse est une étude approfondie de divers algorithmes d'estimation d'itinéraires à haute résolution tels que les algorithmes MUSIC et Root-MUSIC. Ensuite, appliquez-les sur ULA. La deuxième phase consiste à estimer la DOA en utilisant la NLA. Ensuite, décomposez-le en deux ULA pour la matrice co-prime correspondante avant de combiner les deux ULA avec l'algorithme MUSIC. La simulation de l'estimation DOA avec variation des instantanés, du rapport signal sur bruit, des éléments du réseau, de la source du signal en tant que jeu de paramètres d'entrée est réalisée dans la plate-forme MATLAB. Leur performance sous ULA et NLA est testée et l'analyse d'évaluation de leur exactitude et de leur résolution dans l'estimation de la direction d'arrivée (DOA) est vérifiée. L'analyse est basée sur l'évaluation de la performance d'estimation du DOA en utilisant ULA et NLA en présence de bruit Gaussien blanc additif et le calcul des méthodes de sous-espace d'estimation à travers la plate-forme MATLAB. La performance des algorithmes a été analysée en considérant l’erreur quadratique moyenne (EQM) pour huit essais en fonction des éléments du réseau, du rapport signal / bruit et en fonction des instantanés. Grâce à de vastes simulations, l'NLA s'est révélée plus précise et plus efficace dans l'estimation du DOA.

CHAPTER 1

INTRODUCTION

1.1. Background to the study

The direction of arrival (DOA) estimation denotes the angle at which an electromagnetic or acoustic wave arrives at an array of antennas or sensors [1]. Using an array of antennas has an advantage over single antenna in achieving an improved performance. The improved performance involves an increase in the overall gain, provision of diversity reception, reduction of interference from a particular direction, steering the antenna array at desired direction, and determining the DOA of the incoming signals. Furthermore, estimating the DOA of multiple source signals is very important in extracting a useful signal from a noisy and interference-prone environment. Through smart antenna systems based on appropriate spatial filtering, direction finding techniques can be used to separate the direction of the desired signal and interference signal [2].

In the last few decades, the DOA estimation has been among the most active areas of research in signal processing. Practically, it has found many applications in the field of sonar and navigation, radar, radio astronomy, tracking of various objects, rescue missions, mobile communication, internet broadcasting, conferencing and other emergency assistance activities [3]. Due to its convenience and flexibility, DOA estimation has contributed a lot in the popularity of wireless communication systems. Indeed, the need of increasing the wireless capacity is very important because of over usage of low end of the spectrum which has led to the higher frequency band exploitation. With higher frequencies, higher data rate and higher user density, multipath fading and interference pose severe limitations in data transmission [4]. Thus, the localization of a signal source by DOA estimation is vital in the attempt to increase the wireless network capacity by the reduction of multipath effects and other interferences.

1.2. Problem statement

The problem of estimating the angular location of *K* sources using an array of *M* sensors with the distance that is equal to a half wavelength is addressed. The source signals are assumed to be uncorrelated narrowband signals operating at the same center frequency with an uncorrelated white Gaussian noise located in the far field with respect to the physical size of the array. The direction of arrival (DOA) of multiple sources located at $\theta_1, \theta_2, \ldots, \theta_K$ can be easily estimated by identifying the peaks of a MUSIC spatial spectrum which is based on eigen-decomposition of the covariance matrix.

Due to some applications in wireless communication systems that need more than a half wavelength of the distance between the adjacent array elements, the Non-uniform linear array (NLA) in a form of DOA Estimation with Combined MUSIC for Co-prime Array (DECOM) was proposed. Its most remarkable property is that it increases the degrees of freedom. In addition, the autocorrelation of signals can be estimated in a much denser spacing other than the physically sparse sampling spacing, and sinusoids in noise can be estimated in a more effective way. These obviously contribute in improving wireless network capacity.

1.3. Objectives

1.3.1. General objective

The main objective of this research investigation is to build on direction of arrival (DOA) estimation measurement system using uniform linear (ULA) array and non-uniform linear array (NLA) operating at 2.4GHz band which is the normal frequency used in Wi-Fi. The evaluation of sub-space based techniques implementation is given by underlying the factors that affect the accuracy and resolution of the DOA estimation. Those factors include the number of array elements, number of snapshots, number of signal sources, and signal to noise ratio.

1.3.2. Specific objectives

The specific objectives are:

(i) To investigate the potential of the source localization or source tracking by determining the desired signal location.

(ii) To explore the possibility of exploiting MUSIC and Root-MUSIC algorithms in order to reduce the multipath and interference; which contribute to the simplicity of computational complexity and high robustness of the systems.

(iii) To analyze NLA by using DOA estimation with combined MUSIC for co-prime array (DECOM) and evaluate its accuracy versus DOA estimation using ULA.

1.4. Justification of the study

The thesis analyses the accuracy and efficiency of the DOA estimation using uniform linear array and non-uniform linear array for source localization. This research project contributes to the improvement of wireless network capacity by reducing the multipath fading effects and interference as well as reduction of the bit error rate. Therefore, the wireless network designers could employ the techniques to improve network reliability.

1.5. Scope of the work

The scope of this research work is limited to the formulation of a mathematical model of the direction of arrival estimation and simulation of MUSIC and Root-MUSIC algorithms in MATLAB platform. The evaluation is based on MUSIC and Root-MUSIC algorithms for the ULA and NLA using uncorrelated narrowband signal with uncorrelated noise, the signals and noise are also uncorrelated. The source signals operate at the same center frequency and the mutual coupling of the antennas has been ignored.

1.6. Organization of the thesis

The rest of this thesis is organized as follows:

In chapter 2, the antenna arrays - background theory is presented. This chapter also lays down the performance of array antenna structure which include the background of uniform array antenna and non-uniform linear array with some spatial spectrum estimation system which is also known as direction of arrival estimation. In chapter 3, the description of direction of arrival algorithms are given. A mathematical model of DOA estimation for ULA and NLA using MUSIC and Root-MUSIC algorithms is developed. In chapter 4, simulation and discussion of the results are presented. The MATLAB platform is used to simulate the direction arrival estimation by varying some input set parameters such as number of snapshots, signal to noise ratio, number of array elements and number of signal sources; to check on the accuracy and efficiency of the system. Finally, a conclusion and recommendation for further work are highlighted in chapter 5.

1.7. Note on publication

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CHAPTER 2

BACKGROUND THEORY ON ANTENNA ARRAYS

2.1. Historical evolution of antenna arrays

An antenna array is a geometric arrangement of a set of two or more antenna elements used for transmitting and/or receiving electromagnetic waves. These antennas are systematically connected in such a way that their individual currents are in specific amplitude and phase relationship [5].

The development in the fields of electronics, information theory, signal processing and antenna theory have all promoted the quality of wireless communication systems that exist today. However, despite the extreme advances since the days of Marconi in each of these fields, the desire for the advanced wireless communication systems has not been satisfied.

The concept of an antenna array was first initiated for military applications in 1940. This development was significant in wireless communications as it upgraded the reception and transmission patterns of antennas used in these systems. The array also allowed the system of antenna to be electronically directed to receive or transmit information primarily from a particular direction without mechanically moving the structure. As the field of signal processing improved, arrays could be utilized to receive energy (or information) from a particular direction while denying information or nulling out the energy in unwanted directions. Consequently, the arrays could be used to decrease the intentional interference such as jamming or unintentional interference. The main source of unintentional interference is the radiation from other sources not meant for the system in question [6].

Furthermore, the growth in signal processing led to the concept of adaptive antenna arrays. These arrays adapted their radiation or reception pattern based on the operational environment. This raised significantly the capacity available in wireless communication systems [7]. While there has been a large amount of work and development on the signal processing aspects, the physical geometry has had relatively little attention. The reason for this is in mathematical complexity when dealing with the optimization of the element positions for various situations. Array geometry optimization can be therefore expected to support the ongoing advancement performance of wireless communication system [8].

2.2. Performance of array antennas structure

The first articles on development of performance of the array via geometry optimization dates back to the early 1960s. Unz proposed a linear array in 1960 with general arbitrarily distributed elements. The far field pattern was defined by Jacobi expansion [9]. The approach used a matrix form solution which requires the manipulation of matrices of the order of number of elements. It involves expressing the radiation pattern in a series expansion, truncating the expansion and finding the solution of the matrix to find the desired spacing. He found that the additional degree of freedom created by the random distribution of elements allowed him to achieve the same performance as of an equally spaced array with fewer elements. The complexity of this formulation, has limited its practical applications. Unz noted that performance improvement could be obtained by holding the weights constant and varying the element positions.

In 1961, Harrington considered small element perturbations in an attempt to synthesize a desired array pattern [10]. The method of synthesizing a space tapered linear array involves approximation of a desired current distribution over the aperture. For an antenna possessing a continuous aperture, the far-field pattern can be calculated from knowledge of the currents in the aperture. The side lobe structure and beam width of the radiation pattern are controlled by the use of an appropriate illumination function which tapers the aperture current density. In amplitude tapered array, the current sheet of the continuous aperture is approximated by discrete current sources, the elements. The illumination taper is obtained by varying the relative amplitudes of the elements in the array. For arrays, in which the elements are closely spaced (about 0.5 wavelength), there is little difference between the patterns formed by the continuous aperture and by the discrete array aperture. The statistically tapered arrays are useful when the number of elements is large and when it is not practical to employ an amplitude taper to achieve low side lobes.

In the literature of Cetin and Ansari 1987, a new iterative method based on method of projection, for any shaped pattern synthesis for linear antenna arrays has been proposed [11]. The constraints such as number of antenna elements, specific locations of elements and desired power pattern are modeled as a convex set with element vectors which represent the excitation coefficients. Fast Fourier Transform (FFT) algorithm is used to implement the procedure. The computational complexity is more, as each iteration requires two Discrete Time Fourier Transform (DTFT) computations.

Peiging Xia and Mounir Ghogho in 2008 presented Space-time adaptive processing as a powerful tool for interference rejection. However, its performance may significantly degrade in the presence of multipath. Due to the coherence between the multipaths of interference, adaptive methods are not able to satisfactorily remove interference [12].

Arrays of antennas are used to direct radiated power towards a desired angular sector. The number, geometrical arrangement, and relative amplitudes and phases of the array elements depend on the angular pattern that must be achieved. Once an array has been designed to focus towards a particular direction, it becomes a simple matter to steer it towards some other direction by changing the relative phases of the array elements, a process called steering or scanning [13].

2.3. The structure of spatial spectrum estimation system

The spatial spectrum estimation is a common problem in the areas of radar, sonar, astronomy and seismology. The spatial spectrum estimation is a specialized signal estimation technology that uses space arrays to achieve a space signal parameter [14]. The spatial spectrum is an extension of the temporal spectrum into the three physical dimensions. The entire spatial spectrum system should be composed of three parts. These are: the incident signal space, spatial array receiver and parameter estimation. The differences in each of these areas of study are the underlying assumptions for the medium and propagation models [15]. The space can be divided into three corresponding spaces, namely target stage, observation stage, and estimation stage as shown in fig 2.1.

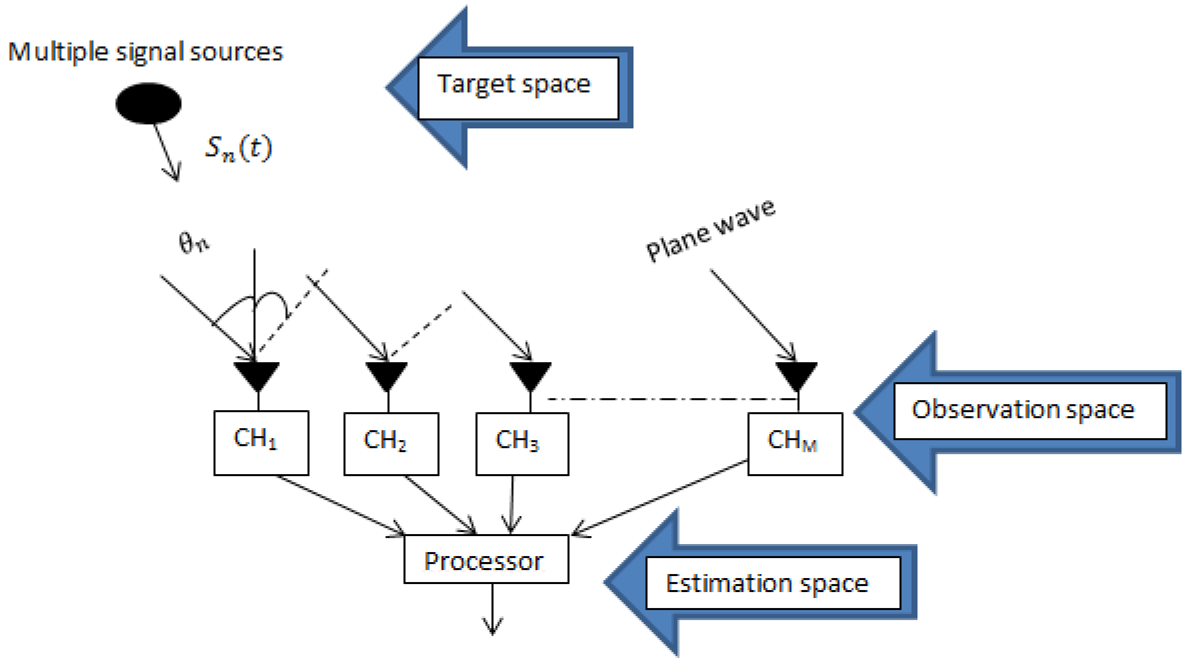

Fig 2.1: The system structure of DOA spatial spectrum estimation

The system illustrated in fig 2.1 consists of:

The target space is the stage where multiple signal sources parameters are found and other complex environment. On this stage, there are some particular spatial filtering methods that are used to estimate the undefined parameters of the signals which are originated from the complex target space.

The observation space is the stage which follows the target stage where it has to receive signal radiations from the target space. According to the complexity of the environment from the target stage, the signal received contains some characteristics such as the distance, the azimuth angle, the elevation angle, the polarization, etc. About the complex space environment characteristics, they include the miscellaneous waves, the interference, noise, etc.

Furthermore, due to the spatial array element influence, the data received in the collected signal contain some characteristics of the space array element such as the channel inconsistency, the frequency band inconsistency, the mutual coupling, etc. At this stage, the system that is supposed to receive data is composed of many channels which make it multidimensional stage and about the time domain processing; it is only one channel which is in charge of it.

Normally, that the channel does not correspond to the array elements; a spatial channel is formed by several or all of the synthetic array elements. There is no doubt that certain array elements in the stage may be contained within different channels.

Finally the estimation space is a stage which uses spatial spectrum estimation techniques including array signal processing techniques such as array correction and spatial filtering techniques, to extract the signal character parameters from the complex environment.

The estimation space is equivalent to the reconstruction of the target stage. The accuracy of reconstruction is determined by many factors, such as the complexity of the environment, the mutual coupling of spatial array, different channels, frequency band inconsistency, etc.
Spatial spectrum expresses the energy distribution of signals in all spatial directions. If one can get the spatial spectrum of the signal, the direction of arrival (DOA) of the signal can be obtained, so spatial spectrum estimation is also referred to DOA estimation [16].

2.4. Direction of arrival estimation methods

Direction of arrival estimation is a key research area in array signal processing and many engineering applications that require to be supported by DOA estimation. In modern society, DOA estimation is normally studied in the field of array processing. The main attraction has been in radio direction finding (RDF). Signal processing as part of smart antenna systems deals with the improvement of efficient techniques for DOA estimation which basically utilize the angle of arrival (AOA) estimation for the desired direction of the signal [17].

Initially, researchers started looking for the direction of arrival estimation using uniform linear array (ULA). Several methods have been applied in DOA estimation; among them the maximum likelihood method of Capon [18], Many researchers take the first category of estimators to consist of the Spectral Estimation Method that covers Minimum Variance Distortionless Response Estimator, Linear prediction method, Maximum Entropy Method, Maximum Likelihood Method (ML) and Beamforming. The second category consists of the Eigen structure Methods that includes the Min-Norm Method, the Estimation of Signal Parameters via

Rotational Invariance Technique (ESPRIT) algorithm, and Burg's maximum entropy method which is among the non-subspace techniques [18]. Although they have been often successful and widely used, these methods have some major limitations especially bias and sensitivity in parameter estimates, generally due to the incorrect model of the measurements. Then, Schmidt in 1979 revised and corrected the measurement model in case of the sensor arrays of arbitrary form and proposes a new subspace technique called Multiple Signal Classification (MUSIC) algorithm [19]. MUSIC technique is based on exploiting the Eigen structure of input covariance matrix. According to MUSIC spatial spectrum, DOA of the multiple source signals can be easily estimated by identifying the peaks.

Huang and Lee proposed the adaptive array beamforming in the presence of errors due to steering vector mismatch and finite sample effect [20]. The method proposed a fully data-dependent loading to overcome the difficulties. No additional sophisticated scheme is needed to choose the required loading. The loading factor can be completely obtained from the received array data.

V. Krishnaveni et al has given a comprehensive survey on beamforming techniques for DOA estimation [21]. The survey was based on studying various beamforming techniques and algorithms to estimate the direction of arrival of a signal. An assessment on the background robust algorithms using Nyquist sampling rate and its compressive sensing alternative was made. Hence, the methods that specifically exploit the spatial sparsity property are advantageous because they use very small number of measurements in the form of random projections of the sensor data along with one full waveform recording at one of the sensors.

J. Revati and D. Ashwinikumar worked with DOA using MUSIC algorithm in uniform linear array antennas [22]. This literature showed how the performance of smart antenna greatly depends on the effectiveness of DOA estimation algorithm. It analyzed the performance of MUSIC (Multiple Signal Classification) algorithm for DOA estimation and simulates results which showed that MUSIC provides better angular resolution for increasing number of array element, distance between array element and number of samples. All the simulations were done using MATLAB.

Youssef Fayad et al proposed Direction of Arrival Estimation Accuracy Enhancement via Displacement Invariance Technique. A new algorithm for improving Direction of Arrival Estimation (DOAE) accuracy has been carried out. Two contributions are introduced. First, Doppler frequency shift that resulted from the target movement is estimated using the displacement invariance technique (DIT). Second, the effect of Doppler frequency is modeled and incorporated into ESPRIT algorithm in order to increase the estimation accuracy. It is worth mentioning that the subspace approach has been employed into ESPRIT and DIT methods to reduce the computational complexity and the model's nonlinearity effect. The simulation results of the proposed algorithm are better than those of the previous estimation techniques leading to the estimator performance enhancement [23].

Most of the methods used to locate the DOA in uniform linear array considered the spacing distance between two adjacent array elements to be a half wavelength. However in wireless communication, there are some cases where such half wavelength minimum spacing is not applicable; for instance many parabola antennas, their physical size are designed to have a large size for enhanced directivity. Also, in an array that operates over a wide spectrum. For example, over-the-horizon radar (OTHR) is a unique radar system that performs wide-area surveillance by exploiting the reflective and refractive nature of high-frequency radio wave propagation through the ionosphere [24].

Recently, non-uniform linear array (NLA) in a form of co-prime array has been proposed. Its most remarkable property is that it increases the degree of freedom. In addition, the autocorrelation of signals can be estimated in a much denser spacing other than the physically sparse sampling spacing, and sinusoids in noise can be estimated in a more effective way. Due to the useful properties of the NLA, its importance has been realized and many researches from different aspects of it have been run in recent four years.

The interest has shifted to the super resolution using the co-prime arrays. Many researchers have discovered many methods to improve on the direction of arrival estimation. For example, P. Pal and P. Vaidyanathan have proposed a novel array structure for the direction of arrival estimation with increased degrees of freedom. This was a new method for a super resolution spectral estimation from the perspective of degree of freedom [24].

Z. Weng and P. Djuric proposed a new search free DOA estimation for co-prime arrays by using a projection-like method to eliminate the phase ambiguities for obtaining the unique estimation of DOA [25].

Z. Chengwei et al proposed a DECOM: DOA Estimation with Combined MUSIC for Co-prime Array. They proposed a combination scheme to obtain the unique, DOA estimation for co-prime array from the MUSIC of the decomposed uniform linear arrays, and give prove of the existence and uniqueness of the solution. They also designed a two-phase adaptive spectrum search scheme to obtain the accurate DOA estimation with low computational complexity [26].

2.5. Factors affecting DOA estimation results

The direction of arrival estimation is not only affected by the incident signal coming from the transmitter but also by the sparse complex environment [27], [28]. The direction of arrival estimation is affected by many factors such as signal to noise ratio, number of array elements, number of snapshots and number of signal source.

2.5.1. Signal to noise ratio (SNR)

Signal to noise ratio measures the difference between the received signal and the background noise level. The $\frac{S}{N}$ ratio can be increased by providing the source with a higher level of signal output power if necessary. In wireless systems, it is always important to optimize the performance of the transmitting and receiving antennas. Thus, the signal to noise ratio affects directly the performance of super-resolution DOA estimation algorithm [29], [30].

2.5.2. *Number of array elements*

Array antennas are a set of two or more individual sensors used for transmitting and /or receiving electromagnetic waves. The desired signals from antennas are combined and processed in order to achieve an improved performance over that of a single antenna. Using the array antenna has an advantage over single antenna of achieving an improved performance when applying Multiple Signal Classification (MUSIC) algorithm [31]. The improved performance involves an increase in the overall gain, improvement of the spatial resolution, provision of diversity reception, reduction of interference from a particular direction, steering the antenna array at desired direction, and determining the DOA of the incoming signals.

Antennas with a given radiation pattern may be arranged in a pattern (line, circle, plane, etc.) to yield a different radiation pattern. For an antenna array, a configuration of multiple antennas (elements) are arranged to achieve a given radiation pattern. Some of the common geometric arrays are:

(i) Linear array: The antenna elements are arranged along a straight line with equal spacing distance between two adjacent antenna elements.

(ii) Non-linear array: The antenna elements arranged along a straight line with spacing distance between two adjacent antenna elements and phase difference.

(iii) Circular array: The antenna elements arranged around a circular ring.

(iv) Planar array - antenna elements arranged over some planar surface (example - rectangular array).

(v) Conformal array - antenna elements arranged to conform to some non-planar surface (such as an aircraft skin).

Basically, the number of array elements can affect the estimation performance for super resolution algorithm. Generally, if the remaining array parameters are the same, the more number of array elements, the better estimation performance for super resolution algorithm.

2.5.3. Snapshots

In the time domain, the number of snapshots is defined as the number of samples. In the frequency domain, the number of snapshots is defined as the number of time sub-segments of discrete Fourier transform (DFT). Generally, due to the direction of arrival the more the number of snapshots, the better estimation performance for super resolution algorithm when other array parameters remain constant [32].

2.5.4. The signal source

The signal source is the signal transmitted from the transmitter to the receiver. This signal source has to be uncorrelated with other source signals otherwise the MUSIC algorithm will fail to perform a super resolution to the direction of arrival estimation [21].

Briefly, there are many factors that can affects the performance of DOA estimation in practical applications, to name some of them the array element amplitude and phase inconsistencies, mutual coupling between array elements, multipath environment and the wrong position of sensors.

To sum up, in this chapter the background theory on antenna arrays was given. Some DOA methods and factors that affect the DOA estimation accuracy were presented. In the next chapter the emphasis is specifically for the DOA estimation algorithms which have shown to be of high resolution and high accuracy that is MUSIC algorithm applied on ULA and NLA and Root-MUSIC algorithm applied on ULA only.

CHAPTER 3

DIRECTION OF ARRIVAL ALGORITHMS

3.1. Introduction

The direction of arrival algorithms applied in this thesis are the MUSIC and Root-MUSIC algorithms. They are known as subspace techniques which use the eigenvectors obtained by an eigendecomposition of sample covariance matrix of the data matrix. These techniques have shown to be effective and high resolution of the system compared to other techniques. They are popular and the most promising to give an accurate DOA estimation with limited sensors, low complexity of computation and with the capacity of identifying multiple targets according to Dhering in [3]. These algorithms allow the systems to get the desired angle of arrival of the incoming narrowband radio signal from the far field source to the receiver arrays. They have the ability to measure multiple signals sources, high precision measurement, and they can be implemented in real time using digital signal processing technology [33].

The uniform linear array and non-uniform linear array based on MUSIC and Root-MUSIC algorithms are used to prove the accuracy and the effectiveness of the DOA estimation.

For uniform linear array, the MUSIC algorithm and Root-MUSIC algorithm are used to find a direction of arrival spectrum. The signal is sent at 2.4GHz frequency band which is the same frequency used in wireless communication (Wi-Fi) [34] with the angle interval in the range: $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

Then, the direction-of-arrival (DOA) estimation method using non-uniform linear array is given. At first, the decomposition of non-uniform linear array into two uniform linear arrays for the corresponding co-prime arrays is done, and then combines the MUSIC results of the two decomposed uniform linear arrays to get DOA estimation [35, 41].

A mathematical model is given and the performance of the direction of arrival estimation based on MUSIC (Multiple Signal Classification) and the Root-MUSIC algorithm simulation are presented in chapter 4 of thesis.

3.2.Mathematical model of DOA estimation for Uniform linear array

Consider a uniform linear array antenna with M antenna elements shown in fig 3.1. The antennas are equally spaced with a distance d that is strictly equal to a half wavelength ($d = \frac{\lambda}{2}$). Assume that there are K narrowband signal sources located at $\theta_1, \theta_2, \dots, \theta_K$ with signal powers $\sigma_1^2, \sigma_2^2, \dots, \sigma_K^2$. The incident signals are at the same center frequency but with uncorrelated noise. Moreover, the incident signals are themselves uncorrelated. Assuming the number of signal sources is less than the number of antenna elements ($K < M$). The steering vector for k^{th} source located at θ_k is $a(\theta_k)$ with k^{th} elements $e^{j(2\pi/\lambda)d_l \sin(\theta_k)}$, where d_l is the antenna location and λ is the wavelength of the incident signal.

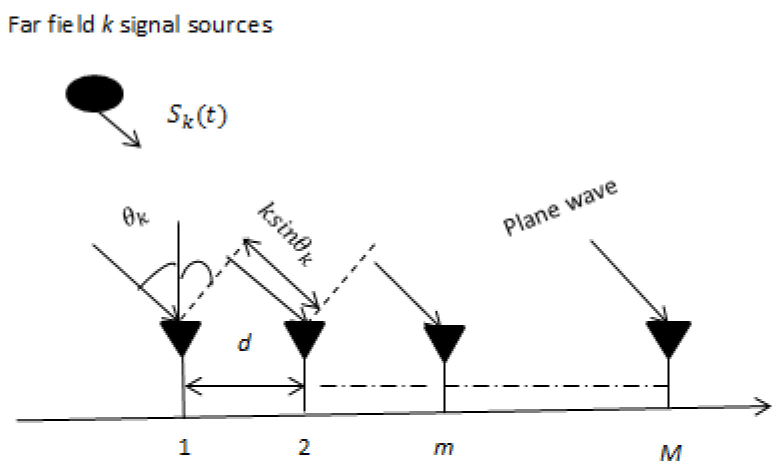

Fig 3.1: A plane wave incident on a ULA

The signal received by all antennas at a given time instant can be expressed as:

$$X(t) = As(t) + n(t) \tag{3.1}$$

Where $X(t) = [x_1(t),\ x_2(t),\ \dots, x_M(t)]^T$ is a vector received by the array antenna, $A = [a(\theta_1),\ a(\theta_2), \dots , a(\theta_K)]$ is a steering vector, $s(t)$ is the signal vector generated by the source and $n(t) = [n_1(t)\ n_2(t)\ \dots\ n_M(t)\ ]^T$ is additive white Gaussian noise.

Assuming the number of signal sources to be k where $1 \leq k \leq K$, the signal source $S_k(t)$ is a narrowband signal as indicated in (3.1) and can be expressed in the following form:

$$S_k(t) = s_k(t)\exp\{j\omega_k(t)\}, \tag{3.2}$$

where $s_k(t)$ is the complex envelope of $S_k(t)$ and ω_k is the angular frequency of $S_k(t)$. As assumed before, all signals have the same center frequency. This means that:

$$\omega_k = \omega_0 = \frac{2\pi c}{\lambda} \tag{3.3}$$

Where c is the speed of the light and λ is the wavelength of the signal.

By considering that the time required by the signal on an antenna array is t_1.
According to the narrowband assumption, the following approximation is valid:

$$S_k(t - t_1) \approx s_k(t) \tag{3.4}$$

Therefore, the delayed wavefront signal is:

$$S_k(t - t_1) = s_k(t - t_1)\exp[j\omega_0(t - t_1)] = s_k(t)\exp[j\omega_0(t - t_1)] \tag{3.5}$$

The output signal of the m^{th} element is:

$$X_m(t) = \sum_{k=1}^{K} s_k(t)\exp\left[-j(m-1)\frac{2\pi d \sin\theta_k}{\lambda}\right] + n_m(t) \tag{3.6}$$

From (3.6) the output steering array vector is:

$$a(\theta_k) = \exp\left[-j(m-1)\frac{2\pi d \sin\theta_k}{\lambda}\right] \tag{3.7}$$

3.3. Eigendecomposition and MUSIC algorithm implementation

Multiple signal classification is a subspace technique based on exploiting the eigenstructure of input covariance matrix suggested by Schmidt in 1979 [36]. The eigenvectors are easily obtained by either an eigendecomposition of sample covariance matrix or a Singular Value Decomposition of the data matrix [37]. By MUSIC algorithm, the powers and cross correlations between the various input signals with a set of input parameters such as number of array elements, number of snapshots, element spacing, angular separation, signal-to-noise ratio can be readily obtained. The Direction of arrival of the multiple incident signals can be estimated by locating the peaks of a MUSIC spatial spectrum at a high resolution. This leads to high quality wireless communication. The multiple signal classification algorithm is normally of high performance in DOA estimation; however at a very small difference of angle of arrival between two adjacent signals and close array elements, it fails to perform well. Therefore, for more accurate estimates at a closer distance between antenna elements and closer signal sources, the Root-MUSIC algorithm is often used [38].

The corresponding covariance matrix of the array output can be computed as shown in (3.8):

$$R_x = E[XX^H] \tag{3.8}$$

Where H is the conjugate transpose of the matrix, the noise is assumed to be zero-mean and additive white Gaussian and is uncorrelated to the signal.
Replacing (3.8) by its value from (3.1), the covariance matrix becomes:

$$\begin{aligned} R_x &= E[(As+N)(As+N)^H] \\ &= AE[ss^H]A^H + E[NN^H] \\ &= AR_S A^H + R_N \end{aligned} \tag{3.9}$$

Where $R_S = E[ss^H]$ is called source signal correlation matrix, $R_N = \sigma^2 I$ is a noise correlation matrix.

If ($\lambda_1, \lambda_2, \dots .\lambda_M$) are eigenvalues of spatial correlation matrix R_x; Then the computation of the eigenvalue associated with a particular eigenvector is given as:

$$R_x - \lambda_i \, \mathrm{I} \; = 0 \tag{3.10}$$

$$AR_S A^H + \sigma^2 I - \lambda_i \, \mathrm{I} = 0 \tag{3.11}$$

$$AR_S A^H + (\sigma^2 - \lambda_i \,)\mathrm{I} = 0 \tag{3.12}$$

Therefore, the eigenvectors v_i of $AR_S A^H$ are obtained using (3.13);

$$v_i \; = \; \sigma^2 \; - \; \lambda_i \tag{3.13}$$

The eigenvalues are sorted in accordance to the size, that is $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M \geq 0$. The larger eigenvalues correspond to signal $(M \; - \; K\;)$ which means all values greater than K, while small eigenvalues corresponds to noise that is all values less than K. Thus, the MUSIC "Spatial spectrum" is defined as:

$$P_{MUSIC}(\theta) = \frac{1}{a^H(\theta) E_N {E_N}^H a(\theta)} \tag{3.14}$$

In short, the multiple signal classification algorithm can be summarized as follow:

Step 1: The data is collected to form the correlation matrix R_x.
Step 2: The eigenstructure of the covariance matrix R_x is decomposed.
Step 3: Assuming the number of signal sources is K.
Step 4: The N columns are chosen to form the noise subspace E_N.
Step 5: P_{MUSIC} versus θ is evaluated.
Step 6: The spectrum function is determined; then an estimation of DOA is obtained by the peak-searching k.

3.4. Root-MUSIC algorithm

The Root- MUSIC algorithm is an improved method of the MUSIC algorithm. It is used to increase resolution and decrease the computational complexity of the MUSIC algorithm [38]. It is based on the idea of polynomial rooting giving higher resolution, although it is possible to use it only with a uniform linear array. For a small number of array elements and with low signal to noise ratio, the Root- MUSIC is more accurate compared to MUSIC [39]. One can simplify the denominator in the expression (3.14) of the MUSIC spectrum by defining the matrix $C = E_N {E_N}^H$. This leads to the Root-MUSIC expression:

$$P_{Root-MUSIC}(\theta) = \frac{1}{a^H(\theta) C a(\theta)} \tag{3.15}$$

For a uniform linear array with the spacing *d* between each element, there exist *m* array elements of the steering vector $a(\theta)$ at the angles of arrival $(\theta_1, \theta_2, \ldots, \theta_K)$ that may be expressed as:

$$a_m(\theta) = \exp(jkd(m-1)\sin(\theta)) \tag{3.16}$$

With $m = 1,2, \ldots, M$

The denominator argument in (3.15) can be written as:

$${P_{Root-MUSIC}}^{-1}(\theta) = \sum_{m=1}^{M} \sum_{n=1}^{M} \exp(-jkd(m-1)\sin(\theta))\, C_{mn} \exp(jkd(n-1)\sin(\theta)) \tag{3.17}$$

$${P_{Root-MUSIC}}^{-1}(\theta) = \sum_{l=-M+1}^{M-1} C_l \exp(jkdl\sin(\theta)) \tag{3.18}$$

Where C_l is the sum of the diagonal elements of C.

We can simplify (3.17) to be in the form of a polynomial whose coefficients are C_l . Thus,

$$D(z) = \sum_{l=-M+1}^{M+1} C_l \, z^{-1} \tag{3.19}$$

Where $z^{-1} = \exp(jkdlsin(\theta))$

Evaluating the Root-MUSIC spectrum becomes the same as evaluating the polynomial $D(z)$ on the unit circle [42]. The root polynomial is of order $2(M-1)$, and therefore has the roots of $z_0, z_2, \ldots, z_{2(M-1)}$. Each root can be complex and polar notation can be written as:

$$z_i = |z_i| \exp\big(jarg(z_i)\big) \tag{3.20}$$

With $i = 0,2, \ldots, 2(M-1)$, and $arg(z_i)$ is the phase of z_i.

Exact zeros in $D(z)$ exist when the root magnitudes $z_i = 1$. Calculating the direction of arrival by comparing $\exp(jarg(z_i))$ to $z^{-1} = \exp(jkdlsin(\theta))$ to get:

$$\theta_i = -sin^{-1}(\frac{1}{kd} \arg(\, z_i)) \tag{3.21}$$

3.5. Non-Uniform linear smart antenna array

Consider a system with non-uniform linear array whereby the array antennas linearly spaced with different array elements spacing and phase difference of the signal [40, 43]. In this case, one non-uniform linear array is assumed to be decomposed into two uniform linear arrays as shown in fig 3.2 and fig 3.3 respectively. The direction of arrival is estimated by combining MUSIC results of two decomposed corresponding co-prime arrays. A prototype co-prime array utilizes a co-prime pair of uniform linear subarrays, where one is of *M* sensors with an interelement spacing of *N* units, whereas the other is of *N* elements with an interelement spacing of *M* units. The numbers *M* and *N* are chosen to be co-prime. The unit interelement spacing, *d*, is typically set as half wavelength. Because the two subarrays share the first sensor at the zeroth position, the corresponding co-prime array has a total number of $M + N - 1$ sensors.

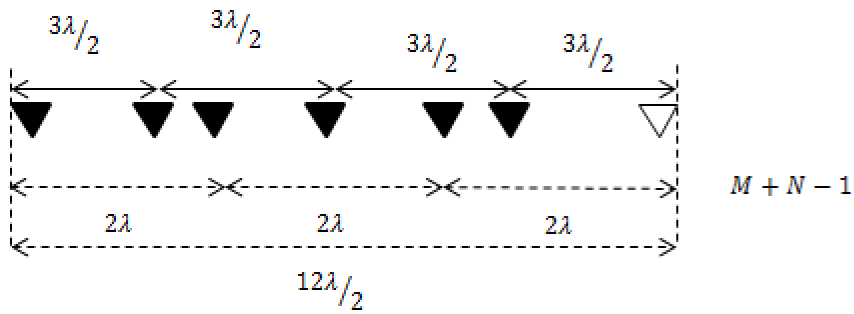

Fig 3.2: Co-prime non-uniform linear array

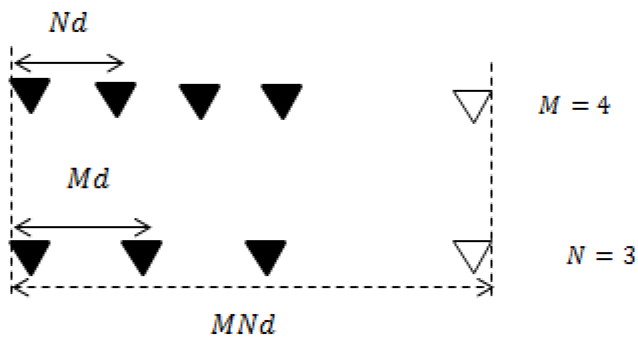

Fig 3.3: Two co-prime uniform linear arrays

The array output vector of the co-prime array is given in (3.1):

$$X(t) = A(t)s(t) + n(t), \tag{3.1}$$

Then, the received signal vector of each sub-array at the *t-th* time slot can be defined as:

$$X_M(t) = A_M s(t) + n(t) \tag{3.22}$$

and;

$$X_N(t) = A_N s(t) + n(t) \tag{3.23}$$

According to the far field assumption, steering vector corresponding to the k^{th} source is:

$$a_k = \left[1, e^{-j\frac{2\pi}{\lambda}d_1 \sin(\theta_k)}, \dots, e^{-j\frac{2\pi}{\lambda}d_l \sin(\theta_k)}, e^{-j\frac{2\pi}{\lambda}d_{(M+N-1)} \sin(\theta_k)}\right]^T \tag{3.24}$$

Where $d_l(l = 1,2, \dots, M + N - 1)$

Steering arrays of each linear array are given by:

$$a_{Mk} = \left[1, e^{-j\pi N sin(\theta_k)}, \dots, e^{-j\pi(M-1)N sin(\theta_k)}\right]^T \tag{3.25}$$

$$a_{Nk} = [1, e^{-j\pi M sin(\theta_k)}, \dots, e^{-j\pi(N-1)M sin(\theta_k)}]^T \tag{3.26}$$

Starting by obtaining each sample of the covariance matrices for (3.22) and (3.23) the two decomposed uniform linear subarrays:

$$R_{xM} = E[X_M(t)X_M(t)^H] \tag{3.27}$$

$$R_{xN} = E[X_N(t)X_N(t)^H] \tag{3.28}$$

Replacing the received signal vector of each sub-array (3.22) and (3.23) by their values gives:

$$\begin{aligned} R_{xM} &= E[(A_M s(t) + n(t))(A_M s(t) + n(t))^H] \\ &= A_M E(s(t)s(t)^H){A_M}^H + E[NN^H] \\ &= A_M R_{SM} {A_M}^H + R_N \end{aligned} \tag{3.29}$$

$$\begin{aligned} R_{xN} &= E[(A_N s(t) + n(t))(A_N s(t) + n(t))^H] \\ &= A_N E(s(t)s(t)^H){A_N}^H + E[NN^H] \\ &= A_N R_{SN} {A_N}^H + R_N \end{aligned} \tag{3.30}$$

Where $R_{SM} = E[s(t)s(t)^H]$ and $R_{SN} = E[s(t)s(t)^H]$ are the source signals correlation matrices of sub-arrays M and N respectively, $R_N = \sigma^2 I$ is a noise correlation matrix for each subarray.

If $(\lambda_1, \lambda_2, \ldots .\lambda_M)$ and $(\lambda_1, \lambda_2, \ldots .\lambda_N)$ are eigenvalues of spatial correlation matrix R_{xM} and R_{xN} respectively. Therefore, the expression of eigenvalue for each sub-array is given as:

$$R_x - \lambda_i \mathrm{I} = 0 \tag{3.31}$$

$$A_M R_{SM} {A_M}^H + (\sigma^2 - \lambda_i)\, \mathrm{I} = 0 \tag{3.32}$$

$$A_N R_{SN} {A_N}^H + (\sigma^2 - \lambda_i)\mathrm{I} = 0 \tag{3.33}$$

The eigenvectors v_i of $A_M R_{SM} {A_M}^H$ and $A_N R_{SN} {A_N}^H$ from (3.29) and (3.30) which correspond to a particular eigenvalues are obtained using (3.34) whereby v_i is:

$$v_i = \sigma^2 - \lambda_i \tag{3.34}$$

Therefore, applying eigendecomposition to the sample covariance matrices in (3.29) and (3.30) yields:

$$R_{xM} = E_{sM} \Lambda_{sM} {E_{sM}}^H + E_{nM} \Lambda_{nM} {E_{nM}}^H \tag{3.35}$$

$$R_{xN} = E_{sN} \Lambda_{sN} {E_{sN}}^H + E_{nN} \Lambda_{nN} {E_{nN}}^H \tag{3.36}$$

Then, according to the orthogonality between the signal subspace and the noise subspace of each sub-array, the MUSIC spatial pseudo-spectrum of the two decomposed linear subarrays M and N respectively, are:

$$P_{MUSIC_M}(\theta) = \frac{1}{a_M(\theta)^H E_{nM} {E_{nM}}^H a_M(\theta)} \tag{3.37}$$

$$P_{MUSIC_N}(\theta) = \frac{1}{a_N(\theta)^H E_{nN} {E_{nN}}^H a_N(\theta)} \tag{3.38}$$

In many practical signal processing problems, the objective is to estimate from measurements a set of constant parameters upon which the received signal depend in the presence of the noise. The Multiple signals processing classification method in non-uniform linear array has the ability to simultaneously measure the multiple signals with fewer antennas as will be shown in chapter 4. It has high precision measurements and high resolution for antenna beam signals.

The angles are initialized at the beginning of the software and many calculations in order to find a sensible subspace matrix follow the initialization. When the number of components is known in advance, the MUSIC in NLA outperforms simple methods such as accurately picking peaks of the DFT spectra in the presence of the noise; because it exploits the knowledge of this number to ignore the noise in its final report as detailed in the next chapter.

CHAPTER 4

SIMULATION RESULTS AND DISCUSSION

In this chapter, computer simulations are provided to investigate the performance analysis of uniform and non-uniform linear arrays. In all cases, the impinging source signals are narrowband signals operating at the same center frequency $2.4GHz$. This frequency is similar to the one used in wireless communication (Wi-Fi) and the azimuth arrival angle interval is set to be in the range: $[-\frac{\pi}{2},\frac{\pi}{2}]$. These signals are uncorrelated with themselves and with the noise as well. The additive background noise is assumed to be spatially and temporally uncorrelated white complex Gaussian with the zero-mean. Through variation of the number of array elements (M), signal to noise ratio (SNR), number of snapshots (N), and number of arriving angles (DOA), the MATLAB simulations can prove the efficiency of the MUSIC and Root-MUSIC algorithms with a ULA and NLA. For a uniform linear array, the space between two adjacent array elements is half wavelength ($d=\frac{\lambda}{2}$), and for a non-uniform linear array, the inter-element spacing is greater than a half wavelength ($d>\frac{\lambda}{2}$).

These simulations are subdivided into six experiments:

(i) The validation of the simulated results
(ii) The variation of the direction of arrival with the number of snapshots
(iii) The variation of the direction of arrival with the signal to noise ratio
(iv) The variation of the direction of arrival with the number of array elements
(v) The variation of the direction of arrival with the number of signal to noise ratio
(vi) The simulation of the direction of arrival with the Root-MUSIC algorithm

The study underlies the performance evaluation and the accuracy of the direction of arrival estimation on both array arrangements. This is achieved by comparing the Mean Square Error (MSE) of the direction of arrival estimation for a ULA and NLA performances through MUSIC and Root-MUSIC algorithms.

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(\theta - \hat{\theta}_i)^2$$

With $i = 1,2, \dots, n$, where n is the number of trials, θ is the true angle of arrival and $\hat{\theta}_i$ is an i^{th} estimated angle of arrival.

4.1. Experiment 1: Validation of results

The simulation results were justified by using existing data in the literature. The dashed line in fig.4.1 is the results presented by Dhering and Bansode [3] and the continuous line is the present work with the increased signal to noise ratio. The fig 4.1 gives a performance of MUSIC spectrum for six different variation of the signal to noise ratio. While keeping the other input parameters such as array size: $M = 20$ elements, number of snapshots: $N = 200$ snapshots, true DOA: $10°, 30°\ and\ 50°$ and the signal frequency: $Freq = 2.4GHz$. In Dhering and Bansode's work, the signal to noise ratio was varied from $15dB, 25dB$ to $35dB$ and for the current work, the signal to noise ratio is varied from $20dB, 30dB$ to $40dB$. As observed, there is a good agreement between the simulated results and the published data by Dhering and Bansode.

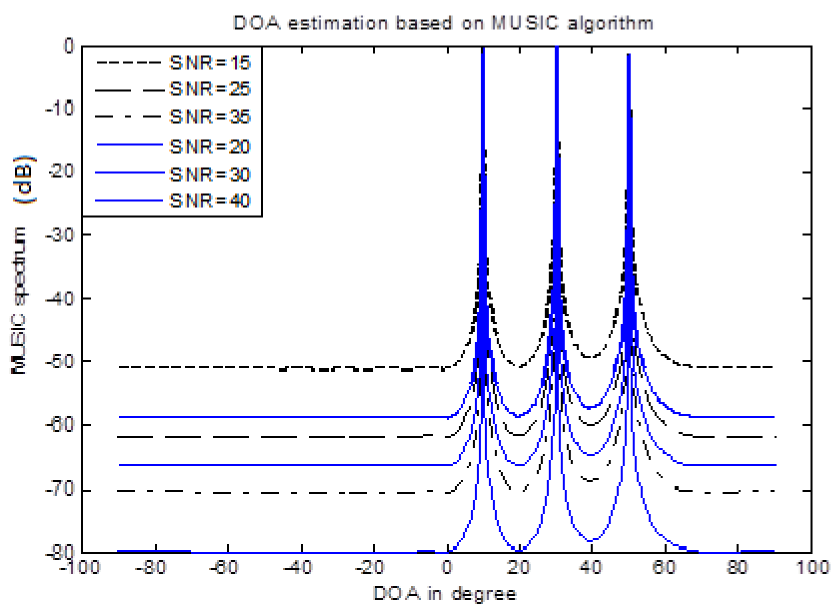

Fig 4.1 MUSIC spectrum for SNR variation for ULA

The simulation results in fig 4.1 show that the signal to noise ratio highly affect the performance of the MUSIC algorithm. It is noted that the more increased the signal to noise ratio, the accurate is the direction of arrival estimation.

The experiment 1 shows the validation of the results obtained in the MATLAB simulations. The results from fig 4.1 give a performance of MUSIC spectrum for six different variation of the signal to noise ratio. The simulation results shows that the signal to noise ratio highly affect the performance of the MUSIC algorithm. Thus, there is a good agreement between the simulated results and the published data by Dhering.

4.2. Experiment 2: Variation of DOA with the number of snapshots

In experiment 2, the variation of the DOA estimation with the number of snapshots is simulated. For uniform linear array, the number of snapshots was varied from $N = 100$ to $N = 1000$ snapshots in steps of 100 snapshots. The results of the DOA for $N = 100,\ 500$ and 1000 snapshots are given in fig 4.2. This simulation is based on MUSIC algorithm. During the simulation of this experiment, the following signal and array parameters are kept constant:

True DOA: $\theta_1 = 60°$ and $\theta_2 = 68°$, signal frequency: $\mathrm{Freq} = 2.4\mathrm{GHz}, \mathrm{SNR} = -5\mathrm{dB}$,
Array size: $\mathrm{M} = 11$ elements.

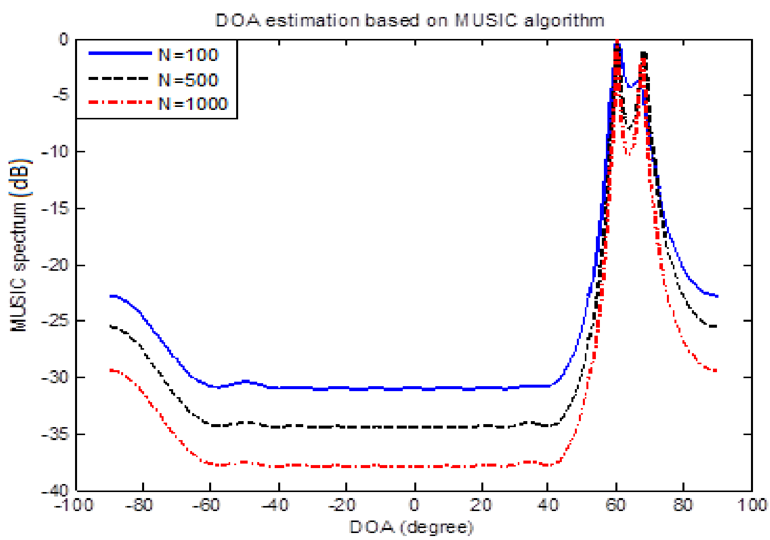

Fig 4.2: DOA estimation with snapshots variation for ULA

The results from fig 4.2 are summarized in Table 4.1 with 8 trials to be able to analyze the error using MSE. The true DOA of arrivals are denoted as θ_1 and θ_2 and then the estimated DOA of arrivals are denoted as $\hat{\theta}_1$ and $\hat{\theta}_2$. It can be noted that with $N = 100$ snapshots, the Mean Square Error: $MSE_1 = 0.543$ and $MSE_2 = 0.636$ for the arrivals at $\theta_1 = 60°$ and $\theta_2 = 68°$ respectively. Also with $N = 500$ snapshots the Mean Square Error: $MSE_1 = 0.063$ and $MSE_2 = 0.125$ for the arrivals at $\theta_1 = 60°$ and $\theta_2 = 68°$ respectively. Then, with $N = 1000$ snapshots the Mean Square Error: $MSE_1 = 0.031$ and $MSE_2 = 0.011$ for the arrivals at $\theta_1 = 60°$ and $\theta_2 = 68°$ respectively.

Table 4.1: Estimated DOA with snapshots and error analysis for ULA

No of snapshots	θ_1	$\hat{\theta}_1$	$Error_1 = (\theta_1 - \hat{\theta}_1)$	θ_2	$\hat{\theta}_2$	$Error_2 = (\theta_2 - \hat{\theta}_2)$
100	$60°$	$60.3°$	-0.3	$68°$	$67.5°$	$+0.5$
	$60°$	$61°$	-1	$68°$	$67°$	$+1$
	$60°$	$61.5°$	-1.5	$68°$	$66.5°$	$+1.5$
	$60°$	$60°$	0	$68°$	$68.3°$	-0.3
	$60°$	$60°$	0	$68°$	$67.5°$	$+0.5$
	$60°$	$60°$	0	$68°$	$67.5°$	$+0.5$
	$60°$	$60°$	0	$68°$	$68°$	0
	$60°$	$61°$	-1	$68°$	$67°$	$+1$
$MSE_1 = 0.543$				$MSE_2 = 0.636$		
500	$60°$	$60°$	0	$68°$	$68.5°$	-0.5
	$60°$	$60°$	0	$68°$	$68°$	0
	$60°$	$60°$	0	$68°$	$67.5°$	$+0.5$
	$60°$	$60°$	0	$68°$	$68°$	0
	$60°$	$60°$	0	$68°$	$68.5°$	-0.5
	$60°$	$59.5°$	$+0.5$	$68°$	$68°$	0
	$60°$	$60.5°$	-0.5	$68°$	$67.5°$	$+0.5$

	$60°$	$60°$	0	$68°$	$68°$	0
$MSE_1 = 0.063$				$MSE_2 = 0.125$		
1000	$60°$	$60°$	0	$68°$	$68°$	0
	$60°$	$60°$	0	$68°$	$68°$	0
	$60°$	$60°$	0	$68°$	$67.7°$	$+0.3$
	$60°$	$60.5°$	-0.5	$68°$	$68°$	0
	$60°$	$60°$	0	$68°$	$68°$	0
	$60°$	$60°$	0	$68°$	$68°$	0
	$60°$	$60°$	0	$68°$	$68°$	0
	$60°$	$60°$	0	$68°$	$68°$	0
$MSE_1 = 0.031$				$MSE_2 = 0.011$		

The simulation for the non - uniform linear array with DECOM (7, 5) that is DOA estmation with combined MUSIC for co-prime array with 7 antenna elements for one sub-arrays then 5 antenna elements for another sub-array. The number of snapshots was varied from $N = 100$ to $N = 1000$ snapshots in steps of 100 snapshots. The results of the DOA for $N = 100, 500$ and 1000 snapshots are given in fig 4.3, fig 4.4 and fig 4.5 respectively.

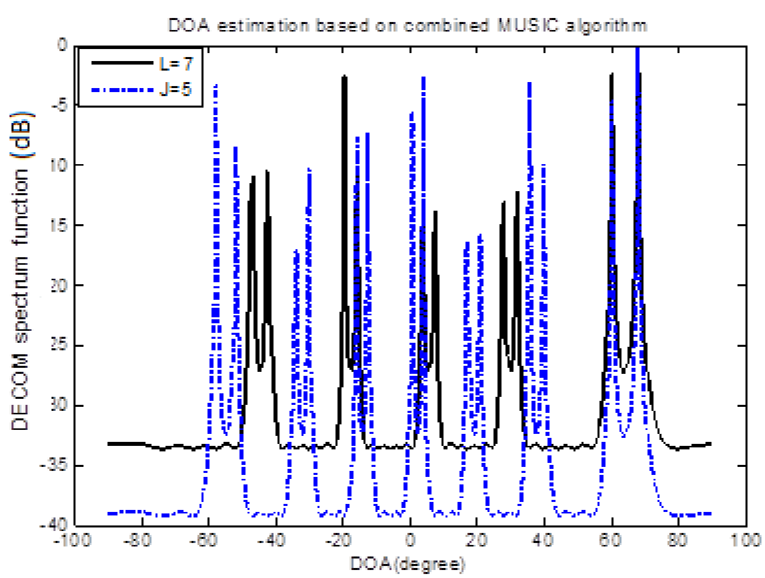

Fig 4.3: DOA estimation with 100 snapshots for DECOM (7, 5)

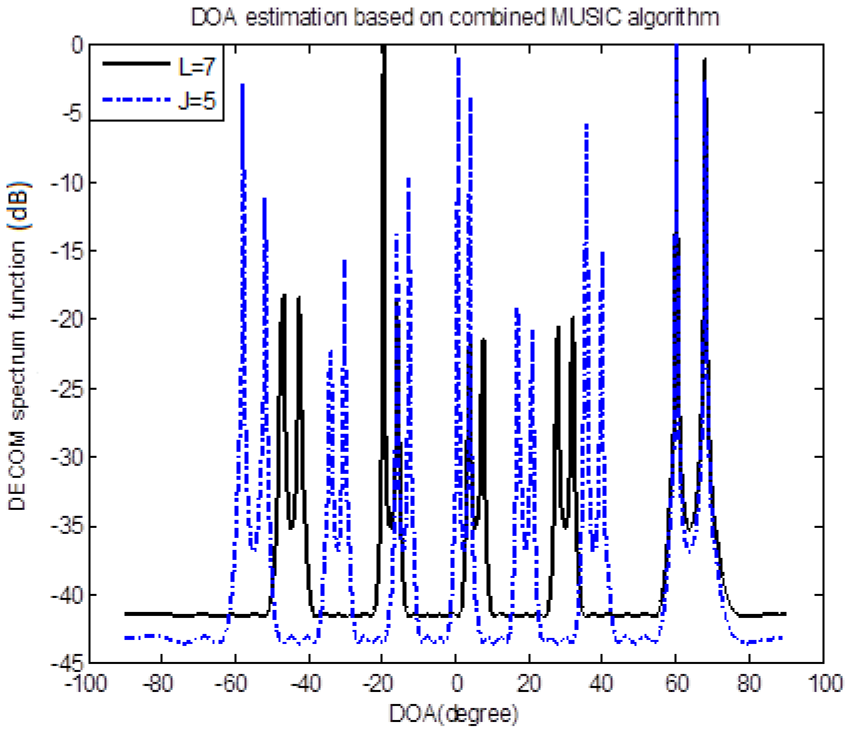

Fig 4.4: DOA estimation with 500 snapshots for DECOM (7, 5)

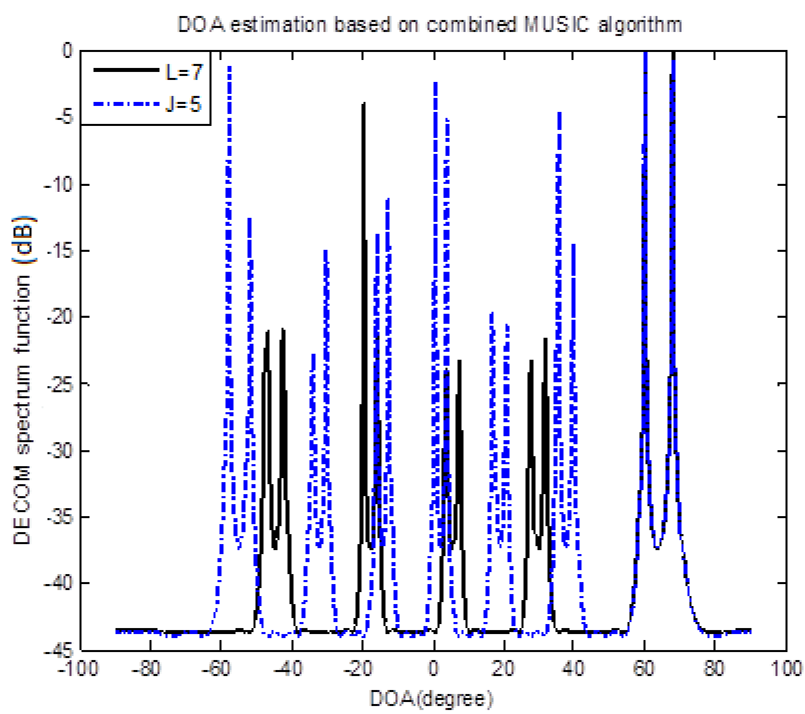

Fig 4.5: DOA estimation with 1000 snapshots for DECOM (7, 5)

The results from fig 4.3, fig. 4.4 and fig. 4.5 are summarized in Table 4.2 with 8 trials to be able to analyze the error using MSE. The true DOA of arrivals are denoted as θ_1 and θ_2. For a uniform linear sub-array formed by 7 elements in DECOM (7, 5), the estimated DOA of arrivals are denoted as $\hat{\theta}_1$ and $\hat{\theta}_2$ and for a uniform linear sub-array formed by 5 elements in DECOM (7, 5), the estimated DOA of arrivals are designated as $\hat{\theta}_1{}'$ and $\hat{\theta}_2{}'$.

It can be noted that with $N = 100$ snapshots, for a uniform linear sub-array formed by 7 elements in DECOM (7, 5), the Mean Square Error: $MSE_1 = 0$ and $MSE_2 = 0$ and for a uniform linear sub-array formed by 5 elements in DECOM (7, 5), the Mean Square Error: $MSE_1{}' = 0$ and $MSE_2{}' = 0$ at the arrivals: $\theta_1 = 60°$ and $\theta_2 = 68°$ respectively. Also with $N = 500$ snapshots, for a uniform linear sub-array formed by 7 elements in DECOM (7, 5), the Mean Square Error: $MSE_1 = 0$ and $MSE_2 = 0$ and for a uniform linear sub-array formed by 5 elements in DECOM (7, 5), the Mean Square Error: $MSE_1{}' = 0$ and $MSE_2{}' = 0$ at the arrivals: $\theta_1 = 60°$ and $\theta_2 = 68°$ respectively. Then, with $N = 1000$ snapshots for a uniform linear sub-array formed by 7 elements in DECOM (7, 5), the Mean Square Error: $MSE_1 = 0$ and $MSE_2 = 0$ and for a uniform linear sub-array formed by 5 elements in DECOM (7, 5), the Mean Square Error: $MSE_1{}' = 0$ and $MSE_2{}' = 0$ at the arrivals: $\theta_1 = 60°$ and $\theta_2 = 68°$ respectively.

Table 4.2: Estimated DOA with snapshots and error analysis for DECOM (7, 5)

$Snapshots$ (N)	θ_1	$\hat{\theta}_1$	$Error_1$	$\hat{\theta}_1{}'$	$Error_1{}'$	θ_2	$\hat{\theta}_2$	$Error_2$	$\hat{\theta}_2{}'$	$Error_2{}'$
100	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0

	60°	60°	0	60°	0	68°	68°	0	68°	0
$MSE_1 = 0$				$MSE_1{}' = 0$		$MSE_2 = 0$			$MSE_2{}' = 0$	
500	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
$MSE_1 = 0$				$MSE_1{}' = 0$		$MSE_2 = 0$			$MSE_2{}' = 0$	
1000	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
$MSE_1 = 0$				$MSE_1{}' = 0$		$MSE_2 = 0$			$MSE_2{}' = 0$	

The experiment 2 shows the variation of DOA with the number of snapshots with the error analysis using MSE. It is noted that the results from fig 4.2 show that the more the snapshots the accurate and narrower is the beam width of the signal. Although when number of snapshots is increased, the more accurate is the DOA estimation for the uniform linear array, the non-uniform linear array has better performance as shown in fig 4.3, fig 4.4 and fig 4.5 according to the results of MSE presented in Table 4.1 and Table 4.2.

4.3. Experiment 3: Variation of DOA with the signal to noise ratio

In experiment 3, the variation of the DOA estimation with the signal to noise ratio is simulated. For uniform linear array, the signal to noise ratio was varied from $SNR = -5dB$ to $SNR = 5dB$ in steps of $1dB$. The results of the DOA for $SNR = -5dB, 0dB\ and\ 5dB$ signal to noise ratio are given in fig 4.6. This simulation is based on MUSIC algorithm. During the simulation of the experiment 3, the following signal and array parameters are kept constant:

True DOA: $\theta_1 = 60°$ and $\theta_2 = 68°$, signal frequency: $\mathrm{Freq} = 2.4\mathrm{GHz}$, snapshots: $\mathrm{N} = 200$,

Array size: $\mathrm{M} = 11$.

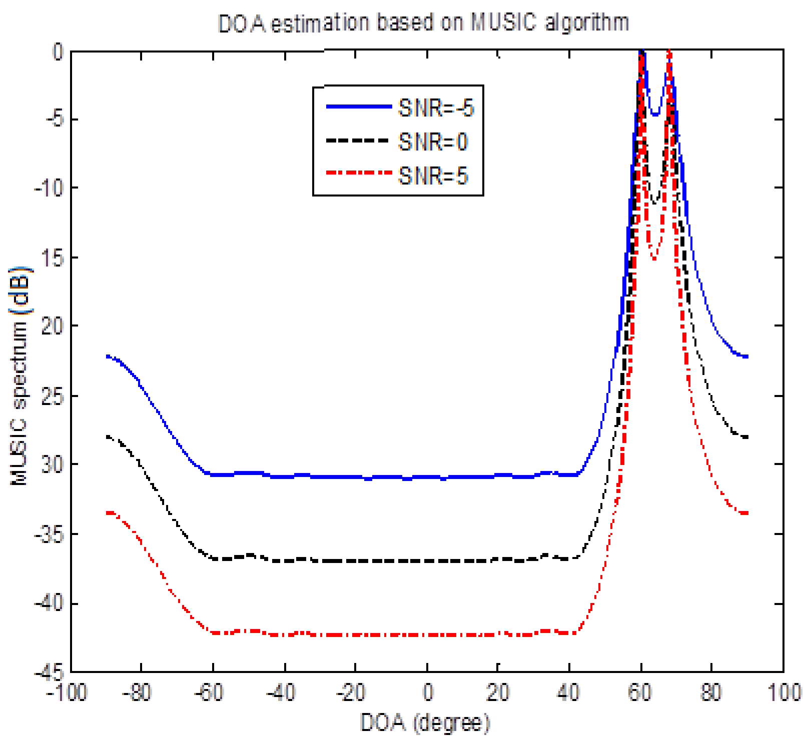

Fig 4.6: DOA estimation with signal to noise ratio variation for ULA

The results from fig 4.6 are summarized in Table 4.3 with 8 trials to be able to analyze the error using MSE. The true DOA of arrivals are denoted as θ_1 and θ_2 and the estimated DOA of arrivals are denoted as $\hat{\theta}_1$ and $\hat{\theta}_2$. It can be noted that with $SNR = -5dB$, the Mean Square

Error: $MSE_1 = 0.125$ and $MSE_2 = 0.511$ for the arrivals at $\theta_1 = 60°$ and $\theta_2 = 68°$ respectively. Also with $SNR = 0dB$, the Mean Square Error: $MSE_1 = 0.063$ and $MSE_2 = 0.031$ for the arrivals at $\theta_1 = 60°$ and $\theta_2 = 68°$ respectively. Then with $SNR = 5dB$, the Mean Square Error: $MSE_1 = 0$ and $MSE_2 = 0$ for the arrivals at $\theta_1 = 60°$ and $\theta_2 = 68°$ respectively.

Table 4.3: Estimated DOA with signal to noise ratio and error analysis for ULA

SNR	θ_1	$\hat{\theta}_1$	$Error_1 = (\theta_1 - \hat{\theta}_1)$	θ_2	$\hat{\theta}_2$	$Error_2 = (\theta_2 - \hat{\theta}_2)$
$-5dB$	$60°$	$60.5°$	-0.5	$68°$	$67.5°$	$+0.5$
	$60°$	$60.5°$	-0.5	$68°$	$68°$	0
	$60°$	$60°$	0	$68°$	$67.7°$	$+0.3$
	$60°$	$60°$	0	$68°$	$67°$	$+1$
	$60°$	$60.5°$	-0.5	$68°$	$66.5°$	$+1.5$
	$60°$	$60°$	0	$68°$	$68.5°$	-0.5
	$60°$	$60°$	0	$68°$	$68°$	0
	$60°$	$60.5°$	-0.5	$68°$	$67.5°$	$+0.5$
$MSE_1 = 0.125$				$MSE_2 = 0.511$		
$0dB$	$60°$	$60°$	0	$68°$	$68.5°$	-0.5
	$60°$	$60.5°$	-0.5	$68°$	$68°$	0
	$60°$	$60°$	0	$68°$	$68°$	0
	$60°$	$60.5°$	-0.5	$68°$	$68°$	0
	$60°$	$60°$	0	$68°$	$68°$	0
	$60°$	$60°$	0	$68°$	$68°$	0
	$60°$	$60°$	0	$68°$	$68°$	0
	$60°$	$60°$	0	$68°$	$68°$	0
$MSE_1 = 0.063$				$MSE_2 = 0.031$		

$5dB$	$60°$	$60°$	0	$68°$	$68°$	0
	$60°$	$60°$	0	$68°$	$68°$	0
	$60°$	$60°$	0	$68°$	$68°$	0
	$60°$	$60°$	0	$68°$	$68°$	0
	$60°$	$60°$	0	$68°$	$68°$	0
	$60°$	$60°$	0	$68°$	$68°$	0
	$60°$	$60°$	0	$68°$	$68°$	0
	$60°$	$60°$	0	$68°$	$68°$	0
$MSE_1 = 0$				$MSE_2 = 0$		

The simulation for the non - uniform linear array with DECOM (7, 5), the signal to noise ratio was varied from $SNR = -5dB$ to $SNR = 5dB$ in steps of $1dB$. The results of the DOA for $SNR = -5dB,\ 0dB\ and\ 5dB$ are given are in fig. 4.7, fig. 4.8 and fig. 4.9 respectively.

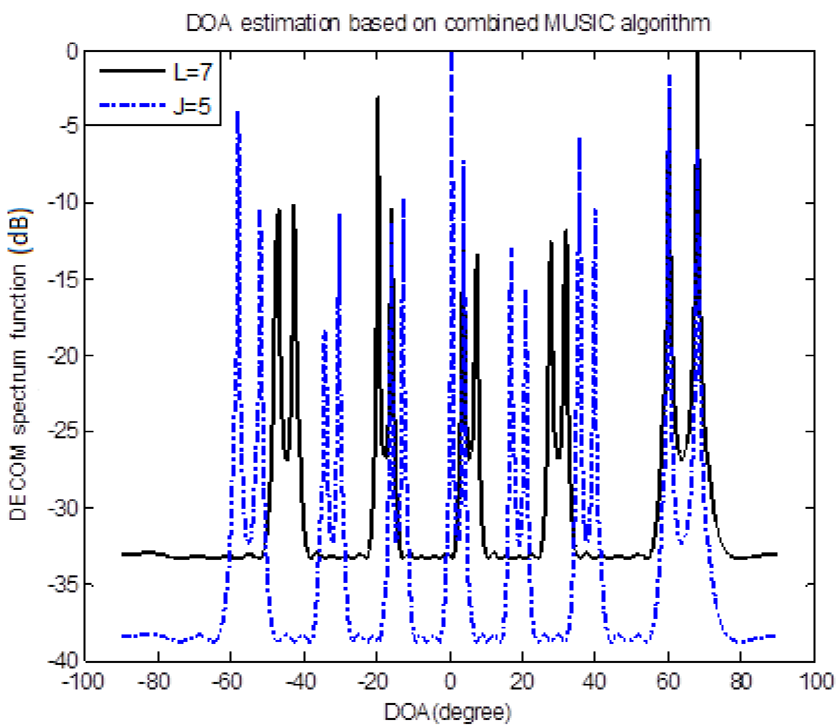

Fig 4.7: DOA estimation with -5dB for DECOM (7, 5)

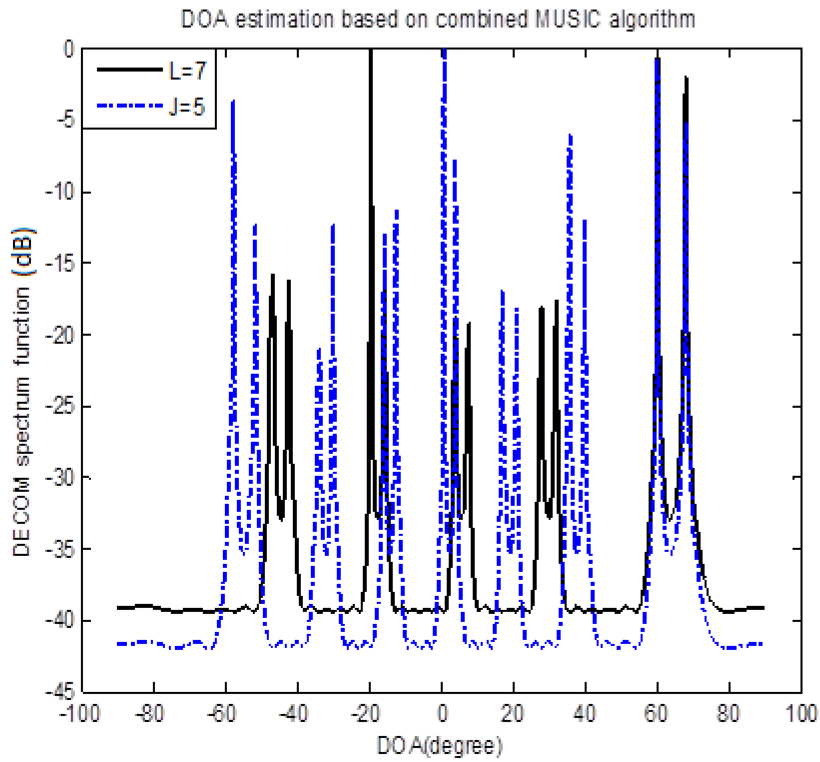

Fig 4.8: DOA estimation with 0dB for DECOM (7, 5)

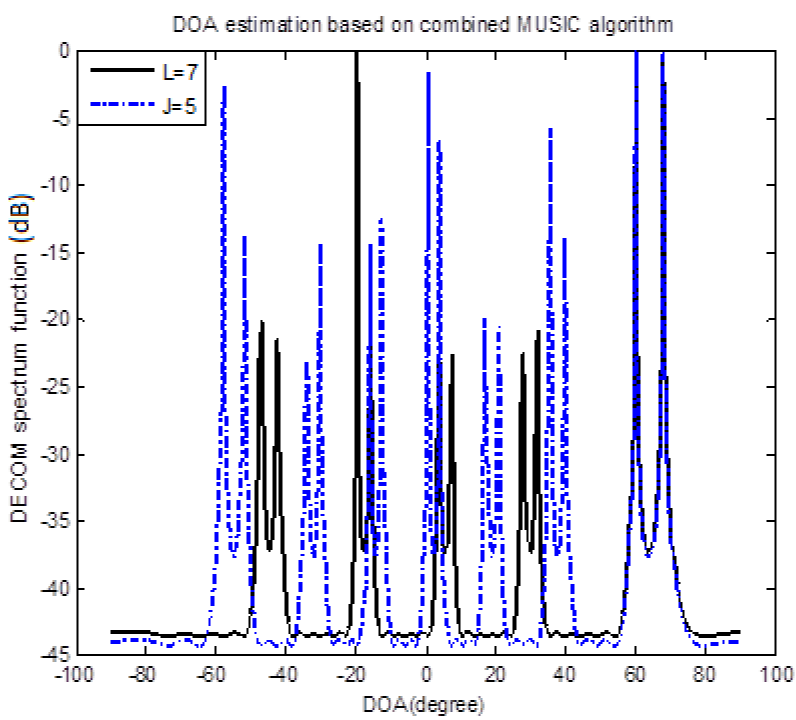

Fig 4.9: DOA estimation with 5dB for DECOM (7, 5)

The results from fig 4.7, fig. 4.8 and fig. 4.9 are summarized in Table 4.4 with 8 trials to be able to analyze the error using MSE. The true DOA of arrivals are denoted as θ_1 and θ_2. For a uniform linear sub-array formed by 7 elements in DECOM (7, 5), the estimated DOA of arrivals are denoted as $\hat{\theta}_1$ and $\hat{\theta}_2$ and for a uniform linear sub-array formed by 5 elements in DECOM (7, 5), the estimated DOA of arrivals are denoted as $\hat{\theta}_1{}'$ and $\hat{\theta}_2{}'$. It can be noted that with $SNR = -5dB$, for a uniform linear sub-array formed by 7 elements in DECOM (7, 5), the Mean Square Error: $MSE_1 = 0.031$ and $MSE_2 = 0.031$ and for a uniform linear sub-array formed by 5 elements in DECOM (7, 5), the Mean Square Error: $MSE_1{}' = 0$ and $MSE_2{}' = 0$ at the arrivals: $\theta_1 = 60°$ and $\theta_2 = 68°$ respectively. Also with $SNR = 0dB$, for a uniform linear sub-array formed by 7 elements in DECOM (7, 5), the Mean Square Error: $MSE_1 = 0$ and $MSE_2 = 0$ and for a uniform linear sub-array formed by 5 elements in DECOM(7, 5), the Mean Square Error: $MSE_1{}' = 0$ and $MSE_2{}' = 0$ at the arrivals: $\theta_1 = 60°$ and $\theta_2 = 68°$ respectively. Then, with $N = 5dB$ for a uniform linear sub-array formed by 7 elements in DECOM (7, 5), the Mean Square Error: $MSE_1 = 0$ and $MSE_2 = 0$ and for a uniform linear sub-array formed by 5 elements in DECOM (7, 5), the Mean Square Error: $MSE_1{}' = 0$ and $MSE_2{}' = 0$ at the arrivals: $\theta_1 = 60°$ and $\theta_2 = 68°$ respectively.

Table 4.4: Estimated DOA with signal to noise ratio and error analysis for DECOM (7, 5)

SNR	θ_1	$\hat{\theta}_1$	$Error_1$	$\hat{\theta}_1{}'$	$Error_1{}'$	θ_2	$\hat{\theta}_2$	$Error_2$	$\hat{\theta}_2$	$Error_2{}'$
$-5dB$	60°	59.5°	+0.5	60°	0	68°	68.5°	−0.5	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
		$MSE_1 = 0.031$		$MSE_1{}' = 0$		$MSE_2 = 0.031$			$MSE_2{}' = 0$	

$0dB$	$60°$	$60°$	0	$60°$	0	$68°$	$68°$	0	$68°$	0
	$60°$	$60°$	0	$60°$	0	$68°$	$68°$	0	$68°$	0
	$60°$	$60°$	0	$60°$	0	$68°$	$68°$	0	$68°$	0
	$60°$	$60°$	0	$60°$	0	$68°$	$68°$	0	$68°$	0
	$60°$	$60°$	0	$60°$	0	$68°$	$68°$	0	$68°$	0
	$60°$	$60°$	0	$60°$	0	$68°$	$68°$	0	$68°$	0
	$60°$	$60°$	0	$60°$	0	$68°$	$68°$	0	$68°$	0
	$60°$	$60°$	0	$60°$	0	$68°$	$68°$	0	$68°$	0
$MSE_1 = 0$				$MSE_1' = 0$		$MSE_2 = 0$			$MSE_2' = 0$	
$5dB$	$60°$	$60°$	0	$60°$	0	$68°$	$68°$	0	$68°$	0
	$60°$	$60°$	0	$60°$	0	$68°$	$68°$	0	$68°$	0
	$60°$	$60°$	0	$60°$	0	$68°$	$68°$	0	$68°$	0
	$60°$	$60°$	0	$60°$	0	$68°$	$68°$	0	$68°$	0
	$60°$	$60°$	0	$60°$	0	$68°$	$68°$	0	$68°$	0
	$60°$	$60°$	0	$60°$	0	$68°$	$68°$	0	$68°$	0
	$60°$	$60°$	0	$60°$	0	$68°$	$68°$	0	$68°$	0
	$60°$	$60°$	0	$60°$	0	$68°$	$68°$	0	$68°$	0
$MSE_1 = 0$				$MSE_1' = 0$		$MSE_2 = 0$			$MSE_2' = 0$	

The experiment 3 gives the variation of DOA with the signal to noise ratio. From fig. 4.6, it is noted that signal to noise ratio affect the accuracy of the DOA estimation. As observed, the more the SNR is increased the accurate is the DOA estimation. With non-uniform linear array, it is even better according to the results obtained in fig 4.7, fig 4.8 and fig 4.9 and according to the MSE expressed in Table 4.3 and Table 4.4. As SNR is improved, the less is the phase difference and also the narrow is the beam width of the signal which is indeed needed to avoid the interference with other undesired signal. It is noted that the more increased the signal to noise ratio, the accurate is the direction of arrival estimation.

4.4.Experiment 4: Variation of DOA with the number of array elements

In experiment 4, the variation of the DOA estimation with the number of array size is simulated. For uniform linear array, the number of array element was varied from $M = 3$ elements to $M = 11$ elements in steps of 2 elements. The results of the DOA for $M = 3, 6\ and\ 11$ elements are given in fig 4.10. This simulation is based on MUSIC algorithm. During the simulation of this experiment, the following signal and array parameters are kept constant:

True DOA: $\theta_1 = 60°$ and $\theta_2 = 68°$, signal frequency: $\mathrm{Freq} = 2.4\mathrm{GHz}$, snapshots: $\mathrm{N} = 100$, $\mathrm{SNR} = -5\mathrm{dB}$.

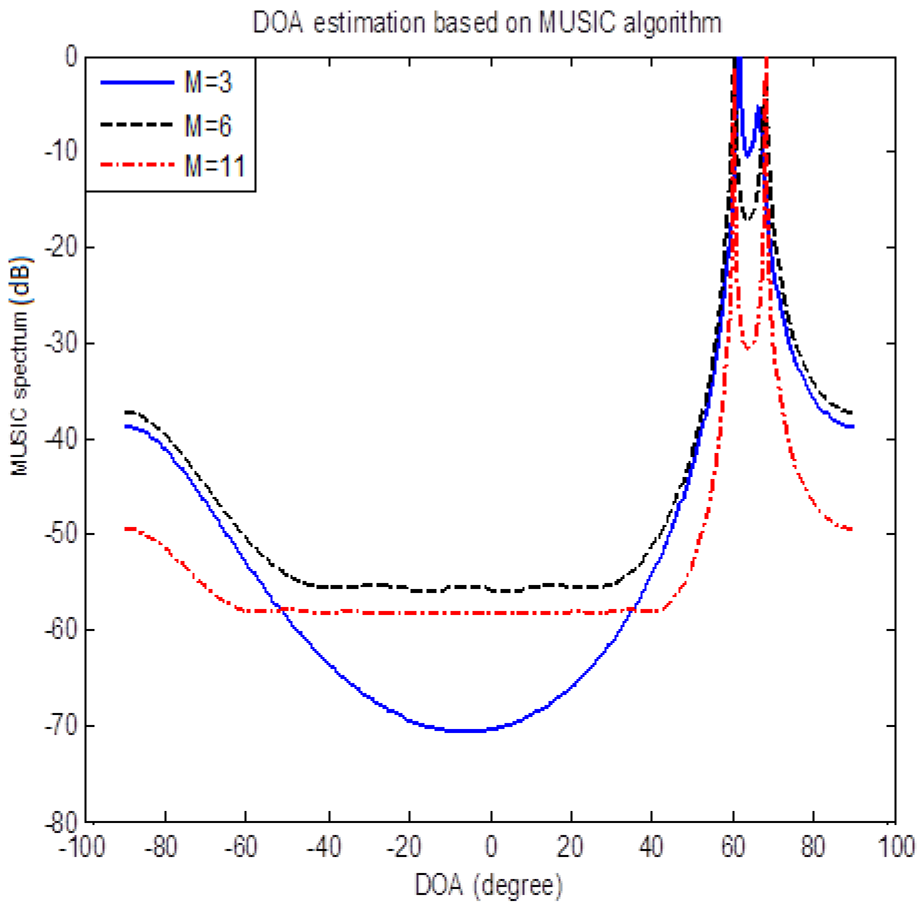

Fig 4.10: DOA estimation with array elements variation for ULA

The results from fig 4.10 are summarized in Table 4.5 with 8 trials to be able to analyze the error using MSE. The true DOA of arrivals are denoted as θ_1 and θ_2 and the estimated DOA of arrivals are denoted as $\hat{\theta}_1$ and $\hat{\theta}_2$. It can be noted that with $M = 3$, the Mean Square Error: $MSE_1 = 0.531$ and $MSE_2 = 0.281$ for the arrivals at $\theta_1 = 60°$ and $\theta_2 = 68°$ respectively. Also with $M = 6$, the Mean Square Error: $MSE_1 = 0$ and $MSE_2 = 0.031$ for the arrivals at $\theta_1 = 60°$ and $\theta_2 = 68°$ respectively. Then with $M = 11$, the Mean Square Error: $MSE_1 = 0$ and $MSE_2 = 0$ for the arrivals at $\theta_1 = 60°$ and $\theta_2 = 68°$ respectively.

Table 4.5: Estimated DOA with the array elements and error analysis for ULA

$Array\ elements$ (M)	θ_1	$\hat{\theta}_1$	$Error_1 = (\theta_1 - \hat{\theta}_1)$	θ_2	$\hat{\theta}_2$	$Error_2 = (\theta_2 - \hat{\theta}_2)$
3	60°	60.5°	−0.5	68°	67.5°	+0.5
	60°	61°	−1	68°	68°	0
	60°	60.5°	−0.5	68°	68°	0
	60°	59.5°	+0.5	68°	67.5°	+0.5
	60°	61°	−1	68°	68.5°	−0.5
	60°	59.5°	+0.5	68°	67°	−1
	60°	60.5°	−0.5	68°	68.5°	−0.5
	60°	59°	+1	68°	68.5°	−0.5
$MSE_1 = 0.531$				$MSE_2 = 0.281$		
6	60°	60°	0	68°	68°	0
	60°	60.5°	−0.5	68°	68.5°	−0.5
	60°	60°	0	68°	68°	0
	60°	60°	0	68°	67.5°	+0.5
	60°	60°	0	68°	68°	0
	60°	60°	0	68°	68°	0
	60°	60°	0	68°	68°	0

	60°	60°	0	68°	68°	0
$MSE_1 = 0.031$				$MSE_2 = 0.125$		
11	60°	60.5°	−0.5	68°	67.5°	+0.5
	60°	60°	0	68°	68°	0
	60°	60°	0	68°	68°	0
	60°	60°	0	68°	68°	0
	60°	60°	0	68°	68°	0
	60°	60°	0	68°	68°	0
	60°	60°	0	68°	68°	0
	60°	60°	0	68°	68°	0
$MSE_1 = 0.031$				$MSE_2 = 0.031$		

The simulation for the non - uniform linear array, the array size was varied. The results of the DOA for $DECOM(4,3)$, $DECOM(9,4)$, $DECOM(11,5)$ are given are in fig 4.11, fig 4.12 and fig. 4.13 respectively.

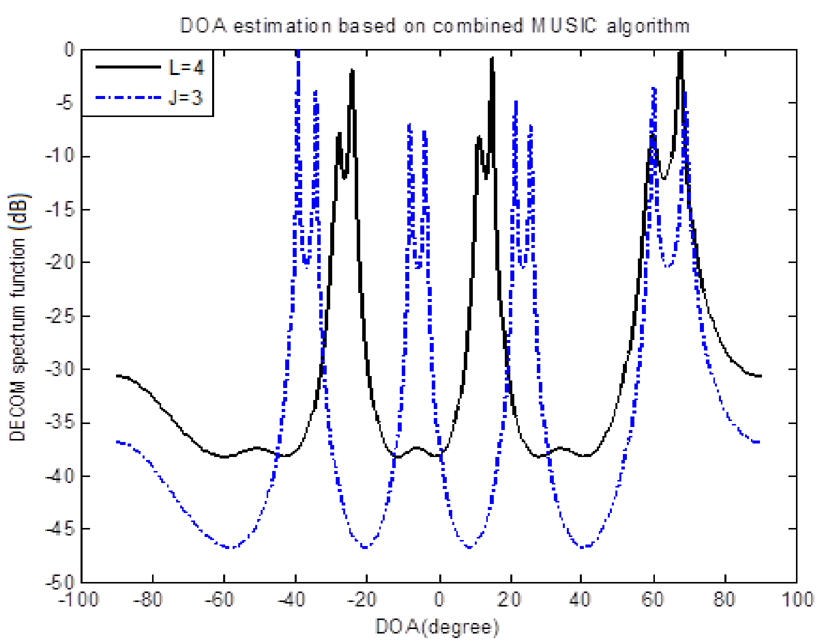

Fig 4.11: Variation of DOA estimation with DECOM (4, 3)

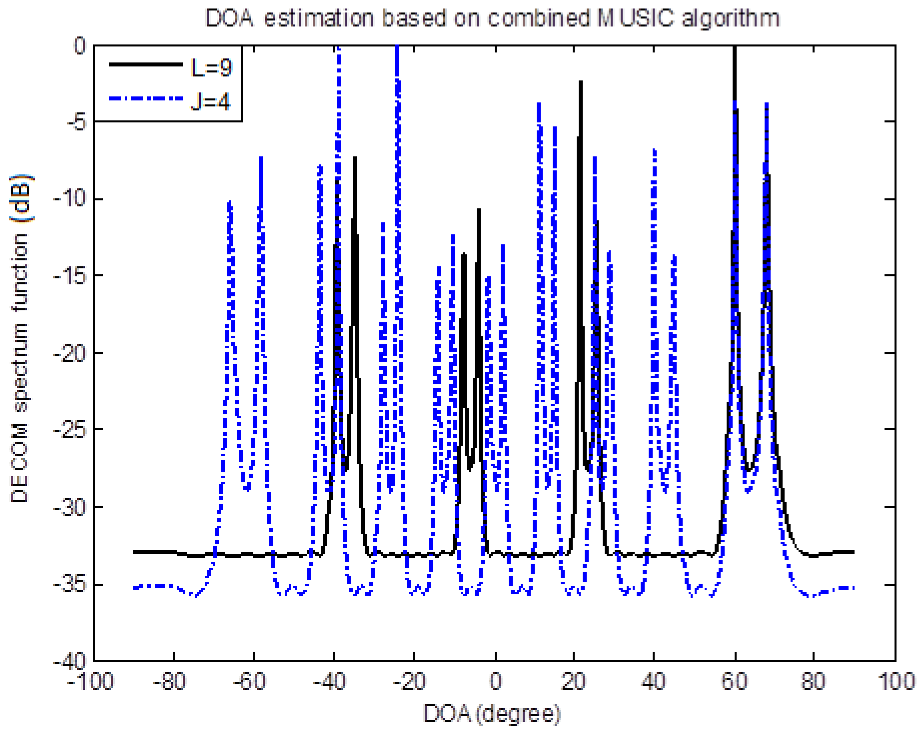

Fig 4.12: Variation of DOA estimation with DECOM (9, 4)

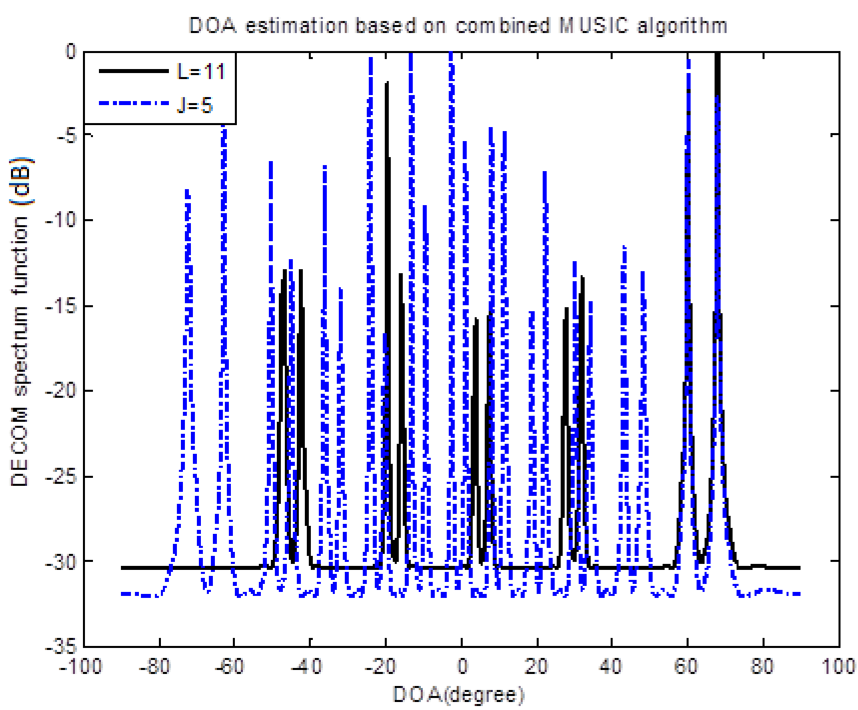

Fig 4.13: Variation of DOA estimation with DECOM (11, 5)

The results from fig 4.11, fig. 4.12 and fig. 4.13 are summarized in Table 4.4 with 8 trials to be able to analyze the error using MSE. The true DOA of arrivals are denoted as θ_1 and θ_2.

For a uniform linear sub-array formed by 4 elements in DECOM (4, 3), the estimated DOA of arrivals are denoted as $\hat{\theta}_1$ and $\hat{\theta}_2$ and for a uniform linear sub-array formed by 3 elements in DECOM (4, 3), the estimated DOA of arrivals are denoted as $\hat{\theta}_1{}'$ and $\hat{\theta}_2{}'$. It can be noted that with for a uniform linear sub-array formed by 4 elements in DECOM (4, 3), the Mean Square Error: $MSE_1 = 0.531$ and $MSE_2 = 0.938$ and for a uniform linear sub-array formed by 3 elements in DECOM (4, 3), the Mean Square Error: $MSE_1{}' = 0.281$ and $MSE_2{}' = 0.750$ at the arrivals: $\theta_1 = 60°$ and $\theta_2 = 68°$ respectively. Also for a uniform linear sub-array formed by 9 elements in DECOM (9, 4), the Mean Square Error: $MSE_1 = 0$ and $MSE_2 = 0$ and for a uniform linear sub-array formed by 4 elements in DECOM (9, 4), the Mean Square Error: $MSE_1{}' = 0$ and $MSE_2{}' = 0$ at the arrivals: $\theta_1 = 60°$ and $\theta_2 = 68°$ respectively. Then for a ULA formed by 11 elements in DECOM (11, 5), the Mean Square Error: $MSE_1 = 0$ and $MSE_2 = 0$ and for a uniform linear sub-array formed by 5 elements in DECOM (11, 5), the Mean Square Error: $MSE_1{}' = 0$ and $MSE_2{}' = 0$ at the arrivals: $\theta_1 = 60°$ and $\theta_2 = 68°$ respectively.

Table 4.6: Estimated DOA with DECOM and error analysis for DECOM

$DECOM$	θ_1	$\hat{\theta}_1$	$Error_1$	$\hat{\theta}_1{}'$	$Error_1{}'$	θ_2	$\hat{\theta}_2$	$Error_2$	$\hat{\theta}_2{}'$	$Error_1{}'$
$DECOM(4,3)$	60°	61°	−1	60°	0	68°	66°	−2	67.5°	+0.5
	60°	60.5°	−0.5	60.5°	−0.5	68°	68°	0	67.5°	+0.5
	60°	60°	0	59.5°	+0.5	68°	69°	−1	67.5°	+0.5
	60°	60.5°	−0.5	59.5°	+0.5	68°	67°	+1	69.5°	−1.5
	60°	59.5°	+0.5	59°	+1	68°	67.5°	+0.5	66.5°	+1.5
	60°	61.5°	−1.5	60°	0	68°	67.5°	+0.5	67.5°	+0.5
	60°	60.5°	−0.5	60.5°	−0.5	68°	68°	0	67.5°	+0.5
	60°	60°	0	59.5°	+0.5	68°	69°	−1	67.5°	+0.5
		$MSE_1 = 0.531$		$MSE_1 = 0.281$			$MSE_2 = 0.938$		$MSE_2 = 0.750$	
$DECOM(9,4)$	60°	60°	0	60°	0	68°	68°	0	68°	0

	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
$MSE_1 = 0$				$MSE_1{}' = 0$		$MSE_2 = 0$			$MSE_2{}' = 0$	
$DECOM(11, 5)$	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
	60°	60°	0	60°	0	68°	68°	0	68°	0
$MSE_1 = 0$				$MSE_1{}' = 0$		$MSE_2 = 0$			$MSE_2{}' = 0$	

The experiment 4 shows the variation of DOA with the number of array elements. For the smart antennas to obtain a reasonable gain, an array antenna with many elements is necessary. As observed in fig 4.10 the beam width of the signal becomes narrower as the number of array elements keeps on increasing. However, the performance is better when using non-uniform linear array as observed in fig 4.11, fig 4.12 and fig 4.13. The non-uniform linear array has better estimates by comparing the MSE in Table 4.5 and Table 4.6. Although, getting a reasonable number of array elements is necessary. Since a very big number of sensors lead to much computational complexity, which demands more time for processing, also it leads to a mutual coupling and may occupy a big space which makes it to be more expensive.

4.5.Experiment 5: Variation of DOA with the number of signal sources

In experiment 5, the variation of the DOA estimation with the number of signal source is simulated. For uniform linear array, the number of signal sources was varied from 10 signals to 1 signal in steps of 1 signal at different angle of arrival. The results of the DOA for $10, 9, 8, 7, 6, 5, 4, 3, 2$ *and* 1 signal sources are given in fig 4.14, fig 4.15, fig 4.16, fig 4.17, fig 4.18, fig 4.19, fig 4.20, fig 4.21, fig 4.22, and fig 4.23 respectively. This simulation is based on MUSIC algorithm. During the simulation of this experiment, the following signal and array parameters are kept constant: $\text{Signal frequency: Freq} = 2.4\text{GHz}, \text{snapshots: N} = 200, \text{SNR} = -5\text{dB}, \text{M} = 11$.

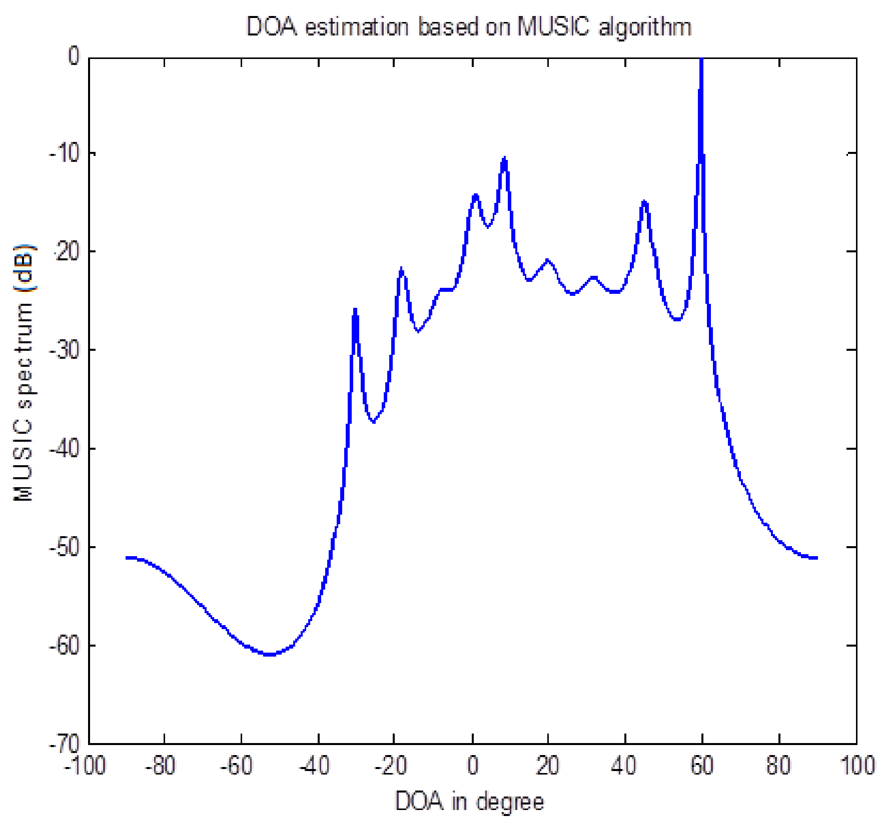

Fig 4.14: DOA estimation with 10 signal sources for ULA

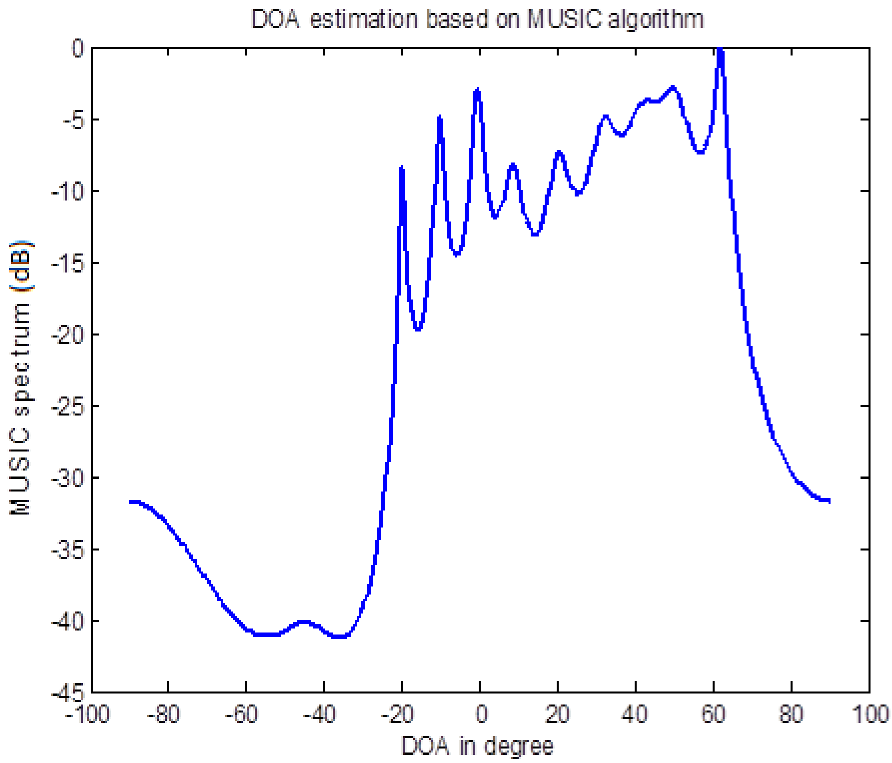

Fig 4.15: DOA estimation with 9 signal sources for ULA

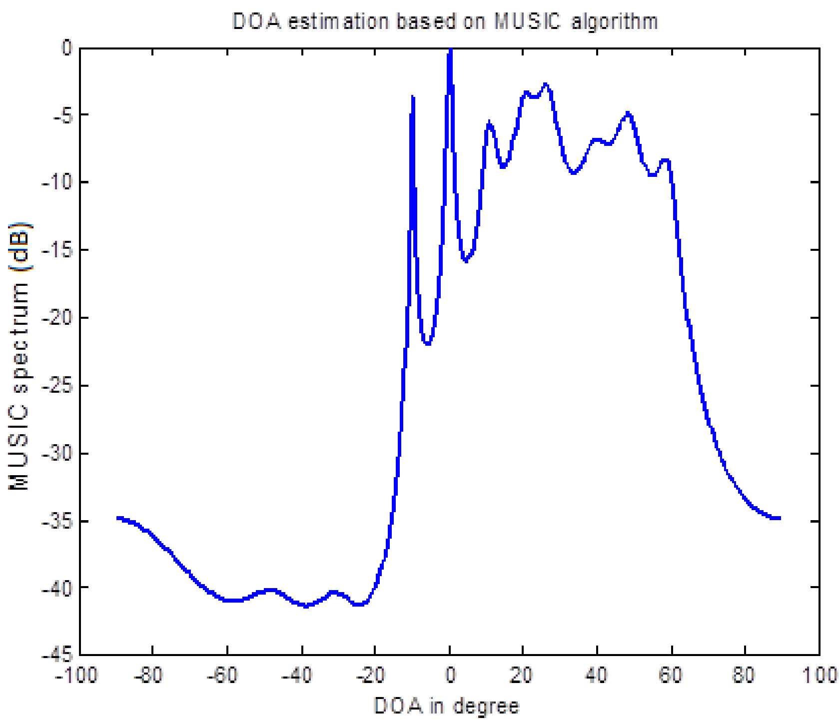

Fig 4.16: DOA estimation with 8 signal sources for ULA

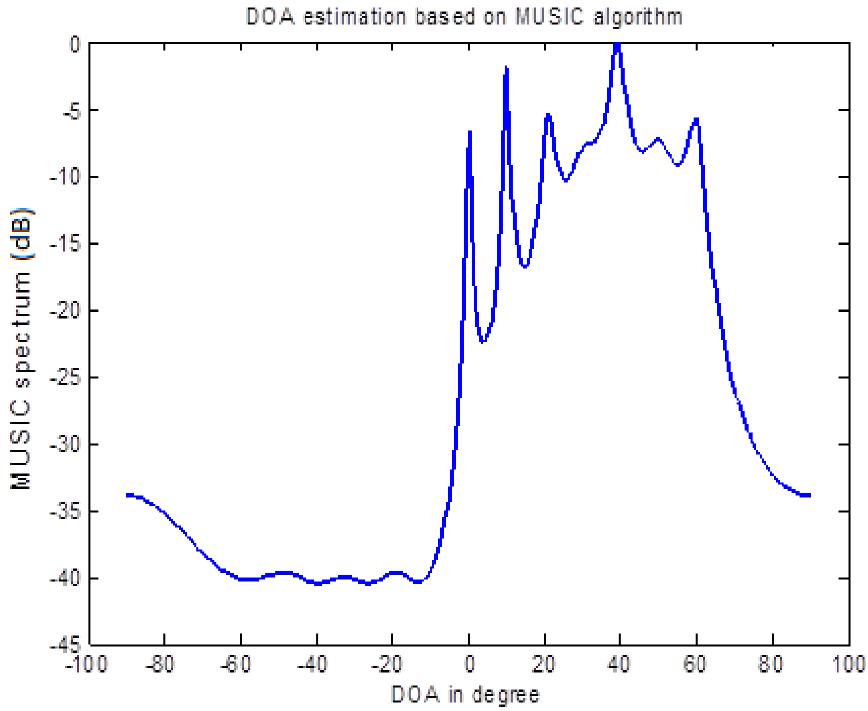

Fig 4.17: DOA estimation with 7 signal sources for ULA

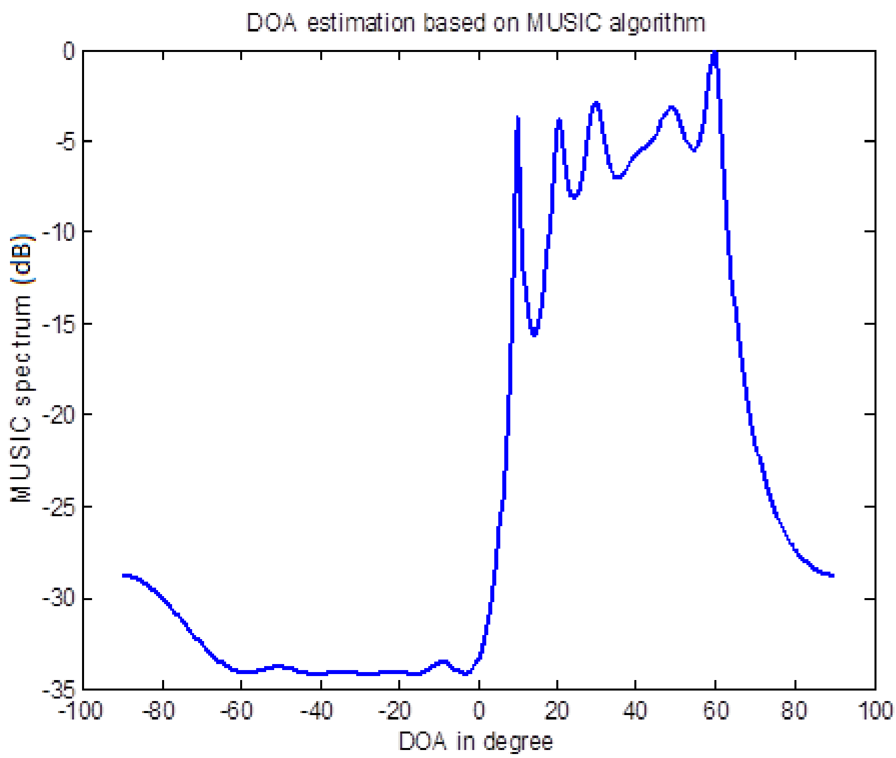

Fig 4.18: DOA estimation with 6 signal sources for ULA

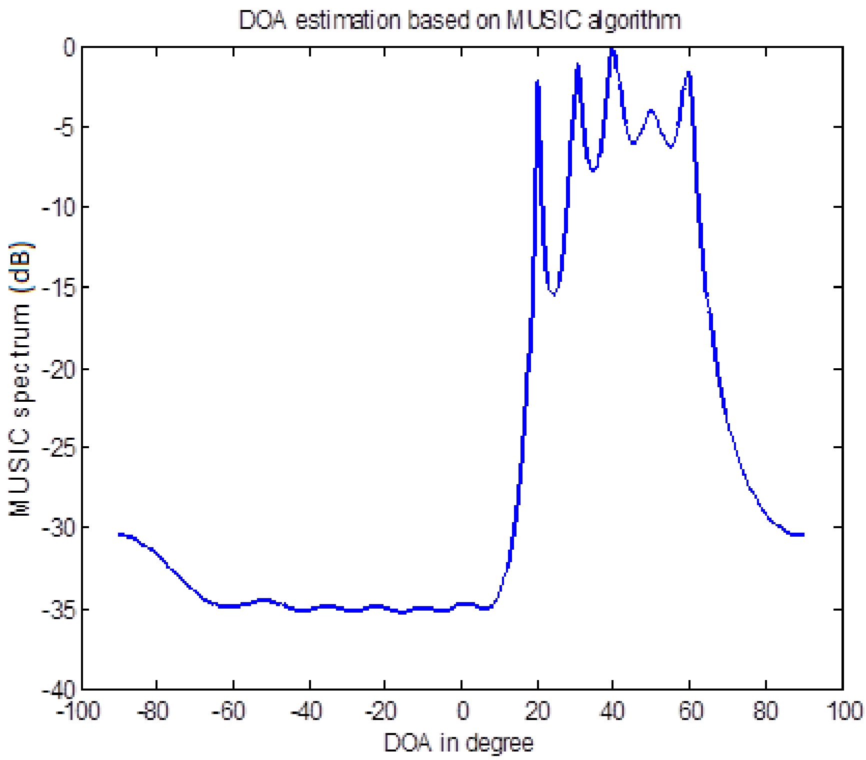

Fig 4.19: DOA estimation with 5 signal sources for ULA

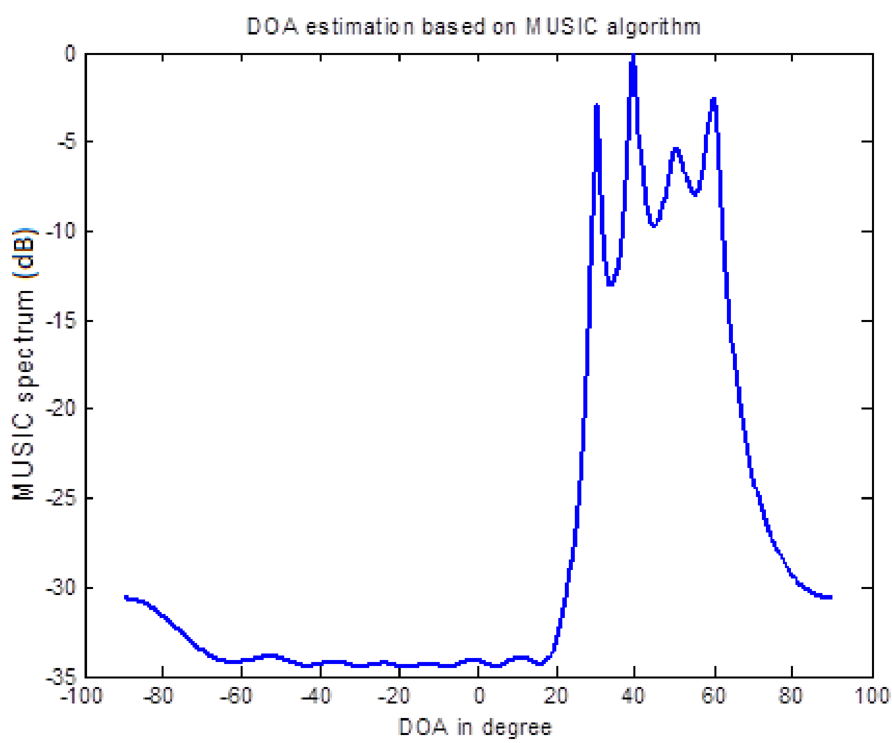

Fig 4.20: DOA estimation with 4 signal sources for ULA

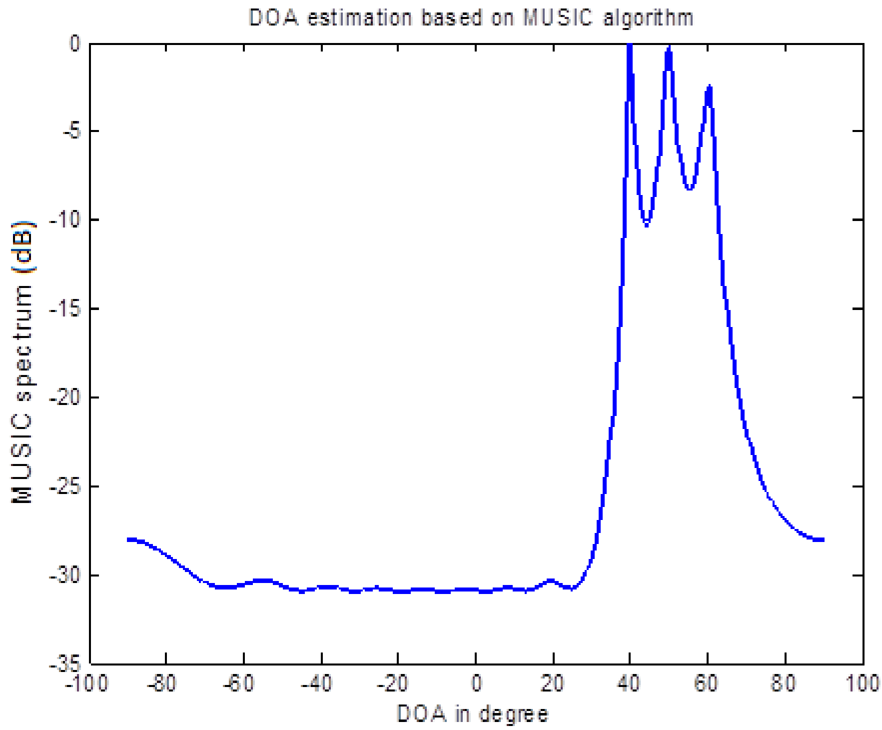

Fig 4.21: DOA estimation with 3 signal sources for ULA

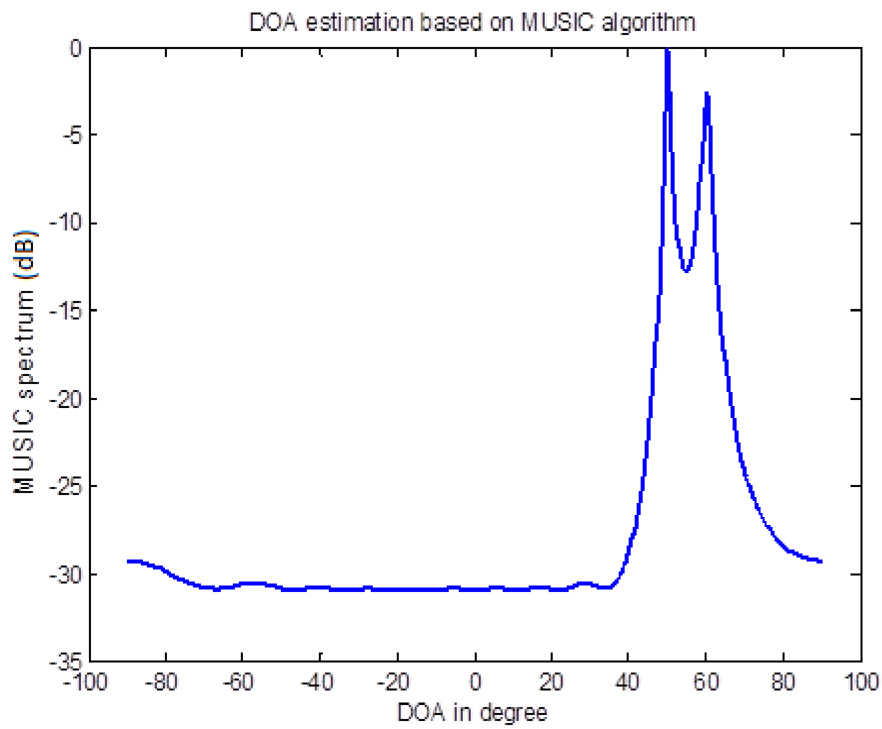

Fig 4.22: DOA estimation with 2 signal source for ULA

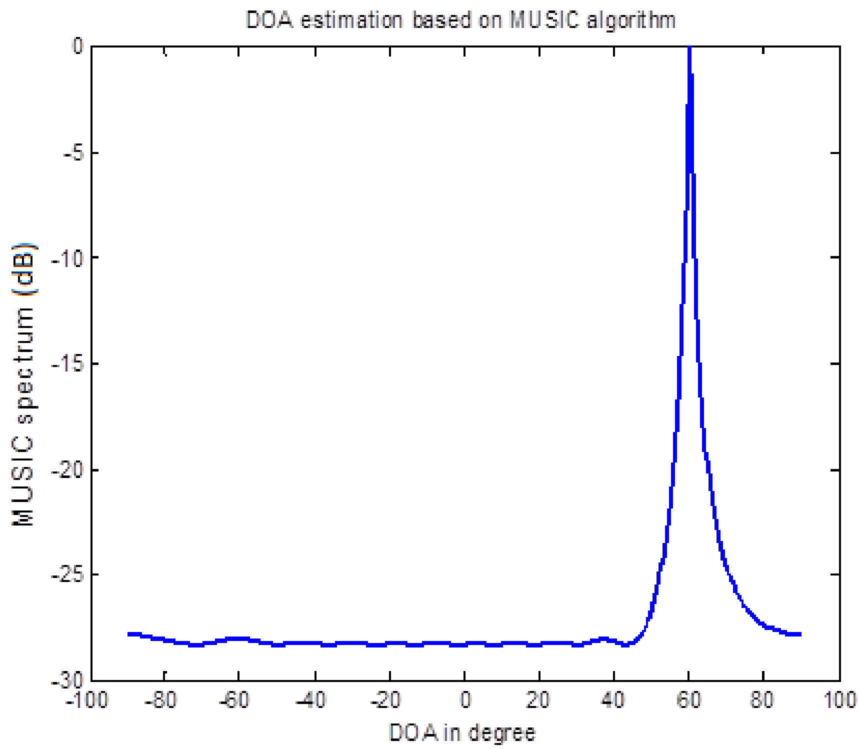

Fig 4.23: DOA estimation with 1 signal source for ULA

The results from fig 4.14, fig 4.15, fig 4.16, fig 4.17, fig 4.18, fig 4.19, fig 4.20, fig 4.21, fig 4.22 and fig 4.23 were given. It is noted that the maximum number of signal sources that can be detected by a uniform linear array is usually less than a number of array elements used in the DOA estimation system. For this case at an array size: $M = 11$ elements, the maximum detected signals are 10 signals.

The simulation for the non - uniform linear array, the signal sources were varied. The results of the DOA for $DECOM(7{,}5)$ are given are in fig. 4.24, fig. 4.25, fig. 4.26, fig. 4.27 and fig. 4.28 respectively. As explained in chapter 3, the non-uniform linear array is decomposed into two uniform linear sub-arrays to form a DECOM (7, 5) with array elements : $L = 7\ and\ J = 5$ elements.

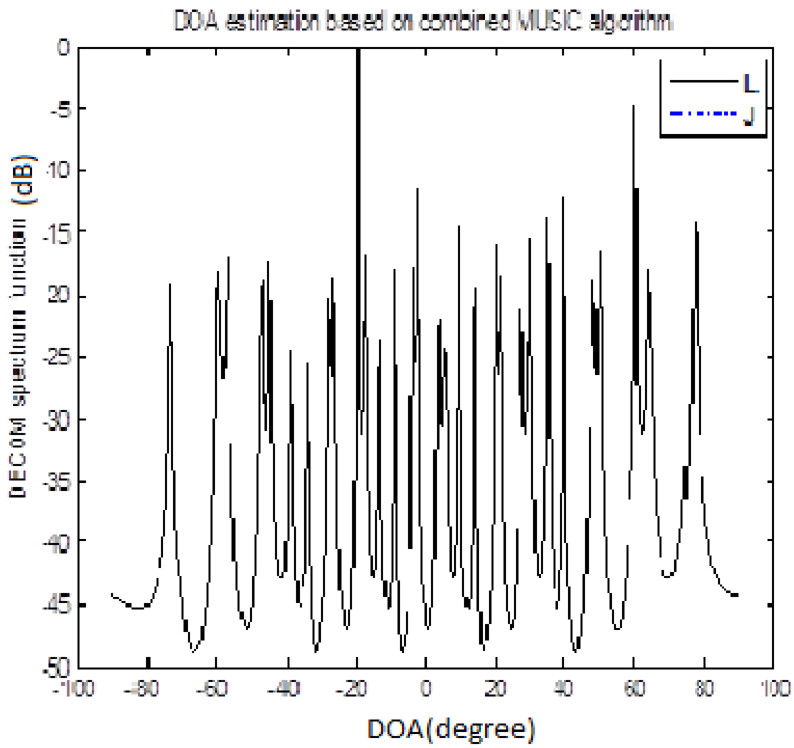

Fig 4.24: DOA estimation with 6 signal sources for DECOM (7, 5)

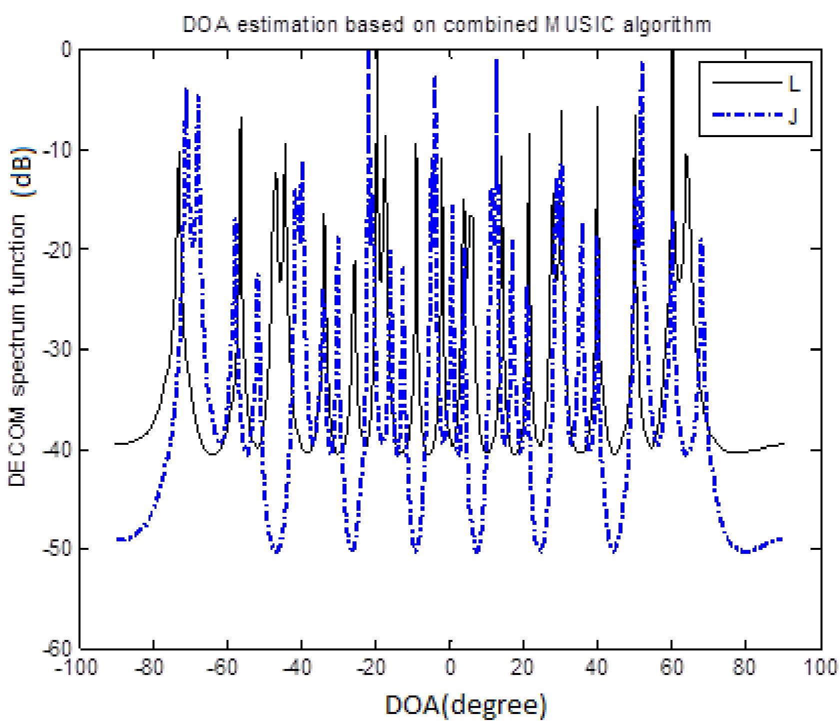

Fig 4.25: DOA estimation with 4 signal sources for DECOM (7, 5)

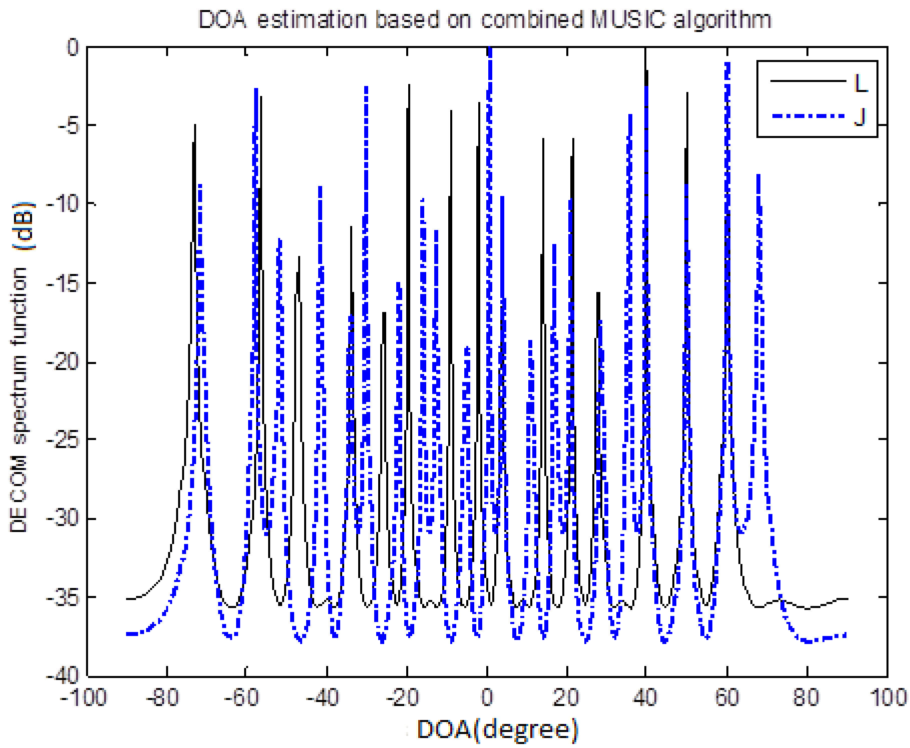

Fig 4.26: Direction of arrival with 3 signal sources for DECOM (7, 5)

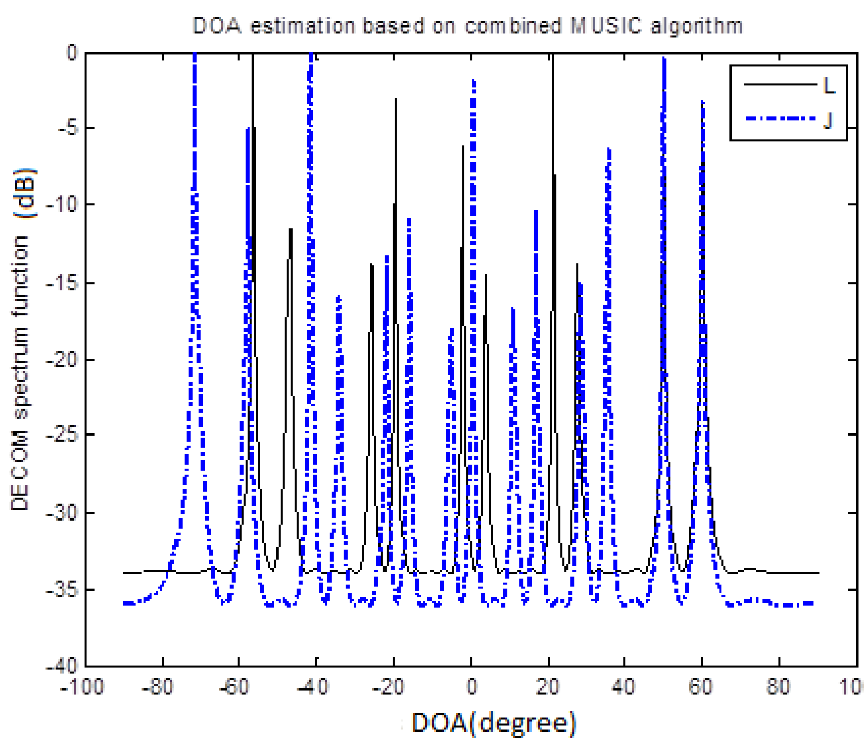

Fig 4.27: DOA estimation with 2 signal sources for DECOM (7, 5

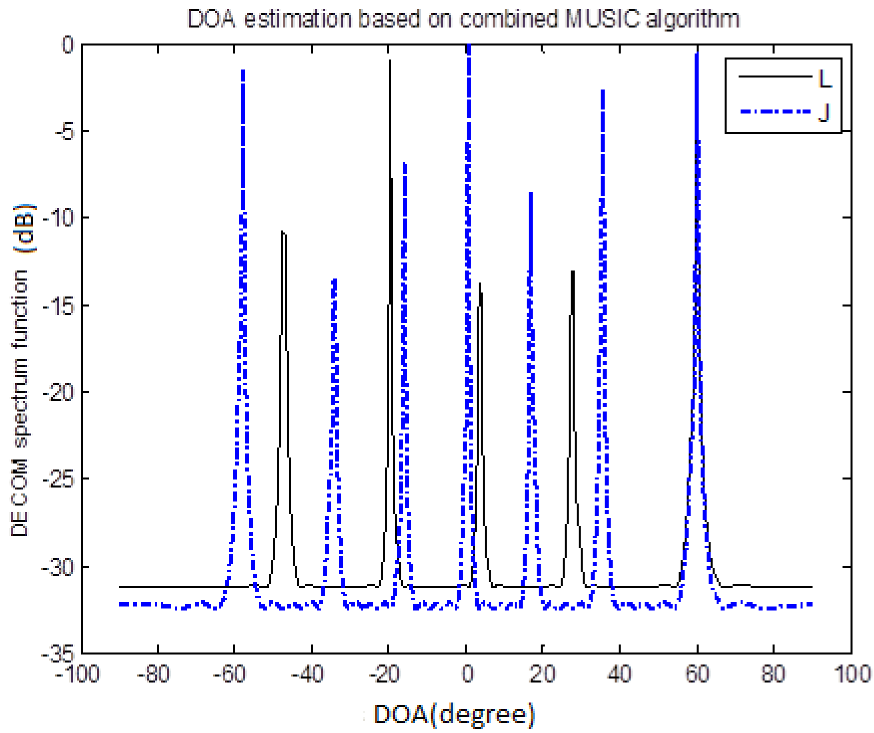

Fig 4.28: DOA estimation with 1 signal source for DECOM (7, 5)

The results from fig. 4.24, fig. 4.25, fig. 4.26, fig. 4.27 and fig. 4.28 were given. It is noted that the maximum number of signal source that can be detected by a non - uniform linear array is usually greater than a number of array elements used in the DOA estimation system. For instance at an array size: $DECOM(7{,}5)$, the maximum signals detected are 24 signals.

The experiment 5 shows the variation of DOA with the number of signal sources. One of the advantages of using MUSIC algorithm is that it is capable of detecting many signals. The results from fig 4.14, fig 4.15, fig 4.16, fig 4.17, fig 4.18, fig 4.19, fig 4.20, fig 4.21, fig 4.22 and fig 4.23 Show that the detected signal has to be less than the antenna elements using uniform linear array ($K < M$) as explained in chapter 3. When using non-uniform linear array, the fig 4.24, fig 4.25, fig 4.26, fig 4.27 and fig 4.28 shows that the signal detected is more than the array elements. The NLA is advantageous because it has more degree of freedom, it allows a larger inter-element spacing than a half wavelength ($d > \frac{\lambda}{2}$), and the signal in the noise background can be estimated in a more effective way.

4.6.Experiment 6: Simulation of DOA with Root-MUSIC compared to MUSIC

In experiment 6, the variation of the DOA estimation with the number of array elements is simulated. For uniform linear array, the number of array elements was varied from $M = 7$ elements to $M = 3$ elements in steps of 2 array elements at a very close angle difference. The results of the DOA for $M = 7$ and $M = 3$ elements are given in fig 4.29, fig 4.30, fig 4.31 and fig. 4.32 respectively. This simulation is based on Root-MUSIC algorithm compared to MUSIC algorithm. During the simulation of this experiment, the following signal and array parameters are kept constant:

Signal frequency: $\text{Freq} = 2.4\text{GHz}$, snapshots: $\text{N} = 100$, $\text{SNR} = -5\text{dB}$.

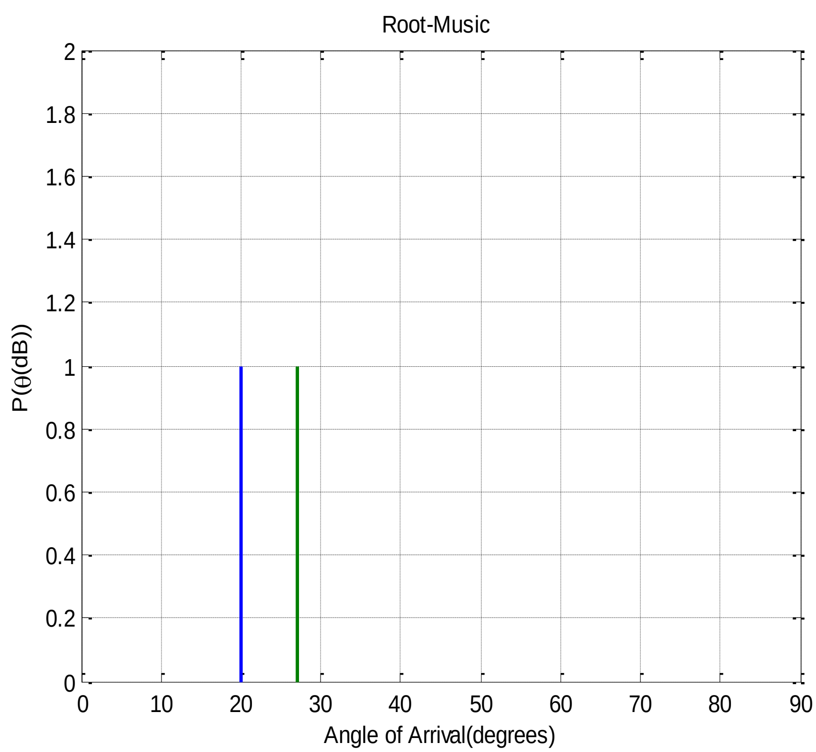

Fig 4.29: DOA estimation with M=7 and DOA= [20^0 27^0] using Root-MUSIC

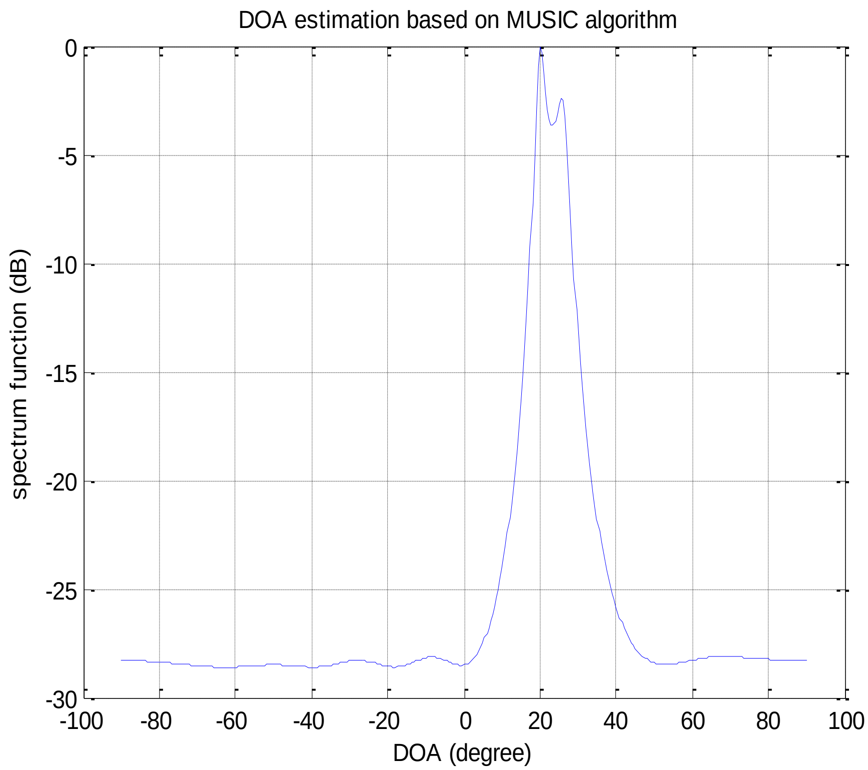

Fig 4.30: DOA estimation with M=7 and DOA= [20^0 27^0] using MUSIC

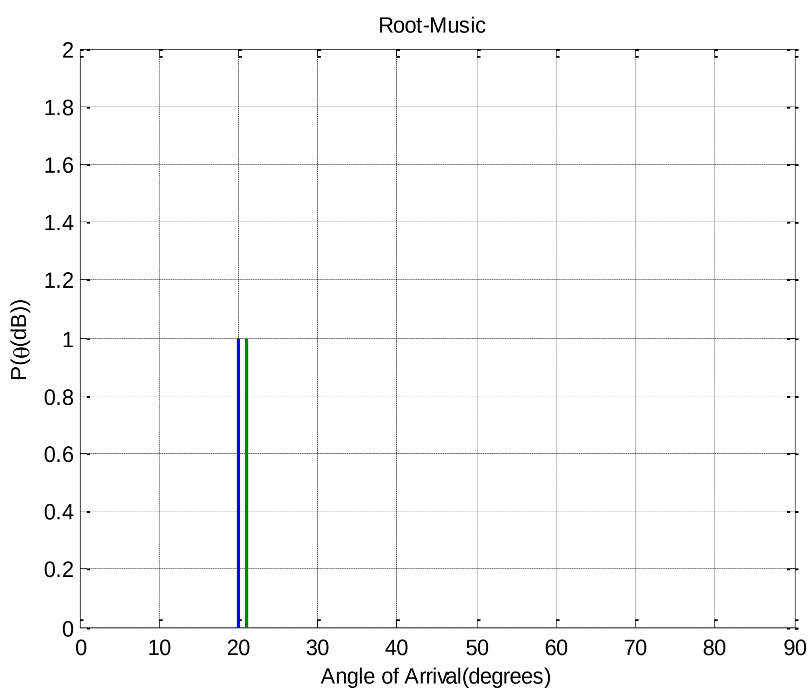

Fig 4.31: DOA estimation with M=3 and DOA= [20^0 21^0] using Root-MUSIC

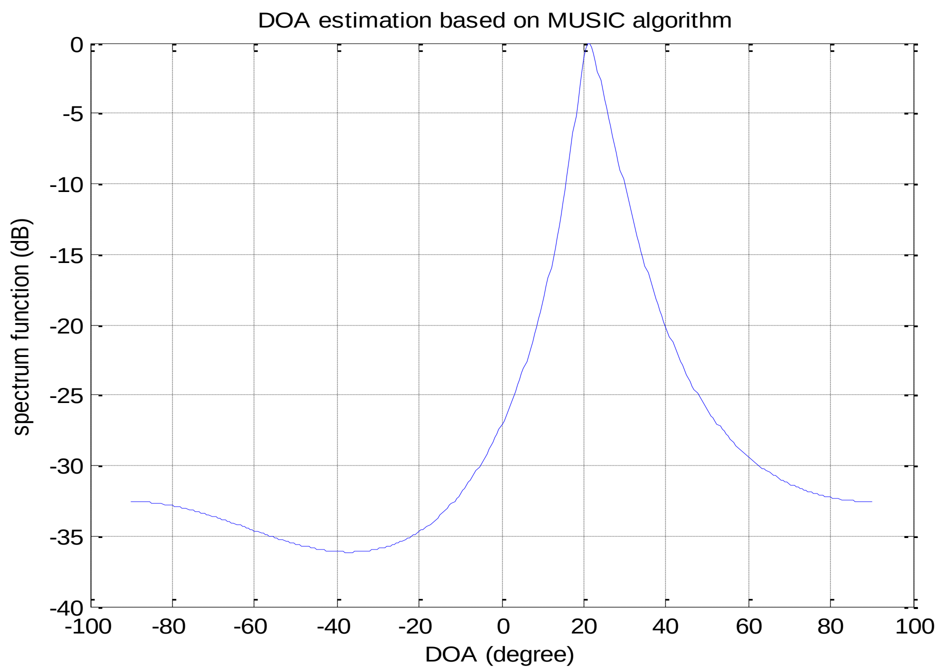

Fig 4.32: DOA estimation with M=3 and DOA= [20^0 21^0] using MUSIC

The results from fig 4.29 and fig 4.31 using Root-MUSIC are summarized as follow. It can be noted that with $M = 7$, the estimated DOA is $20°$ and $27°$ respectively. Also, with $M = 3$ the estimated DOA is $20.004°$ and $21.236°$ respectively. The peaks in the spectrum space correspond to the roots of the polynomial lying close to the unit circle. The results from fig 4.30 and fig 4.32 using MUSIC are summarized as follow. It can be noted that with $M = 7$, the estimated DOA is $20.098°$ and $28.543°$ respectively. Also, with $M = 3$ the estimated DOA is $21.106°$ and $21.539°$ respectively.

The experiment 6 gives the variation of array elements using Root-MUSIC algorithm. As explained in chapter 3 Root-MUSIC implies that the MUSIC algorithm is reduced to finding roots of a polynomial as opposed to merely plotting the pseudospectrum or searching for peaks in the pseudospectrum. It is observed that the Root-MUSIC algorithm is less accurate than MUSIC algorithm, but in some cases the results obtained with Root-MUSIC are acceptable. From fig 4.29 and fig 4.31, it can be noted that Root-MUSIC is able to perform well at very small number of array elements and at a very close signal means with small difference angle of arrival which is not the case for MUSIC algorithm.

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

5.1. Conclusion

The direction of arrival (DOA) plays a very big role in wireless communication systems particularly in array signal processing. It has many application in engineering fields such as radar, sonar, weather forecasting, tracking targets, ocean and geological exploration, seismic survey and biomedical, and communications in general.

The main idea in the DOA estimation is to use an array of antennas to receive a narrowband signal from a far field sources in the diverse directions. The signal received is then processed using a sub-space method that has a high resolution and an accurate DOA estimation. In the past, many directions of arrival (DOA) estimation algorithms have been proposed. Among them are non-subspace methods and subspace methods. In this thesis, the attention has been on Multiple Signal Classification (MUSIC) and Root-MUSIC algorithms because of their popularity, their high performance and mostly their low computational complexity compared to other techniques. Therefore, these algorithms were applied to uniform linear array (ULA) and to Non- uniform linear array NLA to check their performance on the accuracy issue.

The main contents of this research work are summarized as follows: First of all, the description of the DOA estimation and its development was defined. Secondly, the importance of using array antennas was provided, which is mainly to offer a high directivity of the narrowband signal. Thirdly, a mathematical model for a receiving vector matrix of both ULA and NLA was given. Thereafter, the six experiments were performed in MATLAB platform. An implementation of the MUSIC and Root-MUSIC techniques were done to check on factors that mostly affect accuracy such as antenna array elements, signal to noise ratio, number of snapshots, number of signal source, and the frequency of the signal. These input signal parameters were applied on uniform linear array and non-uniform linear array.

From the simulation results, it is clear that the MUSIC algorithm is more accurate and has a higher resolution. This occurs especially when there is a large size of antenna array, a higher SNR, a higher number of snapshots and a substantial difference between the incident angles of the signal source for the ULA. In addition, when comparing the uniform linear array to the non-uniform linear array using the Mean Square Error (MSE), the NLA shows a better estimate, a higher accuracy and higher resolution. Moreover, the NLA has more degree of freedom and allow a larger inter-element spacing than a half wavelength ($d > \frac{\lambda}{2}$) which is not the case for ULA. In addition, the autocorrelation of signals can be estimated in a much denser spacing other than the physically sparse sampling spacing, and sinusoids in noise can be estimated in a more effective way when using non-uniform linear array. The Non-uniform linear array is thus more advantageous than a uniform linear array.

Finally, the answer to some accuracy and directivity problems of direction of arrival estimation by using the MUSIC and Root-MUSIC algorithms has been given. The simulation experiments proved that the non-uniform linear array perform well than uniform linear array.

Briefly, this research work described what DOA estimation is, and gave a mathematical model of DOA estimation. Then, the analysis of the performance for uniform linear array (ULA) and non-uniform linear array (NLA) were given. The estimation DOA based on the multiple classification signal (MUSIC) and Root-MUSIC algorithms has been provided. An extensive simulation has been conducted in MATLAB and the results show that the NLA for co-prime array can achieve an accurate and efficient DOA estimation.

5.2. Recommendations for future work

Although the direction of arrival estimation theory and other technologies in array antennas settings have become well established some further investigations can be conducted as listed below:

(i) Currently, one dimension (1D) of non-uniform linear array setup issues is in place, but little research on two dimension (2D) arrays has been done. The two dimension investigation is more in accordance with the real-time environments where both elevation and azimuth angles are needed.

(ii) Direction of arrival estimation and array calibration can all contribute to the parameters optimization problems. Therefore, a faster algorithm for solving optimization functions is worthy of further research.

(iii) In the spatial spectrum algorithms, most of researchers are concerned with the reception of the narrowband, though there are other applications of the spatial spectrum estimation techniques such as the direction of arrival estimation for wideband signal, correction of path in multi-array conditions and error correction for broadband arrays. Particularly, for the array calibration and angle joint estimation of the parameters.

(iv) Multiple signal classification algorithm as one of the super high algorithms plays a big role in direction of arrival estimation computation, but it still has a large amount of computation complexity. Also enhancing the real time character and robustness will be an important part of the link for spatial spectrum estimation technology.

(v) Current research efforts have paid more attention in removing errors and ambiguities in the spatial spectrum from array amplitude, phase difference, mutual coupling, and positions errors. There are other factors such as near field scattering, electromagnetic interference, channel bandwidth inconsistency, non-uniform linear channel amplifier, quantization and quadrature sampling errors which impact on the estimated errors. All these need to be addressed.

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APPENDIX A: DERIVATION OF MUSIC AND ROOT-MUSIC ALGORITHMS

A.1. Derivation of Multiple Signal Classification (MUSIC) algorithm

Multiple signal classification is the most popular subspace technique based on exploiting the eigenstructure of the input covariance matrix. The direction of arrival of multiple narrowband source signals can be easily estimated by identifying the peaks of a MUSIC spatial spectrum.

If the received signal at sensor 1 is $x_1(t) = s(t)$

Then it is delayed at sensor *i* by: $\Delta_i = \frac{(i-1)dsin\theta}{c}$

The received signal at sensor *i* is $x_i(t) = e^{-j\omega\Delta_i}s_1(t) = e^{-j\omega\Delta_i}s(t) = e^{-j\omega\frac{(i-1)dsin\theta}{c}}s(t)$

The received signals at all *M* sensors together: $X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \dots \\ x_M(t) \end{bmatrix} = \begin{bmatrix} 1 \\ e^{-j\omega\frac{dsin\theta}{c}} \\ \dots \\ e^{-j\omega\frac{(M-1)dsin\theta}{c}} \end{bmatrix} s(t)$

$$= a(\theta)s(t)$$

$a(\theta)$ is called steering vector

If there are K source signals received by the array, a signal model is expressed as follow:

$X(t) = As(t) + n(t)$

$X(t)$ is a received signal vector, $s(t)$ is a source signal vector and $n(t)$ is a noise vector.

$A = [a(\theta_1) \quad \dots \quad a(\theta_K)]$

$s(t) = [s_1(t) \quad \dots \quad s_K(t)]^T$

$S_k(t) = s_k(t)\exp\{j\omega_k(t)\}$

$\omega_k = \omega_0 = \dfrac{2\pi c}{\lambda}$

$S_k(t - t_1) \approx s_k(t)$

$S_k(t - t_1) = s_k(t)\exp[j\omega_0(t - t_1)] = s_k(t)\exp[j\omega_0(t - t_1)]$

The output signal of the m^{th} element is:

$X_m(t) = \sum_{k=1}^{K} s_k(t)\exp\left[-j(m-1)\frac{2\pi d sin\theta_k}{\lambda}\right] + n_m(t)$

The output steering array vector is:

$a_k(t) = \exp\left[-j(m-1)\frac{2\pi d sin\theta_k}{\lambda}\right]$

Therefor the output signal of array element m^{th} is:

$X_m(t) = \sum_{k=1}^{K} s_k(t)\mathrm{a}_k(t) + n_m(t),$

This expression can be described by matrices:

$X = AS + N$

The autocorrelation of the received signal is:

$R_x = E[XX^H]$

$R_x = E[(As + N)(As + N)^H]$

$= AE[ss^H]A^H + E[NN^H]$

$= AR_SA^H + R_N$

Where $R_S = E[ss^H] = diag\{\sigma_1{}^2 \ \dots \ \sigma_K{}^2\}$, and $R_N = \sigma_0{}^2 I$

If $\lambda_i = \sigma_0{}^2$

For $M > K$, the matrix AR_SA^H is singular, $\det[AR_SA^H] = \det[R_x - \lambda_i \mathrm{I}] = 0$,

This implies that λ_i is an eigenvalue of R_x;

Since both R_x and AR_SA^H, there are K other eigenvalues λ_i such that $\lambda_i > \lambda_0 > 0$.

Let e_i be the i^{th} eigenvalues of R_x corresponding to λ_i

$$R_x e_i = [AR_S A^H + \lambda_i I]e_i = \lambda_i e_i; \quad i = 1, 2, \dots, M$$

$$\lambda_i > \lambda_0 > 0; \quad i = 1, 2, \dots, K$$

$$\lambda_i = \lambda_0; \quad i = K + 1, K + 2, \dots, M$$

This implies $AR_S A^H e_i = (\lambda_i - \lambda_0)e_i, \quad i = 1,2, \dots, M$

$$AR_S A^H e_i = \begin{cases} (\lambda_i - \lambda_0)e_i, & i=1,2,\dots,K \\ 0, & i=K+1,\dots,M \end{cases}$$

Partition the *M*- dimensional vector space into the signal subspace E_s and the noise subspace E_n, the signal subspace is orthogonal to the noise subspace. This implies that $a^H(\theta)E_N {E_N}^H a(\theta) = 0$.

Thus, MUSIC algorithm searches through all angles θ, and plots the "Spatial spectrum"

$$P_{music}(\theta) = \frac{1}{a^H(\theta)E_N {E_N}^H a(\theta)}$$

The peaks detection give spatial angles of all incident sources.

A.2. Derivation of Root-MUSIC algorithm

Root-MUSIC is an improved method of MUSIC algorithm which involves finding the roots of a polynomial.

Starting with the pseudospectrum of MUSIC algorithm

$$P_{music}(\theta) = \frac{1}{a^H(\theta)E_N {E_N}^H a(\theta)}$$

Defining $C = E_N {E_N}^H$, the denominator of $P_{music}(\theta)$ can be rewritten as

$$P_{Root-MUSIC}(\theta) = \frac{1}{a^H(\theta)Ca(\theta)}$$

$C = E_N {E_N}^H$

The steering vector at sensor m^{th} is defined as:

$a_m(\theta) = \exp(-jkd(m-1)\sin(\theta))$

with $m = 1,2,\ldots,M$

the denominator, thus can be rewritten as:

$$a^H(\theta)Ca(\theta) = \sum_{m=1}^{M}\sum_{n=1}^{M} \exp(-jkd(m-1)\sin(\theta))\, C_{mn} \exp(jkd(n-1)\sin(\theta))$$

$$a^H(\theta)Ca(\theta) = \sum_{l=-M+1}^{M-1} C_l \exp(jkdl\sin(\theta))$$

Where C_l is the sum of the elements along lth diagonal of C

Letting $z = ex\,p(-jkdsin\theta)$, then the latter expression can be simplified to:

$$D(z) = \sum_{l=-M+1}^{M+1} C_l\, z^{-l}$$

Where $z^{-l} = \exp(jkdlsin(\theta))$

The roots of $D(z)$ that lie closest to the unit circle correspond to the poles of the Root-MUSIC pseudospectrum. These $2(M-1)$ roots can be rewritten as $z_i = |z_i| \exp\big(jarg(z_i)\big)$

with $i = 0,2,\ldots,2(M-1)$.

Choosing those roots inside the unit circle whose magnitude $|z_i| \approx 1,$ and comparing $\exp(jarg(z_i))$ to $\exp(-jkdsin(\theta))$ gives

$$\theta_i = -sin^{-1}\left(\frac{1}{kd}\arg(\, z_i)\right)$$

APPENDIX B: MATLAB PROGRAMS

B.1. Variation of DOA with snapshots for ULA

```
clc
clear all
format long

% Done by KWIZERA EVA on 11 April, 2016
% Department of Electrical Engineering, Option of Telecommunication Engineering
% Pan African University
%--------------------------------------------------------------------------------------------------------------
%THE SIMULATION OF DIRECTION OF ARRIVAL WITH SNAPSHOTS USING MUSIC
%ALGORITHM FOR ULA. THE MULTIPLE SIGNAL CLASSIFICATION ALGORITHM
%AIMS AT VARYING THE NUMBER OF SNAPSHOTS TO CHECK ON ACCURACY
%AND RESOLUTION OF THE SYSTEM.
%--------------------------------------------------------------------------------------------------------------
% with 100, 500 and 1000 snapshots
% M is the array size with 11 elements
% Direction of Arrival (DOA) are 60 and 68 degrees
%spacing between elements is d
% p is the number of signal
% w is the angular frequency
% steering vector is A
% the simulated signal is S
% The incident signal is X
% The covariance matrix is noted R
% NN is the estimated noise subspace

N1 = 100;
N2 = 500;
```

```
N3 = 1000;
doa =  [60 68]/180*pi;
w = [pi/4 pi/3]';
M = 11;
P = length (w);
Lambda = 150;
d = lambda/2;
snr = -5;
D = zeros (P,M);
for k = 1:P
        D(k,:)=exp(-j*2*pi*d*sin(doa(k))/lambda*[0:M-1]);
end

D = D';
xx1 = 2*exp(j*(w*[1:N1]));
xx2 = 2*exp(j*(w*[1:N2]));
xx3 = 2*exp(j*(w*[1:N3]));
x1 = D*xx1;
x2 = D*xx2;
x3 = D*xx3;
x1 = x1+awgn(x1,snr);
x2 = x2+awgn(x2,snr);
x3 = x3+awgn(x3,snr);
R1 = x1*x1';
R2 = x2*x2';
R3 = x3*x3';
[N1,V1] = eig (R1);
[N2,V2] = eig (R2);
[N3,V3] = eig (R3);
NN1 = N1 (:,1:M-P);
NN2 = N2 (:,1:M-P);
```

```
NN3 = N3 (:,1:M-P);
theta = -90:0.5:90;
for ii = 1:length (theta)
        SS = zeros (1,length(M));
        for jj = 0:M-1
                SS(1+jj) = exp (-j*2*jj*pi*d*sin(theta(ii)/180*pi)/lambda);
        end
        PP1 = SS*NN1*NN1'*SS';
        PP2 = SS*NN2*NN2'*SS';
        PP3 = SS*NN3*NN3'*SS';
        Pmusic1(ii) = abs(1/ PP1);
        Pmusic2(ii) = abs(1/ PP2);
        Pmusic3(ii) = abs(1/ PP3);
end

Pmusic1 = 10*log10 (Pmusic1/max(Pmusic1));
Pmusic2 = 10*log10 (Pmusic2/max(Pmusic2));
Pmusic3 = 10*log10 (Pmusic3/max(Pmusic3));
plot (theta,Pmusic1,'b','linewidth',2.0)
hold on
plot(theta,Pmusic2,'g','linewidth',1.0)
hold on
plot(theta,Pmusic3,'r','linewidth',0.1)
hold on
legend ('N=100','N=500','N=1000')
xlabel('angle \theta/degree')
ylabel('spectrum function P(\theta) /dB')
title('DOA estimation based on MUSIC algorithm')
grid off
```

B.2. Variation of DOA with signal to noise ratio for ULA

```
clc
clear all
format long

% Done by KWIZERA EVA on 11th April, 2016
%Department of Electrical Engineering, Option of Telecommunication Engineering
%Pan African University
%-----------------------------------------------------------------------------------------------------------
%SIMULATION OF THE DIRECTION OF ARRIVAL WITH SIGNAL TO NOISE RATIO
%USING MUSIC ALGORITHM FOR ULA. THE MULTIPLE SIGNAL CLASSIFICATION
%ALGORITHM AIMS AT VARYING THE SIGNAL TO NOISE RATIO TO CHECK ON
%ACCURACY AND RESOLUTION OF THE SYSTEM.
%-----------------------------------------------------------------------------------------------------------

% with 11 elements, 2 signals arriving at 60 and 68 degrees
% M is the array size (number of elements)
% Direction of Arrival (DOA) are 60 and 68 degrees
% number of snapshots is 100
% spacing between elements is d
% p is the number of signal
% w is the angular frequency
% steering vector is A
% the simulated signal is S
% the incident signal is X
% the covariance matrix is noted R
% NN is the estimated noise subspace

M = 11;
snr1 = -5;
snr2 = 0;
```

```
snr3 = 5;
 lambda = 0.125;
d = lambda/2;
N = 100;
doa = [60 68]/180*pi;
w = [pi/4 pi/3]';
p = length(w);
A = zeros(p,M);

for k = 1:p
    A(k,:) = exp(-j*2*pi*d*sin(doa(k))/lambda*[0:M-1]);
end

A = A';
S = 2*exp(j*(w*[1:N]));
X1 = A*S;
X2 = A*S;
X3 = A*S;
X1 = X1+awgn(X1,snr1);
X2 = X2+awgn(X2,snr2);
X3 = X3+awgn(X3,snr3);
R1 = X1*X1';
R2 = X2*X2';
R3 = X3*X3';
[N1,V] = eig(R1);
[N2,V] = eig(R2);
[N3,V] = eig(R3);
NN1 = N1(:,1:M-p);
NN2 = N2(:,1:M-p);
NN3 = N3(:,1:M-p);
theta = -90:0.5:90;
```

```
for ii = 1:length(theta)
   SS = zeros(1,length(M));

   for jj = 0:M-1
      SS(1+jj) = exp(-j*2*jj*pi*d*sin(theta(ii)/180*pi)/lambda);
   End

   PP1 = SS*NN1*NN1'*SS';
   PP2 = SS*NN2*NN2'*SS';
   PP3 = SS*NN3*NN3'*SS';
   Pmusic1(ii) = abs (1/PP1);
   Pmusic2(ii) = abs (1/PP2);
   Pmusic3(ii) = abs (1/PP3);
end

Pmusic1 = 10*log10(Pmusic1/max(Pmusic1));
Pmusic2 = 10*log10(Pmusic2/max(Pmusic2));
Pmusic3 = 10*log10(Pmusic3/max(Pmusic3));
plot(theta,Pmusic1,'b','LineWidth',2.0)
hold on
plot(theta,Pmusic2,'--k','LineWidth',2.0)
hold on
plot(theta,Pmusic3,'-.r','LineWidth',2.0)
hold on
legend ('SNR=-5','SNR=0','SNR=5')
xlabel('DOA (degree)')
ylabel('MUSIC spectrum')
title('DOA estimation based on MUSIC algorithm')
grid off
```

B.3. Variation of DOA with array elements for ULA

```
clc
clear all
format long %The data show that as long shaping science and technology

%Done by KWIZERA EVA on 11th April, 2016
%Department of Electrical Engineering, Option of Telecommunication Engineering
%Pan African University
%-------------------------------------------------------------------------------------------------------------
%THE SIMULATION OF THE DIRECTION OF ARRIVAL WITH ARRAY ELEMENTS
%USING MUSIC ALGORITHM FOR ULA. THE MULTIPLE SIGNAL CLASSIFICATION
%ALGORITHM AIMS AT VARYING THE NUMBER OF ARRAY ANTENNA ELEMENTS
%TO CHECK ON ACCURACY AND RESOLUTION OF THE SYSTEM.
%-------------------------------------------------------------------------------------------------------------
% with 3, 6 and 11 elements, -5 dB signal to noise ratio (snr)
% M is the number of elements
% direction of arrival (doa) are 60 and 68 degrees
% number of snapshots is 100
% spacing between elements is d
% p is the number of signal
% w is the frequency
% steering vector is A
% the simulated signal is S
% the incident signal is X
% the covariance matrix is noted R
% NN is the estimated noise subspace

N = 100; %Snapshots
doa = [60 68]/180*pi; %DOA
w = [0.8 1]'; %Frequency
```

```
M1 = 3; %Array elements number
M2 = 6;
M3 = 11;
P = length (w); %Number of signal
Lambda = 150; %Wavelength
d = lambda/2; %Array element spacing
snr = 20; %SNR
D1 = zeros (P,M1);
D2 = zeros (P,M2);
D3 = zeros (P,M3);

for k = 1:P
    D1(k,:) = exp(-j*2*pi*d*sin(doa(k))/lambda*[0:M1-1]); %Assignment matrix
    D2(k,:) = exp(-j*2*pi*d*sin(doa(k))/lambda*[0:M2-1]);
    D3(k,:) = exp(-j*2*pi*d*sin(doa(k))/lambda*[0:M3-1]);
end
D1 = D1';
D2 = D2';
D3 = D3';
xx = 2*exp(j*(w*[1:N])); %Simulate the signal
x1 = D1*xx;
x2 = D2*xx;
x3 = D3*xx;
x1 = x1+awgn(x1,snr); %Add Gaussian white noise
x2 = x2+awgn(x2,snr);
x3 = x3+awgn(x3,snr);
R1 = x1*x1'; %Data covariance matrix
R2 = x2*x2';
R3 = x3*x3';
[N1,V1] = eig(R1); %Find the eigenvalues and eigenvectors of R
[N2,V2] = eig(R2);
```

```
[N3,V3] = eig(R3);
NN1 = N1(:,1:M1-P); ; %Estimate the noise subspace
NN2 = N2(:,1:M2-P);
NN3 = N3(:,1:M3-P);
theta = -90:0.5:90;
%% Search the peak

for ii = 1:length(theta)
        SS1 = zeros(1,length(M1));
        for jj = 0:M1-1
                SS1(1+jj) = exp(-j*2*jj*pi*d*sin(theta(ii)/180*pi)/lambda);
        end
PP1 = SS1*NN1*NN1'*SS1';
Pmusic1(ii) = abs(1/ PP1);
end
for ii = 1:length(theta)
        SS2 = zeros(1,length(M2));
for jj = 0:M2-1
        SS2(1+jj) = exp(-j*2*jj*pi*d*sin(theta(ii)/180*pi)/lambda);
end
PP2 = SS2*NN2*NN2'*SS2';
Pmusic2(ii) = abs(1/ PP2);
end
for ii = 1:length(theta)
        SS3 = zeros(1,length(M3));
        for jj = 0:M3-1
                SS3(1+jj) = exp(-j*2*jj*pi*d*sin(theta(ii)/180*pi)/lambda);
        end
        PP3=SS3*NN3*NN3'*SS3';
        Pmusic3(ii)=abs(1/ PP3);
end
```

```
Pmusic1 = 10*log10(Pmusic1/max(Pmusic1)); %Spatial spectrum function
Pmusic2 = 10*log10(Pmusic2/max(Pmusic2));
Pmusic3 = 10*log10(Pmusic3/max(Pmusic3));
plot(theta,Pmusic1,'b','LineWidth',2.0)
hold on
plot(theta,Pmusic2,'--k','LineWidth',2.0)
hold on
plot(theta,Pmusic3,'-.r','LineWidth',2.0)
hold on
legend ('M=3','M=6','M=11')
xlabel('angle \theta/degree')
ylabel('spectrum function P(\theta) /dB')
title ('DOA estimation based on MUSIC algorithm')
grid off
```

B.4. Variation of DOA using Root-MUSIC for ULA

```
clc
clear all
format long

% Done by KWIZERA EVA on 02nd May, 2016
% Department of Electrical Engineering, Option of Telecommunication
% Pan African University
%------------------------------------------------------------------------------------------------------------
% THE SIMULATION OF DIRECTION OF ARRIVAL USING ROOT-MUSIC ALGORITHM
% ON ULA. THE ROOT-MUSIC ALGORITHM AIMS AT VARYING THE NUMBER OF
% ARRAY ANTENNA ELEMENTS ESPECIALLY WITH FEW ELEMENTS AND WITH
% CLOSE INCIDENT SIGNAL TO CHECK THEIR ACCURACY AND RESOLUTION OF
% THE DOA.
%------------------------------------------------------------------------------------------------------------
```

```
% spacing between elements is d
% w is the frequency
% steering vector is A
% the incident signal is X
% the covariance matrix is noted R
% NN is the estimated noise subspace

w = [pi/4 pi/3]';
P = length(w);
lambda = 150;
d = lambda/2;
snr = input ('The Channel Signal To Noise Ratio : ') ;
N = input ('Enter the number of snapshots: ');
M = input('Enter the number of elements: ');
S = input('Enter the number of signal sources: ');

if    S >= M
     disp ('invalid')
else
    disp ('Enter the angles of arrival')
    for k = 1:S;
        doa(k) = input('Enter the angle: ');
        doa(k) = doa(k)/180*pi;
    end
end
D = zeros(P,M);
for k = 1:P
   D(k,:) = exp(-j*2*pi*d*sin(doa(k))/lambda*[0:M-1]);
end
D = D';
xx = 2*exp(j*(w*[1:N]));
```

```
x = D*xx;
x = x+awgn(x,snr);
R = x*x';
[N,V] = eig(R);
NN = N(:,1:M-P);
C = NN*NN';
[k,l] = size(C);
L = l;
T = zeros(1,2*P-1);
j = 1;
for i = 0:M-1;
        T(j) = trace(C(M-i:M,1:i+1));
        j = j+1;
end
%the coefficients of the polynomial
for i = 1:M-1;
    T(2*M-1) = conj(T(i));
end
% z=exp(kd*sin(theta(m)))
Rx = roots(T);
b = -1:1/10000:1;%to plot a circle
c = sqrt(1-b.^2);%to plot a circle
[r,index] = sort(abs((abs(Rx)-1)));%sorting the roots of the polynomial
rr = Rx(index(1:2*(M-1)));%corresponding roots
rrr = rr(1:2*S);
[u,v] = cart2pol(real(rrr),imag(rrr));
 w = angle(rrr);
 thera = rad2deg(doa' );
 theta = asin(w/pi);
 THETA = rad2deg(theta);
 doa = sort(THETA);
```

```
doa = doa(1:2*S);
[y,z] = size(rrr);
plot(doa,ones(1,y),'s','MarkerSize',10,'MarkerEdgeColor','k','MarkerFaceColor','b');
grid
O = [thera';thera'];
P = [ones(1,S); zeros(1,S)];
n =1:S;
line(O(:,n),P(:,n),'LineWidth',2);
grid
title('Root-Music ');
axis([0 90 0 2]);
xlabel('Angle of Arrival(degrees)')
ylabel('P(\theta(dB))');
grid off
```

B.5. Simulation of DOA for DECOM

```
clc
clear all
format long

%Done by KWIZERA EVA on 03rd August, 2016
%Department of Electrical Engineering, Option of Telecommunication
% Pan African University
%-----------------------------------------------------------------------------------------------------------------
%THE SIMULATION OF DIRECTION OF ARRIVAL ESTIMATION FOR A NON-
%UNIFORM LINEAR ARRAY. THE MULTIPLE SIGNAL CLASSIFICATION
%ALGORITHM AIMS AT PROVING HOW ACCURATE IS THE NLA WHEN
%ESTIMATING THE DOA.
%-----------------------------------------------------------------------------------------------------------------
```

```
% with DECOM (7,5), signal to noise ratio (snr) of -5dB and 2 signals arriving at 60 and 68
%degrees
% L and J are the number of array elements for both ULA respectively
% direction of arrival (doa) are 60 and 68 degrees
% number of snapshots is 100
% spacing between elements is d1 and d2
% p is the number of signal
% w is the frequency
% steering vector are A1 and A2
% the simulated signal is S
% the incident signal is X
% the covariance matrix is noted R1 and R2
% NN is the estimated noise subspace
 N = 100; %Snapshots
doa = [60 68]/180*pi; %DOA
w = [pi/4 pi/3]'; %Frequency
L = 7; %Array elements number
J = 5;
%M3=10;
P = length(w); %Number of signal
lambda = 0.125; %Wavelength
d1 = J*lambda/2; %Array element spacing
d2 = L*lambda/2;
snr = -5;%SNR
%snr2 = -5;
D1 = zeros(P,L);
D2 = zeros(P,J);
%D3=zeros(P,M3);
 for k = 1:P
        D1(k,:) = exp(-j*pi*J*sin(doa(k))*[0:L-1]); %Assignment matrix
        D2(k,:) = exp(-j*pi*L*sin(doa(k))*[0:J-1]);
```

```
    %D3(k,:)=exp(-j*2*pi*d*sin(doa(k))/lambda*[0:M3-1]);
end
 D1 = D1';
D2 = D2';
%D3=D3';
xx = 2*exp(j*(w*[1:N])); %Simulate the signal
x1 = D1*xx;
x2 = D2*xx;
%x3=D3*xx;
x1 = x1+awgn(x1,snr); %Add Gaussian white noise
x2 = x2+awgn(x2,snr);
%x3=x3+awgn(x3,snr);
R1 = x1*x1'; %Data covariance matrix
R2 = x2*x2';
%R3=x3*x3';
[N1,V1] = eig(R1); %Find the eigenvalues and eigenvectors of R1
[N2,V2] = eig(R2);
%[N3,V3]=eig(R3);
NN1 = N1(:,1:L-P); ; %Estimate the noise subspace
NN2 = N2(:,1:J-P);
%NN3=N3(:,1:M3-P);
theta = -90:0.5:90;
%%Search the peak
for ii = 1:length(theta)
    SS1 = zeros(1,length(L));
     for jj = 0:L-1
        SS1(1+jj) = exp(-j*2*jj*pi*d1*sin(theta(ii)/180*pi)/lambda);
    end
    PP1 = SS1*NN1*NN1'*SS1';
    Pmusic1(ii) = abs(1/ PP1);
end
```

```
for ii = 1:length(theta)
        SS2 = zeros(1,length(J));
        for jj = 0:J-1
                SS2(1+jj) = exp(-j*2*jj*pi*d2*sin(theta(ii)/180*pi)/lambda);
        end
        PP2 = SS2*NN2*NN2'*SS2';
        Pmusic2(ii) = abs(1/ PP2);
End
% for ii=1:length(theta)
% SS3=zeros(1,length(M3));
% for jj=0:M3-1
% SS3(1+jj)=exp(-j*2*jj*pi*d*sin(theta(ii)/180*pi)/lambda);
% end
% PP3=SS3*NN3*NN3'*SS3';
% Pmusic3(ii)=abs(1/ PP3);
% end
Pmusic1=10*log10(Pmusic1/max(Pmusic1)); %Spatial spectrum function
Pmusic2=10*log10(Pmusic2/max(Pmusic2));
% Pmusic3=10*log10(Pmusic3/max(Pmusic3));
plot(theta,Pmusic1,'k','LineWidth',2.0)
%legend('Pmusic1')
hold on
plot(theta,Pmusic2,'-.b','LineWidth',2.0)
legend('L=7','J=5')
hold on
% plot(theta,Pmusic3,'r','LineWidth',0.1)
hold off
xlabel('DOA(degree)')
ylabel('DECOM spectrum function')
title('DOA estimation based on combined MUSIC algorithm')
grid off
```

RESEARCH ARTICLE

PERFORMANCE EVALUATION OF DIRECTION OF ARRIVAL ESTIMATION USING UNIFORM AND NON-UNIFORM LINEAR ARRAYS

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Abstract - In signal processing, the direction of arrival (DOA) estimation denotes the direction from which a propagating wave arrives at a point, where a set of antennas is located. Using the array antenna has an advantage over the single antenna in achieving an improved performance by applying Multiple Signal Classification (MUSIC) algorithm. This paper focuses on estimating the DOA using uniform linear array (ULA) and non-uniform linear array (NLA) of antennas to analyze the performance factors that affect the accuracy and resolution of the system based on MUSIC algorithm. The direction of arrival estimation is simulated on a MATLAB platform with a set of input parameters such as array elements, signal to noise ratio, number of snapshots and number of signal sources. An extensive simulation has been conducted and the results show that the NLA with DOA estimation for co-prime array can achieve an accurate and efficient DOA estimation.

Keywords: Co-prime array, Direction of Arrival (DOA) estimation, Multiple Signal Classification (MUSIC), Non-Uniform Linear Array (NLA), Uniform Linear Array (ULA).

1. Introduction

In the past decades, many array models such as the uniform linear array (ULA), uniform circular array, uniform rectangular array, and non-uniform linear array (NLA) have been utilized towards the achievement of the direction of arrival (DOA) estimation [1]. To get an accurate DOA estimation of the narrowband signal sent from a far field source to a receiving array antenna can increase the wireless communication systems capacity [2]. Therefore, putting effort on improving DOA estimation methods is a key to developing the quality of wireless networks.

In many practical signal processing problems, the received narrowband signal depends on a set of constant parameters. The objective of this paper is to estimate those parameters for the effectiveness of the systems. Many methods have been employed to solve such problems among them the maximum likelihood (ML) method of Capon (1969) and Burg's maximum entropy (ME) method which are among the non-subspace techniques [3, 4]. Although they have been often successful and widely used, these methods have certain fundamental limitations especially the bias and sensitivity in parameter estimates, largely because they use an incorrect model of the measurements [5]. Then, Schmidt in 1979 corrected the measurement model in the case of sensor arrays of arbitrary form and proposes the subspace technique termed Multiple Signal Classification (MUSIC) algorithm. This algorithm is based on exploiting the eigenstructure of the input covariance matrix. The direction of arrival of multiple source signals can be easily estimated by identifying the peaks of a MUSIC spatial spectrum [6].

Most of the methods used to locate the DOA consider the spacing distance between two adjacent array elements to be a half wavelength. However in wireless communication, there are some cases where such half wavelength minimum spacing is not applicable; for instance many parabola antennas, their physical size are designed to have a large size for enhanced directivity. Also, in an array that operates over a wide spectrum; for example, over-the-horizon radar (OTHR) is a unique radar system that performs wide area surveillance by exploiting the reflective and refractive nature of high-frequency radio wave propagation through the ionosphere [7].

Recently, a non-uniform linear array in a form of co-prime array has been proposed

[8]. Its most remarkable property is that it increases the degrees of freedom. In addition, the autocorrelation of signals can be estimated in a much denser spacing other than the physically sparse sampling spacing, and sinusoids in noise can be estimated in a more effective way. Due to the useful properties of the NLA, its importance has been realized and has been the object of research in the last few years. Some researchers such as Pal and Vaidyanathan in [9], [10] proposed a new method for a super resolution spectral estimation from the perspective of degree of freedom increase. Further to these efforts, Weng and Petar in [11] proposed a search-free DOA algorithm for co-prime arrays by using a projection-like method to eliminate the phase ambiguities for obtaining the unique estimation of DOA.

In this paper, a method of estimation of DOA using uniform and non-uniform linear array antennas is proposed. It underlies the factors that affect the accuracy and resolution of the system based on multiple signal classification (MUSIC) algorithm. The performance evaluation of the MUSIC algorithm under ULA and NLA is given.

The remainder of this paper is organized as follows. In Section 2, the signal model of the uniform linear array is presented. In Section 3, the description of a detailed MUSIC algorithm implementation is given. Section 4, shows a non-uniform linear array with combined MUSIC algorithm. Simulation and discussion of the results are presented in Section 5. Finally, conclusion and recommendation for future work is highlighted in Section 6.

2. Signal model of uniform linear array

Consider a linear array antenna with M antenna elements that are equally spaced with a distance d that is strictly equal to a half wavelength ($d = \frac{\lambda}{2}$). Assume that there are narrowband signal sources (K) located at $\theta_1, \theta_2, \dots, \theta_K$ with signal powers $\sigma_1^2, \sigma_2^2, \dots, \sigma_K^2$. The incident signals and the noise are uncorrelated. In addition the incident signals are themselves uncorrelated. Let the number of signal sources be less than the number of antenna elements ($K < M$). The steering vector for k^{th} source located at θ_k is with $a(\theta_k) \in \mathbb{C}^{M\times 1}$ with k^{th} element $e^{j(2\pi/\lambda)d_l \sin(\theta_k)}$, where d_l is the antennas location and λ is the wavelength as shown in the fig 1.

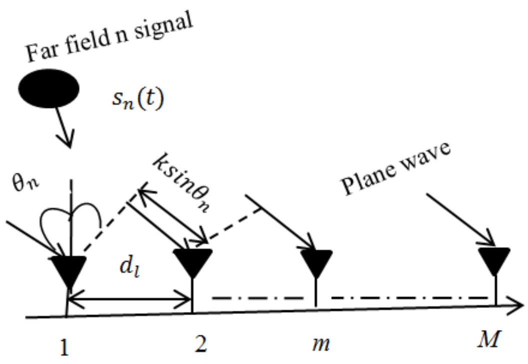

Fig. 2. A plane wave incident on a uniform linear array antenna

The signal collected by all antennas at time can be expressed as:

$$X(t) = As(t) + n(t) \quad (1)$$

Where $X(t) = [x_1(t)\, x_2(t) \ldots x_M(t)]^T$ is a vector received by the array antenna, $A = [a(\theta_1)\ a(\theta_2) \ldots a(\theta_K)]$ is a steering vector. Its expression is shown in (2), $s(t)$ is the signal vector generated by the source, $n(t) = [n_1(t)\, n_2(t)\ \ldots n_M(t)\,]^T$ and is the additive white Gaussian noise.

$$a(\theta_k) = \begin{bmatrix} 1 & 1 & \ldots & 1 \\ e^{-j\varphi_1} & e^{-j\varphi_2} & \ldots & e^{-j\varphi_K} \\ \ldots & \ldots & \ldots & \ldots \\ e^{-j\varphi_1(M-1)} & e^{-j\varphi_2(M-1)} & \ldots & e^{-j\varphi_K(M-1)} \end{bmatrix} \quad (2)$$

$$where\ \varphi_k = \frac{2\pi d}{\lambda} sin\theta_k$$

The narrowband signal from (1) can be expressed as:

$$S_k(t) = s_k(t)\exp\{j\omega_k(t)\}, \quad (3)$$

Where $s_k(t)$ is the complex envelope of $S_k(t)$ and $\omega_k(t)$ is the angular frequency of $S_k(t)$.

As assumed all signals have the same center frequency, therefore:

$$\omega_k = \omega_0 = \frac{2\pi c}{\lambda} \quad (4)$$

Where c is the speed of the light and λ is the wavelength of the signal.

According to the narrowband assumption, the following approximation is valid:

$$S_k(t - t_1) \approx s_k(t) \quad (5)$$

The delayed wavefront signal is:

$$S_k(t - t_1) = s_k(t)\exp[j\omega_0(t - t_1)] = s_k(t)\exp[j\omega_0(t - t_1)] \quad (6)$$

The output signal of the m^{th} element is:

$$X_m(t) = = \sum_{k=1}^{K} s_k(t)\exp\left[-j(m-1)\frac{2\pi dsin\theta_k}{\lambda}\right] + n_m(t) \quad (7)$$

From (6) the output steering array vector is:

$$a_m(t) = \exp\left[-j(m-1)\frac{2\pi dsin\theta_k}{\lambda}\right] \quad (8)$$

3. MUSIC algorithm implementation

Multiple signal classification is a subspace technique based on exploiting the eigenstructure of input covariance matrix [12]. Through a set of input parameters such as number of array elements, number of snapshots, element spacing, angular separation, signal-to-noise ratio and MUSIC algorithm, the DOA is estimated with high resolution. This leads to high quality of wireless communication.

The autocorrelation of the received signal is:

$$R_x = E[XX^H] \tag{9}$$

Where H is the conjugate transpose matrix. The noise is assumed to be zero mean and additive white Gaussian and is uncorrelated to the signal.

Replacing (1) into (9), the covariance matrix is obtained:

$$\begin{aligned} R_x &= E[(As+N)(As+N)^H] \\ &= AE[ss^H]A^H + E[NN^H] \\ &= AR_SA^H + R_N \end{aligned} \tag{10}$$

Where $R_S = E[ss^H]$ is called source signal correlation matrix, $R_N = \sigma^2 I$ is a noise correlation matrix.

If $(\lambda_1, \lambda_2, \ldots .\lambda_M)$ are eigenvalues of spatial correlation matrix R_x; Then the performance of eigenvalue associated with a particular eigenvector is given as:

$$R_x - \lambda_i \mathrm{I} = 0 \tag{11}$$

$$AR_SA^H + \sigma^2 I - \lambda_i \mathrm{I} = 0 \tag{12}$$

$$AR_SA^H + (\sigma^2 - \lambda_i)\mathrm{I} = 0 \tag{13}$$

Therefore, eigenvectors v_i of AR_SA^H are obtained using (14);

$$v_i = \sigma^2 - \lambda_i \tag{14}$$

The eigenvalues are sorted according to their magnitudes $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M \geq 0$. The larger eigenvalues correspond to signal subspace, while small eigenvalues corresponds to noise. The largest eigenvalues are all values greater than K that is $(M - K)$, and the rest are all values that are less than K. Thus, MUSIC "Spatial spectrum" in [12] is defined as:

$$P_{music}(\theta) = \frac{1}{a^H(\theta)E_N{E_N}^H a(\theta)} \tag{15}$$

The multiple signal classification (MUSIC) algorithm can be summarized as follow:

Step 1: The data is collected to form the correlation matrix R_x.

Step 2: The eigenstructure of the covariance matrix R_x is decomposed.

Step3: Let the number of signal sources be K.

Step 4: N columns are chosen to form the noise subspace E_N.

Step 5: P_{music} versus θ is evaluated.

Step 6: The spectrum function is determined; then an estimation of DOA is obtained by the peak-searching k.

4. Non-uniform linear array with combined MUSIC algorithm

A system with the array antennas linearly spaced, the different array elements spacing and the signal phase differences is considered. In this case, one non-uniform linear array can be decomposed into two uniform linear arrays as shown in fig. 2 and fig. 3 respectively. The direction of arrival is estimated by combining MUSIC results from the two co-prime arrays.

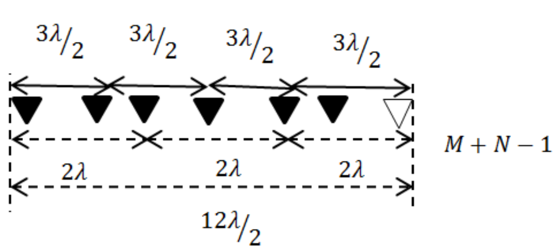

Fig 2. Co-prime non-uniform linear array

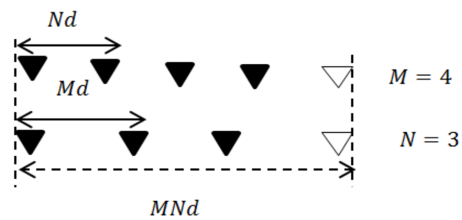

Fig 3. Two co-prime uniform linear arrays

The received signal vector of each sub-array at the *t-th* time slot can be defined as:

$$X_M(t) = A_M s(t) + n(t) \tag{16}$$

$$X_N(t) = A_N s(t) + n(t) \tag{17}$$

The array output vector of the co-prime array is given by:

$$Y(t) = A(t)s(t) + n(t) \tag{18}$$

According to the far-field assumption, the steering vector corresponding to the k^{th} source is:

$$a_k = \left[1, e^{-j\frac{2\pi}{\lambda}d_1 \sin(\theta_k)}, \dots, e^{-j\frac{2\pi}{\lambda}d_i \sin(\theta_k)}, e^{-j\frac{2\pi}{\lambda}d_{(M+N-1)} \sin(\theta_k)}\right]^T \tag{19}$$

Where $d_l(l = 1,2,\dots,M+N-1)$

Steering arrays of each linear array are given by:

$$a_{Mk} = \left[1, e^{-j\pi N sin(\theta_k)}, \dots, e^{-j\pi(M-1)N sin(\theta_k)}\right]^T \tag{20}$$

$$a_{Nk} = \\ = [1, e^{-j\pi M sin(\theta_k)}, \dots, e^{-j\pi(N-1)M sin(\theta_k)}]^T \tag{21}$$

By obtaining each sample of covariance matrices from (16) and (17), the two decomposed uniform linear sub-arrays give:

$$R_{xM} = E[X_M(t)X_M(t)^H] \tag{22}$$

$$R_{xN} = E[X_N(t)X_N(t)^H] \tag{23}$$

Replacing the received signal vector of each sub-array (16) and (17) by their values, the following is obtained:

$$R_{xM} = E[(A_M s(t) + n(t))(A_M s(t) + n(t))^H] \\ = A_M E(s(t)s(t)^H) {A_M}^H + E[NN^H] \\ = \\ A_M R_{SM} {A_M}^H + R_N \tag{24}$$

$$R_{xN} = E[(A_N s(t) + n(t))(A_N s(t) + n(t))^H] \\ = A_N E(s(t)s(t)^H) {A_N}^H + E[NN^H] \\ = \\ A_N R_{SN} {A_N}^H + R_N \tag{25}$$

Where $R_{SM} = E[s(t)s(t)^H]$ and $R_{SN} = E[s(t)s(t)^H]$ are source signal correlation matrices of subarrays M and N respectively, $R_N = \sigma^2 I$ is a noise correlation matrix for each subarray.

If $(\lambda_1, \lambda_2, \dots .\lambda_M)$ and $(\lambda_1, \lambda_2, \dots .\lambda_N)$ are eigenvalues of the spatial correlation matrix R_{xM} and R_{xN}, then the performance of eigenvalue associated with a particular eigenvector for each sub-array is given as:

$$R_x - \lambda_i \mathrm{I} = 0 \tag{26}$$

$$A_M R_{SM} {A_M}^H + (\sigma^2 - \lambda_i)\mathrm{I} = 0 \tag{27}$$

$$A_N R_{SN} {A_N}^H + (\sigma^2 - \lambda_i)\mathrm{I} = 0 \tag{28}$$

The eigenvectors v_i of $A_M R_{SM} {A_M}^H$ and $A_N R_{SN} {A_N}^H$ from (24) and (25) are obtained using (28) whereby v_i is:

$$v_i = \sigma^2 - \lambda_i \tag{29}$$

Therefore applying eigen-decomposition to the sample covariance matrices in (24) and (25) yields:

$$R_{xM} = E_{sM} \Lambda_{sM} {E_{sM}}^H + E_{nM} \Lambda_{nM} {E_{nM}}^H \tag{30}$$

$$R_{xN} = E_{sN} \Lambda_{sN} {E_{sN}}^H + E_{nN} \Lambda_{nN} {E_{nN}}^H \tag{31}$$

Then, according to the orthogonality between the signal subspace and the noise subspace of each sub-array, the MUSIC spatial pseudo-spectrum of the two decomposed linear subarrays M and N respectively, are:

$$P_{MUSIC_M}(\theta) = \frac{1}{a_M(\theta)^H E_{nM} E_{nM}{}^H a_M(\theta)} \qquad (32)$$

$$P_{MUSIC_N}(\theta) = \frac{1}{a_N(\theta)^H E_{nN} E_{nN}{}^H a_N(\theta)} \qquad (33)$$

5. Simulation results and discussion

a. Introduction

The simulation of MUSIC algorithm is carried out on a MATLAB platform. This section is subdivided into four parts.

Firstly, the simulation results are validated using published data in the cited literature.

Secondly in experiment 1, a uniform linear array (ULA) of 13 elements ($M = 13$) is assumed. The same frequency used in wireless communication (Wi-Fi) which is 2.4 GHz (Freq. = 2.4GHz) is considered, with 200 snapshots ($N = 200$), SNR = -5dB and the two uncorrelated estimated signals arriving at angular direction of 60^0 and 68^0, ($DOA = [60° \ 68°]$) at ULA are used.

Thirdly in experiment 2, the simulation with two superimposed uniform linear arrays with the same inputs parameters as in the experiment 1 are assumed.

Lastly in experiment 3, a simulation result of a non-uniform linear array (NLA) is given. The same input parameters such as number of array element, signal to noise ratio, number of snapshots, and number of arriving signal and at the same frequency are considered.

For the first two experiments, the distance between two adjacent antenna elements is assumed to be $d = \frac{\lambda}{2}$; then the third experiment the distance greater than a half wavelength $d > \frac{\lambda}{2}$ is assumed. In all cases, the Gaussian noise is assumed to be zero mean and white and is uncorrelated to the signal. Also, the direction of arrival θ_k is assumed to be within the angle interval $(-\frac{\pi}{2}, \frac{\pi}{2})$.

Here the idea is to vary SNR and number of array element to analyze the performance results of the three experiments and then check on the factors that affect the accuracy and also compare both ULA and NLA in terms of the resolution and accuracy of the system.

b. Validation of results

The simulation results were justified by using existing data in the literature. The dashed lines in fig.4 are results represented by Dhering [7] and the continuous lines are the present work with the increased signal to noise ratio. As observed, there is a good agreement between the simulated results and the published data by Dhering.

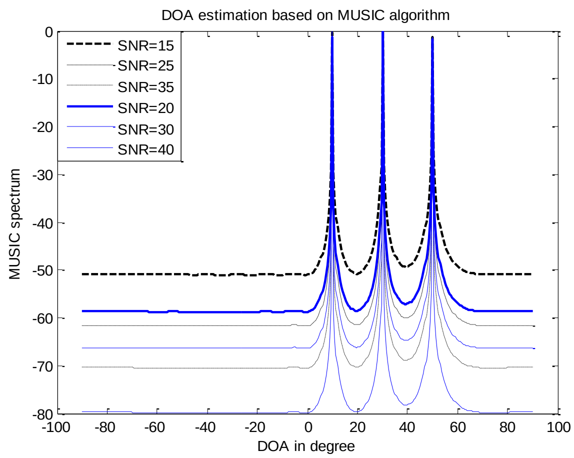

Fig 4. MUSIC spectrum for SNR variation

b. Experiment 1

In this experiment computer simulation, results were obtained using one uniform linear array based on MUSIC algorithm for the input parameter set: $\theta_1 = 60^{\circ}, \theta_2 = 68^{\circ}, SNR = -5dB, M = 13, \quad N = 100, Freq = 2.4GHz, k = 2, d = \frac{\lambda}{2}$. The results are given in fig.5.

c. Experiment 2

In this experiment computer simulation, results were obtained using two uniform linear arrays based on MUSIC algorithm for the input parameter set: $\theta_1 = 60^{\circ}, \theta_2 = 68^{\circ}, SNR = -5dB, L = 9, J = 5, N = 100, Freq = 2.4GHz, k = 2, d = \frac{\lambda}{2}$. The results are given in fig.6.

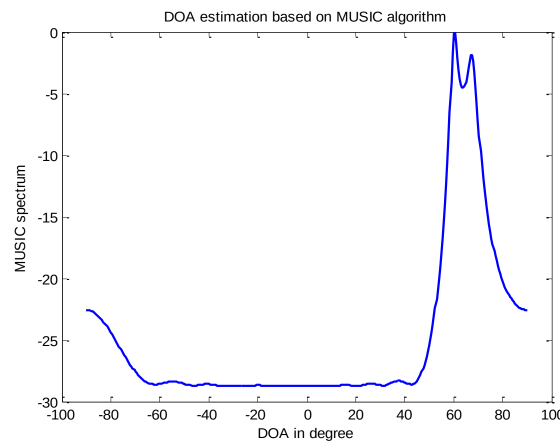

Fig 5. DOA estimation based on MUSIC algorithm for one ULA

d. Experiment 3

In the third experiment, the simulation results of a non-uniform linear array were obtained for the input parameter set: $\theta_1 = 60^{\circ}, \theta_2 = 68^{\circ}, SNR = -5dB, L = 9, J = 5, N = 100, Freq = 2.4GHz, k = 2, d > \frac{\lambda}{2}$. The results are given in fig.7.

e. Discussion

The multiple signal classification method gives a high resolution and is accurate for DOA estimation especially when the number of array elements, the signal to noise ratio and the number of snapshots are high.

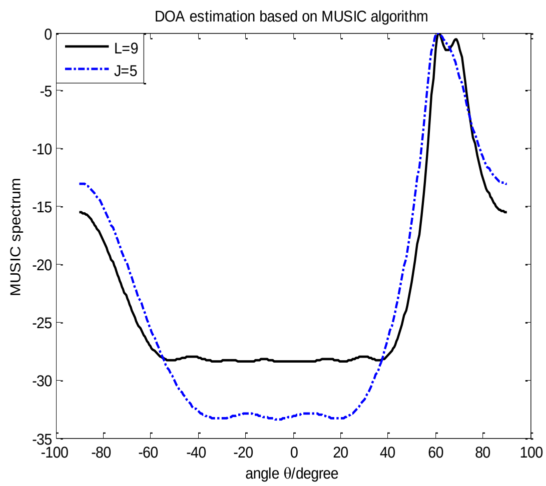

Fig 6. DOA estimation based on MUSIC algorithm for two ULA

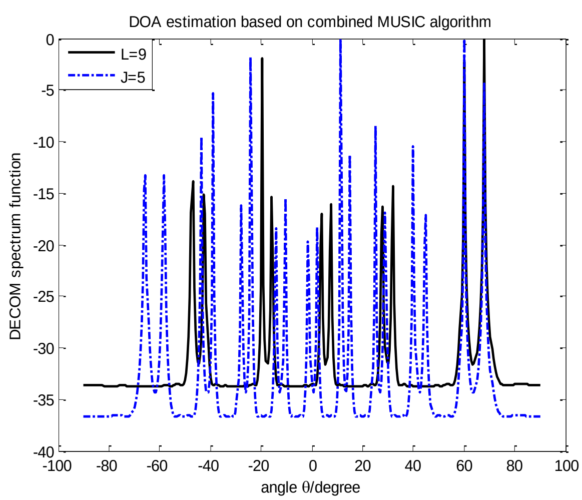

Fig 7. DOA estimation based on combined MUSIC algorithm for non-uniform linear array

However, MUSIC strength varies from one array arrangement to another as indicated in table 1. The simulations experiments were carried out with 13 array elements for one ULA, two superimposed uniform linear arrays of 9 elements and 5 elements, and DECOM (9,5) for two decomposed uniform linear array which form a non-uniform linear array. We assume the DOA at 60^0 and 68^0 in the angle interval $(-\frac{\pi}{2},\frac{\pi}{2})$. Therefore, the observed direction of arrival estimation is shown in Table I.

Table I: variation of DOA estimation via different array arrangement

DOA	Array elements			
	13	9 and 5		(9,5)
		9	5	
60^0	60.5^0	61^0	62^0	60^0
68^0	67.5^0	66.5^0	62^0	68^0

From the observation in experiment 1, Using one ULA of 13 elements ($M = 13$), with the true angles of arrival ($\theta_1 = 60^{\circ}, \theta_2 = 68^{\circ}$), the estimated direction of arrival is 60.5° and 67.5° ($\hat{\theta}_1 = 60.5^{\circ}, \hat{\theta}_2 = 67.5^{\circ}$). Thus, fig. 5 shows that the system is not enough to give an exact estimate. Though it is shown that the more array antennas the better is the resolution.

In experiment 2 simulation, the use of 9 elements and 5 elements [$L = 9,\ J = 5$] for superimposed ULAs is used with the true value of the angle of arrival is $\theta_1 = 60^{\circ}, \theta_2 = 68^{\circ}$. Thus, the observe direction of arrival estimation from L is 61° and

$66.5^{\circ}(\hat{\theta}_1 = 61°, \hat{\theta}_2 = 66.5°)$, and from J is 62° and $62°$ $(\hat{\theta}_1{}' = 62°, \hat{\theta}_2{}' = 62°)$. As fig.6 shows that the higher the number of array element of antennas the more narrow is the signal. Also, the higher the SNR the more accurate the DOA estimation and the higher the resolution of the system as well.

In experiment 3 simulation, the use DECOM (9, 5) that is $[L = 9,\ J = 5]$ in fig. 7. When applying the same characteristics as in the first experiment with the true value of the angle of arrival is $\theta_1 = 60^{\circ}, \theta_2 = 68^{\circ}$, the observed direction of arrival estimation from L is 60° and $68^{\circ}(\hat{\theta}_1 = 60°, \hat{\theta}_2 = 68°)$, then from J is 60° and $68°$ $(\hat{\theta}_1{}' = 60°, \hat{\theta}_2{}' = 68°)$. Thus, the performance of a non-uniform linear array of a decomposed ULA for the corresponding co-prime array with combined MUSIC algorithm has an exact direction of arrival estimation and increases the degree of freedom as shown in fig. 7. This shows how high resolution and accurate is the non-uniform linear array which contribute in improving the wireless communication capacity.

f. Conclusion

In this paper, a method of estimation of DOA using uniform and non-uniform linear array antennas by underlying the factors that affect the accuracy and resolution was given. The simulation of the variation of direction of arrival estimation via different array arrangements showed that non-uniform linear array has better estimate which result in increasing the wireless capacity. A mathematical model of the received signal and the detailed MUSIC algorithm when using ULA and NLA for the direction of arrival has been given.

Through intense simulations in MATLAB, the variation of the inputs parameter sets have shown that the more the increase SNR, array elements, snapshots the more accurate and high resolution is the DOA estimation. Moreover, it is shown that non-uniform linear array based on DOA estimation of co-prime array by combining the MUSIC results of the corresponding two decomposed uniform linear arrays is the best array arrangement due to its increase of the degree of freedom and it is the more accurate and high resolution for the DOA estimation compared to ULA. As observed in the simulation results, the non-uniform linear array helps to get an exact DOA estimation which leads to an improvement of the wireless communication systems capacity.

In the future, the application of the two dimensions of NLA in form of DOA estimation for co-prime array is recommended in more accordance with the real time environments.

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