DETERMINATION OF THE EFFECTS OF ROTATION AND VARIED MAGNETIC FIELD ON UNSTEADY COUETTE FLOW WITH INJECTION.

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Determination of the effects of rotation and varied magnetic field on unsteady Couette flow with injection.

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DECLARATION

This thesis is my original work and has not in part or whole been presented in any other university for a degree award.

Signature:…………………………………… Date:…………………..

Josphat Ndirangu Ndungú.

Declaration by Supervisors

This thesis has been submitted for examination with our approval as university supervisors.

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DEDICATION

I dedicate my thesis work to my family and many friends. A special feeling of gratitude to my loving parents, John and Lydiah whose words of encouragement and push for tenacity ring in my ears. My Brothers Simon, Albert and Kamau have never left my side and are very special.

I also dedicate this thesis to my many friends who have supported me throughout the process. I will always appreciate all they have done.

I dedicate this work and give special thanks to my wife Naomi and my wonderful daughter Keyshiah for being there for me throughout the entire Master of Science course. Both of you have been my best cheerleaders.

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NOMENCLATURE

Roman Symbol	Quantity
B:	Magnetic field vector, [wbm^{-2}]
B_o:	Magnetic flux along the y-axis, [wbm^{-2}]
b:	Magnetic flux along the x-axis, [wbm^{-2}]
g:	Acceleration due to gravity vector, [ms^{-2}]
H:	Magnetic field strength vector, [Am^{-1}]
h:	Magnetic field strength along the x-axis, [Am^{-1}]
H_o:	Magnetic field intensity along the y-axis, [Am^{-1}]
J:	Current density, [AM^{-2}]
d:	Dimension distance between plates, [m]
E:	Electric field, [v]
Re:	Reynolds number
R_m:	Magnetic Reynolds number
M^2:	Magnetic Parameter
P:	Pressure force, [nm^{-2}]
q:	Velocity vector, [ms^{-1}]
i,j,k:	Unit vectors in x, y, z directions respectively.
u,v,w:	Component of velocity vector q, [ms^{-1}]
V_0:	Fluid injection velocity, [ms^{-1}]

U_0:	Velocity of the moving plate, $[ms^{-1}]$
u^*, v^*:	Dimensionless velocity components
x, y, z:	Dimensional Cartesian co-ordinates
F_i:	Body forces tensor, [N]
U_i:	Velocity tensor, $[ms^{-1}]$
K^2:	Rotational parameter
S:	Suction/Injection parameter

Greek Symbol	**Quantity**
ρ:	Fluid density, $[kgm^{-3}]$
μ:	Coefficient of viscosity, $[kgm^{-1}s]$
μ_e:	Magnetic permeability, $[Hm^{-1}]$
σ:	Electrical conductivity [Siemens/meter]
Ω:	Angular velocity, [radians/second]
υ:	Kinematic viscosity, $[m^2s^{-1}]$
ρ_e:	Charge density, $[cm^{-3}]$

Abbreviations

MHD	Magnetohydrodynamics
FDM	Finite Difference Method
PDE	Partial Differential Equations
TE	Truncation Error

ABSTRACT

Unsteady hydromagnetic Couette flow of a viscous incompressible electrically conducting fluid in a rotating system with injection through the lower plate in the presence of a variable transverse magnetic field is studied. The plates are considered porous and fluid flowing past the plates is induced by the movement of the lower plate with the upper plate set stationary. Fluid is injected through the lower plate at a constant velocity. The general solution of the governing equations is obtained from the equation of momentum and magnetic induction equation, which is valid for every value of time *t*. For small values of time t, the solution of the governing equations is obtained by Implicit Finite Difference method of order two. The fluid considered is electrically conducting. The Finite Difference method of order two and a computer program are employed in solving the non-linear equations in order to generate the velocity profiles and induced magnetic field profiles. The effects of the various parameters entering into the flow problem are presented graphically and discussed. It is found that magnetic field accelerates the fluid flow. Rotation and injection retards flow whereas injection accelerates the flow.

CHAPTER ONE

INTRODUCTION AND LITERATURE REVIEW

1.1 Introduction

In this chapter we look at the terms widely used in this study and a detailed explanation of main concepts is made. A review of the literature related to the present work done previously by other researchers in the field of fluid dynamics is given. The problem under study, hypothesis, objectives and justification of the research follows at the end of this chapter.

1.2 Background Information

The study of unsteady Magnetohydrodynamic Couette flow is of considerable importance from practical point of view because fluid transients may be expected in MHD pumps, MHD generators, accelerators, flow meters and nuclear reactors.

Theory of rotating fluids is highly important due to its occurrence in various natural phenomena and for its applications in various technological situations which are directly governed by the action of Coriolis force. Broad subjects of Oceanography, Meteorology, Atmospheric Science and Limnology all contain some important and essential features of rotating fluids.

1.3 Definition of Terms

1.3.1 Hydrodynamics

It is the science that deals with the flows of fluids while electromagnetism is the study of interaction between electric and magnetic fields. Magneto hydrodynamic is a science in which hydrodynamics and electromagnetism interacts. The interaction of the current with the magnetic field changes the motion of the fluid and produces an induced magnetic field.

1.3.2 Couette Flow

It is the laminar flow of a viscous fluid in the space between two parallel plates, one of which is moving relative to the other. The flow is driven by virtue of viscous drag force acting on the fluid and the applied pressure gradient parallel to the plates.

1.3.3 Unsteady-State Flow

It refers to the condition where the fluid properties at a point in the system change over time. Whereas steady flow, all flow variables are independent of time.

1.3.4 Fluid

It is a substance that continually deforms under an applied shear stress. Fluids are a subset of the phases of matter and include liquids and gases.

1.3.5 Viscosity

Viscosity of a fluid is a measure of its resistance to gradual deformation by shear stress or tensile stress. It is a property arising from collisions between neighbouring particles in a fluid that are moving at different velocities. A fluid that has no resistance to shear stress is known as an ideal or inviscid fluid. Zero viscosity is observed only at very low temperatures in superfluids. Otherwise, all fluids have positive viscosity, and are technically said to be viscous or viscid

1.3.6 Nanofluids

Nanofluids are new class of heat transfer fluids which contain a base fluid and nanoparticles. They are characterized by an enrichment of a base fluid like Water, toluene, Ethylene glycol or oil with nanoparticles in variety of types like Metals, Oxides, Carbides and Carbon. Nanofluids are sought to have wide range of applications in medical applications, biomedical industry, detergency, power generation in nuclear reactors and more specifically in any heat removal involved industrial applications.

1.3.7 Boundary layer

A boundary layer is the layer of fluid in the immediate vicinity of a bounding surface where the effects of viscosity are significant. The fluid near the boundary attains the velocity of the boundary. Effects of viscosity reduce as the fluid moves away from the boundary.

1.4 Literarure Review

Seth *et al.,* (2011) studied unsteady MHD Couette Flow of a viscous incompressible electrically conducting fluid, in the presence of a transverse magnetic field, between two parallel porous plates. The fluid flow within the channel was induced due to the impulsive and uniformly accelerated motion of the lower plate of the channel. The magnetic lines of force were assumed to be constant and fixed relative to the moving plate. They concluded that velocity increased with the increase in Magnetic parameter M^2 throughout the channel also the suction exerted retarding influence on the fluid velocity whereas injection had accelerating influence on it.

Ahmad *et al.,* (2015) Studied MHD flow and heat transfer through a porous medium over a shrinking/ stretching surface with suction. They obtained numerical solution for MHD flow over a stretching / shrinking surface with suction and heat transfer. The main findings of their study was: The velocity component for fluid flow over stretching surface decreased with increasing values of magnetic parameter M^2 and suction parameter λ but reverse effects were noticed for flow over shrinking surface. Also the temperature function decreased with increasing values of Prandtl number $\Pr$ and suction parameter λ. Their result hold for fluid flow over stretching/shrinking surface but the temperature distribution was higher for flow over shrinking surface than for stretching surface.

Manyonge *et al.,* (2013) researched on the Steady MHD Poiseuille Flow between Two Infinite Parallel Porous Plates in an Inclined Magnetic Field. They considered electrically conducting and incompressible fluid flowing between two infinite parallel plates the lower plate being porous and under the influence of a transverse magnetic field and constant pressure gradient. They concluded that the velocity was influenced by the four factors under consideration namely, magnetic inclination, suction/injection rates, pressure gradient and Hartmann number. Increase in those quantities decreased the velocity.

Mutua *et al.,* (2013) studied Stokes problem of a convective flow past a vertical infinite plate in a rotating system in presence of variable magnetic field. They concluded that some or all of the parameters affect the primary velocity, secondary velocity and temperature. Consequently their effect alters the rate of heat transfer and skin friction along the x and y axes. Increase in magnetic parameter M and Eckert number Ec lead to an increase in the primary velocity profiles for both free convection cooling and heating at the plate while an increase in the same parameters lead to a decrease in secondary velocity profiles.

Eshetu *et al.,* (2015) studied a steady MHD boundary-layer flow of water-based nanofluids over a moving permeable flat plate. The plate was assumed to move in the same or opposite direction to the free stream. They concluded that the velocity profile decreased with an increase in the magnetic parameter whereas it increased as both the nanoparticle volume fraction and the suction parameters increased. Also increasing the values of magnetic parameter, the suction rate parameter and nanoparticle volume fraction resulted in an increase in the skin friction coefficient.

Naroua *et al.,* (2006) studied computational challenges in fluid flow problems, a MHD Stokes problem of convective flow from a vertical infinite plate in a rotating fluid. A study on Hall current effect on MHD free convection flow past a semi-infinite vertical plate with mass transfer was done by Emad *et al.,* (2001). They discussed the effects of magnetic parameter, Hall parameter and the relative buoyancy force effect between species and thermal diffusion on the velocity, temperature and concentration. The problem of combined heat and mass transfer of an electrically conducting fluid in MHD free convection adjacent to a vertical surface has been analyzed by Chen, (2004) taking into account the effects of Ohmic heating and viscous dissipation.

Young, (2014) investigated The effects of the constant applied magnetic field as a function of its angle with the channel walls was studied using finite elements. This was done for insulating channel walls and for two insulating and two conducting walls forming a short-circuited magnetohydrodynamic generator. He found out that skewing the magnetic induction field does not help but rather hinders the magnetohydrodynamic flow by effectively lowering the Hartmann number.

An investigation of MHD effect on the flow structure and heat transfer characteristics was carried out Li *et al.,* (2005). This was studied numerically for a liquid-gas annular flow under a transverse magnetic field. The results showed that temperature distribution in the liquid film and the Nusselt number distribution in the angular direction were influenced by the flow structures with the side walls.

Osalusi *et al.,* (2007) studied the effects of Ohmic heating and viscous dissipation on unsteady MHD and slip flow over a porous rotating disk with variable properties in the presence of Hall and Ion-slip currents. Plaut *et al.,* (2003) studied the nonlinear dynamics of traveling waves in rotating Rayleigh-Bernard convection in which he examined the effects of the boundary conditions and of the topology.

Ogulu *et al.,* (2004) studied the effect of slip velocity on oscillatory MHD flow with radiative heat transfer and variable suction.

Seth *et al.,* (2009) studied the Effects of Rotation and Magnetic Field on Unsteady Couette Flow in a Porous Channel. They considered the lower plate being stationary with suction on the upper plate. They found that magnetic field has tendency to retard the fluid flow in both the primary and secondary flow directions. Rotation retards primary flow whereas it accelerates secondary flow. Also there exists incipient flow reversal near the stationary plate in primary flow direction on increasing rotation parameter K^2. Suction accelerates primary flow whereas it retards secondary flow. Injection retards both the primary and secondary flows. Fluid flow in both the primary and secondary flow directions increases on increasing time t.

Guria *et al.,* (2009) investigated oscillatory MHD Couette flow of electrically conducting fluid between two parallel plates in a rotating system in the presence of an inclined magnetic field when the upper plate is held at rest and the lower plate oscillates non-torsionally.

Kwanza *et al.,* (2003) investigated Stokes free convection flow past an infinite vertical porous plate subjected to constant heat flux with Ion-slip current and radiation absorption. Kinyanjui *et al* (2001) published work on MHD free convection heat and mass transfers of a heat generating fluid past an impulsively infinite porous plate with Hall currents and radiation absorption.

Singh *et al.,* (2014) studied Hall current effect on visco-elastic MHD oscillatory convective flow through a porous medium in a vertical channel with heat radiation. A magnetic field of uniform strength was applied in the direction normal to the planes of the plates. They concluded that the flow retards as the visco-elasticity of the fluid increases. The velocity increases tremendously with the increase of Grashof number Gr. Physically it means that the buoyancy force enhances the flow velocity. Velocity increased with the increase of Reynolds number Re. Velocity decreased with the increase of Hartmann number M.

Singh *et al.,* (2014) researched on MHD flow and heat transfer for Maxwell fluid over an exponentially stretching sheet through a porous medium in the presence of non-uniform heat source/sink with variable thermal conductivity. The thermal conductivity was assumed to vary as a linear function of temperature. They found out that magnetic parameter M and porosity parameter P decrease the velocity.

Chandran *et al.*, (1993) and Das *et al.*, (2009) studied unsteady hydromagnetic Couette flow of a viscous incompressible electrically conducting fluid in a rotating system when the fluid flow within the channel is induced due to impulsive movement of one of the plates of the channel whereas Singh *et al* (1994) considered this problem when one of the plates of the channel is set into uniformly accelerated motion. Seth *et al.,* (1982) analyzed this problem when the lower plate of the channel moves with time dependent velocity U (t) and the upper plate is kept fixed. They considered two particular cases of interest of the problem, namely, (i) impulsive movement of the plate and (ii) uniformly accelerated movement of the plate. In all these investigations, the channel walls were considered non- porous. However, the study of such fluid flow problem in porous channel may find applications in petroleum, mineral and metallurgical industries, designing of cooling systems with the liquid metals, MHD generators, MHD pumps, MHD accelerators and flow meters, geothermal reservoirs and underground energy transport. Taking into account this fact Muhuri (1963), Rao *et al.,* (1982), Bhaskara *et al.,* (1982), Singh *et al.,* (2004), Abbas *et al.,* (2006) and (Hayat *et al.,* 2007, 2008) considered MHD flow within a parallel plate channel with porous boundaries, under different conditions, in non-rotating/rotating system.

Katagiri, (1962), investigated MHD Couette flow of a viscous incompressible electrically conducting fluid in the presence of a uniform transverse magnetic field when the fluid flow is induced due to the impulsive motion of one of the plates. Muhuri, (1963), studied this problem in a porous channel when the fluid flow is induced due to the accelerated motion of one of the plates. Katagiri, (1962) and Muhuri, (1963), presented their analysis by considering that the magnetic lines of force are fixed relative to the fluid. Singh, (1983) considered the problem studied by Katagiri, (1962) and Muhuri, (1963) in a non-porous channel when the magnetic lines of force are fixed relative to the moving plate. Khan *et al.,* (2006) investigated MHD flow of a generalized Oldroyd-B fluid in a porous space taking Hall current into account whereas Khan *et al.,* (2007) also considered MHD transient flows of an Oldroyd-B fluid in a channel of rectangular cross section in a porous medium. Hayat *et al.,* (2008) studied the influence of Hall current and heat transfer on the steady MHD flows of a generalized Burgers fluid between two eccentric rotating infinite disks of different temperatures. In this case the fluid flow is induced due to a pull with constant velocities of the disks. Khan *et al.,* (2009) considered the effects of variable suction and heat transfer on the oscillatory magnetohydrodynamic flow of a non-Newtonian fluid through a porous medium with slip at the wall. Khan *et al.,* (2008) obtained exact solutions of accelerated flows for a Burgers fluid induced by the accelerating plate by considering two cases of interest viz. (i) constantly accelerating flow and (ii) variable accelerating flow.

The above literature demonstrates that little has been done in MHD Couette flow. However the research that has been done on MHD Couette flow by Seth *et al.,* (2009), the magnetic field was assumed to be a constant. That is why in this research, the problem is modelled mathematically with varying magnetic field and the finite difference method used in solving equations.

1.5 Statement of the problem

Little research has been done in MHD Couette flow with variable magnetic field, hence the need to be investigated. This study is on unsteady MHD Couette flow with variable magnetic field through two infinite porous plates with the upper plate being stationary, the lower plate is moving at a constant velocity with injection at a constant velocity.

1.6 Hypothesis

Rotation parameter K^2, Reynolds number R_e and injection parameter S do not affect unsteady Couette flow past two plates with an injection through the lower plate.

1.7 Objectives

1.4.1 General Objectives

To study effects of rotation and varied magnetic field on unsteady Couette flow past two porous plates with an injection through the lower plate.

1.4.2 Specific Research Objectives

i. To determine the velocity profile.
ii. To determine the induced magnetic field profile.
iii. To determine the effects of various flow parameters on the flow field.

1.8 Justification

The flow of an electrically conducting viscous fluid between two parallel plates in the presence of a transversely applied magnetic field has applications in many devices such as MHD power generators, which relies on moving a conductor through a magnetic field to generate electric current. The MHD generator uses hot conductive plasma as the moving conductor. Research on flows through porous media has lately been applied in the manufacturing machinery and computer disk drives. Natural MHD dynamos are an active area of research in plasma physics and are of great interest to the geophysics and astrophysics communities, since the magnetic fields of the earth and sun are produced by these natural dynamos. Other applications include MHD pumps, accelerators, aerodynamics heating, electrostatic precipitation, polymer technology, petroleum industry, purification of molten metals from non-metallic inclusions and fluid droplets-sprays.

The governing equations are presented and modified subject to the assumption made in order to generate specific equations in the next chapter

CHAPTER TWO

GOVERNING EQUATIONS

2.1 Introduction

In this chapter, the governing equations of an incompressible Newtonian fluid with constant injection velocity v_0, constant viscosity μ, constant angular velocity Ω and constant pressure $\vec{P}$ between two porous infinite plates. Governing equations are presented and modified subject to the assumption made in order to generate specific equations.

In this chapter we consider assumptions made, the general conservation equations of mass and momentum and finally the electromagnetic equations.

2.2 Assumptions

Various set of equations governing the flow problem are derived from the principle of conservation of mass, Newton's 2^{nd} law of motion and the electromagnetic equations.

In order to describe the phenomenon mathematically the following assumptions are made;

i. All velocities are small compared with that of light $\frac{v^2}{c^2} << 1$
ii. Viscosity, Darcy permeability and electrical conductivity are constant.
iii. The force due to electric field is negligible compared to the force $\vec{J} \times \vec{B}$ due to magnetic field.
iv. The induced magnetic field, the external electric field and the electric field due to polarization of charges are negligible.
v. The porous medium is non-magnetic and homogeneous therefore there is no magnetic induction.
vi. No slip conditions are satisfied.
vii. No applied or polarization voltages exist.

2.3 Conservation Equations

Considering an incompressible Newtonian fluid with constant pressure P, constant viscosity μ and velocity vector $\bar{q}$, between two porous infinite plates.

2.3.1 Equation of Continuity

This is derived from the law of conservation of mass which states that mass of the fluid remains constant as the fluid particles flow in the flow field.

It is expressed as;

$$\frac{\partial \rho}{\partial t} + \bar{\nabla}.(\rho \bar{u}_i) = 0 \tag{2.1}$$

Where $i = 1,2,3$ along x, y and z direction respectively. Since the fluid is assumed to be incompressible, fluid density ρ is assumed to be a constant and the equation takes the form below:

$$\bar{\nabla}.(\rho \bar{u}_i) = 0 \tag{2.2}$$

Factoring ρ out and dividing through by ρ leads to;

$$\bar{\nabla}.(\bar{u}_i) = 0 \tag{2.3}$$

2.3.2 Equation of Momentum

This is derived from Newton's second law of motion which states that the sum of resultant forces equal to the rate of change of momentum of the flow.

Momentum equation in tensor form is given by:

$$\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_j} + \upsilon \nabla^2 \bar{u}_i + \bar{F}_i \tag{2.4}$$

2.3.3 Electromagnetic Equations

These equations show the relationship between the electric field intensity $\vec{E}$, the magnetic induction vector $\vec{B}$, the magnetic field strength $\vec{H}$ and the induction current density vector $\vec{J}$ these are;

$$\vec{\nabla} \times \vec{H} = \vec{J} \tag{2.5}$$

From Gauss's law of magnetism

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{2.6}$$

From Farady's law which states that the electromotive force generated around a closed loop equals minus the rate of change of the magnetic flux through the loop.

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{2.7}$$

When an electric current density $\vec{J}$ flows through the fluid, there is a force per unit volume of

$$\vec{J} \times \vec{B} = \frac{\left(\vec{B} \cdot \vec{\nabla}\right)\vec{B}}{\mu_e} - \vec{\nabla}\left(\frac{\vec{B}^2}{2\mu_e}\right) \tag{2.8}$$

Where μ_e is magnetic permeability and the first term on the right hand side of equation (2.8) is the magnetic tension force which is a restoring force that acts to straighten bent magnetic field lines and the second term is the magnetic pressure force which is an energy density associated with a magnetic field

In chapter three, the mathematical analysis of the study is given, governing equations are stated and non-dimensionalised,

CHAPTER THREE

MATHEMATICAL FORMULATION

3.1 Introduction

The equation governing hydrodynamics flows are highly non-linear PDEs thus not possible to obtain analytical solutions.

In this chapter, mathematical analysis of the study is given, governing equations are stated and non-dimensionalised, Implicit finite difference method of order two and a MATLAB algorithm are employed in solving the PDEs. Finally investigations are carried out on how the variation of the injection parameter, rotation parameter and Reynolds number affect the velocity profiles. Also variations of injection/suction parameter, Reynolds number and Magnetic Reynolds number affect the induced magnetic field.

3.2 Model Description.

3.2.1 Formulation of the Problem

Consider the unsteady flow of a viscous incompressible electrically conducting fluid between two parallel porous plates of infinite length, distance d apart in presence of a transverse magnetic field **B** applied parallel to *y*-axis which is normal to the plates. The fluid and the plates are in a state of rotation about *y*-axis with uniform angular velocity Ω.

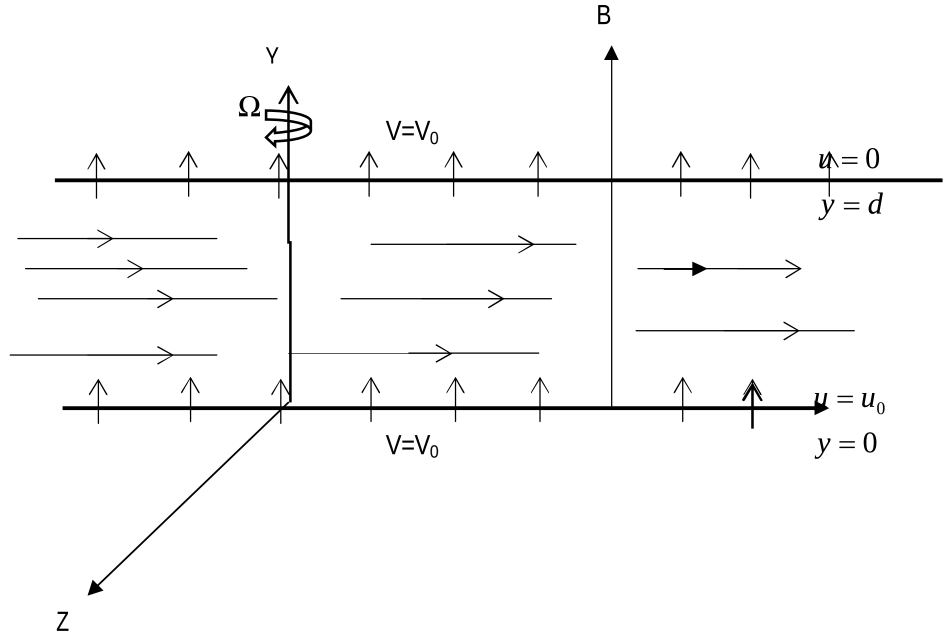

Figure 3. 1: Physical Model of the Problem

Initially when time $t = 0$, the fluid and the plates are assumed to be at rest. When time $t > 0$, the lower plate $(y = 0)$ starts moving with uniform velocity u_0 along x- direction in its own plane while the upper plate $(y = d)$ is kept fixed.

The plates are infinite along x directions and are non-conducting all physical quantities will be functions of *y* and *t* only. Injection of the fluid takes place through the porous walls of the channel with uniform velocity $V_{0,}$ which is greater than zero for injection and less than zero for suction. It is assumed that no applied or polarization voltages exist since the plates are insulated. This corresponds to the case where no energy is being added or extracted from the fluid by electrical means. (I.e. electric field $\boldsymbol{E} = 0$).

In general, the electric current flowing in the fluid gives rise to an induced magnetic field which perturbs the applied magnetic field. Since magnetic Reynolds number is very small for metallic liquids and partially ionized fluids so the induced magnetic field may be neglected in comparison to the applied one.

Under the above assumption fluid velocity $\vec{q}$, is given by

$$\vec{q} = (u, v_0, 0) \tag{3.1}$$

The flow drags the field lines along in the x-direction, so the magnetic field acquires a component in the x-direction. Hence, in general magnetic field strength $\vec{H}$ and magnetic field $\vec{B}$ are given by:

$$\vec{B} = (b, B_0, 0) \quad \text{And} \quad \vec{H} = (h, H_0, 0) \tag{3.2}$$

3.2.2 Lorentz Force

When an electric current density $\vec{J}$ flows through the fluid, there is a force per unit volume of $\vec{J} \times \vec{B}$.

Ohm's Law asserts that the total electric current flowing in a conductor is proportional to the total electric field. In addition to the field $\vec{E}$ acting on a fluid at rest, a fluid moving with velocity $\vec{q}$ in the presence of a magnetic field $\vec{B}$ is subject to an additional electric field $\vec{q} \times \vec{B}$

Ohm's law then gives

$$\vec{J} = \sigma(\vec{E} + \vec{q} \times \vec{B}) \tag{3.3}$$

Where the constant of proportionality σ is called the electrical conductivity, $\vec{E}, \vec{q}$ and $\vec{B}$ are electric field, fluid velocity and magnetic field respectively.

But $\vec{E} = 0$ since it is assumed that no applied or polarization voltages exist since the plates are insulated, therefore equation (3.3) reduces to

$$\vec{J} = \sigma(\vec{q} \times \vec{B}) \tag{3.4}$$

But

$$\vec{B} = \mu_e \vec{H} \tag{3.5}$$

Where μ_e is magnetic permeability and $\vec{H}$ is the magnetic field intensity.

Using equation (3.2) in (3.4) yields:

$$\vec{J} = \sigma \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ u & v_0 & 0 \\ b & B_0 & 0 \end{bmatrix} = \hat{k}\sigma(uB_0 - bv_0) \tag{3.6}$$

From equation (3.2) and (3.6) we get

$$\vec{J} \times \vec{B} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \sigma(uB_0 - bv_0) \\ b & B_0 & 0 \end{bmatrix} = \hat{i}\,\sigma B_0(bv_0 - uB_0) + \hat{j}\,\sigma b(uB_0 - bv_0) \tag{3.7}$$

Therefore resolving $\vec{J} \times \vec{B}$ in the x-direction gives

$$\vec{J} \times \vec{B} = \sigma B_0(bv_0 - uB_0) \tag{3.8}$$

From equation (3.5), we conclude that:

$$B_0 = \mu_e H_0 \text{ and } b = \mu_e h \tag{3.9}$$

Substituting equation (3.9) in (3.8) we get

$$\vec{J} \times \vec{B} = \sigma \mu_e^2 H_0(hv_0 - H_0\vec{u}) \tag{3.10}$$

3.4.3 Equation of Continuity

From the law of conservation of mass which states that mass can neither be created nor destroyed.

It is expressed as;

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}.(\rho \vec{u}_i) = 0 \tag{3.11}$$

Since the fluid is assumed to be incompressible, therefore the density (ρ) is assumed to be a constant. Equation (3.11) takes the form;

$$\vec{\nabla}.(\rho \vec{u}_i) = 0 \tag{3.12}$$

Factoring ρ out and dividing through by ρ leads to;

$$\vec{\nabla}.(\vec{u}_i) = 0 \tag{3.13}$$

Considering two dimensional flow, equation (3.13) becomes:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3.14}$$

But the velocity along y-direction is the injection velocity

$$v = v_0 \tag{3.15}$$

Since the plates are infinitely long on the x-direction, thus the equation of continuity reduces to:

$$\frac{\partial u}{\partial x} = 0 \tag{3.16}$$

3.4.4 Equation of Momentum

This is derived from Newton's second law of motion which states that the sum of resultant forces equal to the rate of change of momentum of the flow.

Momentum equation is given by:

$$\frac{\partial \vec{q}}{\partial t} + \vec{q}\nabla\vec{q} = -\frac{1}{\rho}\nabla\vec{P} + \upsilon\nabla^2\vec{q} + \frac{1}{\rho}\vec{J}\times\vec{B} + \vec{F} \tag{3.17}$$

Following the studies made by Seth *et al.*, (1982, 1988), Chandran *et al.*, (1993), Singh *et al.*, (1994), Singh (2000) and Hayat *et al* (2004) the governing equations for the flow of a viscous incompressible electrically conducting fluid in a rotating frame of reference are:

$$\frac{\partial u}{\partial t} + \bar{u}\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + 2\Omega v = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \frac{1}{\rho}\vec{J}\times\vec{B} \tag{3.18}$$

From the equation of continuity, $\frac{\partial \bar{u}}{\partial x} = 0$ and $\bar{v} = v_0$ since the velocity in y-direction is the injection velocity ,

The fluid motion is induced due to the movement of the lower plate in the x – direction. Therefore the pressure gradient is neglected.

$$\frac{\partial p}{\partial x} = 0 \tag{3.19}$$

Therefore equation (3.18) reduces to

$$\frac{\partial u}{\partial t} + v_0 \frac{\partial u}{\partial y} + 2\Omega v_0 = \upsilon \frac{\partial^2 u}{\partial y^2} + \frac{1}{\rho} \vec{J} \times \vec{B} \tag{3.20}$$

Where v_0 is the injection velocity and Ω is the angular velocity of the fluid.

Substituting equation (3.10) in (3.20) yields:

$$\frac{\partial u}{\partial t} + v_0 \frac{\partial u}{\partial y} + 2\Omega v_0 = \upsilon \frac{\partial^2 u}{\partial y^2} - \frac{\sigma \mu_e^2 H_0}{\rho} \left(H_0 u - h v_0 \right) \tag{3.21}$$

Following the study by Rossow *et al.,* (1958) when magnetic field is fixed with respect to the moving plate, Velocity of the fluid is replaced with the relative velocity of the fluid with respect to the velocity of the moving plate.
Equation (3.21) is replaced by:

$$\frac{\partial u}{\partial t} + v_0 \frac{\partial u}{\partial y} + 2\Omega v_0 = \upsilon \frac{\partial^2 u}{\partial y^2} - \frac{\sigma \mu_e^2 H_0}{\rho} \left[H_0 (u - u_0) - h v_0 \right] \tag{3.22}$$

3.4.5 Magnetic Induction Equation

Since the magnetic field is varied, we need to solve the magnetic induction equation given by

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\vec{q} \times \vec{B} \right) + \frac{1}{\sigma \mu_e} \nabla^2 \vec{B} \tag{3.23}$$

It describes the evolution of the magnetic field $\vec{B}$. The first term on the right-hand side is the advective term that describes the interaction of the field with the flow velocity $\vec{q}$. It is the only term that can generate field. The second term on the right-hand side is a diffusive term. In the absence of a flow velocity $\vec{q}$, the diffusive term leads to a decay of the magnetic field.

$$\vec{q}\times\vec{B}=\begin{bmatrix}\hat{i} & \hat{j} & \hat{k}\\ u & v_0 & 0\\ b & B_0 & 0\end{bmatrix}=\hat{k}\left(uB_0-bv_0\right) \tag{3.24}$$

$$\vec{\nabla}\times\left(\vec{q}\times\vec{B}\right)=\begin{bmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 0 & 0 & uB_0-bv_0\end{bmatrix}=\hat{i}\left(B_0\frac{\partial u}{\partial y}-v_0\frac{\partial b}{\partial y}\right) \tag{3.25}$$

Substituting equations (3.2) and (3.25) in equation (3.23) gives:

$$\frac{\partial b}{\partial t}=B_0\frac{\partial u}{\partial y}-v_0\frac{\partial b}{\partial y}+\frac{1}{\sigma\mu_e}\frac{\partial^2 b}{\partial y^2} \tag{3.26}$$

Using equation (3.9) in equation (3.26) yields:

$$\frac{\partial h}{\partial t}+v_0\frac{\partial h}{\partial y}-H_0\frac{\partial u}{\partial y}=\frac{1}{\sigma\mu_e}\frac{\partial^2 h}{\partial y^2} \tag{3.27}$$

3.4.6 Specific Equations

Specific equations are:

$$\left.\begin{aligned}&\frac{\partial u}{\partial t}+v_0\frac{\partial u}{\partial y}+2\Omega v_0=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma\mu_e^2 H_0}{\rho}\left[H_0(u-u_0)-hv_0\right]\\ &And\\ &\frac{\partial h}{\partial t}+v_0\frac{\partial h}{\partial y}-H_0\frac{\partial u}{\partial y}=\frac{1}{\sigma\mu_e}\frac{\partial^2 h}{\partial y^2}\end{aligned}\right\} \tag{3.28}$$

Specific equations of momentum and the magnetic induction (3.28) are solved simultaneously to generate the velocity profiles and induced magnetic field profiles. Subject to the initial and boundary conditions:

$$\begin{aligned}&h=0,\ u=0; && 0\le y\le d \ \text{ and } t=0\\ &h=0,\ u=0; && y=d\ ;\quad t>0\\ &h=H_0,\ u=u_0; && y=0\ ;\quad t>0\end{aligned} \tag{3.29}$$

3.3 Non Dimensionalization

Dimensional analysis is the practice of checking relations among physical quantities by identifying their dimensions. The dimension of any physical quantity is the combination of the basic physical dimensions that compose it. Some fundamental physical dimensions are length, mass, time, and electric charge. All other physical quantities can be expressed in terms of these fundamental physical dimensions.

It offers a method for reducing complex physical problems to the simplest form prior to obtaining a quantitative answer. The method is of great generality and mathematical simplicity. In our study, dimensional analysis is used in the non-dimensionalization of the governing equations by first selecting certain characteristics quantities and then substituting them in the equations.

3.3.1 Equation of Momentum

In non dimensionalizing equation of momentum (3.22) we introduce the non-dimensional variables;

$$y^* = \frac{y}{d} \qquad u^* = \frac{ud}{\upsilon} \qquad t^* = \frac{t\upsilon}{d^2} \qquad h^* = \frac{h}{H_0} \tag{3.30}$$

$$\frac{\partial \bar{u}}{\partial t} = \frac{\partial \bar{u}}{\partial u^*}\frac{\partial u^*}{\partial t^*}\frac{\partial t^*}{\partial t} = \frac{\vartheta^2}{d^3}\frac{\partial u^*}{\partial t^*} \tag{3.31}$$

$$\frac{\partial \bar{u}}{\partial y} = \frac{\partial \bar{u}}{\partial u^*}\frac{\partial u^*}{\partial y^*}\frac{\partial y^*}{\partial y} = \frac{\vartheta}{d^2}\frac{\partial u^*}{\partial y^*} \tag{3.32}$$

$$\frac{\partial^2 \bar{u}}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial \bar{u}}{\partial y}\right) = \frac{\partial}{\partial y^*}\left(\frac{\partial \bar{u}}{\partial y}\right)\frac{\partial y^*}{\partial y} = \frac{\vartheta}{d^3}\frac{\partial^2 u^*}{\partial y^{*2}} \tag{3.33}$$

Substituting equations (3.30), (3.31), (3.32), and (3.33), equation (3.22) in non-dimensional form becomes:

$$\frac{\partial u^*}{\partial t^*} + \frac{v_0 d}{\upsilon}\frac{\partial u^*}{\partial y^*} + 2\frac{\Omega v_0 d^3}{\upsilon^2} = \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma \mu_e^2 H_0^2 d^2}{\rho \upsilon}\left(u^* - \frac{u_0 d}{\upsilon} - \frac{v_0 d}{\upsilon}h^*\right) \tag{3.34}$$

Thus equation (3.34) becomes:

$$\frac{\partial u^*}{\partial t^*}+S\frac{\partial u^*}{\partial y^*}+2K^2S=\frac{\partial^2 u^*}{\partial y^{*2}}-M^2\left(u^*-R_e-Sh^*\right) \tag{3.35}$$

Where, $S=\frac{v_0 d}{\upsilon}$ is injection parameter, $M^2=\frac{\sigma\mu_e^2 H_0^2 d^2}{\rho\upsilon}$ is magnetic parameter which is the square of Hartmann's number, $R_e=\frac{u_0 d}{\upsilon}$ is the Reynolds number and $K^2=\frac{\Omega d^2}{\upsilon}$ is the rotational parameter which is reciprocal of Ekman number.

3.3.2 Magnetic Induction Equation

In non dimensionalizing magnetic induction equation (3.36) we introduce the non-dimensional variables given in equation (3.39);

$$\frac{\partial\vec{h}}{\partial t}=\frac{\partial\vec{h}}{\partial h^*}\frac{\partial h^*}{\partial t^*}\frac{\partial t^*}{\partial t}=\frac{\vartheta H_0}{d^2}\frac{\partial h^*}{\partial t^*} \tag{3.36}$$

$$\frac{\partial\vec{h}}{\partial y}=\frac{\partial\vec{h}}{\partial h^*}\frac{\partial h^*}{\partial y^*}\frac{\partial y^*}{\partial y}=\frac{H_0}{d}\frac{\partial h^*}{\partial y^*} \tag{3.37}$$

$$\frac{\partial^2\vec{h}}{\partial y^2}=\frac{\partial}{\partial y}\left(\frac{\partial\vec{h}}{\partial y}\right)=\frac{\partial}{\partial y^*}\left(\frac{\partial\vec{h}}{\partial y}\right)\frac{\partial y^*}{\partial y}=\frac{H_0}{d^2}\frac{\partial^2 h^*}{\partial y^{*2}} \tag{3.38}$$

Using equations (3.30), (3.36), (3.37), and (3.38), equation (3.27) in non-dimensional form becomes:

$$\frac{\partial h^*}{\partial t^*}+\frac{v_0 d}{\upsilon}\frac{\partial h^*}{\partial y^*}-\frac{\partial u^*}{\partial y^*}=\frac{1}{\sigma\upsilon\mu_e}\frac{\partial^2 h^*}{\partial y^{*2}} \tag{3.39}$$

Which yields;

$$\frac{\partial h^*}{\partial t^*}+S\frac{\partial h^*}{\partial y^*}-\frac{\partial u^*}{\partial y^*}=\frac{R_e}{R_m}\frac{\partial^2 h^*}{\partial y^{*2}} \tag{3.40}$$

Where, $R_m=\sigma\mu_e u_0 d$ is the magnetic Reynolds number. Which is very small for metallic liquids and partially ionized fluids.

3.3.2 Initial and Boundary Conditions

Introducing non-dimensional variables in equation (3.30), the initial and boundary conditions given by equation (3.29) becomes:

$$
\begin{aligned}
&h^* = 0,\ u^* = 0; && 0 \le y^* \le 1 \ \text{ and } t^* = 0 && \\
&h^* = 0,\ u^* = 0; && y^* = 1; \quad t^* > 0 && (3.41)\\
&h^* = 1,\ u^* = R_e; && y^* = 0; \quad t^* > 0 &&
\end{aligned}
$$

3.3.3 Specific Equations

Specific equations from equations of momentum (3.35) and the magnetic induction (3.39) are:

$$
\left.
\begin{aligned}
&\frac{\partial u^*}{\partial t^*} + S\frac{\partial u^*}{\partial y^*} + 2K^2 S = \frac{\partial^2 u^*}{\partial y^{*2}} - M^2\left(u^* - R_e - Sh^*\right)\\
&And\\
&\frac{\partial h^*}{\partial t^*} + S\frac{\partial h^*}{\partial y^*} - \frac{\partial u^*}{\partial y^*} = \frac{R_e}{R_m}\frac{\partial^2 h^*}{\partial y^{*2}}
\end{aligned}
\right\} \quad (3.42)
$$

Specific equations of momentum and the magnetic induction (3.42) are solved simultaneously to generate the velocity profiles and induced magnetic field profiles. Subject to the initial and boundary conditions (3.41)

3.3.4 Non-Dimensional Numbers

We define the following flow parameters in magnetohydrodynamics which are applicable to this study. They are: Reynolds number R_e, injection parameter S, rotational parameter K^2, magnetic parameter M^2 and magnetic Reynolds number R_m.

3.3.4.1 Reynolds Number

This is a dimensionless quantity that is used to help predict similar flow patterns in different fluid flow situations. Defined as the ratio of inertial forces to viscous forces. Mathematically its given by $R_e = \frac{vl}{\upsilon}$ where v is the mean velocity of the object relative to the fluid, l is the characteristic length and υ is the kinematic viscosity.

3.3.4.2 Magnetic Reynolds Number

This is the ratio of the magnitudes of convective term to diffusive term of the magnetic induction equation. Advective term describes the interaction of the field with the flow velocity $\vec{q}$. It is the only term that can generate field. In the absence of a flow velocity $\vec{q}$, the diffusive term leads to a decay of the magnetic field.

Assuming that the velocity field has a characteristic magnitude u, magnetic field has a characteristic magnitude B and that the lengthscale over which both fields change is d.

Advective term: $\nabla \times (\vec{q} \times \vec{B}) \approx uB/d$

Diffusive term: $\frac{1}{\sigma\mu_e} \nabla^2 \vec{B} = B/\sigma\mu_e d^2$

Therefore:

$$R_m = \sigma u \mu_e d \qquad (2.9)$$

3.3.4.3 Magnetic Parameter.

This is the square of Hartmann's number which gives is the ratio of electromagnetic force to the viscous force $M^2 = \frac{\sigma B^2 d^2}{\mu}$ where B is the magnetic field, d is the characteristic length scale, σ is the electrical conductivity and μ is the dynamic viscosity.

3.3.4.4 Rotational Parameter.

This is the reciprocal of Ekman number. The Ekman number is the ratio of viscous forces to Coriolis forces. When the Ekman number is small, disturbances are able to propagate before decaying owing to frictional effects. The Ekman number describes the order of magnitude for the thickness of an Ekman layer, a boundary layer in which viscous diffusion is balanced by Coriolis effects, rather than the usual convective inertia.

The rotation parameter is therefore given by $K^2 = \frac{\Omega d^2}{\upsilon}$. Where Ω is the angular velocity, d is the characteristic length scale and ϑ is the kinematic viscosity.

3.3.4.5 Injection Parameter.

This is the ratio of inertial to viscous forces. Its given by $S = \frac{vd}{\upsilon}$ where v is the injection velocity of the fluid, d is the characteristic length and υ is the kinematic viscosity.

In Chapter Four, Implicit finite difference method of order two and a MATLAB algorithm are employed in solving the non-dimensionalised PDEs in equation (3.42). Finally investigations are carried out on how the variation of the injection parameter, rotation parameter and Reynolds number affect the velocity profiles and induced magnetic field profiles.

CHAPTER FOUR

METHOD OF SOLUTION

4.1 Introduction

The equations governing hydrodynamics flows are highly non-linear PDEs thus not possible to obtain analytical solutions. Implicit FD method of order two is used solve the PDE (3.42) and involves the following steps:

i. Generate a grid, for example (y_j, t_n) where we want to find an approximate solution.

ii. Substitute the derivatives in a PDE or PDE system of equations with finite difference schemes. The PDE then become a non-linear system of algebraic equations.

iii. Solve the system of algebraic equations.

iv. Implement and debug the computer code.

In a finite difference grid the mesh points or nodal points identified by a double index (j, n) that defines its location with respect to t and y are at the intersections of the straight lines drawn parallel to the y and t axes. The index j refer to spatial points, whereas index n refers to time. as indicated below:

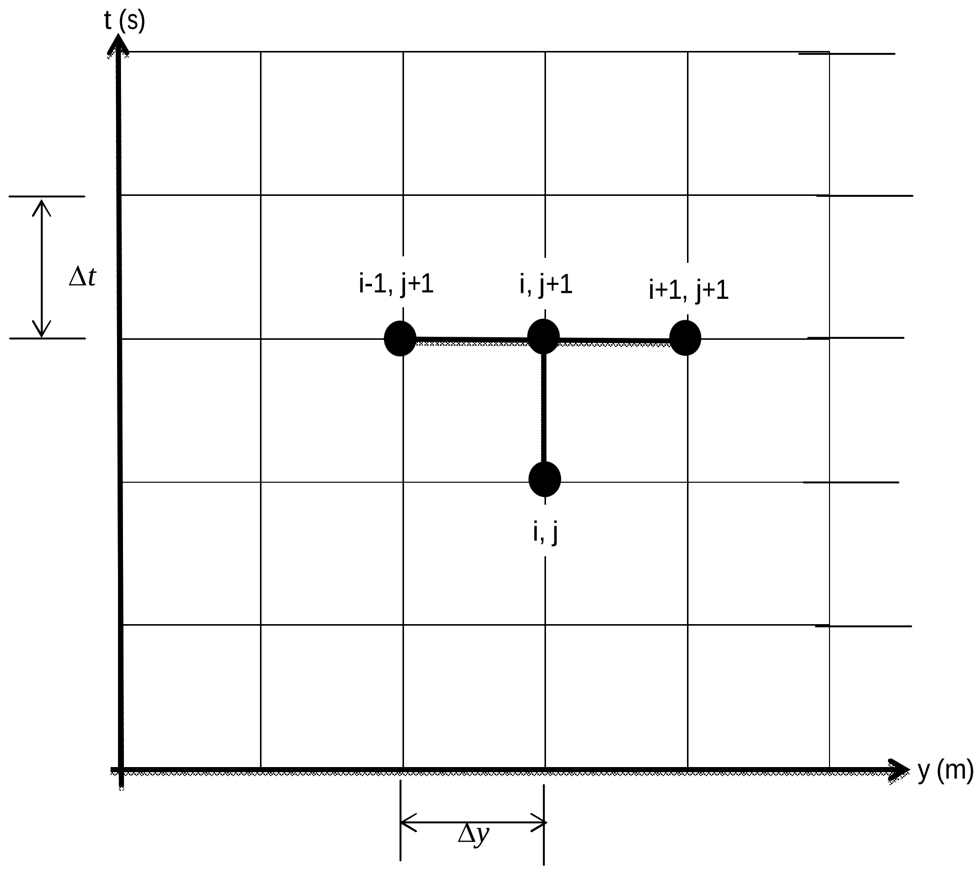

Figure 4. 2: Finite Difference Grid Mesh

If we use the backward difference at time t_n and t_{n+1} and a second-order central difference for the space derivative at position y_j we get the recurrence equations u, u_t, u_y, u_{yy}, H, H_t, H_y and H_{yy} for times n and $n+1$ given by:

$$u = \frac{u_j^{n+1} + u_j^n}{2} \tag{4.1}$$

$$u_t = \frac{u_j^{n+1} - u_j^n}{\Delta t} \tag{4.2}$$

$$u_y = \frac{u_{j+1}^{n+1} - u_{j-1}^{n+1} + u_{j+1}^{n} - u_{j-1}^{n}}{4(\Delta y)} \tag{4.3}$$

$$u_{yy} = \frac{u_{j+1}^{n+1} - 2u_{j}^{n+1} + u_{j-1}^{n+1} + u_{j+1}^{n} - 2u_{j}^{n} + u_{j-1}^{n}}{2(\Delta y)^2} \tag{4.4}$$

$$H = \frac{H_{j}^{n+1} + H_{j}^{n}}{2} \tag{4.5}$$

$$H_t = \frac{H_{j}^{n+1} - H_{j}^{n}}{\Delta t} \tag{4.6}$$

$$H_y = \frac{H_{j+1}^{n+1} - H_{j-1}^{n+1} + H_{j+1}^{n} - H_{j-1}^{n}}{4(\Delta y)} \tag{4.7}$$

$$H_{yy} = \frac{H_{j+1}^{n+1} - 2H_{j}^{n+1} + H_{j-1}^{n+1} + H_{j+1}^{n} - 2H_{j}^{n} + H_{j-1}^{n}}{2(\Delta y)^2} \tag{4.8}$$

To solve specific equations (3.42) for this flow we use Crank-Nicolson method.

4.2 Governing Equations in Finite Difference Form

Equations (3.42) are coupled and non-linear therefore cannot be solved analytically. That is why in this research the finite difference method is used to solve them subject to the initial and boundary conditions (3.41)

4.2.1 Momentum Equation

Using Crank-Nicolson method and setting $y^* = y$, $t^* = t$, $u^* = u$ and $h^* = H$ equation (3.35) becomes

$$u_t + Su_y + 2K^2 S = u_{yy} - M^2(u - R_e - SH) \tag{4.9}$$

Using equations (4.1), (4.2), (4.3) and (4.4) equation (4.9) yields:

$$\frac{u_j^{n+1}-u_j^n}{\Delta t}+S\left(\frac{u_{j+1}^{n+1}-u_{j-1}^{n+1}+u_{j+1}^n-u_{j-1}^n}{4(\Delta y)}\right)+2K^2S=\left(\frac{u_{j+1}^{n+1}-2u_j^{n+1}+u_{j-1}^{n+1}+u_{j+1}^n-2u_j^n+u_{j-1}^n}{2(\Delta y)^2}\right)- M^2\left(\frac{u_j^{n+1}+u_j^n}{2}-R_e-SH\right) \tag{4.10}$$

Thus, Equation (4.10) is one equation in a system of equations for the values of u at the internal nodes of the spatial mesh $(j=2,3,4,....,N-1)$

From equation (4.10) we notice that values of u from time step $n+1$ and time step n appear on the right hand side. Equation (4.10) is used to predict the values of u at time $n+1$, so all values of u at time n are assumed to be known. Rearranging Equation (4.10) so that values of u at time $n+1$ are on the left, and values of u at time n are on the right gives.

$$\begin{aligned}&u_{j+1}^{n+1}\left(\frac{S}{4(\Delta y)}-\frac{1}{2(\Delta y)^2}\right)+u_j^{n+1}\left(\frac{1}{\Delta t}+\frac{1}{(\Delta y)^2}+\frac{M^2}{2}\right)+u_{j-1}^{n+1}\left(\frac{-S}{4(\Delta y)}-\frac{1}{2(\Delta y)^2}\right)=\\&u_{j+1}^{n}\left(\frac{1}{2(\Delta y)^2}-\frac{S}{4(\Delta y)}\right)+u_j^{n}\left(\frac{1}{\Delta t}-\frac{1}{(\Delta y)^2}-\frac{M^2}{2}\right)+u_{j-1}^{n}\left(\frac{1}{2(\Delta y)^2}+\frac{S}{4(\Delta y)}\right)+\left(M^2HS-2K^2S+M^2R_e\right)\end{aligned} \tag{4.11}$$

Multiplying equation (4.11) throughout by $4\,\Delta t(\Delta y)^2$ gives:

$$\begin{aligned}&u_{j+1}^{n+1}\left[S\Delta t\Delta y-2\Delta t\right]+u_j^{n+1}\left[4(\Delta y)^2+4\Delta t+2M^2\Delta t(\Delta y)^2\right]+u_{j-1}^{n+1}\left[-S\Delta t\Delta y-2\Delta t\right]=\\&u_{j+1}^{n}\left[2\Delta t-S\Delta t\Delta y\right]+u_j^{n}\left[4(\Delta y)^2-4\Delta t-2M^2\Delta t(\Delta y)^2\right]+u_{j-1}^{n}\left[2\Delta t+S\Delta t\Delta y\right]+\\&\left[4M^2HS\Delta t(\Delta y)^2+4M^2R_e\Delta t(\Delta y)^2-8K^2S\Delta t(\Delta y)^2\right]\end{aligned} \tag{4.12}$$

The Crank-Nicolson scheme is implicit, and as a result a system of equations for u must be solved at each time step.

If we let the coefficients of interior nodes to be:

$$
\left.
\begin{aligned}
a_j &= \left(-S\Delta t\Delta y - 2\Delta t\right) \\
b_j &= \left(4(\Delta y)^2 + 4\Delta t + 2M^2\Delta t(\Delta y)^2\right) \\
c_j &= \left(S\Delta t\Delta y - 2\Delta t\right) \\
d_j &= u_{j-1}^n\left(2\Delta t + S\Delta t\Delta y\right) \\
e_j &= u_j^n\left(4(\Delta y)^2 - 4\Delta t - 2M^2\Delta t(\Delta y)^2\right) \\
f_j &= u_{j+1}^n\left(2\Delta t - S\Delta t\Delta y\right) \\
g &= \left\{4M^2HS\Delta t(\Delta y)^2 + 4M^2R_e\Delta t(\Delta y)^2 - 8K^2S\Delta t(\Delta y)^2\right\}
\end{aligned}
\right\} \text{For } j = 2,3,4.....N-1 \qquad (4.13)
$$

Therefore using equation (4.13), equation (4.12) becomes:

$$a_j u_{j-1}^{n+1} + b_j u_j^{n+1} + c_j u_{j+1}^{n+1} = d_j + e_j + f_j + g \qquad (4.14)$$

If we let $m = n+1$

For $j = 2$ equation (4.14) becomes:

$$a_2 u_1^m + b_2 u_2^m + c_2 u_3^m = d_2 + e_2 + f_2 + g \qquad (4.15)$$

For $j = 3$ equation (4.14) becomes:

$$a_3 u_2^m + b_3 u_3^m + c_3 u_4^m = d_3 + e_3 + f_3 + g \qquad (4.16)$$

For $j = 4$ equation (4.14) becomes:

$$a_4 u_3^m + b_4 u_4^m + c_4 u_5^m = d_4 + e_4 + f_4 + g \qquad (4.17)$$

If we proceed up to $j = N-1$, the system of equations can be represented in matrix form as:

$$
\begin{bmatrix}
a_2 & b_2 & c_2 & 0 & \cdots & 0 \\
0 & a_3 & b_3 & c_3 & 0 & \vdots \\
\vdots & 0 & \ddots & \ddots & \ddots & 0 \\
0 & 0 & 0 & \ddots a_{N-1} & \ddots b_{N-1} & \ddots c_{N-1}
\end{bmatrix}
\begin{bmatrix} u_1^m \\ u_2^m \\ \vdots \\ u_{N-1}^m \end{bmatrix}
=
\begin{bmatrix} d_2 \\ d_3 \\ \vdots \\ d_{N-1} \end{bmatrix}
+
\begin{bmatrix} e_2 \\ e_3 \\ \vdots \\ e_{N-1} \end{bmatrix}
+
\begin{bmatrix} f_2 \\ f_3 \\ \vdots \\ f_{N-1} \end{bmatrix}
+
\begin{bmatrix} g \\ g \\ \vdots \\ g \end{bmatrix}
\qquad (4.18)
$$

This tridiagonal matrix (4.18) will be solved using MATLAB algorithm inorder to generate the solution.

4.2.2 Induction Equation

Similarly setting $t^* = t$, $u^* = u$ and $h^* = H$ equation (3.40) becomes:

$$H_t + SH_y - u_y = \frac{R_e}{R_m} H_{yy} \tag{4.19}$$

Using equations (4.5), (4.6), (4.7) and (4.8) equation (4.19) becomes:

$$\left(\frac{H_j^{n+1} - H_j^n}{\Delta t}\right) + S\left(\frac{H_{j+1}^{n+1} - H_{j-1}^{n+1} + H_{j+1}^n - H_{j-1}^n}{4(\Delta y)}\right) - u_y = \frac{R_e}{R_m}\left(\frac{H_{j+1}^{n+1} - 2H_j^{n+1} + H_{j-1}^{n+1} + H_{j+1}^n - 2H_j^n + H_{j-1}^n}{2(\Delta y)^2}\right) \tag{4.20}$$

All values of H at time n are assumed to be known. Rearranging Equation (4.20) so that values of H at time $n+1$ are on the left, and values of H at time n are on the right gives.

$$\begin{aligned} &H_{j+1}^{n+1}\left(\frac{S}{4(\Delta y)} - \frac{R_e}{2R_m(\Delta y)^2}\right) + H_j^{n+1}\left(\frac{1}{\Delta t} + \frac{R_e}{R_m(\Delta y)^2}\right) + H_{j-1}^{n+1}\left(\frac{-S}{4(\Delta y)} - \frac{R_e}{2R_m(\Delta y)^2}\right) = \\ &H_{j+1}^{n}\left(\frac{R_e}{2R_m(\Delta y)^2} - \frac{S}{4(\Delta y)}\right) + H_j^{n}\left(\frac{1}{\Delta t} - \frac{R_e}{R_m(\Delta y)^2}\right) + H_{j-1}^{n}\left(\frac{S}{4(\Delta y)} + \frac{R_e}{2R_m(\Delta y)^2}\right) + u_y \end{aligned} \tag{4.21}$$

Multiplying equation (4.21) throughout by $4R_m(\Delta t)(\Delta y)^2$ gives:

$$\begin{aligned} &H_{j+1}^{n+1}[SR_m\Delta t\Delta y - 2R_e\Delta t] + H_j^{n+1}\left[4R_m(\Delta y)^2 + 4R_e\Delta t\right] + H_{j-1}^{n+1}[-SR_m\Delta t\Delta y - 2R_e\Delta t] = \\ &H_{j+1}^{n}[2R_e\Delta t - SR_m\Delta t\Delta y] + H_j^{n}\left[4R_m(\Delta y)^2 - 4R_e\Delta t\right] + H_{j-1}^{n}[2R_e\Delta t + SR_m\Delta t\Delta y] + \\ &\left[4R_m(\Delta t)(\Delta y)^2 u_y\right] \end{aligned} \tag{4.22}$$

If we let the coefficients of interior nodes to be:

$$
\left.\begin{aligned}
l_j &= \left(-SR_m\Delta t\Delta y - 2R_e\Delta t\right) \\
p_j &= \left(4R_m(\Delta y)^2 + 4R_e\Delta t\right) \\
q_j &= \left(SR_m\Delta t\Delta y - 2R_e\Delta t\right) \\
r_j &= H_{j-1}^{n}\left(2R_e\Delta t + SR_m\Delta t\Delta y\right) \\
w_j &= H_j^{n}\left(4R_m(\Delta y)^2 - 4R_e\Delta t\right) \\
v_j &= H_{j+1}^{n}\left(2R_e\Delta t - SR_m\Delta t\Delta y\right) \\
\alpha &= \left(4R_m\Delta t(\Delta y)^2 u_y\right)
\end{aligned}\right\} \text{For } j = 2,3,4.....N-1 \tag{4.23}
$$

Therefore using equation (4.23), equation (4.22) becomes:

$$l_j H_{j-1}^{n+1} + p_j H_j^{n+1} + q_j H_{j+1}^{n+1} = r_j + w_j + v_j + \alpha \tag{4.24}$$

If we let $m = n+1$

For $j = 2$ equation (4.24) becomes:

$$l_2 H_1^m + p_2 H_2^m + q_2 H_3^m = r_2 + w_2 + v_2 + \alpha \tag{4.25}$$

For $j = 3$ equation (4.24) becomes:

$$l_3 H_2^m + p_3 H_3^m + q_3 H_4^m = r_3 + w_3 + v_3 + \alpha \tag{4.26}$$

For $j = 4$ equation (4.24) becomes:

$$l_4 H_3^m + p_4 H_4^m + q_4 H_5^m = r_4 + w_4 + v_4 + \alpha \tag{4.27}$$

If we proceed up to $j = N-1$, the system of equations can be represented in matrix form as:

$$
\begin{bmatrix}
l_2 & p_2 & q_2 & 0 & \cdots & 0 \\
0 & l_3 & p_3 & q_3 & 0 & \vdots \\
\vdots & 0 & \ddots & \ddots & \ddots & 0 \\
0 & 0 & 0 & \ddots l_{N-1} & \ddots p_{N-1} & \ddots q_{N-1}
\end{bmatrix}
\begin{bmatrix} H_1^m \\ H_2^m \\ \vdots \\ H_{N-1}^m \end{bmatrix}
=
\begin{bmatrix} r_2 \\ r_3 \\ \vdots \\ r_{N-1} \end{bmatrix}
+
\begin{bmatrix} w_2 \\ w_3 \\ \vdots \\ w_{N-1} \end{bmatrix}
+
\begin{bmatrix} v_2 \\ v_3 \\ \vdots \\ v_{N-1} \end{bmatrix}
+
\begin{bmatrix} \alpha \\ \alpha \\ \vdots \\ \alpha \end{bmatrix}
\tag{4.28}
$$

This tridiagonal matrix (4.28) will be solved using MATLAB algorithm inorder to generate the solution.

4.2.3 Specific Equation

The momentum matrix (4.18) and magnetic induction matrix (4.28) are solved simultaneously using a MATLAB algorithm.

$$\left.\begin{array}{l}
\begin{bmatrix} a_2 & b_2 & c_2 & 0 & \cdots & 0 \\ 0 & a_3 & b_3 & c_3 & 0 & \vdots \\ \vdots & 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & \ddots a_{N-1} & \ddots b_{N-1} & \ddots c_{N-1} \end{bmatrix} \begin{bmatrix} u_1^m \\ u_2^m \\ \vdots \\ u_{N-1}^m \end{bmatrix} = \begin{bmatrix} d_2 \\ d_3 \\ \vdots \\ d_{N-1} \end{bmatrix} + \begin{bmatrix} e_2 \\ e_3 \\ \vdots \\ e_{N-1} \end{bmatrix} + \begin{bmatrix} f_2 \\ f_3 \\ \vdots \\ f_{N-1} \end{bmatrix} + \begin{bmatrix} g \\ g \\ \vdots \\ g \end{bmatrix} \\
\\
\textit{And} \\
\\
\begin{bmatrix} l_2 & p_2 & q_2 & 0 & \cdots & 0 \\ 0 & l_3 & p_3 & q_3 & 0 & \vdots \\ \vdots & 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & \ddots l_{N-1} & \ddots p_{N-1} & \ddots q_{N-1} \end{bmatrix} \begin{bmatrix} H_1^m \\ H_2^m \\ \vdots \\ H_{N-1}^m \end{bmatrix} = \begin{bmatrix} r_2 \\ r_3 \\ \vdots \\ r_{N-1} \end{bmatrix} + \begin{bmatrix} w_2 \\ w_3 \\ \vdots \\ w_{N-1} \end{bmatrix} + \begin{bmatrix} v_2 \\ v_3 \\ \vdots \\ v_{N-1} \end{bmatrix} + \begin{bmatrix} \alpha \\ \alpha \\ \vdots \\ \alpha \end{bmatrix}
\end{array}\right\} \qquad (4.29)$$

Finally, the resulting coupled block of tridiagonal matrix (4.29) is solved using a MATLAB algorithm and results presented in graphs.

4.3 Error Analysis

Expanding $u(y)$ about the point y_j using Taylor series

$$u(y_j + \Delta y) = u(y_j) + \Delta y \left.\frac{\partial u}{\partial y}\right|_{y_j} + \frac{(\Delta y)^2}{2} \left.\frac{\partial^2 u}{\partial y^2}\right|_{y_j} + \frac{(\Delta y)^3}{6} \left.\frac{\partial^3 u}{\partial y^3}\right|_{y_j} + \ldots\ldots \qquad (4.30)$$

$$u(y_j - \Delta y) = u(y_j) - \Delta y \left.\frac{\partial u}{\partial y}\right|_{y_j} + \frac{(\Delta y)^2}{2} \left.\frac{\partial^2 u}{\partial y^2}\right|_{y_j} - \frac{(\Delta y)^3}{6} \left.\frac{\partial^3 u}{\partial y^3}\right|_{y_j} + \ldots\ldots \qquad (4.31)$$

Subtracting (4.31) from (4.30) yields:

$$u_{j+1} - u_{j-1} = 2\Delta y \left.\frac{\partial u}{\partial y}\right|_{y_j} + \frac{(\Delta y)^3}{3} \left.\frac{\partial^3 u}{\partial y^3}\right|_{y_j} + ... \tag{4.32}$$

Solving for $(\partial u / \partial y)_{y_j}$ gives:

$$\left.\frac{\partial u}{\partial y}\right|_{y_j} = \frac{u_{j+1} - u_{j-1}}{2\Delta y} + \frac{(\Delta y)^2}{3!} \left.\frac{\partial^3 u}{\partial y^3}\right|_{\xi} \tag{4.33}$$

Where $y_j \le \xi \le y_{j+1}$

$$\left.\frac{\partial u}{\partial y}\right|_{y_j} = \frac{u_{j+1} - u_{j-1}}{2\Delta y} + o(\Delta y)^2 \tag{4.34}$$

The term on the right hand side of Equation (4.34) is called the truncation error of the finite difference approximation. It is the error that results from truncating the series in Equation (4.33).

The big O notation can be used to express the dependence of the truncation error on the mesh spacing. Note that the right hand side of Equation (4.34) contain the mesh parameter Δy, which is chosen in finite difference simulation. Since this is the only parameter under the user's control that determines the error, the truncation error is simply written as

$$\frac{(\Delta y)^2}{3!} \left.\frac{\partial^3 u}{\partial y^3}\right|_{\xi} = o(\Delta y)^2 \tag{4.35}$$

The equals sign in this expression is true in the order of magnitude sense. In other words the $o(\Delta y)^2$ on the right hand side of the expression is not a strict equality. Rather, the expression means that the left hand side is a product of an unknown constant and $(\Delta y)^2$. Although the expression does not give us the exact magnitude. It tells us how quickly that term approaches zero as (Δy) is reduced.

Adding equations (4.30) and (4.31) yields:

$$\left.\frac{\partial^2 u}{\partial y^2}\right|_{y_j} = \frac{u_{j+1} - 2u_j + u_{j-1}}{(\Delta y)^2} + o(\Delta y)^2 . \tag{4.36}$$

In general the Crank-Nicolson scheme has a truncation error of

$$o(\Delta t)^2 + o(\Delta y)^2 \tag{4.37}$$

The big o notation expresses the rate at which the truncation error goes to zero. Usually we are only interested in the order of magnitude of the truncation error. For code validation, however, we need to work with the magnitude of the truncation error. Let TE denote the true magnitude of the truncation error for a given (Δt) and (Δy). As $\Delta y \to 0$ and $\Delta t \to 0$ the true magnitude of the truncation error is

$$TE = K_t(\Delta t)^2 + K_y(\Delta y)^2 \tag{4.38}$$

Where K_t and K_y are constants that depend on the accuracy of the Crank Nicolson method and the problem being solved.

To determine whether a reduction in Δt and Δy causes a quadratic reduction in TE, we vary only Δy or Δt.

The result obtained by solving tridiagonal matrix (4.29) and equation (4.38) are discussed in the next chapter.

CHAPTER FIVE

RESULTS AND DISCUSSION

We discuss the results obtained by solving matrix (4.29) using a MATLAB algorithm in the appendix 1. Results are analyzed using graphs and discussed

5.1 Results

5.1.1 Velocity Profiles

To study the effects of rotation and varied magnetic field on the flow-field, velocity profiles are drawn versus the distance between the plates (y) for various values of rotation parameter K^2, magnetic parameter M^2, Reynolds Number R_e and suction/injection parameter S as represented in the figures below.

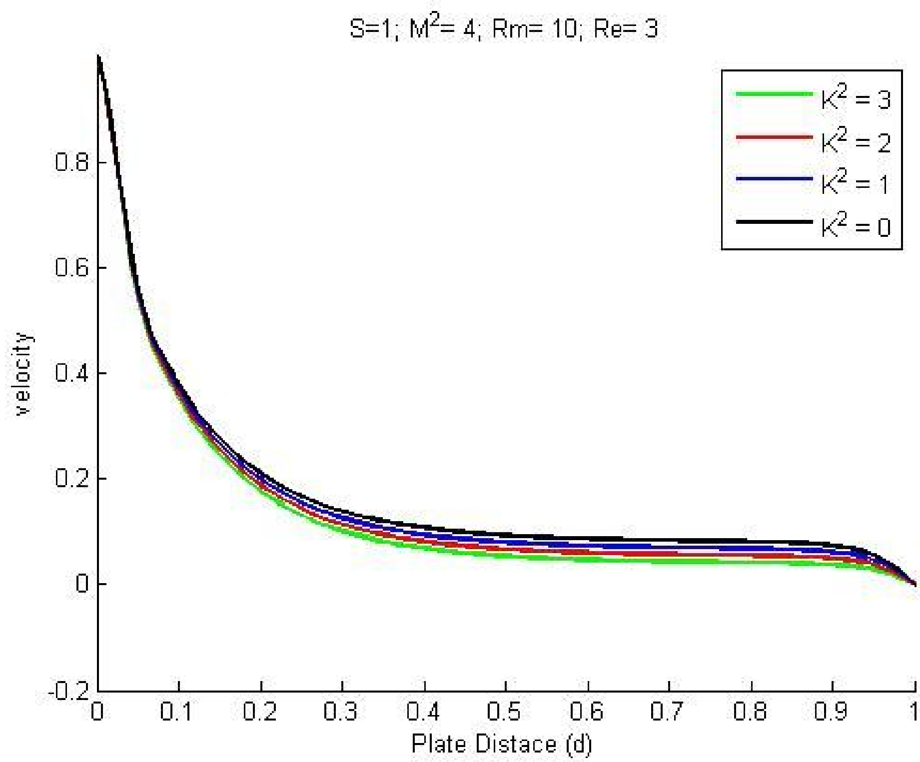

Figure 5. 1: Velocity Profiles varying Rotational Parameter K^2

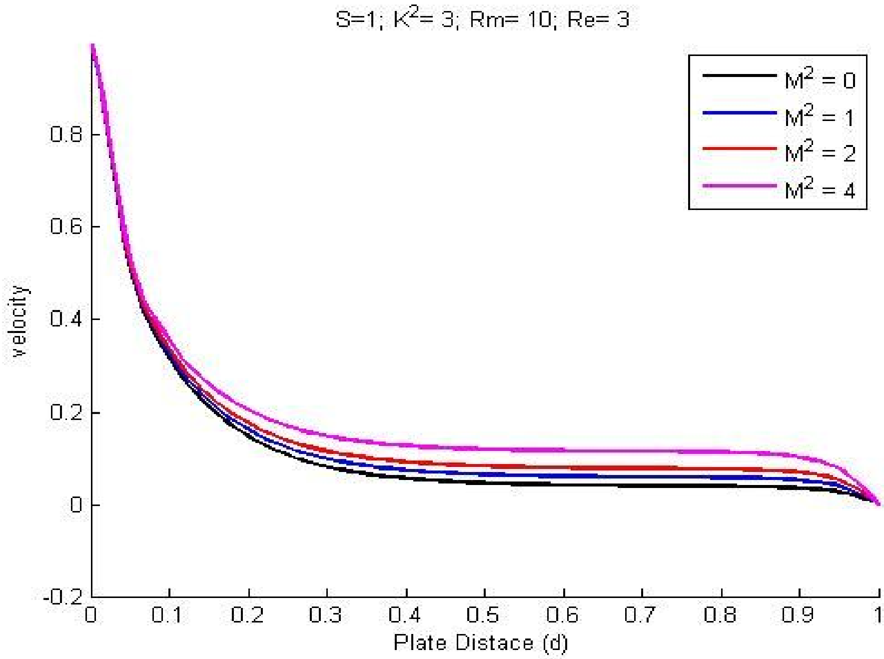

Figure 5. 2: Velocity Profiles varying Magnetic Parameter M^2

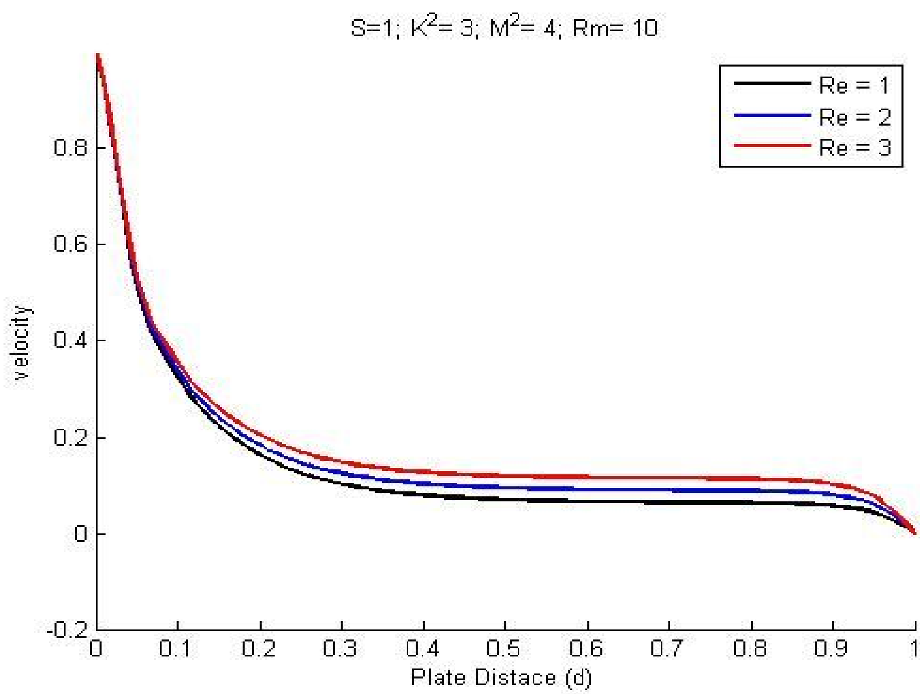

Figure 5. 3: Velocity Profiles varying Reynolds Number R_e

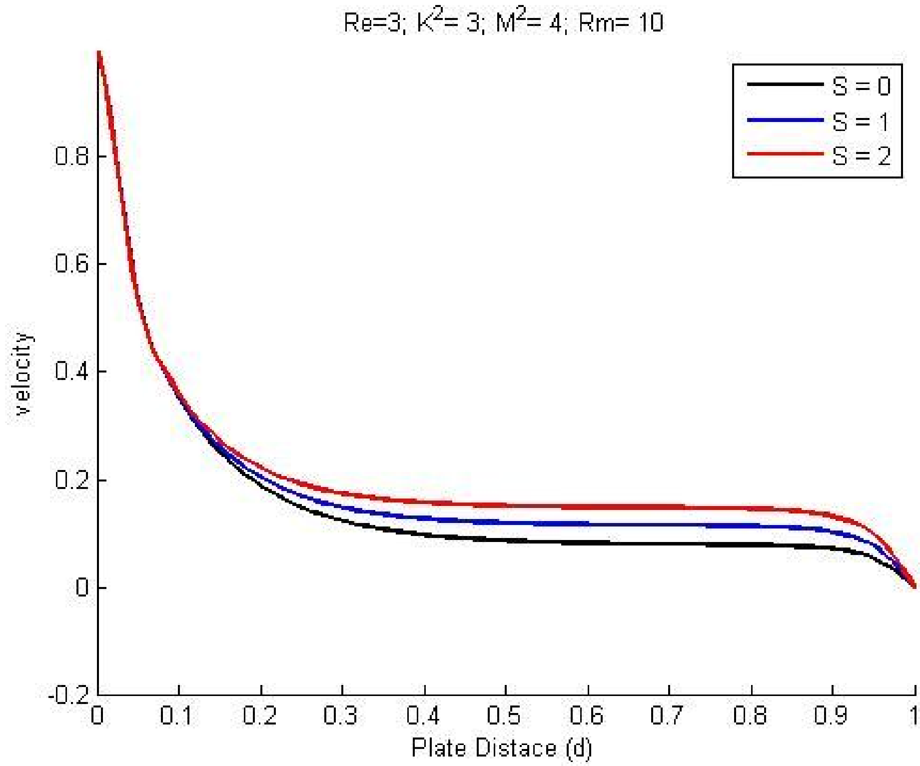

Figure 5. 4: Velocity Profiles varying Injection Parameter S

5.1.2 Induced Magnetic Field Profiles

Induced magnetic field profiles are drawn versus the distance between the plates (y) for various values of injection parameter S, Reynolds Number R_e and Magnetic Reynolds Number R_m as represented in figures below.

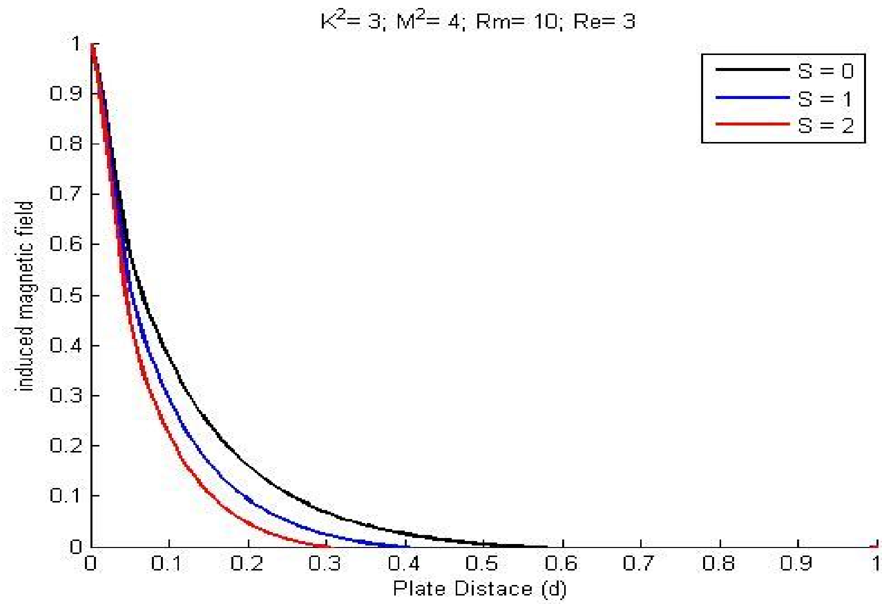

Figure 5. 5: Induced Magnetic Field Profiles varying Suction Parameter S

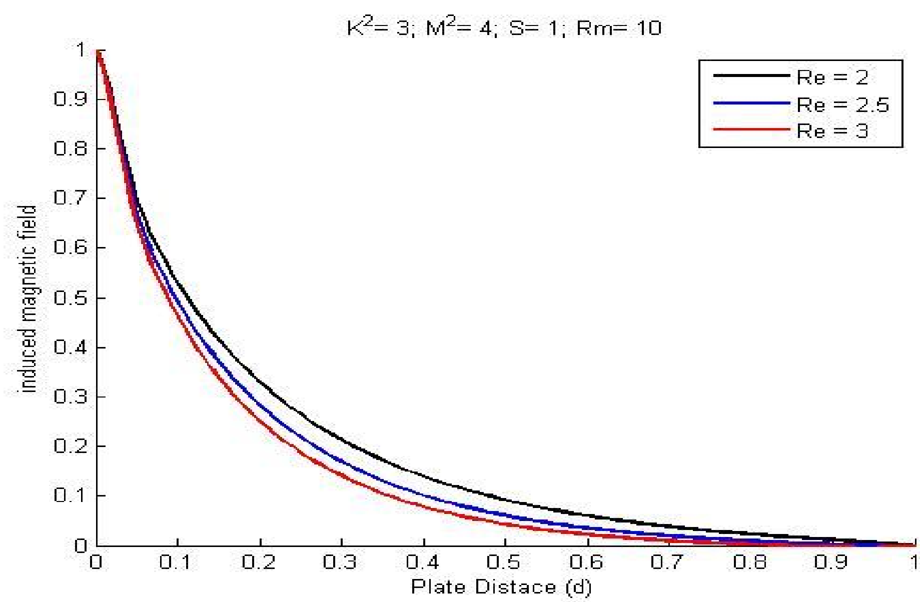

Figure 5. 6: Induced Magnetic Field Profiles varying Reynolds Number R_e

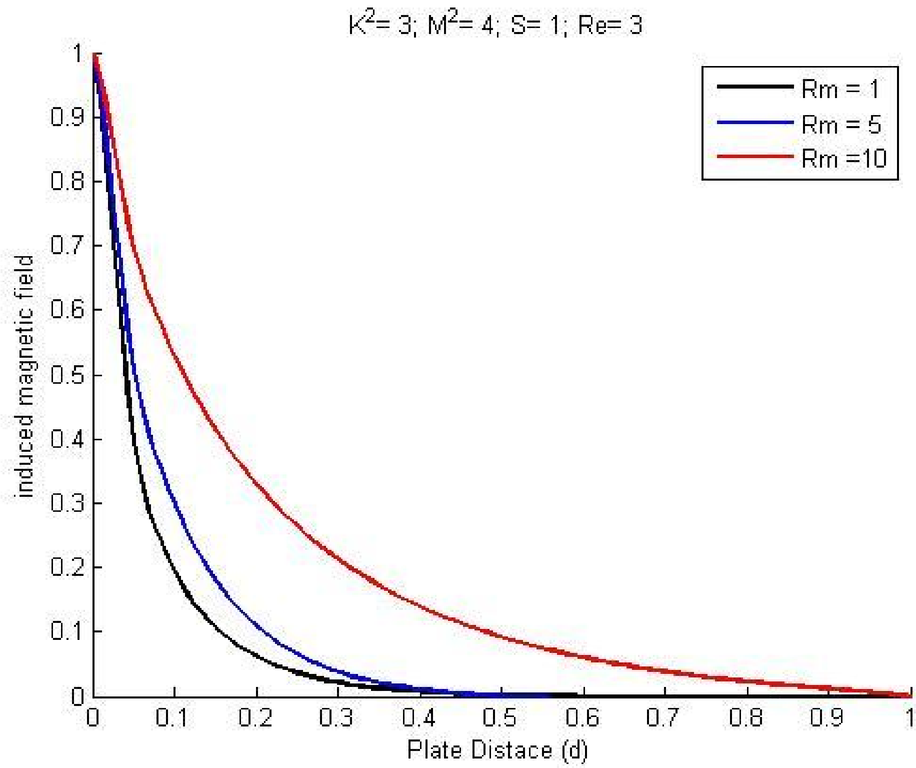

Figure 5. 7: Induced Magnetic Field Profiles varying Magnetic Reynolds Number R_m

5.1.3 Truncation Error

To measure the dependence of TE on Δt, choose a small value of Δy, and hold it constant as Δt is systematically reduced.

To determine whether a reduction in Δt and Δy causes a reduction in TE, we varied only Δy or Δt and the results is as below

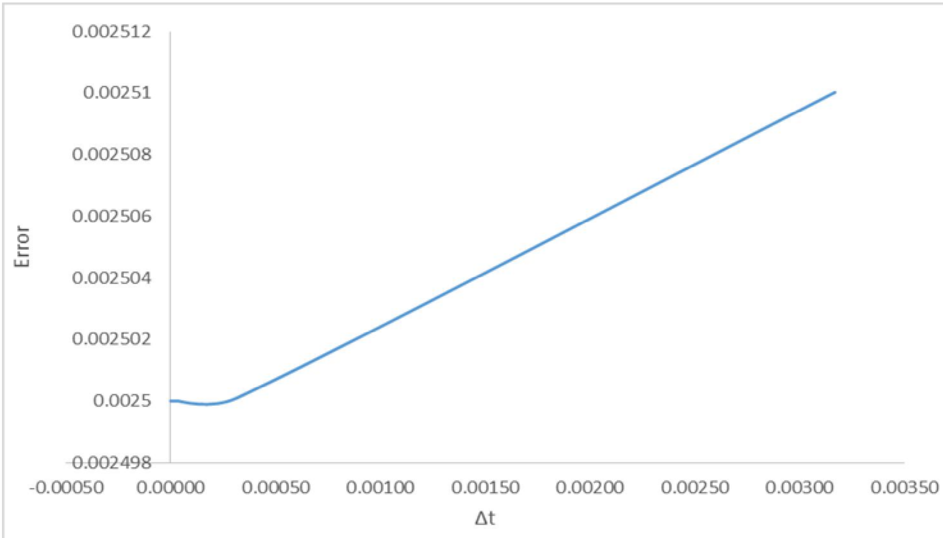

Figure 5. 8: Truncation error as a function of Δt for a fixed $\Delta y = 0.05$.

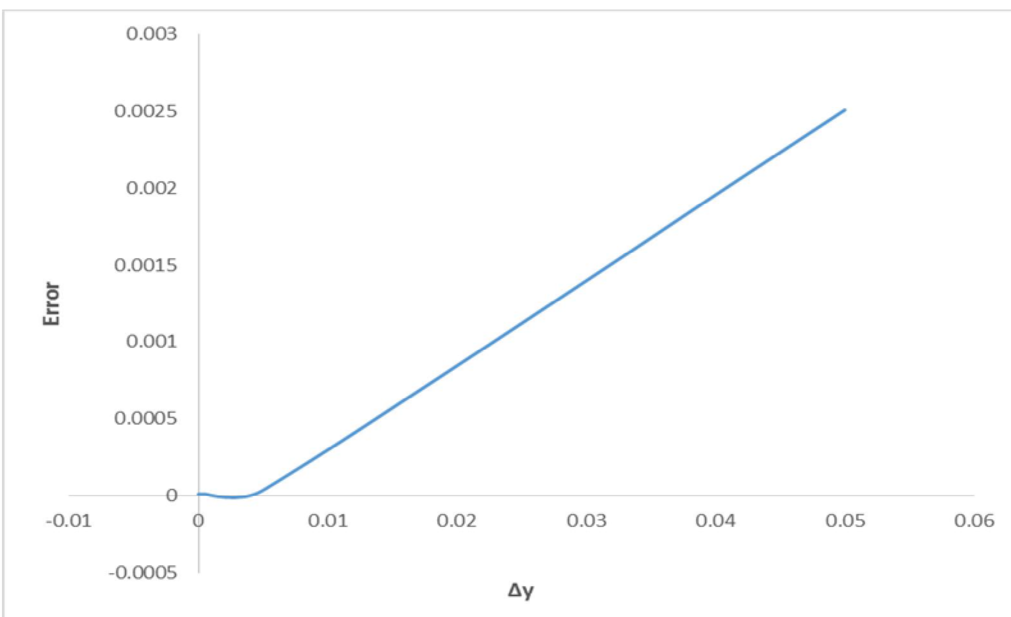

Figure 5. 9: Truncation error as a function of Δy for a fixed $\Delta t = 0.0032$.

5.2 Discussion

5.2.1 Velocity Profiles

From Figure 5.2, Velocity u increases on increasing Magnetic parameter M^2. This is as a result of Lorentz force which acts in the direction of the fluid flow. This force increases the velocity of the fluid. Thus we conclude that the magnetic field has accelerating influence on the fluid flow. Similarly on varying magnetic parameter M^2 and taking magnetic field to be a constant, the result is in agreement with Seth *et al.,* (2011).

From Figure 5.1, Velocity u decreases on increasing rotational parameter K^2. This is justified due to the fact that the Coriolis force acts in a direction perpendicular to the rotation axis which induces secondary flow. Also there exists incipient flow reversal near the stationary plate on increasing K^2 causing the velocity of the fluid to decrease. It is observed that, when magnetic field is taken to be a constant and varying the rotational parameter K^2, the results are in agreement with those obtained by Seth *et al.*, (2009).

In Figure 5.3, Increase in Reynolds number R_e increases the velocity u. Effects of inertial forces are much felt due to decrease in the resistance to shear stress between the layers of the fluid which decreases viscous forces causing the fluid to accelerate.

From Figure 5.4, effects of injection parameter S are much felt in the main stream. This is due to non-slip conditions whereby the layer of fluid in the immediate vicinity of a bounding surface attains the velocity of the boundary. Effects of viscosity reduce as the fluid moves away from the boundary. On increasing injection parameter S, velocity u increases in the case of injection. This is due to decrease in viscous forces which causes the drag between the adjacent fluid layers to reduce hence the fluid accelerates.

5.2.2 Induced Magnetic Field Profiles

From Figure 5.5, increase in injection parameter S, induced magnetic field H decreases in the case of injection this due to decrease in viscous forces which in turn increases the velocity of the fluid particles thus reducing magnetic permeability.

Induced magnetic field H increases in increasing Magnetic Reynolds Number R_m as demonstrated in figure 5.7. This is due to increase in magnetic permeability. This increases the degree of magnetization of the fluid particles in response to an applied magnetic field thus increasing the induced magnetic field.

From Figure 5.6, Increase in Reynolds number R_e decreases the induced magnetic field. At very high Reynolds numbers, inertial forces increases the motion of the fluid and causes eddies to form and give rise to the phenomena of turbulence thus decreasing induced magnetic field.

5.2.3 Truncation Error

From figure 5.8 and figure 5.9, it is observed that Crank Nicolson solutions have truncation error decreases as $o(\Delta t)^2 + o(\Delta y)^2$. As $\Delta y \to 0$ and $\Delta t \to 0$, the truncation errors approach constant values. Further reduction in Δt do not reduce the error because the contribution of the spatial error is fixed (when Δy is fixed).

Based on the results obtained, conclusion is made and future recommendations in the chapter six.

CHAPTER SIX

CONCLUSION AND RECOMMENDATION

6.1 Conclusion

i. Rotation retards the velocity of the fluid. This is due to the presence Coriolis force which induces secondary flow. There also exists incipient flow reversal near the stationary plate

ii. Magnetic field has accelerating influence on the fluid flow, as a result of Lorentz force which acts in the direction of the fluid flow.

iii. On increasing suction/injection parameter S, velocity u increases in the case of injection. This is due to increase in pressure forces which causes the fluid to accelerates.

iv. Increase in Reynolds number R_e increases the velocity u due to decrease in the resistance to shear stress between the layers of the fluid which decreases viscous forces causing the fluid to accelerate.

v. Induced magnetic field H increases in increasing Magnetic Reynolds Number R_m due to increase in magnetic permeability. This increases the degree of magnetization of the fluid particles in response to an applied magnetic field.

6.2 Recommendations

It is recommended that future research should be carried out on:

i. The problem to be studied when varied magnetic field inclined and include secondary flow and temperature profiles.

ii. Studying the problem and solving it using a different numerical technique like shooting method or perturbation method.

iii. The effect of variation of injection velocity.

6.3 Research Paper Published

J. N. Ndung'u, M. N. Kinyanjui, J. K. Sigey and P. R. Kiogora. Determination Of The Effects Of Rotation And Varied Magnetic Field On Unsteady Couette Flow With Injection. International Journal of Engineering Science and Innovative Technology (IJESIT) Volume 4, Issue 1, January 2015, 87-96.

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APPENDICES

a. APPENDIX 1: Publication

Determination of the Effects of Rotation and Varied Magnetic Field on Unsteady Couette Flow with Injection.

J. N. Ndung'u, M. N. Kinyanjui, J. K. Sigey, P. R. Kiogora

Abstract: Unsteady hydromagnetic Couette flow of a viscous incompressible electrically conducting fluid in a rotating system with injection through the lower plate in the presence of a variable transverse magnetic field is studied. The plates are considered porous and fluid flow past the plates is induced by the movement of the lower plate. The upper plate is set stationary. Fluid is injected through the lower plate at a constant velocity. General solution of the governing equations is obtained which is valid for every value of time t. For small values of time t, the solution of the governing equations is obtained by Implicit Finite Difference method of order two. The fluid considered is electrically conducting. The Finite Difference method of order two and a computer program are employed in solving the non-linear equations. The effects of the various parameters entering into the problem are presented graphically and discussed.

Index Terms— Couette Flow, Injection, MHD (Magneto Hydro Dynamic), Suction.

INTRODUCTION

The fundamental concept behind MHD is that magnetic fields can induce currents in a moving conductive fluid, which in turn generates forces on the fluid and the induced electric current changes the magnetic field. The set of equations which describe MHD flow are a combination of the hydrodynamic and Maxwell's equations of electromagnetism. These differential equations have to be solved simultaneously, numerically. Seth *et al* [1] studied the Effects of Rotation and Magnetic Field on Unsteady Couette Flow in a Porous Channel. They considered the lower plate being stationary with suction on the upper plate. They found that magnetic field has tendency to retard the fluid flow in both the primary and secondary flow directions. Guria *et al* [2] investigated oscillatory MHD Couette flow of electrically conducting fluid between two parallel plates in a rotating system in the presence of an inclined magnetic field when the upper plate is held at rest and the lower plate oscillates non-torsionally. Mutua *et al* [3] studied Stokes problem of a convective flow past a vertical infinite plate in a rotating system in presence of variable magnetic field. They concluded that some or all of the parameters affect the primary velocity, secondary velocity and temperature. Consequently their effect alters the rate of heat transfer and skin friction along the x and y axes. Increase in M and Ec leads to an increase in the primary velocity profiles for both free convection cooling and heating at the plate while an increase in the same parameters leads to a decrease in secondary velocity profiles. Naroua *et al* [4] studied computational challenges in fluid flow problems, a MHD Stokes problem of convective flow from a vertical infinite plate in a rotating fluid. Chandran *et al* [5] and Das *et al* [6] studied unsteady hydromagnetic Couette flow of a viscous incompressible electrically conducting fluid in a rotating system when the fluid flow within the channel is induced due to impulsive movement of one of the plates of the channel whereas Singh *et al* [7] considered this problem when one of the plates of the channel is set into uniformly accelerated motion.. From the above cited research analysis of the effects of rotation and varied magnetic field in Couette flow with Injection through the lower plate has not been investigated.

MATHEMATICAL FORMULATION

Consider the unsteady flow of a viscous incompressible electrically conducting fluid between two parallel porous plates of infinite length, distance d apart in presence of a transverse magnetic field **B** applied parallel to *y*-axis which is normal to the plates. The fluid and the plates are in a state of rotation about *y*-axis with uniform angular velocity Ω.

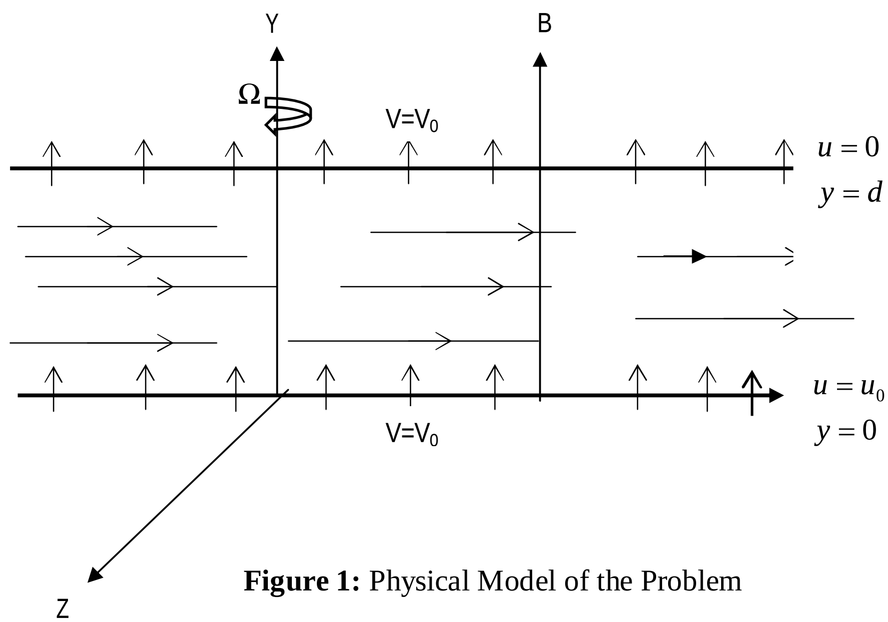

Figure 1: Physical Model of the Problem

Initially when time $t = 0$, the fluid and the plates are assumed to be at rest. When time $t > 0$, the lower plate $(y - 0)$ starts moving with uniform velocity U_0 along x- direction in its own plane while the upper plate $(y = d)$ is kept fixed. Since plates are infinite along x directions and are non-conducting all physical quantities are functions of y and t only. The fluid suction/injection takes place through the porous walls of the channel with uniform velocity V_0 which is greater than zero for suction and is less than zero for injection. It is assumed that no applied or polarization voltages exist since the plates are insulated. This corresponds to the case where no energy is being added or extracted from the fluid by electrical means. (I.e. electric field $\boldsymbol{E} = 0$).The flow drags the field lines along in the x-direction, so the magnetic field acquires a component in the x-direction. Hence, in general Fluid velocity **q** magnetic field intensity **H** and magnetic field ***B*** are given by:

$$\boldsymbol{q} = (u, v_0, 0) \qquad \boldsymbol{B} = (\boldsymbol{b}, \boldsymbol{B_0}, \boldsymbol{0}) \text{ and } \boldsymbol{H} = (\boldsymbol{h}, \boldsymbol{H_0}, \boldsymbol{0}) \tag{1}$$

Following the studies made by Seth *et al* [1], Chandran *et al* [5], Singh *et al* [7], and Hayat *et al* [8] the governing equation for the flow of a viscous incompressible electrically conducting fluid in a rotating frame of reference are:

$$\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} + 2\Omega \bar{v} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} + \upsilon\left(\frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{u}}{\partial y^2}\right) + \frac{1}{\rho}\vec{J} \times \vec{B} \tag{2}$$

Since the plates are infinitely long and fluid motion is induced due to the movement of the lower plate along x-direction, $\frac{\partial u}{\partial x} = 0$, $v = v_0$ and $\frac{\partial p}{\partial x} = 0$, thus (2) reduces to

$$\frac{\partial \bar{u}}{\partial t} + v_0\frac{\partial \bar{u}}{\partial y} + 2\Omega v_0 = \upsilon\frac{\partial^2 \bar{u}}{\partial y^2} + \frac{1}{\rho}\vec{J} \times \vec{B} \tag{3}$$

Following the study by Rossow *et al* [9] when magnetic field is fixed with respect to the moving plate, equation (3) is replaced by:

$$\frac{\partial \bar{u}}{\partial t} + v_0\frac{\partial \bar{u}}{\partial y} + 2\Omega v_0 = \upsilon\frac{\partial^2 \bar{u}}{\partial y^2} - \frac{\sigma\mu_e^2 H_0}{\rho}\left[H_0(\bar{u} - u_0) - \bar{h}v_0\right] \tag{4}$$

The initial and boundary conditions for the problem are

$$\begin{aligned} &u = 0, \quad ; \quad 0 \le y \le d \quad and\ t = 0 \\ &u = 0, \quad ; \quad at\ \ y = d \quad ; \quad t > 0 \\ &u = u_0, \quad ; \quad at\ \ y = 0 \quad ; \quad t > 0 \end{aligned} \tag{5}$$

Introducing the non-dimensional variables;

$$y^* = \frac{y}{d} \qquad u^* = \frac{ud}{\upsilon} \qquad t^* = \frac{t\upsilon}{d^2} \qquad h^* = \frac{h}{H_0} \tag{6}$$

Non dimensionalizing (4) yields:

$$\frac{\partial u^*}{\partial t^*} + S\frac{\partial u^*}{\partial y^*} + 2K^2 S = \frac{\partial^2 u^*}{\partial y^{*2}} - M^2(u^* - R_e - h^* S) \tag{7}$$

Where, $S = \frac{v_0 d}{\upsilon}$ is suction/injection parameter (S<0 for suction and S>0 for injection), $M^2 = \frac{\sigma \mu_e^2 H_0^2 d^2}{\rho \upsilon}$ is magnetic parameter which is the square of Hartman's number, $R_e = \frac{u_0 d}{\upsilon}$ is the Reynolds number and $K^2 = \frac{\Omega d^2}{\upsilon}$ is the rotational parameter which is reciprocal of Ekman number.

The initial and boundary conditions (5) with the help of (6) yields;

$$\begin{aligned} &u^* = 0 \quad ; \quad 0 \le y^* \le 1 \quad and \;\; t^* = 0 \\ &u^* = 0 \quad ; \quad at \;\; y^* = 1 \quad ; \quad t^* > 0 \\ &u^* = R_e \quad ; \quad at \;\; y^* = 0 \quad ; \quad t^* > 0 \end{aligned} \tag{8}$$

Since the magnetic field is variable, we need to solve the induction equation given by

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\mathbf{q} \times \mathbf{B}) + \frac{1}{\sigma \mu_e}\nabla^2 \boldsymbol{B} \tag{9}$$

Expanding equation (9) yields:

$$\frac{\partial h}{\partial t} + v_0\frac{\partial h}{\partial y} - H_0\frac{\partial u}{\partial y} = \frac{1}{\sigma \mu_e}\frac{\partial^2 h}{\partial y^2} \tag{10}$$

The initial and boundary conditions for the problem are

$$h = 0, \quad ; \quad 0 \le y \le d \quad and \;\; t = 0$$

$$h = 0, \quad ; \quad at \;\; y = d \quad ; \quad t > 0 \qquad (11)$$

$$h = H_0, \quad ; \quad at \;\; y = 0 \quad ; \quad t > 0$$

Non-dimensionalizing (10) yields;

$$\frac{\partial h^*}{\partial t^*} + S\frac{\partial h^*}{\partial y^*} - \frac{\partial u^*}{\partial y^*} = \frac{R_e}{R_m}\frac{\partial^2 h^*}{\partial {y^*}^2} \qquad (12)$$

Where, $R_m = \sigma\mu_e u_0 \, d$ is the magnetic Reynolds number. Which is very small for metallic liquids and partially ionized fluids.

The initial and boundary conditions (11) with the help of (6) yields;

$$u^* = 0\,, \;\; h^* = 0 \quad ; \quad 0 \le y^* \le 1 \quad and \;\; t^* = 0$$

$$u^* = 0\,, \;\; h^* = 0 \quad ; \quad at \;\; y^* = 1 \quad ; \quad t^* > 0 \qquad (13)$$

$$u^* = R_e\,, h^* = 1 \quad ; \quad at \;\; y^* = 0 \quad ; \quad t^* > 0$$

COMPUTATIONAL PROCEDURE

Using Crank-Nicolson method and setting $y^* = y, \;\; t^* = t, \;\; u^* = u \;\;\; and \;\; h^* = H$ equation (7) becomes

$$u_t + Su_y + 2K^2S = u_{yy} - M^2(u - R_e - HS) \qquad (14)$$

Thus the expressions for $u,\ u_t,\ u_y\ and\ u_{yy}$ for times n and $n+1$ are:

$$u=\frac{u_j^n+u_j^{n+1}}{2} \tag{15}$$

$$u_t=\frac{u_j^{n+1}-u_j^n}{\Delta t} \tag{16}$$

$$u_y=\frac{u_{j+1}^{n+1}-u_{j-1}^{n+1}+u_{j+1}^n-u_{j-1}^n}{4(\Delta y)} \tag{17}$$

$$u_{yy}=\frac{u_{j+1}^{n+1}-2u_j^{n+1}+u_{j-1}^{n+1}+u_{j+1}^n-2u_j^n+u_{j-1}^n}{2(\Delta y)^2} \tag{18}$$

Therefore equation (14) becomes:

$$\frac{u_j^{n+1}-u_j^n}{\Delta t}+S\left(\frac{u_{j+1}^{n+1}-u_{j-1}^{n+1}+u_{j+1}^n-u_{j-1}^n}{4(\Delta y)}\right)+2K^2S=\left(\frac{u_{j+1}^{n+1}-2u_j^{n+1}+u_{j-1}^{n+1}+u_{j+1}^n-2u_j^n+u_{j-1}^n}{2(\Delta y)^2}\right)-M^2\left(\frac{u_j^n+u_j^{n+1}}{2}-R_e-SH\right) \tag{19}$$

On simplification equation (19) reduces to:

$$u_{j+1}^{n+1}[S\Delta t\Delta y-2\Delta t]+u_j^{n+1}[4(\Delta y)^2+4\Delta t+2M^2\Delta t(\Delta y)^2]+u_{j-1}^{n+1}[-S\Delta t\Delta y-2\Delta t]= u_{j+1}^n[2\Delta t-S\Delta t\Delta y]+u_j^n[4(\Delta y)^2-4\Delta t-2M^2\Delta t(\Delta y)^2]+u_{j-1}^n[2\Delta t+S\Delta t\Delta y]+[4M^2HS\Delta t(\Delta y)^2+4M^2R_e\Delta t(\Delta y)^2-8K^2S\Delta t(\Delta y)^2] \tag{20}$$

If we let the coefficients of interior nodes to be:

$a_j=(-S\Delta t\Delta y-2\Delta t)$

$b_j=(4(\Delta y)^2+4\Delta t+2M^2\Delta t(\Delta y)^2)$

$c_j=(S\Delta t\Delta y-2\Delta t)$

$d_j=u_{j-1}^n(2\Delta t+S\Delta t\Delta y)$ For $j=2,3,\dots,N-1$

$e_j=u_j^n(4(\Delta y)^2-4\Delta t-2M^2\Delta t(\Delta y)^2)$

$f_j=u_{j+1}^n(2\Delta t-S\Delta t\Delta y)$

$g=[4M^2HS\Delta t(\Delta y)^2+4M^2R_e\Delta t(\Delta y)^2-8K^2S\Delta t(\Delta y)^2]$

Therefore equation (20) becomes:

$$a_j u_{j-1}^{n+1} + b_j u_j^{n+1} + c_j u_{j+1}^{n+1} = d_j + e_j + f_j + g \quad (21)$$

If we let $m = n + 1$

The system of equations can be represented in tridiagonal matrix form as:

$$\begin{bmatrix} a_2 & b_2 & c_2 & 0 & 0 & 0 \\ 0 & a_3 & b_3 & c_3 & 0 & 0 \\ 0 & 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & \ddots a_{N-1} & \ddots b_{N-1} & \ddots c_{N-1} \end{bmatrix} \begin{bmatrix} u_1^m \\ u_2^m \\ \vdots \\ u_{N-1}^m \end{bmatrix} = \begin{bmatrix} d_2 \\ d_3 \\ \vdots \\ d_{N-1} \end{bmatrix} + \begin{bmatrix} e_2 \\ e_3 \\ \vdots \\ e_{N-1} \end{bmatrix} + \begin{bmatrix} f_2 \\ f_3 \\ \vdots \\ f_{N-1} \end{bmatrix} + \begin{bmatrix} g \\ g \\ \vdots \\ g \end{bmatrix} \quad (22)$$

Similarly setting $t^* = t, \quad u^* = u \ \ and \ \ h^* = H$ equation (12) becomes:

$$H_t + SH_y - u_y = \frac{R_e}{R_m} H_{yy} \quad (23)$$

The expressions for $H, \ H_t, H_y \ and \ H_{yy}$ for times n and $n + 1$ are:

$$H = \frac{H_j^n + H_j^{n+1}}{2} \quad (24)$$

$$H_t = \frac{H_j^{n+1} - H_j^n}{\Delta t} \quad (25)$$

$$H_y = \frac{H_{j+1}^{n+1} - H_{j-1}^{n+1} + H_{j+1}^n - H_{j-1}^n}{4(\Delta y)} \quad (26)$$

$$H_{yy} = \frac{H_{j+1}^{n+1} - 2H_j^{n+1} + H_{j-1}^{n+1} + H_{j+1}^n - 2H_j^n + H_{j-1}^n}{2(\Delta y)^2}$$

(27)

Therefore equation (23) yields:

$$H_{j+1}^{n+1}(SR_m\Delta t\Delta y - 2R_e\Delta t) + H_j^{n+1}[4R_m(\Delta y)^2 + 4R_e\Delta t] + H_{j-1}^{n+1}(-SR_m\Delta t\Delta y - 2R_e\Delta t) = H_{j+1}^n(2R_e\Delta t - SR_m\Delta t\Delta y) + H_j^n[4R_m(\Delta y)^2 - 4R_e\Delta t] + H_{j-1}^n(2R_e\Delta t + SR_m\Delta t\Delta y) + \left(4R_m(\Delta t)(\Delta y)^2 u_y\right) \quad (28)$$

If we let the coefficients of interior nodes to be:

$$l_j = (-SR_m\Delta t\Delta y - 2R_e\Delta t)$$

$$p_j = [4R_m(\Delta y)^2 + 4R_e\Delta t]$$

$$q_j = (SR_m\Delta t\Delta y - 2R_e\Delta t)$$

$$r_j = H_{j-1}^n(2R_e\Delta t + SR_m\Delta t\Delta y) \qquad \text{For } j = 2,3,\ldots,N-1$$

$$w_j = H_j^n[4R_m(\Delta y)^2 - 4R_e\Delta t]$$

$$v_j = H_{j+1}^n(2R_e\Delta t - SR_m\Delta t\Delta y)$$

$$\alpha = \left(4R_m(\Delta t)(\Delta y)^2 u_y\right)$$

Therefore equation (28) becomes:

$$l_j H_{j-1}^{n+1} + p_j H_j^{n+1} + q_j H_{j+1}^{n+1} = r_j + w_j + v_j + \alpha \qquad (29)$$

If we let $m = n + 1$

This system of equations can be represented in a tridiagonal matrix form as:

$$\begin{bmatrix} l_2 & p_2 & q_2 & 0 & 0 & 0 \\ 0 & l_3 & p_3 & q_3 & 0 & 0 \\ 0 & 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & \ddots l_{N-1} & \ddots p_{N-1} & \ddots q_{N-1} \end{bmatrix} \begin{bmatrix} H_1^m \\ H_2^m \\ \vdots \\ H_{N-1}^m \end{bmatrix} = \begin{bmatrix} r_2 \\ r_3 \\ \vdots \\ r_{N-1} \end{bmatrix} + \begin{bmatrix} w_2 \\ w_3 \\ \vdots \\ w_{N-1} \end{bmatrix} + \begin{bmatrix} v_2 \\ v_3 \\ \vdots \\ v_{N-1} \end{bmatrix} + \begin{bmatrix} \alpha \\ \alpha \\ \vdots \\ \alpha \end{bmatrix} \qquad (30)$$

Matrices (22) and (30) are tridiagonal matrices to be solved simultaneously using MATLAB algorithm subject to initial and boundary conditions (13).

RESULTS AND DISCUSSIONS

Velocity Profiles

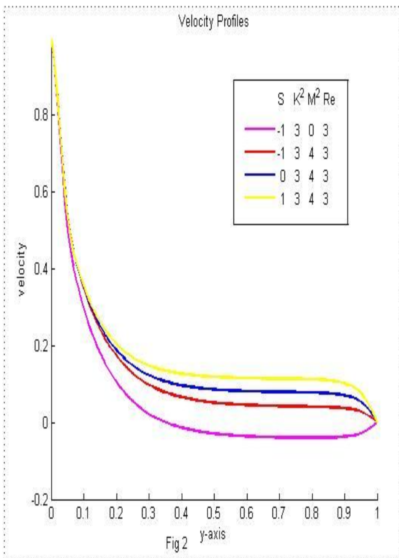

Fig 2

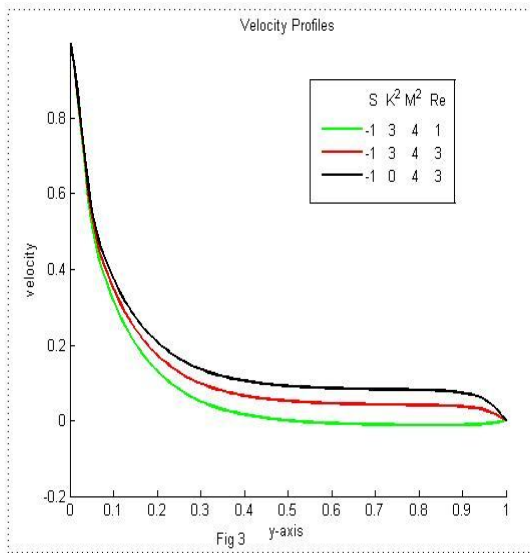

Fig 3

Induced Magnetic Field Profiles

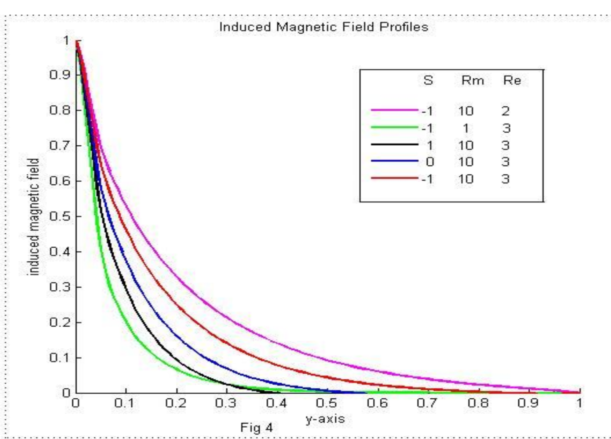

Fig 4

From Fig. 2 Velocity **u** increases on increasing Magnetic parameter $\mathbf{M^2}$. This is as a result of Lorentz force which acts in the direction of the fluid flow. This force increases the velocity of the fluid. Thus we conclude that the magnetic field has accelerating influence on the fluid flow. On increasing suction/injection parameter **S**, velocity **u** increases in the case of suction. This is due to decrease in viscous forces which causes the drag between the adjacent fluid layers to reduce hence the fluid accelerates. While velocity **u** decreases in the case of injection due to increase in shear stress between the layers of the fluid thus reducing the velocity of the fluid.

From Fig 3, Velocity **u** decreases on increasing rotational parameter $\mathbf{K^2}$. This is justified due to the fact that the Coriolis force acts in a direction perpendicular to the rotation axis which induces secondary flow. Also there exists incipient flow reversal near the stationary plate on increasing $\mathbf{K^2}$ causing the velocity of the fluid to decrease. Also increase in Reynolds number **Re** increases the velocity **u** due to decrease in viscous forces which decreases the resistance to shear stress between the layers of the fluid**.**

In Fig 4, it is evident that on increasing suction/injection parameter **S**, induced magnetic field **H** decreases in the case of suction this due to decrease in viscous forces which in turn increases the velocity of the fluid particles thus reducing magnetic permeability. Also induced magnetic field **H** increases in increasing Magnetic Reynolds Number **Rm** due to increase in magnetic permeability. This increases the degree of magnetization of the fluid particles in response to an applied magnetic field thus increasing the induced magnetic field. Increase in Reynolds number **Re** decreases the induced magnetic field. At very high Reynolds numbers, inertial forces increases the motion of the fluid and causes eddies to form and give rise to the phenomena of turbulence thus decreasing induced magnetic field.

CONCLUSION

The effects of rotation and varied magnetic field on unsteady Couette flow with injection was studied. It was found that:

1. Rotation retards the velocity of the fluid. This is due to the presence Coriolis force which induces secondary flow. There also exists incipient flow reversal near the stationary plate
2. Magnetic field has accelerating influence on the fluid flow, as a result of Lorentz force which acts in the direction of the fluid flow.

Authors however recommend future improvement of the model to include secondary flow and temperature profiles.

ACKNOWLEDGEMENT

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NOMENCLATURE

*B***:** Magnetic field strength vector, [wbm^{-2}]

B_o: Magnetic flux intensity along the y-axis, [wbm^{-2}]

b: Magnetic flux intensity along the x-axis, [wbm^{-2}]

g: Acceleration due to gravity vector, [ms^{-2}]

H: Magnetic field intensity vector, [Am^{-1}]

h: Magnetic field intensity along the x-axis, [Am^{-1}]

H_o: Magnetic field intensity along the y-axis, [Am^{-1}]

J: Current density, [AM^{-2}]

d: Dimension distance between plates, [m]

E: Electric field, [v]

ρ_e: Charge density, [cm^{-3}]

Re: Reynolds number

R_m: Magnetic Reynolds number

M^2: Magnetic Parameter

P: Pressure force, [nm^{-2}]

q: Velocity vector, [ms^{-1}]

i,j,k: Unit vectors in x, y, z directions respectively.

u,v,w: Component of velocity vector q, [ms^{-1}]

V_0: Fluid injection velocity, [ms^{-1}]

U_0: velocity of the moving plate, [ms^{-1}]

u^*,v^*: Dimensionless velocity components

x,y,z: Dimensional Cartesian co-ordinates

F_i: Body forces tensor, [N]

U_i: Velocity tensor, [ms^{-1}]

ρ: Fluid density, [kgm^{-3}]

μ: Coefficient of viscosity, [$kgm^{-1}s$]

μ_e: Magnetic permeability, [Hm^{-1}]

σ: Electrical conductivity [Siemens/meter]

K^2: Rotational parameter

S: Suction/Injection parameter

Ω: Angular velocity, [radians/second]

ϑ: Kinematic viscosity, [m^2s^{-1}]

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b. APPENDIX 2: Matlab Computer Code.

```
function JoshNewCode2()
clear all;
clc;
y0=0.0;yJ=1.0;
inJ=20+1;nbc=1;J=inJ+1*nbc;
dy=abs(yJ-y0)/(inJ-1);
y=y0-nbc*dy:dy:yJ;
Crank={};
S=0;K=sqrt(1);M=sqrt(2);Rm=3;Re=3;%to be changed
Crank.S=S;Crank.K=K;Crank.Re=Re;
Crank.M=M;Crank.Rm=Rm;
initVal=1.0;inj=1:2;
clear u
clear h
clear hh
u=zeros(J,1);
h=zeros(J,1);
hh=zeros(J,1);
tend=0.31;
tshow=0.30;
t=0.0;
iteration=0;

cfl=0.95; A3=15;
dt2=cfl*dy/A3;
dt=dt2;
tshow=[tshow inf];
ishow=1;
store_only_special_times=1;
clear t_evolution u_evolution
```

```
if(store_only_special_times==0)
    t_evolution(1)=t;
    u_evolution(:,1)=u;
    h_evolution(:,1)=h;
    hh_evolution(:,1)=hh;
 else
    tstore=tshow;
    tstore=[0 tstore inf];
    ntstore=length(tstore);
    t_evolution=zeros(ntstore,1);
    t_evolution(1,1)=t;
    u_evolution=zeros(J,1,ntstore);
    h_evolution=zeros(J,1,ntstore);
    hh_evolution=zeros(J,1,ntstore);
    u_evolution(:,1,1)=u;
    h_evolution(:,1,1)=h;
    hh_evolution(:,1,1)=hh;
    storestep=2;
end
while(t<tend)
    iteration=iteration+1;
    %time control
    dt=min(dt,tend-t);
    if((t<tshow(ishow))&&(t+dt>tshow(ishow)))
    dt=min(dt,tshow(ishow)-t);
    ishow=ishow+1;
    end
    %update
    [unew hnew hhnew]=CN_solver(Crank,u,h,hh,dy,dt,J,inj);
  if(dt<=dt2)%timestep accepted
    u=unew;
    h=hnew;
```

```
  hh=hhnew;
  t=t+dt;
  dt=dt;
 else%rejected
  dt=dt;
 end

 %boundary update
  u(inj)=initVal;h(inj)=initVal; hh(inj)=initVal;%y0 BC
  u(J)=0; h(J)=0; hh(J)=0;%yJ BC

if(store_only_special_times==0)
      t_evolution(iteration)=t;
      u_evolution(:,1,iteration)=u;
      h_evolution(:,1,iteration)=h;
      hh_evolution(:,1,iteration)=hh;
else
    if(tstore(storestep)<=t)
      t_evolution(storestep)=t;
      u_evolution(:,1,storestep)=u;
      h_evolution(:,1,storestep)=h;
      hh_evolution(:,1,storestep)=hh;
      storestep=storestep+1;
    end
end
  figure(1)
  subplot(1,2,1)
    plot(y(1:J),u(1:J,1),'*-r');grid on
    ylabel('velocity')
    xlabel('y-axis')
  subplot(1,2,2)
    plot(y(1:J),h(1:J,1),'*-b');grid on
```

```
      xlabel('y-axis')
      ylabel('induced magnetic field')
        title(t)
     pause(0.005);

end
    figure(2)
      hold on
      yi=linspace(0,1,120);
      hi=interp1(y,h,yi,'v5cubic','extrap');
      plot(yi,hi,'r','Linewidth',1.5);
      axis([0 1 0 1])
      xlabel('y-axis')
      ylabel('induced magnetic field')
      title('K^2= 3; M^2= 4; Rm= 10; Re= 7 ')
      hold off
  figure(3)
      hold on
      yi=linspace(0,1,120);
      ui=interp1(y,u,yi,'v5cubic','extrap');
      plot(yi,ui,'m','Linewidth',1.5);
      axis([0 1 -0.2 1])
      ylabel('velocity')
      xlabel('y-axis')
      title('K^2= 1; S= 1; Rm= 1; Re= 1')
      hold off
end
function [unew hnew hhnew]=CN_solver(Crank,u,h,hh,dy,dt,J,inj)
K=Crank.K;M=Crank.M;S=Crank.S;Re=Crank.Re;
[e eh ch l dp dhp fp hp]=coeffs(Crank,u,h,hh,dy,dt,J,inj);
unew=zeros(size(u));hnew=zeros(size(u));hhnew=zeros(size(u));
unew(1,1)=u(1);hnew(1,1)=h(1);hhnew(1,1)=h(1);
```

```
unew(J,1)=u(J);hnew(J,1)=h(J);hhnew(J,1)=h(J);
vh(1:2,1)=(unew(2:3,1)-unew(1:2,1)+u(2:3,1)-u(1:2,1))/(4*dy);
vhp(1:2,1)=vh(1:2,1)./l(1:2,1);
for j=inj+1:J-1
unew(j,1)=dp(j,1)-2*S*K*K*fp(j,1)+M*M*Re*fp(j,1)+S*M*M*hp(j,1)-
e(j,1).*unew(j+1,1);
vh(j,1)=(unew(j+1,1)-unew(j-1,1)+u(j+1,1)-u(j-1,1))/(4*dy);
vhp(j,1)=(vh(j,1)+ch(j,1).*vhp(j-1,1))./(l(j,1)+ch(j,1).*eh(j-1,1));
hnew(j,1)=dhp(j,1)+vhp(j,1)-eh(j,1).*hnew(j+1,1);
hhnew(j,1)=(0.5*(h(j,1)+hnew(j,1)));
end
end
function [e eh ch l dp dhp fp hp]=coeffs(Crank,u,h,hh,dy,dt,J,inj)
S=Crank.S;Rm=Crank.Rm;M=Crank.M;Re=Crank.Re;
e=zeros(size(u));eh=zeros(size(h));
d=zeros(size(u));dh=zeros(size(h));
dp=zeros(size(u));dhp=zeros(size(h));
fp=zeros(size(u));hp=zeros(size(u));
%===coefficients for velocity============
b=((S/(4*dy))-(1/(2*dy*dy)))*ones(size(u));
c=((S/(4*dy))+(1/(2*dy*dy)))*ones(size(u));
%===coefficients for magnetic field============
bh=((S/(4*dy))-(Re/(2*Rm*dy*dy)))*ones(size(h));
ch=((S/(4*dy))+(Re/(2*Rm*dy*dy)))*ones(size(h));

miu=(1/(dy*dy))*ones(size(u))+((M*M)/2);
ah=(Re/(Rm*dy*dy))*ones(size(h));
a=(1/dt)*ones(size(miu))+miu;
l=(1/dt)*ones(size(h))+ah;
size(e);
e(1,1)=b(1,1)/a(1,1);
eh(1,1)=b(1,1)/l(1,1);
```

```
for j=inj+1:J-1
e(j,1)=b(j,1)./(a(j,1)+c(j,1).*e(j-1,1));
eh(j,1)=b(j,1)./(l(j,1)+ch(j,1).*eh(j-1,1));
end
d(1,1)=(1/dt-miu(1,1))*u(1,1)-b(1,1)*u(2,1);
dh(1,1)=(1/dt-ah(1,1))*h(1,1)-bh(1,1)*h(2,1);
for j=inj+1:J-1
d(j,1)=((1/dt)*ones(size(miu(j,1)))-miu(j,1)).*u(j,1)+c(j,1).*u(j-1,1)-b(j,1).*u(j+1,1);
dh(j,1)=((1/dt)*ones(size(ah(j,1)))-ah(j,1)).*h(j,1)+ch(j,1).*h(j-1,1)-bh(j,1).*h(j+1,1);
end
dp(1,1)=d(1,1)/a(1,1);
dhp(1,1)=dh(1,1)/l(1,1);
fp(1,1)=1/a(1,1);
hp(1,1)=hh(1,1)/a(1,1);
for j=inj+1:J-1
dp(j,1)=(d(j,1)+c(j,1).*dp(j-1,1))./(a(j,1)+c(j,1).*e(j-1,1));
fp(j,1)=(ones(size(u(j,1)))+c(j,1).*fp(j-1,1))./(a(j,1)+c(j,1).*e(j-1,1));
hp(j,1)=(hh(j,1)+c(j,1).*hp(j-1,1))./(a(j,1)+c(j,1).*e(j-1,1));
dhp(j,1)=(dh(j,1)+ch(j,1).*dhp(j-1,1))./(l(j,1)+ch(j,1).*eh(j-1,1));
end
end
```