HYDROMAGNETIC FLUID FLOW BETWEEN PARALLEL PLATES WHERE THE UPPER PLATE IS POROUS IN PRESENCE OF VARIABLE TRANSVERSE MAGNETIC FIELDS

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Hydromagnetic Fluid Flow between Parallel Plates where the Upper Plate is Porous in Presence of Variable Transverse Magnetic Fields

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DECLARATION

This thesis is my original work and has not been presented for a degree in any other university.

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DEDICATION

This thesis is dedicated to my parents Mr. and Mrs. MBURU, my siblings Mungai, Thuo, Kuria, Nduta, Wairimu, Wambui, Munoru,and Muthoni.

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NOMENCLATURE

ROMAN SYMBOLS

Symbol	Meaning
B	Magnetic flux density, Wbm^{-2}
ℓ	Electric charge Coulombs, m^{-3}
E	Electric field strength, Vm^{-1}
$E_x, E_{y,} E_{z,}$	Components of electric field strength, Vm^{-1}
Ec	Eckert number
F	Body force, N
Fe	Electromagnetic force, N
H	Magnetic field strength, Wbm^{-2}
H_x, H_y, H_z	Components of magnetic field strength, Wbm^{-2}
Ho	Fixed magnetic field intensity, Wbm^{-2}
i, j, k	Unit vectors in the x ,y and z directions
J	Electric current density, Am^{-2}
J_x, J_y, J_z	Components of the electric current density, Am^{-2}
K	Thermal conductivity, $Wm^{-1}K^{-1}$
L	Characteristic length, M
M	Magnetic parameter
P	Pressure of the fluid, Nm^{-2}
Pr	Prandtl number

Q	Test charge
u	Fluid velocity in the x direction, m^2/s
Re	Hydro-magnetic Reynold's number
R	Joules heating parameter
S_o	Dimensional Suction velocity, m/s
T	Dimensional Temperature of the fluid, K
T^*	Dimensionless Temperature of the fluid
t	Dimensional time, s
t^*	Dimensionless time
u, v ,w	Dimensional velocity Components, ms^{-1}
u_o	Velocity of the moving plate, ms^{-1}
$\mathbf{u}^*, \mathbf{v}^*, \mathbf{w}^*$	Dimensionless velocity Components
x, y, z	Dimensional Cartesian coordinates, m
x^*, y^*, z^*	Dimensionless Cartesian coordinates

GREEK SYMBOLS

Symbol	Meaning
β_o	Strength of magnetic field, A/m
β	Coefficient of volumetric expansion
ρ	Density of the fluid, Kg/m^3
μ	Coefficient of Viscosity, $kgm^{-1}s^{-1}$
μ_e	Magnetic permeability, Hm^{-1}

υ	Kinetic viscosity, m^2s^{-1}
σ	Electrical conductivity, $\Omega^{-1}m^{-1}$
Φ	Viscous dissipation function, s^{-1}
∇	Gradient operator, $=\mathbf{i}\frac{\partial}{\partial x}+\mathbf{j}\frac{\partial}{\partial y}+\mathbf{k}\frac{\partial}{\partial z}$
∇^2	Laplacian operator, $=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$
$\frac{D}{Dt}$	Material derivative, $=\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}$
∇t	Time interval, s
∇y	Distance interval, m
$\frac{dp}{dx}$	Pressure gradient in the x-direction

LIST OF ABBREVIATIONS

MHD	Magneto-hydrodynamics
PDE	Partial differential equations
LHD	Left hand side
RHD	Right hand side
HOT	Higher Order Terms
FDM	Finite difference method

ABSTRACT

Analysis of magneto-hydrodynamic fluid flow between parallel plates where the upper plate is porous in presence of variable transverse magnetic fields has been investigated. Both the plates are non-conducting and horizontally placed, constant suction taking place in the upper plate. The upper plate moves in the opposite direction of the fluid flow as the lower plate remains stationary. The electrically conducting Newtonian fluid is unsteady and incompressible. The governing equation for the flow includes the continuity equation, Navier stokes equation and the energy equation. The equations have been formulated and later on non-dimensionalised. The velocity equation is solved simultaneously with the energy equation by the finite difference technique since both the equations are highly non-linear and coupled. The finite difference form of velocity and temperature equations are implemented in MATLAB software version 7.14.0.739 and the results obtained are presented graphically. The effect of varying various parameters on the velocity and temperature profiles has been discussed. These parameters include magnetic parameter M, pressure gradient dp/dx, prandtl number Pr, Eckert number Ec, joules heating parameter R and suction parameter S_0. An increase in suction parameter lead to a decrease in velocity and an increase in temperature profiles of the fluid. The results obtained for the velocity and temperature profiles will provide useful information to the engineers in designing and improving efficiency and performance of machines and especially in the dyeing industry and in extraction of metal industry.

CHAPTER ONE

INTRODUCTION AND LITERATURE REVIEW

1.1 Overview

In this chapter the key terms used in the thesis are defined and an explanation of the main concepts is made. A review of literature related to the present work and the problem statement of the study is outlined. The objectives of the study are discussed and the justification of the present work is stated at the end of this chapter.

1.2 Introduction

There are three classes of matter solid, liquid and gas. Liquids or gases are termed as fluids. There are two types of fluids, that is, Newtonian fluids where viscosity does not change with the rate of deformation and non-Newtonian fluids where viscosity varies with the rate of deformation. A solid is a matter in which the distance between its molecules does not change when a force is applied on it.

Magneto-hydrodynamic (MHD) is the study of motion of electrically conducting fluid in presence of magnetic fields. The flow of an electrically conducting fluid under a magnetic field gives rise to induced electric currents. The magnetic field exerts mechanical forces on the induced electric currents. The induced electric currents flow in the direction perpendicular to both the magnetic field and the direction of motion of the fluid. However the induced currents also generate their own magnetic field, which in turn affects the original magnetic field. The interaction of the electric current and the applied magnetic fields give rise to Lorentz force which affects the velocity of the Newtonian fluid. The hydro-magnetic flow between parallel plates and magnetic field lines applied normal to the moving plate of the channel was first investigated both theoretically and experimentally by Hartman (1937).

This flow has been investigated by many researchers due to its varied applications in dyeing industry, geothermal reservoirs, underground energy transport, petroleum and mineral industries and in purification of crude oil.

1.3 Definition of key terms

1.3.1 Fluid

A fluid is a substance whose constituent particles may continuously change their positions relative to one another when shear force is applied to it. If a fluid is at rest, there are no shearing forces acting on the fluid and therefore, all forces in the fluid are perpendicular to the planes upon which they act. Shear stresses are developed when the fluid is in motion such that the particles of the fluid move relative to each other with different velocities and on the other hand if the velocity of the fluid particles are the same at every point, no shear forces can be produced since the fluid particles are at rest relative to each other.

1.3.2 Ideal and Real fluids

An ideal fluid is one that is incompressible and its flow exhibits no viscous effect. Real fluids are compressible and their flow exhibits viscous effect which means that whenever there is a velocity gradient across the real fluid's flow path, frictional forces arise between the adjacent fluid particles due to the viscosity μ of the fluid. Real fluids obey the Newton's law of viscosity

i.e $$\tau \alpha \frac{\partial u}{\partial y} \tag{1.1}$$

Where τ is the shear stress, u is the fluid velocity y is the transition distance and α is a symbol for proportionality. This means that the shear stress τ in the fluid is proportional to the velocity gradient, which is the rate of velocity across the fluid flow path.

For a Newtonian fluid, we can express shear stress as;

$$\tau = \mu \frac{\partial u}{\partial y} \tag{1.2}$$

The constant of proportionality μ is known as the coefficient of viscosity or viscosity of the fluid. The viscosity of a Newtonian fluid depends on temperature and pressure. A boundary layer is formed in the fluid flow region close to the solid wall and when viscous fluid flows between stationary solid boundary surfaces, the velocity of fluid particles in contact with the solid boundary is zero due to the no slip boundary condition at the solid wall boundary where a type of frictional force called skin friction exists. The thickness of the boundary layer will be dependent on the Reynolds number and other flow variables.

1.3.3 Porous medium

A porous medium is a material containing pores or spaces between which solid materials, liquid or gas can pass through. Porosity refers to a measure of void spaces in a material and can also be defined as the fraction of volume of voids over the total volume. Fluid flow through porous media is of interest to researchers due to its varied applications to the petroleum engineers who are concerned with the movement of oil and gas in pipes and in analysis of the spread of pollutants in groundwater.

1.3.4 Steady and Unsteady flow

Fluid flow can be classified as either steady or unsteady. The flow is said to be steady if the fluid flow variables such as velocity, applied magnetic field and temperature are independent of time while on the other hand if the flow variables are dependent on time the flow is said to be unsteady.

1.3.5 Laminar flow and turbulent Flow

Laminar, flow refers to the motion of the fluid particles in an orderly manner with the fluid particles moving in a straight line parallel to the boundary walls and the fluid particles do not encounter a disturbance along their path. Turbulence in fluid flow occurs when a flowing fluid suddenly encounters a disturbance such as a solid obstruction or a force. As a result, the fluid particles move in a disorderly manner with different velocities and energies. The shape of the velocity curve (the velocity profile across any given section of the flow channel) depends upon whether the flow is laminar or turbulent. For turbulent flow in a pipe a fairly flat velocity distribution exist across the section of the flow field, with the result that the entire fluid flows at a given single value. If the flow is laminar the shape is parabolic with the maximum velocity at the center being about twice the average velocity in the pipe.

1.3.6 Magneto-hydrodynamics flow

Magneto-hydrodynamic (MHD) is the study of an electrically conducting fluid flow in presence of a magnetic field. MHD entails the study of dynamics of the interaction of electrically conducting fluids and electromagnetic field. The fluid can be ionized gases (commonly called plasmas) or liquid metals. When a conducting fluid flows through the magnetic lines of force the positive and negative charges are each accelerated in such a way that their average motion gives rise to an electric current given by;

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{q} \times \mathbf{B}) \tag{1.3}$$

In accordance with the dynamo rule, the voltage drop or electric field which causes this current is at right angles to the direction of the fluid motion and the magnetic field lines. In the case of a fluid conductor flowing in presence of a transverse magnetic field, the ordinary laws of hydrodynamics can easily be extended to cover the effect of magnetic fields and electric fields.

This is done by adding magnetic force to the momentum conservation equation. The magnetic force also referred to Lorentz force is in a direction perpendicular to both **J** and **B** and is proportional to the magnitude of both **J** and **B** and is given by;

$$F_e = \mathbf{J} \times \mathbf{B} \tag{1.4}$$

In MHD this force acts on the fluid particles and is reffered to as electromagnetic force.

1.3.7 Viscosity

This refers to the resistance set up due to shear stresses within the fluid particles and the shear stresses between the fluid particles and the solid surface for a fluid flowing around a solid body.As fluid exerts a shear stress on the boundary,the boundary exerts an equal and opposite force on the fluid called shear resistance. Drag coefficient always depends on the hydrodynamic Reynolds number ($R_{e)}$ and the shape of the body. The work done against the viscous effects usually causes fluid to flow,consequently the energy spent in causing the fluid to flow is converted to heat energy.

1.3.8 Boundary layer

The concept of boundary layer was first introduced by Prandtl and since then it has been applied to several fluid flow problems. The fluid layer in the neighborhood of the solid boundary where effects of fluid friction (viscous effects) are predominant is known as the boundary layer. Boundary layers are thin fluid layers adjacent to the surface of a body or solid wall in which strong viscous effects exist. Flow outside this layer is considered frictionless. The velocity near the boundary is affected by boundary shear stress. At low Reynolds number, viscous forces dominate over the inertial forces.

1.3.9 Velocity Boundary Layer

Velocity boundary layer arises as a result of the velocity difference between the fluid particles adjacent to a solid surface and those in the free stream. The fluid particles adjacent to the solid surface acquire the velocity of that surface due to the assumption of no-slip condition. The latter is a physical requirement that the fluid and solid have equal velocities at their interface. Thus the flow velocity of a fluid is retarded by a fixed solid surface, and a finite, slow-moving boundary layer is formed. For a viscous fluid, velocity boundary layer thickness is defined as the perpendicular distance, measured away from the solid surface, where the velocity of the fluid becomes 0.99 of the free stream velocity. As the fluid moves past the surface of the object, collisions of the fluid molecules within the fluid with those molecules touching the object's surface reduce the kinetic energy of the molecules that are farther away from the solid-fluid interface. Thus a relatively thin layer of fluid is formed near the solid-fluid interface in which there is a rapid change of velocity from zero to the free stream value. This is the layer referred to as the velocity boundary layer.

1.3.10 Thermal Boundary Layer

When temperature difference exists between the solid-fluid interface and the fluid in the free stream, a thermal boundary layer is formed. The fluid particles in contact with the solid- fluid interface acquire the temperature of the interface. If the temperature of the interface is higher than that of the ambient fluid, the kinetic energy of the molecules of the adjacent fluid particles increases. These particles in turn exchange the acquired kinetic energy with those fluid particles in the adjacent fluid layers further away from the interface. This process continues in the adjacent fluid layers and temperature gradients develop in the fluid.

1.3.11 Lift and Drag

The sum of all the forces on a body that acts perpendicularly to the direction of flow is referred to as lift. This force occurs when fluid moves over a stationary solid body. On the other hand, drag is the force parallel and in opposition to the direction of motion of an object moving in the fluid. Drag takes two forms; form drag or pressure drag which is dependent on the shape of the object moving in the fluid and the other form is skin friction which is dependent on the viscous friction between a surface of a moving solid body and a fluid.

1.4 Literature review

The first quantitative observation relating to magnetic and electric fields were made by Faraday (1831) in experiments on the behavior of current in circuits placed in transverse magnetic fields. In this experiment with mercury as the conducting fluid flowing in a glass tube placed in a magnetic field, he observed that a voltage was induced in a direction perpendicular to both the direction of the flow and magnetic field.

Riche (1832) studied analysis of Poiseuille flow of a reactive power law fluid between two parallel plates and discovered that when an electric field is applied to conducting fluid in a direction perpendicular to a magnetic field a force is exerted on the fluid in a direction perpendicular to both the electric field and magnetic field.

Maurice (1843) studied the analysis of boundary conditions of a fluid and showed that the no slip condition was satisfied for the fluids and wall materials tested .He came up with the famous couette flow which refers to the laminar flow of a viscous fluid in the space between parallel plates, one of which is moving relative to the other. The flow is driven by virtue of viscous drag force acting on the fluid and applied pressure gradient parallel to the plates.

He found out that the flow is due to the relative motion of the surfaces and its velocity varies linearly with distance perpendicular to the surface.

Alboussiere (1869). also studied the concept of motion of fluids between two parallel plates by introducing the method of measuring blood pressure. He studied the flow of liquids through tubes and found that the rate of flow depended on the diameter and length of the tube and pressure difference between the ends.

Alfven (1942) studied unsteady hydro-magnetic fluid flow between two parallel plates where he established transverse waves in electrically conducting fluid and explained other astrophysical phenomenon in relation to transverse waves.

Katagiri (1962) investigated unsteady hydro-magnetic coquette flow of a viscous, incompressible and electrically conducting fluid under the influence of a uniform transverse magnetic field when the fluid flow within the channel is induced due to impulsive movement of one of the plates of the channel. He found out that for the case of impulsive as well as accelerated motion of one of the plates, the magnitude of the shear stress component due to the primary flow decrease whereas that due to the secondary flow decrease with increase in hall parameter.

Muhuri (1963) studied unsteady hydro-magnetic couette flow of a viscous, incompressible and electrically conducting fluid between two infinitely long parallel porous plates, taking Hall current into account, in the presence of a transverse magnetic field. Fluid flow within the channel is induced due to impulsive movement of the lower plate of the channel Uniform magnetic fields is in the direction orthogonal to the permeable plates, uniform suction and injection through the plates are applied. He found that when the magnetic field is considered to be fixed relative to the plate, the flow is accelerated by the magnetic field. However, the magnetic field retards the fluid flow when the magnetic field is fixed relative to the flow.

Jain (1968) concentrated on the effect of wall porosity on the stability of hydro-magnetic flow between parallel plates under transverse magnetic field and expressed the idea that the flow is largely influenced by porosity and the flow parameters such as velocity, pressure and temperature.

Singh (1970) investigated unsteady MHD Couette flow of a viscous, incompressible and electrically conducting fluid near an accelerated plate of the channel under constant magnetic field. He compared the unsteady free convection coquette flow at large values of time with the corresponding steady-state problem and found that they are in good agreement. It was also observed that the flow velocity decreases with increasing Prandtl number.

Apere (2000) considered the unsteady MHD free convection coquette flow between two vertical parallel porous plates with uniform suction and injection. The magnetic field is considered fixed relative to the fluid and fixed relative to the moving plate were considered. The velocity and temperature distributions were obtained using the Laplace transform technique. The results revealed that both temperature and velocity decrease with increasing Prandtl number and with increasing suction/injection parameter. The velocity was also found to increase with increasing Grashof number.

Kim (2010) studied unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. The radiative heat flux was described using the Rossel and approximation, the temperature and suction velocity at the plate were taken to be time-dependent. The velocity and temperature distributions were obtained using an asymptotic expansion of velocity for small magnetic number, as well as a similarity transformation. The results showed that a decrease in both velocity and temperature with increasing radiation and suction parameter.

Chauhan and Agrawal (2010) studied Hall effects on MHD slip flow and heat transfer through a porous medium over an accelerated plate in a rotating system and found that interplay of Coriolis force and hydro-magnetic force in the presence of boundary slip and Hall current plays an important role in characterizing the flow behavior.

Seth (2011) studied the problem considered when the fluid flow is confined to porous boundaries with suction and injection considering two cases of interest namely the impulsive movement of the lower plate and the uniformly accelerated movement of the lower plate .Seth concluded that the suction exerted a retarding influence on the flow while the magnetic field, time and injection reduce shear stress at lower plate in both the cases while suction increases shear stress at the lower plate.

Kumar (2012) studied on MHD flow and heat transfer along a porous flat plate with mass transfer and found that the fluid velocity component increased with an increasing value of time and Hall parameter, but decreases owing with an increasing value of transpiration parameter and magnetic field parameter.

Victor (2013) studied unsteady MHD free convection couette flow between two vertical permeable plates in the presence of thermal radiation using galerkin's finite element method. It was found that the radiation parameter and Prandtl number have a greater effect on the temperature than on the velocity. On the other hand, the magnetic parameter and Grash of number have no effect on the fluid temperature.

Venkateswarlu (2013) studied hydro-magnetic unsteady MHD flow of an incompressible, electrically conducting, and viscous fluid past an infinite vertical porous plate along with porous medium of time dependent permeability under oscillatory suction velocity normal to the plate. He found that the velocity of the fluid decreases with the increase of magnetic parameter values and an increase in Prandtl number caused a decrease in Temperature profile.

Gunakala (2014) investigated unsteady MHD couette flow between two infinite parallel porous plates in an inclined magnetic field with heat transfer. The lower plate was considered to be porous and stationary. He found out that an increase in the magnetic number lead to a decrease in the velocity of the fluid.

1.5 Problem statement

This study has considered the analysis of the unsteady hydro-magnetic fluid flow between two parallel plates in presence of variable magnetic fields applied perpendicularly to the top moving plate which is porous and has a constant suction. Initially (when time $t \leq 0$) both the plates are stationary and when time $t \geq 0$ the upper plates starts moving in the opposite direction of the main flow while the lower plate remain stationary. The fluid in consideration is unsteady, viscous, incompressible and electrically conducting between two parallel plates located at a distance y=-h and y=h. The plates are of infinite length in both the x and z directions and the variable transverse magnetic fields are applied parallel to y-axis and the upper porous plate has a constant suction as show in figure 1.1 below.

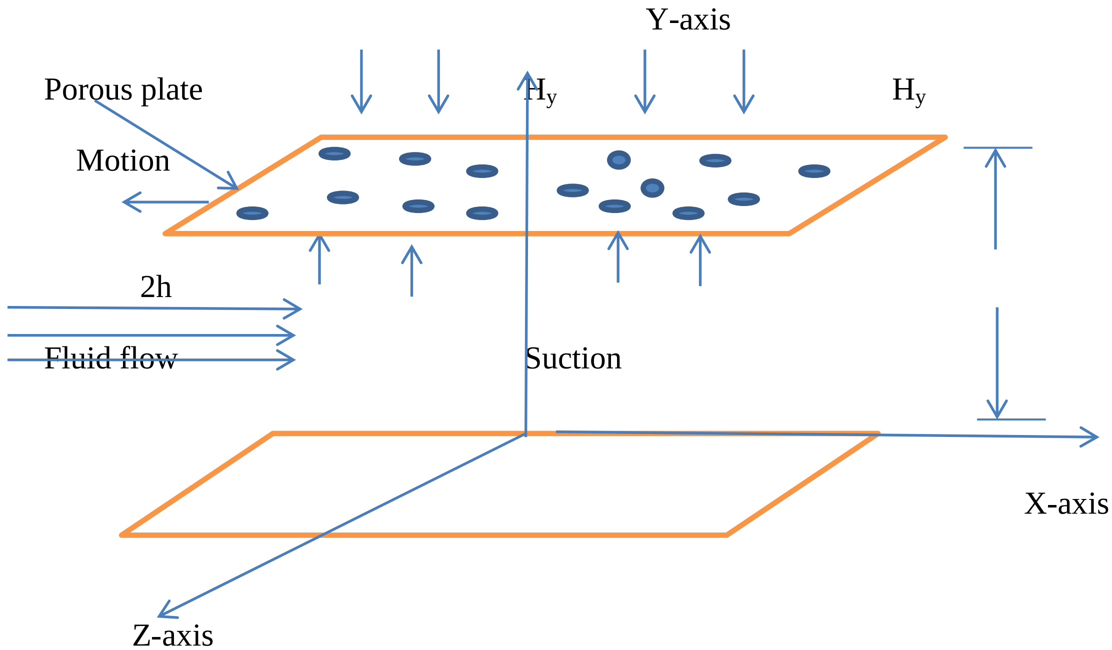

Figure 1.1: Flow configuration

1.6 Justification of the study

The experimental and theoretical research on MHD flows is important to scientific and engineering fields. In particular the influence of a magnetic field on a viscous, incompressible flow of an electrically conducting fluid is encountered in engineering devices such as MHD generators, MHD fluid dynamos, flow meters, heat exchangers and pipes that connect system components. The study of MHD flow through porous media is of fundamental importance in a wide range of disciplines, including natural sciences and technology. MHD flow finds more applications in horticulture and hydrogeology in dealing with ’seepage’ problems in rock mass, sand beds and subterranean aquifers. The applications of MHD in engineering structures such as flow of liquid metals, extrusion of plastics in the manufacture of rayon and nylon, cooling of nuclear reactors, electromagnetic casting, behavior of plasma in fusion reactors, cooling of moving parts in automobile engines, MHD electric current generators and dyeing industry gives this study a practical frame work.

There is need to carry out a study on unsteady hydro-magnetic fluid flow between two parallel plates in presence of variable magnetic fields applied perpendicularly to the top moving plate which is porous and has a constant suction as this will find major applications in dyeing industries, engineering and many other scientific fields useful to the welfare of mankind.

1.7 Null Hypothesis

The applied variable transverse magnetic field has no effect on both the velocity and temperature profiles.

1.8 Objectives of the study

1.8.1 General objective

To determine the effect of applying variable transverse magnetic fields to fluid flow between parallel plates where the upper plate is porous with a constant suction.

1.8.2 Specific objectives

i). To determine the velocity profiles of the flow between the horizontal plates.

ii). To determine the temperature profiles of the flow between the horizontal plates.

iii). To determine the effects of varying Magnetic number M, Reynolds number Re, Pranstl number Pr, Pressure number, Suction parameter So and Eckert number Ec on the velocity and temperature distributions.

The governing equations for the unsteady hydro-magnetic fluid flow between two parallel plates in presence of variable magnetic fields applied normal to the top moving plate which is porous includes the equation of conservation of mass, the equation of momentum, the equation of conservation of energy and the Maxwell's equations are outlined in the next chapter.

CHAPTER TWO

GOVERNING EQUATIONS

2.1 Overview

This chapter has outlined the assumptions and the equations that govern magneto-hydrodynamic fluid flow between the two parallel plates which includes continuity equation, momentum equation, Maxwell's equations and the energy equation. Towards the end of the chapter, non- dimensionalization process is also stated.

2.2 Assumptions

To simplify the equations governing the fluid flow in this study, the following assumptions was made;

i). The fluid is incompressible.

ii). Thermo conductivity, electrical conductivity and coefficient of viscosity are constants.

iii). The flow is unsteady and the plates are of infinite length in x and z directions.

iv). The no-slip condition is satisfied.

v). The flow is two-dimensional.

vi). The fluid does not undergo any chemical change.

vii). The force due to electric field is negligible as compared to Lorenz force due to magnetic force.

viii). The pressure gradient dp/dx is assumed to be a constant.

2.3 Governing equations of the flow

2.3.1 Equation of conservation of mass

This is also referred to as the equation of continuity. The law of conservation of mass states that under normal conditions mass can neither be created nor destroyed. It can also be defined in a mathematical statement as any process where the rate at which mass enters a system is equal to the rate at which mass leaves the system. This implies that inflow into the control volume equals outflow. For an unsteady fluid flow, the tensor form of the equation of continuity is given by;

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho . u_i)}{\partial x_i} = 0 \qquad (2.1a)$$

Where i =1, 2, 3 represent the x, y and z directions respectively. The flow is between two non -conducting horizontal plates with the plates being of infinite length in the x and z directions making the derivative with respect to x and z negligible. Equation (2.1) will therefore reduce to;

$$\frac{\partial \rho}{\partial t} + \frac{\rho \partial v}{\partial y} = 0 \qquad (2.1b)$$

For an incompressible fluid flow density of the fluid is a constant and by differentiating the first term in equation (2.1b) with respect to time is equals to zero and therefore equation (2.1b) reduces to;

$$\frac{\partial v}{\partial y} = 0 \qquad (2.2)$$

The integral of the derivative in equation (2.2) is equals to a constant Vo. This constant represents the suction velocity through the upper porous plate.

2.3.2 Electromagnetic equations

These are the basic equations in electricity and magnetism and they give the relations between the interacting electric and magnetic fields. The electromagnetic equations gives the relationship between the electric field (**E**), the magnetic induction vector (**B**), magnetic field intensity (**H**), the electric displacement **(D)** and induction current density (**J**) in accordance with Griffiths.

The following are the basic Electromagnetic equations;

a) Gauss' law for electricity

This law states that net flux of electric field lines out of a closed surface S is proportional net charge enclosed within the surface

$$\nabla.\mathbf{B} = \mathbf{O} \quad (2.3)$$

b) Faraday's law of induction

The law states that changing magnetic fields produces an electric field. The electromagnetic force induced in a circuit is equal to the rate of change with time of the total magnetic flux through the circuit no matter how the flux changes.

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial \mathbf{t}} \quad (2.4)$$

Where $\mathbf{B} = \mu_e \mathbf{H}$ (2.5)

c) Ampere's law;

The law states that for a constant current flow, flux of the electric current through a surface is proportional to the line integral of the magnetic field (counterclockwise) around its boundary. Ampere's Law states that for any closed loop path, the sum of the length elements times the magnetic field in the direction of the length element is equal to the permeability times the electric current enclosed in the loop.

$$\sum \mathbf{B} \partial L = \mu_0 I \quad (2.6)$$

The magnetic field strength is given by;

$$\mathbf{H} = \frac{\mathbf{B_O}}{\mu_e} \quad (2.7)$$

2.3.3 Lorentz's force law

The law defines the total force resulting from both the electric and magnetic fields.

$$\mathbf{F} = \mathbf{F_e} + \mathbf{F_m} = Q\mathbf{E} + Q\mathbf{v} \times \mathbf{B} \tag{2.8}$$

Where Q is the test charge placed in an electric field $\mathbf{E}$, $\mathbf{F}e = Q\mathbf{E}$ is the electric force and $\mathbf{F}_m = Q\mathbf{v} \times \mathbf{B}$ Which is the magnetic force experienced by the test charge.

2.3.4 Charge conservation

The principle of charge conservation states that under normal conditions, charge is conserved and thus cannot be created nor destroyed.

$$\nabla.\mathbf{J} = \frac{\partial \rho}{\partial \mathbf{t}} \tag{2.9}$$

Where from the generalized Ohm's law,

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{u} \times \mathbf{B}) \tag{2.10}$$

2.3.5 Equations of conservation of momentum

The equation of conservation of momentum is derived from the Newton's second law of motion, which states that, the time rate of change of momentum of a body is equal to the external force applied to the body. This external force is divided into two types of forces i.e. surface forces (e.g. forces due to static pressure and viscous stresses) and body forces (e.g. gravitational force, centrifugal force, magnetic force or electric fields).

The surface forces are due to the interaction between the body and the matter in the immediate contact with it and act on the bounding surfaces. These intensities are expressed in terms of stress and defined as force per unit area.

The body forces are defined as the forces which act on a body from a distance and are usually expressed as forces per unit mass;

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}(\nabla.\mathbf{u}) = -\frac{1}{\rho}\nabla P + \frac{\mu}{\rho}\nabla^2 \mathbf{u} + \mathbf{F}_e \qquad (2.11)$$

The first term is the temporal acceleration while the second term is the convective acceleration. On the right hand side, the first term is the pressure gradient, second term is the force due to viscosity and third term is the body force. The electromagnetic force $\mathbf{F}e$ can be expressed as $\mathbf{F}e = \mathbf{J} \times \mathbf{B}$ from equation (2.8) above, Equation (2.11) can thus be written as;

$$\frac{\partial \mathbf{q}}{\partial t} + \mathbf{u}(\nabla.\mathbf{u}) = -\frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\nabla^2 \mathbf{u} + \frac{1}{\rho}(\mathbf{J} \times \mathbf{B}) \qquad (2.12)$$

From the flow in consideration, all quantities except for the pressure gradient $\frac{dP}{dx}$,

which is assumed to be a constant, do not depend on the x and z coordinates. The velocity vector of the fluid is $\mathbf{u} = \mathbf{u}(y,t)$ Considering equation (2.10)

$\mathbf{J} = \sigma(\mathbf{u} \times \mathbf{B})$ where the electric fields **E** is assumed to be negligible as indicated in assumption (vii) above, we have;

$$\mathbf{J} = \mathbf{u} \times \mathbf{B} = \sigma \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u & 0 & 0 \\ 0 & B_y & 0 \end{vmatrix} = \sigma u \mathbf{B}_y \mathbf{k} \qquad (2.13)$$

$$\mathbf{J} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \sigma u \mathbf{B}_y \\ 0 & B_y & 0 \end{vmatrix} = -\sigma B_o{}^2 u i \qquad (2.14)$$

Since $B_0 = \mu_e H_0$ then,

$$\mathbf{J} \times \mathbf{B} = -\sigma \mu_e^{\,2} H_o^{\,2} u\mathbf{i} \tag{2.15}$$

Since $\mathbf{q} = u(y,t)$ the viscous term $\frac{\mu}{\rho}\nabla^2 u$ takes the form

$$\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}. \tag{2.16}$$

Substituting equations (2.16) and (2.15) in the momentum equation (2.12) we have;

$$\frac{\partial u}{\partial t} + v_0 \frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dP}{dx} + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho}\sigma \mu_e^{\,2} H_o^{\,2} u \tag{2.17}$$

2.3.6 Equation of conservation of energy

The equation of conservation of energy is derived from the First Law of Thermodynamics. It states that energy cannot be created nor destroyed under normal conditions but can be transformed from one form to another. For the flow of an incompressible fluid with constant fluid conductivity k, the energy equation is given by;

$$\rho C_p \frac{DT}{Dt} = k\nabla^2 T + \mu\phi \tag{2.18}$$

For an incompressible two-dimensional fluid flow

$$\phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 \tag{2.19}$$

Which reduces to, $\phi = \left(\frac{\partial u}{\partial y}\right)^2$ since the plates are of infinite length along both x and z directions and thus $\frac{\partial u}{\partial x}$ =0 and $\frac{\partial v}{\partial x} = 0$.

The term $\frac{dv}{dy} = 0$ from the continuity equation (2.2), considering Ohmic heating $\left(\frac{J^2}{\sigma}\right)$ due to electrical resistance of the fluid, equation (2.18) becomes

$$\rho C_p\left[\left(\frac{\partial T}{\partial t}\right) + (\mathbf{u}.\nabla)T\right] = k\nabla^2 T + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{J^2}{\sigma}\right) \tag{2.20}$$

The term $(\mathbf{u}.\nabla)T$ in equation (2.20) simplifies to;

$$(\mathbf{u}.\nabla)T = u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \tag{2.21}$$

From continuity equation (2.2), v=Vo, and $\frac{\partial T}{\partial x} = 0$ equation (2.20) reduces to;

$$\rho c_p\left[\left(\frac{\partial T}{\partial t}\right) + V_o\frac{\partial T}{\partial y}\right] = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{J^2}{\sigma}\right) \tag{2.22}$$

From Ohm's law,

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{u}\times\mathbf{B}) \tag{2.23}$$

Since there is no voltage applied externally then **E**=0. Therefore equation (2.23) reduces to;

$$\mathbf{J} = \sigma(\mathbf{u}\times\mathbf{B}) \tag{2.24}$$

The term $\mathbf{u}\times\mathbf{B}$ of equation (2.24) reduces to;

$$\mathbf{u}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u & 0 & 0 \\ 0 & B_y & 0 \end{vmatrix} = u\mathbf{B}_y\mathbf{k} \tag{2.25}$$

Therefore equation (2.24) reduces to;

$$\mathbf{J} = \sigma u B_y \mathbf{k} \tag{2.26}$$

The joules heating term in equation (2.22) simplifies to;

$$\frac{\mathbf{J}^2}{\sigma} = \sigma \mathbf{B}_y^2 u^2 \tag{2.27}$$

But, $\mathbf{B}_y^2 = \mu_e^2 H_y^2$ thus equation (2.27) reduces to;

$$\frac{\mathbf{J}^2}{\sigma} = \sigma \mu_e^2 H_y^2 u^2 \tag{2.28}$$

Substituting equation (2.28) in equation (2.22) yields;

$$\rho c_p \left[\left(\frac{\partial T}{\partial t} \right) + V_o \frac{\partial T}{\partial y} \right] = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 + \sigma \mu_e^2 H_y^2 u^2 \tag{2.29}$$

The initial and boundary conditions of this problem are:

$$t \le 0: \quad u = 0, \quad T = 0 \quad at \quad -h \le y \ge h$$

$$t > 0: \quad u = 0, \quad T = T_w \quad at \quad y = -h$$

$$t > 0: \quad u = -u_\infty \quad T = T_\infty \quad at \quad y = h$$

2.4 Non-dimensionalization

Non- dimensionalization of the equations governing a particular fluid flow falls under a broad area of study known as dimensional analysis. Dimensional analysis is a method which describes a natural phenomenon by a dimensionally correct equation with certain variables which affect the phenomenon. Dimensional analysis considers how to determine the required set of scales for any given problem. It is a process that starts with selecting a suitable scale against which all dimensions in a given physical model are based.

Non-dimensionalization is basically aimed at ensuring that the results are applicable to other geometrically similar configurations under a similar set of flow conditions. The following non-dimensional transformations have been used:

$$X = LX^* \qquad Y = LY^* \qquad u = Uu^* \tag{2.30}$$

$$P = Pp^*, \qquad t = Tt^* \qquad T^* = \frac{T - T_W}{T} \tag{2.31}$$

2.4.1 Hydrodynamic Reynolds number (Re)

This is a non-dimensional parameter which is defined as the ratio of inertial force to viscous force. It gives the relative significance of inertial force to viscous force in a fluid flow problem. Small Reynolds number corresponds to slow viscous flow where frictional forces are dominant. When Reynolds number increases, a flow are characterized by rapid regions of velocity variation and the occurrence of vortices and turbulence .Reynolds number is expressed as;

$$R_e = \frac{\rho UL}{\mu} \tag{2.32}$$

2.4.2 Prandtl Number (Pr)

Prandtl number (Pr) gives the ratio of the velocity boundary layer thickness and the thermal boundary layer thickness. If the Prandtl number is 1, the two boundary layers are of the same thickness. If the Prandtl number is greater than 1, the thermal boundary layer is thinner than the velocity boundary layer. If the Prandtl number is less than 1, which is the case for air at standard conditions, the thermal boundary layer is thicker than the velocity boundary layer. This non dimensional parameter is a property of the fluid, not of particular flow. Hence, there is a restriction on the transfer of information from experiments with one fluid to those with another. It can also be defined as an approximation of the ratio of momentum diffusivity and thermal of viscous force to the thermal force expressed as;

$$P_r = \frac{C_p \mu}{K} \tag{2.33}$$

2.4.3 The Eckert number (EC)

It is the ratio of kinetic energy of the flow to the thermal energy. It represents the conversion of kinetic energy into internal energy by the work that is done against the viscous fluid stresses. It has been deduced that a positive Eckert number implies loss of heat from the plate to the fluid .This number is expressed as;

$$E_C = \frac{U_\infty^2}{C_p \nabla T} \tag{2.34}$$

2.4.4 Joules heating parameter (R)

Joule heating refers to the ratio between amount of heat released from an electrical resistor to its resistance and the charge passed through it. When a current flows through a conductor, an increase in temperature of the conductor occurs due to its electrical resistance. This phenomenon is called joule heating and is named after the scientist Prescott Joule having been the first scientist to establish Joule's law.This non-dimensional parameter is expressed as;

$$R = \frac{\sigma \mu_e^2 H_y^2}{\rho U C_p \nabla T} \tag{2.35}$$

2.4.5 Magnetic parameter (M)

This number is obtained from the ratio of electromagnetic force to the inertial force and is expressed as;

$$M = \frac{L \sigma \mu_e^2 H_y^2}{\rho U^2} \tag{2.36}$$

2.4.6 Suction Parameter (S_0)

It is expressed as a ratio between the velocity through the network of pores to the main stream velocity.

$$S_O = \frac{V_O}{U} \tag{2.37}$$

In order to transform the equations of momentum and energy into their respective non-dimensional form, the following analysis is carried out:

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial u^*}\frac{\partial u^*}{\partial t^*}\frac{\partial t^*}{\partial t} = \frac{U}{T}\frac{\partial u}{\partial t} \tag{2.38}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial u^*}\frac{\partial u^*}{\partial y^*}\frac{\partial y^*}{\partial y} = \frac{U}{L}\frac{\partial u}{\partial y} \tag{2.39}$$

$$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial p^*}\frac{\partial p^*}{\partial x^*}\frac{\partial x^*}{\partial x} = \frac{P}{L}\frac{\partial P}{\partial x} \tag{2.40}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y^*}\left(\frac{U}{L}\frac{\partial u}{\partial y}\right)\frac{\partial y^*}{\partial y} = \frac{U}{L^2}\frac{\partial^2 u}{\partial y^2} \tag{2.41}$$

$$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial T^*}\frac{\partial T^*}{\partial t^*}\frac{\partial t^*}{\partial t} = (T_w - T_\infty)\frac{U}{T}\frac{\partial T}{\partial t} \tag{2.42}$$

$$\frac{\partial T}{\partial y} = \frac{\partial T}{\partial T^*}\frac{\partial T^*}{\partial y^*}\frac{\partial y^*}{\partial y} = (T_w - T_\infty)\frac{U}{L}\frac{\partial T}{\partial y} \tag{2.43}$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{\partial}{\partial y^*}\left((T_w - T_\infty)\frac{U}{L}\frac{\partial T}{\partial y}\right)\frac{\partial y^*}{\partial y} = (T_w - T_\infty)\frac{1}{L^2}\frac{\partial^2 T}{\partial y^2} \tag{2.44}$$

$$\left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial u^*}\frac{\partial u^*}{\partial y^*}\frac{\partial y^*}{\partial y}\right)^2 = \frac{U^2}{L^2}\left(\frac{\partial u}{\partial y}\right)^2 \tag{2.45}$$

Substituting equations (2.37) - (2.45) in equations (2.17) and (2.29) and dividing equation (2.17) of the resulting terms by $\frac{U^2}{L}$, and dividing equation (2.29) of the resulting terms by $\frac{U\nabla T}{L}$ the two equations of momentum and energy respectively becomes;

$$\frac{\partial u}{\partial t} + \frac{V_0}{U}\frac{\partial u}{\partial y} = -\frac{dP}{dx} + \frac{\mu U}{\rho L^2}\frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho}\frac{L}{U^2}\sigma\mu_e^2 H_y^2 u \tag{2.46}$$

$$\frac{\partial T}{\partial t} + \frac{V_0}{U}\frac{\partial T}{\partial y} = \frac{K}{\rho ULC_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu U}{\rho ULC_p \nabla T}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma\mu_e^2 H_y^2}{\rho UC_p \nabla T}u^2 \tag{2.47}$$

Substituting equations (2.32) - (2.35) in equation (2.46) and (2.47) leads to the final form of the momentum equation and energy equation respectively as follows;

$$\frac{\partial u}{\partial t} + S_0\frac{\partial u}{\partial y} = -\frac{dP}{dx} + \frac{1}{R_e}\frac{\partial^2 u}{\partial y^2} - Mu \tag{2.48}$$

$$\frac{\partial T}{\partial t} + S_0\frac{\partial T}{\partial y} = \frac{1}{R_e P_r}\frac{\partial^2 T}{\partial y^2} + \frac{E_c}{R_e}\left(\frac{\partial u}{\partial y}\right)^2 + Ru^2 \tag{2.49}$$

2.5 Initial and boundary conditions

The initial and boundary conditions of equations (2.47) and (2.48) appear as follows when transformed to their equivalent non-dimensional form:

$t^* \le 0 \quad u^* = 0,\ T^* = o$ At $-L \le y^* \le L$

$t^* > 0 \quad u^* = -1,\ T^* = 0$ At $y^* = L$

$t^* > 0 \quad u^* = 0,\ T^* = 1$ At $y^* = -L$

In the next chapter, the method of solution is outlined and the governing equations are presented in their finite difference forms. The final set of the governing equations of the flow in finite difference form are then implemented in a MATLAB version 7.9.0(R2009b) computer program and the results generated.

CHAPTER THREE

METHODOLOGY

In this chapter, the method of solution is discussed and the governing equations are presented in their finite difference forms. The equations are solved using the numerical method called finite difference method since the equations governing the flow are non-linear in nature and cannot be solved by analytical methods. The final set of the equations are presented in this chapter and later on implemented in a MATLAB version 7.9.0(R2009b) computer program which will generate the results in graphical form.

3.1 Finite difference method

The finite difference approximation method for derivatives is one of the methods used to solve differential equations. The principle of finite difference methods is close to the numerical schemes used to solve ordinary and partial differential equations. It consists in approximating the differential operator by replacing the derivatives in the equation using difference quotients. The domain is partitioned in space and in time and approximations of the solution are computed at the space or time points. Equations (2.48) and (2.49) are non-linear hence cannot be solved analytically. Therefore the finite difference method is used in their solution subject to the initial conditions.

The finite difference approximations of the partial derivatives appearing in equations (2.48) and $(2.49)$are obtained by performing Taylor series expansion of the dependent variable and substituting the truncated expressions into the differential equation. The differentials are approximated by differences in the solution at various points.

By definition,

$$\frac{\partial u}{\partial t} = u_t = \lim_{\Delta y \to 0} \frac{u(y+\Delta y) - u(\Delta y)}{\Delta y} \qquad (3.1)$$

When Δy is small, this formula can be used as an approximation for the derivative of u at y. From Taylor series

$$u(y+\Delta y)=u(y)+\Delta y u_y(y)+\frac{\Delta y^2}{2}u_{yy}+HOT \qquad (3.2)$$

By rearrangement

$$\frac{u(y+\Delta y)-u(y)}{\Delta y}=u_y(y)+\frac{\Delta y}{2}u_{yy}+HOT \qquad (3.3)$$

If Δy is small, the higher order terms in the expansion will be very small values and so it is possible to write equation (3.3) as;

$$u_y(y)=\frac{u(y+\Delta y)-u(y)}{\Delta y}+0(\Delta y) \qquad (3.4)$$

From equation (3.4), the leading term of the error in approximating u_y by the right hand side is of order Δy and so this represents a first order approximation. It is possible to define other difference formula to approximate derivatives and these may have different orders of accuracy.

The above analysis deals with the continuous solution however the objective is to calculate u at a set of discrete points on the mesh, and this is the numerical solution. The numerical solution of equations (2.48) and (2.49) will be approximated at a discrete number of points arranged to form a rectangular grid.

3.2 Definition of the mesh

In a finite difference grid to calculate the values at the mesh points, each nodal point is identified by a double index (j,k) that defines its location with respect to y and t as indicated in the figure 3.1. For this particular flow problem we chose the step value Δt=0.00125 and Δy=0.05 where these step values are chosen so as to bring about convergence, stability and consistency in the values to be obtained. Each corner of the cell forms the mesh or grid point. Considering the y-t plane it is subdivided into uniform rectangular cells of height of Δy=0.05 and width of Δt=0.00125. Considering a reference point (j,k) where j and k represent distance y and time t respectively and using the notation $(k \pm 1)$ for (t ±Δt) and $(j \pm 1)$ for (y ±Δy) we define the adjacent points to y and t, the points that are k and j units from the reference point have the coordinates (jΔy, kΔt) as shown figure 3.1 below.

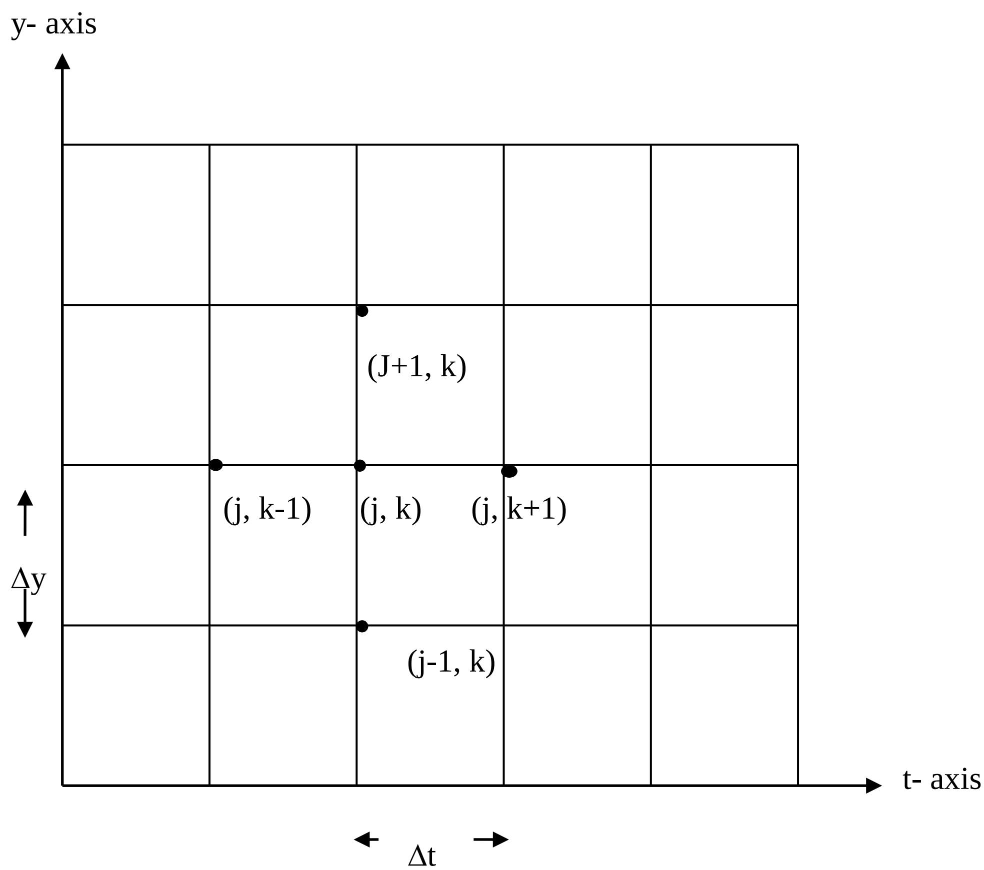

Figure 3.1 : Mesh configuration

The partial derivatives of $U \ and \ T$ at each grid point are expressed using finite difference approximation. U_j^k and T_j^k for each j are calculated directly from the initial value condition. Thus it is convenient to start from the known boundary at the edges of the mesh and working inwards so as to obtain U_j^{k+1} and T_j^{k+1} respectively. The derivatives are approximated using the Forward Time Backward Space finite difference scheme that averages the values velocity profiles at step $j+1$, and j . The FD expressions for U_t ,U_y and U_{yy} averaged for times are given below.

$$\frac{\partial U}{\partial t} = \frac{U_j^{k+1} - U_j^k}{\Delta t} \tag{3.5}$$

$$\frac{\partial U}{\partial y} = \frac{U_j^{k+1} - U_{j-1}^{k+1} + U_j^k - U_{j-1}^k}{2\Delta y} \tag{3.6}$$

$$\frac{\partial^2 U}{\partial y^2} = \frac{U_{j+1}^{k+1} - 2U_j^{k+1} + U_{j-1}^{k+1} + U_{j+1}^k - 2U_j^k + U_{j-1}^k}{2(\Delta y)^2} \tag{3.7}$$

Equation (3.5) represents forward time difference approximation for the partial derivative $\frac{\partial U}{\partial t}$ at U_j^{k+1} having a truncation error of order $0(\Delta t)$ that represent the neglected higher order terms.

$$\frac{\partial U}{\partial t}| j, k+1 = \frac{U_j^{k+1} - U_j^k}{\Delta t} + \text{HOT} \tag{3.8}$$

Equation (3.6) represents forward time difference approximation for the partial derivative $\frac{\partial U}{\partial y}$ at U_j^{k+1} having a truncation error of order $0(\Delta t)$ that represent the neglected higher order terms.

$$\frac{\partial U}{\partial y} = \frac{U_j^{k+1} - U_{j-1}^{k+1} + U_j^k - U_{j-1}^k}{2\Delta y} + \text{HOT} \tag{3.9}$$

Equation (3.7) represents forward time difference approximation for the partial derivative $\frac{\partial^2 U}{\partial y^2}$ at U_j^{k+1} having a truncation error of order $0(\Delta t)^2$ that represent the neglected higher order terms.

$$\frac{\partial^2 U}{\partial y^2} = \frac{U_{j+1}^{k+1} - 2U_j^{k+1} + U_{j-1}^{k+1} + U_{j+1}^{k} - 2U_j^{k} + U_{j-1}^{k}}{2(\Delta y)^2} + \text{HOT} \tag{3.10}$$

Similarly the finite difference approximation T_t, T_y and T_{yy} are:

$$\frac{\partial T}{\partial t} = \frac{T_j^{k+1} - T_j^{k}}{\Delta t} \tag{3.11}$$

$$\frac{\partial T}{\partial y} = \frac{T_j^{k+1} - T_{j-1}^{k+1} + T_j^{k} - T_{j-1}^{k}}{2\Delta y} \tag{3.12}$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{T_{j+1}^{k+1} - 2T_j^{k+1} + T_{j-1}^{k+1} + T_{j+1}^{k} - 2T_j^{k} + T_{j-1}^{k}}{2(\Delta y)^2} \tag{3.13}$$

Substituting the finite difference equations (3.5) - (3.13) in the Momentum equation (2.48) and temperature equation (2.49) above yields;

$$\frac{U_j^{k+1} - U_j^{k}}{\Delta t} + S_0 \frac{U_j^{k+1} - U_{j-1}^{k+1} + U_i^{k} - U_{j-1}^{k}}{2\Delta y} = -\frac{dP}{dx} + \frac{1}{\mathrm{R_e}} \frac{U_{j+1}^{k+1} - 2U_j^{k+1} + U_{j-1}^{k+1} + U_{j+1}^{k} - 2U_j^{k} + U_{j-1}^{k}}{2(\Delta y)^2} - MU_j^{k} \tag{3.14}$$

Multiplying equation (3.14) through by ∇t and making U_j^{k+1} results to;

$$U_j^{k+1} = (U_j^k - \frac{dP}{dx} - \Delta t S_0 \frac{U_j^{k+1} - U_{j-1}^{k+1} + U_j^k}{2\Delta\Delta}$$
$$+ \frac{\Delta t}{R_e} \frac{U_{j+1}^{k+1} - 2U_j^{k+1} + U_{j-1}^{k+1} + U_{j+1}^k - 2U_j^k + U_{j-1}^k}{2(\Delta y)^2} - \Delta t M U_j^k)$$
$$/(1 - \frac{\Delta t S_0}{2\Delta\Delta} + \frac{\Delta t}{2(\Delta y)^2 R_e})$$

(3.15)

Energy equation in finite differences

$$\frac{T_j^{k+1} - T_j^k}{\Delta t} + S_0 \frac{T_j^{k+1} - T_{j-1}^{k+1} + T_j^k - T_{j-1}^k}{2\Delta\Delta} =$$
$$\frac{1}{R_e P_r} \frac{T_{j+1}^{k+1} - 2T_j^{k+1} + T_{j-1}^{k+1} + T_{j+1}^k - 2T_j^k + T_{j-1}^k}{2(\Delta y)^2} +$$
$$\frac{E_c}{R_e} \left(\frac{U_j^{k+1} - U_{j-1}^{k+1} + U_j^k - U_{j-1}^k}{2\Delta\Delta} \right)^2 + R\left(U_j^{k+1}\right)$$

(3.16)

Making T_j^{k+1} in the equation (3.16) the subject of the equation yields;

$$T_j^{k+1} = (T_j^k + \frac{\Delta t}{R_e P_r} \frac{T_{j+1}^{k+1} - 2T_j^{k+1} + T_{j-1}^{k+1} + T_{j+1}^k - 2T_j^k + T_{j-1}^k}{2(\Delta y)^2}$$
$$+ \Delta t S_0 \frac{T_j^{k+1} - T_{j-1}^{k+1} + T_j^k - T_{j-1}^k}{2\Delta\Delta} + \frac{\Delta t E_c}{R_e} \left(\frac{U_j^{k+1} - U_{j-1}^{k+1} + U_j^k - U_{j-1}^k}{2\Delta\Delta} \right)^2$$
$$+ R\left(U_j^{k+1}\right)^2)/(1 + \frac{\Delta t S_0}{2\Delta\Delta} + \frac{\Delta t}{R_e P_r (\Delta y)^2})$$

(3.17)

The equations (3.15) and (3.17) are the final set of equations solved simultaneously using a computer code in MATLAB version 7.9.0(R2009b) computer program.

CHAPTER FOUR

RESULTS AND DISCUSSION

Equations (3.15) and (3.17) are solved using the MATLAB version 7.9.0(R2009b) computer code. The results obtained after running the code are presented graphically and later discussed after varying various parameters which includes the hydrodynamic Reynolds number, Eckert number, Suction parameter, Pressure number, Magnetic number and Pranstl number.

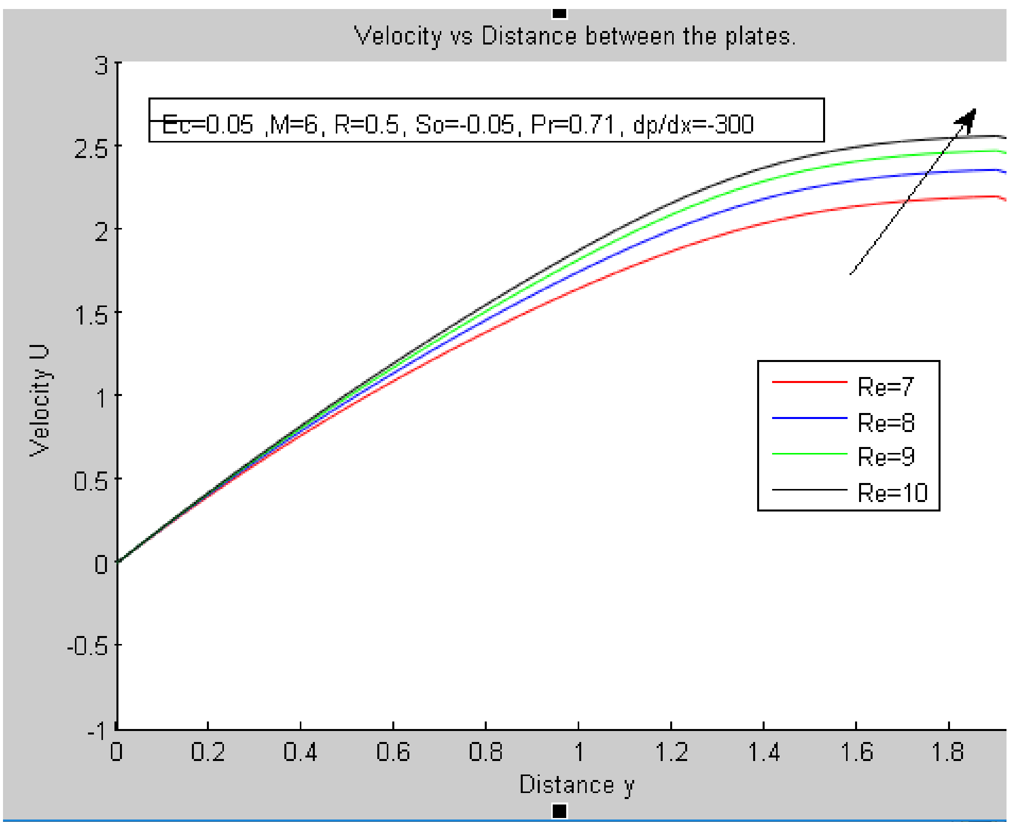

Figure 4.1: Velocity profiles for different values of hydrodynamic Reynolds number (Re).

From figure 4.1 above it is noted that that holding other parameters constant, an increase in hydrodynamic Reynolds number (Re) leads to an increase in the velocity profiles. An increase in the hydrodynamic Reynolds number will lead to a decrease in the viscous forces which is the force that opposes the motion of the fluid which leads to an increase in velocity profiles of the fluid. An increase in the inertia forces would cause an increase in the hydrodynamic Reynolds number and a decrease in the velocity profile. An increase in the hydrodynamic Reynolds number leads to an increase in the flow velocity due to the dominance of inertia forces over the viscous forces.

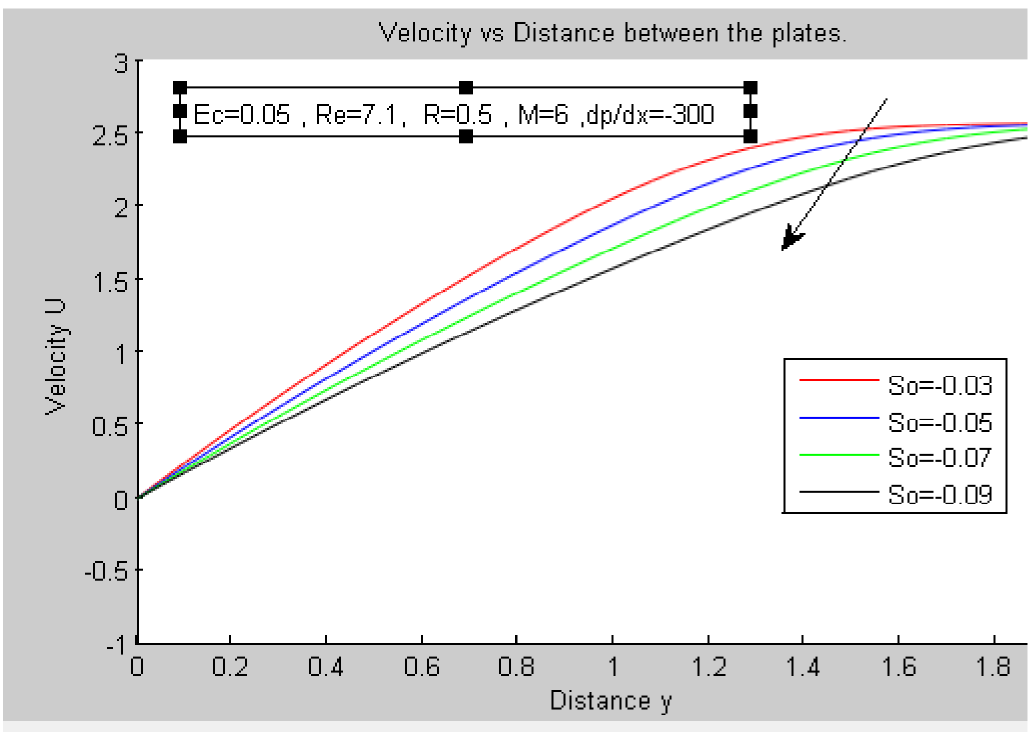

Figure 4.2: Velocity profiles for different values of So.

From figure 4.2 above it is noted that holding other parameters constant an increase in Suction parameter leads to a decrease in the velocity profile of the fluid indicating the usual fact that suction stabilizes the boundary layer growth**.**

Suction reduces the pressure of the fluid inside the conduit**.** This means that the effect of increasing suction parameter retards the fluid flow which can be attributed to the convection of the fluid across the plates.

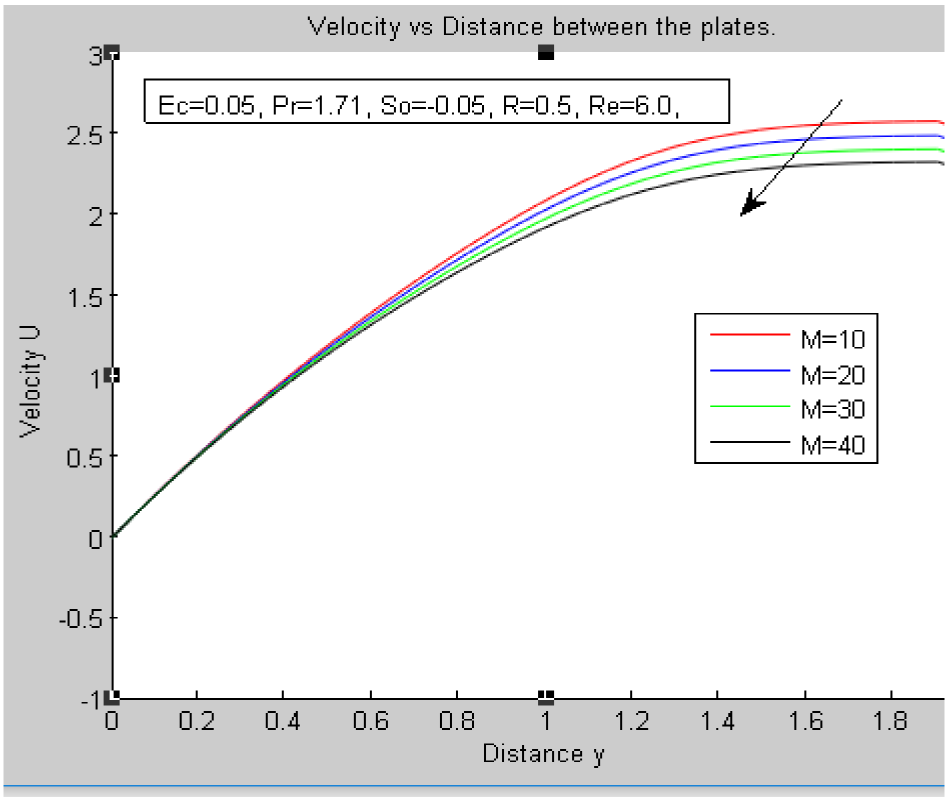

Figure 4.3: Velocity profiles for different values of Magnetic parameter (M)

Figure 4.3 above shows that holding other parameters constant an increase in magnetic parameter (M) leads to a decrease in the velocity profile of the fluid. The presence of a magnetic field in an electrically conducting fluid introduces a force called Lorentz force which acts against the flow if the magnetic field is applied in the normal direction as considered in the present problem. This type of resistive force tends to slow down the flow field.

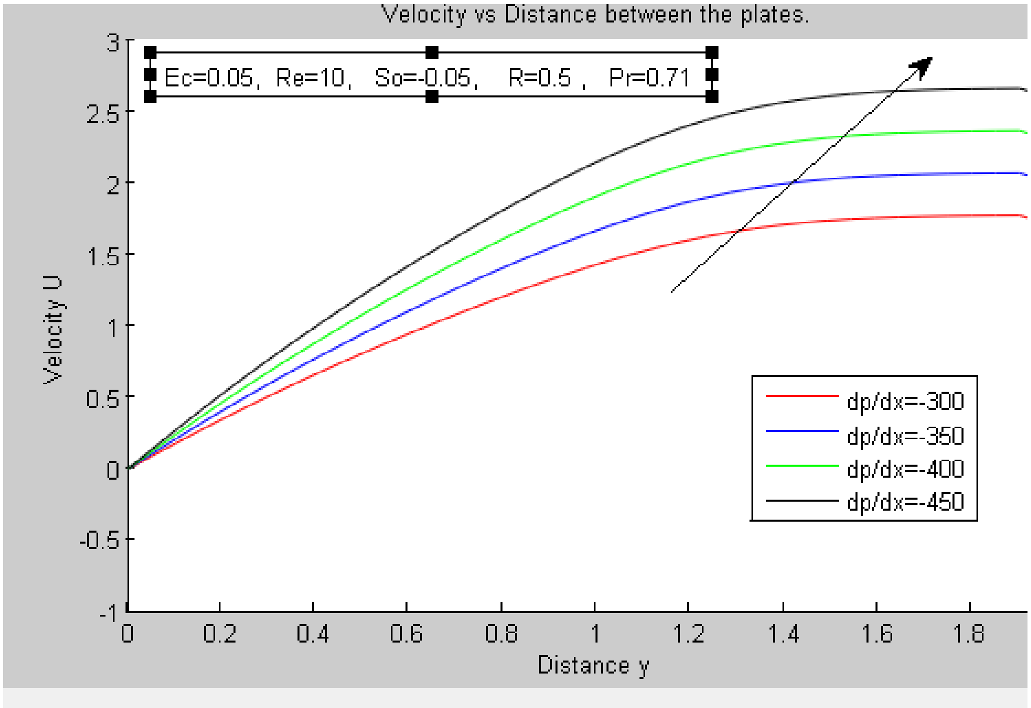

Figure 4.4: Velocity profiles for different values of Pressure gradients (dp/dx)

Figure 4.4 shows that holding other parameters fixed an increase in pressure gradient leads to an increase in the velocity profiles. When the pressure gradient is negative that is ($\frac{dp}{dx} = -300$) ,the pressure force term acts in the same direction as that of the fluid flow hence aiding the fluid flow. It is observed that, at the stationary plate the flow assumes the velocity of the pate. Velocity increases gradually as you move away from the lower plate and reach maximum at the free stream region at the center of the two plates. The velocity starts decreasing gradually as it approaches the upper porous plate and finally the fluid particles that come into contact with the upper plate assumes the velocity of that plate u=-1 due to the no- slip condition.

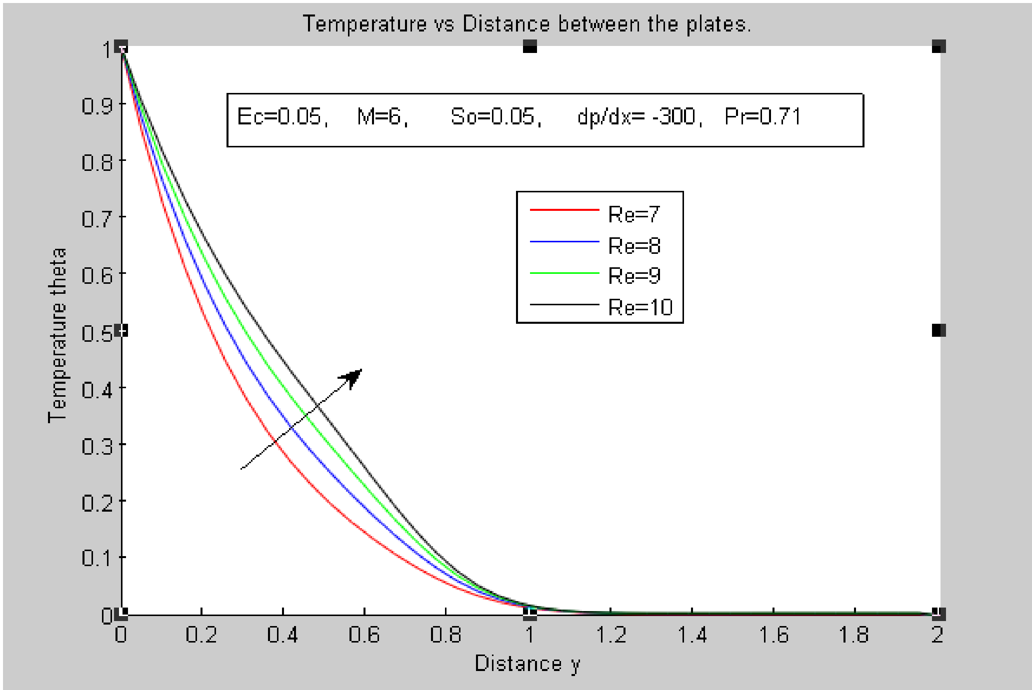

Figure 4.5:Temperature profiles for different values of hydrodynamic Reynolds number

From figure 4.5 above it is noted that keeping other parameters fixed an increase in hydrodynamic Reynolds number (Re) leads to a decrease in the temperature profiles. For an increased in hydrodynamic Reynolds number (Re), the viscous forces reduce and the boundary layer thickness reduces and this in turn reduces the dissipation of heat within the boundary layer. Hence when Re was increased, the boundary layer thickness reduced and the temperature also reduced and when Re was reduced, the boundary layer thickness increased and the temperature also increased. Hence hydrodynamic Reynolds number (Re) is inversely proportional to the boundary layer thickness and temperature.

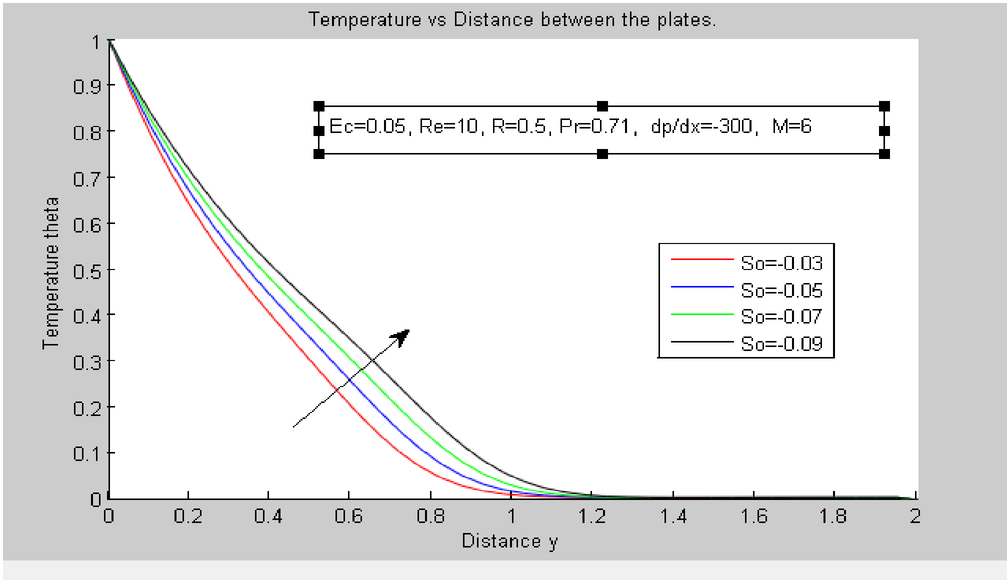

Figure 4.6: Temperature profiles for different values of suction (S_0)

Figure 4.6 above indicates that keeping other parameters fixed an increase in suction parameter (So) leads to an increase in the temperature profiles. This is due to the fact that increasing suction parameter (So) is to decrease the thermal boundary layer thickness and in turn increases the temperature gradient at the surface.

From figure 4.7 below it shows that keeping other parameters fixed an increase in Prandtl (Pr) causes a decrease in temperature profiles. This is due to the fact that a fluid with high Prandtl number has a relatively low thermal conductivity which results in the reduction of the thermal boundary layer thickness and thus a decrease in temperature with an increase in prandtl number.

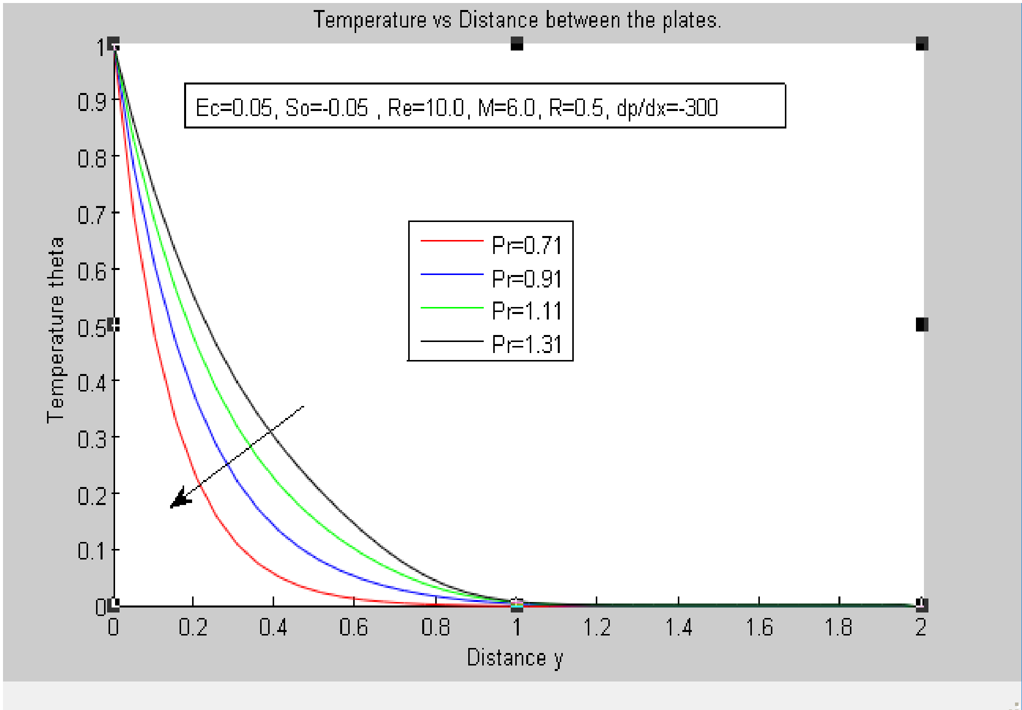

Figure4.7:Temperature profiles for different values of prandtl (Pr)

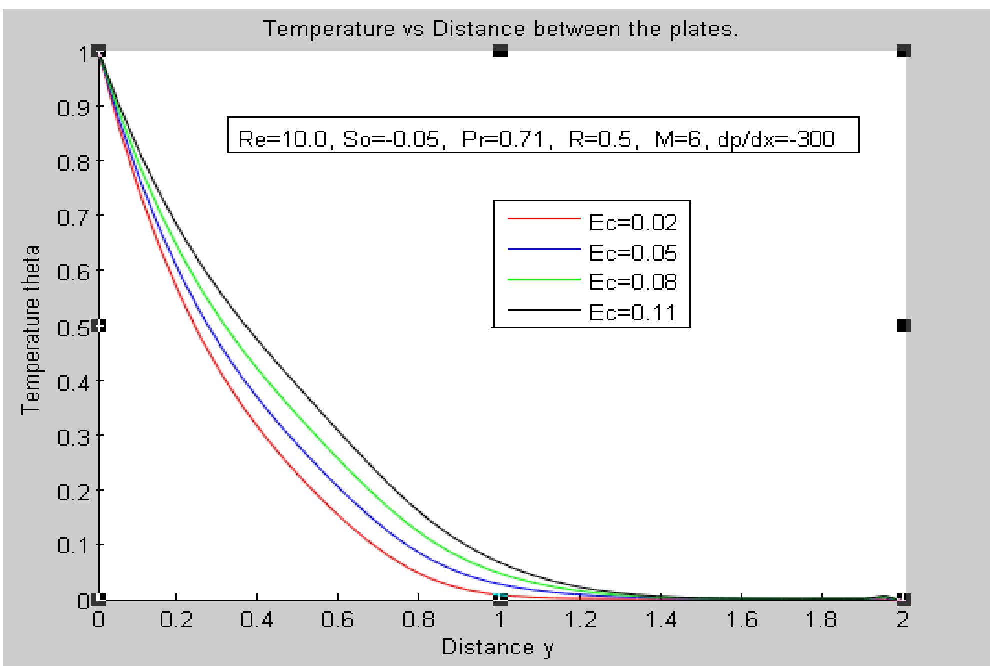

Figure 4.8: Temperature profiles for different values of Eckert (Ec)

From figure 4.8 above it is noted that keeping other parameters fixed an increase in Eckert number (Ec) leads to an increase temperature profiles. This is because for an increase in Eckert number, it implies that the kinetic energy is large and hence the velocities are higher hence when this particles attained high velocity, the vibrations also increases and this leads to increased collision of the particles. This increased collision of particles brings about dissipation of heat in the boundary layer region hence an increase in temperature profiles.

In the next chapter, a conclusion based on the results obtained and the recommendations for further research has been outlined.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

In this chapter, a conclusion based on the results obtained and the recommendations for further research are presented.

5.1 CONCLUSIONS

Magneto-hydrodynamic fluid flow between parallel plates where the upper plate is porous in presence of variable transverse magnetic field has been investigated. The direction of the applied magnetic field is considered to be normal to the direction of the flow. The PDE's governing the flow are highly non-linear and coupled, and the equations have been solved by using the finite difference method. The FD method the spatial mesh sizes used in the computations are reduced and there is no significant difference in the results obtained. Thus the scheme used in the computations is stable.

The results obtained in Chapter 4 show that the rates of heat transfer and mass transfer on the parallel plates is influenced by the Magnetic number M, hydrodynamic Reynolds number Re, Prandtl number Pr, Eckert number Ec, Pressure number and Suction parameter So. For instance the current study has shown that imposing a transverse magnetic field to a flow slows down the velocity of the fluid and decreases the temperature of the fluid. Increasing the value of Eckert number (Ec) leads to an increase temperature profiles. Increasing the value of Suction parameter leads to a decrease in the velocity profile and an increase in the temperature profiles of the fluid. Increasing the value of Prandtl (Pr) causes a decrease in temperature profiles and increasing the value of pressure numbers leads to an increase in the velocity profile of the fluid. The results obtained in this study regarding thermal and mass diffusion effects can be applied in the dyeing industry.

5.2 Validation of the results.

Magneto-hydrodynamic fluid flow between two parallel plates, the top plate being porous with a constant suction and variable magnetic field lines are fixed relative to the top moving plate has been investigated. It is found that an increase in Suction parameter leads to a decrease in the velocity profile and an increase in the temperature profiles respectively. When suction is not considered in the flow the results are similar to those of Gunakala, (2014) who found that increase in the magnetic parameter retarded the motion of the fluid.

Other important findings include:

1) The velocity and temperature of a fluid can be controlled by varying the hydrodynamic Reynold's number whereby an increase in hydrodynamic Reynolds number (Re) leads to an increase in the velocity profiles whereas increase in hydrodynamic Reynolds number (Re) leads to a decrease in the temperature profiles.
2) The velocity of a fluid can be controlled by varying the Magnetic parameter since an increase in magnetic parameter leads to a decrease in velocity.
3) The velocity and temperature of a fluid can be controlled by varying the Prandtl number.
4) The velocity and temperature of a fluid can be controlled by varying the Suction parameter.
5) The velocity and temperature of a fluid can be controlled by varying the Eckert number.

5.3 Recommendations

In this thesis, our study of MHD flow between two parallel plates was not exhaustive but can provide a basis for further research while considering the following areas;

1. Hydro-magnetic fluid flow between porous parallel plates in presence of variable transverse magnetic field when the upper plate is corrugated and vibrating.
2. Two parallel plates inclined at an angle one porous and the other impulsively started under a transverse magnetic field. Fluid flow in an inclined infinite Annulus under a radial magnetic field.
3. Fluid flow under the action of a variable magnetic field inclined at an angle to the flow direction.
4. Fluid flow under the action of variable magnetic fields lines fixed parallel to the plates in a rotating system.
5. Hydro-magnetic fluid flow between porous parallel plates in presence of variable transverse magnetic field in three dimensions.

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APPENDICES

APPENDIX 1: Computer code in MATLAB

In order to solve the governing Equations (3.2.0) and (3.2.1), the following computer program code was developed using MATLAB version 7.9.0(R2009b), subject to the boundary conditions as discussed herein. The results were obtained by varying various flow parameters, notably Reynolds number, magnetic number, prandtl number, Eckert number, pressure gradient and the suction parameter.

```
% NUMERICAL SOLUTION OF MHD PROBLEM

function MBURU ZACHARIAH code()

clear all;

clc

     N = 2.00 ;ITMAX = 1.00;

   % 'Grid

   Pr=0.5;Ec=0.05;Pg=-450;M =30.5;So=-150; R=0.5;Re=3.5;

    Nsteps=40; Tsteps=6000;


Y = linspace(0,2,Nsteps);

U= zeros(N,ITMAX);

T= zeros(N,ITMAX);

delY=N/Nsteps;

delT = ITMAX/Tsteps;
```

```
% Initial condition
for K = 1
for J = 1 : Nsteps

    U( J, 1) = 0;  T( J, 1) = 0;
 end
 end

    % Boundary conditions:
%     stationary plate

     J =1;
    for K = 1 :Tsteps

        U(J, K) = 0;
        T(J, K)= 1;
    end

%     for J=Nsteps-1;

%      for K = 1 :Tsteps
%         U(J,K)=0.5;
%      end
```

```
%     end

%     'porous plate


 J=Nsteps;
     for K = 1 :Tsteps
        U(J,K)=-1.;
        T(J,K)=0;

    end



for J = 2 : Nsteps -1
for K = 2 : Tsteps - 1

U(J, K + 1) = (U(J,K)-((delT*So)*((U(J-1,K+1)-U(J,K)+U(J-
1,K))/(2*delY)))-(Pg*delT)+((delT/Re)*((U(J+1,K+1)+...
    U(J-1,K+1)+U(J+1,K)-2*U(J,K)+U(J-1,K))/(2*delY*delY)))-
(M*U(J,K)*delT))/((1-((delT*So)/(2*delY))+(delT/...
    (2*Re*delY*delY))));
```

```
T( J, K + 1) =(T(J,K)- ((delT*So)*(T(J-1,K+1)-T(J,K)+T(J-
1,K)/(2*delY)))+(((Ec*delT)/Re)*(((U(J,K+1)-...
    U(J-1,K+1)+U(J,K)-U(J-1,K))/(2*delY))*((U(J,K+1)-U(J-
1,K+1)+U(J,K)-U(J-1,K))/(2*delY))))...
    +(delT/(Re*Pr))*((T(J+1,K+1)+T(J-1,K+1)+T(J+1,K)-
2*T(J,K)+T(J-
1,K))/(2*delY*delY))+(R*delT*(U(J,K)*U(J,K))))...
    /(1-((delT*So)/(2*delY))+(delT/(Re*Pr*delY*delY)));
end
end

hold on
figure(1)
hold on
grid off
plot(Y,U(:,50),'r','LineWidth',1,'LineSmoothing','on');
title('Velocity vs Distance between the plates.');
xlabel('Distance y');
ylabel('Velocity U');
hold off

 hold on
figure(2)
 hold on
```

```
grid off
plot(Y,T(:,50),'r','LineWidth',1,'LineSmoothing','on');
title('Temperature vs Distance between the plates.');
xlabel('Distance y');
ylabel('Temperature theta');
hold off
```

APPENDIX 2: PUBLICATION

Z.M.Mbugua, M. N. Kinyanjui, K. Giterere, P. R. Kiogora, (2015). Hydromagnetic fluid flow between parallel plates where the upper plate is porous in presence of variable transverse magnetic fields**.** *International Journal of Engineering Science and Innovative Technology*, **4** (5), 14-22