MODELING FLUID FLOW IN OPEN CHANNEL WITH CIRCULAR CROSS-SECTION

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Abstract
Flow in a closed conduit is regarded as open channel flow, if it has a free surface. This study considers unsteady non-uniform open channel flow in a closed conduit with circular cross-section. We investigate the effects of the flow depth, the cross section area of flow, channel radius, slope of the channel, roughness coefficient and energy coefficient on the flow velocity as well as the depth at which flow velocity is maximum. The Saint-Venant partial differential equations of continuity and momentum governing free surface flow in open channels are highly nonlinear and therefore do not have analytical solutions. The Finite Difference Approximation method is used to solve these equations because of its accuracy, stability and convergence. The results are presented graphically. It is established that for a given flow area, the velocity of flow increases with increasing depth and that the velocity is maximum slightly below the free surface. Moreover, increase in the slope of the channel and energy coefficient leads to an increase in flow velocity whereas increase in roughness coefficient, flow depth, radius of the conduit and area of flow leads to a decrease in flow velocity.

Key words: Open-channel flow, free surface, saint-venant equation, finite difference method

NOMENCLATURE

Symbol	*Quantity*
V	Mean velocity of flow (m/s)
L	Length of the channel (m)
g	Acceleration due to gravity (ms^{-2})
Q	Discharge (m^3s^{-1})
A	Cross- sectional area of flow (m^2)
n	The Manning coefficient of roughness ($sm^{-1/3}$)
S_o	Slope of the channel bottom
S_f	Friction slope
P	Wetted perimeter of the channel cross section (m)
T	Top width of the free surface (m)
y	Depth of the flow (m)
t	Time (s)
q	Lateral/uniform inflow (m^2s^{-1})
R	Hydraulic Radius (m)
x	Distance along the main flow direction (m).
Fr	Froude number (dimensionless)
Re	Reynolds number (dimensionless)
D	Hydraulic depth (m)
ν	Kinematic viscosity (m^2/s)
α	Energy coefficient
r	Channel radius (m)

1.0 Introduction

Water flows more rapidly on a steeper slope, but for a constant slope, the velocity reaches a steady value when the gravitational force is equal to the resistance to flow. Over the years, man has endeavoured to direct water to the desired areas such as farms, where it is used for irrigation. He has also tried to draw water from storage sites such as reservoirs, dams and lakes. To achieve this objective, he has constructed open channels which are physical systems in which water flows with a free surface.

The cross-section of these channels may be open or closed at the top. The structures with closed tops are referred to as closed conduits while those with open tops are called *open channels*. This study focuses on open channel flow in a closed conduit with circular cross-section. The findings of this study will go a long way in providing reference for designers of open channel projects and guidelines for the hydraulic analysis and design of open channel flows. In addition, it will provide an understanding into the propagation of flood wave in natural rivers, originating from torrential rains or of breaking of a control structure.

Open channel flow is a familiar sight, whether in a natural channel like that of a river, or an artificial channel like that of an irrigation ditch. Its flow is a complex when everything is considered, especially with the variability of natural channels, but in many cases the major features can be expressed in terms of only a few variables, whose behavior can be described adequately by a simple theory. The principal forces at work are those of inertia, gravity and viscosity, each of which plays an important role.

Open channels have been a subject of study for a long time. The Chézy equation is one of the earliest procedures developed in 1768 by a French Engineer, Henderson. The development of this equation was based on the dimensional analysis of the friction equation under the assumption that the condition of flow is uniform. Chézy's formula did not provide results that satisfied engineers. The Swiss engineers Ganguillet E. and Kutter W. R. in 1869 showed that much better results could be obtained if the constant C depended on R, S_o and a constant n that was characteristic of the roughness of the channel.

A more practical procedure was presented in 1889 by the Irish engineer Manning R., Chow (1959). Studies in open channel flows have to take into account the coefficient of roughness, called the *Manning coefficient.* The Manning coefficient takes into account the bed materials, degree of channel irregularity, variation in shape and size of the channel and relative effect of channel obstruction, vegetation growing in the channel and meandering, Chadwick (1993). This makes the Manning equation more desirable for the design of open channels.

Akbari G. and Firoozi B. (2010) investigated two different numerical methods, namely; Preissmann and Lax diffusive schemes for numerical solution of Saint-Venant equations that govern the propagation of flood wave, in natural rivers, with the objective of the better understanding of this propagation process. The results showed that the hydraulic parameters play an important role in the flood wave propagation.

Moshirvaziri S. *et al* (2010) examined numerically, the nature of pollutant connectivity between unsealed forest roads and adjacent nearby streams in terms of spatial and temporal patterns of runoff generation, erosion, and sediment transport with an aim of improving our ability to scale-up the impacts of forest roads on catchment water quality in future works. They considered the relative effects of rainfall intensity and duration, surface roughness, infiltration rate, sediment detachment and transport with an objective of identifying the dominant processes and parameters that affect the degree of pollutant connectivity between roads and streams.

Kwanza, J. K. *et al*. (2007) analyzed the effects of channel width, slope of the channel and lateral discharge on fluid velocity and channel discharge for both rectangular and trapezoidal channels. They noted that the discharge increases as the specified parameters are varied upwards. Chagas and Souza (2005) sought to provide solution of Saint Venant's Equation to study flood in rivers through Numerical Methods. They used a discretization, for the equations that governs the propagation of a flood wave, in natural rivers, with the objective of a better understanding of this propagation process. Their results showed that the hydraulic parameters play an important role in the propagation of a flood wave.

Khan A. A. (2000) studied open channel flow over an initially dry bed with the aim of better understanding of flow over islands during rising flood stage, flow downstream of the hydraulic structures (such as dams and gates) during intermittent release of water, and flood wave, either due to natural causes or sudden failure of a hydraulic structure, over an initially dry bed. Tuitoek and Hicks (2001) modeled unsteady flow in compound channels with an aim of controlling floods.

2.0 Mathematical Formulation

The basic equations that describe unsteady one-dimensional fluid flow in an open channel are the Saint Venant equations, which consist of continuity and momentum equations. In the development of the mathematical model, some simplifications are made: the flow is considered one-dimensional, the distribution the pressure in the vertical is a hydrostatic one, the fluid considered is water. The fluid is assumed incompressible and homogeneous. Thus for prismatic channels of arbitrary shape, the model equation, which consists of the continuity and momentum equation, is defined as:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q \tag{1}$$

$$\frac{\partial V}{\partial t} + \alpha V \frac{\partial V}{\partial x} + g \frac{\partial y}{\partial x} = g\left(S_o - S_f\right) \tag{2}$$

Given the velocity, the discharge (Q) is calculated as the product of velocity and cross-sectional area

$$Q = AV \tag{3}$$

Substituting equation (3) in equation (1) above and then differentiating partially w.r.t x yields:

$$V \frac{\partial A}{\partial x} + A \frac{\partial V}{\partial x} + \frac{\partial A}{\partial t} - q = 0 \tag{4}$$

The flow area is assumed to be a known function of the depth; therefore derivatives of A may be expressed in terms of y as:

$$\frac{\partial A}{\partial x} = \frac{dA}{dy}\frac{\partial y}{\partial x} = T\frac{\partial y}{\partial x} \tag{5}$$

$$\frac{\partial A}{\partial t} = \frac{dA}{dy}\frac{\partial y}{\partial t} = T\frac{\partial y}{\partial t} \tag{6}$$

In this discussion, it is assumed that T is as determined by Franz (1982)

$$T = \frac{dA}{dy} \tag{7}$$

Substituting equations (5) and (6) in equation (4) yields:

$$\frac{\partial y}{\partial t} + \frac{A}{T}\frac{\partial V}{\partial x} + V\frac{\partial y}{\partial x} - \frac{q}{T} = 0 \tag{8}$$

3.0 Method of solution

The objective of this paper is to investigate the effects of the various flow parameters on the velocity profiles of a fluid flow in an open channel with circular cross section area as shown in figure A below

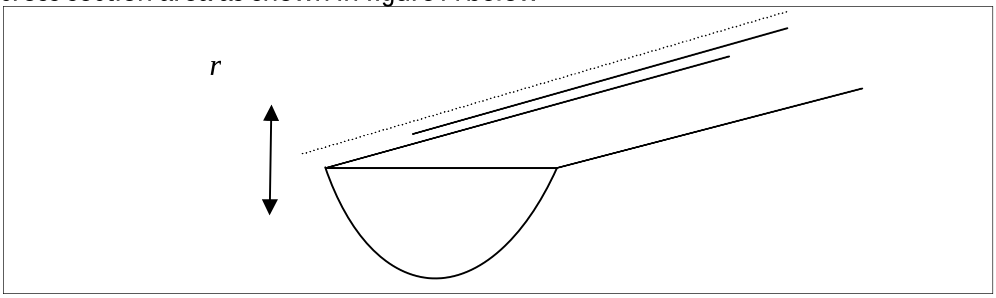

Figure A: Channel cross section

An analysis of the effects of the various parameters on the flow velocity has been carried out. The equations governing the flow considered in the problem are non-linear and therefore to obtain their solutions, an efficient finite difference scheme was developed. The mesh used in the problem considered in this work is divided uniformly.

Equations (2) and (8) are non-linear first order partial differential equations of the hyperbolic type. It is not possible to solve these equations analytically thus the finite difference method is used to obtain approximate solutions. In this technique, the numerical solution of equations (2) and (8) is approximated at a discrete number of points arranged to form a rectangular grid. This rectangular grid is obtained by dividing the (x, t) plane into a network of rectangles of sides Δx and Δt. The nodes or mesh points or grid points of the network occur at the intersections of straight lines drawn parallel to the x and t axes. We are assuming the grid size is uniform along each axis. For brevity, we will call the $i\Delta x$ grid point i

and the $(i+1)\Delta x$ grid point $i+1$. For the time axis, we will call $j\Delta t$ grid point j and the $(j+1)\Delta t$ grid point $j+1$. To refer to different variables at these grid points, we use the number of the spatial grid and that of the time grid as the first and second subscript respectively. We denote the known time level by j and the unknown time level by $j+1$. The primary difficulty with explicit finite difference techniques is the problem of numerically unstable solutions. The partial derivatives in the equations are replaced by their corresponding finite difference approximations. However, Viessman *et al*. (1972) noted that more stable solutions can be obtained if a diffusing difference approximation is used. Using this scheme, equations (8) and (2) become:

$$\frac{y(i,j+1)-0.5(y(i-1,j)+y(i+1,j))}{\Delta t}+\frac{A}{T}\frac{V(i+1,j)-V(i-1,j)}{2\Delta x}+V(i,j)\frac{y(i+1.j)-y(i-1,j)}{2\Delta x}-\frac{q}{T}=0 \quad (9)$$

$$\frac{V(i,j+1)-0.5\big(V(i-1,j)+V(i+1,j)\big)}{\Delta t}+\alpha V(i,j)\frac{V(i+1,j)-V(i-1,j)}{2\Delta x}+g\frac{y(i+1,j)-y(i-1,j)}{2\Delta x}-g\left(S_0-\frac{S_f(i-1,j)+S_f(i+1,j)}{2}\right)=0 \quad (10)$$

From equation (9)

$$y(i,j+1)=0.5[y(i-1,j)+y(i+1,j)]-\Delta t\left\{\frac{A}{T}\frac{V(i+1,j)-V(i-1,j)}{2\Delta x}+V(i,j)\frac{y(i+1,j)-y(i-1,j)}{2\Delta x}-\frac{q}{T}\right\} \quad (11)$$

In the computation of unsteady flow, it is usually assumed that the friction slope S_f can be estimated from either the Manning or Chezy resistance equations. The Manning resistance equation is as follows:

$$S_f=\frac{n^2V^2}{R^{4/3}} \quad (12)$$

Substituting equation (12) in equation (10), it is possible to find

$$V(i,j+1) = 0.5[V(i-1,j) + V(i+1,j)] - \Delta t \left\{ \alpha V(i,j) \frac{V(i+1,j) - V(i-1,j)}{2\Delta x} + g \frac{y(i+1,j) - y(i-1,j)}{2\Delta x} - g \left[S_0 - \frac{n^2}{2R^{4/3}} \left(V^2(i-1,j) + V^2(i+1,j) \right) \right] \right\} \quad (13)$$

In equations (11) and (13), the index i refers to spatial points whereas the index j refers to time. The consecutive terms of depth and velocities $y_{i,j+1}$ and $V_{i,j+1}$ respectively are computed by equations (11) and (13) subject to the initial and boundary conditions

$$V(x,0) = 10\,, \quad y(x,0) = 0.5 \quad \text{for all } x > 0 \quad (14)$$
$$V(0,t) = 10\,, \quad y(0,t) = 0.5 \quad \text{for all } t > 0 \quad (15)$$
$$V(x_l,t) = 10\,, \quad y(x_l,t) = 0.5 \quad \text{for all } t > 0 \quad (16)$$

The computations are performed using small values of Δt. In our research, we set $\Delta t = 0.0012$ and $\Delta x = 0.1$. From equation (11), the depth y at the end of the time step $\Delta t, y_{i,j+1}$ $i = 1,2,3,\dots,40$ is computed in terms of velocities and depths at points on earlier time step. Similarly, $V_{i,j+1}$ is also to be computed from equation (13). The procedure repeated till $j = 50$.

4.0 Discussion of the Results

From figure 1, we observe that for a fixed flow area the flow velocity increases with increase in depth from the bottom of the channel to the free stream and that maximum velocity occurs just below the free surface. The free surface occurs at a depth of 0.5m and the velocity of the fluid layer at this depth is 10m/s. It is also observed that maximum velocity occurs just below the free surface, at a depth of about 0.38m.

The flow velocity in a channel section varies from one point to another due to shear stress at the bottom and at the sides of the channel. The velocity is not maximum at the free surface mainly due to surface tension caused by the strong cohesive forces between the liquid molecules. In the bulk of the liquid, each molecule is pulled equally in every direction by neighboring liquid molecules, resulting in a net force of zero. The molecules at the free surface do not have other molecules surrounding them entirely and are therefore pulled inwards. This

creates some internal pressure and forces liquid surfaces to contract to the minima leading to a reduction in velocity at the free stream. On the other hand, both the atmospheric pressure and gravity acting in a direction that is perpendicular to the free surface creates some internal pressure causing the contraction of the liquid surface. This contraction lowers the movement of the fluid particles at the free surface resulting in reduced velocities. In addition, the wind blowing over the free surface also affects the velocity in the free stream due to frictional resistance particularly when wind blows over the free surface at high velocities and in the opposite direction to the main direction of flow.
From figure 2, we observe that a reduction in the slope from 0.02 m/m to 0.004 m/m leads to a decrease in the flow velocity as shown from curve I to curve II. An increase in the cross sectional area of flow from 0.6144 m^2 to 1.5712 m^2 results to a decrease in the flow velocity from curve I to curve III.

Manning's velocity formula shows a direct relationship between flow velocity and the slope. Thus a decrease in slope results in a decrease in the flow velocity. An increase in the cross-sectional area of flow leads to an increase in the wetted perimeter. A large wetted perimeter results in high shear stresses at the sides of the channel which results in a reduction in the flow velocity.

From figure 3, we observe that increasing the radius from 1 m to 3 m results in a decrease in the flow velocity from curve I to curve II. Moreover, an increase in the roughness coefficient from 0.012 to 0.029 also results in a reduction in the flow velocity as shown from curve I to curve III.

An increase in the radius results in an increase in the wetted perimeter because the fluid will spread more in the conduit. A large wetted perimeter will result in large shear stresses at the sides of the channel and therefore the flow velocity will be reduced. An increase in the roughness coefficient results in large shear stresses at the sides of the channel. This means that the motion of fluid particles at or near the surface of the conduit will be reduced. The velocity of the neighbouring molecules will also be lowered due to constant bombardment with the slow moving molecules leading to an overall reduction in the flow velocity.

From figure 4, we observe that an increase in the energy coefficient from 1 to 2 leads to an increase in the flow velocity from curve I to curve II. In addition, an increase in the flow depth from 0.5m to 1m results in a reduction in the flow velocity from curve I to curve III.

An increase in the energy of the fluid results in the molecules attaining high energy which leads to more random motion. This random motion causes constant bombardment between the fluid particles resulting in an increase in velocities of the molecules and in general, of the fluid. The increase in flow depth leads to an increase in the wetted perimeter. This leads to large shear stresses at the sides of the channel and therefore the flow velocity will be reduced.

5.0 Conclusion

The various flow parameters were varied, one at a time while holding the other parameters constant. This was repeated for all the flow parameters and the results presented graphically. It was established that for a fixed flow area, the flow velocity increases with increase in depth from the bottom of the channel to the free stream and that maximum velocity occurs just below the free surface. Reduction in the slope leads to a decrease in the flow velocity. An increase in the cross sectional area of flow results in a decrease in the flow velocity.
An increase in the radius of the conduit results in a reduction of the flow velocity. This is because, as the radius is increased, so is the wetted perimeter as the fluid spreads more in the conduit. Moreover, an increase in the roughness coefficient results in a decrease in the flow velocity due to large shear stresses at the sides of the channel.

An increase in the energy coefficient leads to an increase in the flow velocity due to an increase in the energy of the fluid resulting in an increase in the molecular energy which leads to more random motion. Finally, an increase in the flow depth results in a reduction in the flow velocity. The increase in flow depth leads to an increase in the wetted perimeter resulting in large shear stresses at the sides of the channel with an effect of reduced velocities.

References

Akbari G., Firoozi B. Implicit and Explicit Numerical Solution of Saint-Venant Equations for Simulating Flood Wave in Natural Rivers. *5th National Congress on Civil Engineering*. May 4-6, 2010, Ferdowsi University of Mashhad, Mashhad, Iran.

Chadwick A. and Morfet J. (1993). *Hydraulics in Civil Engineering and Environmental Engineering*, Chapman & Hall, pp. 187-200.

Chagas Patricia and Souza Raimundo. Solution of Saint Venant's Equation to Study Flood in Rivers, through Numerical Methods. Paper_1 *Conf AGU Hydrology Days* 2005. Mar7-Mar 9, 2005.

Chow V. T. (1959). *Open-Channel Hydraulics.* McGraw Hill. New York.

Franz D. D. (1982). Tabular representation of cross-sectional elements. Proceedings of the American Society of Civil Engineers. *Journal of the Hydraulics Division*. **108** no. HY10, pp. 1070-1081.

French R.H. (1985). Open-Channel Hydraulics. Mc-Graw Hill, Inc. New York.

Ganguillet E., and Kutter, W.R. (1869). Versuch zur Augstellung einer neuen allgeneinen Furmel für die gleichformige Bewegung des Wassers in Canalen und Flussen. *Zeitschrift des Osterrichischen Ingenieur – und Architecten-Vereines*,**21**, (1), pp 6-25 and (2), pp 46-59. Translated into English by: Hering, R. and Trautwine, J.C. (1888 and 1891). *A General Formula for the Uniform Flow of Water in Rivers and Other Channels*. New York, NY: John Wiley & Sons.

Khan Abdul A. Modeling flow over an initially dry bed. *Journal of Hydraulic Research*., **38**, 2000, NO. 5.

Kwanza J. K., Kinyanjui M. and Nkoroi, J.M. (2007). *Modeling fluid flow in rectangular and trapezoidal open channels*. Advances and Applications in Fluid Mechanics., **2**, pp. 149-158.

Moshirvaziri S., Sheridan G., Lane P., Jones O. Modelling connectivity between pollutant source areas and streams.
http://www.iemss.org/iemss2010/index.php?n=Main.Proceedings.

Trautwine J. C. *The Civil Engineer's Pocket-Book* (Philadelphia: Wm. Butter & Co., 1888). pp. 236-281. Kutter's formula, together with much practical information.

Tuitoek D. K. and Hicks F. E. (2001). Modelling of unsteady flow in compound channels. *African Journal Civil Engineering*, **4**, pp. 45-53.

Viessman W., Jr., Knapp J. W., Lewis G. L., and Harbaugh T. E., Introduction to hydrology, 2nd ed., Harper & Row, New York, 1972.

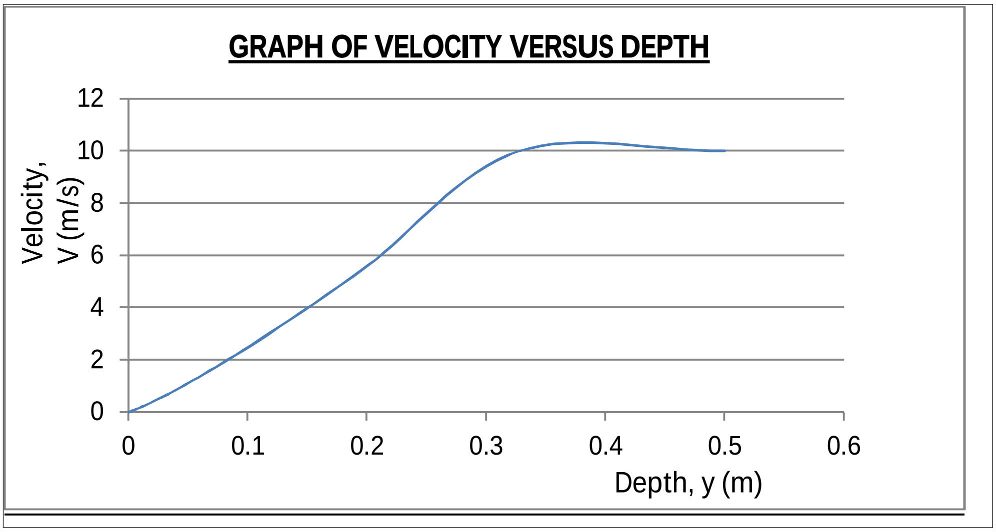

Figure 1: Velocity profiles for A = 0.6144, S_o =0.02, r = 1, n = 0.012, α = 1, P = 2.0947

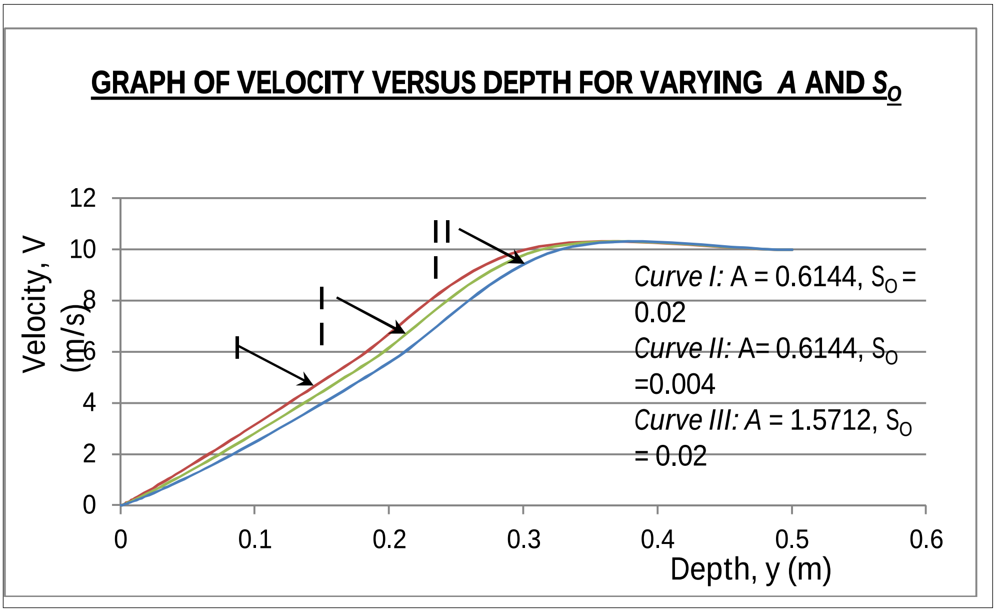

Figure 2: Velocity profiles for r = 1, n = 0.012, α = 1, P = 2.0947

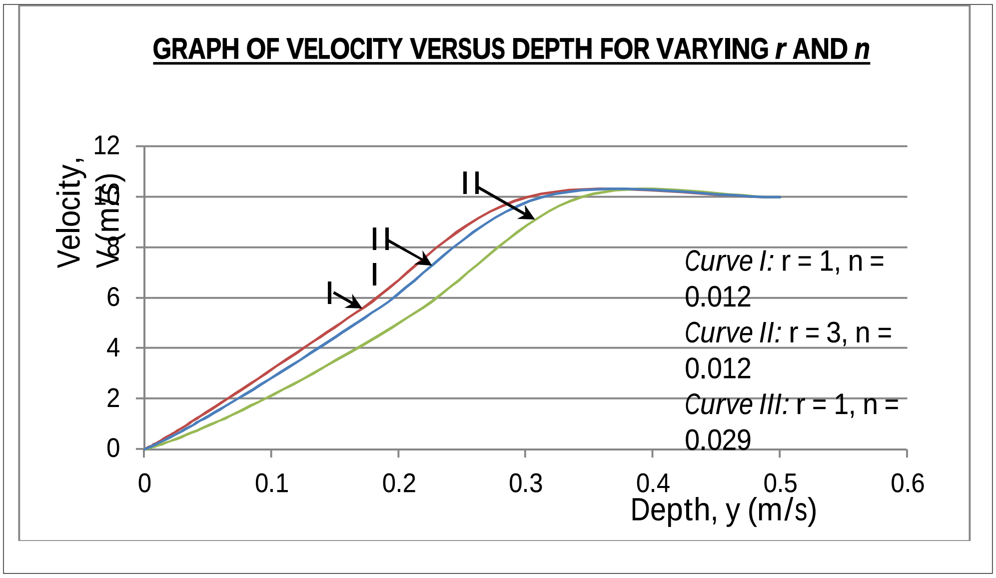

Figure 3: Velocity profiles for A = 0.6144, S_o =0.02, α = 1, P = 2.0947

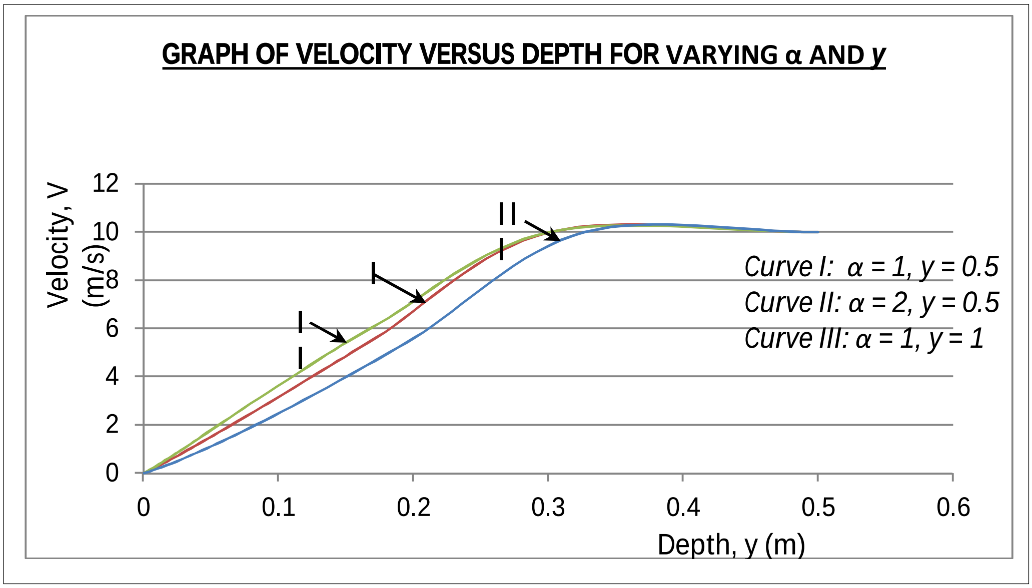

Figure 4: Velocity profiles for A = 0.6144, S_o =0.02, r = 1, n = 0.012