PREDICTION OF WIDTH AND DEPTH OF CUT IN LASER MACHINING OF PMMA USING A CO$_2$ LASER

E. Wangui, B.W. Ikua, G.N. Nyakoe and T. Mulembo

Abstract—Lasers are gaining popularity in micro-machining of polymers in the production of micro-devices. Polymer micro-fluidic systems can be produced in several ways, some of which are based on replication from a micro-machined master tool, while others rely on direct machining. The replication methods which include hot embossing and micro-injection moulding are not only relatively complex but also expensive compared to laser micro-machining which may account for the rise in popularity.

Micro-fluidic systems have so far been mostly made using silicon. As a material, silicon is a good choice when small numbers of long-lasting systems have to be produced. However, especially in clinical applications, single-use systems are desired in order to avoid contamination. Polymers are a good alternative to silicon, since they are cheaper and the commercially available selection of polymers represents a very wide range of physical and chemical properties that for any given application an appropriate polymer is likely to exist.

While factors that affect the quality of laser machined features have been extensively researched, in most of the studies the laser beam has been normal to the work piece. The beam is not always perpendicular to the surface being machined as is the case when working with freeform profiles. Parameters that can be neglected in products of a macro size can significantly affect the functionality of micro-devices and it is important to carefully analyze all factors that may affect their intended functionality.

This paper presents analytical models for the prediction of the width and depth of cut of laser machined polymethlymethacrylate, PMMA, under different angles of incidence of a CO$_2$ laser beam. The validation of the model results with experimental results will be presented at a later publication.

Keywords—CO$_2$ laser, beam incidence angle, kerf width, depth of cut, analytical models.

I. INTRODUCTION

ODELLING in laser beam machining aids in getting a better understanding of this complex process. Dubey and Yadava [1] reported that experimental results indicated that the effect of process parameters on process performance do not have a definite pattern that can be applied to different operating ranges. In such cases more scientific experimental study was needed for different ranges of operating parameters for the prediction of process behavior. Dubey and Yadava [1] found in many of the experimental results discussed by various researchers, the optimum range of process parameters had been obtained based on variation of one factor at a time. Simultaneous effect of variation of more than one parameter at a time had not been studied extensively.

A comprehensive, scientific, methodology-based study was thus needed for laser beam machining of different advanced engineering materials with all possible input parameters as well as single or multiple performance measure.

On the basis of their origin, models can be divided in three categories i.e. experimental or empirical models, analytical models, and artificial intelligence (AI) based models.

In this paper, two analytical models are used to predict the depth of cut of laser machined polymethylcrylate, PMMA, under different angles of incidence of the laser beam. A simple model predicting the width of cut will also be presented. The laser used is a 60 W, continuous wave, sealed CO$_2$ laser.

II. WIDTH OF CUT

The width of cut or the kerf width is the distance between the edges of a laser cut channel. Snakenborg et al. [2] and Yuan and Das [3] have reported that the width of cut depends upon the spot size of the beam as defined by the diameter of the spot where the irradiance is greater than the threshold for ablation. This means that any change in the diameter of the beam will cause a corresponding change in the width of cut. By simple geometrical relations, it can be deduced that the spot diameter of the beam as it hits the work piece depends on the angle of incidence of the beam as illustrated in Figs 1 and 2.

Fig. 1 shows a case where the beam is normal to the workpiece. d is the diameter of the beam as it exits the focusing lens and h is the focal length. Ideally, the width of cut, w, should be equal to the beam diameter, i.e. $w = d$.

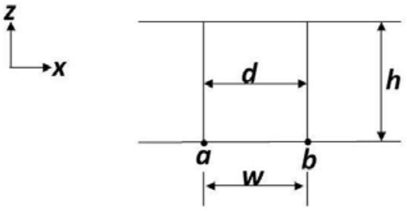

Fig. 1. Width of cut due to a perpendicular beam

For any angle of incidence θ, the spot diameter of the beam when it hits the workpiece is dependent on θ as illustrated in Fig. 2, where $w = d/\cos\theta$.

III. DEPTH OF CUT

Two models will be used to predict the depth of cut when applying oblique angles of incidence of the beam. The two models discussed will be those presented by Snakenborg et al. [2] and that from Yuan and Das [3].

E. Wangui, Department of Mechatronic Engineering, JKUAT (+254724667947; e-mail: wangui_es@eng.jkuat.ac.ke)

B.W. Ikua, School of Mechanical, Manufacturing and Materials Engineering, JKUAT (+254722286264; e-mail: ikua_bw@eng.jkuat.ac.ke)

G.N. Nyakoe, Department of Mechatronic Engineering, JKUAT (+254721456661; e-mail: nyakoe@eng.jkuat.ac.ke)

T. Mulembo, Department of Mechatronic Engineering, JKUAT (+254729061526; e-mail: titusmulembo@gmail.com)

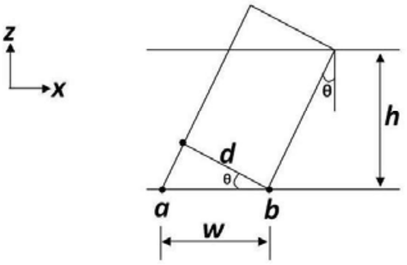

Fig. 2. Width of cut due to an oblique beam

A. Snakenborg's model

Snakenborg et al. [2] developed a simple model to predict depth of cut when machining PMMA. The model has been used by other researchers [4]–[6] and their experimental results closely agree with the model's output.

The model proposed in this paper assumes the removed material m to be proportional to the amount of heat Q available to vaporize the polymer [7]. This heat is given as the absorbed energy Q_{in} coming in from the laser that is higher than a certain threshold energy Q_{th}

$$m = kQ = k(Q_{in} - Q_{th}) \tag{1}$$

k is a proportionality constant associated with the chemical binding energy of the PMMA polymer. The threshold heat, (Q_{th}), represents energy losses in the ablation process. The two major loss terms are the energy needed to heat up the irradiated spot and its surroundings before ablation begins as well as the energy that is lost due to heat conduction into the surroundings [2], [7].

It is further assumed that the laser is programmed to cut a channel into a PMMA block at a steady velocity v with a given laser power ϕ_{in}. The irradiated spot has an area $A = a^2$, that is a square beam is assumed while in reality the area is circular, so a factor of π is neglected [6].

The absorbed energy is equal to the product of incident power and irradiation time, Δ_t

$$Q_{in} = \alpha\, \phi_{in}\, \Delta_t \tag{2}$$

α is the absorptance of PMMA. The irradiation time is related to the cutting speed by

$$\Delta_t = \frac{a}{v} \tag{3}$$

As mass is the product of volume and density the depth is given as

$$D = \frac{m}{\rho\, A} \tag{4}$$

Substituting equations 1, 2 and 3 into 4 gives

$$D = \frac{k\, \alpha\, \phi_{in}}{v\, \rho a} - \frac{k\, Q_{th}}{\rho\, a^2} \tag{5}$$

Although Snakenborg et al. [2] used the model to study the the effect of power and cutting speed on the depth, here, for a particular speed and a laser power of 60 W, the only variant will be a changing beam diameter due to the changing beam incidence angle.

The values of the constants are given by Snakenborg et al. [2] as

$$k = 0.55 \times 10^{-6}\ \text{Kg/J} \tag{6}$$

$$Q_{th} = 0.48\ \text{mJ} \tag{7}$$

Re-writing equation 5 for a speed $v = 0.8333\text{x}10^{-3}$ m/s (50 mm/min) and a power of $\phi_{in} = 60$ W where the density of PMMA is $1190\ \text{kg/m}^3$ and the absorptance $a = 25000\text{m}^{-1}$ [8]

$$D = \frac{0.8323}{a} - \frac{0.2285 \times 10^{-12}}{a^2} \tag{8}$$

Now a is half the beam diameter and the diameter varies with different angles of incidence of the beam. Taking the half-diameter a as the effective half-width measured at each angle of incidence varied from 0° to 80° and substituting in Equation 8 gives the depth of cut.

B. Yuan and Das' model

Yuan and Das [3] investigated the micromachining of channels in PMMA and using a low power cw CO_2 laser and low cutting speeds. They developed physical models for predicting the depth and profile of laser-ablated channels. The range of parameters tested were 0.45 - 1.35W for laser power and 2 - 14 mm/s for the cutting speed. Within this range, the model results were in agreement with their experimental results.

In their model, Yuan and Das based the CO_2 laser ablation on heat balance and the Gaussian mode of propagation. They considered a Gaussian laser beam striking the surface of a substrate having semi-infinite thickness and an infinite length. The beam moved in the positive x direction with a velocity v. The laser power is absorbed at the surface and once the material reaches evaporation temperature T_v, it consumes some latent heat. The following three assumptions were made

1) The CO_2 laser beam can be described by the Gaussian intensity distribution as follows

$$I(x, y, z) = \frac{P}{\pi w^2(z)} \exp^{-[(x^2+y^2)/w^2(z)]} \tag{9}$$

$$w^2(z) = R^2[1 + (\lambda z/2\pi R^2)^2] \tag{10}$$

Where P was the laser power, $w(z)$ is the laser beam radius at a distance z from the focal waist, λ is the wavelength and R is the laser beam radius at the focal waist. Because $\lambda z/2\pi R^2 << 1$, $w(z)$ can be treated as equal to constant R.

2) Volatiles would not affect the CO_2 laser.
3) Energy losses due to radiation and conduction are negligible.

After derivation, the channel depth $D(0)$ was expressed as

$$D(0) = \frac{aP}{\overline{\pi} R v \rho [L + c_p(T_v - T_o)]} \tag{11}$$

Where a is the absorptance of PMMA at the CO_2 wavelength, L the latent heat of decomposition from PMMA to the momomer methyl metha crylate, MMA, ρ the density of PMMA and T_o the room temperature. The values for the constants are chosen as [8]

a	25000 m^{-1}
L	1.358×10^6J/Kg
c_p	$1.84 \times 10^3 + 2.38(T_v - T_o)$ J/Kg.K
ρ	1190 Kg/ m^3

The decomposition temperature given by Yuan and Das [3] is $T_v = 360°$ and the room temperature is taken as $T_o = 20°$. Then Equation 11 becomes

$$D(0) = \frac{0.3778}{R} \tag{12}$$

where R is the effective measured half-beam diameter at each angle of incidence.

The Gaussian profile of the channel in the y direction was described as

$$D(y) = D(0)\exp^{-y^2/R^2} \tag{13}$$

The width of the channel was taken as the diameter of the laser beam. However this is only applicable for high laser power density which can ablate the PMMA across the entire area of the laser beam width. For the low power case, the laser input energy is not sufficient to ablate all the PMMA material exposed to the laser beam because of the Gaussian intensity distribution. Beyond a given beam radius, the energy intensity is below the threshold fluence required to initiate ablation of PMMA and consequently the channel profiles have a smaller width than the diameter of the laser beam. The formula for channel profile was modified by replacing R with the threshold beam radius for ablation to occur, y_{th} which was obtained by experimentally determining the threshold ablation intensity and time. The adjusted channel profile was therefore given by:

$$D(y) = D(0)\exp^{-y^2/{y_{th}}^2} \tag{14}$$

IV. RESULTS

A. Width of cut

Figure 3 shows a variation of the values of the predicted width of cut as a function of the angle of incidence and the values measured from channels cut at the various angles of incidence. The measurements were taken from channels cut on 5 mm thick PMMA machined with a 60 W, cw, sealed gas CO_2 laser.

The plot of Fig. 3 shows some disparity between the two sets of values with the predicted values being lower than the actual width of cut obtained. It is possible that the underestimation occurs due to a larger minimum diameter that is observed for an oblique beam than for a beam normal to the surface. As shown in Fig. 4, when there is oblique incidence, achieved by inclining the work piece, the diameter of the area impacted by the beam is larger, $d_2 > d_1$.

As θ increases, the difference between the predicted and measured width values reduces. This is probably due to a reduction in the beam intensity normal to the surface as the θ increases correspondingly reducing the energy available for ablation.

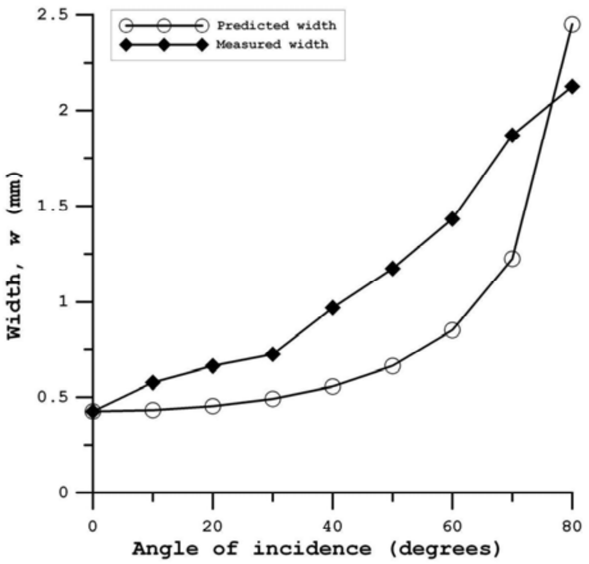

Fig. 3. Variation of the width of cut with the angle of incidence of the beam

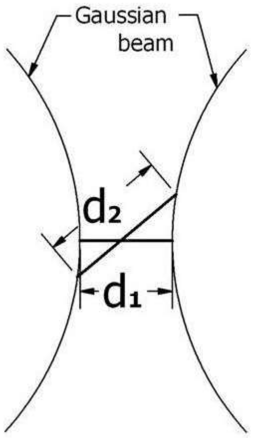

Fig. 4. Diameters of perpendicular and oblique beams

Romoli et al. [4] found that the width of cut tended to approach a limit value corresponding to the spot diameter. The spot diameter in their case was varied by varying the working focal length distance. They formulated an empirical equation for predicting the groove width which related the width to the spot diameter and a correlation index. The index corresponded to the focal length used. The indices were obtained by a nonlinear regression of the acquired data of the different spot diameters at different focal lengths and there were therefore different indices for each focal length.

B. Depth of cut

The depth vs angle of incidence for the Snakenborg's model depth is plotted in Fig. 5.

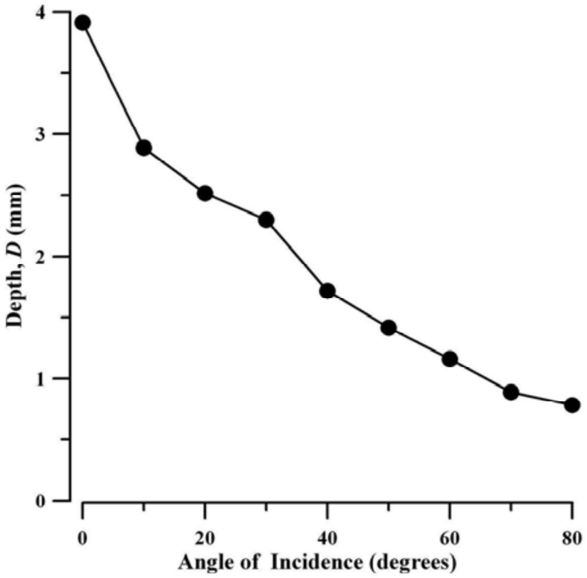

Fig. 5. Predicted depth from Snakenborg's model

The depth of cut values as predicted by Yuan and Das' model [3] are shown in Fig. 6.

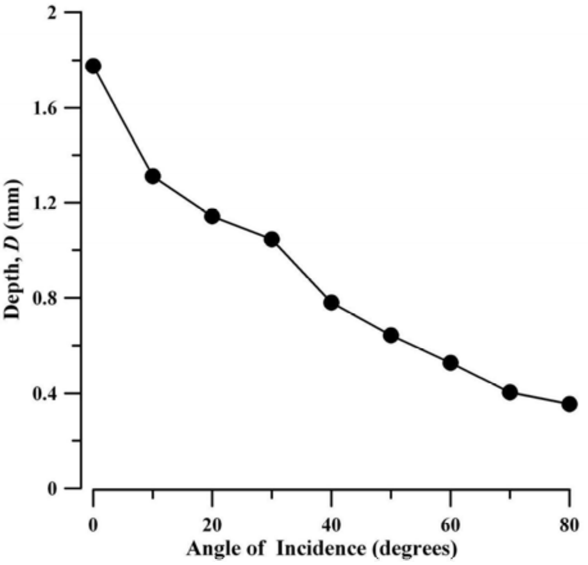

Fig. 6. Predicted depth from Yuan and Das' model

From Figures 5 and 6, both models produce similar plot trends for the depth values. However, Snakenborg et al. [2] depth values are higher than those predicted by Yuan and Das [3]. In developing their model, Snakenborg et al. used a range of values for power and cutting speed where removal temperature is reached quickly and the irradiated material evaporates before it can heat up the surrounding material. In that case the evaporative heat term is much larger than the conductive or the warm-up heat losses, since the incident laser power is very high, while the irradiated area as well as the heat conduction and heat capacity of PMMA are small. However, for low input powers and slow moving beams, the energy balance shifts and a larger volume in the surroundings of the moving irradiated spot is warmed up whereby a higher percentage of heat is lost than in the case of a powerful and fast moving beam. Snakenborg et at. [2] predicted that their model would overestimate the depths for velocities less than 100 mm/s and power values between 34 and 48 W. Yuan and Das' model [3] on the other hand, was developed for low input powers of laser and this may account for the difference in depth values between the results produced by the two models.

V. CONCLUSION

In this paper, two analytical models predicting the depth of cut in laser machining at various angles of beam incidence have been analyzed. The analytical models used were those developed by Snakenborg et al. [2] and Yuan and Das [3]. The models produce a similar trend in the predicted depth of cut values for different angles of incidence although the values differ in magnitude, possibly attributed to the use of parameters outside the range for which the models were developed. A simple geometry-based model for the prediction of the width of cut was also presented.

In a later publication, the depth of cut results from the models will be validated against values obtained from experiments.

ACKNOWLEDGEMENT

This work is supported by Jomo Kenyatta University of Agriculture and Technology, JKUAT. The authors would like to acknowledge Mr. Joshua Owiyo and Mr George Kariuki of the engineering workshops, JKUAT and Mr. Boniface Kariuki of the Department of Mechatronic Engineering, JKUAT, for their assistance.

REFERENCES

[1] Dubey A. K. and Yadava V., "Laser beam machininga review," *INTERNATIONAL JOURNAL OF MACHINE TOOLS & MANUFACTURE, DESIGN, RESEARCH AND APPLICATION*, 2007.

[2] Snakenborg D., Klank H., and Kutter J.P., "Microstructure fabrication with a CO_2 laser system," *Journal of mechanics and microelectronics*, 2004.

[3] Yuan D. and Das S., "Experimental and theoretical analysis of direct-write laser micromachining of polymethyl methacrylate by CO_2 laser ablation," *Journal of Applied Physics*, 2007.

[4] Romoli L., Tantussi G., and Dini G., "Experimental approach to the laser machining of pmma substrates for the fabrication of microfluidic devices," *Optics and Lasers in Engineering*, 2010.

[5] Romoli L., Tantussi G., and Dini G., "Layered laser vaporization of pmma manufacturing 3d mould cavities," *Annals of the CIRP*, 2007.

[6] Jensen M. F., *Laser Micromachining of Polymers*. PhD thesis, Technical University of Denmark, 2004.

[7] Jia F., Galea E.R., and Patel M.K., "The numerical simulation of the non-charring pyrolysis process and fire development within a compartment," *Applied Mathematical Modeling*, 1999.

[8] Bulgakova N. M., Zakharov L. A., Onischuk A. A., Kiselev V. G., and Baklanov A. M., "Thermal and gasdynamic analysis of ablation of poly(methyl methacrylate) by pulsed ir laser irradiation under conditions of nanoparticle formation," *Journal of Physics D: Applied Physics*, 2009.