SUITABILITY OF AGGREGATION METHODS OF INDUCTION MOTOR MODELS FOR PARAMETER IDENTIFICATION LOAD MODELING

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Abstract

This paper focusses on the aggregation of multiple induction motors connected in parallel on the same bus bars into a single equivalent model using two methods of aggregations.This has been necessitated by the strong effects that the non linear loads have on power system characteristics and therefore erroneous modeling of these devices continues to be an area of greater uncertainty. Though various methods of aggregations have been used, comparison on their suitablity and accuracy on identification of aggregation model parameters has not been extensively explored. Appropriate dynamic load model aggregation reduces the computation time and provides a faster and efficient model derivation and parameters identification that are most sensitive to load dynamics. The simulations and analysis are implemented using MATLAB.The accuracy of these methods is compared to identify the most suitable aggregation method of IM model. Their performance is validated by evaluating the results obtained from industrial and standard individual induction motor and the aggregation model on IEEE 30 Bus standard system. The results obtained are suitable and practical.

Key words: Aggregation methods, induction motor, load modeling, MATLAB, parameters identification

1.0 Introduction

Different aggregation methods have been applied for induction motor load representation by several researchers Abdel *et al*., (1976) and Zhao *et al*., (2010), as a single equivalent model for ease and speed of power system analysis. However, comparison on the accuracy of these methods has not been meticulous. The modeling of a group of induction motors is paramount in the dynamic analysis of induction motor (IM) since they contribute the biggest percentage of power system loads. This high percentage of induction motor loads in the power system causes delay during normal voltage recovery under fault conditions. It is however not practical to model every individual induction motors and especially large number of individual IM during the simulation studies and this can be highly time-consuming; therefore, aggregate models (single-unit equivalent models) are often employed. The accuracy of the results obtained with aggregate models depends in part on the assumptions made when deriving the aggregate motor and varies from method to method; grouping criterion is used to classify homogeneous motors Pillay *et al* (1996). Further, the accuracy of the results depends on how good the models are.

It is well known that load modeling on system dynamics is crucial; however it is still a big challenge. This complexity is brought about by the fact that load consists of various components with various characteristics, which nevertheless has to be represented as an equivalent single model. Further, it is the consumer of power who decides the order in which to connect their power consuming devices, thus making it even more intricate.

The goal of this paper is to represent and compare two methodologies of aggregation of the nonlinear characteristic of induction motor loads from common bus bars namely; transformer-type equivalent and aggregation of a group of IM loads based on two operating conditions i.e. no-load and locked rotor condition. These are achieved by Simulation of a group of induction motor model using a single equivalent motor model and analyze their suitability on parameter identification of aggregation load model. Matlab-based software is utilized in the simulations and analysis. The test results clearly demonstrates that, aggregation methods are of varying degrees of accuracy and are dependent on the assumptions made on derivation of the aggregate motors. However, in this research, it has been proven that, the transformer equivalent type model is of low accuracy in the analysis of parameter identification as compared to (Franklin *et al.,* 1976). The result further, demonstrates that, the rate of convergence for the latter method is prompt compared to the latter. The efficiency of the aggregated and individual IM is estimated using the IEEE 30 bus standard system.

2.0 Methodologies

2.1 Aggregation of Multiple Induction Motor Loads

Generally, large portion of power system loads are induction motors and their aggregation for parameter identifications and transient stability study is critical. The simulation of large group of IM takes time; therefore, in order to reduce the computation time, reduced order modeling is suggested to represent a group of motors with one or more aggregate motors. There are different aggregation methods proposed in the literature Abdel *et al*., (1976) and Zhao *et al*., (2010), and their accuracy depends on the assumptions made.

In this paper, an aggregation method based on steady state theory of induction motor modeling is used (Franklin *et al.,* 1976), and a transformer-type equivalent circuit Pillay *et al*., (1996). is used to represent induction motors. Four IM have been used in this research to obtain an aggregate motor model and their parameter identification. Aggregation without making some assumptions can prove to be an intricate venture and therefore, in this paper the following were assumptions made:

(i) All the motors are of the same type with similar ratings
(ii) All the motors are connected in parallel and at the same bus with no other load types
(iii) The output power for each sizes of motor is maintained for ease of comparison under the two aggregation methods of IM
(iv) Same number of poles is maintained

Total bus load=3.5Mw

total number of 3kw motors in 3.5MW

$=3.5*10^3/(3*0.746)=1564$ Number of motors to be aggregated **(1)**

It therefore, means that there are 1564 number of motors to be aggregated and whose total output powers remains the same 3.5Mw. The same procedure is applied to other sizes of motors.

(v) K is assumed to be a constant (0.98) while $\angle k = 0$ for the case of transformer type Aggregation

2.2 Transformer-Type Equivalent Circuit Model

It is a common practice to represent IM in a conventional equivalent circuit model as shown in fig 1; however, a transformer-type equivalent circuit model shown in Figure 2 has been used. The equations used to obtain the aggregate motor model and their parameters can be found in [4].

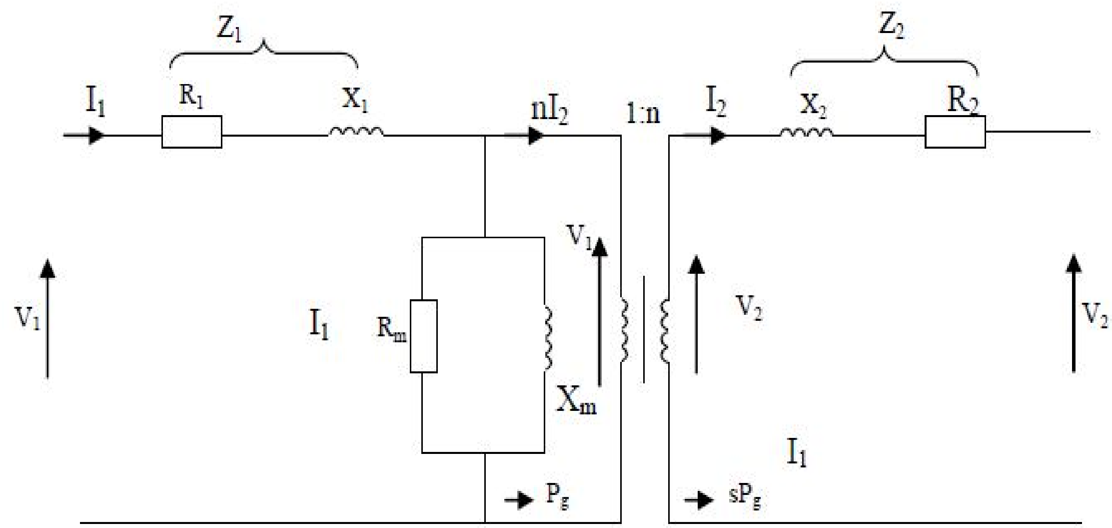

Figure 1 Conventional equivalent circuit model

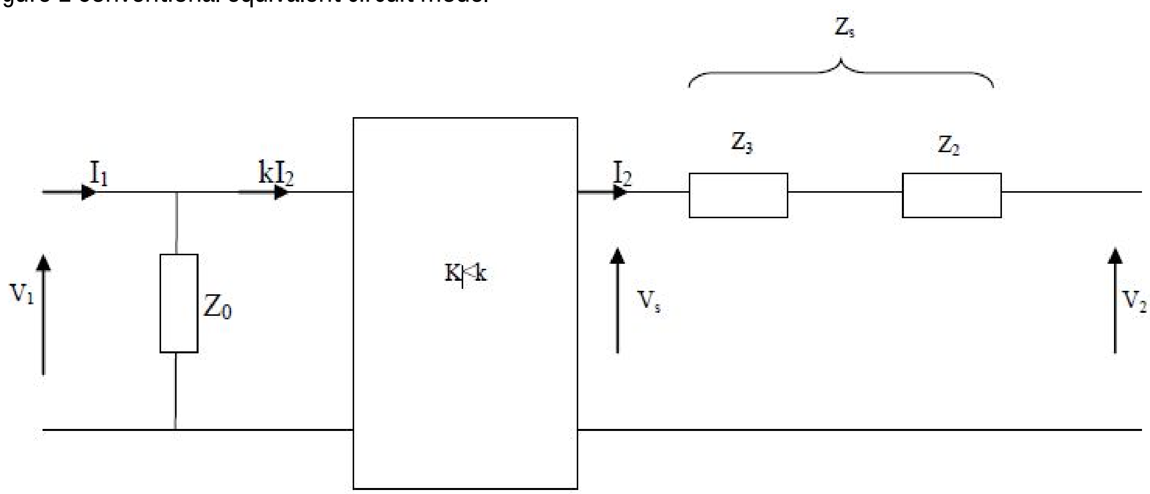

Figure 2: Transformer-type equivalent circuit model

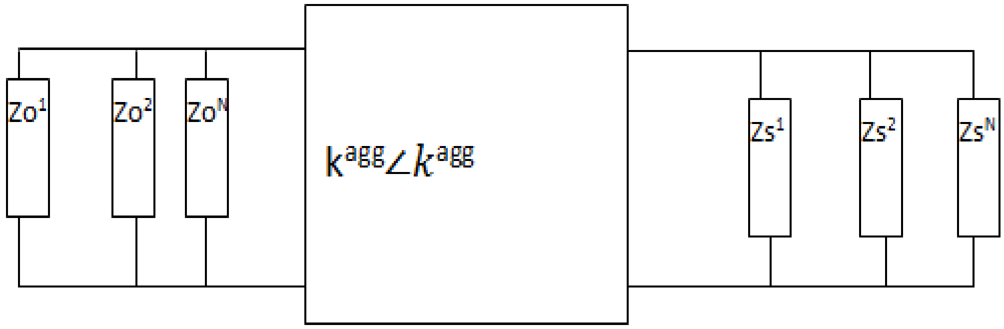

Figure 3: Multimachine connected to the same bus

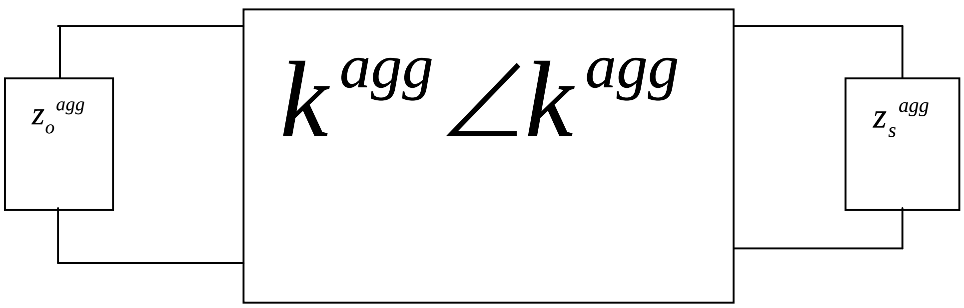

Figure 4: The aggregate induction motor

n this paper k is taken as 0.98 and $\angle k = 0^{\circ}$ for the aggregate motor.
From fig .4, the following parameters can be obtained as shown below:

$$Z_O^{agg} = \frac{1}{\sum_{i=1}^{N} \frac{1}{\left[Z_1 + Z_M\right]^i}}, where Z_1 + Z_M = Z_O, Z_1 = R_1 + X_1, Z_M = R_C // X_M \qquad (2)$$

From fig 2 above, the equation below is derived

$$Z_S^{agg} = \frac{1}{\sum_{i=1}^{N} \frac{1}{Z_S^{\,i}}} where, Z_S = Z_2 + Z_3, \quad (3)$$

Other derivations can be found from equations (3.3)-(3.9) in [4] respectively.

2.3 Steady-State Theory of Induction Motor Aggregation using Two Special Conditions

This method is proposed by Arif *et al*., (2009), where the parameters of the aggregate induction motor are determined from two operating conditions, i.e., no-load and locked-rotor conditions. However, the above method was first proposed by Uchida *et al*., (2000), and in this paper, the equivalent circuit parameters of the aggregation model are determined based on the same procedure. Figure 5 was been used for identification of model parameters of the aggregated IM. In the no-load operating conditions it is assumed that slips of all the induction motors are equal to zero while in the locked-rotor conditions the slips of all induction motors are equal to unity. The equations used to obtain the aggregate model can be found in Arif et al., (2009), from (1)-(21) respectively. Figure 5 shows the equivalent circuits of the aggregate induction motor load, where R_S-stator resistance, X_S-stator reactance, Rr-rotor resistance, Xr-rotor reactance, Xm-magnetizing reactance and S- Slip of the induction motor respectively whose parameters of the aggregated model are identified.

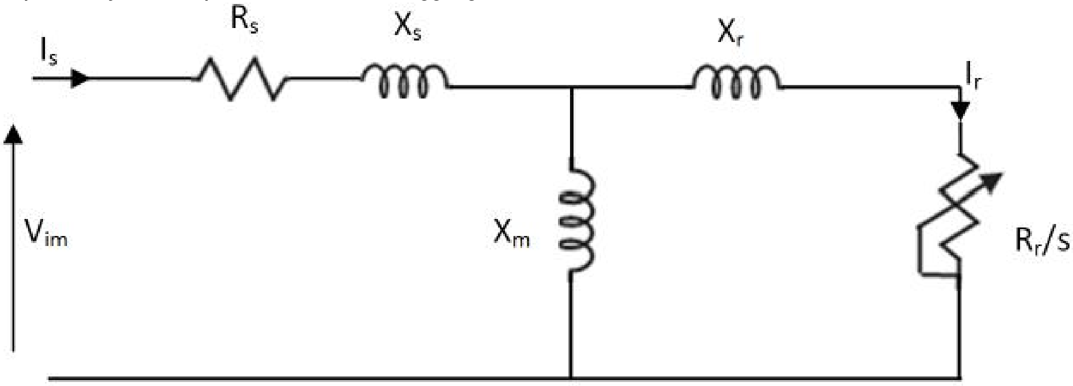

Figure 5: Classical equivalent circuit model of an induction motor

2.4 Grouping Criterion

Generally, the above method is used to identify and group homogeneous motors. The inertia and open circuit time constant are often used to classify motors. In Uchida *et al*., (2000), the authors have developed a grouping criterion that may be expressed as:

$$G = a \times b \times H \quad (4)$$

$$a = Xm/R2 \quad (5)$$

$$b = (X1+X2)/R1+R2 \quad (6)$$

The group is homogeneous if $1 \leq \frac{Gmax}{G\min} \leq 2.5$ ... (7)

Using the above grouping criterion, the different sizes of motors are classified into different groups. Aggregation based on this method is then done for different motor groups separately to find aggregate motors from each group.

Based on Uchida *et al*., (2000), table 1 below shows the industrial load model parameters on individual induction motors.

Table 1: Industrial per unit individual induction motor parameters

HP	R_S	Rr	X_S	Xr	Xm	H	RPM
3	0.02	0.04	0.03	0.03	1.21	0.71	1469
25	0.02	0.05	0.05	0.05	1.95	0.78	1435
50	0.02	0.04	0.05	0.05	2.31	0.79	1465
100	0.01	0.05	0.05	0.05	2.51	1.06	1485

Table 2: Typical Parameters for individual induction motor parameters

HP	RS	Rr	XS	Xr	Xm	H	RPM
3	0.02	0.037	0.035	0.035	1.21	0.707	1760
25	0.022	0.047	0.05	0.05	1.95	0.528	1695
50	0.0153	0.0402	0.053	0.053	2.31	0.79	1750
100	0.011	0.047	0.053	0.053	2.51	1.06	1705

2.4.1 Non Linear Model of Aggregated Power System

This model is used for analysis of large disturbance. The aggregated multi-machine power system can be represented by a set of first order nonlinear differential equations [9] in the form below:

$$\dot{X}_2 = X_2 \quad \text{.........(8)}$$

$$\dot{X}_2 = C_6X_3 + C_2X_2 + (C_3V_{d1} + C_4V_{q1})\ X_3 + C_5X_{32} \quad \text{.........(9)}$$

$$\dot{X}_3 = C_6X_3 + C_7X_4 + C_8V_{d2} + C_9V_{q1} \quad \text{.........(10)}$$

$$\dot{X}_4 = C_{10}X_2 + C_{11} \quad \text{.........(11)}$$

$$\dot{X}_6 = C_{12}T_{L2} + C_{13}X_6 + (C_{14}V_{d2} + C_{15}V_{q2})X_7 + C_{16}X_{7^2} \quad \text{.........(12)}$$

Where:

$X = [\delta g \omega_{rg}\ \acute{E}_{qg}\ E_{fd}\ \delta_m\ \omega_{rm}\ \acute{E}_{qm}]^T$

and the C'S are constant coefficients in terms of the system parameters and the operating point. Normally, nonlinear equations are solved iteratively using the Runge Kutta Merson integration technique with typical step length of one Ms, additionally, the transient stability analysis of the multi-machine power system are performed using the nonlinear transient simulation program. From the simulation study it is possible to analyze voltage stability of the aggregated nonlinear model such as induction motor load.

2.4.2 Linearized Model of Aggregated Power System

This model is used to analyze small signal stability of the power system. The signal of the multi-machine system in the matrix form is derived from the equations of the individual machines in the system after being linearized and combined to represent a multi-machine, multi-load system. Below are the system equations:

$$\dot{X} = Ax + Bu \quad \text{.........(13)}$$

$$Y_0 = Cx + Du \quad \text{.........(14)}$$

The above two general equations are used to simulate the response of the system to small disturbances such as sudden change of load. Furthermore, Eigenanalysis can be used to analyze small signal stability.

The eigenvalue analysis allows for the computation of modal sensitivities with respect to generator or voltage controllers, reactive power compensating devices etc.

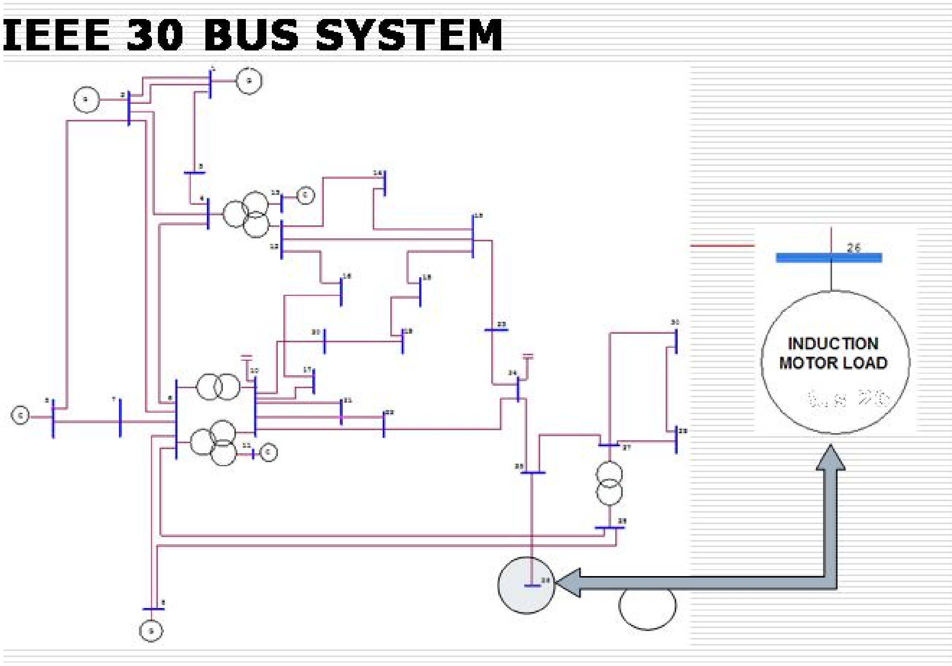

Figure 6: IEEE 30 bus standard systems

2.4.3 Tabulation of Results Using the Two Methods of Aggregation

Aggregation of Im Based on Two Special Operating Conditions

Table 3: Aggregated IM parameters of different sizes of an industrial consumer

Bus	P_{agg}	R_{1agg}	R_2agg	X_{1agg}	X_{2agg}	XM_{agg}	V p.u	$SLIP_{agg}$	$RPM_{ag}g$
26	4692	0.02	0.04	0.03	0.03	1.21	1	0.0268	1751
26	4700	0.02	0.05	0.05	0.05	1.95	1	0.0129	1777
26	4700	0.02	0.04	0.05	0.05	2.31	1	0.0073	1787
26	4700	0.01	0.05	0.05	0.05	2.51	1	0.0078	1786

Table 4: Typical aggregated IM parameters

Bus	P_{agg}	R_{1agg}	R_2agg	X_{1agg}	X_{2agg}	XM_{agg}	V p.u	$SLIP_{agg}$	$RPM_{ag}g$
26	4692	0.02	0.037	0.035	0.035	1.21	1	0.0248	1755
26	4700	0.022	0.047	0.050	0.050	1.95	1	0.0121	1778
26	4700	0.013	0.0402	0.0530	0.0530	2.31	1	0.0074	1787
26	4700	0.011	0.047	0.053	0.053	2.51	1	0.0074	1787

Transformer-Type Equivalent Circuit Method of Aggregation

Table 5: Aggregation of IM parameters of an industrial consumer

Bus	P_{agg}	R_{1agg}	R_2agg	X_{1agg}	X_{2agg}	XM_{agg}	V p.u	$SLIP_{agg}$	$RPM_{ag}g$
26	4692	0.02	2	0.03	0.03	1.47	1	1	-
26	4700	0.02	2.5773	0.05	0.05	2.45	1	1	-
26	4700	0.02	4.253	0.05	0.05	2.45	1	1	-
26	4700	0.01	5.3191	0.05	0.05	2.451	1	1	-

Table 6: Typical induction motor parameters aggregation

Bus	P_{agg}	R_{1agg}	R_2agg	X_{1agg}	X_{2agg}	XM_{agg}	V p.u	$SLIP_{agg}$	$RPM_{ag}g$
26	4692	0.02	1.85	0.035	0.035	1.715	1	1	-
26	4700	0.022	2.427	0.05	0.05	2.45	1	1	-
26	4700	0.0153	4.2766	0.053	0.053	2.597	1	1	-
26	4700	0.011	5	0.053	0.053	2.597	1	1	-

2.4.4 Simulation Results

In this paper, an IEEE-30 bus standard network was used to show suitability and accuracy of aggregation methods in identification of motor parameters for the two methods of motors aggregations. Four IM connected in parallel on bus 26 are considered for a case study. From the analysis of the two methods of aggregation it demonstrates that aggregation of IM based on two special operating conditions yields comparatively better results of the aggregated parameters; see table 3 and 4 respectively, as opposed to the transformer-type equivalent method of aggregation as in table 5 and 6 respectively. The latter method gives good results for all the IM aggregated motor parameters except R_2agg, $SLIP_{agg}$ and $RPM_{ag}g$. It should be noted that when the rotor is not turning the slip is 100% and at no-load, any increase in mechanical load will result in slip increase.

As seen from figures 7 to 10; Small induction motors can handle small loads and therefore, the aggregated group of IM gives rise to a higher value of the slip (i.e., Rotor speed decreases) such that the induced voltage and current produce the torque required by the load. It is also seen that as the size of the motor increases, the aggregate slip also reduces (i.e., Rotor speed increases) such that the induced voltage and current produce the torque required by the load; the aggregate stator resistance reduces while the aggregate rotor resistance and the aggregate stator and rotor reactance increase significantly. This is contributed by the fact that the conversion of electrical power to mechanical power takes place in the rotating part of an electric motor (rotor). It should be noted that the slip increase of IM normally depends on the mechanical torque demand of the induction motor and the motor inertia. This is clearly demonstrated in Figure 7. As a matter of fact, large motors operate at low slip and therefore consume less amount of reactive power.

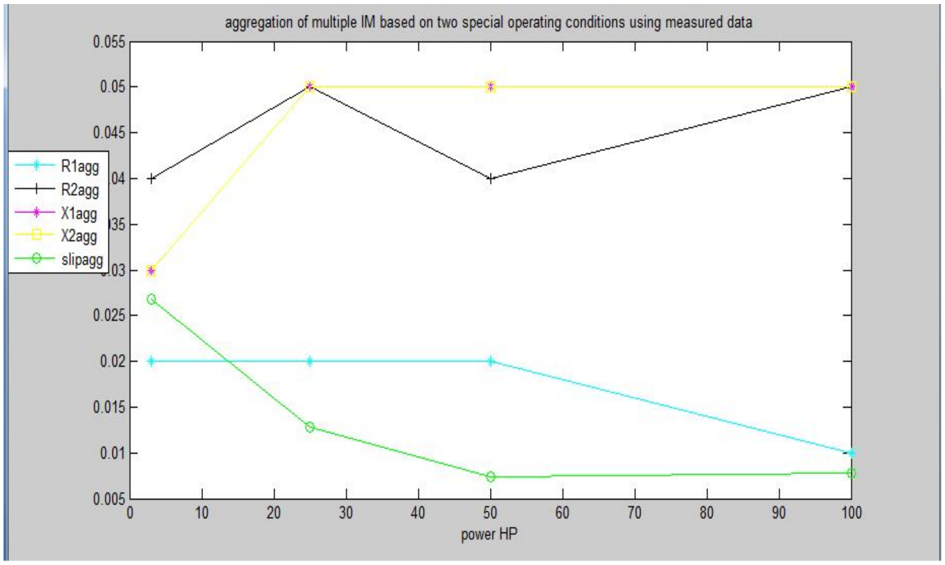

Figure 7: Aggregations of IM parameters based on two special operating conditions using industrial measured data

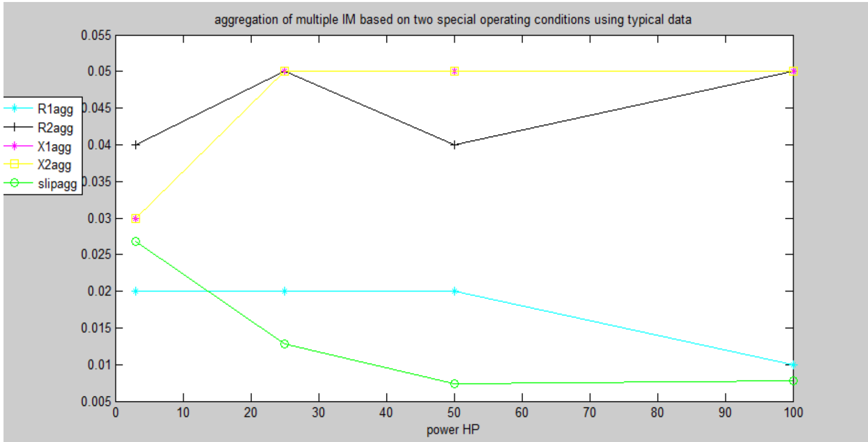

Figure 8: Aggregations of IM parameters based on two special operating conditions using typical data

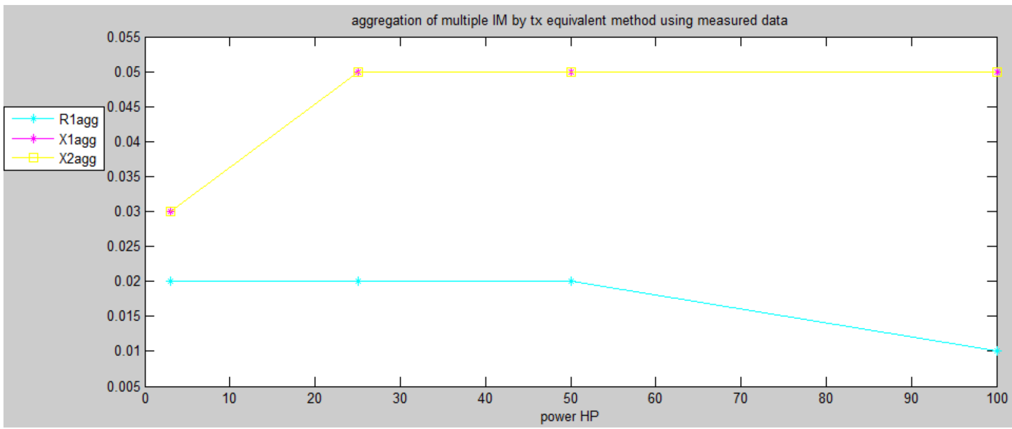

Figure 9: Aggregations of IM parameters based on TX equivalent-circuit method using measured data

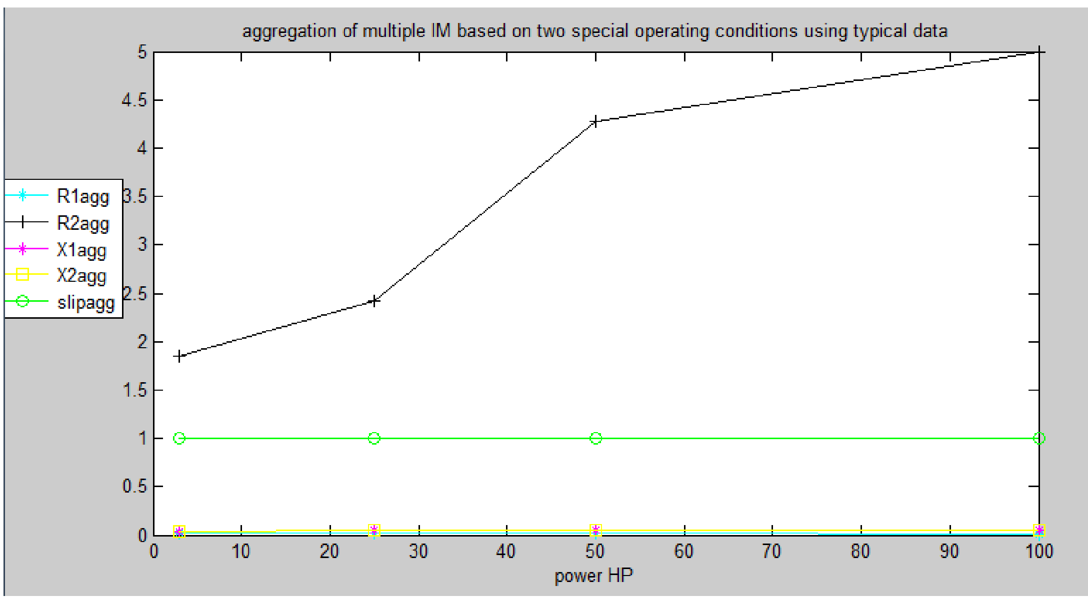

Figure 10: Aggregations of IM parameters based on TX equivalent-circuit method using typical data

5.0 Conclusion

It was found out that the transformer-type equivalent method of aggregation is inferior in identification of some of the aggregated motor parameters. This method was compared to aggregation of multiple induction motors based on two special operating conditions, i.e., no-load and locked-rotor conditions which yielded better results that are comparable to individual motor parameters. This validated the latter method of aggregation employed. The objective of this research has been achieved. It has also been realized that unless a suitable method of aggregation is selected the results is bound to generate some errors. This does not resonate well with system control operators whose responsibilities is to ensure that the power system is run with minimal disturbances. It was also found out that small motors have higher slip as opposed to big motors whose slip is low. Therefore, for better analyses of power system, aggregation of IM is crucial for parameters identification and their sensitivity to various power system operating conditions.

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APPENDIX A: ACCURACY OF PARAMETER AGGREGATION OF IM USING DIFFERENT METHODS

```
%aggregation of multiple IM based on two special operating conditions using measured data
power=(IM_data(:,1))';
R1agg=(IM_agg(:,3))';
R2agg=(IM_agg(:,4))';
X1agg=(IM_agg(:,5))';
X2agg=(IM_agg(:,6))';
%Xmagg=(IM_agg(:,7))';
slipagg=(IM_agg(:,9))';
plot(power,R1agg,'c-*',power,R2agg,'k-+',power,X1agg,'m-*',power,X2agg,'ys-',...
 power,slipagg,'go-')
title ('aggregation of multiple IM based on two special operating conditions using measured data')
xlabel('power HP')
%ylabel('R1agg and R2agg')
legend('R1agg','R2agg','X1agg','X2agg','slipagg')


%aggregation of multiple IM based on two special operating conditions using typical data
power=(IM_data(:,1))';
R1agg=(IM_agg(:,3))';
R2agg=(IM_agg(:,4))';
X1agg=(IM_agg(:,5))';
X2agg=(IM_agg(:,6))';
%Xmagg=(IM_agg(:,7))';
slipagg=(IM_agg(:,9))';
plot(power,R1agg,'c-*',power,R2agg,'k-+',power,X1agg,'m-*',power,X2agg,'ys-',...
 power,slipagg,'go-')
title('aggregation of multiple IM based on two special operating conditions using typical data')
xlabel('power HP')
%ylabel('R1agg and R2agg')
legend('R1agg','R2agg','X1agg','X2agg','slipagg')

   %aggregation of multiple IM using tx equivalent method using measured data
   power=(IM_data(:,1))';
   R1agg=(IM_agg(:,3))';
   R2agg=(IM_agg(:,4))';
   X1agg=(IM_agg(:,5))';
   X2agg=(IM_agg(:,6))';
   %Xmagg=(IM_agg(:,7))';
   %slipagg=(IM_agg(:,9))';
   plot(power,R1agg,'c-*',power,X1agg,'m-*',power,X2agg,'ys-')
   title('aggregation of multiple IM by tx equivalent method using measured data')
   xlabel('power HP')
   %ylabel('R1agg and R2agg')
  legend('R1agg','X1agg','X2agg')



   % aggregation of multiple IM using tx equivalent method using typical data
   power=(IM_data(:,1))';
   R1agg=(IM_agg(:,3))';
   R2agg=(IM_agg(:,4))';
```

```
X1agg=(IM_agg(:,5))';
X2agg=(IM_agg(:,6))';
%Xmagg=(IM_agg(:,7))';
%slipagg=(IM_agg(:,9))';
plot(power,R1agg,'c-*',power,X1agg,'m-*',power,X2agg,'ys-')
title('aggregation of multiple IM using tx equivalent method using typical data')
xlabel('power HP')
%ylabel('R1agg and R2agg')
legend('R1agg','X1agg','X2agg')
```

APPENDIX B: IDENTIFICATION OF VARIOUS PARAMETERS OF THE AGGREGATED INDUCTION MOTORS

```
%aggregation of IM parameters based on two operating conditions: no load and locked-rotor aggregation method
%industrial data
R1=0;R2=0;X1=0;X2=0;Xm=0;Z1=0;Z2=0;
s=IM_data(:,8);
k=IM_data(:,9);
Hp=IM_data(:,1);
l=numel(s);
IM_agg=zeros(l,10);
Pmagg=Hp.*k;
clear w
for w=1:l

j=sqrt(-1);
   R1=IM_data(w,2)*Pmagg(w)/Hp(w);
   R2=IM_data(w,3)*Pmagg(w)/Hp(w);
   X1=IM_data(w,4)*Pmagg(w)/Hp(w);
   X2=IM_data(w,5)*Pmagg(w)/Hp(w);
   Xm=IM_data(w,6)*Pmagg(w)/Hp(w);
   Z1=R1+j*X1;
   Z2=R2/s(w)+j*X2;
   Znl=R1+j*(X1+Xm);
   Zrl=R1+R2+j*(X1+X2);
   Z=Z1+(Xm*Z2)/(Xm+Z2);

   Znlsum=k(w)*1/Znl;
   Zrlsum=k(w)*1/Zrl;
   Zsum=k(w)*1/Z;

Znlagg=1/Znlsum;
Zrlagg=1/Zrlsum;
Zagg=1/Zsum;
R2agg=real(Zrlagg)-real(Znlagg);
R1agg=real(Znlagg);
%n=X1agg/X2agg
n=1;
X1agg=(n/(n+1))*imag(Zrlagg);
X2agg=(1/(n+1))*imag(Zrlagg);
Xmagg=abs(imag(Znlagg)-(n/(n+1))*imag(Zrlagg));

Rs=R1agg;
Rr=R2agg;
```

```
Xls=X1agg;
Xlr=X2agg;
Xm=Xmagg;
i=1;
   Vm=1;%bus voltage
   Pm=1;
   %slip equation a*S^2+b*s+c=0, a,b and c are coefficients
   a=1;
   b=(Rr(i)*(2*Pm(i)*Rs(i)*Xm(i)^2-Vm^2*Xm(i)^2))/(Pm(i)*....
      (Xm(i)^2-Xlr(i)*Xls(i))^2+(Rs(i)*Xlr(i))^2-(Vm^2*Rs(i)*Xlr(i)^2));
   c=(Rr(i)^2*((Rs(i)^2+Xls(i)^2)*Pm(i)-Vm^2*Rs(i)))/(Pm(i)*(Xm(i)^2-....
      Xlr(i)*Xls(i))^2+(Rs(i)*Xlr(i))^2-(Vm^2*Rs(i)*Xlr(i)^2));
   coeff=[a b c];

   rt=roots(coeff);
   if any(imag(rt))%check if slip is real
      error('MOTORS:slip must be real numbers')
   end

   motoring=0;%flag for checking negative slip values.
   if rt(1)<=0&&rt(2)<=0
    motoring=1;
   end
   if motoring==1
      error('MOTORS:motors are in generation mode(negative slip)')
   end
   if rt(1)<=0||rt(2)<=0
      slip=max(rt);
   else
      slip=min(rt);
   end

   sagg=slip;
%sagg=min(rtt);
nragg=1800*(1-sagg);
IM_agg(w,1)=IM_data(w,10);IM_agg(w,2)=Pmagg(w);IM_agg(w,3)=R1agg;
IM_agg(w,4)=R2agg;IM_agg(w,5)=X1agg;IM_agg(w,6)=X2agg;IM_agg(w,7)=Xmagg;
IM_agg(w,8)=1;IM_agg(w,9)=sagg;IM_agg(w,10)=nragg;
clear R1agg R2agg X1agg X2agg Xmagg nragg slip i sagg
end

% aggregation of IM parameters based on tx-type equivalent-circuit model
% aggregating induction motors
s=IM_data(:,2);
l=numel(s);
IM_agg=zeros(l,10);
% HP Rs Rr Xs Xr Xm  Vp.u slip n bus
for i=1:length(IM_data(:,1))
HP_agg=IM_data(i,1)*IM_data(i,9);
s=IM_data(i,8);
Z1=(IM_data(i,2)+j*IM_data(i,4))*HP_agg/IM_data(i,1);
Z2=(IM_data(i,3)/s+j*IM_data(i,5))*HP_agg/IM_data(i,1);
Zm=(j*IM_data(i,6))*HP_agg/IM_data(i,1);
```

```
Z0=(Z1+Zm)*HP_agg/IM_data(i,1);
%K=Zm/(Zm+Z1);

Z3=Z1*Zm*s/(Z1+Zm);
Zs=Z2+Z3;

Z0agg=1/(IM_data(i,9)*1/Z0);
Zsagg=1/(IM_data(i,9)*1/Zs);
Z2agg=1/(IM_data(i,9)*1/Z2);
Z3agg=1/(IM_data(i,9)*1/Z3);
Z1agg=1/(IM_data(i,9)*1/Z1);
K=0.98+j*0;
Zmagg=K/(1-K)*Z1agg;

R2agg=real(Zsagg-Z3agg);
K1r=real(Z3agg);
K2r=real(Zsagg);
%sagg=(K2r-sqrt[(K2r)^2-4*K1r*R2agg]/(2*K1r);
% slip equation a*Sagg^2+b*Sagg+c=0, a,b and c are coefficients
 a=K1r;
 b=-K2r;
 c=R2agg;
 coeff=[a b c];
 rt=roots(coeff);
   if any(imag(rt))%check if slip is real
      error('MOTORS:slip must be real numbers')
   end

 motoring=0;%flag for checking negative slip values.
 if rt(1)<=0&&rt(2)<=0
 motoring=1;
   end
   if motoring==1
      error('MOTORS:motors are in generation mode(negative slip)')
   end
   if rt(1)<=0||rt(2)<=0
     slip=max(rt);
   else
      slip=min(rt);
   end

   sagg=slip;

   nragg=(1-sagg)*1800;

  %EQUIVALENT AGGREGATE MOTOR PARAMETERS: RESULTS
IM_agg(i,1)=IM_data(i,10);IM_agg(i,2)=HP_agg;IM_agg(i,3)=real(Z1agg);
IM_agg(i,4)=real(Z2agg);IM_agg(i,5)=imag(Z1agg);IM_agg(i,6)=imag(Z2agg);
  IM_agg(i,7)=imag(Zmagg);
  IM_agg(i,8)=1;IM_agg(i,9)=sagg;IM_agg(i,10)=nragg;
  clear motoring coeff rt nragg slip  sagg
  end
```