Generation of Ultra Wideband Waveforms based on Chaos Theory

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Abstract— Radar systems have been used extensively over the past decade for variety of applications and in a multitude of configurations. Radar engineers wishing to improve the resolution of resultant imagery cannot evade the underlying principle of inverse relationship between a radar signals bandwidth and a minimum featured dimension in range co-ordinate. When it is desired not only to detect but also to identify the target as belonging to a certain category, high resolution of obtained radar images is of much importance.

Conventionally UltraWideBand (UWB) radars were based on generation and coherent reception e.g via matched filtering and coherent filtering of ultrashort pulses e.g Gaussian pulses. This approach provides high resolution if the pulse duration is short enough. However it has the advantage of low spectral efficiency and ease of signal repeatability which can make these imaging systems susceptible to certain types of electronic counter measures (ECM).

This paper discusses the generation of pseudo noise radar waveform using chaos theory but which has a better chance of combating electronic counter measures and high spectral efficiency. Simulations and discussions on the suitability of the chaotic waveforms as radar waveforms are presented.

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Keywords—Bandwidth, Chaos theory, Radar waveforms,

1.0 INTRODUCTION

Radar systems have been used extensively over the past decade for a variety of applications and in a multitude of configurations. Imaging radars are used to obtain visual information about the environment of interest, often with a goal of discerning particular objects concealed in the background. These radars can be geared towards certain scenarios such as discovery of buried mines and unexplored ordnance or as a surveillance and target tracking tool in a reconnaissance operation. In all scenarios radar engineers wishing to improve the resolution of resultant imagery cannot evade the underlying principle of inverse relationship between a radar signals bandwidth and a minimum imaged feature dimension in range coordinates. Thus to properly distinguish between target components of particular size, one needs to select the bandwidth of imaging radar signal accommodating range resolution that corresponds to that size. If the radar waveform is a rectangular pulse, it is known that the resolution

$$\Delta R = \frac{cT_p}{2}$$

(1)

The bandwidth of a rectangular pulse is $B = \frac{1}{T_p}$. Hence the range resolution ΔR is related to the signal bandwidth and propagation velocity c by

$$\Delta R = \frac{c}{2B},$$

(2)

where the c/2 factor converts the two-way travel time to range in meters.

2.0 TYPICAL RADAR WAVEFORMS

Continuous-Wave (CW)

A radar system that transmits continuously is termed continuous-wave (CW) radar. The unmodulated, single frequency waveform has been used for Doppler radars for a long time due to its capability of testing target motion relative to the radar. The receiver of CW radar mixes (homodynes) the received signal with a replica of the transmitted signal. After low pass filtering, the only remaining component is the Doppler shift which can be used for velocity measurements. Since B=0 for a CW signal, the range resolution according to equation 1.2 is infinity. Therefore no target range information is available in CW radar, and a combination of pulse and CW radar is often used for practical purposes. This is termed pulsed-Doppler radar.

Short pulse (impulse)

The most obvious and straight forward high resolution radar waveform is the impulse or short pulse. If the pulses are transmitted without a carrier, they are termed *carrierless* impulses or baseband video pulses. In many cases it's advantageous to remove the DC content of the pulses by differentiation or high pass filtering. The resulting pulses are often called monocycle pulses. A popular short duration waveform is the *Ricker wavelet* that can be described mathematically as the negative of a second derivative of a Gaussian pulse

$$p(t) = -\frac{d^2\left(e^{-at^2}\right)}{dt^2} = -2ae^{-at^2}\left(2at^2 - 1\right) \quad \textbf{(3)}$$

where a is a constant that determines the time duration and amplitude of the wavelet.

Chirp (Linear FM)

A common way of increasing the pulse energy while still maintaining the high resolution of radar is to apply a *chirp* waveform which is a sine wave with a linearly increasing/decreasing frequency. This is the most common way of generating radar waveforms. The frequency of a chirp f(t) is given by

$$f(t) = f_0 + \alpha t, \quad \textbf{(4)}$$

where f_0 is the start frequency and α is the chirp rate [Hz/s].Since the frequency is related to the phase ϕ by

$$f(t) = \frac{1}{2\pi}\frac{d\phi}{dt}, \quad \textbf{(5)}$$

and the phase of the chip waveform can be written as

$$\phi(t) = 2\pi\int_0^t f(\tau)d\tau = 2\pi\left(f_o t + \frac{\alpha}{2}t^2\right) \quad \textbf{(6)}$$

Using this quadratic phase function a chirp with length T_c can be written as complex exponential

$$p(t) = e^{j(\omega_0 t + \pi\alpha t^2)} \qquad 0<t<T_c. \quad \textbf{(7)}$$

The spectrum of the chirp can be derived by rewriting p(t) in equation 1.7 as

$$p(t) = rect\left(\frac{t - \frac{T_C}{2}}{T_C}\right) \times e^{j(\omega_o t + \pi\alpha t^2)} \quad \textbf{(8)}$$

Taking the real part of equation (1.8), the transmitted chirp pulse can be expressed as

$$p(t) = rect\left(\frac{T - \frac{T_c}{2}}{T}\right)\cos\left(2\pi\left[f_o t + \frac{1}{2}\alpha t^2\right]\right) \quad \textbf{(9)}$$

Figure 1 illustrates the time frequency behavior of a chirp

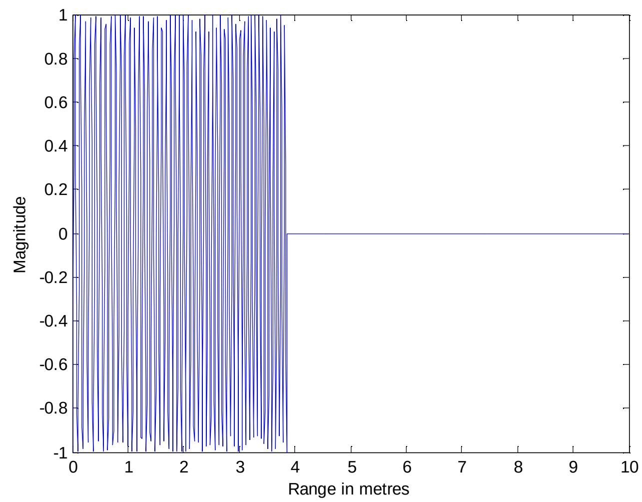

Fig.1

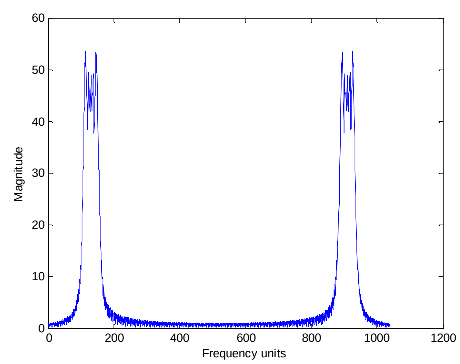

Fig.2

As is the case with the discrete Fourier transform, the N/2 samples contain the positive frequency components and the negative frequency components appear in the second half of the array as indicated in Fig.2.

3.0 CHAOS THEORY

Dynamical systems

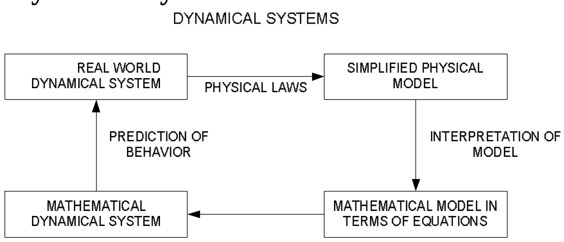

Fig.3

Mathematically, a dynamical system is a description of possible evolutions over time of points in some space called state space of the system. In applied mathematics the points in state space are often identified with pairs of positions and velocities and some physical configuration. If time t is measured continuously over some interval of real numbers, the possible evolution of a system will often be determined by a differential equation with t as an independent variable. Figure 3 shows the interplay between a differential equation and the physical system .Specialists may find themselves focusing on this diagram at particular points. For example a theoretical physicist might focus across the top, a pure mathematician across the bottom and applied mathematician or scientists across the vertical direction [6]

Chaos system

Chaos is the word used to describe deterministic behavior for which even if the initial conditions were known to an arbitrary degree of precision, the long term behavior cannot be accurately predicted. This is certainly the case with many natural systems for which we cannot know the initial conditions to an arbitrary degree of precision. A classic example, first considered by E.N Lorenz is the weather [Lorenz 1963]. Edward Lorenz, pioneer in using computers accidentally found in 1960, that in certain kinds of non-linear equations, the result displayed sensitive dependence on initial conditions. This is a distinguishing feature of chaos systems. At the end of the nineteenth century Henry Poincarẻ was aware of the fact that orbits of three bodies moving under a central force due to gravity are quite complicated and change drastically with a change in initial conditions. He tried to find a theorem to explain more generally the phenomenon and to establish a theory related to the chaotic paths in a system of differential equations. He showed that three dimensional paths in a system of non linear differential equations can be chaotic. Lorenz developed mathematical models which could predict the behavior of weather. However these equations have now been found to predict the behavior of many natural phenomenon.

NON LINEAR DYNAMICS

Any nonlinear system which can be expressed by a set of mathematical equations includes two types of variables- *dynamic* and *static*. Dynamic variables are the quantities which changes with time whereas the static variables often referred to as the control parameters, remain constant until changed by an outside force[7]. When studying the non linear system, the control parameters are often changed so as to learn how the behavior of the system changes in response. The act of changing a control parameter to change the system behavior is known as perturbation.

State space or phase *space* is the space of the dynamic variables and might in some cases include their derivatives. A point in the state space represents a state of the system at a given time. As the system evolves with time, the state of the system moves from point to point in the state space thus defining a trajectory. A trajectory therefore displays the history of the states of the system.

Chaos is an aperiodic long term behavior in a deterministic system that exhibits sensitive dependence on initial conditions. The three components of the definition are classified as follows:

1. Aperiodic long term behavior means that the system trajectory in phase space does not settle down to any fixed points (steady state), periodic orbits, or quasi-periodic solutions as time tends to infinity. This part of the definition differentiates aperiodicity of for example, a periodically oscillating system that has been momentarily perturbed.
2. “Deterministic” systems can have no stochastic systems (meaning probabilistic) parameters. It is a common misconception that chaotic systems are noisy systems driven by random processes. The irregular behavior of chaotic systems arises from intrinsic non-linearity rather than noise.
3. “Sensitive dependence on initial conditions” requires that trajectories originating from nearly identical conditions will diverge exponentially quickly.[2]

The mathematical model developed, now called, the Lorenz system has been used as a paradigm for chaotic systems satisfy the above definition. The Lorenz system consists of just three coupled first-order differential equations.

$$\frac{dx}{dt} = -\sigma x + \alpha y$$

$$\frac{dy}{dt} = -xz + rx - y$$

$$\frac{dz}{dt} = xy - bz$$

(9)

Lorenz chose parameter values α=10, b=8/3 and r=28.With these choice for the parameters, Lorenz system is chaotic exhibiting the traits described in the definition given for chaos[8].For parameter values for which the Lorenz system demonstrates chaotic dynamics, all solutions, regardless of their initial conditions, converge to a set called the strange attractor. The strange attractor can be observed in state space (also called phase space),where each state variable is assigned a respective axis in the x-y-z plane space. Figure 2 illustrates a solution to the Lorenz system tracing out the strange attractor in state space. Since the Lorenz system satisfies the uniqueness theorem [4], no points of intersection appears on the strange attractor.Nontheless all solutions eventually converge to the butterfly shaped attractor. Specifically the attractor is composed of two wings, and each wing of the attractor encircles one of the two nontrivial fixed points. The shape

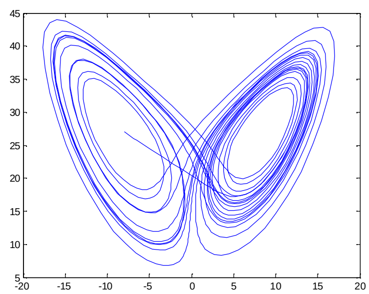

Figure 4: Lorenz strange attractor

of the strange attractor varies for varying parameter values. A time series of x, y and z is shown in Figure 2.The initial conditions can be chosen arbitrarily. From direct observation of the time series, it is reasonable to say that the y and z variables are periodic.

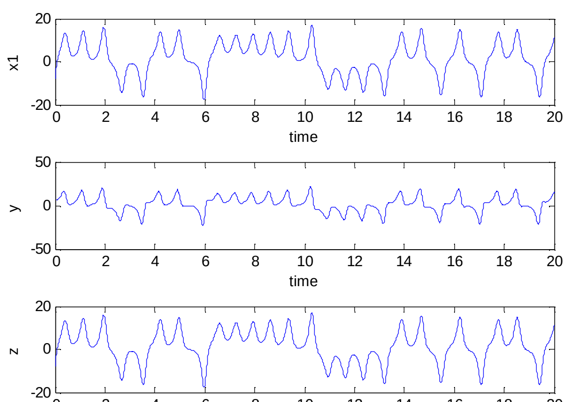

Figure 3: Time series of x, y and z

Chaotic systems exhibit sensitivity to initial conditions. For chaotic systems, two solutions with nearby initial conditions exponentially diverge. To demonstrate that the Lorenz system has this sensitivity, the x state variable from two distinct solutions with nearby initial conditions are shown in Figure 3.As can be seen from this figure, the two signals begin nearby and rapidly diverge from each other. Although only x (t) is shown the same behavior can be observed from both y (t) and z(t).

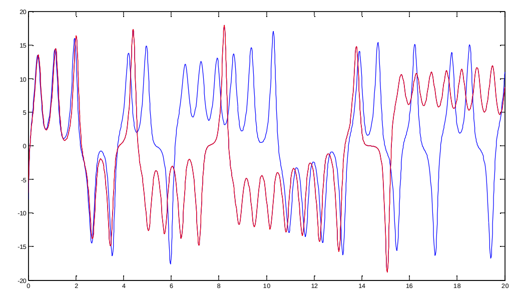

Figure 5: Time series plot for x with a variation in initial conditions

NON LINEAR DYNAMICS

This section focuses on numerically and analytically exploring the Lorenz parameters to determine how various radar waveform metrics vary as the parameters are varied

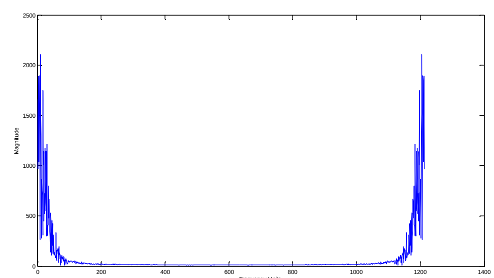

Fig. 6

The FFT of the chirp pulse is shown in Figure 6.As in the case with the discrete Fourier Transform, the first N samples contain the positive frequency components and the negative frequency components appear in the second half of the array. The setting of the bandwidth of the system can be analytically discussed by considering the time scaling of the Lorenz equations

Time-Scaling the Lorenz Equations

Define the variable $x(t)$ as shown in equation2.0

where x, y and z denote the state variable of the Lorenz system:

$$\underline{x}(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix}$$

(14)

Next, the $f\left(\left(\underline{x}\right)\right) = \begin{pmatrix} \alpha y(t) - x(t) \\ rx(t) - y(t) - x(t)z(t) \\ x(t)y(t) - bz(t) \end{pmatrix}$

Therefore, the Lorenz system can be written as shown in equation 2.2.

$$\dot{\underline{x}}(t) = f\left(\underline{x}(t)\right)$$

Let $\underline{x}(t)$ denote the solution to equation 2.3.Also let $\underline{\tilde{x}}(t)$ denote the solution to equation 2.3 where a is a constant greater than zero.

$$\underline{x}(t) = f\left(\underline{x}\right) \quad \textbf{(11)}$$

$$\underline{\tilde{x}}(t) = af\left(\underline{\tilde{x}}\right)$$

(12)

If both s If both systems have identical initial conditions then ,then

$$x(t) = \underline{\tilde{x}}(at)$$

(13)

In other words scaling the Lorenz equations by **a** has the effect of time scaling $\underline{x}(t)$ by **a**. Scaling $\underline{x}(t)$ by **a** in time also scales the bandwidth of x(t) by a.

CONCLUSION

Observation of the waveforms in Fig.3 shows its pseudorandom nature of the chaotic waveforms which means it cannot be susceptible to any form of electronic countermeasure. Compared to the more commonly used radar waveform, it can be seen from Fig.2 and Fig.5. that by variation of the Lorenz parameters, appropriate signal spectrum can be obtained.

REFERENCE

[1]Thomson J.M.T, "*Non Linear Dynamics and Chaos*, John Wiley and Sons, New York, 2002.

[2] Arrow, D and Place, C.N *Introduction to Dynamical Systems,* Cambridge University Press, Cambridge, 1990.

[3] Peterson I.*: Newton's Clock*: *Chaos in the solar System*.W.H Freeman, San Francisco

[4]John Guckenheimer and Philip Holmes. *Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector fields,* volume 42 of Applied Mathematical Sciences.Springer-Verlag, New York, 1983

[5]T.Kapitaniak, CHAOS for Engineers, Theory, Applications, and Control,

[6]Richard E Williams. *Introduction to differential equations and dynamical systems.* The McGraw-Hill Companies. New York, 1997

[7]May, R, Simple Mathematical models with very complicated dynamics, Nature, Springer-Verlag, New York ,1976

[8]Ott,E,Chaos in Dynamical Systems,Cambrige University Press,Cambrige,1993