HYDRODYNAMIC RADIATING FLUID FLOW PAST AN INFINITE VERTICAL POROUS PLATE IN PRESENCE OF CHEMICAL REACTION AND INDUCED MAGNETIC FIELD

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MASTER OF SCIENCE

(Applied Mathematics)

JOMO KENYATTA UNIVERSITY OF AGRICULTURE AND TECHNOLOGY

2016
Hydrodynamic Radiating Fluid Flow Past an Infinite Vertical Porous Plate in Presence of Chemical Reaction and Induced Magnetic Field

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A Thesis Submitted in Partial Fulfilment for the Degree of Master of Science in Applied Mathematics in The Jomo Kenyatta University of Agriculture and Technology

2016
DECLARATION

This research is my original work and has not been presented for a degree award in any other University.

Signature........................................       Date........................................

Kiprop Kibet

This thesis has been submitted for examination with our approval as University Supervisors.

Signature........................................       Date........................................

Professor Mathew Ngugi Kinyanjui
JKUAT, Kenya

Signature........................................       Date........................................

Professor Jackson Kwanza
JKUAT, Kenya.
DEDICATION

To my loving parents, Mr and Mrs Ben Tanui and my siblings Jepchirchir, Jebiwott, Jeptoo and Kipkoech.
ACKNOWLEDGMENTS

My sincere thanks, praise and honor go to the almighty God for his unfailing love, protection, provision and sound health through my study period.

I acknowledge my supervisors, Professor Mathew Kinyanjui, and Professor Jackson Kwanza for their guidance, advice and inspiration throughout my master program at the Jomo Kenyatta University of Agriculture and Technology (JCUAT). I thank the staff of Pure and Applied Mathematics (PAM) department for their assistance and guidance during my studies.

I also thank my family especially my parents for the financial support and prayers throughout my studies.

Further, my regards goes to my colleagues, Richard, Ochieng, Kaiga, Leah, Zachary, Thomas and Mwai for many informative discussions we have held. More so to Dr. Mark who assisted me in computer coding program.

Last, let me extend my regards to Jomo Kenyatta University for giving me a chance to undertake this course. Thank you.
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NOMENCLATURE

\( q \quad \text{Velocity vector of fluid, (} ms^{-1} \text{)} \)

\( H_0 \quad \text{Externally applied transverse magnetic field, (} wb m^{-2} \text{)} \)

\( H_x \quad \text{Induced magnetic field along x-direction, (} wb m^{-2} \text{)} \)

\( C^* \quad \text{Species concentration, (mole/kg)} \)

\( C_p \quad \text{Specific heat at constant pressure, (} JKg^{-1}K^{-1} \text{)} \)

\( C_{\infty}^* \quad \text{Species concentration in free stream, (mole/kg)} \)

\( C_w^* \quad \text{Species concentration at the plate, (mole/kg)} \)

\( D \quad \text{Chemical molecular diffusivity, (} m^2s^{-1} \text{)} \)

\( g \quad \text{Acceleration due to gravity, (} ms^{-2} \text{)} \)

\( Gr \quad \text{Thermal Grashof number} \)

\( Gr_m \quad \text{Mass Grashof number} \)

\( M \quad \text{Magnetic parameter} \)

\( a \quad \text{Absorption coefficient} \)

\( k_l \quad \text{Chemical reaction parameter} \)

\( Pr_m \quad \text{Magnetic Prandtl number} \)

\( Pr \quad \text{Prandtl number} \)

\( \sigma^* \quad \text{Stefan-Boltzmann constant (} W/m^2K^4 \text{)} \)

\( Sc \quad \text{Schmidt number} \)

\( T^* \quad \text{Temperature, (} K \text{)} \)

\( T_w^* \quad \text{Temperature of the fluid at the plate, (} K \text{)} \)

\( T_{\infty}^* \quad \text{Temperature of the fluid in the free stream, (} K \text{)} \)

\( u \quad \text{Velocity components in x-direction, (} ms^{-1} \text{)} \)

\( v_0^* \quad \text{Dimensional Injection velocity, (} ms^{-1} \text{)} \)

\( v_0 \quad \text{Dimensionless injection velocity} \)

\( J \quad \text{Current density, (} Am^{-2} \text{)} \)

\( q_r \quad \text{Radiative heat flux, (} Wm^{-2} \text{)} \)

\( K \quad \text{Thermal conductivity , (} wm^{-1}k^{-1} \text{)} \)

\( K_p \quad \text{Darcy permeability} \)

\( Ec \quad \text{Eckert number} \)
\( \beta \) Coefficient of volume expansion for heat transfer, \( (K^{-1}) \)

\( \beta^* \) Coefficient of volume expansion for mass transfer, \( (K^{-1}) \)

\( \eta \) Magnetic diffusivity

\( \mu_e \) Magnetic permeability, \( (Hm^{-1}) \)

\( \mu \) Viscosity of fluid, \( (kgm^{-1}s) \)

\( \theta \) Dimensionless fluid temperature

\( \kappa \) Thermal viscosity, \( (wm^{-1}k^{-1}) \)

\( \nu \) Kinematic viscosity, \( (m^2s^{-1}) \)

\( \rho \) Density, \( (kg/m^3) \)

\( \sigma \) Electrical conductivity, \( (\Omega^{-1}m^{-1}) \)

\( \tau \) Shearing stress \( (N.m^{-2}) \)

\( \phi \) Dimensionless species concentration

\( \delta_{ij} \) Kronecker delta

\( \sigma_{ij} \) The stress tensor

\( \nabla \) Gradient operator \( \left( i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \right) \)

\( \rho_q \) Volumetric charge density \( (C/m^3) \)

\( E \) Specific internal energy \( (Jkg^{-1}k^{-1}) \)
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ABSTRACT

An investigation of the unsteady magnetohydrodynamic fluid flow, heat and mass transfer characteristic in a viscous, incompressible, electrically conducting and Newtonian fluid past a vertical plate embedded in a porous medium taking into account induced magnetic field, first order chemical reaction and thermal radiation effect was carried out. The dimensionless governing, non-linear boundary layer partial differential equations were solved by an efficient and unconditionally stable finite difference scheme of the Crank-Nicholson type. A computer software is used to iteratively solve the partial differential equations. The numerical solutions for fluid velocity, induced magnetic field, species concentration and fluid temperature are depicted graphically. The effect of various non-dimensional parameters on the fluid flow variables are discussed and physical interpretation given. Hydromagnetic flows, heat transfer and mass transfer have become more important in recent years because of its varied applications in agricultural engineering, petroleum industries, magnetohydrodynamic generators, plasma propulsion in astronautics, nuclear reactor thermal dynamics and ionized-geothermal energy systems.
CHAPTER ONE
INTRODUCTION AND LITERATURE REVIEW

1.1 Introduction

A fluid is a substance that continually deforms under an applied shear stress. Fluids are a subset of the states of matter and include three of the four states—liquids, gases, and plasma.

A fluid flow is steady if its velocity and the thermodynamic properties at each point in the flow region are independent of time, otherwise unsteady if the flow variables are dependent on time.

Fluid flow may be termed as laminar or turbulent. The term laminar is used to refer to a fluid flow in which fluid particles downstream move in an orderly manner in lamina or layers parallel to the solid boundary as opposed to turbulent whereby fluid velocity components have random turbulent fluctuations imposed upon their mean values. Turbulent fluid motion is an irregular condition of flow in which various quantities like velocity and pressure show random variation with time and space. Turbulent flow is also characterised by eddies that causes mixing of layers of the fluid until the layers are no longer distinguishable. This mixing and collision of fluid particles produces heat and the greater the turbulence the larger the amount of heat transfer, as these increased collisions leads to increased dissipation of heat.

The fluid flow depends on the geometry of the surface. For example, flow in pipes and channels are controlled by geometry of cross-section, surface roughness and velocity distribution. For geometrical cross-sections, pipes are generally circular while in open channels they could be triangular, rectangular, trapezoidal, parabolic, elliptic etc. As such the maximum velocity in pipes is at the centre while for channels it is just below the surface of the fluid.

In fluid flow, fluid currents which could be regular or irregular could equalize temperature in the entire fluid. There are two types of convection heat transfer. These are free and forced convection. Free convection flow takes place when density varies due to pressure, concentration and temperature gradients. In forced convection heat transfer is due external forces.

1.2 Magnetohydrodynamics

Magnetohydrodynamics (MHD) is the branch of continuum mechanics which deals with the flow of electrically conducting fluids in electric and magnetic fields. The fluids can be ionized gases (commonly called plasmas) or liquid metals. The flow of
an electrically conducting fluid under a magnetic field in general gives rise to induced electric currents. The magnetic field exerts mechanical forces on the induced electric currents.

1.3 Heat Transfer

Heat transfer involves energy in transit as a result of temperature gradient in the medium. This temperature gradient may arise from various causes such as viscous effects, release of latent heat as fluid vapour condenses and absorption of thermal radiation or radioactivity. Heat transfer takes place mainly in three modes; conduction, convection and radiation.

1.4 Mass Transfer

The relative motion of species as a result of concentration gradients is termed as mass transfer. Thus, mass transfer is caused by concentration difference of the species in a mixture.

1.5 Radiation Heat Transfer

Heat transfer through radiation takes place in form of electromagnetic waves mainly in the infrared region. Radiation emitted by a body is due to thermal agitation of its molecules. Radiation heat transfer can be described in reference to the so called ‘black body’. A black body is defined as a hypothetical body that completely absorbs all wavelengths of thermal radiation incident to it. The emission spectrum of such a black body was first fully described by Max Planck in 1918. All black bodies heated to a given temperature emit thermal radiation. Radiation energy per unit time from a black-body is proportional to the fourth power of the absolute temperature and can be expressed by Stefan-Boltzmann Law as $q_r = \sigma^* T^4 A$. For objects other than ideal black bodies (‘gray bodies’) the Stefan-Boltzmann Law can be expressed as $q_r = \varepsilon \sigma^* AT^4$ where $\sigma^* = 5.6703 \times 10^8 \text{ (W/M}^2\text{K}^4)$ is the Stefan-Boltzmann constant and $\varepsilon$ is emissivity of a black body.

For the gray body the incident radiation (also called irradiation) is partly reflected, absorbed or transmitted. The emissivity coefficient lies in the range $0 < \varepsilon < 1$ depending on the type of material and the temperature of the surface. The fluid is considered to be a gray body and the Rosseland approximation is used to describe the radiative heat
flux in the energy equation, Rohsenow, et al (1998). In this study, the respective radiative heat flux in the y directions is considered. By using the Rosseland approximation, radiation energy per unit time is

\[ q_r = -\frac{4\sigma^*}{3K^*} \frac{\partial T^4}{\partial y} \]  

(1.5.1)

where \( K^* \) is the mean absorption coefficient.

1.6 Viscosity

Viscosity is the measure of resistance to gradual deformation by shear stress or tensile stress. It is a property arising from collision between neighbouring particles in a fluid that is moving at different velocities. A fluid that has no resistance to shear stress is said to be an ideal or inviscid fluid.

1.7 Porous medium

A porous medium is a solid permeated by an interconnected network of pores filled with a fluid (liquid or gas). The pore network is usually assumed to be continuous so as to form two interpenetrating continua, such as in a sponge. Many natural substances such as rocks, soils, bones; and man-made materials such as cement slabs, foam and ceramics are some examples of porous media.

1.8 Boundary layer

The fluid layer in the neighbourhood of solid boundary where effects of fluid friction (viscous effect) are predominant is known as boundary layer. The region outside this layer is known as free stream region where the flow is unaffected by viscous forces.

1.9 Velocity Boundary layer

The velocity boundary layer thickness is defined as the distance away from the surface where the velocity reaches 0.99 that of the free-stream velocity.

1.10 Thermal Boundary layer

Thermal boundary layer develops if the temperature of the fluid at the surface of the plate and the free stream temperature differ. Fluid particles that come into contact
with the plate attain the same temperature as the temperature of the surface. In turn these particles exchange heat energy with those in the adjacent fluid layers, and the temperature gradients develops in the fluid. The region in the fluid in which these temperature gradients exist is the thermal boundary layer.

1.11 Concentration Boundary layer

When concentration of some species at the surface differs from that of the free stream, a concentration boundary layer will develop; the region of the fluid in which the concentration gradients exist is the concentration boundary layer.

1.12 Literature Review

1.12.1 Overview

In this section, we review relevant literature related to chemical reaction, porous medium, thermal radiation, induced magnetic field.

1.12.2 Studies Involving Chemical Reaction

In the cases of chemical reaction, the reaction rate depends on the concentration of the species itself. A reaction is said to be first order if the rate of reaction is directly proportional to concentration itself. ? analyzed the diffusion of chemically reactive species in a laminar boundary layer flow it was deduced that the concentration at a fixed point within the boundary layer decrease with increasing values of Schmidt number. ? presented an analytical solution for heat and mass transfer by laminar flow of a Newtonian, viscous, electrically conducting fluid and heat generation/absorption. The result indicate that increasing the chemical reaction parameter produces decreasing effect on skin friction coefficient and the couple stress coefficient at the wall, while the opposite is true when permeability parameter is increased. ? presented heat and mass transfer effects on a continuously moving isothermal vertical surface with uniform suction by taking into account the homogeneous chemical reaction of first order. ? investigated the chemical reaction effects on vertical oscillating plate with variable temperature and mass diffusion. It is observed that the velocity increases with decreasing phase or chemical reaction parameter.
1.12.3 **Studies Involving Porous Medium**

There has been a renewed interest in studying magnetohydrodynamic (MHD) flow and heat transfer in porous and non-porous media due to the effect of magnetic fields on the boundary layer flow control and on the performance of many systems using electrically conducting fluids. Obtained an approximate solution to the problem of an unsteady flow past an infinite vertical plate with constant suction and embedded in a porous medium with oscillating plate. It is found that increase in porosity parameter depresses the fluid velocities and shear stress in the regime. Also it has been found that, when the conduction-radiation increased, the fluid velocities as well as temperature profiles were decreased. The unsteady flow through a highly porous medium in the presence of radiation was studied by investigated the effect of transverse periodic permeability oscillating with time on the heat transfer flow of a viscous incompressible fluid through a highly porous medium bounded by an infinite vertical porous plate, by means of series solution method.

Carried out an analytical study of a two dimensional unsteady MHD free convective flow past a vertical porous plate immersed in a porous medium with Hall currents, thermal diffusion and heat source. The influence of certain flow parameters on velocity, temperature, species concentration, and shearing stress at the plate were investigated. This study concluded that the concentration at the surface of the plate increases under the Soret effect; and the Soret effect causes the main flow shear stress to rise and the crossflow shear stress to fall. A decrease in the Soret effect leads to an increase in the main flow and crossflow velocities.

1.12.4 **Studies Involving Thermal Radiation**

Over the past years, this problem attracted the attention of several researchers. Studied the effect of transpiration on combined heat and mass transfer in mixed convection along a vertical plate. For thermally assisted flow, he found that the local surface shear, heat and mass transfer rates decrease owing to suction and this trend reversed for blowing of fluid. investigated the effects of radiation on the oscillatory flow of a gray gas, absorbing-emitting in presence induced magnetic field and analytical solutions were obtained with help of perturbation technique. They found out that the mean velocity decreases with the Hartmann number, while the mean temperature decreases as the radiation increases. In all above studies, the stationary vertical plate is considered. studied the effects of thermal radiation and free convection flow past a moving vertical plate. The governing equations were solved analytically analyzed
the effects of mixed convection and mass transfer of three-dimensional oscillatory flow of a viscous incompressible fluid past an infinite vertical porous plate in presence of transverse sinusoidal suction velocity oscillating with time and a constant free stream velocity.

? investigated the flow of fluid through porous medium bounded by vertical channel with slip-flow condition and in the presence of thermal radiation and the fluid is of optically thin with relatively low-density. It is found that velocity increase with increase in permeability of porous plate and decrease with increase of radiation parameter. ? investigated the effects of radiation and chemical reaction on a steady mixed convective heat and mass transfer past an infinite vertical permeable plate with constant suction taking into account the induced magnetic field. It is found that the effect of magnetic parameter and magnetic Prandtl number is to increase the velocity and temperature fields while the induced magnetic field is negatively increasing with them. ? investigated the effects of porosity and magnetohydrodynamic on a horizontal channel flow of a viscous incompressible electrically conducting, Newtonian and radiating fluid through a porous medium in the presence of thermal radiation and transverse magnetic field. It is found that an increase in porosity parameter is found to depress the fluid velocities and shear stress in the regime. Also it has been found that, when the conduction-radiation increased, the fluid velocities as well as temperature profiles were decreased. It has been found that, when the chemical reaction parameter increased, the fluid velocities as well as concentration profiles were decreased. . ? studied the problem of an unsteady flow past an infinite vertical permeable plate with constant suction and transverse magnetic field with oscillating plate temperature.

? studied the effects of radiation, heat generation and viscous dissipation on MHD free convection flow along a stretching sheet. The study found that larger values of buoyancy parameter can be used to control the temperature and concentration boundary layers; and that suction stabilizes the boundary layer growth. The boundary layers were found to be highly influenced by the Prandtl number. They concluded that magnetic field can be used to control the flow characteristics and has significant effect on heat and mass transfer. Increasing radiation reduced the momentum boundary layer and the thermal boundary layer thicknesses. The presence of a heavier species (large Sc) decreased the fluid velocity, heat transfer and the concentration in the boundary layer. Large values of heat source parameter Q had a significant effect on the velocity and temperature distributions whereas such large values reduced the concentration distribution in the boundary layer. Eckert number was found to have a significant effect on the boundary layer growth.

Analytical and numerical solutions of a non-linear MHD flow with heat and mass trans-
fer characteristics of an incompressible, viscous, electrically conducting and Boussi-
nesq’s fluid over a vertical oscillating plate embedded in a Darcian porous medium in
the presence of thermal radiation effect have been presented by ? They found that an
increase in porosity parameter depresses fluid velocities and shear stress in the regime.
Also it has been found that, when the conduction-radiation is increased, the fluid ve-
locity and the temperature profiles decreased.

1.12.5 Studies involving Induced Magnetic Field

? studied viscous fluid flow past a hot vertical porous plate. The flow parameters in this
study were analyzed under the assumptions that the suction velocity was constant and
the wall temperature was spanwise cosinusoidal. The solutions for the velocity, the
temperature, skin friction and rate of heat transfer were obtained using perturbation
method. The study observed that both the velocity and the skin friction decrease as
the Magnetic parameter increases. The values of all flow quantities in the magnetic
case were less than the values in the non-magnetic case. The study also found that
the velocity and the skin friction increased with increasing suction.? investigated the
Hydromagnetic flow of a viscous incompressible fluid due to uniformly accelerated
motion of an infinite flat plate in the presence of a magnetic field fixed relative to the
plate and he found that velocity at any point and at any instant decreases when the
strength of the magnetic field is increased. ? presented the model for Darcian and
non-Darcian effects of the porous medium and the Hall effects of magneto hydrody-
namics. It is assumed that the magnetic Reynolds number is small so that the induced
magnetic field is neglected. The flow is assumed unsteady, laminar, and incompress-
able. ? studied the MHD Stokes problem for a vertical infinite plate in a dissipative
rotating fluid with Hall currents.? presented work on MHD free convection heat and
mass transfer of a heat generating fluid past an impulsively started infinite porous plate
with Hall currents and radiation absorption. Subhas et al. (2001) analyzed the effect of
magnetic field on the visco-elastic fluid flow and heat transfer over a non-isothermal
stretching sheet with internal heat generation. The solutions for heat transfer charac-
teristics were evaluated numerically for different parameters such as Prandtl number,
magnetic field, suction and visco-elasticity. The study concluded that visco-elasticity
decreased the temperature profiles in the flow field for small values of the Prandtl num-
ber; and that the temperature profiles decreased with increase in the strength of the
magnetic field. Magneto hydrodynamic mixed free–forced heat and mass convective
steady incompressible laminar boundary layer flow of a gray optically thick electric-
cally conducting viscous fluid past a semi-infinite vertical plate for high temperature
and concentration differences have studied by? considered an exact solution for the hydro magnetic natural convection boundary layer flow past an infinite vertical flat plate under the influence of a transverse magnetic field with magnetic induction effects and the transformed ordinary differential equations are solved exactly. studied the thermal radiation effects on flow past an impulsively started infinite vertical plate with uniform temperature and variable mass diffusion in the presence of transverse applied magnetic field. The governing equations are solved by the Laplace-transform technique. presented the magneto hydrodynamic transient convective radiative heat transfer in an isotropic, homogenous porous regime adjacent to a hot vertical plate using the Laplace transform technique. considered the natural convection flow from an inclined, semi-infinite, impermeable flat plate embedded in a variable porosity porous medium due to solar radiation and in the presence of an externally applied magnetic field. have presented the magnetohydrodynamic natural convective flow of an electrically conducting and viscous incompressible fluid in a vertical channel due to symmetric heating in the presence of induced magnetic field. It is clear that when M increases the velocity decreases in central region and reverse trend is observed near the channel wall.

1.13 Statement of the Problem

Despite the numerous investigations done on various areas of MHD by the above mentioned scientists and mathematicians, no study has been done on the effects of mass and heat transfer on unsteady MHD free convective flow past a vertical porous plate with induced magnetic field, first order chemical reaction, thermal radiation and injection. An approximate solution to the problem using finite difference method (FDM) is obtained. The velocity, temperature, concentration and induced magnetic field is obtained as a variable of time \((t)\) and space\((y)\).

1.14 Hypothesis

The null hypothesis of this study is that the radiation, concentration, permeability and magnetic field has no affect the flow past a vertical porous plate.
1.15 Justification

It is difficult to fully explain many physical phenomena. Even when an explanation exists, the mathematical prediction may not be exact since there are many assumptions. In most MHD studies, the assumption is that the flow is laminar and streamlined. The laminar flow is what we consider in our case. We also take into account porosity of the plate as well as effects of radiation, concentration and induced magnetic field.

Free convection flow involving coupled heat and mass transfer occurs frequently in nature and in industrial processes. A few representative fields of interest in which combined heat and mass transfer plays an important role are designing chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and groves of fruit trees, crop damage due to freezing, and environmental pollution.

Hydromagnetic flows and heat transfer have become more important in recent years because of its varied applications in agricultural engineering and petroleum industries. Recently, considerable attention has also been focused on new applications of magnetohydrodynamics (MHD) and heat transfer such as metallurgical processing. Melt refining involves magnetic field applications to control excessive heat transfer rate. Other applications of MHD heat transfer include MHD generators, plasma propulsion in astronautics, nuclear reactor thermal dynamics and ionized-geothermal energy systems.

The viscous dissipation heat in the natural convective flow is important, when the flow field is of extreme size or at low temperature or in high gravitational field. Such effects are also important in geophysical flows and also in certain industrial operations and are usually characterized by the Eckert number. Whenever the temperature of surrounding fluid is high, the radiation effects play an important role and this situation does exist in space technology.

The study of heat and mass transfer with chemical reaction is of great practical importance to engineers and scientists because of its almost universal occurrence in many branches of science and engineering. Possible applications of this type of flow can be found in many industries like power industry and chemical process industries.
1.16 Objectives

1.16.1 General Objective

To determine the effects of induced magnetic field, thermal radiation, concentration and injection velocity on the magnetohydrodynamic fluid flow past an infinite vertical porous plate.

1.16.2 Specific Objectives

1. To develop the final set of governing equations for the MHD problem which are the induction equation, momentum equation, concentration equation and energy equation.

2. To determine the velocity distribution of the fluid flow past a vertical plate.

3. To determine effects of injection parameter, Schmidt number, radiation parameter, Eckert number, Magnetic parameter, Permeability parameter, Grashof number and modified Grashof number on the velocity, the temperature, induction and the concentration distributions on the flow field variable.

The assumptions and equations governing MHD flow past a vertical plate embedded in a porous medium are given in the next chapter.
CHAPTER TWO
GOVERNING EQUATIONS

2.1 Introduction

In this chapter, equations governing the flow of an incompressible, Newtonian fluid past a vertical infinite plate are given taking into consideration assumptions of the fluid flow. The fundamental equations to be considered include mass conservation equation, momentum conservation equation, induction equation and equation of energy.

2.2 Assumptions

The following assumptions are used for the research problem:

1. The fluid is incompressible.
2. The plate is non-conducting.
3. Fluid is Newtonian.
4. Fluid flow is assumed to be laminar.
5. The fluid flow is unsteady.

2.3 General Governing Equations

2.3.1 Equation of continuity

The continuity equation is based on the principle of conservation of mass. It states that in any steady state the rate at which mass enters the system equals to rate at which mass leaves the system.

The equation can be expressed in tensor form as;

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} (\rho u_j) = 0 \quad (2.3.1)$$

If \( \rho \) is assumed to be constant, as in a case of incompressible flow, the mass continuity equation simplified to;

$$\frac{\partial}{\partial x_j} (u_j) = 0 \quad (2.3.2)$$
2.3.2 Equation of Conservation of Momentum

The equation of conservation of momentum is derived from the Newton’s second law of motion, which states that the time rate of change of momentum of a body matter is equal to the net external forces applied to the body. The external force is divided into two types of forces i.e. surface forces (e.g. forces due to static pressure and viscous stresses) and body forces (e.g. gravitational force, centrifugal force, magnetic force or electric fields). The momentum of the body is defined as product of mass and velocity. Thus, when a force is applied to an incompressible fluid of any given mass its velocity changes. This is expressed mathematically in tensor form as follows;

\[ \rho \left( \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) = \rho F_i + \frac{\partial \sigma_{ij}}{\partial x_j} \]  

(2.3.3)

Since this study involves viscous fluids, from Newton’s constitutive law we have

\[ \sigma_{ij} = - \left[ P + \frac{2}{3} \eta \text{div} u \right] \delta_{ij} + \eta \left[ \frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right] \]  

(2.3.4)

Where \( \sigma_{ij}, \delta_{ij} \) and \( \eta \) are the stress tensor, kronecker delta and dynamic viscosity respectively. For incompressible flow, the divergence of \( u \) is negligible (continuity equation) hence equation (2.3.4) reduces to

\[ \sigma_{ij} = -P \delta_{ij} + \eta \left[ \frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right] \]  

(2.3.5)

On substitution of equation (2.3.5) into (2.3.3) and using the fact that the flow is incompressible with invariant viscosity the equation becomes;

\[ \rho \left( \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) = - \frac{\partial P}{\partial x_j} + \mu \nabla^2 u_i + \rho F_i \]  

(2.3.6)

The equation (2.3.6) is the general momentum equation;

2.4 Energy Equation

The equation of conservation of energy is derived from the First Law of Thermodynamics, which states that the amount of heat added to a system \( dQ \) equals to change in internal energy \( dE \) plus the work done i.e. \( dW = pdv \) \( dE = dQ - dW = dQ - pdv. \)

In tensor form, the energy equation can be expressed mathematically as;

\[ \rho \frac{\partial H}{\partial t} + \rho u_j \frac{\partial H}{\partial x_j} = \frac{\partial P}{\partial t} + \mu \nabla^2 u_i + \rho F_i + \phi \]  

(2.4.1)
Where $\phi$ denotes the dissipation function and it represents the time rate at which energy is being dissipated per unit volume through viscosity. Hence we have

$$
\phi = 2 \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 \right] + \left[ \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial z} + \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right)^2 \right] - \frac{2}{3} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)^2
$$

(2.4.2)

The term $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$ reduces to zero since it represents the equation of continuity. Since the fluid is flowing past the plate in the direction of $x$-axis, the contribution of the terms $\frac{\partial v}{\partial x}$ and $\frac{\partial u}{\partial z}$ to viscous dissipation is assumed to be negligible and the terms are therefore dropped from the equation. The viscous dissipation equation (2.4.2) thus reduces to

$$
\phi = \left( \frac{\partial u}{\partial y} \right)^2
$$

(2.4.3)

By Rosseland approximation, radiative heat flux of an optically thin gray gas $q_r$ is expressed as;

$$
\frac{\partial q_r}{\partial y} = -\frac{4\sigma^*}{3K^*} \frac{\partial}{\partial y} \left( 4TT_\infty^3 - 3T^4 \right) = -\frac{16\sigma^* T_\infty^3}{3K^*} \frac{\partial^2 T}{\partial y^2}
$$

(2.4.4)

Following ? we assume that the temperature differences within the flow are sufficiently small so that the term $T^4$ in equation (2.4.4) can be expressed as a linear function of the temperature $T$, using a truncated Taylor series about the free stream temperature $T_\infty$. The order of $T$ and $T_\infty$ are assumed to be more or less equal. So that any product of two respective temperature whose order is higher than four is neglected. This results into the following approximation;

$$
f(T) = T^4 \approx T_\infty^4 + \frac{(T - T_\infty)^2}{2!} - 2T_\infty + \ldots \ldots
$$

(2.4.5)

$$
f(T) = T_\infty^4 + 4TT_\infty^3 - 4T_\infty^4 + 6T_\infty^2 \left( 2TT_\infty + T_\infty^2 \right) + \ldots \ldots
$$

(2.4.6)

$$
f(T) = T^4 \cong 4TT_\infty^3 - 3T_\infty^4
$$

(2.4.7)

The equation (2.4.1) can be simplified using the definition of $H$ which is given as
\[ H = E + \frac{P}{\rho} \]  
(2.4.8)

in which \( E \) is specific internal energy. In differential form (2.4.8) can be written as

\[ dH = dE + \frac{1}{\rho} dP + (Pd) \left[ \frac{1}{\rho} \right] \]  
(2.4.9)

Applying the first and second laws of thermodynamics the equation (2.4.9) yields

\[ dE = T dS - Pd \left[ \frac{1}{\rho} \right] \]  
(2.4.10)

In this case \( S \) is specific enthalpy. Substituting equation (2.4.10) into (2.4.9) yields

\[ dH = TdS + \frac{1}{\rho} dP \]  
(2.4.11)

Since enthalpy is a fluid property it can be expressed as \( S = S(P,T) \) so that on differentiating both sides of this equation yields

\[ dS = \left[ \frac{\partial S}{\partial P} \right]_T dP + \left[ \frac{\partial S}{\partial T} \right]_P dT \]  
(2.4.12)

On substitution of the following generalized thermodynamic relations \( \left[ \frac{\partial S}{\partial P} \right]_T = \frac{-\beta^*}{\rho}, \left[ \frac{\partial S}{\partial T} \right]_P = \frac{C_p}{T} \) and \( \left[ \frac{\partial^2 S}{\partial T^2} \right]_P = \frac{-\beta}{\rho} \) to equation (2.4.12) yields

\[ dS = -\frac{\beta^*}{\rho} dP + \frac{C_p}{T} dT \]  
(2.4.13)

In which \( \beta^* \) is the volumetric coefficient of expansion and \( C_p \) is specific heat at constant pressure. Substituting equation (2.4.13) into (2.4.11) we obtain

\[ dH = C_p dT + \frac{1}{\rho} (1 - \beta^* T) dP \]  
(2.4.14)

Making use of equations (2.4.14) and (2.4.11) and further substituting into equation (2.4.10), the energy equation can be expressed mathematically as

\[ \rho C_p \frac{DT}{Dt} = K_T \nabla^2 T + \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2 + \phi - \frac{\partial q_r}{\partial y} \]  
(2.4.15)

In this study we consider the flow of viscous incompressible fluid with \( C_p \) being a constant, then considering the local radiant equation (2.4.4) the energy equation yield;

\[ \rho C_p \frac{DT}{Dt} = K_T \nabla^2 T + \mu \left( \frac{\partial u}{\partial y} \right)^2 + \frac{16 \sigma^* T^3 \partial^2 T}{3 K^* \partial y^2} \]  
(2.4.16)
2.5 The Mass Transfer Equation

The equation of species concentration is based on the law of conservation of mass. This equation is used when the porous medium is saturated with fluid and obeys Darcy’s law. Convection is one of the major modes of mass transfer. Convective mass transfer takes place through both diffusion, the random Brownian motion of individual particles in the fluid, and advection, in which dissolved substances or heat are carried along with bulk fluid flow. In advection, the species spread out from the path expected to be followed by the advection alone. The equation of species concentration is given as

\[
d\frac{dC}{dt} = D \nabla^2 C - k_f C
\]  

(2.5.1)

Where \(D\) is the molecular diffusion coefficient and the last term represents change in species concentration due to chemical reaction.

The species concentration is assumed to be a function of \(y\) and \(t\). Equation (2.5.1) assumes that the thermal and molecular diffusion rates are equal. Since this study assumes there is chemical reaction, equation (2.5.1) becomes

\[
\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial y^2} - v \frac{\partial C}{\partial y} - k_f C
\]  

(2.5.2)

2.6 Electromagnetic Equations

These equations show the relationship between electric field intensity \(\vec{E}\), the magnetic induction vector \(\vec{B}\), the magnetic field strength \(\vec{H}\) and the induction current density \(\vec{J}\), these are;

\[
\nabla \times \vec{H} = \vec{J}
\]  

(2.6.1)

From Gauss’s law of magnetism

\[
\nabla \cdot \vec{B} = 0
\]  

(2.6.2)

From Faraday’s law which states that the electromotive force generated around a closed loop equals minus the rate of change of magnetic flux through the loop.

\[
\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}
\]  

(2.6.3)

When electric current density \(\vec{J}\) flows through a fluid, there is a force per unit volume
\[ \vec{J} \times \vec{B} = \left( \frac{\vec{B} \cdot \nabla}{\mu_e} \right) \vec{B} - \nabla \left( \frac{B^2}{2\mu_e} \right) \] (2.6.4)

Where \( \mu_e \) is magnetic permeability and the first term on the right hand side of the equation (2.6.4) is magnetic tension force which is restoring force that acts to straighten bent magnetic field lines and the second term is the magnetic pressure force which is energy density associated with magnetic field.

In the next chapter, mathematical formulation is done where the equations of momentum, energy, induction and species concentration are non-dimensionalised and finite difference method is used to solve the equations numerically.
CHAPTER THREE
MATHEMATICAL FORMULATION

3.1 Model Description

The two-dimensional unsteady magnetohydrodynamic heat and mass transfer flow of a Newtonian, electrically-conducting and viscous incompressible fluid past a porous vertical infinite plate with induced magnetic field and radiation has been considered as in Figure (3.1.1). The vertical plate is permeable to allow for possible blowing or suction, and is infinite in the x-axis direction which is the direction of fluid flow and y-axis is normal to it. Let \( \vec{q} = (u(y), v, 0) \) be the fluid velocity and \( \vec{H} = (\vec{H}_x(y), \vec{H}_y, 0) \) be the magnetic induction vector at a point \((x, y, z)\) in the fluid. The x-axis is taken along the plate in the upward direction, y-axis is normal to the plate into the fluid region. Since the plate is infinite in length in x-direction, therefore all the physical quantities are assumed to be independent of x.

Injection of the fluid takes place through the porous wall of the vertical plate with uniform velocity \( v_0 \) which is greater than zero. It is assumed that no polarization voltages exists since the plate is insulated. The wall is maintained at constant temperature \( T_w \) and concentration \( C_w \) higher than the ambient temperature \( T_\infty \) and concentration \( C_\infty \) respectively. The plate is non-conducting and the applied magnetic field is of uniform strength \( H_0 \) and applied transversely to the direction of the main stream taking into account the induced magnetic field;

![Figure 3.1.1: Fluid Flow Configuration](image)

3.1.1 Momentum Equation

From equation (2.3.6) the momentum equation is
\[ \rho \left( \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) = -\frac{\partial P}{\partial x} + \mu \nabla^2 u_i + \rho g + \mu_e \left( \vec{J} \times \vec{H} \right) \]  

(3.1.1)

Density has been assumed to be a linear function of the temperature and species concentration, and therefore the Boussinesq approximation has been used. The buoyancy force also contributes towards driving the fluid.

\[ \rho = \rho_\infty + \left( \frac{\partial \rho}{\partial T} \right)_p (T - T_\infty) + \left( \frac{\partial \rho}{\partial C} \right)_p (C - C_\infty) \]  

(3.1.2)

The volumetric coefficient of thermal expansion is given as

\[ \beta = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial T} \right)_p \]  

(3.1.3)

The coefficient of thermal expansion due to concentration gradient is given as

\[ \beta^* = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial C} \right)_p \]  

(3.1.4)

From equation (3.1.4) and (3.1.3) the equation (3.1.2) become

\[ \rho_\infty - \rho = \rho \beta (T - T_\infty) + \rho \beta^* (C - C_\infty) \]  

(3.1.5)

To determine the pressure gradient term, the momentum equation is evaluated at the edge of the boundary layer where \( \rho \rightarrow \rho_\infty \). When the fluid is at momentum equilibrium, the pressure gradient balanced due to the variation of the fluid density. Thus

\[ -\frac{\partial P}{\partial x} = \rho_\infty g \]  

(3.1.6)

The body force term in the momentum equation along the x-axis direction is

\[ -\nabla P - \rho g = -\frac{\partial P}{\partial x} - \rho g \]  

(3.1.7)

Substituting the value of \( \frac{\partial P}{\partial x} \) given in equation (3.1.6) into equation (3.1.5): The overall pressure gradient is given as

\[ -\frac{\partial P}{\partial x} = \rho_\infty g - \rho g = \rho \beta (T - T_\infty) + \rho \beta^* (C - C_\infty) \]  

(3.1.8)

Due to the porosity of the medium and considering the Brinkman-Forchheimer-extended Darcy model the general macroscopic momentum equation for the saturated porous medium is considered. Considering equations (3.1.8) for the pressure gradient, the porosity of the plate, the viscous dissipation of the fluid and the magnetic field the
equation of momentum is given as;

\[
\left( \frac{\partial u}{\partial t} + v_0 \frac{\partial u}{\partial y} \right) = g \beta (T - T_w) + g \beta^* (C - C_w) + v \frac{\partial^2 u}{\partial y^2} - u \frac{\nu}{K_p} + \frac{\mu_e H_0}{\rho} \frac{\partial H_x}{\partial y}
\]  

(3.1.9)

3.1.2 Energy Equation

From equation (2.4.16) the energy equation for the fluid flow past a vertical plate in consideration of radiation is given as;

\[
\frac{\partial T}{\partial t} + v_0 \frac{\partial T}{\partial y} = \frac{K_T \partial^2 T}{\rho C_p \partial y^2} + \frac{\mu}{\rho C_p} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{1}{\rho C_p} \frac{16 \sigma^* T_\infty^3 \partial^2 T}{3K^* \partial y^2}
\]  

(3.1.10)

3.1.3 Concentration Equation

Considering the equation of species concentration (2.5.2) for the fluid flow past a vertical plate is given as:

\[
\frac{\partial C}{\partial t} + v_0 \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - K_1 (C - C_\infty)
\]  

(3.1.11)

3.1.4 Induction Equation

Since the magnetic field is varied we have to solve the magnetic induction equation as given from equation (2.6.4)

\[
\frac{\partial \vec{B}}{\partial t} = \nabla \times \left( \vec{q} \times \vec{B} \right) + \frac{1}{\sigma \mu_e} \nabla^2 \vec{B}
\]  

(3.1.12)

Equation (3.1.12) describes the evolution of magnetic field. The first term on the right-hand side is the advective term that describes the interaction of the field with the fluid flow velocity. The second term on the right hand side is a diffusive term.

\[
\vec{q} \times \vec{B} = \begin{vmatrix} i & j & k \\ u & v_0 & 0 \\ B_x & B_y & 0 \end{vmatrix} = \tilde{k} (u B_y - v_0 B_x)
\]  

(3.1.13)
\[ \nabla \times (\vec{q} \times \vec{B}) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & (uB_y - v_0B_x) \end{vmatrix} = \tilde{i} \left( B_y \frac{\partial u}{\partial y} - v_0 \frac{\partial B_x}{\partial y} \right) - j \frac{\partial}{\partial x} (uB_y - v_0B_x) \]  

(3.1.14)

Substituting equation (3.1.14) into (3.1.12) we get

\[ \frac{\partial \vec{B}}{\partial t} = \tilde{i} \left( B_y \frac{\partial u}{\partial y} - v_0 \frac{\partial B_x}{\partial y} \right) + \frac{1}{\sigma \mu_e} \nabla^2 \vec{B} \]  

(3.1.15)

\[ B = \mu_e H, \quad \vec{B} = (B_x, B_y, 0) and \quad \vec{H} = (H_x, H_0, 0) \]  

(3.1.16)

Substituting equation (3.1.16) in (3.1.15) we get

\[ \frac{\partial H_x}{\partial t} + v_0 \frac{\partial H_x}{\partial y} = H_0 \frac{\partial u}{\partial y} + \frac{1}{\sigma \mu_e} \frac{\partial^2 H_x}{\partial y^2} \]  

(3.1.17)

The equations (3.1.9), (3.1.11), (3.1.10) and (3.1.17) are the specific equations for the problem and are solve simultaneously. Subject to initial and boundary conditions

For \( t > 0, y = 0 : u = 0, v = v_0, T = T_w, C = C_w, H_x = 0 \)

\( t > 0, y = \infty : u \to U_0, T \to T_\infty, C \to C_\infty, H_x \to 0 \)

3.2 Non-Dimensionalization

Dimensional analysis is a mathematical technique which makes use of the study of the dimensions for solving several engineering problems. Each physical phenomenon can be expressed by an equation giving relationship between different quantities, such quantities are dimensional and non-dimensional. Dimensional analysis helps in determining a systematic arrangement of the variables in the physical relationship, combining dimensional variables to form non-dimensional parameters. Dimensional analysis is used to plan model tests and present experimental results in a systematic manner; thus making it possible to analyse the complex fluid flow phenomenon.

3.2.1 Non-dimensional Parameters.

The following non-dimensional numbers plays a significant role in our study;
3.2.1.1 Prandtl number, $Pr$

Prandtl number is the ratio of viscous force to thermal force. It throws light on the relative importance of viscous dissipation to the thermal dissipation. A high prandtl number indicates that heat diffuses very slowly relative to momentum and vice versa. The Prandtl number is given by;

$$Pr = \frac{\rho C_p \nu}{K_T}$$

(3.2.1)

3.2.1.2 Local Temperature Grashof number, $Gr$

This is a non-dimensional number which normally occurs in natural convection problem. It is the ratio of thermal buoyancy forces to viscous hydrodynamic forces.

$$Gr = \frac{\nu g \beta (T_w - T_\infty)}{U_0^3}$$

(3.2.2)

3.2.1.3 Local mass Grashof number, $Gr_m$

It is the ratio of species buoyancy forces to viscous hydrodynamic forces.

$$Gr_m = \frac{\nu g \beta^* (C_w - C_\infty)}{U_0^3}$$

(3.2.3)

3.2.1.4 Eckert number, $Ec$

This expresses the relationship between the kinetic energy in the fluid flow and the enthalpy. It represents the conversion of kinetic energy into internal energy by work that is done against the viscous fluid stresses.

$$Ec = \frac{U_0^2}{C_p (T_w - T_\infty)}$$

(3.2.4)

3.2.1.5 Schmidt number, $Sc$

The Schmidt number quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic (velocity) and the concentration (species) boundary layers. It physically relates the relative thickness of the hydrodynamic layer and mass transfer boundary layer.

$$Sc = \frac{\nu}{D_m}$$

(3.2.5)
3.2.1.6 Hartmann Number

This is the ratio of magnetic force to viscous force

\[ M = \sqrt{\frac{\mu_e H_0}{U_0 \rho}} \]  

(3.2.6)

3.2.1.7 Magnetic Prandtl Number

This approximates ratio of momentum diffusivity and magnetic diffusivity

\[ Pr_M = \sigma \mu_e v \]  

(3.2.7)

3.2.1.8 Permeability Parameter

Permeability is a measure of the ability of a porous material to allow fluids to pass through it.

\[ \chi_i = \frac{v^2}{U_0^2 K_p} \]  

(3.2.8)

3.2.2 Non-dimensionalizing equations governing fluid flow past a vertical porous plate.

We define the following non-dimensional variables for the the present MHD problem.

\[ y^* = \frac{y U_0}{v} \quad u^* = \frac{u}{U_0} \quad \theta = \frac{T - T_\infty}{T_w - T_\infty} \quad \phi = \frac{C - C_\infty}{C_w - C_\infty} \quad t^* = \frac{t U_0^2}{v} \quad v_0^* = \frac{v_0}{U_0} \quad H^* = \sqrt{\frac{\mu_e H_x}{\rho U_0}} \]  

(3.2.9)

In order to transform the equations of continuity, momentum, energy, and concentration into their respective non-dimensional form, the following analysis is first carried out:

\[ \frac{\partial u}{\partial t} = \frac{\partial u}{\partial u^*} \frac{\partial u^*}{\partial t^*} = \frac{U_0^3}{v} \frac{\partial u^*}{\partial t^*} \]  

(3.2.10)

\[ v_0 \frac{\partial u}{\partial y} = U_0 v_0^* \frac{\partial u}{\partial u^*} \frac{\partial u^*}{\partial y^*} \frac{\partial y^*}{\partial y} = \frac{U_0^3 v_0^*}{v} \frac{\partial u^*}{\partial y^*} \]  

(3.2.11)
Substituting equations (3.2.10), (3.2.11), (3.2.12) into equation (3.1.19); and divide equation (3.2.23) by 
3 \frac{\partial \nu}{\partial t^2} \frac{\partial y^*}{\partial y} = \frac{(T_w - T_\infty) U_0^2}{\nu} \frac{\partial \theta^*}{\partial t^*} \quad (3.2.13)
\frac{\partial T}{\partial y} = \frac{\partial T}{\partial \theta} \frac{\partial \nu^*}{\partial t} = (T_w - T_\infty) \frac{U_0^2}{\nu} \frac{\partial \theta}{\partial y^*} \quad (3.2.14)
\frac{\partial^2 T}{\partial y^2} = \frac{\partial}{\partial y^*} \left( \frac{\partial T}{\partial y} \right) \frac{\partial y^*}{\partial y} = (T_w - T_\infty) \frac{U_0^2}{\nu} \frac{\partial^2 \theta}{\partial y^2} \quad (3.2.15)
\frac{\partial u}{\partial y} = \frac{\partial u}{\partial u^*} \frac{\partial \nu^*}{\partial y} = \frac{U_0^2}{\nu} \frac{\partial u^*}{\partial y^*} \quad (3.2.16)
\frac{\partial C}{\partial t} = \frac{\partial C}{\partial \phi} \frac{\partial \nu^*}{\partial t} = (C_w - C_\infty) \frac{U_0^2}{\nu} \frac{\partial \phi}{\partial t^*} \quad (3.2.17)
\frac{\partial C}{\partial y} = \frac{\partial C}{\partial \phi} \frac{\partial \nu^*}{\partial y} = (C_w - C_\infty) \frac{U_0}{\nu} \frac{\partial \phi}{\partial y^*} \quad (3.2.18)
\frac{\partial^2 C}{\partial y^2} = \frac{\partial}{\partial y^*} \left( \frac{\partial C}{\partial y} \right) \frac{\partial y^*}{\partial y} = (C_w - C_\infty) \frac{U_0^2}{\nu^2} \frac{\partial^2 \phi}{\partial y^2} \quad (3.2.19)
\frac{\partial H_x}{\partial t} = \frac{\partial H_x}{\partial H^*} \frac{\partial H^*}{\partial y^*} \frac{\partial y^*}{\partial y} = \frac{2H^* \rho U_0^3}{\mu_c \nu} \frac{\partial H^*}{\partial y^*} \quad (3.2.20)
\frac{v_0}{\nu} \frac{\partial H_x}{\partial y} = v_0 U_0 \frac{\partial H_x}{\partial H^*} \frac{\partial H^*}{\partial y^*} \frac{\partial y^*}{\partial y} = \frac{2H^* \rho U_0^3}{\mu_c \nu} \frac{\partial H^*}{\partial y^*} \quad (3.2.21)
\frac{\partial^2 H_x}{\partial y^2} = \frac{\partial}{\partial y^*} \left( \frac{\partial H_x}{\partial y} \right) \frac{\partial y^*}{\partial y} = \frac{2H^* \rho U_0^3}{\mu_c \nu^2} \frac{\partial^2 H^*}{\partial y^2} \quad (3.2.22)

Substituting equations (3.2.10), (3.2.11), (3.2.12) into equation (3.1.19);

\frac{U_0^3}{\nu} \frac{\partial u^*}{\partial t^*} + U_0 v_0 \frac{U_0^2}{\nu} \frac{\partial u^*}{\partial y^*} = \nu \theta \beta (T_w - T_\infty) + \nu \phi \beta^* (C_w - C_\infty) + \frac{v}{\nu^2} \frac{U_0^3}{\nu^2} \frac{\partial^2 u^*}{\partial y^2} + \frac{2H^* \mu_c H_0 U_0^3}{\nu^2} \frac{\partial H^*}{\partial y^*} = \left( \frac{U_0^3}{\nu} \frac{\partial u^*}{\partial y^*} \frac{v}{K_p} \right) \quad (3.2.23)

and divide equation (3.2.23) by \frac{U_0^3}{\nu} and let v_0^* = S we get;
\[
\frac{\partial u^*}{\partial t^*} + S \frac{\partial u^*}{\partial y^*} = \frac{v g \beta (T_w - T_\infty)}{U_0^3} + \frac{v g \phi \beta^* (C_w - C_\infty)}{U_0^3} + \frac{\partial^2 u^*}{\partial y^{*2}} + M \frac{\partial H^*}{\partial y^*} - \frac{u^* v^2}{U_0^3 K_p}
\]

(3.2.24)

With non-dimensional quantities defined above, \(Gr_T = \frac{v g \beta (T - T_w)}{U_0^3}\) is the local temperature Grashof number \(Gr_C = \frac{v g \beta^* (C - C_w)}{U_0^3}\) is the local mass Grashof number \(\chi_i = \frac{\nu^2}{U_0^3 K_p}\) is permeability parameter \(M = \sqrt{\frac{\mu H_0}{\rho U_0^3}}\) is the magnetic parameter.

\[
\frac{\partial u^*}{\partial t^*} + S \frac{\partial u^*}{\partial y^*} = Gr_T + Gr_m \phi + \frac{\partial^2 u^*}{\partial y^{*2}} + M \frac{\partial H^*}{\partial y^*} - \chi_i u^* \quad (3.2.25)
\]

Substituting equations (3.2.13), (3.2.14) and (3.2.15) into (3.1.10):

\[
(T_w - T_\infty) \frac{U_0^2}{\nu} \frac{\partial \theta}{\partial t^*} + U_0 v_0 (T_w - T_\infty) \frac{\partial \theta}{\partial y^*} = \frac{K_T}{\rho C_p} (T_w - T_\infty) \frac{U_0^2}{\nu} \frac{\partial^2 \theta}{\partial y^{*2}}
\]

\[
+ \frac{\mu}{\rho C_p} \left( \frac{U_0^2}{\nu} \frac{\partial u^*}{\partial y^*} \right)^2 + \frac{1}{\rho C_p} \frac{16 \sigma^* T_\infty^3}{3 k^*} (T_w - T_\infty) \frac{U_0^2}{\nu} \frac{\partial^2 \theta}{\partial y^{*2}}
\]

(3.2.26)

divide equation (3.2.26) by \((T_w - T_\infty) \frac{U_0^2}{\nu}\) we get:

\[
\frac{\partial \theta}{\partial t^*} + S \frac{\partial \theta}{\partial y^*} = \frac{K_T}{\rho C_p \nu} \frac{\partial^2 \theta}{\partial y^{*2}} + \frac{\mu U_0^2}{\rho C_p \nu (T_w - T_\infty)} \left( \frac{\partial u^*}{\partial y^*} \right)^2 + \frac{16 \sigma^* T_\infty^3}{\rho C_p 3 k^* \nu} \frac{\partial^2 \theta}{\partial y^{*2}}
\]

(3.2.27)

With the non-dimensional parameters, \(Pr = \frac{\rho C_p \nu}{k_T}\) is Prandtl number \(Ec = \frac{U_0^3}{C_p (T_w - T_\infty)}\) is Eckert number and \(R = \frac{16 \sigma^* T_\infty^3}{3 k^*}\) is the radiation parameter.

\[
\frac{\partial \theta}{\partial t^*} + S \frac{\partial \theta}{\partial y^*} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^{*2}} + Ec \left( \frac{\partial u^*}{\partial y^*} \right)^2 + R \frac{\partial^2 \theta}{\partial y^{*2}}
\]

(3.2.28)

Also substituting equations (3.2.17), (3.2.18), (3.2.19), (3.2.20) and (3.2.21) into equation (3.1.11):

\[
(C_w - C_\infty) \frac{U_0^2}{\nu} \frac{\partial \phi}{\partial t^*} + v_0 U_0 (C_w - C_\infty) \frac{\partial \phi}{\partial y^*} = D_m (C_w - C_\infty) \frac{U_0^2}{\nu^2} \frac{\partial^2 \phi}{\partial y^{*2}} - K_l (C - C_\infty)
\]

(3.2.29)

and divide equation (3.2.29) by \((C_w - C_\infty) \frac{U_0^2}{\nu}\) we get;

24
\[ \frac{\partial \phi}{\partial t^*} + S \frac{\partial \phi}{\partial y^*} = \frac{D_m}{v} \frac{\partial^2 \phi}{\partial y^{*2}} - \frac{K_i v}{U_0^2} \frac{C - C_\infty}{C_w - C_\infty} \]  

(3.2.30)

With non-dimensional parameters \( Sc = \frac{v}{\nu_m} \) and \( K = \frac{K_i v}{U_0} \). The equation (3.2.30) simplifies to:

\[ \frac{\partial \phi}{\partial t^*} + S \frac{\partial \phi}{\partial y^*} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^{*2}} - k \phi \]  

(3.2.31)

Finally substituting equations (3.2.20), (3.2.21), (3.2.22) and (3.2.16) into equation (3.1.17) which is magnetic induction equation we get:

\[ \frac{2H^* \rho U_0^2}{\mu_e v} \frac{\partial H^*}{\partial y^*} + \frac{2H^* \rho U_0^3 \nu_0}{\mu_e v} \frac{\partial H^*}{\partial y^*} = \frac{1}{\sigma \mu_e} \frac{2H^* \rho U_0^3}{\mu_e v^2} \frac{\partial^2 H^*}{\partial y^{*2}} + H_0 \frac{U_0^2}{v} \frac{\partial u^*}{\partial y^*} \]  

(3.2.32)

then we divide equation (3.2.32) by \( \frac{2H^* \rho U_0^2}{\mu_e v} \) we get;

\[ \frac{\partial H^*}{\partial t^*} + \nu_0 \frac{\partial H^*}{\partial y^*} = \frac{1}{\sigma \mu_e} \frac{\partial^2 H^*}{\partial y^{*2}} + \frac{\mu_e H_0}{2U_0 H^*} \frac{\partial u^*}{\partial y^*} \]  

(3.2.33)

With non-dimensional parameters, \( Pr_M = \frac{\sigma \mu_e v}{v_0} \) is magnetic Prandtl Number and \( v_0^* = S \) is the injection velocity which we substitute in equation (3.3.33) we get;

\[ \frac{\partial H^*}{\partial t^*} + S \frac{\partial H^*}{\partial y^*} = \frac{1}{Pr_M} \frac{\partial^2 H^*}{\partial y^{*2}} + M \frac{\partial u^*}{\partial y^*} \]  

(3.2.34)

The corresponding initial and boundary conditions are;

\[ t^* \geq 0 ; \ y^* = 0 : \ u^* = 0, \ \theta = 1, \ \phi = 1, \ H^* = 0 \]

\[ y^* \to \infty : \ u^* \to 1, \ \theta \to 0, \ \phi \to 0 \ H^* \to 0 \]  

(3.2.35)

We develop numerical scheme corresponding to the final set of equations in view of boundary conditions.

### 3.3 Method of Solution

The final set of governing equations (3.2.25), (3.2.28), (3.2.31) and (3.2.34) cannot be solved analytically since they are highly coupled and nonlinear. The finite difference method is used to obtain an accurate and efficient solution to the boundary value
problem under consideration. To begin discretization process, we define the mesh. This will be a rectangular mesh with the space co-ordinate along the horizontal axis and temporal coordinates along the vertical axis. The space coordinates are subdivided into N-1 intervals of equal length $\Delta y$ so that there are N nodal points and the temporal coordinate is subdivided into K-1 intervals of equal length $\Delta t$ so that there are K nodals. Each nodal is labelled by a pair of indices $i$ and $j$.

![Finite Difference Grid Mesh](image)

**Figure 3.3.1: Finite Difference Grid Mesh**

The Crank-Nicolson schemes are independent of time and space. Since the final sets of governing equations are highly coupled and non-linear, this method is preferred in solving the MHD problem. This method is easiest to use provided the flow variables are initially known. It is also appropriate for highly non-linear and highly coupled equations since all future variables are determined from already known variables. The fully explicit method has a shortcoming in that it is likely to be unstable for large step sizes. This means that the space has to be finely subdivided for convergence.

We evaluate the finite difference schemes for the fluid properties in the sections below

\[ u = \frac{u_{i,j+1} + u_{i,j}}{2} \]  \tag{3.3.1}

\[ u_t = \frac{u_{i,j+1} - u_{i,j}}{\Delta t} \]  \tag{3.3.2}

\[ u_y = \frac{u_{i+1,j+1} - u_{i-1,j+1} + u_{i+1,j} - u_{i-1,j}}{4\Delta y} \]  \tag{3.3.3}

\[ u_{yy} = \frac{u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1} + u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{2(\Delta y)^2} \]  \tag{3.3.4}
\[ \theta = \frac{\theta_{i,j+1} + \theta_{i,j}}{2} \]  
\[ \theta_t = \frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta t} \]  
\[ \theta_y = \frac{\theta_{i+1,j+1} - \theta_{i-1,j+1} + \theta_{i+1,j-1} - \theta_{i-1,j-1}}{4\Delta y} \]  
\[ \theta_{yy} = \frac{\theta_{i-1,j} - 2\theta_{i,j} + \theta_{i+1,j} + \theta_{i-1,j+1} - 2\theta_{i,j+1} + \theta_{i+1,j+1}}{2(\Delta y)^2} \]  
\[ \phi = \frac{\phi_{i,j+1} + \phi_{i,j}}{2} \]  
\[ \phi_t = \frac{\phi_{i,j+1} - \phi_{i,j}}{\Delta t} \]  
\[ \phi_y = \frac{\phi_{i+1,j+1} - \phi_{i-1,j+1} + \phi_{i+1,j-1} - \phi_{i-1,j-1}}{4\Delta y} \]  
\[ \phi_{yy} = \frac{\phi_{i-1,j} - 2\phi_{i,j} + \phi_{i+1,j} + \phi_{i-1,j+1} - 2\phi_{i,j+1} + \phi_{i+1,j+1}}{2(\Delta y)^2} \]  
\[ H_t = \frac{H_{i,j+1} - H_{i,j}}{\Delta t} \]  
\[ H_y = \frac{H_{i+1,j+1} - H_{i-1,j+1} + H_{i+1,j-1} - H_{i-1,j-1}}{4\Delta y} \]  
\[ H_{yy} = \frac{H_{i-1,j} - 2H_{i,j} + H_{i+1,j} + H_{i-1,j+1} - 2H_{i,j+1} + H_{i+1,j+1}}{2(\Delta y)^2} \]

To solve equations (3.2.25), (3.2.28), (3.2.31) and (3.2.34) for the fluid flow problem we use the Crank-Nicolson Method.

### 3.3.0.1 Momentum Equation

Using Crank-Nicolson method and setting \( y^* = y \), \( t^* = t \), \( u^* = u \), \( T^* = T \), \( C^* = C \), \( H^* = H \). Then momentum equation (3.2.25) will be:

\[ u_t + Su_y = Gr\theta + Gr_m\phi + u_{yy} + MH_y - \chi_i u \]
Substituting equations (3.3.1), (3.3.2), (3.3.3) and (3.3.4) into equation (3.3.16) above we get

\[
\frac{u_{i,j+1} - u_{i,j}}{\Delta t} + S \left( \frac{u_{i+1,j+1} - u_{i-1,j+1} + u_{i+1,j} - u_{i-1,j}}{4\Delta y} \right) = Gr\theta + Gr_m\phi + 
\]

\[
\left( \frac{u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1} + u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{2(\Delta y)^2} \right) + MH_y - \chi_i \left( \frac{u_{i,j+1} + u_{i,j}}{2} \right),
\]

(3.3.17)

From equation (3.3.17) the values of \(u\) at time step \(j+1\) and \(j\) are appearing on both sides of the equation. This equation (3.3.17) is used to predict the values of \(u\) at time step \(j+1\), so all values of \(u\) at time step \(j\) are assumed to be known. We rearrange equation (3.3.17) above so that values of \(u\) at time step \(j+1\) are on the left and values of \(u\) at time step \(j\) so that we get;

\[
u_{i-1,j+1} \left( \frac{S}{4\Delta y} + \frac{1}{2(\Delta y)^2} \right) + u_{i,j+1} \left( \frac{1}{\Delta t} + \frac{1}{(\Delta y)^2} + \frac{\chi_i}{2} \right) + u_{i+1,j+1} \left( -\frac{S}{4\Delta y} - \frac{1}{2(\Delta y)^2} \right) = 
\]

\[
u_{i-1,j} \left( \frac{1}{4\Delta y} + \frac{1}{2(\Delta y)^2} \right) + \chi_i \left( \frac{1}{(\Delta y)^2} - \frac{\chi_i}{2} \right) + u_{i+1,j} \left( -\frac{S}{4\Delta y} + \frac{1}{2(\Delta y)^2} \right) + Gr\theta + Gr_m\phi + MH_y
\]

(3.3.18)

Multiplying equation (3.3.18) by \(4\Delta t(\Delta y)^2\) gives

\[
u_{i-1,j+1} \left( -S\Delta t\Delta y - 2\Delta t \right) + u_{i,j+1} \left( 4(\Delta y)^2 + 4\Delta t + 2\chi_i\Delta t(\Delta y)^2 \right) + u_{i+1,j+1} \left( S\Delta y\Delta t - 2\Delta t \right) = 
\]

\[
u_{i-1,j} \left( \Delta t\Delta y + 2\Delta t \right) + \chi_i \left( 4(\Delta y)^2 - 4\Delta t - 2\chi_i\Delta t(\Delta y)^2 \right) + u_{i+1,j} \left( 2\Delta t - S\Delta y\Delta t \right) + 
\]

\[
\left( (Gr\theta + Gr_m\phi + MH_y)4\Delta t(\Delta y)^2 \right)
\]

(3.3.19)

The Crank-Nicolson scheme is implicit, hence a system of equations for \(u\) must be solved at each time step.
If we let coefficients of interior nodes be:

\[ a_i = (-S\Delta t \Delta y - 2\Delta t) \]  \hspace{1cm} (3.3.20)

\[ b_i = \left( 4(\Delta y)^2 + 4\Delta t + 2\chi_i \Delta t (\Delta y)^2 \right) \]  \hspace{1cm} (3.3.21)

\[ c_i = (S\Delta y \Delta t - 2\Delta t) \]  \hspace{1cm} (3.3.22)

\[ d_i = u_{i-1,j}(\Delta t \Delta y + 2\Delta t) \]  \hspace{1cm} (3.3.23)

\[ e_i = u_{i,j} \left( 4(\Delta y)^2 - 4\Delta t - 2\chi_i \Delta t (\Delta y)^2 \right) \]  \hspace{1cm} (3.3.24)

\[ f_i = u_{i+1,j}(2\Delta t - S\Delta y \Delta t) \]  \hspace{1cm} (3.3.25)

\[ g = \left( (Gr\theta + Gr_m\phi + MH_y) 4\Delta t (\Delta y)^2 \right) \]  \hspace{1cm} (3.3.26)

For \( i = 2, 3, 4, 5...N - 1 \)

Substituting equations from (3.3.20) to (3.3.26) into equation (3.3.19) we get;

\[ a_i u_{i-1,j+1} + b_i u_{i,j+1} + c_i u_{i+1,j+1} = d_i + e_i + f_i + g \]  \hspace{1cm} (3.3.27)

if we let \( m = j + 1 \)

For \( i = 2 \) the equation (3.3.27) becomes

\[ a_2 u_{1,m} + b_2 u_{2,m} + c_2 u_{3,m} = d_2 + e_2 + f_2 + g \]  \hspace{1cm} (3.3.28)

For \( i = 3 \) the equation (3.3.27) becomes

\[ a_3 u_{2,m} + b_3 u_{2,m} + c_3 u_{4,m} = d_3 + e_3 + f_3 + g \]  \hspace{1cm} (3.3.29)

If we proceed upto \( i = N - 1 \), the system of equations can be represented in matrix form as
\[
\begin{bmatrix}
    a_2 & b_2 & c_2 & 0 & \ldots & 0 \\
    0 & a_3 & b_3 & c_3 & 0 & \ldots \\
    \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & 0 & a_{N-1} & b_{N-1} & c_{N-1}
\end{bmatrix}
\begin{bmatrix}
    u_{1,m} \\
    u_{2,m} \\
    \vdots \\
    u_{N-1,m}
\end{bmatrix}
= \begin{bmatrix}
    d_2 \\
    d_3 \\
    \vdots \\
    d_{N-1}
\end{bmatrix}
+ \begin{bmatrix}
    e_2 \\
    e_3 \\
    \vdots
\end{bmatrix}
\]

\[
\begin{bmatrix}
    f_2 \\
    f_3 \\
    \vdots \\
    f_{N-1}
\end{bmatrix}
+ \begin{bmatrix}
    g \\
    g \\
    \vdots \\
    g
\end{bmatrix}
\]

(3.3.30)

### 3.3.0.2 Energy Equation

Setting \( u^* = u \), equation (3.2.28) becomes;

\[
\theta_i + S\theta_y = \frac{1}{P_r} \theta_{yy} + E c(u_y)^2 + \frac{R}{P_r}
\]

(3.3.31)

Using equations (3.3.5) to (3.3.8) becomes

\[
\left[ \frac{\theta_{i+1,j+1} - \theta_{i,j}}{\Delta t} \right] + S \left[ \frac{\theta_{i+1,j+1} - \theta_{i,j+1} + \theta_{i+1,j} - \theta_{i,j}}{4\Delta y} \right] =
\]

\[
\left( \frac{1}{P_r} \right) \left[ \frac{\theta_{i,j} - 2\theta_{i,j} + \theta_{i+1,j} + \theta_{i-1,j+1} - 2\theta_{i,j+1} + \theta_{i+1,j+1}}{2(\Delta y)^2} \right] + u_y + \frac{R}{P_r}
\]

(3.3.32)

All values of \( \theta \) at time \( j \) are assumed to be known. Rearrange equation (3.3.32) so that values of \( \theta \) at time \( j+1 \) are on the left and others on the right.

\[
\theta_{i-1,j+1} \left[ \frac{-S}{4\Delta y} - \frac{1}{2P_r(\Delta y)^2} \right] + \theta_{i,j+1} \left[ \frac{1}{\Delta t} + \frac{1}{P_r(\Delta y)^2} \right] +
\]

\[
\theta_{i+1,j+1} \left[ \frac{S}{4\Delta y} - \frac{1}{2P_r(\Delta y)^2} \right] = \theta_{i-1,j} \left[ \frac{S}{4\Delta y} + \frac{1}{2P_r(\Delta y)^2} \right] + \theta_{i,j} \left[ \frac{1}{\Delta t} - \frac{1}{P_r(\Delta y)^2} \right] +
\]

\[
\theta_{i+1,j} \left[ \frac{-S}{4\Delta y} + \frac{1}{2P_r(\Delta y)^2} \right] + Ec u_y + \frac{R}{P_r}
\]

(3.3.33)
Multiplying equation (3.3.33) by $4Pr \triangle t (\triangle y)^2$ we get

$$\theta_{i-1,j+1}[-SPr \triangle t \triangle y - 2\triangle t] + \theta_{i,j+1} \left[ 4Pr (\triangle y)^2 + 4\triangle t \right] +$$

$$\theta_{i+1,j+1} [SPr \triangle t \triangle y - 2\triangle t] = \theta_{i-1,j} [SPr \triangle y \triangle t + 2\triangle t] + \theta_{i,j} \left[ 4Pr (\triangle y)^2 - 4\triangle t \right] +$$

$$\theta_{i+1,j} [-SPr \triangle y \triangle t + 2\triangle t] + 4PrEC (u_y)^2 (\triangle y)^2 \triangle t + R4Pr \triangle t (\triangle y)^2 \quad (3.3.34)$$

If we let the coefficients of nodes be

$$l_i = -SPr \triangle t \triangle y - 2\triangle t \quad (3.3.35)$$

$$p_i = 4Pr (\triangle y)^2 + 4\triangle t \quad (3.3.36)$$

$$r_i = SPr \triangle t \triangle y - 2\triangle t \quad (3.3.37)$$

$$w_i = \theta_{i-1,j} [SPr \triangle y \triangle t + 2\triangle t] \quad (3.3.38)$$

$$v_i = \theta_{i,j} \left[ 4Pr (\triangle y)^2 - 4\triangle t \right] \quad (3.3.39)$$

$$k_i = \theta_{i+1,j} [-SPr \triangle y \triangle t + 2\triangle t] \quad (3.3.40)$$

$$z = 4PrEC (u_y)^2 (\triangle y)^2 \triangle t \quad (3.3.41)$$

$$q = R4Pr \triangle t (\triangle y)^2 \quad (3.3.42)$$

Therefore using equations (3.3.35) to equation (3.3.42) equation (3.3.34) becomes

$$l_i \theta_{i-1,j+1} + p_i \theta_{i,j+1} + r_i \theta_{i+1,j+1} = w_i + v_i + k_i + z + q \quad (3.3.43)$$

If we let $m = j + 1$

For $j=2$ equation (3.3.43) becomes;
\[ l_2 \theta_{1,m} + p_2 \theta_{2,m} + r_2 \theta_{3,m} = w_2 + v_2 + k_2 + z + q \]  
(3.3.44)

For \( j = 3 \) equation (3.3.43) becomes;

\[ l_3 \theta_{2,m} + p_3 \theta_{3,m} + r_3 \theta_{4,m} = w_3 + v_3 + k_3 + z + q \]  
(3.3.45)

For \( j = 4 \) equation (3.3.43) becomes;

\[ l_4 \theta_{3,m} + p_4 \theta_{4,m} + r_4 \theta_{5,m} = w_4 + v_4 + k_4 + z + q \]  
(3.3.46)

If we proceed upto \( j = N-1 \), the system of equations can be represented in matrix form as;

\[
\begin{bmatrix}
  l_2 & p_2 & 0 & \ldots & 0 \\
  0 & l_3 & p_3 & 0 & \ldots \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  0 & 0 & 0 & 0 & l_{N-1}
\end{bmatrix}
\begin{bmatrix}
  \theta_{1,m} \\
  \theta_{2,m} \\
  \vdots \\
  \theta_{N-1,m}
\end{bmatrix}
= 
\begin{bmatrix}
  w_2 \\
  w_3 \\
  \vdots \\
  w_{N-1}
\end{bmatrix}
+ 
\begin{bmatrix}
  v_2 \\
  v_3 \\
  \vdots \\
  v_{N-1}
\end{bmatrix}
+ 
\begin{bmatrix}
  k_2 \\
  k_3 \\
  \vdots \\
  k_{N-1}
\end{bmatrix}
\begin{bmatrix}
  z \\
  z \\
  \vdots \\
  z
\end{bmatrix}
+ 
\begin{bmatrix}
  q \\
  q \\
  \vdots \\
  q
\end{bmatrix}
\]  
(3.3.47)

### 3.3.0.3 Concentration Equation

The equation (3.2.31) becomes

\[ \phi_t + S \phi_y = \frac{1}{Sc} \phi_{yy} - K \phi \]  
(3.3.48)

Substituting equations (3.3.9) to equation (3.3.12) into equation (3.3.47) we get;

\[
\frac{\phi_{i,j+1} - \phi_{i,j}}{\Delta t} + S \frac{\phi_{i+1,j+1} - \phi_{i-1,j+1} + \phi_{i,j+1} - \phi_{i,j}}{4\Delta y} = 
\frac{1}{Sc} \left[ \phi_{i-1,j} - 2\phi_{i,j} + \phi_{i+1,j} + \phi_{i-1,j+1} - 2\phi_{i,j+1} + \phi_{i+1,j+1} \right] - K \frac{\phi_{i,j+1} + \phi_{i,j}}{2}
\]  
(3.3.49)
All values of $\phi$ at time $j$ are assumed to be known. Rearrange equation (3.3.49) so that values of $\phi$ at time $j+1$ are on the left and others on the right.

$$
\phi_{i-1,j+1} \left[ -\frac{S}{4\Delta y} - \frac{1}{2Sc(\Delta y)^2} \right] + \phi_{i,j+1} \left[ \frac{1}{\Delta t} + \frac{K}{2} + \frac{1}{Sc(\Delta y)^2} \right] + \phi_{i+1,j+1} \left[ \frac{S}{4\Delta y} - \frac{1}{Sc(\Delta y)^2} \right] =
$$

$$
\phi_{i-1,j} \left[ \frac{S}{4\Delta y} + \frac{1}{2Sc(\Delta y)^2} \right] + \phi_{i,j} \left[ \frac{1}{\Delta t} - \frac{1}{Sc(\Delta y)^2} - \frac{K}{2} \right] + \phi_{i+1,j} \left[ -\frac{S}{4\Delta y} + \frac{1}{2Sc(\Delta y)^2} \right]
$$

Multiply equation (3.3.50) by $4Sc\Delta t (\Delta y)^2$ we get;

$$
\phi_{i-1,j+1} [-SSc\Delta y\Delta t - 2\Delta t] + \phi_{i,j+1} \left[ 4(\Delta y)^2 Sc + 2KS\Delta t (\Delta y)^2 + 4\Delta t \right] +
$$

$$
\phi_{i+1,j+1} [SSc\Delta y\Delta t - 4\Delta t] = \phi_{i-1,j} [SSc\Delta y\Delta t + 2\Delta t] + \phi_{i,j} \left[ 4Sc(\Delta y)^2 - 4\Delta t - 2KS\Delta t (\Delta y)^2 \right]
$$

$$
+ \phi_{i+1,j} [-SSc\Delta t \Delta y + 2\Delta t]
$$

(3.3.51)

If we let the coefficient of nodes be

$$
\alpha_i = -SSc\Delta y\Delta t - 2\Delta t
$$

(3.3.52)

$$
\beta_i = 4(\Delta y)^2 Sc + 2KS\Delta t (\Delta y)^2 + 4\Delta t
$$

(3.3.53)

$$
\gamma_i = SSc\Delta y\Delta t - 4\Delta t
$$

(3.3.54)

$$
\theta_i = \phi_{i-1,j} [SSc\Delta y\Delta t + 2\Delta t]
$$

(3.3.55)

$$
\vartheta_i = \phi_{i,j} \left[ 4Sc(\Delta y)^2 - 4\Delta t - 2KS\Delta t (\Delta y)^2 \right]
$$

(3.3.56)

$$
\phi_i = \phi_{i+1,j} [-SSc\Delta t \Delta y + 2\Delta t]
$$

(3.3.57)
Substituting equations (3.3.52) to (3.3.57) into equation (3.3.50) we get;

\[ \alpha_i \phi_{i-1,j+1} + \beta_i \phi_{i,j+1} + \gamma_i \phi_{i+1,j+1} = \theta_i + \vartheta_i + \phi_i \]  
(3.3.58)

If we let \( m = j + 1 \)

For \( i = 2 \) then equation (3.3.58) becomes

\[ \alpha_2 \phi_1, m + \beta_2 \phi_{2,m} + \gamma_2 \phi_{3,m} = \theta_2 + \vartheta_2 + \phi_2 \]  
(3.3.59)

For \( i = 3 \) then equation (3.3.59) becomes

\[ \alpha_3 \phi_{2,m} + \beta_3 \phi_{3,m} + \gamma_3 \phi_{4,m} = \theta_3 + \vartheta_3 + \phi_3 \]  
(3.3.60)

For \( i = 4 \) then equation (3.3.58) becomes;

\[ \alpha_4 \phi_{3,m} + \beta_4 \phi_{4,m} + \gamma_4 \phi_{5,m} = \theta_4 + \vartheta_4 + \phi_4 \]  
(3.3.61)

If we proceed upto \( i = N - 1 \) then we get

\[
\begin{bmatrix}
\alpha_2 & \beta_2 & \gamma_2 & 0 & \cdots & 0 \\
0 & \alpha_3 & \beta_3 & \gamma_3 & 0 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \alpha_{N-1} & \beta_{N-1} & \gamma_{N-1}
\end{bmatrix}
\begin{bmatrix}
\phi_{1,m} \\
\phi_{2,m} \\
\vdots \\
\phi_{N-1,m}
\end{bmatrix}
= 
\begin{bmatrix}
\theta_2 \\
\theta_3 \\
\vdots \\
\theta_{N-1}
\end{bmatrix}
+ 
\begin{bmatrix}
\vartheta_2 \\
\vartheta_3 \\
\vdots \\
\vartheta_{N-1}
\end{bmatrix}
+ 
\begin{bmatrix}
\phi_2 \\
\phi_3 \\
\vdots \\
\phi_{N-1}
\end{bmatrix}
\]  
(3.3.62)

**3.3.0.4 Induction Equation**

Using Crank-Nicolson method and setting \( y^* = y, \ t^* = t, \ u^* = u, \ C^* = C, \ h^* = H \). The equation (3.2.34) will now become;

\[ H_t + S H_y = \frac{1}{Pr_m} H_{xy} + M u_y \]  
(3.3.63)

Substituting equations (3.3.13) to (3.3.15) into equation (3.3.63) we get;

\[ \frac{H_{i,j+1} - H_{i,j}}{\Delta t} + S \left[ H_{i+1,j+1} - H_{i-1,j+1} + H_{i+1,j} - H_{i-1,j} \right]/4\Delta y = \right] 
\]

\[ \frac{1}{Pr_m} \left[ \frac{H_{i-1,j} - 2H_{i,j} + H_{i+1,j}}{2(\Delta y)^2} + H_{i+1,j+1} \right] + M u_y \]  
(3.3.64)
From equation (3.3.64) the values of H at time step \( j + 1 \) and \( j \) are appearing on both sides of the equation. This equation (3.3.64) is used to predict the values of H at time step \( j + 1 \), so all values of H at time step \( j \) are assumed to be known. We rearrange equation (3.3.64) above so that values of H at time \( j + 1 \) are on the left and values of H at time \( j \) so that we get;

\[
H_{i-1,j+1} \left[ -\frac{S}{4\Delta y} - \frac{1}{2Pr_m (\triangle y)^2} \right] + H_{i,j+1} \left[ \frac{1}{\Delta t} + \frac{1}{Pr_m (\triangle y)^2} \right] + H_{i+1,j+1} \left[ \frac{S}{4\Delta y} - \frac{1}{2Pr_m (\triangle y)^2} \right] =
\]

\[
H_{i-1,j} \left[ \frac{S}{4\Delta y} + \frac{1}{2Pr_m (\triangle y)^2} \right] + H_{i,j} \left[ \frac{1}{\Delta t} - \frac{1}{2Pr_m (\triangle y)^2} \right] + H_{i+1,j} \left[ -\frac{S}{4\Delta y} + \frac{1}{2Pr_m (\triangle y)^2} \right] + u_y
\]

Divide equation (3.3.65) by \( 4\Delta tPr_m (\triangle y)^2 \) we get

\[
H_{i-1,j+1} \left[ -SPr_m \triangle y \Delta t - 2\Delta t \right] + H_{i,j+1} \left[ 4Pr_m (\triangle y)^2 + 4\Delta t \right] + H_{i+1,j+1} \left[ SPr_m \triangle y \Delta t - 2\Delta t \right] =
\]

\[
H_{i-1,j} \left[ SPr_m \triangle y + 2\Delta t \right] + H_{i,j} \left[ 4Pr_m (\triangle y)^2 - 4\Delta t \right] +
\]

\[
H_{i+1,j} \left[ - SPr_m \triangle y \Delta t + 2\Delta t \right] + Mu_y \left[ 4\Delta tPr_m (\triangle y)^2 \right]
\]

(3.3.66)

If we let coefficient of interior nodes be;

\[
A_i = - SPr_m \triangle y \Delta t - 2\Delta t
\]

(3.3.67)

\[
B_i = 4Pr_m (\triangle y)^2 + 4\Delta t
\]

(3.3.68)

\[
C_i = SPr_m \triangle y \Delta t - 2\Delta t
\]

(3.3.69)

\[
D_i = H_{i-1,j} \left[ SPr_m \triangle y + 2\Delta t \right]
\]

(3.3.70)

\[
E_i = H_{i,j} \left[ 4Pr_m (\triangle y)^2 - 4\Delta t \right]
\]

(3.3.71)
\[ F_i = H_{i+1,j} \left[ -SP_{rm} \Delta y \Delta t + 2 \Delta t \right] \]  
(3.3.72)

\[ G = Mu_y \left[ 4 \Delta t Pr_m (\Delta y)^2 \right] \]  
(3.3.73)

Therefore substituting equations (3.3.67) to (3.3.73) into (3.3.66) we get;

\[ A_i H_{i-1,j+1} + B_i H_{i,j+1} + C_i H_{i+1,j+1} = D_i + E_i + F_i + G \]  
(3.3.74)

If we let \( j + 1 = m \)
For \( i = 2 \)

\[ A_2 H_{1,m} + B_2 H_{2,m} + C_2 H_{3,m} = D_2 + E_2 + F_2 + G \]  
(3.3.75)

For \( i = 3 \)

\[ A_3 H_{2,m} + B_3 H_{3,m} + C_3 H_{4,m} = D_3 + E_3 + F_3 + G \]  
(3.3.76)

For \( i = 4 \)

\[ A_4 H_{3,m} + B_4 H_{4,m} + C_4 H_{5,m} = D_4 + E_4 + F_4 + G \]  
(3.3.77)

If we proceed upto \( i = j + 1 \), the system of equations can represented in matrix form as:

\[
\begin{bmatrix}
A_2 & B_2 & C_2 & 0 & \cdots & 0 \\
0 & A_3 & B_3 & C_3 & 0 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & A_{N-1} & B_{N-1} & C_{N-1}
\end{bmatrix}
\begin{bmatrix}
H_{1,m} \\
H_{2,m} \\
\vdots \\
H_{N-1,m}
\end{bmatrix}
= 
\begin{bmatrix}
D_2 \\
D_3 \\
\vdots \\
D_{N-1}
\end{bmatrix}
+ 
\begin{bmatrix}
E_2 \\
E_3 \\
\vdots \\
E_{N-1}
\end{bmatrix}

\]  
(3.3.78)

Finally, the block of tridiagonal matrices (3.3.78), (3.3.62), (3.3.47) and (3.3.31) are solved. A computer program written in Matlab is used to solve the differential equations. The initial and boundary conditions were set. The program has graphical output
for the velocity profile, temperature profile, concentration profile and magnetic induction profile for various fluid parameter value. In the next chapter these results are presented and discussed.
CHAPTER FOUR
RESULTS AND DISCUSSION

4.1 Overview

In this chapter we present the numerical results obtained upon employing computer program. The trends observed upon varying various fluid flow parameters are discussed and explained.

4.2 Fluid Property Default Values

Since the problem has been non-dimensionalized, the default values are chosen which are used to determine the changes which will affect the fluid flow. In the present study we adopted the following default parameter values of finite difference computations:

\[ S = 10, \ Gr = Gr_m = 1 \times 10^3, \ \chi_i = 2, \ M = 2, \ Pr_m = 0.07, \ Pr = 0.35, \ R = 20, \]

\[ Ec = 0.01, \ Sc = 0.04, \ K = 1 \] (4.2.1)

4.3 Effects of Magnetic parameter

While keeping all fluid properties at their default values, the magnetic parameter is varied as \( M = 1, 2, 3 \) and 4. The results obtained are presented in figures below:

Figure 4.3.1: Effect of magnetic parameters on fluid flow variables

Subfigure (a) Sub figure (b)
From figure (4.3.1) we can observe that magnetic parameter affects both velocity and induced magnetic field. With increase in $M$ from $M=1$ through $M=2$ to $4$ as from figure (4.3.1 a) there is a strong deceleration in the flow. The presence of magnetic field in an electrically conducting fluid introduces a force called Lorentz force, which acts against the flow if the magnetic field is applied normal direction as considered in present problem. We also notice that there is a fall in induced magnetic field as magnetic parameter increases as in figure (4.3.1b). The induced magnetic flux peaks a short distance from the plate, and then decays to zero in free stream. The induced magnetic field is parabolic type in upward and downward direction in all cases. The strength of induced magnetic field is directly proportional to the strength of the magnetic parameter in the region $0 < y < 0.25$ while inversely proportional in the region $0.25 < y < 3$. In this case there exists a point in the flow domain where the induced magnetic field is independent of magnetic field applied. Also, the point of inflection where the induced magnetic field changes its character is strongly dependent on the nature of working fluid.

4.4 Effects of Injection Velocity

While keeping all fluid properties at their default values, the Injection parameter was varied as $S = 5, 10, 15$ and $20$. The results obtained are presented in figure below
From the figure (4.4.1) it is observed that injection parameter(s) has a significant effect on velocity, temperature, concentration and induced magnetic field profiles.

From figure (4.4.1a) we can observe that an increase in injection parameter leads to an increase in velocity of the fluid. This is because blowing destabilizes the boundary layer leading to decrease in viscous forces hence increase in motion of the fluid. Velocity near the plate increases owing to presence of foreign gases in fluid flow field.

From figure (4.4.1c) we can observe that an increase in injection causes a rise in concentration profiles. Injection means an increase in molecular diffusivity which consequently result in the rise of concentration. This also leads to increase in temperature profile as seen in figure (4.4.1d) this is due to presence of foreign particles leading to reaction.

From figure (4.4.1b) we can observe that increase in injection parameter leads to increased in induced magnetic field at a region $0 < y < 1$ and vice versa from $1 < y < 3$. 

Figure 4.4.1: Effect of injection on fluid flow variables
4.5 Effects of Prandtl Number

While maintaining the other fluid properties at their default values, the Prandtl number was varied as $Pr = 0.17, 0.35, \text{ and } 0.71$. From $Pr = 0.17$ to $Pr = 0.71$ is for conducting air. The results obtained are shown in the figure below.

![Figure 4.5.1: Effect of Prandtl numbers on Fluid flow variables](image)

From the figure (4.5.1) it is observed that increase in Prandtl number leads to decrease in velocity, temperature and induced magnetic field. No effects on the concentration.

From figure (4.5.1a) we can observe that increase in Prandtl number leads to decrease in velocity of fluid. This is due to increase in the viscosity of the fluid which makes fluid thick and hence causes a decrease in velocity of the fluid.

In figure (4.5.1b) we can observe that increase in Prandtl number leads to decrease in temperature of fluid flow. Physically, increase in Prandtl number leads to decrease in thermal boundary layer and lower average temperature within boundary layer. The smaller Prandtl number is equivalent to increase in thermal conductivity of fluid and
heat is able to diffuse away from heated surface more rapidly for higher values of Prandtl number. As a result the flow is decelerated.

From figure (4.5.1c) it is evident that the induced magnetic field increases as well as decreases with increase in Prandtl number. For $Pr=0.71$ the induced magnetic field increasingly become negative. When $Pr$ increases the induced magnetic field decreases absolutely.

### 4.6 Effects of Eckert Number

While maintaining the other fluid properties at their default values, the Eckert number was varied as $Ec = 0.002, 0.01, 0.03$ and $0.05$. The results obtained are shown in the figure below.

![Figure 4.6.1: Effects of Eckert Numbers on Fluid flow variables](image)

From figure (4.6.1) we can observe that Eckert number has significant effect on temperature, velocity and induced magnetic field. The Eckert number affects the temperature...
profiles the most because from the governing equations it directly affect the energy equation as observed in figure (4.6.1c). When Eckert number is increased the temperature of the fluid flow increases hence increases the velocity of the fluid flow. This is because when the value of Eckert number is large, the kinetic energy dominates the boundary layer enthalpy which implies that that the particles or molecules of the fluid have increased velocities. The effects of viscous dissipation on the flow field is to increase the energy, yielding a greater temperature and as a consequence greater buoyancy force.

From figure (4.6.1b) we can observe that induced magnetic field increases and reduces when Eckert number is increased.

### 4.7 Effects of Schmidt Number

While maintaining the other fluid properties at their default values, the Schmidt number was varied as \( Sc = 0.02, 0.04, 0.1 \) and 0.2. The results obtained are shown in the figure below.
We observe from figure (4.7.1) that Schmidt number affects mostly the velocity, induced magnetic field and concentration profiles. There is no observable effects on temperature profiles. Increase in Schmidt number leads to reduction in velocity and concentration profiles and also reduction of induced magnetic field near the plate and increase in the free stream.

Mathematically, these observations can be explained from the relation in the governing equations above. We notice that there exists an inverse relationship between the concentration and schmidt number, therefore the inhibition of concentration profile on increasing Schmidt number. The effects on velocity and induced magnetic field is affected by the concentration.

Physically, Schmidt number represents the ratio of momentum diffusivity to mass diffusivity. Schmidt number quantifies the relative effectiveness of momentum and mass transport by diffusion in hydrodynamic and concentration boundary layer. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The larger value of Sc means a presence of a heavier fluid and this implies a lower
velocity of fluid.

### 4.8 Effects of Radiating Parameter

While maintaining the other fluid properties at their default values, the Radiating parameter was varied as $R = 10, 20$ and $30$. The results obtained are shown in the figure below.

![Sub Figure (a)](image1)

![Sub Figure (b)](image2)

![Sub Figure (c)](image3)

Figure 4.8.1: Effects of Radiating parameters on Fluid flow variables

We can observe from figure (4.8.1) that as we increase Radiating parameter the velocity, temperature and induced magnetic field increases. Physically, a large value of Radiating parameter corresponds to an increased dominance of thermal radiation over conduction. Thermal radiation supplements thermal diffusion and increases the overall thermal diffusivity of the regime. Since the local radiant diffusion flux model adds radiation conductivity to the convectional thermal conductivity. As a result the temperature in the fluid region are significantly increased hence increased velocity of the fluid flow.
4.9 Effects of Grashof Parameters

While maintaining the other fluid properties at their default values, the Grashof parameter was varied as $Gr = 1 \times 10^1, 1 \times 10^2$ and $1 \times 10^3$. The results obtained are shown in the figure below.

![Graphs showing effects of Grashof numbers on fluid flow variables](image)

**Figure 4.9.1: Effects of Grashof numbers on fluid flow variables**

We can observe from figure 4.9.1 some significant change in velocity, temperature and induced magnetic field but no much change in concentration. We notice that when Grashof Number is increased the velocity and the temperature of fluid flow increases. For the induced magnetic field it increases near the plate and and decays to the relevant free stream.

The Grashof number $Gr_c$ defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases and the peak value is more distinctive due to increase in the species buoyancy force. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases.
properly to approach the free stream value. The thermal Grashof number signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is observed that there is a rise in the velocity due to the enhancement of thermal buoyancy force. Here, the positive values of $Gr$ correspond to cooling of the plate. Also, as $Gr$ increases, the peak values of the velocity increases rapidly near the porous plate and then decays smoothly to the free stream velocity.

### 4.10 Effects of Magnetic Prandtl Number

While maintaining the other fluid properties at their default values, the Magnetic Prandtl number was varied as $Pr_m = 0.05, 0.07, and 0.1$. The results obtained are shown in the figure below.

![Figure 4.10.1: Effects of Magnetic Prandtl Numbers on Fluid flows variables](image)

From figure (4.10.1) we can observe slight changes only on induced magnetic field. In this figure magnetic prandtl number is set as less than unity, which implies that the magnetic diffusion rate exceeds the viscous diffusion rate. As such $Pr$ increases, momentum diffusivity will be increased. Therefore when $Pr_m$ increases from 0.05 to 0.1 the induced magnetic field is found to increase absolutely in the boundary layer $0 \leq y < 0.5$ but this trend is opposite in the region $0.5 < y < 3$.

In chapter five we conclude and recommend on further research problems.
CHAPTER FIVE
CONCLUSIONS, RECOMMENDATIONS

5.1 Introduction

In this chapter conclusions of the research carried out were outlined and thereafter recommendations for future research work were also made.

5.2 Conclusion

From the present numerical investigation, following conclusions have been drawn:

- It is found that the velocity decreases with increasing magnetic parameter (M). This is due to the Lorenz force which opposes the fluid motion. It was also observed that increasing magnetic Prandtl number effects elevated the induced magnetic field near the plate, while this trend is reversed away from the plate.

- A decrease in concentration with increasing Schmidt number as well as chemical reaction parameter is observed. It is marked that the rate of concentration transfer increases with increasing values of chemical reaction parameter ‘K’ and Schmidt number ‘Sc’. This also reduces the induced magnetic field.

- Velocity components distributions were reduced by the increased values of the permeability of the plate.

- The fluid temperature was found to increase as the Eckert number, magnetic strength, radiation and surface permeability increase.

- Radiation has significant effects on the velocity as well as temperature distributions.

- It was also observed that increasing magnetic Prandtl number effects elevated the induced magnetic field near the plate, while this trend is reversed away from the plate.

- The velocity increases with the increase Grashof number.

5.3 Recommendations

There remains a lot to be done from this research in order to be able to move closer to the realisation of real life situation and results. In order to achieve this, based on
this research the following recommendations will be helpful to assist in further deeper exploration of this topic area:

- Flow that involves non-Newtonian fluids.
- Three dimensional flows.
- Flow of a compressible fluid.
- The magnetic field to be inclined at an angle
- Consider rotation of the plate.
- Turbulent flow.

5.4 Research Paper Published

REFERENCES
APPENDICES

.1 MATLAB CODE

```matlab
function kiprop()
clear all; clc;

M=9.2; S=0.8; X=3; Grt=2*10^3; Grc=2*10^3; Prm=2; Pr=0.07; R=0.5; Ec=0.01; Sc=0.07; K=0.5;
color='r-

ylow=yup-4; ny=49;
dy=(yup-ylow)/(ny-1);
y=ylow; dy=yup;
tinit=0; tend=1; nt=49;
dt=(tend-tinit)/(nt-1);
t=tinit:dt:tend;
u=zeros (ny,nt); T=zeros (ny,nt); C=zeros (ny,nt); H=zeros (ny,nt);
Urh=0; Ch=0; Trh=0; Crh=0;

% INITIAL CONDITIONS
% u(:,1)=0; T(:,1)=0; C(:,1)=0; H(:,1)=0; % at t=0
% INITIAL CONDITIONS
% BOUNDARY CONDITIONS
u (1,:)=0; T (1,:)-1; C (1,:)-1; H (1,:)-0; % at y=0
u (ny,:)-1; T (ny,:)-0; C (ny,:)-0; H (ny,:)-0; % at y=inf

% mesh(C)
% BOUNDARY CONDITIONS
% H-trimatrix
au=diag (diag (bu*ones (ny+1), 0) + diag (cu*ones (ny, 1), 1));

% T-trimatrix
ah=diag (diag (bh*ones (ny+1), 0) + diag (ch*ones (ny, 1), 1));

% C-trimatrix
at=diag (diag (bt*ones (ny+1), 0) + diag (ct*ones (ny, 1), 1));
```

51
for j=1:nt-1
    for i=2:ny-1
        Urhs(i,j)=Un(u(i+1,j),u(i,j),u(i-1,j),dt,dy,N,X,S,T(i,j),C(i,j),H(i,j),H(i,j+1),H(i,j+1),Gr,Gr);
        Urhs(i,j)=Hn(H(i,j),H(i,j),H(i,j),H(i,j),dt,dy,Prm,u(i+1,j+1),u(i-1,j+1),S,u(i+1,j),u(i-1,j));
        Trhs(i,j)=Tn(T(i+1,j),T(i,j),T(i-1,j),dt,dy,Fr,u(i+1,j+1),u(i,j+1),S,u(i+1,j),u(i-1,j),Ec,R);
        Crhs(i,j)=Cn(C(i+1,j),C(i,j),C(i-1,j),dt,dy,Sc,S,K);
    end
end

%%%% compute A
Au2(:,j)=Au:Urhs(:,j);
u(2:ny-1,j)=Au2(2:ny-1,2:ny-1,j)
Urhs(2:ny-1,j);
%%%% compute B
Ah2(:,j)=Ah:Urhs(:,j);
H(2:ny-1,j)=Ah2(2:ny-1,2:ny-1,j)
Trhs(2:ny-1,j);
%%%% compute T
At2(:,j)=At:Trhs(:,j);
\begin{verbatim}
T(2:ny-1,j)=T(2:ny-1,2:ny-1,j)\textbackslash rhs(2:ny-1,j);
%%%compute C
Ac2(:,j)=Ac\textbackslash rhs(:,j);
C(2:ny-1,j)=Ac2(2:ny-1,2:ny-1,j)\textbackslash rhs(2:ny-1,j);
end
figure(1)
mesh(t(2:nt-1),y(1:ny),H(1:ny,2:nt-1));
title('induced magnetic')
figure(2)
mesh(t(2:nt-1),y(1:ny),T(1:ny,2:nt-1));
title('temperature')
figure(3)
mesh(t(2:nt-1),y(1:ny),C(1:ny,2:nt-1));
title('concentration')
figure(4)
mesh(t(2:nt-1),y(1:ny),u(1:ny,2:nt-1));
title('velocity')
figure(5)
hold on
plot(y(1:ny),H(1:ny,nt-1),color,'linewidth',2);
hold off
function Urhs=0n(ur,uc,ul,dt,dx,dy,M,X,S,Tjp,Tj,Cjp,Cj,Hjp,Hj,Grt,Grc)
drhs=(dt*dy+2*dt)*ul;
ehrs=(4*dt*dy^4+dt*2*dt*dy*M*M-2*X*dt*dy*dy)*uc;
frhs=(2*dt+S*dy*dt)*ur;
grhs=(Grt*(0.5*(Tjp+Tj))+Grc*(0.5*(Cjp+Cj))+M*M*S*0.5*(Hjp+Hj)))*4*dt*dy*dy;
Urhs=drhs+erhs+frhs+grhs;
end
function Hrhs=0n(hr,hc,hl,dt,dy,Frm,urjp,uijp,S,urj,ulj)
d2rhs=(S*Frm*dy+2*dt)*hl;
e2rhs=(4*Frm*dy*dy+4*dt)*hc;
f2rhs=(2*dt+S*Frm*dy*dt)*hr;
g2rhs=(4*dt*Frm*dy^4+(urjp-uijp+urj+ulj))/(4*dt);
Hrhs=d2rhs+e2rhs+f2rhs+g2rhs;
\end{verbatim}
function
\[ U_{r\text{h}s} = U_n(u_r, u_c, u_l, d_t, d_y, M, X, S, T_j, T_j, C_j, C_j, H_j, H_j, G_{r}, G_{c}) \]
\[ d_{r\text{h}s} = (d_t \times d_y + 2 \times d_t) \times s_{ul}; \]
\[ e_{r\text{h}s} = (4 \times d_t \times d_y - 4 \times d_t + 2 \times d_t \times d_y + M \times M - 2 \times X \times d_t \times d_y) \times u_c; \]
\[ f_{r\text{h}s} = (2 \times d_t + s \times d_y) \times u_r; \]
\[ g_{r\text{h}s} = (G_{r} \times (0.5 \times (T_j + T_j)) + G_{c} \times (0.5 \times (C_j + C_j)) + M \times M \times S \times (0.5 \times (H_j + H_j))) \times 4 \times d_t \times d_y; \]
\[ U_{r\text{h}s} = d_{r\text{h}s} \times e_{r\text{h}s} + f_{r\text{h}s} + g_{r\text{h}s}; \]
end

function
\[ H_{r\text{h}s} = H_n(h_r, h_c, h_l, d_t, d_y, P_r, u_r_j, u_l_j, S, u_r_j, u_l_j) \]
\[ d_{2\text{r}\text{h}s} = (s \times P_r \times d_y + 2 \times d_t) \times h_l; \]
\[ e_{2\text{r}\text{h}s} = (4 \times P_r \times d_y - 4 \times d_t) \times h_c; \]
\[ f_{2\text{r}\text{h}s} = (2 \times d_t + s \times P_r \times d_y) \times h_r; \]
\[ g_{2\text{r}\text{h}s} = 4 \times d_t \times P_r \times d_y \times (u_r_j - u_l_j + u_r_j + u_l_j) / (4 \times d_y); \]
\[ H_{r\text{h}s} = d_{2\text{r}\text{h}s} + e_{2\text{r}\text{h}s} + f_{2\text{r}\text{h}s} + g_{2\text{r}\text{h}s}; \]
end

function
\[ T_{r\text{h}s} = T_n(t_r, t_c, t_l, d_t, d_y, P_r, u_r_j, u_l_j, S, u_r_j, u_l_j, E_{c}, R) \]
\[ d_{1\text{r}\text{h}s} = (-s \times P_r \times d_y + d_t + 2 \times R \times P_r \times d_t) \times t_l; \]
\[ e_{1\text{r}\text{h}s} = (4 \times P_r \times d_y - 4 \times d_t + 2 \times R \times P_r \times d_t) \times t_c; \]
\[ f_{1\text{r}\text{h}s} = (2 \times d_t + s \times P_r \times d_y + 2 \times R \times P_r \times d_t) \times t_r; \]
\[ g_{1\text{r}\text{h}s} = 4 \times d_t \times P_r \times E \times d_y \times ((u_r_j - u_l_j + u_r_j - u_l_j) / (4 \times d_y))^2); \]
\[ T_{r\text{h}s} = d_{1\text{r}\text{h}s} + e_{1\text{r}\text{h}s} + f_{1\text{r}\text{h}s} + g_{1\text{r}\text{h}s}; \]
end

function
\[ C_{r\text{h}s} = C_n(c_r, c_c, c_l, d_t, d_y, S_{c}, S_{c}, K) \]
\[ d_{4\text{r}\text{h}s} = (-s \times S_{c} \times d_y + d_t + 2 \times d_t) \times c_l; \]
\[ e_{4\text{r}\text{h}s} = (4 \times S_{c} \times d_y - 4 \times d_t - 2 \times K \times S_{c} \times d_y) \times c_c; \]
\[ f_{4\text{r}\text{h}s} = (2 \times d_t - s \times S_{c} \times d_y) \times c_r; \]
\[ C_{r\text{h}s} = d_{4\text{r}\text{h}s} + e_{4\text{r}\text{h}s} + f_{4\text{r}\text{h}s}; \]
end
end