OPTIMIZING LQR TO CONTROL BUCK CONVERTER BY MESH ADAPTIVE SEARCH ALGORITHM

M. M. Kanai, J. N. Nderu and P. K. Hinga
Department of Electrical and Electronics Engineering, Jomo Kenyatta University of Agriculture and Technology, Nairobi, Kenya
E-mail: cheptora@yahoo.com

Abstract

In this study, control method to control Buck converters by Linear Quadratic Regulator (LQR) controllers is employed. Systems with conventional LQR controllers present good stability properties and are optimal with respect to a certain performance index. However, LQR control does not assure robust stability when the system is highly uncertain. In this paper, a convex model of converter dynamics is obtained taking into account uncertainty of parameters. In order to apply the LQR control in the uncertain converter case, the performance index is optimized by using Mesh Adaptive Search (MADS). As a consequence, a new robust control method for dc–dc converters is derived. This LQR-MADS control is compared with normal LQR design. All the analysis and simulations on the above converter is by MATLAB software. The simulation results show the improvement in voltage output response.

Key words: Linear quadratic regulator (LQR), mesh adaptive search (MADS), DC-DC converter, voltage control

1.0 Introduction

The DC converter is a device which transforms AC power to DC. This device is also known as an AC to DC converter. A Chopper can be considered as a DC equivalent of an AC transformer with a continuously convertible constant. Like a transformer, the converter can be employed for stepwise increase or reduction of DC source voltage. The converters are wildly used for the control of motor voltage in electric cars, ceiling elevators, mine excavation etc. Their specific features are the precise control of acceleration with high efficiency and fast dynamic response. Converters are also employed in DC motors to return the energy to its source. In this way, it results in the saving of energy in the transportation systems in prolonged stoppage. Converters are also used in DC voltage regulators along with an inductor to produce a DC current source especially for the current source inverters.

Some control methods have stated the issue of control through pole placement as with Kelly and Rinne, (2005). Another method is the use of state feedback in the control of DC-DC converters as stated by Keller, et al 2005). In modeling area of DC-DC converters, a variety of models are presented which comprise desirable responses by administration of control methods. Most of the articles have concentrated on design of PI and PID controllers as in Uran and Milanovic, (2003) and Namnabat, *et al.*, (2007)]. The feedback loop is another control method used by He and Luo, (2004). The use of LQR method for the improvement of Buck converter function is the subject presented by Leung, *et al*., (1993), Bayati *et al*., (2007) and Mohammad. (2007)]. Linear state feedback controls are among the simplest way of feedback control scheme especially for system with multi outputs. When the model is obtained in state space, then the state feedback control can be designed based on it. Commonly, the state feedback control gains can be determined by means of linear quadratic regulator (LQR) method via solution of Riccati equation or pole placement method as indicated by Ogata, (2002). However, these approaches still possess trial and error approach of parameter adjustment. Particularly, choosing elements of Q and R matrices in the feedback control design using LQR method has to be done by trial.

In this paper an optimum LQR is designed that can improve the Buck converter response. There is no specific method in LQR design which is based on trial and error. The best constant values for state feedback matrix are laboriously obtained through trial and error, although time consuming. Genetic algorithm is employed to find the best values for LQR controller in a very short time. Therefore, a new method is presented for optimizing the systems with two factors of the least response time and the highest precision.

2.0 Materials and Methods

2.1 Buck Converter Circuit Model

The Buck converter circuit model is depicted in Figure1.

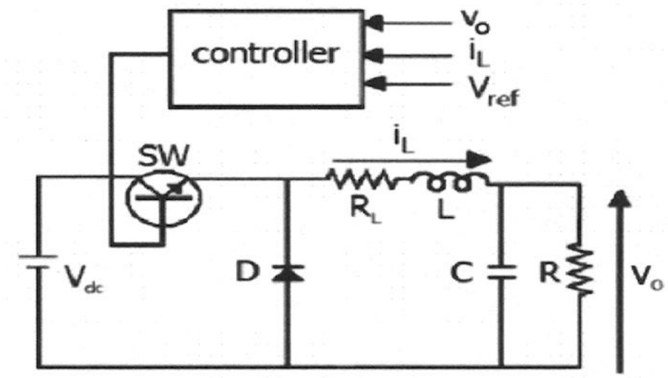
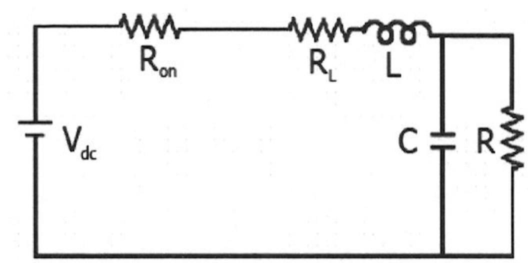


Figure.1: Buck converter

In this model, Vo is the system output voltage and Vref , is the converter voltage. To obtain the converter state equations in low-frequency state, it is required that the system state be studied in two states of on and off as shown in Fig. 2, and Fig.3

2.1 Switch ON:



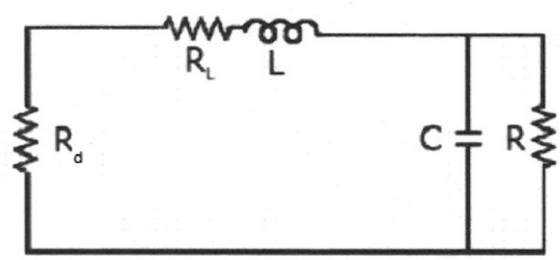


Figure.2: Circuit topology during T_{on}. *Figure.3: Circuit topology during T_{off}*

$Vdc - V_0 = (Rs + RL)iL + L(diL / dt)$,
$i_L = C(dV_0 / dt) + V_0 / R$, ... (1)

$$x_1 = i_L \ , \quad x_2 = V_o \ , \quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$X = A_I X + B_I V_{dc}$ '

2.2 Switch OFF:

$(R_d + R_L)i_L + L(di_L/dt) + V_o = 0$,
$i_L = C(dVo/dt) + Vo/R$,
$X = A_2X + B_2V_{dc}$ .... (2)

$$A_2 = \begin{bmatrix} \frac{-(R_L + R_d)}{L} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} , \ B2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} .$$

Now it is required to show the effect of on and off durations of switch in (1) and (2) to obtain the mean values of state equations.
$X = AX + BV_{dc}$
$A = mA_I + (I - m)A_2$

$B = mB_l + (l - m)B_2$ .... (3)

$$m = \frac{t_{on}}{T_s}$$.

2.2 Linear Quadratic Regulator (LQR) Designing Methodology

The LQR design problem has been extensively investigated for the past four decades. It is possible to improve the converter response by employing the LQR control method. Application of the LQR involves choosing the positive definite state and control input matrices, *Q* and *R* that provide satisfactory closed-loop performance. The closed-loop eigenvalues are related to these weighting matrices. Many methods are available for determining weighting matrices, with the closed loop poles placed in a specified region of the complex plane. A sequential procedure which selects the weighting matrix *Q* and degree of relative stability to position individually and arbitrarily the real parts of the eigenvalues of the optimal LQR system has been presented by Qi. Feng, et al (*2002*). Many methods are available for determining weighting matrices, with the closed-loop poles placed in a specified region of the complex plane. R. L. Haupt and S. E. Haupt,(2004) used sequential method with classical root-locus techniques has been developed for determining the weighting matrices in the frequency domain to retain closed-loop eigenvalues in a desired region in the complex plane. But the main method is based on trial and error, although time consuming. In this method, the feedback gain matrix is determined if *J* energy function is optimized. To achieve equilibrium among range control parameters, response speed, settling time, and proper overshoot rate, all of which guarantee the system stability, the LQR is employed.

2.2.1 LQR Algorithm

For a system in the form of

$X=AX+BU$ ....(4)

The LQR Method determines the K matrix of the equation

$U(t)=-KX(t)$..(5)

to minimize

$J = \int (X^TQX +U^TRU)\, dt$ ....(6)

Function .R and Q matrices express the relation between error and energy expense rate. R and Q are also the definite positive matrices. From the above equation, we have:

$J= \int(X^TQX+X^TK^TRKX)dt$ .... (7)

$=\int(X^T(Q+K^TRK)X)dt$

Subsequent to the solution stages of the equation and optimization of the parameters of the following equation:

$X^T(Q + K^TRK)X =d/dt(X^TPX)$

The following equation is presented:

$X^T(Q+ K^TRK)X=-X^TPX -X^TPX$

$=X^T[(A \_BK)^T P +P(A -BK)]X$ .... (8)

With regard to the values on the both sides of the above equation and with regard to this fact that these equations are true for every X, then the following equation is obtained:

$(A - BK)^TP+ P (A - BK) = -(Q+ K^TRK)$..(9)

If $R= T^T.T$ matrix is positive and definite and T matrix is also gross:

$A^TP+PA+ [(TK-T^T)-^1B^TP]^T*[TK-(T^T)^{-1}B^TP]-PBR^{-1}B^TP+Q=0$.. (10)

For quantifying J in relation to K we have:

$X^T[TK - T^{-T} B^TP]T\ TK - T^{-T} B^TP]X$ ....(11)

This equation is non-negative and the minimum amount takes place when it is zero or when:

$TK=T^{-1}TB^TP$

So;

$K = T^{-1}T^{-T} B^TP= R^{-1}B^T P$ ....(12)

We can also obtain the control matrix for U input.

$U(t) = -KX(t) = -R^{-1}B^TPX(t)$..(13)

And P should be true in the following Riccati equation.

$A^TP+PA-PBR^{-1}B^Tp+Q=O$ .... (14)

In LQR design, Rand *Q* weight matrix which determines the quotient related to the closed loop feedback system within the least time is determined. The selection of R and *Q* has the least dependence on the specification of system administration and requires a long range of trial and error.

2.3 Mesh Adaptive Search Algorithm (MADS)

Mesh Adaptive Search Algorithm **(**MADS**)** optimization routine is an evolutionary technique that is suitable to solve a variety of optimization problems that lie outside the scope of the standard optimization methods. Generally, MADS has the advantage of being very simple in concept, and easy to implement and computationally efficient algorithm. Unlike other heuristic algorithms, such as GA, MADS possesses a flexible and well-balanced operator to enhance and adapt the global and fine tune local search. A historic discussion of direct search methods for unconstrained optimization is presented by R. M. Lewis, V. Torczon, and M. W. Trosset(2000). The authors gave a modern prospective on the classical family of derivative-free algorithms, focusing on the development of direct search methods. The algorithm proceeds by computing a sequence of points that may or may not approaches to the optimal point. The algorithm starts by establishing a set of points called *mesh*, around the given point. This current point could be the initial starting point supplied by the user or it could be computed from the previous step of the algorithm. The mesh is formed by adding the current point to a scalar multiple of a set of vectors called a *pattern*. If a point in the mesh is found to improve the objective function at the current point, the new point becomes the current point at the next iteration. This maybe better explained by the following:

First: The Pattern search begins at the initial point 0 *X* that is given as a starting point by the user. At the first iteration, with a scalar =1 called *mesh size,* the pattern vectors are constructed as [0 1], [1 0], [−1 0] and [0 −1], they may be called direction vectors. Then the Pattern search algorithm adds the direction vectors to the initial point X_0 to compute the following mesh points:X_0 + [1 0],X_0 + [0 1],X_0 +[−1 0] and X_0 + [0 −1].

Figure.4 illustrates the formation of the mesh and pattern vectors. The algorithm computes the objective function at the mesh points in the order shown.

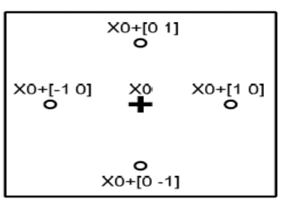


Figure 4: Mesh points and the Pattern illustration

The algorithm polls the mesh points by computing their objective function values until it finds one whose value is smaller than the objective function value of X_0. If there is such point, then the poll is successful and the algorithm sets this point equal to *X*1.After a successful poll, the algorithm steps to iteration 2 and multiplies the current mesh size by 2, (this is called the *expansion factor* and has a default value of 2). The mesh at iteration 2 contains the following points: 2*[1 0] + X_1, 2*[0 1] + X_1, 2*[-1 0] + *X*1 and 2*[0 -1] + *X*1. The algorithm polls the mesh points until it finds one whose value is smaller the objective function value of X_1. The first such point it finds is called X_2, and the poll is successful. Because the poll is successful, the algorithm multiplies the current mesh size by 2 to get a mesh size of 4 at the third iteration because the expansion factor =2.

Second: Now if iteration 3, (mesh size= 4), ends up being unsuccessful poll, i.e. none of the mesh points has a smaller objective function value than the value at X_2 , so the poll is called an unsuccessful poll. In this case, the algorithm does not change the current point at the next iteration. That is, $X_3 = X_2$. At the next iteration, the algorithm multiplies the current mesh size by 0.5, a contraction factor, so that the mesh size at the next iteration is smaller. The algorithm then polls with a smaller mesh size, this is shown by The Math works (2010),A.K.Al-Othman, et al,(2008) and R. M. Lewis, and V. Torczon (2007)].The optimization algorithm will repeat the illustrated steps until it finds the optimal solution for the minimization of the objective function.

2.3.1 Tuning R and Q using MADS

It is not a trivial problem to find the optimal *K* since the control performance depends on choosing weighting matrices. In this paper, weighting matrices are decided by the genetic search to obtain the best *K* for the optimal LQR design.

The target function is as follows:

$$Fobj = \{tr^{0.2} + ts^{0.5} + Ess^{5} + Mp^{2}\} \quad (15)$$

That tr is rise time, ts is settling time, Mp is overshoot and Ess is steady state error.

3.0 Results and Discussions

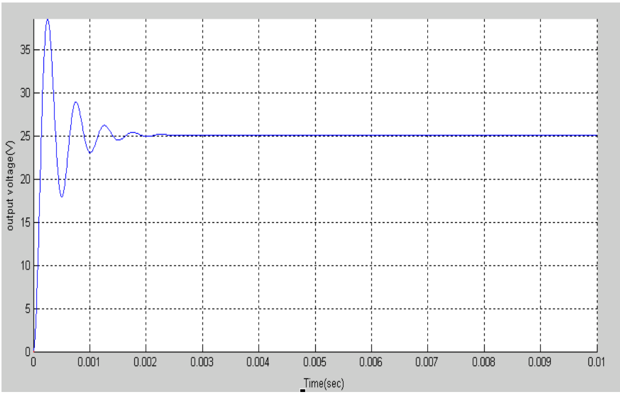


Figure .5: Open loop output voltage response

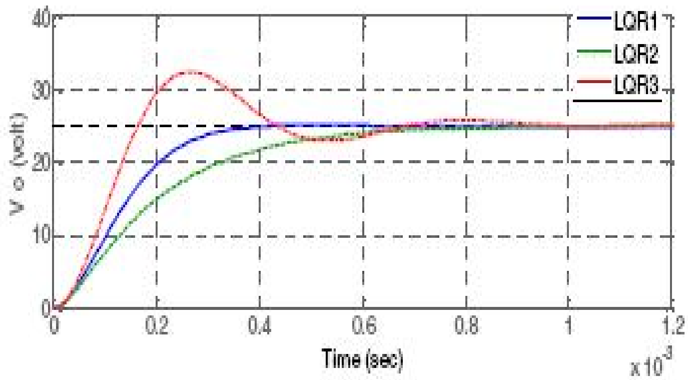


Figure.6:simple LQR output voltage responses

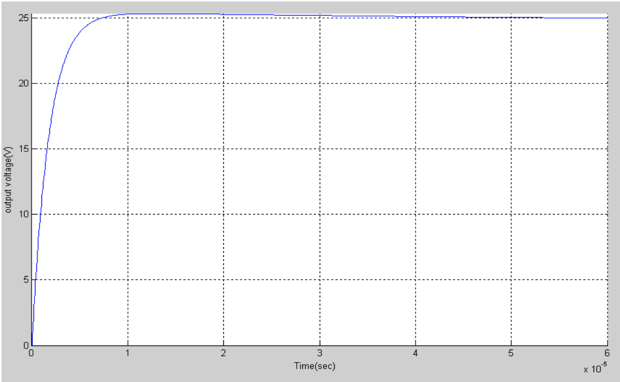


Figure.6: LQR-MADS output voltage response

Table 1: LQR Parameters

	Q11	Q22	R11
LQR1	1.01	0.001	7.95
LQR2	663.9	0.016	1999.4
LQR3	19.1	0.007	1105.6
LQR-MADS	257.5	0.002	1823

Table 2: Simulation results

	rise time(s)	over-shoot (%)	steady state error(%)	settling time(s)
OPEN-LOOP	94.9ms	53.5	0.1	1.56ms
LQR1	0.25ms	1.50	0	0.324ms
LQR2	0.423ms	0	0.05	0.741ms
LQR3	0.15ms	29.1	0.2	0.849ms
LQR-MADS	1.9µs	1.2	0	0.05ms

4.0 Conclusion

The reduction of output voltage ripple of the converter is very important. The optimum design method for linear controller is able to control the dynamic behavior of the converter. The using of mesh adaptive algorithm for the calculation of optimum coefficients of the matrices in the design of LQR controllers can bring about optimum dynamic response. In this paper, a LQR controller is designed to improve the Buck converter performance, in this way, mesh adaptive algorithm is used to optimize the LQR matrices. The results of simulation prove the improvement of the functioning of this converter compared with simple LQR method as there is a significant reduction in rise time and settling time which indicate that the system is quite fast.

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