Effects of Temperature Dependent Viscosity on Magnetohydrodynamic Natural Convection Flow past an Isothermal sphere

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DECLARATION

This Thesis is my original work and has not been presented for a degree award in any other university.

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This Thesis has been submitted for examination with our approval as university supervisors.

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DEDICATION

This Thesis is dedicated to my parents Mr. Ndirangu Mwangi and Mrs. Rahab Njeri Mwangi for their continued support in my studies. It also goes to my Uncle Joseph Kimama Mwangi and Auntie Grace Wambui Mwangi for their relentless support they have given towards my study.

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NOMENCLATURE

β Volumetric coefficient of thermal expansion,$[(C^0)^{-1}]$

δ_0 Electric conduction, $[S/m]$

$\hat{\triangledown}$ Gradient operator $\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

$\hat{B}$ Strength of magnetic field, $[A/m]$

$\hat{F}$ Body forces vector in x and y directions, $[N]$

$\hat{g}$ Acceleration due to gravity, $[m/s^2]$

$\hat{x}, \hat{y}$ Dimensional Axis direction$[M]$

μ Viscosity of the fluid,$[Ns/m^2]$

ρ Density of the fluid,$[kg/m^3]$

τ_w Shearing stress,$[N/m^2]$

θ Dimensionless temperature function

ϑ Reference Kinematic Viscosity, $[m^2/k]$

a Radius of the sphere,$[m]$

C_f Skin friction coefficient,$[Ns^2/kg]$

C_p Specific heat at constant pressure, $[Jkg^{-1}K^{-1}]$

C_{fx} Local skin friction coefficient, $[Ns^2/kg]$

f Dimensionless stream function

Gr Grashof number

h_m Magnetic field intensity, $[T]$

K Thermal conductivity of the fluid,$[W/mK]$

M Magnetic Parameter

Nu Nusselt number

o The center of the sphere

Pr Prandtl number

Q Heat generation parameter

q_w Heat flux at the surface,$[W/m^2]$

$r(\hat{x})$ Radial distance from the symmetrical axis to the surface of the sphere, $[m]$

T Temperature of the fluid, $[K]$

T_∞ Temperature of the ambient fluid,$[K]$

T_w Temperature at the surface,$[K]$

u, v Dimensionless velocity component

V Fluid velocity in the x,y direction, $[m/s]$

x, y Axis direction

ABBREVIATIONS

MHD   Magnetohydrodynamics

DNS   Direct Numerical Scheme

PDEs   Partial Differential Equations

MATLAB   Matrix Laboratory

HOTs   Higher Order Terms

ABSTRACT

In this study, the effects of temperature dependent viscosity on magnetohydrodynamic natural convection flow past an isothermal sphere are determined. The uniformly heated sphere is immersed in a viscous and incompressible fluid where viscosity of the fluid is taken as a non-linear function of temperature. The Partial Differential Equations governing the flow are nonlinear in nature, thus,they are transformed into non dimensional form and solved using the Direct Numerical Scheme and implemented in matrix Laboratory. The numerical results obtained are presented graphically and in a table and are discussed. In this study, it has been observed that increasing the Magnetic parameter leads to a decrease in velocity, temperature, skin friction and the rate of heat transfer. It has also been noted that increase in the Grashof number leads to an increase in velocity and temperature whereas increase in the values of viscous variation parameter leads to an increase in temperature but there is a decrease in velocity. These results are useful in engineering, technology and biomedical fields.

Chapter 1

INTRODUCTION AND LITERATURE REVIEW

1.1 Introduction

Fluid is a substance that can flow. It can also be defined as matter which deforms continuously when subjected to a given amount of external shearing stress . Fluids are classified as;a). Liquid, Gas and Vapour and b).Ideal Fluids and Real Fluids. A liquid is fluid that possesses a definite volume which varies slightly with temperature and pressure. A gas possesses no definite volume and is compressible. Vapour is a gas whose temperature is such that it is very near the liquid state. In liquids, molecules are close together compared to the molecules in gases which are not close to one another and are in a haphazard movement in all directions which make them to collide with each other. Ideal fluid is one which is incompressible, has no viscosity and has no surface tension whereas a real fluid is one which has viscosity, surface tension and are compressible.

Fluid flow can be classified as being steady ,unsteady,Uniform, Non Uniform, Rotational, Irrotational, Laminar, Turbulent, Compressible or incompressible flows. A steady flow is one in which the fluid variables like velocity, pressure and temperature change with time whereas unsteady flow is one in which the velocity, pressure or temperature changes with respect to time. Fluid flow is said to be uniform if fluid variables at any given time does not change with respect to space and non-uniform when the velocity at any given time varies with respect to space.

A fluid flow is rotational if the fluid particles while moving in the direction of flow rotate about their mass centres and is irrotational if the fluid particles while moving in the

direction of flow do not rotate. A laminar fluid flow is one in which paths taken by the individual particles do not cross one another and move along well defined paths. This flow can also be referred to as viscous flow or streamline flow. A turbulent flow is the one in which fluid particles do not move along well defined paths but they move in a haphazard way.

Laminar and turbulent flows are characterized on the basis of Reynolds number where for Reynolds number $(Re) < 2000$, the flow in pipes is said to be laminar; for Reynolds number$(Re) > 4000$, the flow in pipes is turbulent and for (Re) between 2000 and 4000, the flow in pipes may be laminar or turbulent.

Compressible flow is one in which the density(ρ) of the fluid changes from point to point or density is not a constant whereas an incompressible flow is the one in which density is assumed to be a constant.

1.1.1 Isothermal Sphere

A sphere is a geometrical object in three dimensional space which can be referred as a surface of a complete round ball which is analogous to a circular object in two dimensions. An Isothermal process is one that takes place at constant temperature. Therefore, in this study, a sphere which is completely immersed in a ferrofluid and maintained at constant temperature is referred to as an Isothermal sphere.

1.1.2 Temperature Dependence of Fluid Viscosity

This is a phenomenon in which viscosity of fluid decreases as its temperature increases and increases as the temperature of the fluid decreases. In other words, it can be explained that the fluidity tends to increase as temperature increases.

1.1.3 Natural Convection Fluid Flow

In natural convection, the fluid motion is due to natural means such as buoyancy. The heat transfer coefficient encountered in natural convection is low since the fluid velocity associated with natural convection is relatively low. In this mechanism, consider a hot object exposed to cold air where temperature of the air outside the object will drop and the temperature of the air adjacent to the object will rise as a result of heat transfer in the cold air. Therefore,the object will be surrounded with a thin layer of warmer air and heat will be transferred from this layer to the outer layers of air. Since the temperature of the air adjacent to the hot object is higher,then the density is lower which makes the heated air to rise. This movement illustrates natural convection current.

1.1.4 Magnetohydrodynamics

This word is derived from the words, Magneto (meaning magnetic field), Hydro (meaning water) and Dynamics (meaning movement). Magnetohydrodynamics (MHD) is the study of the dynamics of electrically conducting fluids which include Plasmas, Liquid metals and salt water or electrolytes. The fundamental concept behind MHD is that magnetic fields induces electric current in a moving conductive fluid which in turn polarizes the fluid and reciprocally changes the magnetic field. Electromagnetism is the study of interaction between magnetic fields and electric current whereas Hydrodynamics is the study of the flow of fluids.

1.1.5 Dimensional Analysis

Dimensional Analysis is a mathematical technique which makes use of the study of dimensions in solving several engineering problems.It is a method of dimensions and a technique used in research work for design and for conducting model tests. It's based on the principle of dimensional homogeneity which states that every term in an equation when reduced to

fundamental dimensions must contain identical powers of each dimension. Each physical phenomenon can be expressed by an equation giving a relationship between different dimensional and non-dimensional quantities. Dimensional analysis helps in determining a systematic arrangement of variables in the physical relationship combining dimensional variables to form non-dimensional parameters.Therefore, dimensional analysis is used in;

1. Testing the dimensional homogeneity of any equation of fluid motion.

2. Deriving rational formulae for a flow phenomenon.

3. Deriving equations expressed in terms of non-dimensional parameters to show the relative significance of each parameter.

4. Planning model tests and present experimental results in a systematic manner, thus making it possible to analyze the complex fluid flow phenomenon.

1.2   Literature Review

Temperature is one of the factors that may cause variation in the viscosity of a given fluid. Soares et al. (2010) studied the effects of temperature dependent viscosity on forced convection heat transfer from a cylinder in cross flow of power-law fluids and deduced that the variation of viscosity with temperature have an effect on both the local and the surface averaged values of the Nusselt number. They concluded that the velocity and temperature of the fluid changes with the variation of the fluid flow parameters. This means that for higher values of Prandtl number (Pr), both velocity and temperature decreases such that there exists a local maximum value of velocities. Alam et al. (2006) studied the effects of viscous dissipation on MHD natural convection fluid flow over a sphere in the presence of heat generation and concluded that velocity increases as the values of viscous parameter increases and velocity distribution increase as the values of heat generation parameter increases.

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Shrama and Singh (2010) studied the effect of temperature dependent electrical conductivity on steady natural convection flow of viscous incompressible low Prandtl number ($Pr << 1$) electrical conducting fluid along an isothermal vertical plate in the presence of transverse magnetic field and exponentially decaying heat generation. They deduced that fluid velocity increases in the presence of heat generation due to increase in the electrical conductivity parameter while it decreases due to increase in the magnetic field intensity. They also concluded that fluid temperature increases in the presence of volumetric rate of heat generation or due to increase in magnetic field intensity while it decreases due to increase in electrical conductivity parameter. They also explained that the skin friction coefficient increases with the increase in electrical conductivity parameter or in the presence of volumetric rate of heat generation while it decreases due to increase in the Prandtl number.

Khan et al. (2012) studied the unsteady MHD free convection boundary layer flow of nanofluid along a stretching sheet with thermal radiation and viscous dissipation effects and deduced that larger values of the Grashof number showed a significant effect momentum boundary layer. In their study, they concluded that the effect of Brownian motion and thermophoresis stabilizes the boundary layer growth and that boundary layers are highly affected by the Prandtl number. Khan et al. (2012) concluded that thermal boundary layer thickness increases as a result of increasing radiation and the concentration in the boundary layer decreases in the presence of heavier species (large Lewis number).

Chaudhary and Jain (2007) studied combined heat and mass transfer effects on MHD free convection flow past an oscillating plate embedded in porous medium and explained that the concentration decreases with an increase in Schmidt number and in case of cooling of the plate ($Gr > 0$), the velocity decreases with an increase in the phase angle, magnetic parameter, Schmidt number and Prandtl number while it increases with an increase in the value of Grashof number and modified Grashof number, permeability parameter and time. In case of heating of the plate ($Gr < 0$), the velocity increases with an increase in magnetic

parameter, Schmidt number and Prandtl number while it decreases with an increase in the value of Grashof number and modified Grashof number , permeability parameter and time. The Skin friction increases with an increase in Schmidt number, Prandtl number and Magnetic parameter while it decreases with an increase in the value of Grashof number, modified Grashof number, permeability parameter and time. They also concluded that Nusselt number increase with an increase in the Prandtl number while temperature decreases with an increase in the value of Prandtl number.

Mebine and Adigio (2011) studied the effects of Thermal Radiation on Transient MHD Free convection flow over a vertical surface embedded in a porous medium with the periodic boundary temperature and concluded that the temperature of the fluid decreases with increasing radiation and this signified that reduction in the maximum velocity for the temperature decreases the thermal boundary layer thickness thus leading to higher heat transfer to the plate and increasing magnetic parameter reduces the magnitude of velocity. Velocity increases with increasing porosity parameters, and this implied that the presence of a porous medium increases the resistance to flow and greater velocity is experienced in the flow field when the porosity parameter vanishes. The maximum velocity decreases with an increase in phase angle whereas temperature decreases with an increase in the phase angle. They concluded that increase in radiation parameter decreases the skin friction and the heat flux.

Molla et al. (2005) investigated Magnetohydrodynamic natural convection flow from an isothermal sphere with temperature dependent heat generation. Molla et al. (2005) concluded that when the values of heat generation parameter Q increases, there is an increase in the local skin friction coefficient C_{fx} but there is a decrease in the local rate of heat transfer. Both the velocity and temperature profiles increase significantly when the value of heat generation parameter increases. The local rate of heat transfer and the local skin friction coefficient decreases when the value of magnetic parameter increases. Finally, they concluded that increased values of magnetic parameter leads to decrease in the velocity

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distribution whereas there is an increase in the temperature distribution.

Alam et al. (2006) studied viscous dissipation effects on MHD natural convection flow along a sphere and deduced that an increase in the values of magnetic parameter causes both the local skin friction and the local heat transfer to decrease the velocity causing an increase in temperature. These researchers concluded that increase in viscous dissipation parameter leads to an increase in velocity and temperature and increasing the values of Prandtl number leads to a decrease in velocity, temperature, local skin friction coefficient and the local rate of heat transfer.

Natural convection flow along an isothermal vertical plate with temperature dependent viscosity and heat generation has been studied by Molla et al. (2014) and they deduced that the effect of viscosity variation parameter and Rayleigh number decreases the skin friction coefficient whereas increasing the local average rate of heat transfer. They also explained that the momentum and thermal boundary layer becomes thinner when the values of viscosity -variation parameter increases and that viscosity and velocity distribution increases with the effect of Rayleigh number. This has also led to significant decrease in temperature distributions whereas the thickness of momentum boundary layer is enhanced.

Miraj et al. (2011) studied the effects of viscous dissipation and radiation on MHD free convection flow along a sphere with joule heating and heat generation. They deduced that velocity profiles increases with the increasing values of radiation parameter, heat generation parameter, magnetic parameter, joule heating parameter and viscous dissipation parameter whereas the profiles decreases with increasing values of Prandtl number. Skin friction coefficient increases with the increasing values of heat generation parameter and decreases for the increasing values of magnetic parameter. In conclusion,Miraj et al. (2011) deduced the rate of heat transfer increases for increasing values of radiation parameter and decreases for increasing values of heat generation parameter, joule heating parameter and viscous dissipation parameter.

Molla et al. (2012) have studied the MHD natural convection flow from an isothermal

cylinder under the consideration of temperature dependent viscosity and explained that increasing the values of magnetic parameter M and viscosity variation parameter leads to decrease in the local skin friction coefficient, Grashof number and the local Nusselt number. It was concluded that velocity distribution decreases and temperature distribution increases with the increasing values of the magnetic parameter M and viscosity variation parameter.

Haque et al. (2014) studied the effects of viscous dissipation on MHD natural convection flow over a sphere with temperature dependent thermal conductivity in presence of heat generation and they concluded that the velocity and temperature of the fluid within the boundary layer increases with increasing thermal conductivity variation parameter, heat generation parameter and viscous dissipation parameter. They also noted that the skin friction along the surface of the sphere increases with increasing thermal conductivity variation parameter, heat generation parameter and viscous dissipation parameter but decreases for the increasing values of M. They explained that the rate of heat transfer from the surface decreases with the increasing value of magnetic parameter, thermal conductivity variation parameter, heat generation parameter and viscous dissipation parameter.

From the research work cited above, it can be concluded that extensive research work has been carried out on MHD natural convection fluid flow past a surface. However, no emphasis has been given to the problem of MHD natural convection flow past an isothermal sphere considering viscosity as a non-linear function of temperature. Therefore, this work presents findings obtained from carrying a study on the effects of temperature dependent viscosity on the MHD natural convection flow past an isothermal sphere taking viscosity as a non-linear function of temperature and analysis of the results using Direct Numerical Scheme.

1.3　Problem statement

This study considers a two-dimensional MHD Laminar free convectional fluid flow past a uniformly heated sphere Centre at o and radius a which is immersed in a viscous and incompressible fluid as shown in figure 1.1 below. Viscosity of the fluid is taken as a non-linear function of temperature and thus, viscosity varies inversely proportional to temperature. In figure 1.1, **B** is the magnetic field which impends the motion of the fluid flow.The direction of the fluid flow is along the normal X-axis but $\hat{x}$ in the figure shows that the fluid flow is past the isothermal sphere.
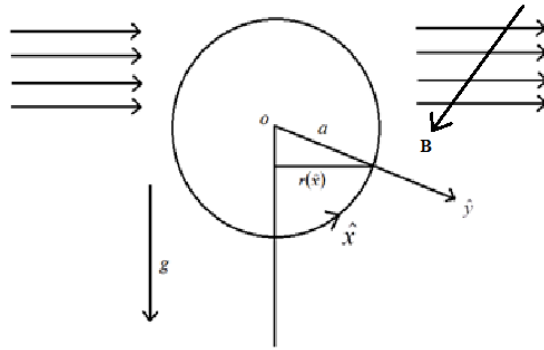
Figure 1.1: Physical Model of Problem

1.4 Justification of the study

MHD natural convection fluid flow is a phenomenon mainly applied in various engineering plants. Other applications of ferrofluids are technological, materials research and biomedical applications. Technological applications of ferrofluids include; Dynamic sealing, Damping, Heat Dissipation and doping of technological materials. In material research, ferrofluids are used in the study of magnetic colloids which are used in doping liquid crystals and in doping of lyotropic liquid crystals with magnetic properties.

In biomedical applications, the main categories include; magnetic drug targeting, hyperthermia and contrast enhancement for Magnetic Resonance Imaging(MRI). Therefore, there is need to carry out a study on the effect of temperature dependent viscosity on MHD natural convection flow past an isothermal sphere.

1.5 Hypothesis

Temperature dependent viscosity has no effect on MHD natural convection fluid flow past an isothermal sphere.

1.6 Objectives

1.6.1 General Objective

To analyse the effects of temperature dependent viscosity on MHD natural convection fluid flow past an isothermal sphere.

1.6.2 Specific Objectives

1. To determine the flow variables.

2. To determine the effects of magnetic field, Grashof number and viscosity on the flow variables.

3. To determine the effects of magnetic field on the rate of skin friction on the surface of the sphere.

4. To determine the effect of magnetic field on the rate of heat transfer on the surface of the sphere.

In the next chapter, governing equations are considered.

Chapter 2

THE GOVERNING EQUATIONS

2.1   Introduction

The equations governing the flow of an incompressible electrically conducting fluid in presence of transverse magnetic field lines past an isothermal sphere are presented in this chapter. First, the assumptions used in this study are highlighted. The equations of conservation of mass, equation of motion and equation of energy are considered in general forms, non-dimensional parameters are defined and non-dimensionalization of the resulting equations is also discussed.

2.2   Assumptions

The following assumptions are made in this study;

1. The viscosity of the fluid is a non-linear function of temperature.

2. The fluid flow is two-dimensional and steady.

3. All velocities are small compared with that of light $v^2/c^2 << 1$.

4. The fluid flow is considered to be laminar.

5. The fluid is incompressible.

6. The force due to electric field is negligible compared to the force $\hat{J} \times \hat{B}$ due to magnetic field.

7. The electric field, external electric field and induced magnetic field due to polarization charges are negligible.

The universal laws that govern fluid flows such as equation of conservation of mass, equation of motion and equation of energy form the basis for the fundamentals of fluid dynamics.

2.3   The Governing Equations

2.3.1   Equation of conservation of mass

The law of conservation of mass states that mass can neither be created or destroyed and it forms the basis for the equation of continuity. This equation is derived by taking a mass balance of fluid entering and leaving a volume in the flow field. The general equation of conservation of mass is given by;

$$\frac{\partial \rho}{\partial t} + \hat{\bigtriangledown} \cdot (\rho \hat{q}) = 0 \qquad (2.1)$$

where $\hat{q}$ is the velocity in x, y and z directions($\hat{q} = u\hat{i} + v\hat{j} + w\hat{k}$) and $\hat{\bigtriangledown} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

2.3.2   Equation of motion

This equation describe the behaviour of physical system in terms of its motion as a function of time. The main descriptions of motion are dynamics and kinematics. In dynamics, momenta, forces and energy are put into consideration whereas kinematics deals with variables derived from the positions of objects and time.The expression of the equation is given as;

$$\frac{\partial \hat{q}}{\partial t} + (\hat{q} \cdot \hat{\bigtriangledown}) \cdot \hat{q} = -\frac{1}{\rho}\hat{\bigtriangledown}P + \vartheta \bigtriangledown^2 \hat{q} + \hat{F} \qquad (2.2)$$

where $\frac{\partial \hat{q}}{\partial t}$ is the temporal acceleration, $(\hat{q} \cdot \hat{\bigtriangledown}) \cdot \hat{q}$ is the convective term, $\hat{\bigtriangledown}P$ is the pressure gradient, $\vartheta \bigtriangledown^2 \hat{q}$ is the force due to viscosity and $\hat{F}$ represents the body forces vector in x and Y directions.

2.3.3   The Energy Equation

This equation is derived by applying the first law of thermodynamics to an arbitrary control volume in the flow field. The general equation in two dimensions is given as:

$$\rho C_p \left(\hat{u} \frac{\partial T}{\partial \hat{x}} + \hat{v} \frac{\partial T}{\partial \hat{y}} \right) = K \left(\frac{\partial^2 T}{\partial \hat{x}^2} + \frac{\partial^2 T}{\partial \hat{y}^2} \right) + \mu \phi \tag{2.3}$$

where the viscous dissipation term ϕ is defined as:

$$\phi = 2 \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right] + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)^2 \tag{2.4}$$

2.4   Non-Dimensional Numbers

Dimensionless numbers which are also called the non-dimensional parameters are obtained by dividing the inertia force which is always present when any mass is in motion by viscous force or gravity force or pressure force or surface tension or elastic force. These parameters allow the application of the results obtained in a model to any other dynamically similar case. In this study, there are two numbers that are used.

These numbers are;

- Grashof Number, Gr

- Prandtl Number, Pr

2.4.1 Grashof Number

This number occurs in natural convection problem and is usually defined as:

$$Gr = \frac{g\beta \left(T_w - T_\infty\right) a^3}{\vartheta^2} \tag{2.5}$$

Where g is acceleration due to gravity, β is the coefficient of Thermal Expansion, a is the radius of the sphere, ϑ is the Reference Kinematic Viscosity, T_w is the temperature of the sphere and T_∞ is the temperature of the ambient fluid.This number gives the relative importance of buoyancy force to viscous force.

2.4.2 Prandtl Number

This number gives the ratio of viscous force to the thermal force and is defined as:

$$Pr = \frac{\mu C_p}{K} = \frac{\vartheta}{\left(\frac{K}{\rho C_p}\right)} \tag{2.6}$$

Where $\left(\dfrac{K}{\rho C_p}\right)$ is the thermal diffusivity and viscosity. Fluids that are more viscous have a higher value of ϑ and thus it follows that they have large Prandtl number. Prandtl number (Pr) is large when K is small and small when viscosity is small. This number gives the relative importance of viscous dissipation to thermal dissipation.

In order to develop the empirical laws in engineering, the process of non-dimensionalization is widely used. This helps in writing the important parameters in a problem as a functional relationship between them. In the next section,mathematical formulation is carried in order to obtain the specific equations used in this study.

2.5 Mathematical Formulation

2.5.1 Dimensional Equation of Conservation of Mass

In this study, fluid flow is considered to be steady and two dimensional laminar flow. The fluid in consideration is incompressible,thus, density (ρ) is assumed to be a constant and thus the term $\dfrac{\partial \rho}{\partial t} = 0$.

Thus, equation (2.1) reduces to $\hat{\nabla} \cdot (\rho \hat{q}) = 0$. Substituting the values of $\hat{\nabla}$ and $\hat{q}$, this equation becomes;

$$\rho \left(\hat{i} \frac{\partial}{\partial \hat{x}} + \hat{j} \frac{\partial}{\partial \hat{y}} + \hat{k} \frac{\partial}{\partial \hat{z}} \right) \cdot \left(\hat{i} \hat{u} + \hat{j} \hat{v} + \hat{k} \hat{w} \right) = 0 \tag{2.7}$$

or

$$\rho \left(\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\partial \hat{y}} + \frac{\partial \hat{w}}{\partial \hat{z}} \right) = 0 \tag{2.8}$$

But $\hat{r}(x)$ is the radial distance from the symmetric axis to the surface of the sphere and thus, $\hat{r}(x)$ is included in equation (2.8) which becomes;

$$\rho \left(\frac{\partial r\hat{u}}{\partial \hat{x}} + \frac{\partial r\hat{v}}{\partial \hat{y}} + \frac{\partial r\hat{w}}{\partial \hat{z}} \right) = 0 \tag{2.9}$$

Since the flow is two dimensional, the term $\dfrac{\partial r\hat{w}}{\partial \hat{z}} = 0$ Thus, equation (2.9)becomes;

$$\rho \left(\frac{\partial r\hat{u}}{\partial \hat{x}} + \frac{\partial r\hat{v}}{\partial \hat{y}} \right) = 0 \tag{2.10}$$

Since ρ then equation (2.10) becomes;

$$\frac{\partial r\hat{u}}{\partial \hat{x}} + \frac{\partial r\hat{v}}{\partial \hat{y}} = 0 \tag{2.11}$$

Equation (2.11) is the final dimensional equation of conservation of mass governing the flow.

2.5.2   Dimensional Equation of Motion

From the general equation (2.2), the term $\dfrac{\partial \hat{q}}{\partial t} = 0$ since the fluid flow is steady.

The term

$$(\hat{q} \cdot \hat{\bigtriangledown}) \cdot \hat{q} = \hat{u}\frac{\partial \hat{u}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{u}}{\partial \hat{y}} \tag{2.12}$$

since the flow is two dimensional.

For Laminar fluid flow; $\hat{\bigtriangledown}P = -\dfrac{\partial P}{\partial x}$ and $\dfrac{\partial \tau}{\partial \hat{y}} = \dfrac{\partial P}{\partial x}$. This is because the pressure gradient in the direction of flow is equal to the shear gradient in the direction normal to the direction of flow. But $\tau = \dfrac{1}{\mu}\dfrac{\partial \hat{u}}{\partial \hat{y}}$. Therefore, the pressure gradient term becomes;

$$\hat{\bigtriangledown}P = \frac{\partial \tau}{\partial \hat{y}} = \frac{\partial}{\partial \hat{y}}\left(\frac{1}{\mu}\frac{\partial \hat{u}}{\partial \hat{y}}\right) \tag{2.13}$$

The term;

$$\vartheta\hat{\bigtriangledown}^2 \hat{q} = \frac{\mu}{\rho}\left(\frac{\partial^2 \hat{u}}{\partial \hat{x}^2} + \frac{\partial^2 \hat{v}}{\partial \hat{y}^2}\right) = 0 \tag{2.14}$$

This is because from the boundary layer approximations, the equations $\left(\dfrac{\partial^2 \hat{u}}{\partial \hat{x}^2} + \dfrac{\partial^2 \hat{v}}{\partial \hat{y}^2}\right)$ is negligible in this study. The sphere is immersed in the fluid and it is subjected to an upward force which tends to lift it up. This force is known as Buoyancy which is the tendency of an immersed body to be lifted up in the fluid. Therefore, from bousinessq approximation, this force in this study is given as;

$$g\beta(T - T_\infty)Sin\left(\frac{\hat{x}}{a}\right) \tag{2.15}$$

The fluid in consideration is a ferrofluid and thus the Magnetic Force $\hat{F}$ is given as;

$$\hat{F} = \hat{J} \times \hat{B} \tag{2.16}$$

The generalized Ohms law neglecting the hall effects can be expressed as:

$$\hat{J} = \delta_0 \left[E + (\hat{q} \times \hat{B})\right]. \tag{2.17}$$

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. E in equation (2.17) is the thermal electric effect and it is equivalent to 0 since there is no applied electrc field internally. Therefore, equation (2.17) can be written as:

$$\hat{J} = \delta_0 \left[\hat{q} \times \hat{B} \right].$$ (2.18)

The velocity and Magnetic fields are given as: $\hat{q} = (u, v, 0)$ and $\hat{B} = (0, 0, B_0)$, Therefore;

$$\hat{q} \times \hat{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u & v & 0 \\ 0 & 0 & B_0 \end{vmatrix} = B_0 v \hat{i} - B_0 u \hat{j}$$ (2.19)

Therefore, $\hat{J} = \delta_0 B_0 v \hat{i} - \delta_0 B_0 u \hat{j}$

$$\hat{J} \times \hat{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \delta_0 v B_0 & -\delta_0 u B_0 & 0 \\ 0 & 0 & B_0 \end{vmatrix} = -\delta_0 u B_0^2 \hat{i} - \delta_0 v B_0^2 \hat{j}$$ (2.20)

Therefore, taking the force in the x-direction, $\hat{F} = -\delta_0 u B_0^2 \hat{i}$

Substituting equations (2.12),(2.13),(2.14),(2.15),(2.20) in the general equation of motion(2.2), the dimensional equation of motion governing the flow in this study becomes;

$$\hat{u}\frac{\partial \hat{u}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{u}}{\partial \hat{y}} = \frac{1}{\rho}\frac{\partial}{\partial \hat{y}}\left(\frac{1}{\mu}\frac{\partial \hat{u}}{\partial \hat{y}}\right) + g\beta(T - T_\infty)Sin\left(\frac{\hat{x}}{a}\right) - \frac{\delta_0 B_0^2 \hat{u}}{\rho}$$ (2.21)

2.5.3 Dimensional Energy Equation

From the general equation (2.3), $\mu\phi$ is the Viscous Energy dissipation term and it is very small since the sphere in consideration is isothermal. Hence, it is neglected in this study. The term $(\hat{\nabla}^2 T) = \frac{\partial^2 T}{\partial \hat{x}^2} + \frac{\partial^2 T}{\partial \hat{y}^2}$ is considered along the y-axis due to boundary layer approximations where its considered that $\frac{\partial^2 T}{\partial \hat{x}^2} << \frac{\partial^2 T}{\partial \hat{y}^2}$

Therefore, the energy equation governing the fluid flow in this study is given as;

$$\hat{u}\frac{\partial T}{\partial \hat{x}} + \hat{v}\frac{\partial T}{\partial \hat{y}} = \frac{K}{\rho C_p}\frac{\partial^2 T}{\partial \hat{y}^2}$$ (2.22)

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Equations (2.11),(2.21)and (2.22) are the equations governing the flow with boundary conditions defined as;

$$\hat{u} = \hat{v} = 0, \; T = 1 \text{ at } \hat{y} = 0$$
$$\hat{u} \to 0, \; T \to 0 \text{ as } \hat{y} \to \infty$$

(2.23)

In the next section, non-dimensionalization of the governing equation (2.11), (2.21) and (2.22) is carried out.

2.6   Non-Dimensionalization

Non-dimensionalization is a process that aims at ensuring that the results obtained from a study are applicable to other geometrically similar configurations under similar set of equations.The characteristic dimensionless quantities are selected which are used in the non-dimensionalization of the governing equations. The independent variables are non-dimensionalized according to the following dimensionless quantities;

$x = \dfrac{\hat{x}}{a} = \dfrac{L}{L}$ (Dimensionless)

$y = \dfrac{\hat{y}}{a} = \dfrac{L}{L}$ (Dimensionless)

$u = \dfrac{\rho a}{\mu} Gr^{-\frac{1}{2}} \hat{u} = \dfrac{ML^{-3}LLT^{-1}}{ML^{-1}T^{-1}}$ (Dimensionless)

$v = \dfrac{\rho a}{\mu} Gr^{-\frac{1}{2}} \hat{v} = \dfrac{ML^{-3}LLT^{-1}}{ML^{-1}T^{-1}}$ (Dimensionless)

$\theta = \dfrac{T - T_\infty}{T_w - T_\infty} = \dfrac{T}{T}$ (Dimensionless)

The dimensionless quantities above together with the boundary condition defined in the

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above section are used in non-dimensionalization of the governing equations (2.11), (2.21) and (2.22) Non-dimensionalizing equations (2.11), (2.21) and (2.22)is done by replacing $\hat{x}, \hat{y}, \hat{u}, T$ and $\hat{v}$ with $\hat{x} = xa, \hat{y} = ya, \hat{u} = \dfrac{\mu}{\rho a} Gr^{\frac{1}{2}} u, \hat{v} = \dfrac{\mu}{\rho a} Gr^{\frac{1}{2}} v, T = \theta(T_w - T_\infty) + T_\infty.$

Therefore, from equation (2.11) ;

$$\frac{\partial\left(r\hat{u}\right)}{\partial\hat{x}} = \frac{\partial\left(r\frac{\mu}{\rho a}Gr^{\frac{1}{2}}u\right)}{\partial(xa)} = \frac{\mu Gr^{\frac{1}{2}}}{\rho a^2}\frac{\partial\left(ru\right)}{\partial x} \tag{2.24}$$

$$\frac{\partial\left(r\hat{v}\right)}{\partial\hat{y}} = \frac{\partial\left(r\frac{\mu}{\rho a}Gr^{\frac{1}{2}}v\right)}{\partial(ya)} = \frac{\mu Gr^{\frac{1}{2}}}{\rho a^2}\frac{\partial\left(rv\right)}{\partial y} \tag{2.25}$$

Substituting equations (2.24) and (2.25) in (2.11), the equation becomes;

$$\frac{\mu Gr^{\frac{1}{2}}}{\rho a^2}\frac{\partial\left(ru\right)}{\partial x} + \frac{\mu Gr^{\frac{1}{2}}}{\rho a^2}\frac{\partial\left(rv\right)}{\partial y} = 0 \tag{2.26}$$

Multiplying equation (2.26) both sides by $\dfrac{\rho a^2}{\mu Gr^{\frac{1}{2}}}$, the equation becomes;

$$\frac{\partial\left(ru\right)}{\partial x} + \frac{\partial\left(rv\right)}{\partial y} = 0 \tag{2.27}$$

Equation (2.27) is the non-dimensionalized equation of conservation of mass.

Replacing the values of $\hat{x}, \hat{y}, \hat{u}, T$ and $\hat{v}$ with $\hat{x} = xa, \hat{y} = ya, \hat{u} = \frac{\mu}{\rho a}Gr^{\frac{1}{2}}u,$

$\hat{v} = \frac{\mu}{\rho a}Gr^{\frac{1}{2}}v$ and $T = \theta(T_w - T_\infty) + T_\infty$ in equation (2.21),the terms are given as;

$$\hat{u}\frac{\partial\hat{u}}{\partial\hat{x}} = \frac{\mu Gr^{\frac{1}{2}}}{\rho a^2}\frac{\partial\left(\frac{\mu}{\rho a}Gr^{\frac{1}{2}}u\right)}{\partial(xa)} = \frac{\mu^2 Gr}{\rho^2 a^3}u\frac{\partial u}{\partial x} \tag{2.28}$$

$$\hat{v}\frac{\partial\hat{v}}{\partial\hat{y}} = \frac{\mu Gr^{\frac{1}{2}}}{\rho a^2}\frac{\partial\left(\frac{\mu}{\rho a}Gr^{\frac{1}{2}}v\right)}{\partial(ya)} = \frac{\mu^2 Gr}{\rho^2 a^3}v\frac{\partial v}{\partial y} \tag{2.29}$$

$$\frac{1}{\rho}\frac{\partial}{\partial\hat{y}}\left(\frac{1}{\mu}\frac{\partial\hat{u}}{\partial\hat{y}}\right) = \frac{1}{\rho}\frac{\partial}{\partial\hat{y}}\left(\frac{1}{\mu_\infty}(1+\gamma\theta)\frac{\partial\hat{u}}{\partial\hat{y}}\right) = \frac{\gamma}{\rho\mu_\infty}\frac{\partial\theta}{\partial\hat{y}}\frac{\partial\hat{u}}{\partial\hat{y}} + \frac{1}{\rho\mu_\infty}(1+\gamma\theta)\frac{\partial^2\hat{u}}{\partial\hat{y}^2} \tag{2.30}$$

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Equation (2.30) can be written as;

$$\frac{\gamma}{\rho\mu_\infty}\frac{\partial\theta}{\partial(ya)}\frac{\partial\left(\frac{\mu}{\rho a}Gr^{\frac{1}{2}}u\right)}{\partial(ya)} + \frac{1}{\rho\mu_\infty}(1+\gamma\theta)\frac{\partial^2\left(\frac{\mu}{\rho a}Gr^{\frac{1}{2}}u\right)}{\partial(ya)^2} \tag{2.31}$$

$$\frac{\gamma\mu Gr^{\frac{1}{2}}}{\rho^2 a^3\mu_\infty}\frac{\partial\theta}{\partial y}\frac{\partial u}{\partial y} + \frac{\mu Gr^{\frac{1}{2}}}{\rho^2 a^3\mu_\infty}(1+\gamma\theta)\frac{\partial^2 u}{\partial y^2} \tag{2.32}$$

$$g\beta(T-T_\infty)Sin\left(\frac{\hat{x}}{a}\right) = g\beta(T-T_\infty)Sin\left(\frac{xa}{a}\right) = g\beta(T-T_\infty)Sin\left(x\right) \tag{2.33}$$

$$\frac{\delta_0 B_0^2\hat{u}}{\rho} = \frac{\delta_0 B_0^2}{\rho}\left(\frac{\mu Gr^{\frac{1}{2}}}{\rho a}\right)u = \frac{\delta_0 B_0^2\mu Gr^{\frac{1}{2}}}{\rho^2 a}u \tag{2.34}$$

Substituting equations (2.28),(2.29),(2.32),(2.33),(2.33) and (2.34)in (2.21)and multiplying the equation both sides by $\frac{\rho^2 a^3}{\mu^2 Gr}$, the equation becomes;

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \frac{\gamma}{\mu\mu_\infty Gr^{\frac{1}{2}}}\frac{\partial\theta}{\partial y}\frac{\partial u}{\partial y} + \frac{(1+\gamma\theta)}{\mu\mu_\infty Gr^{\frac{1}{2}}}\frac{\partial^2 u}{\partial y^2} + \frac{\rho^2 a^3 g\beta(T-T_\infty)}{\mu^2 Gr}Sinx - \frac{\delta_0 a^2 B_0^2}{\mu Gr^{\frac{1}{2}}}u \tag{2.35}$$

In this study, μ is the viscosity of the fluid and is equal to the viscosity of the free stream. $Gr = \frac{\rho^2 g\beta(T-T_\infty)a^3}{\mu_\infty^2}$ and therefore $1/Gr = \frac{\mu_\infty^2}{\rho^2 g\beta(T-T_\infty)a^3}$

$\theta = \frac{T-T_\infty}{T_w-T_\infty}$ and $M = \frac{\delta_0 B_0^2 a^2}{\mu Gr^{\frac{1}{2}}}$

Substituting the values of μ, Gr and M in equation (2.35), this equation reduces to;

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\gamma}{\mu^2 Gr^{\frac{1}{2}}}\frac{\partial\theta}{\partial y}\frac{\partial u}{\partial y} + \frac{(1+\gamma\theta)}{\mu^2 Gr^{\frac{1}{2}}}\frac{\partial^2 u}{\partial y^2} + \theta Sinx - Mu \tag{2.36}$$

In equation(2.36), let $\eta = \frac{\gamma}{\mu^2 Gr^{\frac{1}{2}}}$ to represent the Viscosity Variation Parameter in this

study.

Therefore, equation (2.36) can be written as;

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \eta\frac{\partial \theta}{\partial y}\frac{\partial u}{\partial y} + \frac{\eta}{\gamma}(1+\gamma\theta)\frac{\partial^2 u}{\partial y^2} + \theta Sinx - Mu \quad (2.37)$$

Equation (2.37) is the non-dimensionalized equation of motion governing the flow.
Non-dimensionalizing the values of $\hat{x}, \hat{y}, \hat{u}, T$ and $\hat{v}$ with $\hat{x} = xa, \hat{y} = ya, \hat{u} = \frac{\mu}{\rho a}Gr^{\frac{1}{2}}u,$
$\hat{v} = \frac{\mu}{\rho a}Gr^{\frac{1}{2}}v, T = \theta(T_w - T_\infty) + T_\infty$ in equation (2.22),the terms are given as;

$$\hat{u}\frac{\partial T}{\partial \hat{x}} = \frac{\mu}{\rho a}Gr^{\frac{1}{2}}u\frac{\partial \theta}{\partial(xa)} = \frac{\mu}{\rho a^2}Gr^{\frac{1}{2}}u\frac{\partial \theta}{\partial x} \quad (2.38)$$

$$\hat{v}\frac{\partial T}{\partial \hat{y}} = \frac{\mu}{\rho a}Gr^{\frac{1}{2}}v\frac{\partial \theta}{\partial(ya)} = \frac{\mu}{\rho a^2}Gr^{\frac{1}{2}}v\frac{\partial \theta}{\partial y} \quad (2.39)$$

$$\frac{K}{\rho C_p}\frac{\partial^2 T}{\partial \hat{y}^2} = \frac{K}{\rho C_p}\frac{\partial^2 \theta}{\partial(\hat{ya})^2} = \frac{K}{\rho C_p a^2}\frac{\partial^2 \theta}{\partial y^2} \quad (2.40)$$

Substituting equations (2.38),(2.39) and(2.40) in equation (2.22) gives;

$$\frac{\mu}{\rho a^2}Gr^{\frac{1}{2}}u\frac{\partial \theta}{\partial x} + \frac{\mu}{\rho a^2}Gr^{\frac{1}{2}}v\frac{\partial \theta}{\partial y} = \frac{K}{\rho C_p a^2}\frac{\partial^2 \theta}{\partial y^2} \quad (2.41)$$

Multiply both sides of equation (2.41) by $\frac{\rho a^2}{\mu Gr^{\frac{1}{2}}}$, gives;

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{K}{\mu C_p Gr^{\frac{1}{2}}}\frac{\partial^2 \theta}{\partial y^2} \quad (2.42)$$

But Prandtl number is given as; $Pr = \frac{\mu C_p}{K}$,thus equation (2.42) can be written as;

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{PrGr^{\frac{1}{2}}}\frac{\partial^2 \theta}{\partial y^2} \quad (2.43)$$

Equation (2.43) can also be written as;

$$Gr^{\frac{1}{2}}u\frac{\partial \theta}{\partial x} + Gr^{\frac{1}{2}}v\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} \quad (2.44)$$

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Equation (2.44)is the non-dimensionalized equation of energy governing the fluid flow. The boundary conditions from the previous section becomes;

$$u = v = 0,\ \theta = 1 \text{ at } y = 0$$
$$u \to 0,\ \theta \to 0 \text{ as } y \to \infty$$

$$(2.45)$$

In the next chapter,the governing equations (2.27),(2.37) and (2.44) together with the boundary conditions (2.45) are written in finite difference form.

Chapter 3

METHOD OF SOLUTION AND ANALYSIS

3.1 Introduction

In this chapter, the method of solution is discussed and the governing equations (2.27),(2.37) and (2.44) are transformed using the Direct Numerical Scheme(DNS) which is consistent with the method of solution and final set of equations are presented in finite difference form.The DNS method is applied since the governing equatons obtained are non-linear in nature.

3.2 Direct Numerical Scheme

To apply the Direct Numerical Scheme (DNS) method, a new set of transformations are introduced. These are;

$X = x$, $Y = y$, $U = \dfrac{u}{x}$, $V = \dfrac{v}{y}$. Therefore, $u = Ux = UX$ and $v = Vy = VY$

But $r(x)$ is the radial distance from the centre of the sphere in consideration and is given as $r(x) = Sinx$. Using the transformations above, the radial distance from the sphere can be written as $r(x) = SinX$. The non-dimensionalized equation (2.27) of continuity can be transformed as follows using the transformations above;

$$\frac{\partial(ru)}{\partial x} = \frac{\partial(UXSinX)}{\partial X} = XSinX\frac{\partial U}{\partial X} + USinX + UXCosX \tag{3.1}$$

$$\frac{\partial(rv)}{\partial y} = \frac{\partial(VYSinX)}{\partial Y} = YSinX\frac{\partial V}{\partial Y} + VSinX \tag{3.2}$$

Substituting equations (3.1)and (3.2) in equation (2.27), gives;

$$X \, SinX \frac{\partial U}{\partial X} + U \, SinX + UX \, CosX + V \, SinX + Y \, SinX \frac{\partial V}{\partial Y} = 0 \tag{3.3}$$

Dividing equation (3.3) by $SinX$, the equation can be written as;

$$X \frac{\partial U}{\partial X} + \left[1 + X \frac{CosX}{SinX} \right] U + V + Y \frac{\partial V}{\partial Y} = 0 \tag{3.4}$$

Substituting the transformations above in the non-dimensionalized equation of motion, we obtain the following to transformations;

$$u \frac{\partial u}{\partial x} = UX \frac{\partial (UX)}{\partial X} = UX^2 \frac{\partial U}{\partial X} + U^2 X \tag{3.5}$$

$$v \frac{\partial u}{\partial y} = VY \frac{\partial (UX)}{\partial Y} = VXY \frac{\partial U}{\partial Y} \tag{3.6}$$

$$\eta \frac{\partial \theta}{\partial y} \frac{\partial u}{\partial y} = \eta \frac{\partial \theta}{\partial Y} \frac{\partial (UX)}{\partial Y} = \eta X \frac{\partial \theta}{\partial Y} \frac{\partial U}{\partial Y} \tag{3.7}$$

$$\frac{\eta}{\gamma} (1 + \gamma\theta) \frac{\partial^2 u}{\partial y^2} = \frac{\eta}{\gamma} (1 + \gamma\theta) \frac{\partial^2 (UX)}{\partial Y^2} = \frac{\eta}{\gamma} (1 + \gamma\theta) X \frac{\partial^2 U}{\partial y^2} \tag{3.8}$$

$$\theta \, Sinx = \theta \, SinX \tag{3.9}$$

$$Mu = MUX \tag{3.10}$$

Substituting equations(3.5),(3.6),(3.7)(3.8),(3.9) and (3.10) in equation (2.37) and dividing both sides of the equation by X, gives;

$$UX \frac{\partial U}{\partial X} + U^2 + VY \frac{\partial U}{\partial Y} = \frac{\eta}{\gamma} (1 + \gamma\theta) \frac{\partial^2 U}{\partial y^2} + \eta \frac{\partial \theta}{\partial Y} \frac{\partial U}{\partial Y} + \theta \frac{SinX}{X} + MU \tag{3.11}$$

Substituting the transformations $X = x$, $Y = y$, $U = \dfrac{u}{x}$, $V = \dfrac{v}{y}$ in equation (2.44), the following equations are obtained;

$$Gr^{\frac{1}{2}} u \frac{\partial \theta}{\partial x} = Gr^{\frac{1}{2}} UX \frac{\partial \theta}{\partial X} \tag{3.12}$$

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$$Gr^{\frac{1}{2}}v\frac{\partial\theta}{\partial y} = Gr^{\frac{1}{2}}VY\frac{\partial\theta}{\partial Y} \qquad (3.13)$$

$$\frac{1}{Pr}\frac{\partial^2\theta}{\partial y^2} = \frac{1}{Pr}\frac{\partial^2\theta}{\partial Y^2} \qquad (3.14)$$

Substituting equations (3.11),(3.12) and (3.13) in the non dimensionalized equation of energy,the equation becomes;

$$Gr^{\frac{1}{2}}UX\frac{\partial\theta}{\partial X} + Gr^{\frac{1}{2}}VY\frac{\partial\theta}{\partial Y} = \frac{1}{Pr}\frac{\partial^2\theta}{\partial Y^2} \qquad (3.15)$$

In order to determine the physical quantities, namely the shearing stress and the rate of heat transfer the following dimensionless relations are used in this study:

$$\frac{C_f Gr^{\frac{1}{4}}}{2(1+\gamma)} = X\left(\frac{\partial U}{\partial Y}\right)_{Y=0} \qquad (3.16)$$

$$NuGr^{\frac{-1}{4}} = -\left(\frac{\partial\theta}{\partial Y}\right)_{Y=0} \qquad (3.17)$$

Using the transformations $X = x$, $Y = y$, $U = \frac{u}{x}$, $V = \frac{v}{y}$, the boundary conditions represented in equation 2.45 in section 2.6 can be written as;

$$U = V = 0, \ \theta = 1 \text{ at } X = 0 \text{ any } Y$$

$$U = V = 0, \ \theta = 1 \text{ at } Y = 0, \ X > 0 \qquad (3.18)$$

$$U \rightarrow 0, \ \theta \rightarrow 0 \text{ as } Y \rightarrow \infty, \ X > 0$$

Equations (3.4), (3.11), (3.15), (3.16) and (3.17)are non-linear in nature and this is the main reason why these equations together with the boundary conditions are solved numerically.Therefore, these equations together with the boundary conditions above are written into finite differences as shown in the next section so as to solve them.

3.3 Finite Difference Technique

The finite difference approximations for derivatives is one of the methods that can be used to solve differential equations. This entails approximating the differential operator when

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the derivative in the equations are replaced by difference quotients.The solution obtained by the use of the finite difference method approaches the true solution of the partial differential equation as the increments on the mesh are minimized and approaches zero and hence the method converges.

In this method, the (t, y) plane is divided into a network of rectangles of sides $\triangle t = h$ and $\triangle y = k$ by drawing the set of lines $t = ih$ and $y = jk$ where $i, j = 0, 1, 2, ...$ Each nodal point is identified by double index (i, j) that define its location with respect to t and y as shown in the figure below:
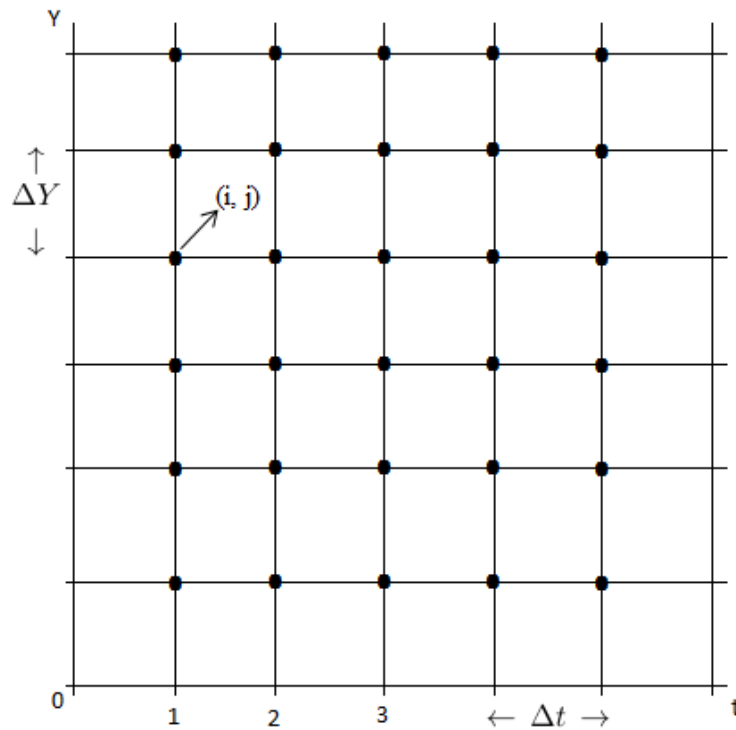

Figure 3.1: Finite Difference Mesh

The finite difference mesh is applied in dividing the physical flow domain into finite number

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of discrete approximation for space and time domains to be used in the finite difference method.From the plane figure represented in the figure above, each corner of the cell forms the mesh or the grid point. Point (i, j) in the diagram is the reference point where i and j represent t and y respectively. The points adjacent to y and t are defined using the notation $(i \pm 1)$ for $(t \pm \triangle t)$ and $(j \pm 1)$ for $(x \pm \triangle x)$ whereas the points that$i$ and j units from the reference point have the coordinates $(i \triangle t, j \triangle x)$

In the finite difference approximation method, the derivatives are replaced with the finite differences. These finite differences are obtained from the Taylor's series.

$U = u(t, x)$ and $t = t(t, x)$ are called mesh points or grid points. The first and second order derivatives with respect to t are obtained in finite difference form using differencing. A finite difference mesh is used to express the unknown functional values at the $(i, j)^{th}$ interior mesh using the unknown boundary points.

The finite differences arising from the equations governing this flow are obtained by replacing the derivatives in the governing equations by the corresponding difference approximation putting into consideration the initial values and the boundary values set as shown below;

$$\frac{\partial U}{\partial X} = \frac{U_{i+1}^j - U_{i-1}^j}{2 \triangle X} \tag{3.19}$$

$$\frac{\partial U}{\partial Y} = \frac{U_i^{j+1} - U_i^{j-1}}{2 \triangle Y} \tag{3.20}$$

$$\frac{\partial V}{\partial Y} = \frac{V_i^{j+1} - V_i^{j-1}}{2 \triangle Y} \tag{3.21}$$

$$\frac{\partial^2 U}{\partial Y^2} = \frac{U_i^{j+1} - 2U_i^j + U_i^{j-1}}{(\triangle Y)^2} \tag{3.22}$$

$$\frac{\partial \theta}{\partial X} = \frac{\theta_{i+1}^j - \theta_{i-1}^j}{2 \triangle X} \tag{3.23}$$

$$\frac{\partial \theta}{\partial Y} = \frac{\theta_i^{j+1} - \theta_i^{j-1}}{2 \triangle Y} \tag{3.24}$$

$$\frac{\partial^2 \theta}{\partial Y^2} = \frac{\theta_i^{j+1} - 2\theta_i^j + \theta_i^{j-1}}{(\triangle Y)^2} \tag{3.25}$$

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Equations (3.20) to (3.25) are the derivatives of the equations governing the fluid flow written in finite difference form which are substituted in the respective equations as shown in the next section.

3.4   Governing Equations in Finite Difference Form

Equations (3.19),(3.20),(3.21),(3.22),(3.23),(3.24)and (3.25) will be substituted for the derivatives in equations (3.3),(3.11),(3.15),(3.16)and (3.17).

The derivatives are written in Central Finite Difference Form as shown above in equations (3.20) to (3.25).

Equation (3.4) transforms into;

$$X_i \left[\frac{U_{i+1}^j - U_{i-1}^j}{2(\triangle X)} \right] + U_i^j \left[1 + X_i \frac{Cos X_i}{Sin X_i} \right] + \left[\frac{V_i^{j+1} - V_i^{j-1}}{2} \right] + \left[\frac{V_i^{j+1} - V_i^{j-1}}{2(\triangle Y)} \right] = 0 \quad (3.26)$$

Making U_i^j the subject of the formula in equation (3.26), we obtain;

$$U_i^j = - \left[X_i \left[\frac{U_{i+1}^j - U_{i-1}^j}{2(\triangle X)} \right] + \left[\frac{V_i^{j+1} - V_i^{j-1}}{2} \right] + \left[\frac{V_i^{j+1} - V_i^{j-1}}{2(\triangle Y)} \right] \right] \div \left[1 + X_i \frac{Cos X_i}{Sin X_i} \right]$$
$$(3.27)$$

Equation (3.11) can be written as;

$$x_i \left[\frac{U_i^{j+1} - U_i^{j-1}}{2} \right] \left[\frac{U_{i+1}^j - U_{i-1}^j}{2(\triangle X)} \right] + Y_j \left[\frac{V_i^{j+1} - V_i^{j-1}}{2} \right] \left[\frac{U_i^{j+1} - U_i^{j-1}}{2(\triangle Y)} \right] + \left[\frac{U_{i+1}^j - U_{i-1}^j}{2} \right]^2 =$$
$$\frac{\eta}{\gamma}(1 + \gamma \theta_i^j) \left[\frac{U_i^{j+1} - 2U_i^j + U_i^{j-1}}{(\triangle Y)^2} \right] + \eta \left[\frac{U_i^{j+1} - U_i^{j-1}}{2 \triangle Y} \right] \left[\frac{\theta_i^{j+1} - \theta_i^{j-1}}{2 \triangle Y} \right] + \theta_i^j \frac{Sin X_i}{X_i} - MU_i^j$$
$$(3.28)$$

Consider the term below from equation (3.28);

$$\frac{\eta}{\gamma}(1 + \gamma \theta_i^j) \left[\frac{U_i^{j+1} - 2U_i^j + U_i^{j-1}}{(\triangle Y)^2} \right] \qquad (3.29)$$

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Considering term by term and opening the brackets in the term (3.29), it can also be written as;

$$-\frac{2}{(\triangle Y)^2}(1+\gamma\theta_i^j)\frac{\eta}{\gamma}U_i^j + \frac{\eta}{\gamma}(1+\gamma\theta_i^j)\left[\frac{U_i^{j+1}+U_i^{j-1}}{(\triangle Y)^2}\right] \tag{3.30}$$

Substituting the term (3.30) into (3.28) gives;

$$X_i\left[\frac{U_i^{j+1}-U_i^{j-1}}{2}\right]\left[\frac{U_{i+1}^j-U_{i-1}^j}{2(\triangle X)}\right]+Y_j\left[\frac{V_i^{j+1}-V_i^{j-1}}{2}\right]\left[\frac{U_i^{j+1}-U_i^{j-1}}{2(\triangle Y)}\right]+\left[\frac{U_{i+1}^j-U_{i-1}^j}{2}\right]^2=$$

$$-\frac{2}{(\triangle Y)^2}(1+\gamma\theta_i^j)\frac{\eta}{\gamma}U_i^j+\frac{\eta}{\gamma}(1+\gamma\theta_i^j)\left[\frac{U_i^{j+1}+U_i^{j-1}}{(\triangle Y)^2}\right]+\eta\left[\frac{U_i^{j+1}-U_i^{j-1}}{2\triangle Y}\right]\left[\frac{\theta_i^{j+1}-\theta_i^{j-1}}{2\triangle Y}\right]$$

$$+\theta_i^j\frac{SinX_i}{X_i}-MU_i^j \tag{3.31}$$

From equation (3.31), making U_i^j the subject of the formulae, we obtain;

$$U_i^j=\frac{\eta}{\gamma}(1+\gamma\theta_i^j)\left[\frac{U_i^{j+1}+U_i^{j-1}}{(\triangle Y)^2}\right]+\eta\left[\frac{U_i^{j+1}-U_i^{j-1}}{2\triangle Y}\right]\left[\frac{\theta_i^{j+1}-\theta_i^{j-1}}{2\triangle Y}\right]+\theta_i^j\frac{SinX_i}{X_i}-$$

$$X_i\left[\frac{U_i^{j+1}-U_i^{j-1}}{2}\right]\left[\frac{U_{i+1}^j-U_{i-1}^j}{2(\triangle X)}\right]-Y_j\left[\frac{V_i^{j+1}-V_i^{j-1}}{2}\right]\left[\frac{U_i^{j+1}-U_i^{j-1}}{2(\triangle Y)}\right]+\left[\frac{U_{i+1}^j-U_{i-1}^j}{2}\right]^2$$

$$\div\left[\frac{2}{(\triangle Y)^2}(1+\gamma\theta_i^j)\frac{\eta}{\gamma}+M\right] \tag{3.32}$$

Equation (3.32)is the final equation of motion written in finite difference form.

Writing equation (3.15) in Finite differences, we obtain;

$$Gr^{\frac{1}{2}}X_i\left[\frac{U_i^{j+1}-U_i^{j-1}}{2}\right]\left[\frac{\theta_{i+1}^j-\theta_{i-1}^j}{2(\triangle X)}\right]+Gr^{\frac{1}{2}}Y_j\left[\frac{V_i^{j+1}-V_i^{j-1}}{2}\right]\left[\frac{\theta_i^{j+1}-\theta_i^{j-1}}{2(\triangle Y)}\right]=$$

$$\frac{1}{Pr}\left[\frac{\theta_i^{j+1}-2\theta_i^j+\theta_i^{j-1}}{(\triangle Y)^2}\right] \tag{3.33}$$

From equation (3.33), consider the term;

$$\frac{1}{Pr}\left[\frac{\theta_i^{j+1}-2\theta_i^j+\theta_i^{j-1}}{(\triangle Y)^2}\right] \tag{3.34}$$

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Opening the brackets in the term (3.34) and considering term by term,the equation can also be written as;

$$-\frac{2}{(\triangle Y)^2)}\frac{1}{Pr}\theta_i^j + \frac{1}{Pr}\left[\frac{\theta_i^{j+1}+\theta_i^{j-1}}{(\triangle Y)^2}\right] \tag{3.35}$$

Substituting equation (3.35) in equation (3.33), we obtain;

$$Gr^{\frac{1}{2}}X_i\left[\frac{U_i^{j+1}-U_i^{j-1}}{2}\right]\left[\frac{\theta_{i+1}^j-\theta_{i-1}^j}{2(\triangle X)}\right] + Gr^{\frac{1}{2}}Y_j\left[\frac{V_i^{j+1}-V_i^{j-1}}{2}\right]\left[\frac{\theta_i^{j+1}-\theta_i^{j-1}}{2(\triangle Y)}\right] =$$
$$-\frac{2}{(\triangle Y)^2)}\frac{1}{Pr}\theta_i^j + \frac{1}{Pr}\left[\frac{\theta_i^{j+1}+\theta_i^{j-1}}{(\triangle Y)^2}\right] \tag{3.36}$$

From equation (3.36), making θ_i^j the subject of the formulae, gives;

$$\theta_i^j = \frac{1}{Pr}\left[\frac{\theta_i^{j+1}+\theta_i^{j-1}}{(\triangle Y)^2}\right] - Gr^{\frac{1}{2}}X_i\left[\frac{U_i^{j+1}-U_i^{j-1}}{2}\right]\left[\frac{\theta_{i+1}^j-\theta_{i-1}^j}{2(\triangle X)}\right] -$$
$$Gr^{\frac{1}{2}}Y_j\left[\frac{V_i^{j+1}-V_i^{j-1}}{2}\right]\left[\frac{\theta_i^{j+1}-\theta_i^{j-1}}{2(\triangle Y)}\right] \div \frac{1}{Pr}\left[\frac{2}{(\triangle Y)^2}\right] \tag{3.37}$$

The physical quantities to be obtained are the shearing stress(rate of skin friction) and the rate of heat transfer. The finite difference equations used to obtain these results are;

$$\frac{C_f Gr^{\frac{1}{4}}}{2(1+\gamma)} = X_i\left(\frac{U_i^{j+1}-U_i^{j-1}}{2(\triangle Y)}\right)_{Y=0} \tag{3.38}$$

$$NuGr^{\frac{-1}{4}} = -\left(\frac{\theta_i^{j+1}-\theta_i^{j-1}}{2(\triangle Y)}\right)_{Y=0} \tag{3.39}$$

Therefore, equations(3.27),(3.32),(3.37),(3.38) and (3.39) are the Final Set of Equations and are solved using a computer code in MATLAB software.

The results obtained from the simulation of equations (3.27),(3.32),(3.37),(3.38) and (3.39) in the Computer code and discussions are presented in the next chapter.

Chapter 4

RESULTS AND DISCUSSIONS

4.1   Introduction

In this chapter, the results of the simulations are presented, followed by discussions at each step.

4.2   Results and Discussions

In this study, the numerical solutions start from the lower stagnation point $x = 0$ round the sphere to the upper stagnation point where $x = \pi$.
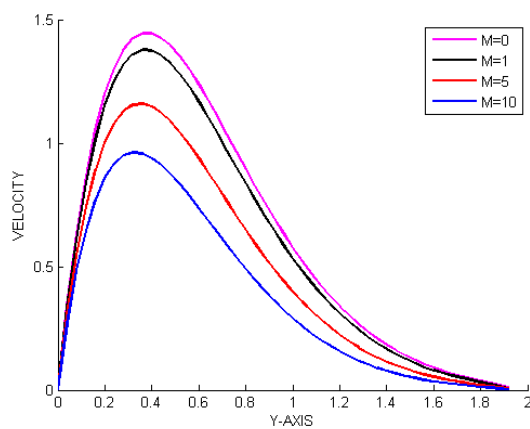

Figure 4.1: Velocity distribution for different values of Magnetic Parameter M

Figure (4.1) shows the results for the velocity for different values of Magnetic Parameter $M(= 0.0, 1.0, 5.0, 10.0)$ plotted against Y at $x = \frac{\pi}{3}$ having Prandtl number $Pr = 0.73$, Grashof number $Gr = 4$ and Viscosity Variation Parameter $\eta = 0.015$ It is observed that velocity profiles decreases with increase in the Magnetic Parameter. The reason behind this is that the interaction of the magnetic field and the moving electric charge carried by the fluid induces a force which opposes the motion of the fluid.Near the surface of the sphere , the velocity increases and then decreases slowly and finally approaches zero according to the outer boundary condition. This implies that there exists local maximum of the velocity within the boundary layer due to the effect of the viscosity of the fluid.It is also observed that the velocity sharply increases and the decreases exponentially. This is because of the no-slip condition which implies that the velocity of the fluid at a solid boundary must be the same as that of boundary itself. Thus a layer of fluid which cannot slip away from the boundary surface undergoes retardation; thus this retarded layer further causes retardation for the adjacent layers of the fluid thereby developing a small region in the immediate vicinity of the boundary surface in which the velocity of the flowing fluid increase rapidly from zero at the boundary surface and approaches the velocity of the mainstream. After reaching the critical point, the velocity of the fluid starts to decrease as shown in the figure above.
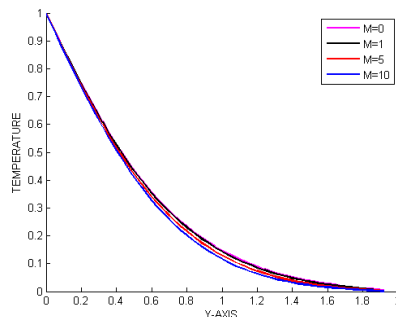

Figure 4.2: Temperature distribution for different values of Magnetic Parameter M

Figure (4.2) shows the results for the Temperature for different values of Magnetic Parameter $M(= 0.0, 1.0, 5.0, 10.0)$ plotted against Y at $x = \frac{\pi}{3}$ having Prandtl number $Pr = 0.73$, Grashof number $Gr = 4$ and Viscosity Variation Parameter $\eta = 0.015$

It is observed that temperature profiles decreases with increase in the Magnetic Parameter. This is because the interaction of the magnetic field and the moving electric charge carried by the fluid induces a force which opposes the motion of the fluid.
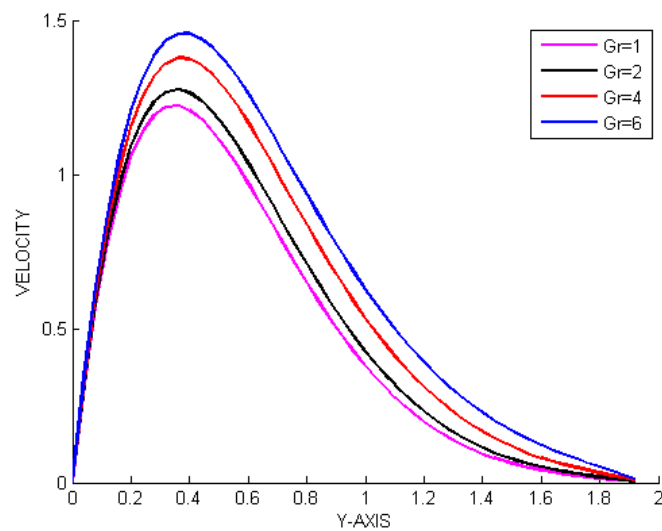

Figure 4.3: Velocity distribution for different values of Grashof number Gr

Figure (4.3) depicts velocity distribution against variable Y for different values of Grashof number $Gr = (1.0, 2.0, 4.0, 6.0)$ at $x = \frac{\pi}{3}$ while $Pr = 0.73$, $\eta = 0.015$ and $M = 1$

It can be observed that the velocity of the fluid increases with increase in the Grashof number and decreases with decrease in Grashof number. Grashof number gives the relative importance of Bouyancy force to viscous force. Increase in Grashof number leads to decrease in viscous force in the fluid which leads to increase in the velocity of the fluid. Thus,the increase in the velocity profiles with increase in Grashof Number Gr.
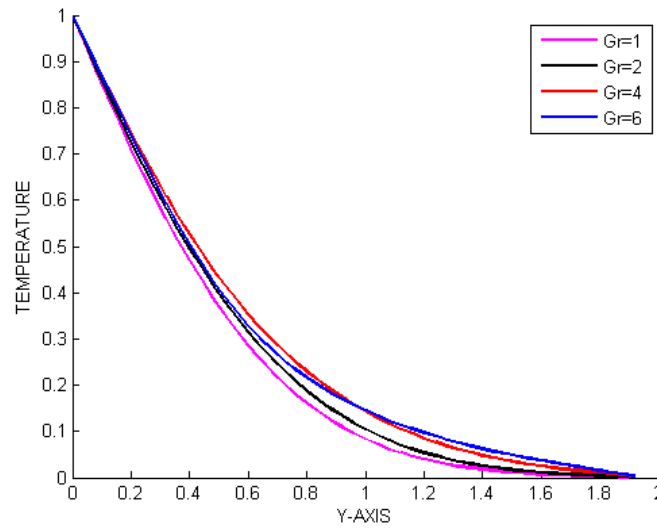

Figure 4.4: Temperature distribution for different values of Grashof number Gr

Figure (4.4) depicts temperature distribution against variable Y for different values of Grashof number $Gr = (1.0, 2.0, 4.0, 6.0)$ at $x = \frac{\pi}{3}$ while $Pr = 0.73$, $\eta = 0.015$ and $M = 1$ Grashof number gives the relative importance of Bouyancy force to viscous force and therefore, increase in Grashof number leads to increase in Bouyancy force which leads to increase in the temperature profiles of the fluid flow. Thus,the reason behind increase in Temperature profiles with increase in Grashof Number Gr.

Figure 4.5: Velocity distribution for different values of Viscous Variation Parameter η

Figure (4.5) represents velocity distributions for different values of Viscosity Variation Parameter $\eta = (0.015, 0.02, 0.025, 0.03)$ with $Pr = 0.73$, $M = 1$ and $Gr = 4$. It can be observed that velocity profiles decreases with increase in the viscous Variation Parameter (η). This is because increase in η leads to decrease in the viscous force of the fluid. This leads to decrease in the velocity of the fluid.

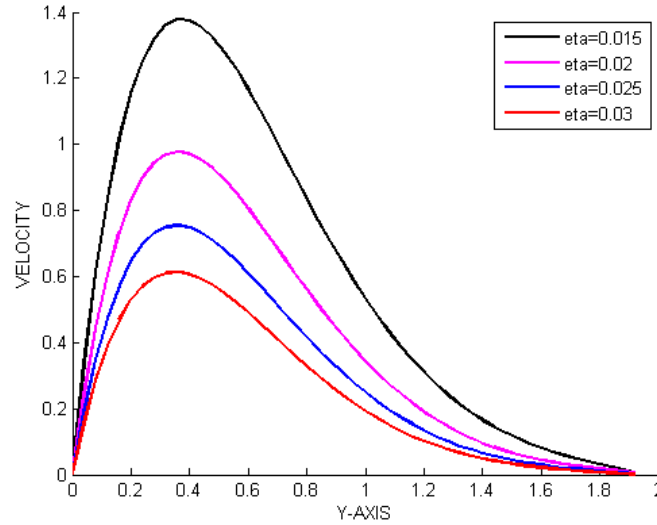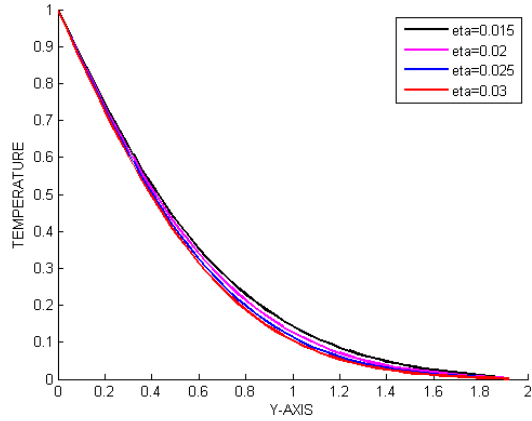Figure 4.6: Temperature distribution for different values of Viscous Variation Parameter η

Figure (4.6) represents Temperature distributions for different values of Viscosity Variation Parameter $\eta = (0.015, 0.02, 0.025, 0.03)$ with $Pr = 0.73$, $M = 1$ and $Gr = 4$. It can be observed that temperature profiles decreases with increase in the viscous Variation Parameter (η). This is because increase in η leads to decrease in the Bouyancy force and thus decrease in the temperature profiles of the fluid.
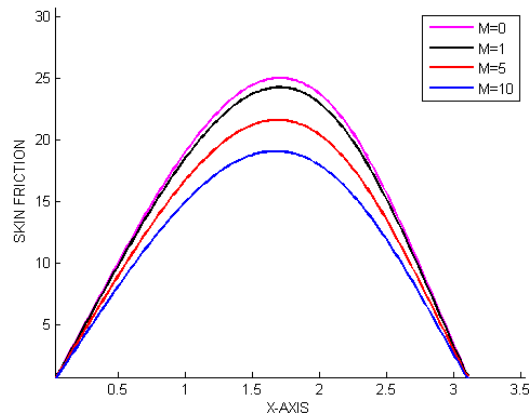

Figure 4.7: Skin Friction Coefficient for different values of Magnetic Parameter M

Figure (4.7) shows the effect of different values of Magnetic Parameter $M = (0.0, 1.0, 5.0, 10.0)$ on local skin friction coefficient with Viscosity Variation Parameter $\eta = 0.015$, $Pr = 0.73$ and $Gr = 4$. It can be observed that increase in the Magnetic Parameter M leads to decrease in the local skin friction coefficient $(C_f Gr^{\frac{1}{4}})$. This is because increase in the values Magnetic parameter leads to increase in the Lorentz force which opposes the fluid flow. This means that there is a decrease in velocity gradient of the fluid flow. Therefore, leading to a decrease in local skin friction.
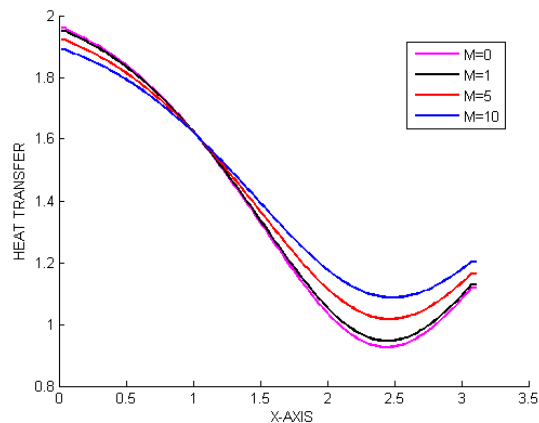
Figure 4.8: Heat transfer for different values of Magnetic Parameter M

Figure (4.8) shows the effect of different values of Magnetic Parameter $M = (0.0, 1.0, 5.0, 10.0)$ on the Local Nusselt number $(NuGr^{\frac{-1}{4}})$ with Viscosity Variation Parameter $\eta = 0.015$, $Pr = 0.73$ and $Gr = 4$. It is observed that increase in the Magnetic Parameter M leads to decrease in the values of local Nusselt number. This is because increase in the values Magnetic parameter leads to increase in the Lorentz force which opposes the fluid flow. This means that there is a decrease in temperature gradient of the fluid flow.Therefore, leading to a decrease in local Nusselt number.

It can be observed that there is a decrease in Nusselt number along the X-axis and a

slight increase after X=2.5 which leads to an observation of an inverse behaviour of the profiles in figure (4.8). This is because viscosity of the fluid is taken to vary inversely proportional to temperature. The table below shows the results of skin friction and Rate of heat transfer varying M. In the next chapter, validation of the results obtained,conclusions and recommendations are discussed.

Table 4.1: The results of skin friction and Rate of heat Transfer (Nusselt number) varying Magnetic parameter while $Pr = 0.073$, $\eta = 0.015$, $Gr = 4$

M=0			M=1		
X^0	$(C_f Gr^{\frac{1}{4}})$	$(NuGr^{\frac{-1}{4}})$	X^0	$(C_f Gr^{\frac{1}{4}})$	$(NuGr^{\frac{-1}{4}})$
0.02	0.4015	1.9581	0.02	0.3914	1.9498
0.4	8.0984	1.8704	0.4	7.905	1.8642
0.79	15.4196	1.7279	0.79	15.0242	1.725
1.17	21.3508	1.5315	1.17	20.7584	1.5336
1.55	24.7245	1.2949	1.55	23.9855	1.3039
1.94	24.2875	1.0665	1.94	23.5241	1.083
2.32	19.393	0.9343	2.32	18.7799	0.9546
2.71	10.9739	0.9644	2.71	10.643	0.9816
3.09	1.263	1.1182	3.09	1.2298	1.1281
M=5			M=10		
X^0	$(C_f Gr^{\frac{1}{4}})$	$(NuGr^{\frac{-1}{4}})$	X^0	$(C_f Gr^{\frac{1}{4}})$	$(NuGr^{\frac{-1}{4}})$
0.02	0.3558	1.9202	0.02	0.3198	1.889
0.4	7.222	1.8417	0.4	6.5319	1.8181
0.79	13.6387	1.7143	0.79	12.2564	1.7029
1.17	18.7002	1.5409	1.17	16.6756	1.5481
1.55	21.4388	1.3359	1.55	18.9665	1.3679
1.94	20.9063	1.1409	1.94	18.3863	1.1987
2.32	16.6764	1.0262	2.32	14.6496	1.0972
2.71	9.499	1.0428	2.71	8.3833	1.1045
3.09	1.1125	1.1642	3.09	0.9942	1.2023

Chapter 5

CONCLUSION AND RECOMMENDATIONS

5.1   Introduction

This chapter presents the validation of the results of this research study, conclusions and recommendations on the areas that require further research.The published paper of this study is also included in this chapter.

5.2   Validation of the Results

According to the results discussed in the previous chapter, we note that the Magnetic Field and Viscous Variation Parameter has a retarding influence whereas Grashof number has an accelerating influence on the fluid velocity and Temperature.

In absence of viscosity being a non-linear function of temperature, the results are in agreement with the results of the study by Molla et al. (2012) which noted that Viscous Variation parameter and Magnetic Field has a retarding influence on the fluid velocity whereas Grashof number have an accelerating influence on both the velocity and temperature of the fluid.

5.3   Conclusions

The results of this study leads to the conclusion that the Magnetic Field and Viscous Variation Parameter has a retarding influence whereas Grashof number has an accelerating influence on the fluid velocity and Temperature. Therefore, increase in Magnetic Parameter M leads to decrease in velocity and temperature profiles of the fluid flow.

It can also be concluded that velocity and temperature of the fluid increases with increase in the Grashof number and decreases with increase in the Viscous Variation Parameter.Skin friction and the rate of heat transfer in the fluid decreases with increase in the Magnetic Field and thus viscosity Variation exerts a retarding influence in the fluid velocity and temperature.

5.4   Recommendations

In this study, the effects of temperature dependent viscosity on Magnetohydrodynamic natural convection flow past an Isothermal sphere has been investigated. Its recommended that further research should be carried out:

1. When the fluid flow is unsteady.

2. When the fluid flow is turbulent.

3. When the sphere which is immersed in the fluid is non-uniformly heated.

4. When the fluid is taken to be compressible.

REFERENCES

Alam, M. M., Alim, M., and Chowdhury, M. M. (2006). Viscous dissipation effects on mhd natural convection flow along a sphere. *Journal of Mechanical Engineering*, 36:44–48.

Chaudhary, R. and Jain, A. (2007). Combined heat and mass transfer effects on mhd free convection flow past an oscillating plate embedded in porous medium. *Rom. J. Phys*, 52(5-7):505–524.

Haque, M. R., Alam, M. M., Ali, M., and Sheikh, M. N. (2014). Effects of viscous dissipation on mhd natural convection flow over a sphere with temperature dependent thermal conductivity in presence of heat generation. *European Scientific Journal*, 10(15).

Khan, M. S., Karim, I., Ali, L. E., and Islam, A. (2012). Unsteady mhd free convection boundary-layer flow of a nanofluid along a stretching sheet with thermal radiation and viscous dissipation effects. *International Nano Letters*, 2(1):1–9.

Mebine, P. and Adigio, E. M. (2011). Effects of thermal radiation on transient mhd free convection flow over a vertical surface embedded in a porous medium with periodic boundary temperature. *Math. Aeterna*, 1(4):245–261.

Miraj, M., Alim, M., and Andallah, L. (2011). Effects of viscous dissipation and radiation on magnetohydrodynamic free convection flow along a sphere with joule heating and heat generation. *Thammasat Int. J. Sc. Tech*, 16(4).

Molla, M., Saha, S. C., and Khan, M. (2012). Mhd natural convection flow from an isothermal horizontal circular cylinder under consideration of temperature dependent viscosity. *Engineering Computations*, 29(8):875–887.

Molla, M. M., Biswas, A., Al-Mamun, A., and Hossain, M. A. (2014). Natural convection flow along an isothermal vertical flat plate with temperature dependent viscosity and heat generation. *Journal of Computational Engineering*, 2014.

Molla, M. M., Rahman, A., and Rahman, L. T. (2005). Natural convection flow from an isothermal sphere with temperature dependent thermal conductivity. *Journal of Naval Architecture and Marine Engineering*, 2(2):53–64.

Shrama, P. and Singh, G. (2010). Steady mhd natural convection flow with variable electrical conductivity and heat generation along an isothermal vertical plate. *Tamkang J. of Science and Engineering*, 13(3):235–242.

Soares, A., Ferreira, J., Caramelo, L., Anacleto, J., and Chhabra, R. (2010). Effect of temperature-dependent viscosity on forced convection heat transfer from a cylinder in crossflow of power-law fluids. *International Journal of Heat and Mass Transfer*, 53(21):4728–4740.

1: COMPUTER CODE IN MATLAB

The governing equations (3.27),(3.32),(3.37),(3.38) and (3.39) in matrix notation in finite difference form are simulated in the following computer programme code developed usng MATLAB software subject to the boundary conditions as discussed here in.

The results are obtained by varying various flow parameters notably, Magnetic number,Grashof number and Viscosity variation parameter.

```
function LucyCode()

clear all;clc;

x0=0;xend=pi; dx=pi/180;

x=x0:dx:xend;

nx=floor((xend-x0)/dx);

y0=0;yend=2; dy=0.04;

y=y0:dy:yend;

ny=(yend-y0)/dy;

M=10;Pr=0.73;eta=.015;Gr=4;color='-b';

uk=zeros(nx,ny);vk=zeros(nx,ny);thetak=zeros(nx,ny);

uk(1:2,:)=0;vk(1:2,:)=0;thetak(1:2,:)=0;%BC at X=0

uk(nx-1:nx,:)=0;thetak(nx-1:nx,:)=1;%BC at X=pi

uk(:,1:2)=0;vk(:,1:2)=0;thetak(:,1:2)=1;%BC at Y=0

uk(:,ny-1:ny)=0;thetak(:,ny-1:ny)=0;%BC at Y tend to Inf

xk=zeros(nx,ny);

matrix=ones(nx,ny);

for i=2:nx-1

    for j=2:ny-1

        x(i,j)=x(i).*matrix(i,j);

   vk(i,j)=vk(i,j-1)-0.5*dy*(uk(i,j-1)+uk(i,j)).*(1+xk(i,i).*

    (cos(xk(i,j))/sin(xk(i,j))))-xk(i,j)*(dy/dx)*(uk(i,j)-uk(i-1,i)));

uk(i,j)=(dy*dy/(2+M*(dy*dy)))*((1/(dy*dy))*(uk(i,j+1)+uk(i,j-1))+(eta/(dy*dy)).

*(thetak(i,j)).*(uk(i,j)+uk(i,j-1))-(1/(dy*dy))*(thetak(i,j)).*(uk(i+1,j)+uk(i,j))+

(0.25/(dy*dy))*(thetak(i,j+1)-thetak(i,j)).*(uk(i,j+1)-uk(i,j))+(sin(xk(i,j))./(xk(i,j))).*

thetak(i,j)-(0.25/dy)*(vk(i,j+1)+vk(i,j-1)).*(uk(i,j+1)-uk(i,j)) -(0.25/dx)*xk(i,j)*
```

```
(uk(i+1,j)+uk(i-1,j)).*(uk(i+1,j)-uk(i-1,j))-(0.25*(uk(i+1,j)+uk(i,j-1))).^2);
 thetak(i,j)=(0.5*Pr*dy*dy)*((1/(Pr*dy*dy))*(thetak(i,j+1)+thetak(i,j-1))-
    Gr*(vk(i,j)+vk(i,j)).*((thetak(i,j+1)-thetak(i,j))/(2*dy))-(xk(i,j)).
    *Gr*((uk(i,j-1)+uk(i,j))).*((thetak(i+1,j)-thetak(i-1,j))/(2*dx)));
   SFk(i,j)=2*(1+eta)*xk(i,j).*((uk(i,j)-uk(i,j+1))/(2*dy));
   HTk(i,j)=-((thetak(i,j+1)-thetak(i,j))/(2*dy));
    end
end
figure(1)
subplot(2,1,1)
mesh(y(2:ny-1),x(2:nx-1),thetak(2:nx-1,2:ny-1))
subplot(2,1,2)
mesh(y(2:ny-1),x(2:nx-1),uk(2:nx-1,2:ny-1))
figure(2)
plot(y(1:ny-1),thetak(ceil(0.85*nx),1:ny-1),color,2)
xlabel('Y-AXIS')
ylabel('TEMPERATURE')
figure(3)
plot(y(1:ny-1),uk(ceil(0.85*nx),1:ny-1),color)
xlabel('Y-AXIS')
ylabel('VELOCITY')
end
```

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2: PUBLICATION

Mwangi Wanjiku Lucy, Mathew Ngugi Kinyanjui, Surindar Mohan Uppal.Effects of Temperature Dependent Viscosity on Magnetohydrodynamic Natural Convection Flow Past an Isothermal Sphere. *American Journal of Applied Mathematics* 2016; 4(1): 53-61. Published online February 25, 2016 (http://www.sciencepublishinggroup.com/j/ajam) doi: 10.11648/j.ajam.20160401.15