TURBULENT HYDROMAGNETIC FLOW WITH RADIATIVE HEAT OVER MOVING VERTICAL POROUS PLATE IN A ROTATING SYSTEM

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Turbulent Hydromagnetic Flow With Radiative Heat Over Moving Vertical Porous Plate In A Rotating System

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DECLARATION

This thesis is my original work and has not been presented for a degree in any other University.

Signature…………………………………………… Date……………………

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This thesis has been submitted for examination with our approval as university

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DEDICATION

This thesis is dedicated to my loving parents Mr. and Mrs. Ngari and my siblings Patrick, Nancy and Mary. Thank you for the enduring support you have given me throughout this study period. God bless you all.

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF APPENDICES

NOMECLATURE

Roman Symbol	**Quantity**
D	Electric flux density, [$N\,m^2C^{-1}$]
e	Unit electric charge, [C]
E	Electric field vector, [v]
$\mathbf{E_e}$	Charge density, [cm^{-3}]
g	Acceleration due to gravity vector, [ms^{-2}]
Gr_θ	Local temperature Grashoff number
j	Current density, [Am^{-2}]
m	Hall parameter
M	Magnetic parameter
$\mathbf{q_r}$	Radiative heat flux, [Wm^{-2}]
$\mathbf{C_p}$	specific heat at a constant pressure, [$Jkg^{-1}K^{-1}$]
k	Darcy permeability, [m^2]
Pr	Prandtl number
Ec	Eckert number

X	Permeability parameter
R	Rotation parameter
P	Pressure force, [N m^{-2}]
K	Thermal conductivity of porous medium, [$wm^{-1}K^{-1}$]
q	Velocity vector, [ms^{-1}]
i, j, k	Unit vectors in the x, y, z directions respectively
u, v, w	Components of velocity vector, [ms^{-1}]
x, y, z	Dimensional Cartesian co-ordinates
F	Body force tensor, [N]
$\mathbf{U_i}$	Velocity tensor, [ms^{-1}]
T	Absolute free temperature of the fluid, [K]

GREEK SYMBOL	QUANTITY
β	Volume coefficient of thermal expansion, [K^{-1}]
ρ	Fluid density, [kgm^{-3}]
μ	coefficient of viscosity, [$kgm^{-1}s$]
ϕ	Viscous dissipation functions, [s^{-1}]
Ω	Angular velocity, [s^{-1}]
σ	Electrical conductivity, [$\Omega^{-1}m^{-1}$]
T_e	Electron collision time, [s]
ω_e	Electron frequency, [s^{-1}]
∇	Gradient operator $\mathbf{i}\frac{\partial}{\partial x}+\mathbf{j}\frac{\partial}{\partial y}+\mathbf{k}\frac{\partial}{\partial z}$

LIST OF ABBREVIATIONS

RANS	Reynolds Averaged Navier Stokes
HOT	Higher Order Terms
MHD	Magneto hydrodynamics
PDEs	Partial Differential Equations
FD	Finite difference

ABSTRACT

In this study the combined effects of magnetic fields, buoyancy force, thermal radiation, viscosity, porosity, rotation and Joule's heating on flow variables have been investigated numerically. The x-axis is along the plate in upward direction, z-axis is normal to the plane of the plate and y-axis is perpendicular to x-z-plane. A uniform transverse magnetic field is applied in a direction which is parallel to z–axis. The equations governing turbulent flow are modelled using Reynolds-averaged Navier–Stokes equations. The final set of equations are non linear and highly coupled and thus no analytical method can be used to obtain their solution. These equations are converted to a system of linear equations and numerical techniques are used to obtain the approximate solutions. In that case the partial derivatives in the equations are re-written into their corresponding finite differences. The solution is generated by computer generated program code developed in MATLAB version 7.90.529 (R2009b). Graphical results showing the effects of varying various flow parameters on the velocity and temperature profiles are presented and discussed. The results obtained are useful in engineering, industries and many other scientific fields.

CHAPTER ONE

INTRODUCTION AND LITERATURE REVIEW

1.1 Introduction

In this chapter the main terms used in this study have been defined and the literature review on MHD rotating flow over a moving vertical porous plate in a rotating system has been cited. The chapter ends by stating the objectives and the applications of this study.Magneto-hydrodynamic (MHD) flow is the study of the flow of an electrically conducting fluid in presence of a magnetic field. MHD studies the dynamics of the interaction of an electrically conducting fluids and electro-magnetic fields. A conducting fluid becomes magnetized when flowing in presence of a magnetic field and the fluid is referred to as a magnetic fluid.

Turbulence is a phenomenon of fluid flow that occurs when momentum effects dominate viscous effects resulting to high Reynolds number. Turbulent flow regimes are characterized by chaotic variation in the fluid properties, such as low momentum diffusion, high momentum convection, and rapid variation of pressure and velocity in space and time. In turbulent flow, drag due to boundary layer increases and the unsteady vortices appear on many scales and interact with each other. Turbulence causes the formation of eddies of many different length scales. Studies related to turbulent flow and heat transfer not only present a mathematical challenge but find several applications in many industrial, engineering and technological processes.

1.2 Definition of terms

1.2.1 Fluids

A fluid is a substance that is capable of flowing and it undergoes continuous deformation when it is acted upon by shear stress.When a fluid is at rest, there is no shearing forces acting on it and thus all forces acting on the fluid must be orthogonal to the planes which they act on. Shear stress is developed when the fluid is in motion and the fluid particles are moving relative to each other so that their velocities are different. If the fluid particles move at the same velocity, no shear stresses can be produced since the particles are not moving relative to each other. If the fluid motion is such that the viscous stress associated with it is linearly dependent on the instantaneous rate of deformation it is called a Newtonian fluid. When two adjacent fluid particles situated at distance dy apart move at velocities $v + \delta v_0$ and v_0 respectively, then the force F acting on the fluid particles is directly proportional to $\frac{dv_0}{dy}$, i.e.

$$F = \mu \frac{dv_0}{dy}$$

The proportionality constant μ is called the coefficient of fluid viscosity. The above equation is called Newtonian's law. Fluids can be classified as either incompressible or compressible fluids. For compressible fluids, the fluid density changes significantly with variation in temperature and pressure while in incompressible fluids their density does change by significant amount when temperature and pressure are varied.

1.2.2 Fluid flows

If for a given fluid flow, the flow variables are time dependent, then the fluid flow is said to be unsteady otherwise the flow is termed as steady. Fluid flows can further be classified as lamina or turbulent flows. In lamina flows, the fluid particles move in an orderly manner with streamlines parallel to the solid boundary. Turbulent flow is a flow regime that features disorderly changes in fluid properties such as high momentum convection and rapid variation in pressure and velocity in time and space. In turbulent flow, unsteady vorticities emerge on many scales and fluid particles interact with each other results in formation of eddies.

1.2.3 Viscosity

Viscosity is a scientific term that refers internal friction in fluid that causes resistance to flow. The resistance is as a result of friction produced by the fluid molecules and determines the extent to which the fluid would oppose its deformation, movement of the bodies in it and amount of pressure that is required to make it move through a passage.

1.2.4 Magnetohydrodynamics

The term magnetohydrodynamics is derived from the word 'magneto' which means magnetic fields, 'dynamics' which mean the forces causing the movement are considered and 'hydro' which means liquids. Magnetohydrodynamics is a branch of fluid mechanics that deals with study of flowof an electrically conducting fluid under the influence of applied magnetic field. The electrically conducting fluids can be molten metals or plasmas (ionized gases). The flow of an electrically conducting fluid under the influence of magnetic field gives rise to an induced electric current in which mechanical forces are exerted by the magnetic field. The induced electric current flows in direction transverse to both magnetic field and the direction of motion. The interaction between the induced current and the magnetic field give rise to Lorentz force. Also the induced electric current produces its own magnetic which in turn disorients the original magnetic field.

1.2.5 Joule heating

Joule heating, also referred to as resistive heating or ohmic heating, is a phenomenon in which passage of electric current through a current carrying conductor results to increase temperature of the conductor. The rise in temperature is as a result of interaction between the atomic ions that constitute conductor and the moving particles that form the current. The charged particles in the electric circuit are speeded up by the electric field but they lose some kinetic energy whenever they clash with an ion. The increase in vibration energy of the ions manifests itself as heat that is depicted by the rise of temperature of the fluid. The increased fluid temperature results to non- uniform changes in fluid properties like fluid density and conductivity. Thus energy is converted from electrical power supply to the fluid or any other medium that is in thermal contact.

1.2.6 Coriolis Effect

A rotating system is a systemwhose axes are rotating as seen from an inertial coordinate system. When fluid flows through such a system, the fluid flow is said to occur in a rotating system. Colioris effect is the apparent deflection of an object in motion from a straight path when they are observed from a rotating frame of reference.

The effect is an inertial force described by a French mathematician Gustave-Gaspard Coliolis in 1835. He showed that if the normal Newtonian laws of motion of bodies are to be applied in a rotating frame of reference, an inertial force acting to the right of the direction of body motion for anticlockwise rotation frame of reference, or to the left for clockwise rotation, must be included in the equation of motion.

1.2.7 Porous Media

A porous medium is a solid with interconnected network of uniformly distributed pores through which fluid passes freely. Many naturally occurring substances like soil, bones, rocks and artificial materials such as concrete slabs, foam and ceramics are examples of porous media. Porosity of a media is defined as the fraction of the total volume of the media that is occupied by the pores; as a fraction it ranges between 0 and 1 while in percentage form it ranges from 0 to 100. The voids in the pores may be occupied by water, air or hydrocarbons. Hydrodynamic permeability determines the ability of the fluids to pass though the porous media.

1.2.8 Radiation Heat Transfer

Radiation is a mode of heat transfer by which heat is transmitted by electromagnetic waves propagation. The transmission may occur in vacuum thus a medium may not required. Radiation emitted by a body is due to thermal agitation of its molecules.

1.3Literature review

The studies related to hydromagnetic flows past moving vertical porous plate have been carried out by other authors before. Some of them include:

Kinyanjui *et al.,* (2003) presented their work in MHD free convection heat and mass transfer of a heat generating fluid past an impulsively started infinite vertical porous plate with Hall current and radiation absorption and revealed that change in the parameters is observed to either increase, decrease or to have no effects on the velocity, temperature, and concentration profiles respectively.

Ferdows *et al.,* (2008) studied MHD Free Convection and Mass Transfer Flow in a Porous Media with Simultaneous Rotating Fluid. They observed that as porous parameter, K increases the primary velocity decreases and the secondary velocity increases.

Rao *et al.,* (2012) Finite element solution of heat and mass transfer in MHD Flow of a viscous fluid past a vertical plate under oscillatory. They noted that velocity increases with the increase dimensionless porous medium parameter. The effect of the dimensionless porous medium K becomes smaller as K increase. Physically, this result can be achieved when the holes of the porous medium are neglected.

Kinyanjui *et al.,* (2012) studied Hydro magnetic turbulent flow of a rotating system past a semi-infinite vertical plate with Hall Current The results revealed that thermal variant of the Grashof number enhances both primary and secondary velocities but has a diminished effect on the temperature profiles. Also rotational parameter diminishes the primary velocity profiles while accelerating the secondary velocity profiles and also due to the strong magnetic field the presence of the Hall current affected.

Halima *et al.,* (2012) studied Effect of thermal conductivity on MHD heat and mass transfer: low past an infinite vertical plate with Soret and Dufour effects. The results revealed that increase in suction had very little effects on velocity of the fluid while it significantly reduced the temperature and concentration profiles of the fluid flow. Seth *et al*., (2012) studied effects of Hall current and rotation on unsteady MHD Couette flow in the presence of an inclined angle of inclination of magnetic field. The results showed that magnetic field has accelerating influence on the fluid velocity in both the primary and secondary flow directions.

Ngesa *et al.,* (2012) studied Magnetohydrodynamic (MHD) free convective flow past an infinite vertical porous plate with joule heating and reveal that an increase in joules heating parameter causes an increase in the velocity and temperature profiles uniformly near the plate but remain constantly distributed away from the plate.

Kimeu *et al*., (2012) studied hydromagnetic free convection flow past a semi-infinite vertical porous plate subjected to constant heat flux with radiation absorption. MHD Stokes problem of convective flow from a vertical infinite plate in a rotating fluid and found that In the absence of Turbulent and the magnetic field being constant, results agreed with those of Kinyanjui *et al.,* (2000) who investigated the MHD free convection heat and mass transfer of a heat generating fluid past an impulsively started infinite vertical porous plate with Hall current and radiation absorption.

Jana *et al.,* (2012) studied Unsteady Couette flow through a porous medium in a rotating system. It was observed that the presence of porous medium produces a resisting force in the flow field. So, the resistance in the flow decreases as the porosity parameter decrease.

Idowu *et al.*,(2013) Heat and Mass Transfer of Magneto hydrodynamic (MHD) And Dissipative Fluid Flow Past A Moving Vertical Porous Plate With Variable Suction revealed that as permeability increases the peak value of velocity tends to increase. These results could be very useful in deciding the applicability of enhanced oil recovery in reservoir engineering.

Sandeep *et al*.,(2014) studied Radiation and Inclined Magnetic Field Effects on Unsteady Hydromagnetic Free Convection Flow past an Impulsively Moving Vertical Plate in a Porous Medium. From the study it was noted that porosity of the plate causes the velocity to increase. Also noted was that velocity increases with time. Bala *et al*. ,(2012) investigated Radiation effects on MHD flow past an exponentially accelerated isothermal vertical plate with uniform mass diffusion in the presence of heat source. They noted that an increase in the radiation parameter results in decreasing velocity and temperature within the boundary layer

Mayaka *et al.*, (2014) examined MHD Turbulent Flow in a Porous Medium with Hall Currents, Joule's Heating and Mass Transfer the mass transfer velocity, accelerates all the flow variables. This is because increased injection rates enhance transfer from the plate to the rest of the fluid which leads to enhanced boundary layers. Dawit *et al. ,(*2014) examined turbulent hydromagnetic flow with radiative heat over a moving vertical plate in a rotating system and found that combined increase in the fluid rotation, Hall current, viscous heating and plate cooling due to buoyancy force increases the secondary skin friction while an increase in the Ohmic heating, thermal radiation and Prandtl number decreases the secondary skin friction at the plate surface.

Most of the above studies dealt with analysis of turbulent hydromagnetic flow with radiative heat over a moving vertical plate considering various geometries. None of the studies have comprehensively considered turbulent hydromagnetic flow with radiative heat past a moving vertical porous plate in a rotating system. This prompted the present study

1.4Statement of the problem

This study considers turbulent hydromagnetic flow with radiative heat past a moving vertical porous plate in a rotating system. The x-axis is along the plate in upward direction, z-axis in the fluid and y-axis perpendicular to x-z plane. A uniform transverse magnetic field B_0 is applied in a direction which is parallel to z-axis but normal to plane of the plate. It is assumed that no applied or polarization voltages exist so electric field is zero. The induced magnetic field produced by fluid motion is negligible in comparison to applied one. The plate moves along x-axis with velocity U_0 and both the plate and the fluid rotate in uniform angular velocity Ω about z–axis as shown in the diagram;

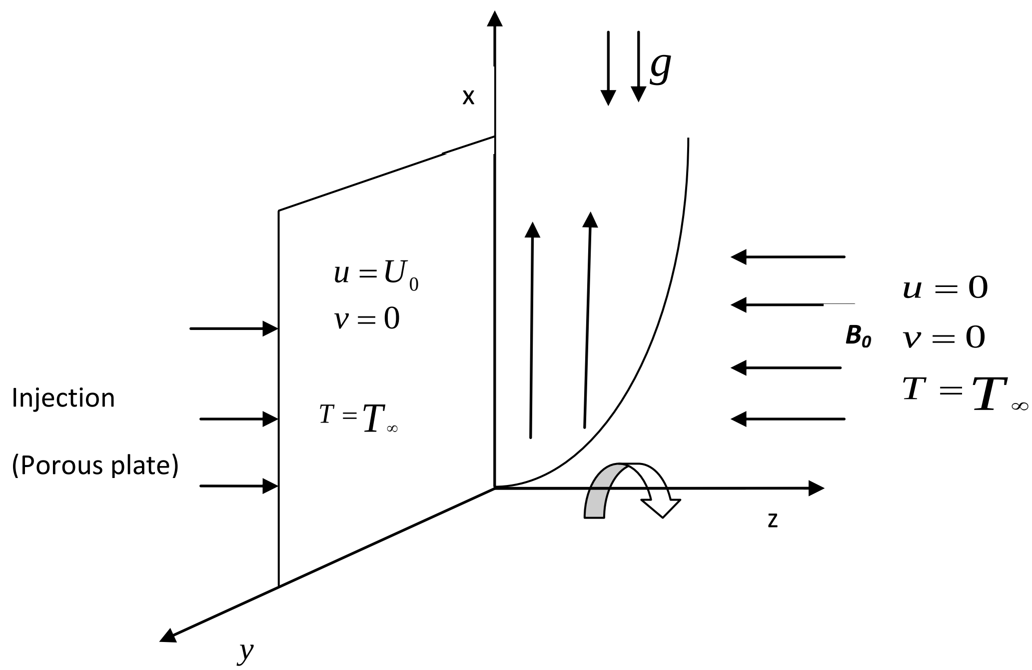

Figure 1.1: Flow configuration

1.5Justification

Fluid flows past rotating surfaces have many applications in engineering, mining, industries and many other scientific fields. Turbulent flows has developed revolutionary technology in the form of a portfolio of devices for the mixing, separation and the homogenization of liquids with liquids, liquids with gasses and gasses with gasses. The mixing technology may be chemicals, pharmaceuticals, cosmetics, foods, agricultural, water treatment with purification and hybrid fuels. In chemical engineering and ceramic engineering studies of flows through a porous media are used in filtration of harmful particles in contaminated fluid and seepage properties. Magnetohydromagnetic flows have applications in regulating fluid flows through reactors so as to maintain a uniform temperature throughout the bed of reactors.

1.6Hypothesis

Porosity of the plate and injection do not have effects on the velocity and temperature profiles of the fluid flow

1.6.0Objectives

1.6.1 General Objectives

To study the combined effects of magnetic fields, thermal radiation, viscous, porosity and Ohmic heating on turbulent hydromagnetic flow past a moving vertical porous plate in a rotating system.

1.6.1 Specific research objectives

1. To determine velocity profiles of the flow.
2. To determine temperature profiles of the flow.
3. To determine the effects of varying dimensionless numbers on the flow variables.

CHAPTER TWO

MATHEMATICAL FOMULATION

2.1 Introduction

In this chapter, the equations governing the MHD flow of an electrically conducting fluid with radiative heat past a moving vertical porous plate in a rotating system have been formulated. The chapter begins by listing the assumptions made in regard to this flow problem. The basic conservation laws taken into consideration are the momentum conservation, mass conservation and energy conservation. The various terms in the equations are discussed and refined to satisfy the flow geometry in their general dimensional forms. This is followed by expressing the governing equations in terms of non-dimensional parameters to show the relative significance of each parameter in the flow. The chapter ends with the introduction of the finite difference method used to approximate the solutions to the non-dimensional equations governing the flow.

2.2 Assumptions

In order to describe the phenomenon mathematically the following assumptions are made on the fluid and fluid flow. They include;

1. The velocity of the fluid is very small compared to that of light i.e. $\frac{q^2}{c^2} << 1$. This implies that the velocity scales considered in this study are not relativistic.
2. The fluid flow is restricted to turbulent domain.
3. Electrical conductivity, thermal conductivity, viscosity, Darcy permeability, and diffusion coefficient are constant.
4. The force due to electric field is negligible compared with the Lorenz force due to applied magnetic field.

5. The induced magnetic field, the external electric field and the electric field due to the polarization of charges are negligible.
6. The porous plate is isentropic, homogeneous and non-magnetic, therefore there is no magnetic induction.
7. The porous medium and the fluid are in local thermal equilibrium.
8. The no slip condition applies.2.3.0 Equations governing the fluid flow

Under the above assumptions the governing equations are:

2.3.1 The Equation of Continuity

The equation of conservation of mass is also called the equation of continuity and it's derived from the law of conservation of mass. The equation of continuity lays its foundation on two propositions namely;

i. Mass can neither be created nor destroyed but can be transformed from one form to another. Thus the mass of the fluid is conserved.
ii. The fluid flow is continuous i.e. empty spaces between particles in contact do not occur and thus for a steady flow process, mass stored in a control volume is constant.

Therefore for a given control volume, influx equals the outflow. The general equation of continuity for unsteady fluid flow in tensor form is given as;

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho . u_i)}{\partial x_j} = 0$$

Where i =1, 2, 3 represent the x, y and z directions respectively. Since fluid is assumed to have constant density, the equation of continuity in tensor notation takes the form;

$$\frac{\partial u_i}{\partial x_j} = 0$$

(2.1)

2.3.2 Equation of Motion

The equation of conservation of momentum is derived from Newton's second law of motion which requires that the sum of all the resultant forces acting on a control volume must be equal to the rate of increase of the momentum of the fluid within the control volume. This implies that the total momentum in a closed system is a constant. The momentum of an object is the product of its mass and the velocity in which it moves. Thus the velocity changes when a force is applied to an incompressible fluid of a particular mass. The general equation of motion in tensor form is written as;

$$\rho\left(\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}+2\Omega u_i\right)=-\frac{\partial p}{\partial x_j}+\mu\frac{\partial^2 u_i}{\partial x_j^2}+\vec{F}-\mu\frac{u_i}{k} \tag{2.2}$$

Where i=1, 2, 3 and j= 1, 2, 3 are the summation variables along the x, y and z directions respectively. From equation (2.2), the terms $u_j\frac{\partial u_i}{\partial x_j}$ and $2\Omega u_i$ represent the convective and coriollis acceleration respectively. $\vec{F}_i$ represents the body forces acting on the fluid. In this study, body forces considered to obtain the volumetric density are electromagnetic force and gravitational force thus $\vec{F}_i=\rho g+J\times B$

In the equation of momentum, the rate of change of momentum balance with the body forces and the surface forces. Body forces are forces that are proportional to the volume element and act on the fluid element from external force field e.g. gravitational force and centrifugal force while surface forces are proportional to area and they result from stresses such as static pressure and viscous stresses acting on the surface of the volume element.

For a three dimensional fluid flow under the influence of gravity, Darcy's law takes the following form;

$$-\nabla p+\rho g=\frac{\mu}{k}u_i \tag{2.3}$$

To obtain the pressure gradient, the equation of motion at the end of boundary layer has the conditions: $\rho \to \rho_\infty$ and $u \to 0$. From the change in elevation, the pressure term in x direction is given by $-\frac{\partial p}{\partial x}=\rho_\infty g$. The body force term in the momentum equation along the negative x-direction is $-\rho g$.

Combining the two terms yields;

$$-\rho g-\frac{\partial p}{\partial x}=\rho_\infty g-\rho g$$

$$=g(\rho_\infty-\rho)$$

Volumetric coefficient β of thermal expansion is expressed mathematically as

$$\beta=\frac{1}{\rho}\left(\frac{\nabla\rho}{\nabla T}\right)_p=-\frac{1}{\rho}\left(\frac{\rho_\infty-\rho}{T_\infty-T}\right)_p=\frac{1}{\rho}\left(\frac{\rho_\infty-\rho}{T-T_\infty}\right)_p$$

Where $\nabla\rho=\rho_\infty-\rho$ and $\nabla T=T_\infty-T$ are changes in density and temperature respectively.

Also $\rho\beta(T-T_\infty)=\rho_\infty-\rho$ (2.4)

Substituting equation (2.4) into equation (2.3) yields:

$$-\frac{\partial p}{\partial x}-\rho g=\beta g\left(T-T_{\infty}\right) \tag{2.5}$$

2.3.2a Coriolis Effect

This is scenario where when moving objects are viewed from a rotating frame of reference seem to apparently deflect from a straight path. The effect is caused by Coriolis force, which appears in the momentum equation in a rotating system, Persson (1998). For a system rotating at a constant angular velocity Ω along about z direction, the formula for the magnitude and direction of the Coriolis acceleration is given by

$$2\Omega\times q=\begin{vmatrix} i & j & k \\ 0 & 0 & 2\Omega \\ u & v & w \end{vmatrix}=-2\Omega v i+2\Omega u j \tag{2.6}$$

Where u, v and w are velocities in the x, y and z directions respectively.

2.3.2b Lorentz Force

The geometry of the flow is that constant magnetic field $\boldsymbol{B_0}$ is applied in the direction perpendicular to that of fluid flow. The assumption that the induced magnetic field is negligible is made. This is justified from the fact that Magnetic Reynolds number is very small for partially ionized fluid. From the equation of conservation of electric charge $\nabla\bullet\mathbf{J}=0$ shows that $\vec{J}_z=0$ since the plate is electrically non-conducting. Where $\mathbf{J}=(J_x,J_y,J_z)$ are components of the current in x, y and z directions respectively. For strong magnetic field, Ohm's law is modified to include Hall currents. When the ion slips and thermoelectric are neglected, the Ohm's takes the form; (Cowling, 1957).

$$\vec{J}+\frac{m}{H_0}(\vec{J}\times \mathrm{H})=\sigma(\mathrm{E}+\mu_e q\times H+\frac{1}{e\eta_e}\nabla \mathrm{P}_e) \tag{2.7}$$

Where σ , m, μ_e, η_e and P_e is the electrical conductivity, hall parameter, the magnetic permeability, the number of density electron and electron pressure respectively and m<<1. Since there is no applied electric field **E**=0 and for partially ionized gases, the electron pressure gradient are neglected and the generalized Ohm's law takes the form;

$$\left(J_x + J_y + J_z\right) + \frac{m}{H_0}\begin{vmatrix} i & j & k \\ J_x & J_y & J_z \\ 0 & 0 & H_0 \end{vmatrix} = \sigma\mu_e \begin{vmatrix} i & j & k \\ u_0 & v & w \\ 0 & 0 & H_0 \end{vmatrix}$$

$$\left(J_x + J_y + J_z\right) + \frac{m}{H_0}\left(H_0 J_y - H_0 J_x + 0\right) = \sigma\mu_e\left(vH_0 - u_0 H_0 + 0\right)$$

$$\left.\begin{array}{l} J_x + mJ_y = \sigma\mu_e v H_0 \\ J_y - mJ_x = \sigma\mu_e u_0 H_0 \end{array}\right\} \tag{2.8}$$

Solving equations (2.8) simultaneously yields

$$J_x = \frac{\sigma B_0}{1+m^2}(mv - u) \tag{2.9}$$

$$J_y = \frac{\sigma B_0}{1+m^2}(v + mu) \tag{2.10}$$

Thus the force due to electromagnetism is given as

$$\mathbf{J}\times\mathbf{B} = \begin{vmatrix} i & j & k \\ j_x & j_y & 0 \\ 0 & 0 & \mathbf{B}_0 \end{vmatrix}$$

From equations (2.9)and (2.10) the components of Lorentz force are given by

$$(J \times B)_x = \frac{\sigma B^2{}_0}{1+m^2}(u - mv) \tag{2.11}$$

$$(J \times B)_y = \frac{\sigma B^2{}_0}{1+m^2}(v + mu) \tag{2.12}$$

2.3.2c Momentum equation in the x-direction

Substituting equations 2.3, 2.5, 2.6 and 2.11 in the equation 2.2, the equation of motion along the x direction takes the form;

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - 2v\Omega = v\nabla^2 u + \beta g(T - T_\infty) - \frac{\sigma B_0^2(u - mv)}{\rho(1+m^2)} - \frac{\upsilon}{k}u \tag{2.13}$$

2.3.2d Momentum equation in the y-direction

Substituting equations 2.3, 2.6 and 2.12 in the equation 2.2, the equation of motion along the y direction takes the form;

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + 2u\Omega = v\nabla^2 u - \frac{\sigma B_0^2(v + mu)}{\rho(1+m^2)} - \frac{\upsilon}{k}v \tag{2.14}$$

2.3.3The Equation of energy

This equation is derived from the first law of thermodynamics. The law states that energy is conserved in processes involving a thermodynamic system and its surrounding. This implies that the increase in internal energy dE of a system is equal to the amount of the amount of energy added by heating the system dQ less the amount lost as a result of work done by the system to its surroundings *dW=pdv.*

Therefore *dE=dQ-dw* or *dW=dQ-pdv.*

Mathematically the law is defined as;

$$C_p\left(\frac{D\rho}{Dt} + \rho\nabla T\right) = k\nabla^2 T + \mu\phi + Q' - Q'' \qquad (2.15)$$

Where Q' and Q" are the Joules heating and thermal radiations respectively, Dawit, (2012) and $\frac{D}{Dt}$ is the material derivative and is expressed as

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} \qquad (2.16)$$

From the law of conservation of mass and assuming that the fluid is of constant density, the material derivative of density equates to zero. The term $\mu\phi$ represents the internal heating due to viscous dissipation. In three dimensional fluid flows, viscous dissipation function ϕ for an incompressible fluid is expressed as

$$\phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \left[\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)^2 \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)^2\right]$$
$$- \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2$$

The term $\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$ vanishes since it represents the equation of continuity. All the partial derivatives of w vanish because there is no flow along the z-direction. Since the plate is moving parallel to the x-direction, the $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial x}$ have minimal contribution to viscous dissipation and thus neglected. Also, all the partial derivatives with respect to y are dropped from the equation and thus the viscous term reduces to

$$\phi = \left(\frac{du}{dz}\right)^2 + \left(\frac{dv}{dz}\right)^2 \tag{2.17}$$

The term $\frac{J^2}{\sigma}$ represents Joules' dissipation. From the equations (2.9) and (2.10) the electric current density is given as

$$\mathbf{J} = J_x + J_y$$

$$J = \frac{\sigma B_0}{1+m^2}(mv-u)i + \frac{\sigma B_0}{1+m^2}(v+mu)j \tag{2.18}$$

Therefore

$$J^2 = J \bullet J = \left(\frac{\sigma B_0}{1+m^2}(mv-u)\right)^2 + \left(\frac{\sigma B_0}{1+m^2}(v+mu)\right)^2 \tag{2.19}$$

Joules' heating is expressed as

$$\frac{J^2}{\sigma} = \frac{1}{\sigma}\left[\left(\frac{\sigma B_0}{1+m^2}(mv-u)\right)^2 + \left(\frac{\sigma B_0}{1+m^2}(v+mu)\right)^2\right] \tag{2.20}$$

$$= \sigma B_0^2\left(\frac{(mv-u)^2 + (v+mu)^2}{(1+m^2)^2}\right) \tag{2.21}$$

The term $Q'' = \frac{\partial q_r}{\partial z}$ represents thermal radiation.

Substituting equations 2.14 and 2.18 in equation 2.12 the energy equation takes the form

$$\rho C_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}-w_0\frac{\partial T}{\partial w}\right)=k\nabla^2 T+\mu\phi-\frac{\partial q_r}{\partial z}-\sigma B_0^2\left(\frac{(mv-u)^2+(v+mu)^2}{(1+m^2)^2}\right). \tag{2.22}$$

2.4.0Turbulent Modeling

Flows encountered in engineering applications become unstable when a certain Reynolds number is exceeded. In turbulent flows, flow variables undergo chaotic fluctuation and hence they are highly irregular in time and space. In RANS mode of approach, the equations are obtained by decomposing the flow variables of the equations into time mean and the fluctuating part and there after averaging the entire equations.

2.4.1 Time Averaged Continuity Equation

Equation 2.1 can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial w}=0 \tag{2.23}$$

Where $u=\bar{u}+u', v=\bar{v}+v'$ and $w=\bar{w}+w'$ (2.24a)

$\bar{u}$, $\bar{v}$ and $\bar{w}$ are the mean velocities in the x, y and z respectively given by

$$\bar{u}=\frac{1}{\Delta t}\int^{\Delta t} u\,dt,\ \bar{v}=\frac{1}{\Delta t}\int^{\Delta t} v\,dt \text{ and } \bar{w}=\frac{1}{\Delta t}\int^{\Delta t} w\,dt \tag{2.24b}$$

Substituting equations (2.24a) and (2.24b) in equation (2.23) yields;

$$\frac{\partial(\bar{u}+u')}{\partial x}+\frac{\partial(\bar{v}+v')}{\partial y}+\frac{\partial(\bar{w}+w')}{\partial w}=0 \tag{2.24}$$

Averaging over the period $0 \rightarrow \Delta t$ results to

$$\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y} + \frac{\partial \overline{w}}{dw} = 0 \qquad \text{`} \tag{2.25}$$

2.4.2 Time averaged Momentum Equation

X-direction momentum equation is:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - w_0\frac{\partial u}{\partial w} - 2v\Omega = \nu\nabla^2 u + \beta g\left(T - T_\infty\right) - \frac{\sigma B_0^2\left(u - mv\right)}{\rho\left(1 + m^2\right)} - \frac{\nu}{k}u \tag{2.26}$$

Multiplying continuity equation by u as: $u.\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial w} = 0\right)$ and adding the results to the above equation yields:

$$\frac{\partial u}{\partial t} + u\frac{\partial u^2}{\partial x} + v\frac{\partial (uv)}{\partial y} - w_0\frac{\partial (uw)}{\partial w} - 2v\Omega = \nu\nabla^2 u + \beta g\left(T - T_\infty\right) - \frac{\sigma B_0^2\left(u - mv\right)}{\rho\left(1 + m^2\right)} - \frac{\nu}{k}u \tag{2.27}$$

Integrating over period $0 \rightarrow \Delta t$ results to for an unsteady case

$$\frac{\partial u}{\partial t} + u\frac{\partial \overline{u}^2}{\partial x} + v\frac{\partial \left(\overline{uv}\right)}{\partial y} - w_0\frac{\partial \left(\overline{uv}\right)}{\partial w} - 2\overline{v}\Omega = \nu\nabla^2 \overline{u} + \beta g\left(\overline{T} - T_\infty\right) - \frac{\sigma B_0^2\left(\overline{u} - m\overline{v}\right)}{\rho\left(1 + m^2\right)} - \frac{\nu}{k}\overline{u} \tag{2.28}$$

Making the substitutions $u = \overline{u} + u'$, $v = \overline{v} + v'$, $w = \overline{w} + w'$ and $T = \overline{T} + T'$ in the above equations yields

$$\frac{\partial(\bar{u}+u')}{\partial t}+u\frac{\partial(\bar{u}+u')^2}{\partial x}+v\frac{\partial(\bar{u}+u')(\bar{v}+v')}{\partial y}-w_0\frac{\partial(\bar{u}+u')(\bar{v}+v')}{\partial w}-2(\bar{v}+v)\Omega=\nu\nabla^2(\bar{u}+u')+$$
$$\beta g\left(\bar{T}+T'-T_\infty\right)-\frac{\sigma B_0^2\left((\bar{u}+u')-m(\bar{v}+v')\right)}{\rho(1+m^2)}-\frac{\nu}{k}(\bar{u}+u')$$

(2.29)

$$\frac{\partial\bar{u}}{\partial t}+\left[\frac{\partial\overline{u^2}}{\partial x}+\frac{\partial(u')^2}{\partial x}\right]+\left[\frac{\partial\overline{(uv)}}{\partial y}+\frac{\partial(u'v')}{\partial y}\right]-\left[\frac{\partial\overline{(uw)}}{\partial z}+\frac{\partial(u'w')}{\partial z}\right]-2\bar{v}\Omega=\nu\nabla^2\bar{u}+\beta g\left(\bar{T}-T_\infty\right)$$
$$-\frac{\sigma B_0^2(\bar{u}-m\bar{v})}{\rho(1+m^2)}-\frac{\nu}{k}\bar{u}$$

(2.30)

From the direct chain rule $\frac{\partial\overline{u^2}}{\partial x}=2\bar{u}\frac{\partial\bar{u}}{\partial x}$ and from product rule $\frac{\partial\overline{(uv)}}{\partial y}=\bar{u}\frac{\partial\overline{(v)}}{\partial y}+\bar{v}\frac{\partial\overline{(u)}}{\partial y}$ and $\frac{\partial\overline{(uw)}}{\partial z}=\bar{w}\frac{\partial\overline{(u)}}{\partial z}+\bar{u}\frac{\partial\overline{(w)}}{\partial z}$ thus

$$\left[2\bar{u}\frac{\partial\bar{u}}{\partial x}+\frac{\partial(u')^2}{\partial x}\right]+\left[\bar{u}\frac{\partial\overline{(v)}}{\partial y}+\bar{v}\frac{\partial\overline{(u)}}{\partial y}+\frac{\partial\overline{(u'v')}}{\partial y}\right]-\left[\bar{w}\frac{\partial\overline{(u)}}{\partial z}+\bar{u}\frac{\partial\overline{(w)}}{\partial z}+\frac{\partial\overline{(u'w')}}{dz}\right]$$
$$-2\bar{v}\Omega=\nu\nabla^2\bar{u}+\beta g\left(\bar{T}-T_\infty\right)-\frac{\sigma B_0^2(\bar{u}-m\bar{v})}{\rho(1+m^2)}-\frac{\nu}{k}\bar{u}\quad.$$

(2.31)

Multiplying (2.56) by $\bar{u}$ and subtracting the results from equation (2.31) yields

$$\frac{\partial\bar{u}}{\partial t}+\left[\bar{u}\frac{\partial\bar{u}}{\partial x}+\frac{\partial(u')^2}{\partial x}\right]+\left[\bar{v}\frac{\partial\overline{(u)}}{\partial y}+\frac{\partial(u'v')}{\partial y}\right]-\left[w_0\frac{\partial\overline{(u)}}{\partial z}+\frac{\partial(u'w')}{\partial z}\right]-2\bar{v}\Omega=$$
$$\nu\nabla^2\bar{u}+\beta g\left(\bar{T}-T_\infty\right)-\frac{\sigma B_0^2(\bar{u}-m\bar{v})}{\rho(1+m^2)}-\frac{\nu}{k}\bar{u}$$

(2.32)

Then

$$\bar{u}\frac{\partial\bar{u}}{\partial x}+\bar{v}\frac{\partial\overline{(u)}}{\partial y}-w_0\frac{\partial\overline{(u)}}{\partial z}-2\bar{v}\Omega=\nu\nabla^2\bar{u}+\beta g\left(\bar{T}-T_\infty\right)-\frac{\sigma B_0^2\left(\bar{u}-m\bar{v}\right)}{\rho\left(1+m^2\right)}-\frac{\nu}{k}\bar{u}-\frac{\partial\overline{(u')^2}}{\partial x}$$
$$-\frac{\partial\overline{(u'v')}}{\partial y}-\frac{\partial\overline{(u'w')}}{\partial z}$$

(2.33)

Similarly the momentum equation along the y-direction is:

$$\frac{\partial\bar{v}}{\partial t}+\left[\bar{u}\frac{\partial\overline{(v)}}{\partial x}+\frac{\partial\overline{(u'v')}}{\partial x}\right]+\left[\bar{v}\frac{\partial\overline{(u)}}{\partial y}+\frac{\partial\overline{(v')^2}}{\partial y}\right]-\left[w_0\frac{\partial\overline{(v)}}{\partial z}+\frac{\partial\overline{(u'w')}}{\partial z}\right]-2\bar{u}\Omega=\nu\nabla^2\bar{v}$$
$$-\frac{\sigma B_0^2\left(\bar{v}+m\bar{u}\right)}{\rho\left(1+m^2\right)}-\frac{\nu}{k}\bar{v}$$

Or

$$\frac{\partial\bar{v}}{\partial t}+\bar{u}\frac{\partial\overline{(v)}}{\partial x}+\bar{v}\frac{\partial\overline{(v)}}{\partial y}-w_0\frac{\partial\overline{(v)}}{\partial z}+2\bar{u}\Omega=\nu\nabla^2\bar{v}-\frac{\sigma B_0^2\left(\bar{v}+m\bar{u}\right)}{\rho\left(1+m^2\right)}-\frac{\partial\overline{(u'v)}}{\partial x}-\frac{\partial\overline{(v')^2}}{\partial y}-\frac{\partial\overline{(v'w')}}{\partial z}$$

(2.34)

2.4.3 Time averaged Energy Equation

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}-w_0\frac{\partial T}{\partial w}=\frac{k\nabla^2 T}{\rho C_p}+\frac{\mu}{\rho C_p}\phi-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial z}-\frac{\sigma B_0^2}{\rho C_p}\left(\frac{(mv-u)^2+(v+mu)^2}{(1+m^2)^2}\right)$$

(2.35)

Using the normal time averaging and RANS equation the above equation reduces to:

$$\frac{\partial \overline{T}}{\partial t}+\bar{u}\frac{\partial \overline{T}}{\partial x}+\bar{v}\frac{\partial \overline{T}}{\partial y}-w_0\frac{\partial \overline{T}}{\partial w}=\frac{k\nabla^2\overline{T}}{\rho C_p}+\frac{\mu}{\rho C_p}\phi-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial z}-\frac{\sigma B_0^2}{\rho C_p}\left(\frac{(m\bar{v}-\bar{u})^2+(\bar{v}+m\bar{u})^2}{(1+m^2)^2}\right) \tag{2.36}$$

For two dimensional turbulent boundary time-averaged equations (2.25), (2.33), (2.34) and (2.36) and taking into account the geometry of the problem all the derivatives with respect to x and those with respect to y vanish and the set of equations (2.25), (2.33), (2.34) and (2.36) reduces to:

Continuity equation

$$\frac{\partial \bar{u}}{\partial x}+\frac{\partial \bar{u}}{\partial y}+\frac{\partial \bar{u}}{\partial z}=0 \tag{2.37}$$

Momentum equation along the x-direction

$$\frac{\partial \bar{u}}{\partial t}-w_0\frac{\partial \bar{u}}{\partial z}-2\bar{v}\Omega=\nu\frac{\partial^2\bar{u}}{\partial z^2}-\frac{\partial(u'w')}{\partial z}+\beta g\left(\overline{T}-T_\infty\right)-\frac{\sigma B_0^2(\bar{u}-m\bar{v})}{\rho(1+m^2)}-\frac{\upsilon}{k}\bar{u}$$

or

$$\frac{\partial \bar{u}}{\partial t}-w_0\frac{\partial \bar{u}}{\partial z}-2\bar{v}\Omega=\frac{1}{\rho}\frac{\partial}{\partial z}\left(\mu\frac{\partial \bar{u}}{\partial z}-\rho u'w'\right)+\beta g\left(\overline{T}-T_\infty\right)-\frac{\sigma B_0^2(\bar{u}-m\bar{v})}{\rho(1+m^2)}-\frac{\upsilon}{k}\bar{u}$$

Momentum equation along the y-direction

$$\frac{\partial \bar{v}}{\partial t}-w_0\frac{\partial \bar{v}}{\partial z}+2\bar{u}\Omega=\frac{1}{\rho}\frac{\partial}{\partial z}\left(\mu\frac{\partial \bar{v}}{\partial z}-\rho u'w'\right)-\frac{\sigma B_0^2(\bar{v}+m\bar{u})}{\rho(1+m^2)}-\frac{\upsilon}{k}\bar{v} \tag{2.39}$$

The energy equation

$$\frac{\partial \overline{T}}{\partial t} - w_0 \frac{\partial \overline{T}}{\partial z} = \frac{k}{\rho C_p}\frac{\partial^2 \overline{T}}{\partial z^2} + \frac{\mu}{\rho C_p}\phi - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial z} - \frac{\sigma B_0^2}{\rho C_p}\left(\frac{(m\overline{v} - \overline{u})^2 + (\overline{v} + m\overline{u})^2}{(1+m^2)^2}\right)$$

(2.40)

The above set of equations are incompressible Reynolds-Averaged Navier Stokes. The molecular shear stress; $\mu \frac{\partial \overline{v}}{\partial z}$ and $\mu \frac{\partial \overline{u}}{\partial z}$, the eddy shear stress: $-\rho u' w' \equiv \rho \varepsilon_m \left(\frac{\partial \overline{u}}{\partial z}\right)$, ε_m is a flow field property called momentum eddy diffusivity.

The molecular heat flux $-\rho T' w' \equiv \rho \varepsilon_H \left(\frac{\partial \overline{T}}{\partial z}\right)$, ε_H is the eddy diffusivity.

2.5 Boussinesq approximation

From Boussinesq approximation

$$\tau_t = -\rho u' v' = \rho \varepsilon_m \left(\frac{\partial \overline{u}}{\partial z}\right) \tag{2.41}$$

Empirical methods are used to resolve the Reynolds shear stress terms in the equations above and this leads to adoption of Prandtl mixing length hypothesis. The Reynolds shear stress $\rho u' v'$ represents the flux of x-momentum in the z-direction. Prandtl made the assumption that the momentum is transported by eddies that moved in the z-direction over the distance l without interaction and then mixed with the existing fluid at the new locations McComb (2009)

From the experiments Prandtl deduced that:

$$\rho u' w' = -\rho l^2 \left(\frac{\partial \overline{u}}{\partial z}\right)^2 \tag{2.42}$$

Further assumptions were made i.e. $l = nz$ where n is the Von Karman constant, $n = 0.4$ McComb (2009). Finally the stress is given by:

$$\rho u'v' = -\rho n^2 z^2 \left(\frac{\partial \bar{u}}{\partial z} \right)^2$$

Thus the approximations of the terms as a result of turbulence effects for the model for the momentum and energy equations are;

$$-\rho u'w' \approx \rho \varepsilon_m \frac{\partial \overline{u}}{\partial z} \approx \rho n^2 z^2 \left(\frac{\partial \bar{u}}{\partial z} \right)^2 \tag{2.43}$$

$$-\rho u'w' \approx \rho \varepsilon_m \frac{\partial \bar{v}}{\partial z} \approx \rho n^2 z^2 \left(\frac{\partial \bar{v}}{\partial z} \right)^2 \tag{2.44}$$

$$-T'w' \approx \varepsilon_m \frac{\partial \overline{T}}{\partial z} \varepsilon_H = \frac{\varepsilon_m}{\Pr_t} = \frac{n^2 z^2}{\Pr_t} \left(\frac{\partial \bar{u}}{\partial z} \right)$$

$$\varepsilon_m = n^2 z^2 \frac{\partial \bar{u}}{\partial z} = n^2 z^2 \frac{\partial \bar{v}}{\partial z} \tag{2.46}$$

(2.47)

$$-T'w' \approx \frac{n^2 z^2}{\Pr_t} \frac{\partial \overline{T}}{\partial z} \tag{2.48}$$

From the above assumptions and approximations, modeled turbulent equations take the form

$$\frac{\partial \bar{u}}{\partial t} - w_0 \frac{\partial \bar{u}}{\partial z} - 2\bar{v}\Omega = \nu \frac{\partial^2 \bar{u}}{\partial z^2} + \frac{\partial}{\partial z}\left(n^2 z^2 \left(\frac{\partial \bar{u}}{\partial z} \right)^2 \right) + \beta g\left(\overline{T} - T_\infty\right) - \frac{\sigma B_0^2 \left(\bar{u} - m\bar{v}\right)}{\rho\left(1 + m^2\right)} - \frac{\nu}{k}\bar{u}$$

$$\frac{\partial\bar{v}}{\partial t}-w_0\frac{\partial\bar{v}}{\partial z}+2\bar{u}\Omega=\nu\frac{\partial^2\bar{v}}{\partial z^2}+\frac{\partial}{\partial z}\left(n^2z^2\left(\frac{\partial\bar{v}}{\partial z}\right)^2\right)-\frac{\sigma B_0^2\left(\bar{v}+m\bar{u}\right)}{\rho\left(1+m^2\right)}-\frac{\nu}{k}\bar{u} \tag{2.50}$$

$$\frac{\partial\bar{T}}{\partial t}-w_0\frac{\partial\bar{u}}{\partial z}=\bar{u}\frac{k\nabla^2\bar{T}}{\rho C_p}+\frac{\partial}{\partial z}\left(\frac{n^2z^2}{\Pr t}\frac{\partial\bar{u}}{\partial z}\frac{\partial\bar{T}}{\partial z}\right)-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial z}+\frac{\mu}{\rho C_p}\left(\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2\right)$$

$$-\frac{\sigma B_0^2}{\rho C_p}\left(\frac{(m\bar{v}-\bar{u})^2+(\bar{v}+m\bar{u})^2}{(1+m^2)^2}\right) \tag{2.51}$$

2.6Non-dimesionalisation

The principal use of dimensional analysis is to deduce from a study of the dimensions of the variables in any physical system certain limitations on the form of any possible relationship between those variables. The method is of great generality and mathematical simplicity. This is a process that starts with selecting a suitable scale against which all dimensions in a given physical model are scaled. This process aims at ensuring that the results obtained are applicable to other geometrically similar configurations under similar set of flow conditions. The non-dimensionalized boundary layer equations have a solution that is bounded, where the solution values lie between 0 and 1 inclusive. The characteristic velocity is taken as the free stream velocityU_∞ . The non-dimensional parameters are defined as follows:

$$U=\frac{\bar{u}}{U_0},\ V=\frac{\bar{v}}{U_0},\ \eta=\frac{zU_0}{\nu},\ t=\frac{\bar{t}U_0^2}{\nu},\ m=w_e\tau_e,\ M=\frac{\sigma B_0^2\nu}{\rho U_0},\ \theta=\frac{\bar{T}-T_\infty}{\bar{T}_w-T_\infty},\ R=\frac{\Omega\nu}{U_0^2},$$

$$\Pr=\frac{\nu}{\alpha},\ Ec=\frac{U_0^2}{C_p\left(T_w-T_\infty\right)},\ Gr=\frac{g\beta\nu(T_w-T_\infty)}{U_0^3},Nr=\frac{4\varepsilon^2\nu}{\rho C_pU_0^2}\ \text{and}\ X=\frac{\upsilon^2}{kU_0^2}$$

In order to re-write the equations of continuity, momentum, and energy into their respective dimensionless form, the following analysis is carried out first:

$$\frac{\partial \bar{u}}{\partial t} = \frac{\partial \bar{u}}{\partial U}\frac{\partial U}{\partial \bar{t}}\frac{\partial \bar{t}}{\partial t} = U_0 U_0^2 \frac{\partial U}{\partial \bar{t}} = U_0^3 \frac{\partial U}{\partial t} \tag{2.52}$$

$$-2\Omega \bar{v} = -\frac{2RU_0^2 V U_0}{\nu} = -\frac{2RU_0^3 V}{\nu} \tag{2.53}$$

$$\nu \frac{\partial^2 u}{\partial z^2} = \nu \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial U}\frac{\partial U}{\partial \eta}\frac{\partial \eta}{\partial z}\right) = \nu \frac{\partial}{\partial \eta}\left(\frac{U_0^2}{\nu}\frac{\partial U}{\partial \eta}\right)\frac{\partial \eta}{\partial z} = \frac{U_0^3}{\nu}\frac{\partial^2 U}{\partial \eta^2} \tag{2.54}$$

$$\frac{\partial}{\partial z}\left(n^2 z^2 \left(\frac{\partial \bar{u}}{\partial z}\right)^2\right) = \frac{\partial}{\partial \eta}\left(n^2 \left(\frac{\eta \nu}{U_0}\right)^2 \left(\frac{\partial u}{\partial U}\frac{\partial U}{\partial \eta}\frac{\partial \eta}{\partial z}\right)^2\right)\frac{\partial \eta}{\partial z} = \frac{\partial}{\partial \eta}\left(\frac{n^2 \eta^2 \nu^2}{U_0^2}\frac{U_0^4}{\nu^2}\left(\frac{\partial U}{\partial \eta}\right)^2\right)$$

$$= \frac{U_0^3}{\nu}\frac{\partial}{\partial \eta}\left(n^2 \eta^2 \left(\frac{\partial U}{\partial \eta}\right)^2\right) \tag{2.55}$$

$$\theta = \frac{T - T_\infty}{T_w - T_w} \quad , \; T - T_\infty = \theta (T - T_\infty)$$

$$Gr = \frac{g\beta\nu (T_w - T_w)}{U_0^3}$$

$$g\beta (T - T_w) = \theta g\beta (T_w - T_\infty)$$

(2.56)

$$\frac{\sigma B_0^2}{\rho}\left(\frac{\bar{u} - m\bar{v}}{1 + m^2}\right) = \frac{\sigma B_0^2}{\rho}\left(\frac{UU_0 - mVU_0}{1 + m^2}\right) = \frac{\sigma B_0^2 U_0}{\rho}\left(\frac{U - mV}{1 + m^2}\right) \tag{2.57}$$

$$\frac{\partial \bar{v}}{\partial t}=\frac{\partial \bar{v}}{\partial U}\frac{\partial V}{\partial \bar{t}}\frac{\partial \bar{t}}{\partial t}=U_0U_0^2\frac{\partial V}{\partial \bar{t}}=U_0^3\frac{\partial V}{\partial \bar{t}} \tag{2.58}$$

$$-2\Omega\bar{u}=-\frac{2RU_0^2UU_0}{\nu}=-\frac{2RU_0^3U}{\nu} \tag{2.59}$$

$$\nu\frac{\partial^2\bar{v}}{\partial z^2}=\nu\frac{\partial}{\partial z}\left(\frac{\partial \bar{v}}{U}\frac{\partial U}{\partial \eta}\frac{\partial \eta}{dz}\right)=\nu\frac{\partial}{\partial \eta}\left(\frac{U_0^2}{\upsilon}\frac{\partial U}{\partial \eta}\right)\frac{\partial \eta}{\partial z}=\frac{U_0^3}{\upsilon}\frac{\partial^2 U}{\partial \eta^2} \tag{2.60}$$

$$\frac{\partial}{\partial z}\left(n^2z^2\left(\frac{\partial \bar{v}}{\partial z}\right)^2\right)=\frac{\partial}{\partial \eta}\left(n^2\frac{\eta^2\upsilon^2}{U_0^2}\left(\frac{\partial \bar{v}}{\partial V}\frac{\partial V}{\partial \eta}\frac{\partial \eta}{\partial z}\right)^2\right)\frac{\partial \eta}{\partial z}=\frac{\partial}{\partial \eta}\left(n^2\frac{\eta^2\upsilon^2}{U_0^2}U^2\frac{U_0^2}{\upsilon^2}\left(\frac{\partial \eta}{\partial z}\right)^2\right)\frac{U_0}{\upsilon}$$

$$\frac{\sigma B_0^2}{\rho}\left(\frac{\bar{v}+m\bar{u}}{1+m^2}\right)=\frac{\sigma B_0^2}{\rho}\left(\frac{VU_0+mUU_0}{1+m^2}\right)=\frac{\sigma B_0^2U_0}{\rho}\left(\frac{V+mU}{1+m^2}\right) \tag{2.61}$$

$$\frac{\partial \bar{T}}{\partial t}=\frac{\partial \bar{T}}{\partial U}\frac{\partial T}{\partial \bar{t}}\frac{\partial \bar{t}}{\partial t}=U_0U_0^2\frac{\partial T}{\partial \bar{t}}=U_0^3\frac{\partial T}{\partial \bar{t}} \tag{2.62}$$

$$\alpha\frac{\partial^2\bar{T}}{\partial z^2}=\alpha\frac{\partial}{\partial \eta}\left(\frac{\partial \bar{T}}{\partial \theta}\frac{\partial \theta}{\partial \eta}\frac{\partial \eta}{\partial z}\right)\frac{\partial \eta}{\partial z}=\alpha\frac{\partial}{\partial \eta}\left(\frac{U_0}{\upsilon}\left(T_w-T_\infty\right)\frac{\partial \theta}{\partial \eta}\right)\frac{U_0}{\upsilon}=\frac{U_0^2}{\Pr}\left(T_w-T_\infty\right)\frac{\partial^2\theta}{\partial \eta^2}$$

(2.63)

$$\frac{\partial}{\partial z}\left(\frac{n^2z^2}{\Pr_t}\frac{\partial \bar{u}}{\partial T}\frac{\partial \bar{T}}{\partial z}\right)=\frac{\partial}{\partial \eta}\left(\frac{n^2z^2\eta^2}{U_0^2\Pr_t}\frac{\partial \bar{u}}{\partial U}\frac{\partial U}{\partial \eta}\frac{\partial \eta}{\partial z}\frac{\partial T}{\partial \theta}\frac{\partial \theta}{\partial \eta}\frac{\partial \eta}{\partial z}\right)=$$

$$\frac{\partial}{\partial \eta}\left(\frac{n^2\upsilon^2\eta^2}{U_0^2\Pr_t}\frac{U_0^2}{\upsilon}\left(T_w-T_\infty\right)\frac{U_0^2}{\upsilon}\frac{\partial U}{\partial \eta}\frac{\partial T}{\partial \eta}\right)$$

$$=\left(T_w-T_\infty\right)\frac{U_0^2}{\upsilon}\frac{\partial}{\partial \eta}\left(\frac{n^2\eta^2}{\Pr_t}\frac{\partial U}{\partial \eta}\frac{\partial T}{\partial \eta}\right) \tag{2.64}$$

$$\frac{1}{\rho C_p}\frac{\partial q_r}{\partial z} = -4\varepsilon^2\left(T - T_\infty\right) = \frac{-4\varepsilon^2\theta}{\rho C_p}\left(T_w - T_\infty\right) \tag{2.65}$$

$$\frac{\upsilon}{C_p}\left(\left(\frac{\partial \bar{u}}{\partial z}\right)^2 + \left(\frac{\partial \bar{v}}{\partial z}\right)^2\right) = \frac{\upsilon}{C_p}\left(\left(\frac{\partial \bar{u}}{\partial U}\frac{\partial U}{\partial \eta}\frac{\partial \eta}{\partial z}\right)^2 + \left(\frac{\partial \bar{v}}{\partial U}\frac{\partial U}{\partial \eta}\frac{\partial \eta}{\partial z}\right)^2\right)$$

$$= \frac{\upsilon}{C_p}\left(\frac{U_0^4}{\upsilon^2}\left(\frac{\partial U}{\partial \eta}\right)^2 + \frac{U_0^4}{\upsilon^2}\left(\frac{\partial v}{\partial \eta}\right)^2\right)$$

$$= \frac{U_0^4}{C_p \upsilon}\left(\left(\frac{\partial U}{\partial z}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2\right) \tag{2.66}$$

$$\frac{\sigma B_0^2}{\rho C_p}\left(\frac{\left(\bar{u} - m\bar{v}\right)^2 + \left(\bar{v} + m\bar{u}\right)^2}{\left(1 + m^2\right)^2}\right) = \frac{\sigma B_0^2}{\rho C_p}\left(\frac{\left(UU_0 - mVU_0\right)^2 + \left(VU_0 + mUU_0\right)^2}{\left(1 + m^2\right)^2}\right)$$

$$= \frac{\sigma B_0^2 U_0^2}{\rho C_p}\left(\frac{\left(U - mV\right)^2 + \left(V + mU\right)^2}{\left(1 + m^2\right)^2}\right) \tag{2.67}$$

Substituting equations (2.43) to (2.50) into equations (2.40) and (2.41), the two equations of momentum respectively become:

$$\frac{U_0^3}{\nu}\frac{\partial U}{\partial t} - \frac{U_0^3}{\nu}w_0\frac{\partial U}{\partial \eta} - \frac{2RU_0^3 V}{\nu} = \frac{U_0^3}{\nu}\frac{\partial^2 U}{\partial \eta^2} + \frac{U_0^3}{\nu}\frac{\partial}{\partial \eta}\left(n^2\eta^2\left(\frac{\partial U}{\partial \eta}\right)^2\right) + \theta g\beta\left(T_w - T_\infty\right)$$
$$+ \frac{\sigma B_0^2 U_0}{\rho}\left(\frac{U - mV}{1 + m^2}\right) - \frac{\upsilon}{k}UU_0$$

$$\frac{U_0^3}{\nu}\frac{\partial U}{\partial t} - \frac{U_0^3}{\nu}w_0\frac{\partial U}{\partial \eta} - \frac{2RU_0^3 U}{\nu} = \frac{U_0^3}{\nu}\frac{\partial^2 V}{\partial \eta^2} + \frac{U_0^3}{\nu}\frac{\partial}{\partial \eta}\left(n^2\eta^2\left(\frac{\partial U}{\partial \eta}\right)^2\right) + \frac{\sigma B_0^2 U_0}{\rho}\left(\frac{V + mU}{1 + m^2}\right) - \frac{\upsilon}{k}VU_0$$

Dividing each term of the above equations by $\frac{U_0^3}{\upsilon}$ yields:

$$\frac{\partial U}{\partial t} - w_0 \frac{\partial U}{\partial \eta} - 2RV = \frac{\partial^2 U}{\partial \eta^2} + \frac{\partial}{\partial \eta}\left(n^2\eta^2\left(\frac{\partial U}{\partial \eta}\right)^2\right) + \frac{g\beta\upsilon(T_w - T_\infty)}{U_0^3}\theta + \frac{\sigma B_0^2 \upsilon}{\rho U_0^2}\left(\frac{U - mV}{1+m^2}\right) - \frac{\upsilon^2}{kU_0^2}U$$

(2.68)

$$\frac{\partial U}{\partial t} - w_0 \frac{\partial U}{\partial \eta} - 2RV = \frac{\partial^2 U}{\partial \eta^2} + \frac{\partial}{\partial \eta}\left(n^2\eta^2\left(\frac{\partial U}{\partial \eta}\right)^2\right) + Gr\theta - M\left(\frac{U - mV}{1+m^2}\right) - XU$$

(2.69)

Similarly the momentum equation in the y direction becomes:

$$\frac{\partial V}{\partial t} - w_0 \frac{\partial V}{\partial \eta} - 2RU = \frac{\partial^2 V}{\partial \eta^2} + \frac{\partial}{\partial \eta}\left(n^2\eta^2\left(\frac{\partial V}{\partial \eta}\right)^2\right) - M\left(\frac{V + mU}{1+m^2}\right) - XV \quad (2.70)$$

From equations (2.57) and (2.58), R is the rotation parameter, $Gr_\theta = \frac{g\beta\upsilon(T_w - T_\infty)}{U_0^3}$ is the local temperature Grashoff's number, M is the magnetic field parameter and X_i is the permeability parameter.

Similarly substituting equations (2.51) to (2.54) into equation (2.42) and dividing each of the resulting terms by $\frac{(T_w - T_\infty)U_0^2}{\upsilon}$, equation (2.42) becomes:

$$\frac{\partial \theta}{\partial t} - wo\frac{\partial \theta}{\partial \eta} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial \eta^2} + \frac{\partial}{d\eta}\left(\frac{n^2\eta^2}{\Pr_t}\frac{\partial U}{\partial \eta}\frac{\partial \theta}{\partial \eta}\right) + Nr\theta + Ec\left(\left(\frac{\partial U}{\partial \eta}\right)^2 + \left(\frac{\partial V}{\partial \eta}\right)^2\right) +$$

$$MEc\left(\frac{(U-mV)^2+(V+mU)^2}{(1+m^2)^2}\right) \tag{2.71}$$

Initial conditions of the problem

$$\left.\begin{aligned} U(0,0)=U(\eta,0) &= 0\\ V(0,0)=V(\eta,0) &= 0\\ T(0,0)=T(\eta,0) &= 0 \end{aligned}\right\} t\le 0 \tag{2.72}$$

Boundary conditions at the plate.

$$\left.\begin{aligned} U(0,t) &= 1\\ V(0,t) &= 0\\ T(0,t) &= 1 \end{aligned}\right\} \quad t>0 \tag{2.73}$$

Boundary conditions at infinity (RHS boundary conditions)

$$\left.\begin{aligned} U(\eta,t) &= 0\\ V(\eta,t) &= 0\\ T(\eta,t) &= 0 \end{aligned}\right\} \quad t>0 \tag{2.74}$$

From equation (2.71), Pr is the Prandtl number, $\Pr_t$ is turbulent Prandtl number, *Nr* is the radiation parameter, *Ec* is the Eckert number and M is the magnetic field parameter.

The non-dimensionalized equations (2.69), (2.70) and (2.71) contain Grasoff, Prandtl, Eckert and non-dimensional numbers.

Prandtl Number

The Prandtl number (Pr) is the ratio of fluid properties controlling the velocity and the temperature distributions. It is the ratio of viscous force to thermal force.

$$\Pr = \frac{C_p \mu}{k} = \frac{\rho C_p \upsilon}{k} = \frac{\upsilon}{\alpha}$$

where $\alpha = \frac{k}{\rho C_p}$ is the thermal diffusivity and $\upsilon = \frac{\mu}{\rho}$ is the viscosity of the fluid. Fluids that are particularly viscous have a relatively large value of and correspondingly a large Prandtl number, for instance lubricating oils have large Prandtl numbers which can exceed 10^4. A fluid with a Prandtl number close to unity has thermal and momentum boundary layers of similar (nearly equal) thickness. For instance water vapour at $700k$ has a Prandtl number of 1. Fluids which are good conductors of heat have a relatively large value of α ; and occurrence is found in liquid metals whose Prandtl numbers are correspondingly small, such as Mercury (Pr = 0.023).

The local temperature Grashof number

The local temperature Grashof number Gr_θ signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. A positive value of Gr_θ corresponds to cooling of the porous plate (or heating the fluid in contact with the porous plate) while a negative value corresponds to heating the porous plate (or cooling the fluid).

Eckert number Ec

The Eckert number is the ratio between the kinetic energy and thermal energy and its mathematically expressed as;

$$Ec = \frac{U^2}{C_p (T_w - T_\infty)} = \frac{U^2}{C_p \Delta T}$$

For incompressible fluid, $Ec << 1$

When Ec >0, the heat is transferred from the fluid to the surrounding.

When Ec <0, the heat is transferred from the surrounding to the fluid.

When Ec =0 there is no heat transfer.

Equations 2.69, 2.70and 2.71are then simultaneously solved numerically using finite difference iteration method. The final solution is obtained after successive iterations, beginning with an initial values indicated in the boundary conditions.

CHAPTER THREE

METHOD OF SOLUTION

3.1 Introduction

In this chapter, the method of solution is discussed and the governing equations are presented in their finite difference form. The solutions to these equations are obtained using a computer program implemented in a MATLAB version 7.90.529 (R2009b) computer program.

Equations 2.69, 2.70 and 2.71 are coupled non-linear partial differential equations. It is therefore not possible to solve these equations analytically and thus finite difference methods are used to obtain approximate solutions. To solve these equations, the partial derivatives in these equations are first replaced by their corresponding finite difference approximations and this result to a series of algebraic equations that are solved iteratively in a computer generated program. Many techniques are available for numerical simulation work and in order to quantify how well a particular numerical technique performs in generating a solution to a problem, there are four fundamental criteria that can be applied to compare and contrast different methods. The concepts are accuracy, consistency, stability and convergence.

Accuracy is a measure of how well the discrete solution represents the exact solution of the problem. Two quantities exist to measure this – the local or truncation error, which measures how well the difference equations match the differential equations, and the global error which reflects the overall error in the solution. This is not possible to find unless the exact solution is known.

A technique is consistent if the truncation error decreases as the step size is reduced, that is to as Δt , $\Delta \eta$ tend to zero then the discretized equations should tend towards the differential equations. A technique is stable if any errors in the solution remain bounded. In practice if an unstable method is used then the solution will tend towards infinity. Numerical technique is convergent if a numerical solution approaches the exact solution as the grid spacing is reduced to zero.

3.2Finite difference method

The finite difference approximations of the partial derivatives in equations (2.57), $(2.58)$and $(2.59)$are obtained by performing Taylor series expansion of the dependent variable and substituting the truncated expressions into the differential equation. The partial dervatives are approximated by finite differences in the solution at various points. By definition,

$$\frac{\partial u}{\partial \eta} = u_\eta = \lim_{\Delta\eta \to 0} \frac{u(\eta + \Delta\eta) - u(\Delta\eta)}{\Delta\eta}$$

(3.1)When $\Delta\eta$ is small, this formula can be used as an approximation for the derivative of u at η. From Taylor series

$$u(\eta + \Delta\eta) = \frac{u(\eta)}{0!} + \frac{\Delta\eta\, u_\eta(\eta)}{1!} + \frac{\Delta\eta^2}{2!} u_{\eta\eta} + HOT \qquad (3.2)$$

By rearrangement

$$\frac{u(\eta + \Delta\eta) - u(\eta)}{\Delta\eta} = u_\eta(\eta) + HOT \qquad (3.3)$$

If $\Delta\eta$ is small, the higher order terms in the expansion will be very small values and so it is possible to write equation (3.3) as;

$$u_\eta(\eta) = \frac{u(\eta + \Delta\eta) - u(\eta)}{\Delta\eta} + 0(\Delta\eta)^2 \qquad (3.4)$$

From equation (3.4), the leading term of the error in approximating u_η by the right hand side is of order $\Delta\eta$ and so this represents a first order approximation. It is possible to define other difference formula to approximate derivatives and these may have different orders of accuracy.

Similarly the second order derivatives are approximated by rearranging the first three terms of the equation (3.2) and neglecting the higher order terms. This yield:

$$u_{\eta\eta}(\eta)=\frac{u(\eta+\Delta\eta)-2u(\eta)+u(\eta-\Delta\eta)}{(\Delta\eta)^{2}}+0(\Delta\eta)^{3}$$

The above analysis deals with the continuous solution however the objective is to calculate u at a set of discrete points on the mesh, and thus the numerical solution is used. The numerical solution of equations (2.69), (2.70) and (2.71) is approximated at a discrete number of points arranged to form a rectangular grid. This rectangular grid is obtained by dividing the (η, t) plane into a network of rectangles of sides $\Delta\eta$ and Δt by drawing the set of lines:

$$\left.\begin{array}{ll}\eta = i\Delta\eta = ih & i = 0, 1, 2, \ldots\\ t = i\Delta t = ik, & k = 0, 1, 2, \ldots\end{array}\right\}$$

The nodes or mesh points or grid points of the network occur at the intersections of straight lines drawn parallel to the η and t axes. The lines parallel to the vertical axis represent spatial distance away from the plate while those drawn parallel to the horizontal axis represent the times steps. The location lines are drawn with spacing h while the time lines are drawn with spacing k .

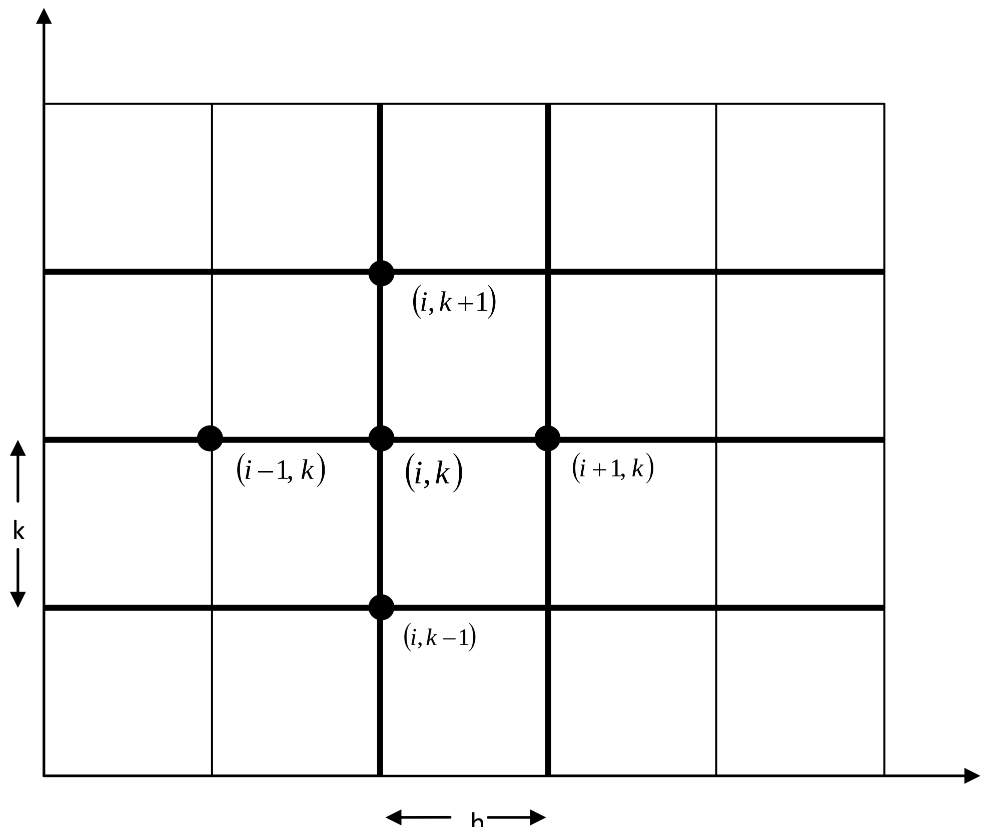

Figure 2.2: Finite difference mesh

The partial derivatives of U, V *and* T at each grid point are expressed using finite difference approximation. U_i^k , V_i^k and T_i^k for each i are obtained directly from the initial value conditions. Thus it is convenient to start from the known boundary at the edges of the mesh and working inwards so as to obtain U_i^{k+1} , V_i^{k+1} and T_i^{k+1} respectively. The derivatives are approximated using the Forward Time Backward Space (FTBS) finite difference scheme that averages the values velocity profiles at step $k+1$, and k .The FD expressions for U ,U_η and $U_{\eta\eta}$ and averaged for times $k+1$, and k are as given below.

$$\frac{\partial U}{\partial t}=\frac{U_i^{k+1}-U_i^k}{\Delta t} \tag{3.6}$$

$$\frac{\partial U}{\partial \eta}=\frac{U_i^{k+1}-U_{i-1}^{k+1}+U_i^k-U_{i-1}^k}{2\Delta\eta} \tag{3.7}$$

$$\frac{\partial^2 U}{\partial \eta^2}=\frac{U_{i+1}^{k+1}-2U_i^{k+1}+U_{i-1}^{k+1}+U_{+1i}^k-2U_i^k+U_{i-1}^k}{2(\Delta\eta)^2} \tag{3.8}$$

Equation (3.6) represents forward time difference approximation for the partial derivative $\frac{\partial U}{\partial t}$ at U_i^{k+1} having a truncation error of order $0\left(\Delta t\right)$ that represent the neglected higher order terms.

$$\frac{\partial U}{\partial t}\Big| i,k+1=\frac{U_i^{k+1}-U_i^k}{\Delta t}+\text{HOT}$$

(3.9)Equation (3.7) represents forward time difference approximation for the partial derivative $\frac{\partial U}{\partial \eta}$ at U_i^{k+1} having a truncation error of order $0\left(\Delta\eta\right)$ that represent the neglected higher order terms.

$$\frac{\partial U}{\partial \eta}=\frac{U_i^{k+1}-U_{i-1}^{k+1}+U_i^k-U_{i-1}^k}{2\Delta\eta}+\text{HOT}$$

(3.10)Equation (3.8) represents forward time difference approximation for the partial derivative $\frac{\partial^2 U}{\partial \eta^2}$ at U_i^{k+1} having a truncation error of order $0\left(\Delta\eta\right)^2$ that represent the neglected higher order terms.

$$\frac{\partial^2 U}{\partial \eta^2}=\frac{U_{i+1}^{k+1}-2U_i^{k+1}+U_{i-1}^{k+1}+U_{i+1}^k-2U_i^k+U_{i-1}^k}{2(\Delta\eta)^2}+\text{HOT} \tag{3.11}$$

Similarly the finite difference approximation for V, V_η and $V_{\eta\eta}$ and T, T_η and $T_{\eta\eta}$ are:

$$\frac{\partial V}{\partial t} = \frac{V_i^{k+1} - V_i^k}{\Delta t} \tag{3.12}$$

$$\frac{\partial V}{\partial \eta} = \frac{V_i^{k+1} - V_{i-1}^{k+1} + V_i^k - V_{i-1}^k}{2\Delta\eta} \tag{3.13}$$

$$\frac{\partial^2 V}{\partial \eta^2} = \frac{V_{i+1}^{k+1} - 2V_i^{k+1} + V_{i-1}^{k+1} + V_{i+1}^k - 2V_i^k + V_{i-1}^k}{2(\Delta\eta)^2} \tag{3.14}$$

$$\frac{\partial T}{\partial t} = \frac{T_i^{k+1} - T_i^k}{\Delta t} \tag{3.15}$$

$$\frac{\partial T}{\partial \eta} = \frac{T_i^{k+1} - T_{i-1}^{k+1} + T_i^k - T_{i-1}^k}{2\Delta\eta} \tag{3.16}$$

$$\frac{\partial^2 T}{\partial \eta^2} = \frac{T_{i+1}^{k+1} - 2T_i^{k+1} + T_{i-1}^{k+1} + T_{i+1}^k - 2T_i^k + T_{i-1}^k}{2(\Delta\eta)^2} \tag{3.17}$$

Initial conditions

$$\left.\begin{array}{l} U(0,0) = U(\eta,0) \quad = 0 \\ V(0,0) = V(\eta,0) \quad = 0 \\ T(0,0) = T(\eta,0) \quad = 0 \end{array}\right\} \qquad t \le 0 \tag{3.18}$$

Boundary conditions at the wall (LHS boundary conditions)

$$\left.\begin{aligned} U(0,k) &= 1 \\ V(0,k) &= 0 \\ T(0,k) &= 1 \end{aligned}\right\} \quad k > 0 \tag{3.19}$$

Boundary conditions at infinity (RHS boundary conditions)

$$\left.\begin{aligned} U(\eta,k) &= 0 \\ V(\eta,k) &= 0 \\ T(\eta,k) &= 0 \end{aligned}\right\} \quad k > 0 \tag{3.20}$$

Momentum equation along the x axis in finite differences

$$\begin{aligned} &\frac{U_i^{k+1} - U_i^k}{\Delta t} - Wo\frac{U_i^{k+1} - U_{i-1}^{k+1} + U_i^k - U_{i-1}^k}{2\Delta\eta} - 2R\frac{U_i^{k+1} + U_i^k}{2} = \\ &\frac{U_{i+1}^{k+1} - 2U_i^{k+1} + U_{i-1}^{k+1} + U_{+1i}^k - 2U_i^k + U_{i-1}^k}{2(\Delta\eta)^2} + N^2\eta_i\left(\frac{U_i^{k+1} - U_{i-1}^{k+1} + U_i^k - U_{i-1}^k}{2\Delta\eta}\right)^2 + (N\eta_i)^2 \\ &\left(\frac{U_i^{k+1} - U_{i-1}^{k+1} + U_i^k - U_{i-1}^k}{2\Delta\eta}\right)\left(\frac{U_{i+1}^{k+1} - 2U_i^{k+1} + U_{i-1}^{k+1} + U_{+1i}^k - 2U_i^k + U_{i-1}^k}{2(\Delta\eta)^2}\right) + Gr\theta\frac{T_i^{k+1} + T_i^k}{2} \\ &- \frac{M}{2(1+m^2)}\left(\left(U_i^{k+1} + U_i^k\right) - m\left(V_i^{k+1} + V_i^k\right)\right) - \frac{X}{2}\left(U_i^{k+1} + U_i^k\right) \end{aligned} \tag{3.21}$$

Making U_i^{k+1} in equation (3.20) the subject of the formula yields

$$U_i^{k+1} = (U_i^k + \frac{\Delta t Wo}{2\Delta\eta}\left(-U_{i-1}^{k+1} + U_i^k - U_{i-1}^k\right) + \Delta t R\left(V_i^{k+1} + V_i^k\right) + \frac{\Delta t}{2(\Delta\eta)^2}$$

$$\left(U_{i+1}^{k+1} + U_{i-1}^{k+1} + U_{+1i}^k - 2U_i^k + U_{i-1}^k\right) + \frac{N^2\eta_i\Delta t}{8(\Delta\eta)^2}\left(U_{i+1}^{k+1} - U_{i-1}^{k+1} + U_{i+1}^k - U_{i-1}^k\right)^2 + \frac{(N\eta_i)^2\Delta t}{4(\Delta\eta)^3}$$

$$\left(U_i^{k+1} - U_{i-1}^{k+1} + U_i^k - U_{i-1}^k\right)\left(U_{i+1}^{k+1} + U_{i-1}^{k+1} + U_{+1i}^k - 2U_i^k + U_{i-1}^k\right) + \frac{Gr\theta\Delta t}{2}\left(T_i^{k+1} + T_i^k\right)$$

$$-\frac{\Delta t M}{2(1+m^2)}\left(\left(U_i^k\right) - m\left(V_i^{k+1} + V_i^k\right)\right) - \frac{X\Delta t}{2}U_i^k)/(1 - \frac{\Delta t Wo}{2\Delta\eta} + \frac{\Delta t}{(\Delta\eta)^2} - \frac{(N\eta_i)^2\Delta t}{2(\Delta\eta)^3}$$

$$\left(U_i^{k+1} - U_{i-1}^{k+1} + U_i^k - U_{i-1}^k\right) + \frac{\Delta t M}{2(1+m^2)} + \frac{X\Delta t}{2})$$

.

, (3.21)

Momentum equation along the y- axis is

$$\frac{V_i^{k+1} - V_i^k}{\Delta t} - Wo\frac{V_i^{k+1} - V_{i-1}^{k+1} + V_i^k - V_{i-1}^k}{2\Delta\eta} - 2R\frac{U_i^{k+1} + U_i^k}{2} =$$

$$\frac{V_{i+1}^{k+1} - 2V_i^{k+1} + V_{i-1}^{k+1} + V_{+1i}^k - 2V_i^k + V_{i-1}^k}{2(\Delta\eta)^2} + N^2\eta_i\left(\frac{V_i^{k+1} - V_{i-1}^{k+1} + V_i^k - V_{i-1}^k}{2\Delta\eta}\right)^2 + (N\eta_i)^2$$

$$\left(\frac{V_i^{k+1} - V_{i-1}^{k+1} + V_i^k - V_{i-1}^k}{2\Delta\eta}\right)\left(\frac{V_{i+1}^{k+1} - 2V_i^{k+1} + V_{i-1}^{k+1} + V_{+1i}^k - 2V_i^k + V_{i-1}^k}{2(\Delta\eta)^2}\right) + \tag{3.22}$$

$$-\frac{M}{2(1+m^2)}\left(\left(V_i^{k+1} + V_i^k\right) + m\left(U_i^{k+1} + U_i^k\right)\right) - \frac{X}{2}\left(V_i^{k+1} + V_i^k\right)$$

Making V_i^{k+1} in equation (3.22) the subject of the formula yields

$$V_i^{k+1} = (V_i^k + \frac{\Delta t Wo}{2\Delta\eta}\left(-V_{i-1}^{k+1} + V_i^k - V_{i-1}^k\right) + \Delta t R\left(U_i^{k+1} + U_i^k\right) +$$

$$\frac{\Delta t}{2(\Delta\eta)^2}\left(V_{i+1}^{k+1} + V_{i-1}^{k+1} + V_{+1i}^k - 2V_i^k + V_{i-1}^k\right) + \frac{N^2\eta_i \Delta t}{8(\Delta\eta)^2}\left(V_{i+1}^{k+1} - V_{i-1}^{k+1} + V_{i+1}^k - V_{i-1}^k\right)^2$$

$$+ \frac{(N\eta_i)^2 \Delta t}{4(\Delta\eta)^3}\left(V_i^{k+1} - V_{i-1}^{k+1} + V_i^k - V_{i-1}^k\right)\left(V_{i+1}^{k+1} + V_{i-1}^{k+1} + V_{+1i}^k - 2V_i^k + V_{i-1}^k\right)$$

$$- \frac{\Delta t M}{2(1+m^2)}\left(m\left(U_i^{k+1} + U_i^k\right) + \left(V_i^k\right)\right) - \frac{X\Delta t}{2} V_i^k) /(1 - \frac{\Delta t Wo}{2\Delta\eta} + \frac{\Delta t}{(\Delta\eta)^2}$$

$$- \frac{(N\eta_i)^2 \Delta t}{2(\Delta\eta)^3}\left(V_i^{k+1} - V_{i-1}^{k+1} + V_i^k - V_{i-1}^k\right) + \frac{\Delta t M}{2(1+m^2)} + \frac{X\Delta t}{2})$$

$$(3.23)$$

The energy equation in finite differences

$$\frac{T_i^{k+1} - T_i^k}{\Delta t} - Wo\frac{T_i^{k+1} - T_{i-1}^{k+1} + T_i^k - T_{i-1}^k}{2\Delta\eta} - 2R\frac{T_i^{k+1} + T_i^k}{2} =$$

$$\frac{T_{i+1}^{k+1} - 2T_i^{k+1} + T_{i-1}^{k+1} + T_{+1i}^k - 2T_i^k + T_{i-1}^k}{2\Pr(\Delta\eta)^2} + Nr\theta\frac{T_i^{k+1} + T_i^k}{2} +$$

$$\frac{N^2\eta_i}{4\Pr t(\Delta\eta)^2}\left(U_i^{k+1} - U_{i-1}^{k+1} + U_i^k - U_{i-1}^k\right)\left(T_{i+1}^{k+1} - T_{i-1}^{k+1} + T_{i+1}^k - T_{i-1}^k\right)$$

$$+ \frac{(N\eta_i)^2}{4(\Delta\eta)^3}\left(U_i^{k+1} - U_{i-1}^{k+1} + U_i^k - U_{i-1}^k\right)\left(T_{i+1}^{k+1} - 2T_i^{k+1} + T_{i-1}^{k+1} + T_{+1i}^k - 2T_i^k + T_{i-1}^k\right) .$$

$$+ \frac{(N\eta_i)^2}{4(\Delta\eta)^3}\left(T_i^{k+1} - T_{i-1}^{k+1} + T_i^k - T_{i-1}^k\right)\left(U_{i+1}^{k+1} - 2U_i^{k+1} + U_{i-1}^{k+1} + U_{i+1}^k - 2U_i^k + U_{i-1}^k\right)$$

$$+ \frac{Ec}{4(\Delta\eta)^2}\left[\left(U_i^{k+1} - U_{i-1}^{k+1} + U_i^k - U_{i-1}^k\right)^2 + \left(V_i^{k+1} - V_{i-1}^{k+1} + V_i^k - V_{i-1}^k\right)^2\right]$$

$$\frac{MEc}{4(1+m^2)^2}\left[\left(\left(U_i^{k+1} + U_i^k\right)^2 - m\left(V_i^{k+1} + V_i^k\right)^2\right)\right]$$

$$(3.24)$$

Making T_i^{k+1} in equation (3.24) the subject of the formula yields

$$
\begin{aligned}
T_i^{k+1} = \quad & (T_i^k + \frac{Wo}{2\Delta\eta}\left(-T_{i-1}^{k+1} + T_i^k - T_{i-1}^k\right) + \\
& \frac{\Delta t}{2\Pr(\Delta\eta)^2}\left(T_{i+1}^{k+1} + T_{i-1}^{k+1} + T_{+1i}^k - 2T_i^k + T_{i-1}^k\right) + \frac{Nr\theta\Delta t}{2}T_i^k + \\
& \frac{N^2\eta_i\Delta t}{4\Pr t(\Delta\eta)^2}\left(U_i^{k+1} - U_{i-1}^{k+1} + U_i^k - U_{i-1}^k\right)\left(T_{i+1}^{k+1} - T_{i-1}^{k+1} + T_{i+1}^k - T_{i-1}^k\right) \\
& + \frac{(N\eta_i)^2\Delta t}{4\Pr t(\Delta\eta)^3}\left(U_i^{k+1} - U_{i-1}^{k+1} + U_i^k - U_{i-1}^k\right)\left(T_{i+1}^{k+1} + T_{i-1}^{k+1} + T_{+1i}^k - 2T_i^k + T_{i-1}^k\right) \\
& + \frac{(N\eta_i)^2\Delta t}{8\Pr t(\Delta\eta)^3}\left(T_{i+1}^{k+1} - T_{i-1}^{k+1} + T_{i+1}^k - T_{i-1}^k\right)\left(U_{i+1}^{k+1} - 2U_i^{k+1} + U_{i-1}^{k+1} + U_{i+1}^k - 2U_i^k + U_{i-1}^k\right) \\
& + \frac{Ec\Delta t}{4(\Delta\eta)^2}\left[\left(U_i^{k+1} - U_{i-1}^{k+1} + U_i^k - U_{i-1}^k\right)^2 + \left(V_i^{k+1} - V_{i-1}^{k+1} + V_i^k - V_{i-1}^k\right)^2\right] \\
& + \frac{MEc\Delta t}{4\left(1+m^2\right)^2}\left[\left(\left(U_i^{k+1} + U_i^k\right)^2 - m\left(V_i^{k+1} + V_i^k\right)^2\right)\right]) /(1 - \frac{Wo\Delta t}{2\Delta\eta} + \frac{\Delta t}{\Pr(\Delta\eta)^2} - \\
& \frac{(N\eta_i)^2\Delta t}{2\Pr t(\Delta\eta)^3}\left(U_i^{k+1} - U_{i-1}^{k+1} + U_i^k - U_{i-1}^k\right) - \frac{Nr\theta\Delta t}{2})
\end{aligned}
\tag{3.25}
$$

,

The equations (3.21), (3.23) and (3.25) are the final set of equations and are solved simultaneously using a computer code in MATLAB application software version 7.90.529 (R2009b).

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Introduction

A MATLAB computer program is written to solve equations (3.21), (3.23) and (3.25) and run for different values of non-dimensional parameters to determine velocity profiles and temperature profiles as the plate is cooled. The Non-dimensional parameters used include rotational parameter, Prandtl number, Grashof number, Hall parameter, Eckert number, radiation parameter, magnetic parameter, permeability parameter and injection parameter.

The velocities are classified as primary and secondary velocity along x and y-axes respectively. Numerical computations for the velocities (both primary and secondary) profiles and temperature profiles are obtained and the unsteady flow results obtained are presented in form of graphs as in figures 4.1 to 4.9. Gr>0 corresponds to cooling of the plate since the plate is at a higher temperature than the surrounding.

4.2 Results and Discussion

From figures 4.1 and 4.2 it is noted that increase in permeability parameter leads to a decrease in magnitude of both the primary and secondary velocity profiles of the flow. From figure 4.3 it is noted that increase in permeability parameter results to a decrease in temperature of the fluid. The Permeability parameter X is inversely proportional to the actual permeability k of the porous medium and thus increase in permeability parameter increases the porosity of the plate. Increased porosity reduces the acceleration of the flow and as a result primary and secondary velocities decreases. Reduced acceleration decreases the movement of particles and thus reduces transfer of heat and temperature leading to decrease in primary velocity profiles and temperature profiles. Radiation is the loss of heat in form of electromagnetic waves. Radiation emitted by a body is due to thermal agitation of its molecules. Heat loss due to radiation results in temperature decrease and thus thermal boundary layer of the fluid reduces. Reduced temperature of the fluid flow due to radiation leads to decreased kinetic energy of the fluid particles. Reduced kinetic energy of the fluid particles is reflected by reduction of the velocity profiles of the fluid. Increase in the magnitude of the Injection parameter w_0 leads to an increase in the velocity and temperature profiles respectively. Crossoverswere identified in the curves for w_0 shows that injection destabilizes the flow profiles.

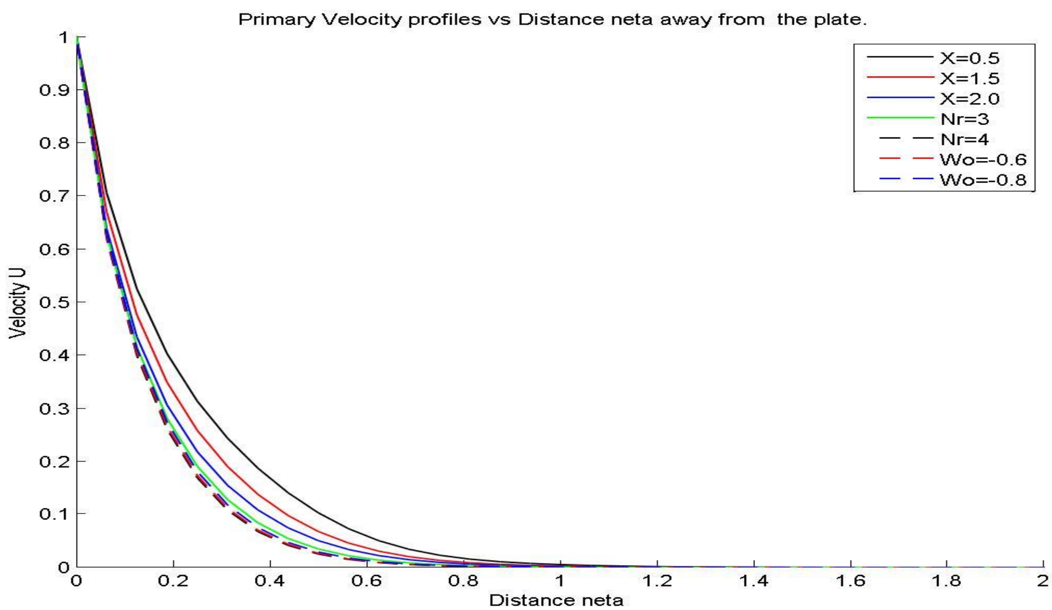

Figure 4.1: plot of primary velocity profiles verses distance

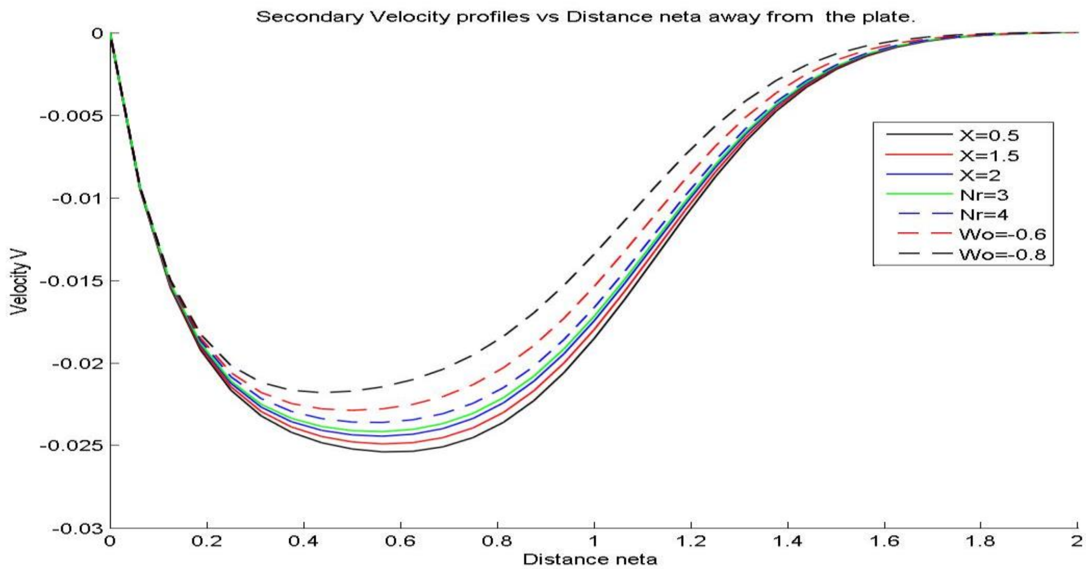

Figure 4.2: plot of secondary velocity profiles verses distance

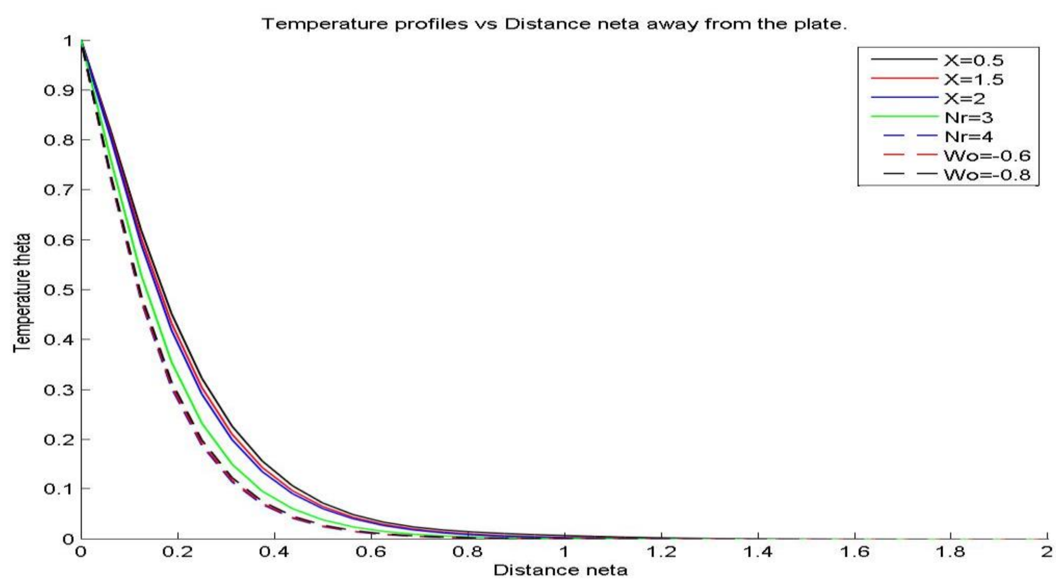

Figure 4.3: plot of temperature profiles verses distance

From figures 4.4, 4.5 and 4.6 it is observed that; increase in Eckert number results to increase in both primary and secondary velocity profiles as well as temperature profiles.It have been deduced that a positive Eckert number implies cooling of the porous plate. Increase in Eckert number is attributed to increased kinetic energy when the fluid absorbs more heat energy that is released from the internal viscous forces. Increase in temperature of the fluid increases the kinetic energy of the fluid particles which results to increase in primary and secondary velocity profiles of the fluid. Also as heat energy is released into the fluid, temperature increases is reflected by the increase in temperature profiles. Increase in Hall parameter m slightly decreases both primary and secondary velocity profiles. This is due to the fact that the effective conductivity decreases with the increase in Hall parameter which reduces the magnetic damping force hence the decrease in velocity. An increase in the Hall parameter leads to a very slight decrease in the temperature profiles. This is due to a decrease in the Joule dissipation as a result of conduction.

From figure 4.4 we note that increase in the value of rotation parameter R results to decrease in the magnitude of primary velocity profiles and secondary velocity profiles. This is because increase in rotation parameter implies that angular velocity is increased which more or less disorients and retards the fluid motion decreasing the fluid's velocity profiles.

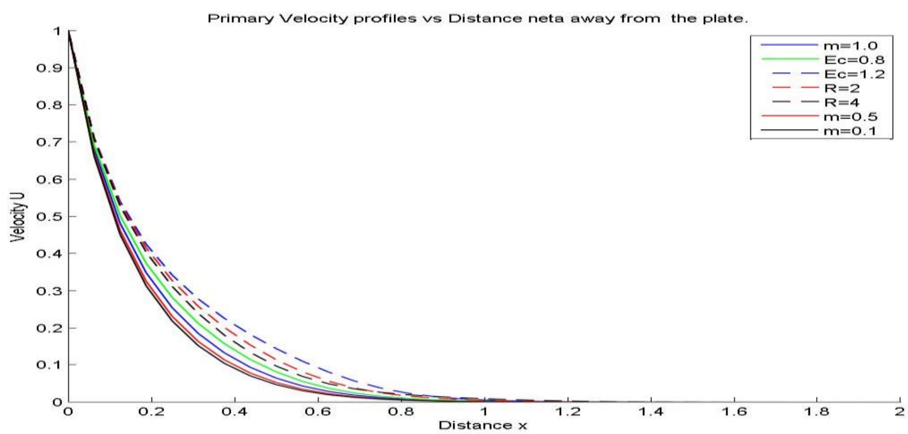

Figure 4.4: plot of primary velocity profiles verses distance

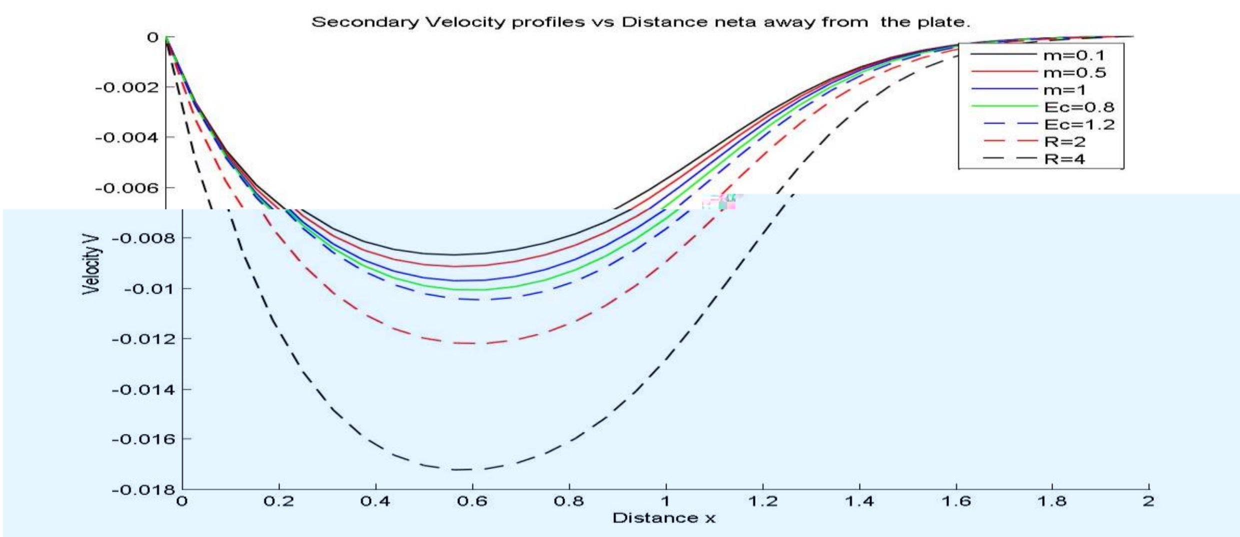

Figure 4.5: plot of secondary velocity profiles verses distance

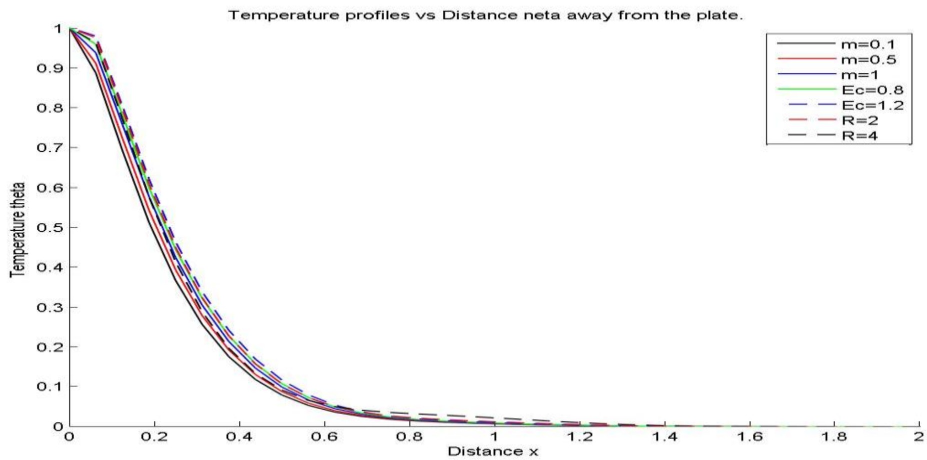

Figure 4.6: plot of Temperature profiles verses distance

From figures 4.7, 4.8 and 4.9 it is noted that variation of primary velocity profiles, secondary velocity profiles and temperature with change in local Grashoff number Gr, Magnetic parameter M, Prandtl number Pr and time t. In 4.7we observe that as Grashoff number increases the velocity increases. Grashoff number gives the ratio of buoyancy forces to the viscous forces. When the Grashoff number increases velocity increases too showing the buoyancy forces are more significant in this flow. Velocity of the fluid increases because the fluid flow is assisted by the free convection currents. The increase in the velocity profiles is partly due to the enhancement by the thermal buoyancy forces.

In figure 4.8 it is noted that the secondary mean flow reversal and its peak values increase with cooling ($Gr > 0$). The curves show that the peak value of the velocity increases rapidly near the wall of the porous plate.This is because the effects of the buoyancy forces are higher near the porous plate. Figure 4.9 shows that the fluid temperature within the boundary layer regime is enhanced by plate cooling. This is because heat is transferred from the plate to the fluid by buoyancy forces during cooling leading to a rise in the mean temperature. Figures 4.7 and 4.8 shows that primary and secondary velocity profiles diminish with increase in magnetic parameter while the thermal boundary layer is enhances by temperature increase. Introduction of strong magnetic field normal to the direction of electrically conducting fluid gives rise to induced electric current flows in direction transverse to both magnetic field and the direction of motion. The interaction between the induced current and the magnetic field give rise to Lorentz force. Lorentz force resists the flow of fluid resulting to deceleration of the fluid and thus the fluid velocity profiles reduce. Figure 4.9 shows that increase in magnetic parameter M results to slight rise in temperature of the fluid. The rise in temperature is as a result of interaction between the atomic ions that constitute conductor and the moving particles that form the current. The charged particles in the electric circuit are speeded up by the electric field but they lose some kinetic energy whenever they collide with ions. The increase in vibration energy of the ions manifests itself as heat that is depicted by the rise of temperature of the fluid. The increased fluid temperature results to non- uniform changes in fluid properties like fluid density and conductivity. Thus energy is converted from electrical power supply to the fluid or any other medium that is in thermal contact. From figures 4.7 and figure 4.8 it is noted that both velocity profiles decrease with time while figure 4.9 shows that temperature profiles increase with time. From figures 4.7, 4.8 and 4.9, it is noted that velocity and temperature profiles increase with increase in Prandtl number. This is because as the plate is cooled by the fluid, viscous forces are released thus resulting to rise in temperature as well as increase in kinetic energy of the fluid particles.

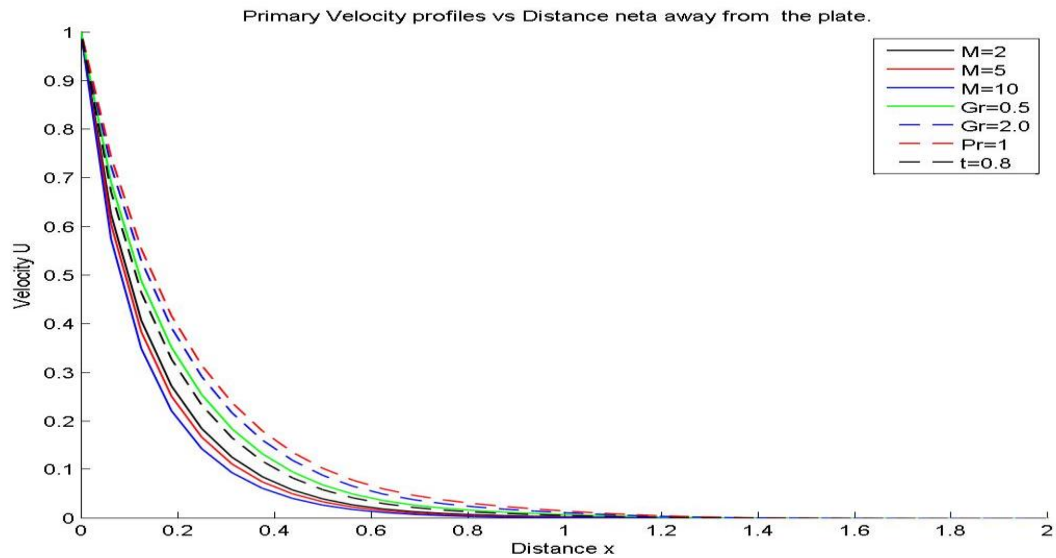

Figure 4.7: plot of Primary velocity profiles verses distance

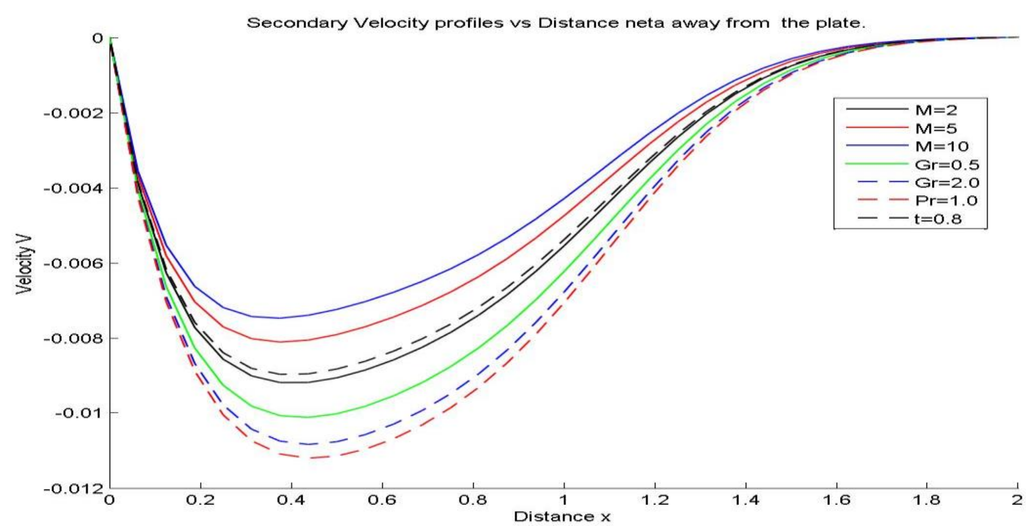

Figure 4.8: plot of Secondary profiles verses distance

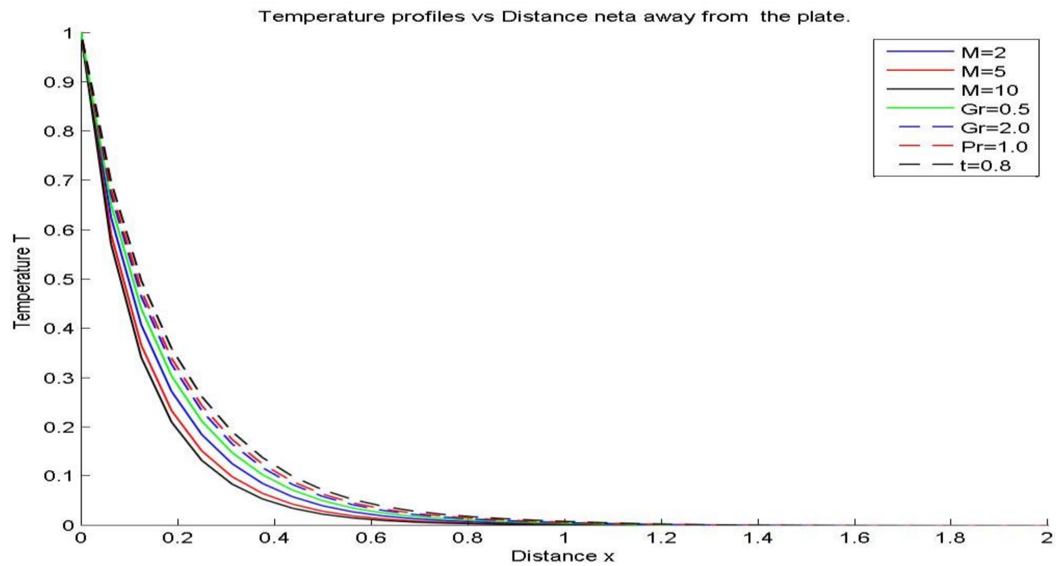

Figure 4.9: plot of Temperature profiles verses distance

4.3 Validation of the results.

Turbulent hydromagnetic flow with radiative heat past a moving vertical porous plate in a rotating system has been investigated. It is found an increase in injection parameter leads to a decrease in the velocity profiles and an increase in the temperature profiles respectively. Also noted is increase in the magnitude of the Injection parameter w_0 leads to a decrease in the velocity and temperature profiles respectively. When these two parameters are not accounted for in the flow the results are similar with those Dawit et al (2014).

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

In this chapter, conclusions based on the results obtained from the previous chapter, validation of the results and the recommended future research works are presented.

5.1Conclusions

The combined effects of magnetic fields, thermal radiation, viscous, porosity and Ohmic heating on turbulent hydromagnetic flow past a moving vertical porous plate in a rotating system have been investigated. The conclusions that have been arrived at and the contributions added are, herein, presented:

Increase in Eckert number, Grashoff number and suction leads to increase in primary velocity profiles. This implies that there is a direct variation in velocity with each of these parameters. Increase in rotation parameter R, magnetic parameter M, Hall parameter m, Radiation parameter Nr, Prandtl number Pr and permeability parameter Xi all lead to decrease in primary velocity profiles. This shows that the velocity inversely varies with these parameters.

Increasing the values of Eckert number, magnetic parameter, Grashoff number, suction and rotation lead to an increase in temperature profiles of the flow.This is attributed to combined effects of fluid rotation, viscous and Ohmic heating, and leading to a rise in the mean temperature.

A decrease in the thermal boundary layer thickness is observed with increasing parameter values of injection parameter w_0, Radiation parameter *Nr*, Hall parameter *m a*nd Prandtl number *Pr*, consequently decreases the mean temperature of the fluid. This proves the fact that blowing (injection) destabilizes the growth of the velocity and temperature the boundary layers respectively.

Flow reversal is generally observed with its peak value within the boundary layer regime and zero secondary mean velocity at both the plate surface and free stream satisfying the prescribed boundary conditions. As the values of parameter X_i, R, Nr,w_0, M and Pr increase due to combined effects of fluid rotation, magnetic field and thermal radiation, the fluid flows toward the plate surface with a decrease in the reverse flow intensity

The secondary velocity profiles increase with cooling $Gr > 0$ of the plate surface by convectional current due to buoyancy force. Increase viscous heating Ec increases the secondary velocity of the flow.

5.2Recommendations

From this study, there are areas that arise for further analysis and development. These may be theoretical or experimental and specific areas of study include:

1. An extension of this study to incorporate ion slips.
2. Flow involving variable magnetic field applied at an angle, variable suction/injection, variable viscosity and thermal conductivity.
3. Flow of fluid which is compressible.
4. Study of hydro magnetic flows that are bounded.
5. Solve hydro magnetic flow problem in three dimensions.
6. An extension of the difference method to problems involving mass transfer.

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APPENDICES

Appendix 1

Computer code in matlab

In order to solve the governing equations (3.56), (3.57) and (3.58), the following computer program code was developed using MATLAB application software version 7.90.529 (R2009b), subject to the boundary conditions as discussed herein. The results were obtained by varying various flow parameters,

```
% NUMERICAL SOLUTION OF MHD PROBLEM

function NJAGI CAMERA code()

clear all;

clc

X=0.5; Pr = .71 ; Ec =1.2; Prt=(Pr)^0.5; M =2; Nr=2 ; Gr_theta =2.; N =0.4; Wo =0.4; R =.8; m=0.11;

 nmax= 2.0; Tmax = 1; nsteps=32; Tsteps=6400;

Tp =450;     %Time at which plotting is done

neta = linspace(0,2,nsteps);

%time = linspace(0,15,Tsteps)

U= zeros(nsteps,Tsteps);

V= zeros(nsteps,Tsteps);

T= zeros(nsteps,Tsteps);
```

```
%neta= zeros(nsteps,Tsteps);

%spacial and time steps

delneta = nmax/nsteps;

delT = Tmax/Tsteps;

%initial conditions

for K=1

for I =  1: nsteps

  U(I,  1) = 0;

  V(I,  1) = 0;

  T(I,  1) = 0;

end

end

% LHS boundary conditions

for I = 1 ;

for K = 1 : Tsteps

    U(I,  K) = 1; V(I,  K) = 0; T(I,  K) = 1;

end

end

%RHS boundary condations

  I =nsteps;
```

```
for K = 1 : Tsteps

   U(I,  K) = 0; V(I,  K) = 0; T(I,  K) = 0;

end

% 'Solving for velocities

for I = 2 :nsteps  - 1

for K = 2 : Tsteps - 1



U(I,  K + 1) = ((U(I,  K) +(((delT* Wo) / (2 * delneta))  * ( -U(I - 1,  K + 1)+ U(I,  K) - U(I -1,  K))) +( (R
* delT) * (V(I,  K + 1) + V(I,  K))) +              ((delT / (2  * (delneta)^2 )) * (U(I + 1,K + 1)+ U(I -1,K+1
)+U(I + 1,  K) - 2 * U(I,  K) + U(I - 1,  K)))- ((delT * X / 2) * U(I,  K))     + ((50*delT * Gr_theta ) * (T(I,
K + 1) + T(I,  K)))+((delT / (8* (delneta)^2)) * ((N^2)*neta(I))*(U(I+1, K + 1) - U(I - 1, K + 1) +
U(I+1,  K) - U(I - 1,  K))^2)+((delT / (2* (delneta)^3)) * (((N*neta(I))^2)*(U(I+1, K + 1) - U(I - 1, K + 1)
+ U(I+1,  K) - U(I - 1,  K))*          (U(I+1, K + 1)+U(I - 1, K + 1) + U(I+1,  K) - 2 * U(I,  K)+ U(I - 1,
K))))-((M*delT/(2*(1+m*m)))*(U(I,  K)-m*(V(I,  K + 1) + V(I,  K))))) / (1 -((Wo * delT) / (2 *
delneta))+ (delT / ( delneta * delneta))  +( delT * X / 2) +((delT / ( (delneta)^3 )) *
(2*((N*neta(I))^2)*(U(I+1, K + 1) -U(I - 1, K + 1) + U(I+1,  K) - U(I - 1, ))))+(M*delT/(2*(1+m*m)))));

V(I,  K + 1) = (V(I,  K) +(delT / (2 * delneta)) * Wo * (-V(I - 1,  K + 1) + V(I,  K) - V(I -1,  K)) -R * delT
* (U(I,  K + 1) + U(I,  K)) +    (delT / (2  * delneta * delneta)) * (V(I + 1,K + 1)+ V(I -1,K+1 )+V(I + 1,
K) - 2 * V(I,  K) + V(I - 1,  K))-(delT * X / 2) * V(I,  K)        +((delT / (8 * delneta^2)) *
(N*N*neta(I))*(V(I+1, K + 1) - V(I - 1, K + 1) +V(I+1,  K) - V(I - 1,  K))^2)+((delT / (8 * (delneta)^3))
* (4*((N*neta(I))^2)*(V(I+1, K + 1) - V(I - 1, K + 1) + V(I+1,  K) - V(I - 1,  K))* (V(I+1, K + 1)+V(I - 1,
K + 1) + V(I+1,  K) - 2 * V(I,  K)+ V(I - 1,  K))))-((M*delT/(2*(1+m*m)))*(V(I,  K)+(U(I,  K + 1) + U(I,
K)))) / (1 -Wo * delT / (2 * delneta)   + delT / ( delneta^2)  + delT * X / 2  +((delT / ( (2*delneta)^3)) *
(((N*neta(I))^2)* (V(I+1, K + 1) - V(I - 1, K + 1) + V(I+1,  K) - V(I - 1,  K))))+(M*delT/(2*(1+m*m)))));

  T(I,  K + 1) = ((T(I,  K) +(delT / (2 * delneta)) * Wo * (-T(I - 1,  K + 1) + T(I,  K) - T(I -1,  K))+(delT /
(2  * Pr*delneta ^2)) *   (T(I + 1,K + 1)+ T(I -1,K+1 )+T(I + 1,  K) - 2 * T(I,  K) + T(I - 1,  K))+((delT /
(4*Prt* (delneta)^2)) * ((N^2)*neta(I))(T(I+1, K 1) - T(I - 1, K + 1) + T(I+1,  K) - T(I - 1,  K))*(U(I, K +
1) - U(I - 1, K + 1) + U(I,  K) - U(I - 1,  K)) )+((delT / (4 * Prt*(delneta)^3)) * N*neta(I)*neta(I)))*(U(I, K
```

```
+ 1) - U(I - 1, K + 1) + U(I,  K)         - U(I - 1,  K))*(T(I+1, K + 1)+T(I - 1, K + 1) + T(I+1,  K) - 2 * T(I,
K)+ T(I - 1,  K))))+((delT / (8 *Prt* (delneta)^3))  * (((N*neta(I))^2)* (T(I+1, K + 1) - T(I - 1, K + 1) +
T(I+1,  K) - T(I - 1,  K))*  (U(I+1, K + 1)- 2 * U(I,  K+1)+U(I - 1, K + 1) + U(I+1,  K) - 2 * U(I,  K)+ U(I
- 1,  K))) -   (delT * Nr / 2) * T(I,K) +( (Ec * delT / ( 4 * delneta ^2)) * (U(I, K + 1) - U(I - 1, K + 1) + U(I,
K)  1,  K)) ^ 2    +  (Ec * delT / ( 4 * delneta ^2)) * (V(I, K + 1) - V(I-1, K + 1) + V(I,  K) - V(I-1,  K)) ^
2)  +((0.25*M*Ec*delT)/(((1+m*m)^2)))*(((U(I,  K + 1) + U(I,  K))-m*(V(I,  K + 1) + V(I,
K))))^2+(m*(U(I,  K + 1) + U(I,  K))+(V(I,   K + 1) + V(I,  K)))^2)/(1-(delT / (2 * delneta)) * Wo
+(delT / (2  * Pr*delneta * delneta))+( delT * Nr / 2)+(((delT*(N*neta(I) /Prt)*(1/2*(delneta)^3)*(U(I+1,
K + 1)+ U(I-1,  K+1)+U(I + 1, K ) + U(I-1,  K)  ))));

end

end
```

Appendix 2

PUBLICATION

D. N Ngari, M. N Kinyanjui, K Giterere and P. R Kiogora, (2015). Turbulent Hydro magnetic Flow with Radiative Heat over Moving Vertical Porous Plate in a Rotating System, *International Journal of Engineering Science and Innovative Technology* **4**(5) 74-82.