HYDROMAGNETIC FLUID FLOW OVER AN IMMERSED AXI-SYMMETRICALBODY WITH CURVED SURFACE IN PRESENCE OF HEAT TRANSFER

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Hydromagnetic Fluid Flow over an Immersed Axi-Symmetrical Body with Curved Surface in Presence of Heat Transfer

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DECLARATION

This thesis is my original work and has not been presented for a degree in any other university.

Signature: ____________ Date: ____________

Mariam Wanjiru Kariuki

This thesis has been submitted for examination with our approval as the university supervisors.

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Dr. Jeconia Okelo,
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DEDICATION

This research thesis is dedicated to my mother Elena Muthoni, my husband Moses and our children Stephanie, Hellenlita and Amoslionel.

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NOMENCLATURE

Romans Symbol	Quantity
A	Area of the curved surface
B	Magnitude of magnetic field
C_p	Specific heat at constant pressure. $Jkg^{-1}\ K^{-1}$
E	External work done
J	Current
L	Reference length, m
m	Real number
O	Order
P	Pressure, Pa
E_c	Eckert number
Pr	Prandtl number
Pe	Peclet number
Re	Reynolds number
q	Quantity of heat added to the system, Joules (J)
q_s	Local wall heat flux, W/m^2
T	Temperature, K
T_s	Temperature of the body’s surface, K
	Free stream velocity, m/s

h	Heat transfer coefficient. h= (-), W/m^2K
u	Outer flow fluid velocity in the x-direction, ms^{-1}
v	Reference fluid velocity in the y-direction, ms^{-1}
w	Fluid velocity in z-direction, ms^{-1}
W	Work done by the system
Fx, Fy	Body forces along the x and y directions respectively
x,y,z	Cartesian co-ordinates
i,j,k	Unit vectors in the x,y and z directions respectively
	Material derivative $\left(= \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right.$
	Gradient operator $\left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y}\right.$
	Laplacian operator $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right.$

GREEK SYMBOLS

	Kinematic viscosity, m^2s^{-1}
	Absolute viscosity (dynamic viscosity coefficient), kg/ms
	Fluid density, kgm^{-3}
	Viscous dissipation function
	Boundary layer thickness, m
	Fluid stresses, Nm^{-2}

ABSTRACT

In this study, convective heat transfer in a magnetohydrodynamics flow over an immersed axi-symmetrical body with curved surface is investigated. The study is aimed at determining the velocity distribution, temperature variation within the thermal boundary layer of hydromagnetic fluid and the effect of heat generated within the boundary of an immersed axi-symmetrical body with curved surface. The magnetohydrodynamic flow in consideration is unsteady and the fluid is assumed to be of constant density. Convective heat transfer is caused by different temperature profiles which bring about temperature gradient. The temperature difference is due to the frictional forces on and within the surface of the body when fluid flows over it. The equations governing the flow over curved surfaces are highly non-linear and a suitable numerical method; finite difference method is used. This method is used because of its stability, convergence and consistency. A computer code is used to obtain results. The results are presented graphically and discussed. It was observed that when magnetic field parameter is increased there was a decrease in both velocity and temperature profiles. These results are applicable in designing devices requiring high maneuverability and low resistance to motion e.g. aerofoil and cooling fans.

CHAPTER ONE
INTRODUCTION AND LITERATURE REVIEW

1.1 Introduction

The theory of convective heat transfer is very important in analysis of thermodynamics of fluid. It involves the natural transfer of heat within the fluid. Depending on the conditions on which the fluid flow is occurring, different fluids have different rates of transfer of dissipated heat. This dissipation of heat is brought about by viscosity of the fluid, density gradients and the nature of the surface of the body in the fluid flow region.

Magnetohydrodynamics is the study of motion of electrically conducting fluids in the presence of a magnetic field. When a conducting fluid or an ionized gas (plasma) flows in magnetic field, an electric field is generated, and electric current is induced at a right angle to the magnetic field. The interaction of the current with the magnetic field changes the motion of the fluid and produces an induced magnetic field.

In this study, a laminar hydro-magnetic fluid is considered. The fluid is flowing over an axi-symmetrical body with curved surfaces. Studies have been done on fluid flows on curved surfaces with axi-symmetrical orientation for instance cylindrical bodies, cones, spheres whereby forces acting on these bodies are investigated. This study concentrates in the boundary layer region.

1.01 Heat Transfer

Heat transfer involves energy in transit as a result of temperature gradient in the medium. This temperature gradient may arise from various causes such as viscous effects, release of latent heat as fluid vapour condenses, and absorption of thermal radiation or radioactivity. Heat transfer takes place mainly in three modes; conduction, convection and radiation. This study is only concerned with convective heat transfer over an axi- symmetrical body with curved surfaces.

1.02 Convection

Convection refers to the heat transfer by circulation or movement of the heated parts of a liquid or a gas. Convective heat transfer is due to the superposition of energy transport by random molecular motion (diffusion) and by advection (the bulk or macroscopic fluid motion). The contribution due to bulk fluid motion originate from the fact that boundary layer grows as the flow advances. Convection laws rely on the fundamental principles of both heat transfer and fluid flow which include law of conservation of mass, law of conservation of momentum and law of conservation of energy.

Convective heat transfer depends on viscosity, thermal conductivity, specific heat and density. Viscosity influences the velocity profiles of the fluid flow. Convective heat transfer may be categorized as either natural convection or forced convection whereby in forced_convection, flow is caused by some external force like the case of a fan, a pump or atmospheric winds while in natural convection the flow is induced by buoyancy forces resulting from density gradients as a result of temperature gradients in the fluid. The density of the fluid on the boundary layer changes on heating, causing the fluid to rise and be replaced by cooler fluid that also will heat and rise. This continues and is a phenomenon called natural convection. In free convection the driving force for the fluid motion is gravity field acting on the density difference. The density gradients are due to temperature and concentration gradients existing in the fluid while the body force is due to the gravitational field. When the body forces act on the fluid there results a buoyancy force that induces free convectional currents. Forced and natural type of heat convection may occur together in a phenomenon called mixed convection.

1.03 Fluid

Fluid is a state of matter which under given thermodynamic conditions and in absence of external forces takes the shape of the container. Fluid motion may be constrained by geometrical boundaries to be predominantly parallel to the sides. When flow variables (pressure, velocity and temperature) at all successive cross sections are identical at any instant, the flow is uniform otherwise it is non-uniform. Fluid is considered incompressible if the density is assumed to be invariant otherwise is compressible if its density is a variable.

A fluid flow is steady if its velocity and the thermodynamic properties at each point in the flow region do not change and is independent of time; otherwise it will be unsteady if the flow is dependent of time. Fluid flow may be termed as laminar or turbulent. The term laminar refers to a fluid flow in which fluid particles downstream of the leading edge moves in an orderly manner in laminas or layers parallel to the solid boundary as opposed to turbulent whereby fluid velocity components have random turbulent fluctuations imposed upon their mean values. Turbulent fluid motion is an irregular condition of flow in which various quantities like velocity and pressure show random variation with time and space. Turbulent flow is also characterized by eddies that causes mixing of layers of fluid until the layers are no longer distinguishable. This mixing and collision of fluid particles produces heat and the greater the turbulence the larger the amount of heat transfer, as these increased collisions lead to increased dissipation of heat. A fluid can also be ideal or real, whereby if it is assumed that if there exists no frictional effect between the fluid layers and the boundary walls then it is regarded as ideal, otherwise real.

1.04 Viscosity

Viscosity is the resistance that occurs due to shear stresses within the fluid particles and the shear stresses between the fluid particles and the solid surface for a fluid flowing around a solid body. As fluid exerts a shear stress on the boundary, the boundary exerts an equal and opposite force on the fluid called shear resistance (frictional drag). Drag coefficient (C_d) always depends on the Reynolds number (Re) and the shape of the body. The work done against the viscous effects usually causes fluid flow, and consequently the energy spent in doing so is converted to heat. At low values of Reynolds number, the fluid is highly viscous causing deformation drag, the fluid is deformed on a wide zone around the body which brings about pressure force and frictional force. At large values of Reynolds number, the fluid is less viscous, for example in water and air, and the viscous effects is limited to the boundary layer thickness. In this case deformation drag is exclusively friction drag. The shear force exerted on the surface of the body due to the formation of boundary layer results into friction drag.

1.05 Boundary Layer

Boundary layer is a thin layer of fluid particles adjacent to the surface of a body or solid wall in which viscous forces exist. The fluid particles in contact with the solid body surface attain the velocity of the body. The region outside this layer is called freestream region where the flow is unaffected by viscous forces. Boundary layer thickness theory is important in analyzing flow problems involving convective heat transfer. The physical significance of the boundary layer is that it is the region that determines the magnitude of the surface friction and convective heat transfer in a fluid.

1.06 Velocity Boundary Layer

When fluid particles of a real fluid are in contact with a flat surface, their velocities are retarded gradually. These particles then act to retard the motion of the particles of the adjoining fluid layer which in turn acts to retard the motion of the particles in the next layer. The process continues until the effect is negligible. The velocity boundary thickness is defined as the distance away from the plate's surface where the velocity reaches 99% that of free –stream velocity.

1.07 Thermal Boundary Layer

Thermal boundary layer develops if the temperature of the fluid at the surface of the immersed body and the free stream temperature differ. Fluid particles that come into contact with the solid body attain the same temperature as the temperature of the surface of the solid. In turn these particles exchange heat energy with those in the adjacent fluid layers and the temperature gradients develops in the fluid. The region in the fluid in which these temperature gradients exist is the thermal boundary layer.

1.08 Lift and Drag

The sum of all forces on a body that acts perpendicularly to the direction of the flow is referred to as lift. This force occurs when fluid flows over stationary solid body. On the other hand, drag is the force parallel and opposite to the direction of motion of an object immersed in a flowing fluid. Drag takes two forms; pressure drag which is dependent on the shape of the object immersed in the flowing fluid and other form of skin friction which is dependent on the viscous friction between a surface of a solid body and flowing fluid.

1.09 Joule Heating

Joule heating is also known as ohmic or resistive heating. It is the process by which the passage of an electric current through a conductor releases heat. Joule heating was first studied by James Prescott Joule in 1841. When an electric current passes through an electrolyte, it causes joule heating. The increase in kinetic or vibrational energy of particles manifests itself as heat, and heat causes a rise in temperature of the fluid. The rise in temperature of the fluid translates to non-uniform properties of the fluid.

1.10 Literature Review

The concept of magnetohydrodynamics was first introduced by Hartman (1938) when he studied the effects of a conductor in an electrically conducting fluid. The important point is that the flow of electrically conducting fluid such as mercury under a magnetic field, in general, gives rise to an induced electric current. Much of the work in hydromagnetic was done by Alfven, H (1942) who established transverse waves in electrically conducting fluid and explained many astrophysical phenomena with it.

Lundquist (1949) performed laboratory experiments that produced electromagnetic-hydromagnetic waves in a magnetized mercury, with a velocity that approximated Alfven's formula. The theory of convective heat transfer strongly emerged in 20^{th} century. By its nature convective energy transfer is closely related to fluid particles motion and therefore is a fundamental part of fluid mechanics study. Advancement in research in fluid mechanics have greatly influenced the theory of heat and mass transfer in moving media such as air ,water, and oil. The relationship between the intensities of turbulent momentum and heat transfer process is one of the problems of heat transfer theory.

A German aero dynamist Prandtl (1904), established that a flow of large Reynolds number means that it has a low viscosity (low frictional forces associated during flow). If the viscosity of the fluid is low then the effects of friction will be confined to a very thin layer known as the boundary layer near the solid body while the region outside the boundary layer can be considered frictionless or ideal i.e. in this region the fluid is assumed to be inviscid or non-viscous. For a flow with high Reynolds number, the viscous forces in region near the boundary layer will dominate over the inertia forces and the effects of the viscosity will be very important in the boundary region, and as a result shear forces will be very high due to the extremely high velocity gradients at and near the boundary layer.

Barenblatt (2002), in their study on the model of the turbulent boundary layer with non-zero pressure gradient observed that the turbulent boundary layer at large Reynolds number consists of two separate layers upon which the structure of the vortex fields is different, although both exhibit similar characteristics. In the first layer, vertical structure is common to all developed bounded shear flows and the mean flows. The influence of viscosity is transmitted to the main body of flow via streaks separating the viscous sub layer. The second layer occupies the remaining part of the intermediate region of the boundary layer. The upper part of the boundary layer is covered with statistical regularity by large scale “humps” and the upper layer is influenced by the external flow via the pressure drag of these humps as well as by the shear stress. In their earlier studies, it is indicated that the mean velocity profile is affected by the intermittency of the turbulence and as the humps affects intermittency, the two seeking regions are visible. On the basis of these considerations, the effective Re, which determines the flow structure in the first layer (and is affected in turn by the viscous sub layer), was identified as one set of such parameters. The other parameters that influence the flow in the upper layer include pressure gradient $_xP$; dynamic (friction) viscosity ; velocity u; fluid’s kinematic viscosity and density .

In recent past, many researchers have been attracted to solving the boundary layer equations. Smith (1963) in one of their papers presented a method for solving the complete incompressible laminar boundary layer equations; both for two dimensional and axi-symmetrical laminar flow, essentially full generality and with speed. In subsequent papers (1970, 1972), Smith discussed application potential flow and the boundary layer theory in the hydrodynamics, they also provided a solution technique of the laminar boundary layers by means of the differential difference method.

Wehrle (1986) presented a paper on analytical shears for the determination of the separation point in the laminar boundary layer. Unlike conventional approaches the scheme does not require the full-filed solution of the governing partial difference equations, but rather the solution of a first order set of boundary layer equations defined in the neighborhood of the leading edge. Continuing interest in flows and heat transfer over flat plate, concave , convex surfaces stems from their possible effects in the turbine blades of jet engines, vehicle aerodynamics ,aircraft wings, submarines, spaceship, cooling plants power plants e.t.c. flow phenomenon are mainly subjected to pressure gradients (favorable or adverse), surface curvature and a wide range of Reynolds number.

There have been many previous investigation of flow and heat transfer on flat plate boundary layers with pressure gradients. Fukagata (2002) were concerned with transition to turbulent flow and the Reynolds stress distribution, while Umur and Karagoz (1999) dealt with augmentation of heat transfer with or without stream wise pressure gradients. Filippova, and Hanel (1998) developed a curved boundary treatment using Taylor's series expansion in both space and time for single particle distribution near the wall. This boundary condition satisfies the no-slip condition to the second order in a space step and preserves the geometrical integrity of the wall boundary.

Mei, ,Luo & Shyy (1999) and Bouzidi (2001) proposed some other boundary treatment methods. In all those methods, the boundary conditions were treated separately for some specific steps when some variations occurs in the specified steps while dealing with curved boundaries, and an abrupt change in the single particle mass distribution was caused. In the turbo machinery applications; a variations in the rate of heat transfer due to a small flow disturbance can lead to an increase in the thermal stress and decrease the effective working life span of such a component. On a highly curved wall, the change in heat transfer rate is mainly due to an increase or decrease of the turbulent mixing by effect of streamline curvature. It has been indicated in Karman's (1934) stability argument that the convex wall has a stabilizing effect on the fluid particles, while concave wall has a de-stabilizing effect with reference to a flat plate.

The measurement and prediction of the rate of heat transfer for a two dimensional boundary layer on a concave surface were presented by Mayle (1979). It was indicated that the heat transfer on the convex surface was less than that of a flat surface having the same free stream, Re and turbulence. Concave surface heat transfer was augmented when compared to the flat surface. One area of practical interest to researchers is on the degradation of aerofoils .Aerofoils form a crucial part of aviation and air conditioning systems.

Omboro(2009) in his study on the convection heat transfer in a fluid flow over a curved surface established that as fluid flows over an immersed curved surface, some work is done against viscous effects and energy spent is converted into heat and also vorticies formed in the boundary layer due to high velocity gradient is swept towards the edge. Mugambi (2008) in their research, an investigation of forces produced by fluid motion on a submerged finite curved plate, they established a relationship between geometrical shape of the curvature and the variation of drag force of specific velocities of the viscous fluid.

Kioi(2011) studied convective heat transfer in homogenous fluid flow of Reynolds number of order less than 2000 over an immersed axi-symmetrical body with curved surface. In their study, it was noted that when Reynolds number is increased, the dissipation also increased. When the curvature of the surface was increased, the heat dissipation also increased. However they did not establish the effect of magnetism on velocity and temperature over immersed curved surface which will be investigated in this study.

1.11 Statement of Problem

This study is on the analysis of convective heat transfer in magnetohydrodynamics over an immersed axi-symmetrical body with curved surface. A stationary curved body is immersed in an ambient fluid with surface temperature which is the same as the surrounding fluid. The flow of an electrically conducting fluid is horizontal along x- axis. There is application of constant magnetic field along the y-axis. The hydrodynamic flow field is axi-symmetrical and the fluid possesses constant thermophysical property with the exceptional of those caused by density changes which generate the buoyancy forces. The energy converted into heat within the boundary layer is transferred from this boundary layer through convection into the rest of the region

A lot has been done with regard to heat transfer but less has been done in regard to how magnetism affects velocity and temperature profiles in the boundary layer of immersed bodies with curved surfaces. Convective heat transfer forms the basis of this research.

1.12 Justification

In our everyday day life people encounter cost maintenance brought about by degradation of equipment and machines whose parts come into contact with a fluid and this has become a major concern. Heat produced due to viscosity on the body surfaces leads to the degradation of equipment and machines which has led to high cost of maintenance.

Magnetohydrodynamic convection flow has many important applications in the design of power generators, heat exchangers, pumps and flow meters, in solving space vehicle propulsion, control and re-entry problems, in designing communications and radar systems. Heat injection or heat withdrawal on immersed curved surface enhance velocity variations in the hydrodynamic flow thereby improving the maneuverability of such bodies in the fluid as in the case of submarines in water, wings of flying planes, fan blades in the air conditioning system and in computer cooling appliances.

1.13 Hypothesis

There exist no relationship between convective heat transfer and the shape of an axi-symmetrical body with curved surface.

1.14 Objective of the Study

1.14.1. General Objective

The general objective of this study is to determine the velocity regimes and temperature profiles that occur when hydromagnetic fluid flows past an immersed axi-symmetrical body with a curved surface.

1.14.2 Specific Objective

1. To determine the velocity profiles of hydromagnetic fluid flow past an immersed axi-symmetrical body with curved surface.
2. To determine the temperature profiles within the thermal boundary layer of the hydromagnetic fluid flow past the immersed axi-symmetrical body with curved surface due to velocity variation.
3. To determine the effect of heat generated on boundary of an immersed axi-symmetrical body with curved surfaces on drag and lift

CHAPTER TWO

MATHEMATICAL ANALYSIS

2.1 Introduction

In this chapter, equations governing the flow of an incompressible, Newtonian fluid over an axi symmetrical body with curved surface are discussed. The fundamental equations that are considered include; mass conservation equation, Maxwell equations and equation of energy. Also description of flow and dimensional analysis of equations that govern this fluid flow problem is done in this chapter.

2.2 Assumptions and Approximations

In order to describe the flow problem mathematically the following approximations and assumptions are made;

1. All velocities are small compared with that of light   <<1
2. Flow is restricted to laminar domain, i.e., the region being considered is the laminar boundary layer.
3. Fluid is incompressible (density   assumed constant)
4. Fluid has constant thermal conductivity, constant electrical conductivity, and constant coefficient of viscosity.
5. The fluid flow is unsteady
6. Force $\rho_e E$ due to electric field is negligible compared to the force ***JxB*** due to magnetic field.
7. The velocity component u along the surface of the body is much larger than the velocity component v normal to the surface of the body.
8. Radius of the curvature is greater than zero.

9. The length of the boundary layer is large compared to boundary layer thickness.

2.3 Equation Governing the Fluid Flow

The fundamental equations of fluid dynamics are based on the following universal laws of conservation. i.e. conservation of mass, momentum, and energy.

2.3.1 Equation of Continuity

The equation of continuity is a mathematical statement in any process where the rate at which mass enters a system is equal to the rate at which mass leaves the system. This equation combines the law of conservation of mass and that of transport theorem.

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho$$

(2.1)

Where and are fluid density and fluid velocity vector respectively.

In Cartesian coordinate form, equation (2.1) is expressed as;

$$\frac{\partial \rho}{\partial t} + \rho \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial}{} \right. \qquad (2.2)$$

For an incompressible two-dimensional fluid flow, w = 0 and hence (2.2) reduces to;

$$\frac{\partial u}{\partial x} + \qquad (2.3)$$

2.3.2 Equation of Conservation of Momentum

The equation of conservation of momentum is derived from the Newton's second law of motion, which states that the time rate change of momentum of a body matter is equal to the net external forces applied to the body. This external force is divided into two types of forces; surface forces (e.g. force due to static pressure and viscous stresses) and body forces (e.g. gravitational force, centrifugal force, magnetic force or electric fields)

The momentum along the x-axis

$$\frac{\rho \partial u}{\partial t} + \rho \left(u \frac{\partial u}{\partial x} + \; v \frac{\partial u}{\partial y} \right) = - \frac{\partial P}{\partial x} + \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} \right) + \; \rho \, F_x \qquad (2.4)$$

The momentum equation along the y-axis become;

$$\rho \frac{\partial v}{\partial t} + \; \rho \left(\; u \frac{\partial v}{\partial x} + \; v \frac{\partial v}{\partial y} \right) = - \frac{\partial p}{\partial y} + \left(\frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} \right) \qquad (2.5)$$

The viscous stresses and shear stresses in two dimensions are defined by;

$$\sigma_{xx} = 2\mu \frac{\partial u}{\partial x} - \; \frac{2}{3} \mu \left(\frac{\partial u}{\partial x} + \; \frac{\partial v}{\partial y} \right) \qquad (2.5a)$$

$$\sigma_{yy} \; = 2\mu \; \frac{\partial v}{\partial y} - \; \frac{2}{3} \; \mu \; \left(\frac{\partial u}{\partial x} + \right. \qquad (2.5b)$$

$$\tau_{xy} = \; \tau_{yx} = \; \mu \left(\frac{\partial u}{\partial y} + \; \frac{\partial v}{\partial x} \right) \qquad (2.5c)$$

Substituting equations (2.5a,b,c) into equations (2.4) and (2.5) the following momentum equation along the x-axis and y-axis was obtained.

Along x-axis;

$$\rho \frac{\partial u}{\partial t} + \; \rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right)$$

$$= \; - \frac{\partial P}{\partial x} + \frac{\partial}{\partial x} \left\{ \mu \left[2 \frac{\partial u}{\partial x} - \frac{2}{3} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right] \right\} + \frac{\partial}{\partial y} \left[\mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right. \qquad (2.6a)$$

Along the y-axis;

$$\rho\frac{\partial v}{\partial t}+\rho\left(u\frac{\partial v}{\partial x}\right.$$

$$=-\frac{\partial P}{\partial y}+\frac{\partial}{\partial y}\left\{\mu\left[2\frac{\partial v}{\partial y}-\frac{2}{3}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right]\right\}+\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right.\right. \quad (2.6b)$$

Since; $\frac{\partial u}{\partial x}+$ equations (2.6a) and (2.6b) reduce to

$$\rho\frac{\partial u}{\partial t}+\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial P}{\partial x}+2\mu\frac{\partial^2 u}{\partial x^2}+\mu\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 v}{\partial x\partial y}\right. \quad (2.7a)$$

$$\rho\frac{\partial v}{\partial t}+\rho(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial P}{\partial y}+2\mu\frac{\partial^2 v}{\partial x^2}+\mu\left(\frac{\partial^2 u}{\partial y\partial x}+\frac{\partial^2 v}{\partial y^2}\right)+ \quad (2.7b)$$

From the boundary layer approximations in this study, the distance under consideration is very small than the boundary thickness to the extent that the velocity component in the direction along the surface is much larger than that normal to the surface. Hence the gradients normal to the surface were larger than those along the surface. i.e. $\frac{\partial u}{\partial y}\gg\frac{\partial}{\partial}$ and . From this approximation (2.7a) and (2.7b) reduced to

$$\frac{\partial u}{\partial t}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\vartheta \quad (2.8a)$$

and

$$0=- \quad (2.8b)$$

respectively.

From the Bernoulli's equation;

$$P+\frac{1}{2}\rho u^2=c \quad (2.9)$$

The curved surfaces provides both adverse and favourable pressure gradient (i.e this is where the pressure decreases in the direction of the flow the physical effect is to accelerate the flow, the boundary layer remains attached to the surface and tends to reduce in thickness and this is termed as favourable pressure gradient) whose tangential component of the velocity of the outer flow reveals a power law dependence on the streamwise x measured along the curved surface boundary as;

$$u \tag{2.10}$$

where c is a positive velocity coefficient and m is an integer obtained from the angle of inclination. This integer m is given as n where is the angle in radians of the inclination at a given point from the horizontal plane. Let denote the angle. Then r differentiating partially equation (2.9) with respect to x, we obtained

$$\frac{\partial P}{\partial x} + \rho u \tag{2.11}$$

this implied that

$$-\frac{1}{\rho}\frac{\partial}{\partial} \tag{2.12}$$

But for the power law dependence

$$u\frac{\partial u}{\partial x} = c^2 \tag{2.13}$$

Hence equation (2.8a) became

$$\frac{\partial u}{\partial t} = P_t + \vartheta\frac{\partial}{\partial} \tag{2.14}$$

where $P_t = c^2$

The body in consideration has a concave surface. The curvature effect on the fluid flow has to be taken in consideration. The concave surface brought about an unstable effect which was determined by . The curved surface is a curve and was defined by a quadratic equation of the form

$$y = ax^2 \tag{2.15}$$

Where O<a<1 is set to ensure surface radius of the curvature is large enough and the end points are set at a specific co-ordinates values when solving for a particular case where length of the plate curvature is determined analytically. The concave wall exerts a destabilizing influence on the momentum exchange. Prandtl proposed to account for curvature effect by multiplying the length of the curved surface by a factor f which was a function of dimensionless curvature parameter, that is

$$f = 1 \tag{2.16}$$

The boundary layer equation on the curved surface is written as

$$\frac{K_r u^2}{h_1} \tag{2.17}$$

where K_r are curvature parameters which are defined as

$$K_r(x) \tag{2.18}$$

$$h_1 = \tag{2.19}$$

where r(x) is the radius of the curved surface

Equation (2.8b) is written as

$$\frac{1}{\rho} \tag{2.20}$$

A comparison is done between equations (2.17) and (2.20) which yields

$$\frac{K_r}{r} \tag{2.21}$$

The body forces under consideration and are partly due to gravitational pull which is assumed to be a constant in both cases, hence an important assumption that

(2.22)

On comparing equations (2.21) and (2.22) it is resolved that

$$\frac{K_1}{l}$$ (2.23)

Equation (2.23) is substituted in the equation of conservation of momentum along the x-axis equation (2.14) results to a generalized equation of conservation of momentum for fluid flow over an axi-symmetrical body with curved surfaces, the equation become

$$\frac{\partial u}{\partial t} = P_t + \vartheta \frac{\partial^2}{\partial y}$$ (2.24)

but since $h_1 =$, then the term is written in Taylor series as

$$K_r u^2 (1 + K_r y)^{-1} = K_r u^2 (1 - K_r y + K_r$$

Since the flow along the x-axis, and for a very small values of momentum equation (2.24) become

$$\frac{\partial u}{\partial t} = P_t + \vartheta \frac{\partial^2 u}{\partial y^2}$$ (2.25)

In this study, there was application of a constant magnetic field along the y-axis. The equation of momentum along the y axis become;

$$\frac{\partial u}{\partial t} = P_t + \vartheta \frac{\partial^2 u}{\partial y^2} + K_r u$$ (2.26)

Where is the Lorentz force.

On taking $\boldsymbol{J} = (J_x, J_y, 0)$, $\boldsymbol{B} = (0, B_0, 0)$, $\boldsymbol{q} = ($

The term can be expressed as

$$\boldsymbol{q} x \boldsymbol{B} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ u & 0 & 0 \\ 0 & B_0 & 0 \end{vmatrix} = \qquad (2.26a)$$

But $\boldsymbol{J} =$ (therefore the Lorentz force term ***J**x**B*** is give as

$$\mathrm{J} x \boldsymbol{B} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & 0 & \sigma u B_0 \\ 0 & B_0 & 0 \end{vmatrix} = - \qquad (2.26b)$$

Upon Substituting the ***J**x**B***, the equation of momentum (2.26) takes the form

$$\frac{\partial u}{\partial t} = P_t + \vartheta \frac{\partial^2 u}{\partial y^2} + K_r u \qquad (2.27)$$

2.3.3 Equation of Conservation of Thermal Energy

The equation of energy is derived from the first law of thermodynamics which state that the amount of heat added to the system, dQ is equal to the sum of the change in the internal energy, dE of the system and the external work done dW by the system. Mathematically the law is expressed as

$$dQ = dE + dW \qquad (2.28)$$

where $dW = PdV =$ for a unit mass. Equation 2.28 yields

$$dQ = dE + \qquad (2.29)$$

The first law of thermodynamics for fluid flow with constant thermal conductivity K, zero internal generation and negligible compressibility effect the equation is given by;

$$\rho C_p \frac{Dh}{Dt} = K\nabla \qquad (2.30)$$

where is the internal heating due to the viscous dissipation while for an incompressible two dimensional fluid flow the viscous dissipation function is;

$$\emptyset = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial}{\partial}\right. \tag{2.31}$$

By considering unsteady incompressible flow in a control volume, the standard thermal energy equation for the thermal boundary layer is given by

$$\rho\frac{\partial h}{\partial t} + \rho u\frac{\partial h}{\partial x} + \rho v\frac{\partial h}{\partial y} = \frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right) + \left(u\frac{\partial P}{\partial x} + v\frac{\partial P}{\partial y}\right) + \tag{2.32}$$

where h is enthalpy and is the rate of heat generation.

The enthalpy h is given by

$$h = E \tag{2.33}$$

The first order derivative of enthalpy become

$$dh = dE + \frac{1}{\rho}dP \tag{2.34}$$

But $dQ = dE + dW = dE +$ and for a unit mass and single species fluid, d,

therefore

$$dE = Tds \tag{2.35}$$

Substituting equation (2.35), the equation (2.34) become

$$dh = Tds + \frac{1}{\rho}dP + Pd\left(\frac{1}{\rho}\right) - \tag{2.36}$$

Hence,

$$dh = Td\colon \tag{2.37}$$

Assuming that the flow is fully developed, and are negligible and dh, then equation (2.32) reduces to

$$C_p\rho\frac{\partial T}{\partial t} + C_p\rho\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = K\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial}{\partial}\right. \tag{2.38}$$

For fluid flow over a body with curved surfaces the convective heat transfer due to the viscosity in the thermal boundary layer is modeled to the equation of conservation of energy. Increase in flow cross-sectional area increases the adverse pressure gradient that opposed the buoyancy induced acceleration. The convection equation is expressed as

$$\dot{q} \tag{2.39}$$

Where $dT = (T$ is the temperature difference between the surface and the bulk fluid and is the area of the surface. In this case the area of the surface is the length of the curved surface. The effect of the curved surface is taken into account by multiplying area (A) by a dimensionless factor given by equation (2.39) which results to

$$\dot{q} = \tag{2.40}$$

Where is the heat transferred per unit time. On replacing f, with $\left(1 -\right.$ equation (2.40) reduced to

$$\dot{q} = K\left(1 - \frac{1}{4}\frac{K_r u}{\left(\frac{\partial u}{\partial y}\right)}\right)A(T \tag{2.41}$$

From Newton's law of cooling the local heat flux is given by

$$q_s'' = h(T \tag{2.42}$$

where h is the local convection coefficient. Since the flow conditions varies from one point to another on the curved surface both and *h* also varies along the curved surface. At any distance x from the leading edge of the curved surface local heat flux is obtained by applying the Fourier law to the fluid at y=0 as

$$q_s' \tag{2.43}$$

The local convection heat transfer is then expressed as

$$h = \tag{2.44}$$

In the thermal boundary layer the rate of heat conduction along the y-direction is larger than that along the x-axis i.e $(\frac{\partial}{\partial}$. The equation of first law of thermodynamics (2.38) reduces to;

$$C_p\rho\frac{\partial T}{\partial t} + C_p\rho(u\frac{\partial T}{\partial x} + v\left(\frac{\partial T}{\partial y}\right) = K\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial}{\partial}\right. \tag{2.45}$$

from the above approximations the above equation (2.45) reduce to;

$$C_p\rho\frac{\partial T}{\partial t} = K\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial}{\partial}\right. \tag{2.46}$$

but is substituted as per above in order to take curvature effects the equation yields;

$$C_p\rho\frac{\partial T}{\partial t} = K\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + K\left(1 - \frac{1}{4}\frac{K_r u}{\left(\frac{\partial u}{\partial y}\right)}\right)A(T \tag{2.47}$$

In this study, heat generated due to electrical resistance of the fluid to the flow of induced electric current is also considered. This is given by

$$\frac{|J^2|}{\sigma} = \tag{2.48}$$

On substitution in equation (2.47) it yields,

$$C_p\rho\frac{\partial T}{\partial t} = K\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + K\left(1 - \frac{1}{4}\frac{K_r u}{\left(\frac{\partial u}{\partial y}\right)}\right)A(T_\infty - T_S) + \qquad (2.49)$$

2.4 Non-Dimesionalisation of Equations Governing the Flow

The subject of dimensional analysis considers how to determine the required set of scales for any given problem. It is a process that starts with selecting a suitable scale against which all dimensions in a given physical model are based. Non-dimesionalisation is aimed at ensuring that the results are applicable to other geometrically similar configurations under a similar set of flow conditions.

For this research, we let H, ,P, and T to be the characteristic length, velocity, pressure, and temperature respectively. The following transformations are used to non-dimensionalise the equations governing the flow.

$$x, \quad u = \quad y, \quad v =, \quad P, \quad T^* =, \quad t^* = \qquad (2.50)$$

In order to transform the equations of continuity, momentum and energy into their non-dimensional equation form, the following analysis was first carried out:

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial u^*}\frac{\partial u^*}{\partial x^*}\frac{\partial x^*}{\partial x} = \qquad (2.51)$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial u^*}\frac{\partial u^*}{\partial y^*}\frac{\partial y^*}{\partial y} = \qquad (2.52)$$

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial u^*}\frac{\partial u^*}{\partial t^*}\frac{\partial t^*}{\partial t} = \qquad (2.53)$$

$$\vartheta\frac{\partial^2 u}{\partial y^2} = \vartheta\frac{\partial}{\partial y^*}\left(\frac{\partial u}{\partial u^*}\frac{\partial u^*}{\partial y^*}\frac{\partial y^*}{\partial y}\right)\frac{\partial y^*}{\partial y} = \frac{\vartheta}{H^2} \qquad (2.54)$$

$$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial T^*}\frac{\partial T^*}{\partial t^*}\frac{\partial t^*}{\partial t} = \frac{U_\infty}{H}(T_\infty - \qquad (2.55)$$

$$K\frac{\partial^2 T}{\partial y^2} = K\frac{\partial}{\partial y^*}\left(\frac{\partial T}{\partial T^*}\frac{\partial T^*}{\partial y^*}\frac{\partial y^*}{\partial y}\right)\frac{\partial y^*}{\partial y} = \frac{K}{H}(T_\infty - \qquad (2.56)$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial u^*}\frac{\partial u^*}{\partial y^*}\frac{\partial y^*}{\partial y} = \qquad (2.57)$$

$$\mu\left(\frac{\partial u}{\partial y}\right)^2 = \mu\left(\frac{\partial u}{\partial u^*}\frac{\partial u^*}{\partial y^*}\frac{\partial y^*}{\partial y}\right)^2 = \mu\frac{U_\infty}{H^2} \qquad (2.58)$$

Substituting equations (2.51) and (2.52) in the equation (2.3), the following was obtained

$$\frac{U_\infty}{H}\left(\frac{\partial u^*}{\partial x^*} + \frac{\partial}{\partial}\right. \qquad (2.59)$$

Or

$$\left(\frac{\partial u^*}{\partial x^*} + \frac{\partial}{\partial}\right. \qquad (2.60)$$

The equation of conservation of momentum for this study was given as (2.27), on substituting equations (2.53) and (2.54) the equation of momentum become

$$\frac{U_\infty{}^2}{H}\frac{\partial u^*}{\partial t^*} = PP_t^* + \frac{\vartheta U_\infty}{H^2}\frac{\partial^2 u^*}{\partial y^{*2}} + K_r U_\infty{}^2 u^{*2} - \frac{\sigma}{} \qquad (2.61)$$

Dividing this equation throughout by , the following equation was obtained

$$\frac{\partial u^*}{\partial t^*} = \frac{PH}{U_\infty{}^2}P_t^* + \frac{\vartheta}{HU_\infty}\frac{\partial^2 u^*}{\partial y^{*2}} + K_r H u^{*2} - \qquad (2.62)$$

Equation (2.62) gives the equation of momentum in non-dimensional form

But $= \frac{\rho H U_\infty}{\mu}$, M hence the above equation reduces to

$$\frac{\partial u^*}{\partial t^*} = \frac{HP}{U_\infty{}^2}P_t^* + \frac{1}{Re}\frac{\partial^2 u^*}{\partial y^{*2}} + K_r H u^{*2} \qquad (2.63)$$

In the equation of conservation of energy (2.49), equations (2.55), (2.56) ,(2.57) and (2.58) are substituted and dividing all through by the following equation is obtained;

$$\frac{U_\infty (T_\infty - T_S)}{H}\frac{\partial T^*}{\partial t^*} = \frac{K}{C_p\rho}\frac{(T_\infty - T_S)}{H^2}\frac{\partial^2 T^*}{\partial {y^*}^2} + \frac{\mu U_\infty{}^2}{C_p\rho H^2}\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \frac{KA}{C_p\rho}(T_{\infty}$$

$$\left(1 - \frac{1}{4}\frac{K_r u^* H}{\left(\frac{\partial (u^*)}{\partial (y^*)}\right)}\right) + \frac{\sigma u}{} \qquad (2.64)$$

Dividing equation (2.64) throughout by $\frac{U_\infty}{}$ the following equation is obtained

$$\frac{\partial T^*}{\partial t^*} = \frac{K}{C_p\rho H U_\infty}\frac{\partial^2 T^*}{\partial {y^*}^2} + \frac{\mu U_\infty}{C_p\rho H (T_\infty - T_S)}\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \frac{KHA}{C_p\rho U_\infty}\left(1 - \frac{1}{4}\frac{K_r u^* H}{\left(\frac{\partial u^*}{\partial (y^*)}\right)}\right) +$$

$$\frac{\sigma {u^*}^2 U_\infty{}^2 B_0^2}{\rho C_p}\frac{H}{U_\infty (T_\infty - T_S)}$$

(2.65)

Multiplying the term $\frac{\mu U_\infty}{C_p\rho H(T_\infty - T_S)}$ by in the numerator and the denominator, the following is obtained;

$$\frac{\partial T^*}{\partial t^*} = \frac{K}{C_p\rho H U_\infty}\frac{\partial^2 T^*}{\partial {y^*}^2} + \frac{\mu U_\infty . U_\infty}{C_p\rho H . U_\infty (T_\infty - T_S)}\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \frac{KHA}{C_p\rho U_\infty}\left(1 - \frac{1}{4}\frac{K_r u^* H}{\left(\frac{\partial u^*}{\partial (y^*)}\right)}\right) +$$

$$\frac{\sigma {u^*}^2 U_\infty B_0^2}{\rho C_p}\frac{H}{\Delta t}$$

(2.66)

The equation (2.66) is the equation of conservation of energy in non-dimensional form.

But, $Re = \frac{\rho H U_\infty}{\mu}$, $Pe = RePr = \frac{\rho U_\infty HCp}{K}$, $Pr = \frac{\vartheta}{\alpha} = \frac{\mu\rho}{K\rho C_p}$,

$Ec = \frac{}{C_p}$ and R

Hence the equation for conservation of energy reduces to

$$\frac{\partial T^*}{\partial t^*} = \frac{1}{Pe}\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{Ec}{Re}\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \frac{HA}{Pe}\left(1 - \frac{1}{4}\frac{K_r u^* H}{\left(\frac{\partial u^*}{\partial (y^*)}\right)}\right) + \qquad (2.67)$$

2.5 Non-Dimesional Numbers

2.5.1 Reynolds number, Re

This number was named after a scientist Osborne Reynolds; it is defined as the ratio of the inertia forces to the viscous forces. It is given by

$$Re = \frac{\rho H U_\infty}{\mu} = \frac{U_\infty H}{\vartheta} = \frac{inerti}{viscoi}$$

2.5.2 Eckert number, Ec

This is the measure of kinetic energy of flow to the boundary layer enthalpy difference across thermal boundary, given by;

$$Ec = \frac{}{C_p(}$$

2.5.3 The Prandtl number, Pr

This number was named after Ludwig Prandtl (1904) a German aero dynamist who was closely associated with the conception of boundary layer theory. It is the parameter which relates the relative thickness of the hydrodynamic and thermal boundary layers. The Prandtl number provided the link between the velocity field and the temperature field. It is expressed as

$$Pr = \frac{\vartheta}{\alpha} = \frac{\mu\rho}{K\rho C_l}$$

2.5.4 Peclet number,Pe

This number is named after a French physicist Jean Claude Peclet, in context of transport of heat, the Peclet number is equivalent to the product of the Reynolds number and the Prandtl number and is given by;

$$\boldsymbol{Pe = RePr = \frac{\rho UH}{\kappa}}$$

2.6 Boundary and Initial Conditions

The boundary conditions for hydromagnetic fluid flow over axi-symmetrical body with curved surface are stated below. Equation of conservation of momentum (2.63) is solved subject to the following boundary and initial conditions.

$$u(t, \tag{2.68}$$

$$u(t,\infty \tag{2.69}$$

$$u(0, \tag{2.70}$$

On non-dimensionalising the boundary and initial conditions

$$u^*U_\infty \left(\frac{t^*H}{U_\infty}, \right. \tag{2.71}$$

$$u^*U_\infty \left(\frac{t^*H}{U_\infty},\infty \right. \tag{2.72}$$

$$u^*U_\infty \,(0,y^* \tag{2.73}$$

On simplifying the above boundary and initial conditions the following is obtained

$$u^*(t^*, \tag{2.74}$$

$$u^*(t^*, \tag{2.75}$$

$$u^*(0, \tag{2.76}$$

The equation of conservation of energy (2.67) is solved subject to the following boundary and initial conditions

$$T(t, \tag{2.77}$$

$$T(t,\infty \tag{2.78}$$

$$T(0, \tag{2.79}$$

On non-dimensionalising the boundary and initial conditions;

$$T^*(T_\infty - T_S) + T_s\left(\frac{t^*H}{U_\infty}, 0\right) \quad (2.80)$$

$$T^*(T_\infty - T_S) + T_s\left(\frac{t^*H}{U_\infty}, \infty\right) \quad (2.81)$$

$$T^*(T_\infty - T_S) + T_s(0, y^* \quad (2.82)$$

On simplifying the conditions above

$$T^*(t^*, \quad (2.83)$$

$$T^*(t^*, \quad (2.84)$$

$$T^*(0, \quad (2.85)$$

2.7 Method of Solution

Explicit relation between the partial derivatives and the functional values at the adjacent nodal points are obtained using mesh system. The rectangular regions are subdivided into smaller and equal square elements whose length is and width is and the time variation along the horizontal axis is represented by . The mesh system can thus be described as shown in the the figure below.

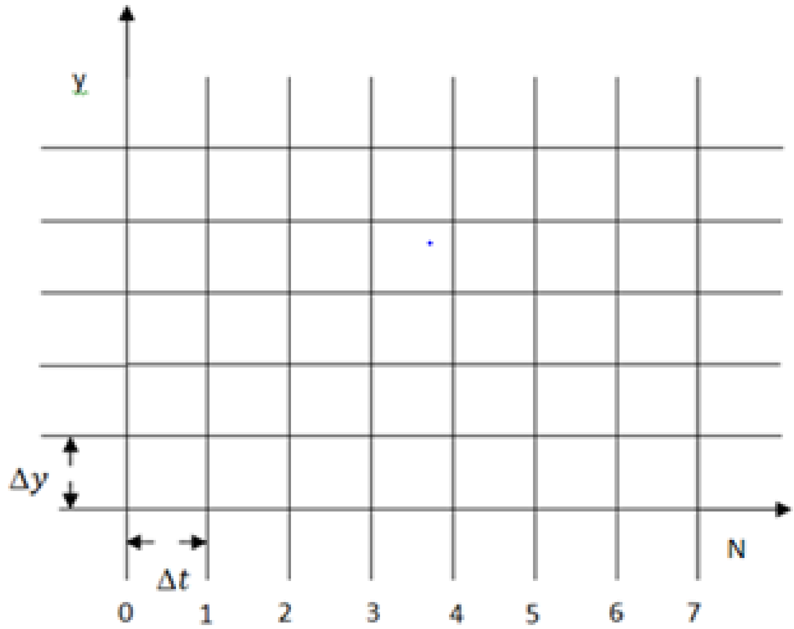

Figure 2.1 finite difference mesh

Considering a reference point (i,j) where i and j represent t and y respectively. Using the notation (for (and for (the adjacent points to y and t are defined, the points that are i and j units from the reference point have the coordinates (i. In finite difference approximation, the derivatives are replaced with finite differences. If $u =$ and T=T (t,y), their first derivatives with respect to t are written in finite difference form as

$$U_t = \frac{U_{i+1,j} - U_{i,j}}{\Delta t} + 0(\qquad (2.86)$$

$$T_t = \frac{T_{i+1,j} - T_{i,j}}{\Delta t} + 0(\qquad (2.87)$$

The first order derivatives with respect to y are written in forward finite difference form as

$$U_y = \frac{U_{i+1,j} - U_{i,j}}{\Delta y} + 0(\qquad (2.88)$$

$$Ty = \frac{T_{i+1,j} - T_{i,j}}{\Delta y} + 0(\qquad (2.89)$$

The second order derivatives with respect to y by central difference method are as follows;

$$U_{yy} = \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta y)^2} + 0(\qquad (2.90)$$

$$Tyy = \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{(\Delta y)^2} + 0(\qquad (2.91)$$

Upon substitution in the equation of momentum (2.63), this is obtained

$$\frac{u^*_{i+1,j} - u^*_{i,j}}{\Delta t} = \frac{HP}{U_\infty{}^2} P_t + \frac{1}{Re}\left[\frac{u^*_{i,j+1} - 2u^*_{i,j} + u^*_{i,j-1}}{(\Delta y)^2}\right] + K_r H u^*_{i,j}{}^2 \qquad (2.92)$$

On making the subject we obtained;

$$u^*_{i+1,j} = u^*_{i,j} + \frac{\Delta t HP}{U_\infty{}^2} P_t + \frac{\Delta t}{Re}\left[\frac{u^*_{i,j+1} - 2u^*_{i,j} + u^*_{i,j-1}}{(\Delta y)^2}\right] + \Delta t K_r H u^*_{i,} \qquad (2.93)$$

Using crank Nicolson approximation for increased accuracy and convergence, the first and second derivative becomes;

$$\frac{\partial u^*}{\partial t^*} = \frac{u^*_{i+1,j+1} - u^*_{i,j+1} + u^*_{i\cdot}}{2\Delta t} \qquad (2.94)$$

$$\frac{\partial^2 u^*}{\partial y^2} = \left[\frac{\{u^*_{i+1,j+1} - 2(u^*_{i+1,j}) + u^*_{i+1,j-1}\} + \{u^*_{i,j+1} - 2u^*_{i,j} +}{2(\Delta y)^2}\right. \qquad (2.95)$$

The equation of momentum thus yields:

$$\frac{u^*_{i+1,j+1} - u^*_{i,j+1} + u^*_{i+1,j} - u^*_{i,j}}{2\Delta t} =$$

$$\frac{HPP_t^*}{U_\infty{}^2}\frac{1}{Re}\left[\frac{\{u^*_{i+1,j+1} - 2(u^*_{i+1,j}) + u^*_{i+1,j-1}\} + \{u^*_{i,j+1} - 2u^*_{i,j} + u^*_{i,j-1}\}}{2(\Delta y)^2}\right] + K_r H u^*_{i,j}{}^2 - M u^*_{ij}$$

(2.96)

This equation is solved explicitly after making the term the subject of the equation.

$$u^*{}_{i+1,j} = \left\{u^*{}_{i,j} + \frac{\Delta tHPP_t^*}{U_\infty{}^2} + \frac{\Delta t}{Re}\left[\frac{\{u^*{}_{i+1,j+1}+u^*{}_{i+1,j-1})+u^*{}_{i,j+1}-2u^*{}_{i,j}+u^*{}_{i,j-1}\}}{2(\Delta y)^2}\right] + \Delta tK_rHu^*{}_{(i,j)}{}^2 - \Delta tMu_{ij}^*\right\} \div \left(1+\frac{\Delta t}{Re(\Delta y)^2}\right)$$

(2.97)

The boundary conditions are given as follows:

$$u^*(t^*,$$
$$u^*(t^*, \qquad (2.98)$$
$$u^*(0,$$

The equation of energy (2.67) is written in finite difference form, the partial derivatives are written in forward for time and central for space;

$$\frac{\partial T^*}{\partial y} = \frac{T^*{}_{i+1,j-}T^*{}_{i,j}}{\Delta y} + 0(\qquad (2.99)$$

$$\frac{\partial T^*}{\partial t} = \frac{T^*{}_{i+1,j-}T^*{}_{i,j}}{\Delta t} + 0 \qquad (3.0)$$

$$\frac{\partial^2 T^*}{\partial y} = \frac{T^*{}_{i,j+1}-2T^*{}_{i,j}+T^*{}_{i,j-1}}{(\Delta y)^2} + 0(\qquad (3.1)$$

on substitution of the derivatives this is got

$$\frac{T^*{}_{i+1,j-}T^*{}_{ij}}{\Delta t} = \frac{1}{Pe}\left[\frac{T^*{}_{ij+1}-2T^*{}_{i,j}+T^*{}_{i,j-1}}{(\Delta y)^2}\right] + \frac{Ec}{Re}\left[\frac{u^*{}_{i+1,j}-u^*{}_{ij}}{\Delta y}\right]^2 + \frac{H^2A}{Pe}\left[1-\frac{K_ru^*{}_{ij}H\Delta y}{u^*{}_{i,j+1}-u^*{}_{i,j}}\right] + Ru_{ij}^*{}^2H$$

(3.2)

On implementing the crank Nicolson approximation, the above equation became;

$$\frac{T^*_{i+1,j+1} - T^*_{i,j+1} + T^*_{i+1,j} - T^*_{i,j}}{2\Delta t} = \frac{1}{Pe}\left[\frac{T^*_{i+1,j+1} - 2T^*_{i+1,j} + T^*_{i+1,j-1} + T^*_{i,j+1} - 2T^*_{i,j} + T^*_{i,j-1}}{2(\Delta y)^2}\right] + \frac{Ec}{Re}\left[\frac{u^*_{i+1,j+1} - u^*_{i+1,j} + u^*_{i,j+1} - u^*_{i,j}}{2\Delta y}\right]^2 + \frac{H^2 A}{Pe}\left[1 - \frac{K_r u^*_{ij} H \Delta y}{u^*_{i+1,j+1} - u^*_{i+1,j} + u^*_{i,j+1} - u^*_{i,j}}\right] + R u^{*\,2}_{ij} H$$

(3.3)

Making the subject so that the equation can be solved explicitly

$$T^*_{i+1,j} = T^*_{i,j} + \frac{\Delta t}{Pe}\left[\frac{T^*_{i+1,j-1} + T^*_{i+1,j+1} + T^*_{i,j+1} - 2T^*_{i,j} + T^*_{i,j-1}}{2(\Delta y)^2}\right] + \frac{\Delta t Ec}{Re}\left[\frac{u^*_{i+1,j+1} - u^*_{i+1,j} + u^*_{i,j+1} - u^*_{i,j}}{2\Delta y}\right]^2 + \frac{\Delta t H^2 A}{Pe} - \left[\frac{K_r u^*_{ij} A H^3 \Delta t}{4Pe\left(\frac{u^*_{i+1,j+1} - u^*_{i+1,j} + u^*_{i,j+1} - u^*_{i,j}}{2\Delta y}\right)}\right] + \Delta t R u^{*\,2}_{ij} H \div \left(1 + \frac{\Delta t}{Pe(\Delta y)^2}\right)$$

(3.4)

The boundary conditions are as follows;

$$T^*(t^*,$$
$$T^*(t^*,$$
$$T^*(0,$$

(3.5)

The values of velocity obtained in the momentum equation are used to compute for temperature values in energy equation, this is done iteratively and different values are obtained when various flow parameters are varied.

CHAPTER THREE

DATA REPRESENTATION AND DISCUSSION.

3.0 Introduction

In this chapter data representation is done using graphs obtained from a computer code. The various parameters that have been varied include M, Re, Kr, Ec, Pe and R.

3.1 Data Representation and Discussion.

The following data representation is obtained after solving the equations governing the fluid flow

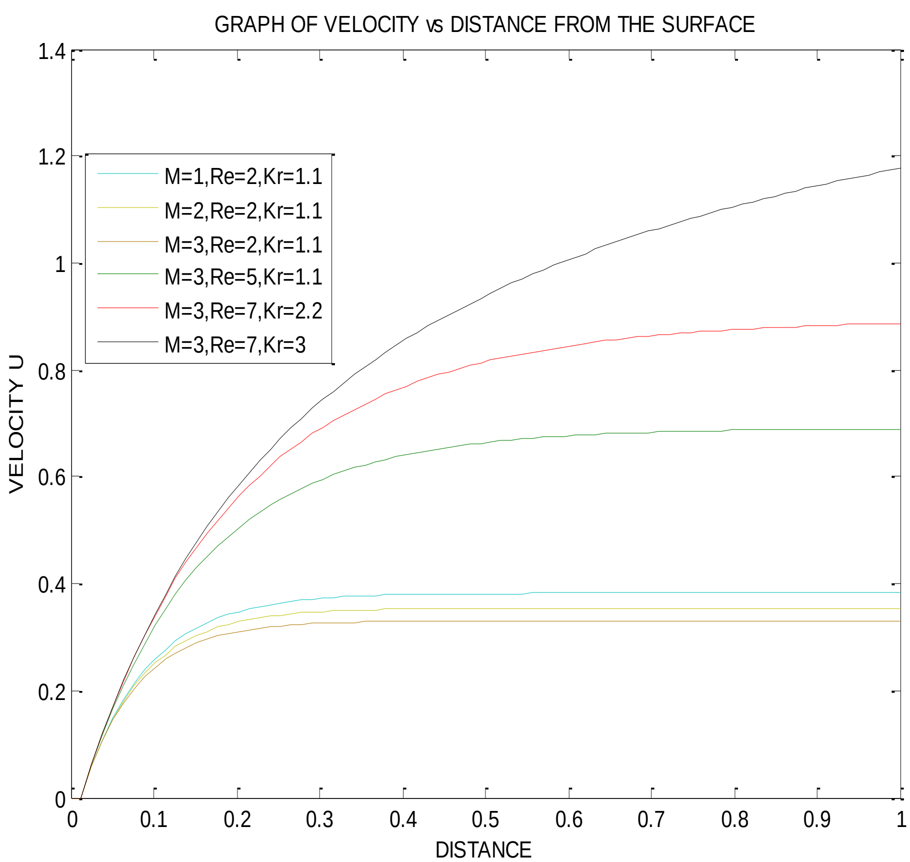

Figure 3.1: Velocity profiles for different values of magnetic number, Reynolds number and curvature.

From Figure 3.1 above it was observed that an increase in magnetic field parameter M causes a decrease in magnitude of velocity profiles. This implies that increase in M has a tendency to slow down the velocity of the fluid. Application of a transverse magnetic field to an electrically conducting fluid give rise to a resistive type of force called Lorentz force. This force has a tendency to slow down the motion of the fluid in the boundary layer. From the figure it was noted that an increase in Reynolds number causes a decrease in the magnitude of velocity profiles. This is because when Re is increased, inertia forces increases and these forces oppose the fluid from accelerating hence reduced velocities. From figure 3.1, when the radius of curvature Kr is increased the free stream velocity of the fluid particle also increases. This is because increase in curvature increases the velocity gradient. Thus increasing the speed of the fluid flow.

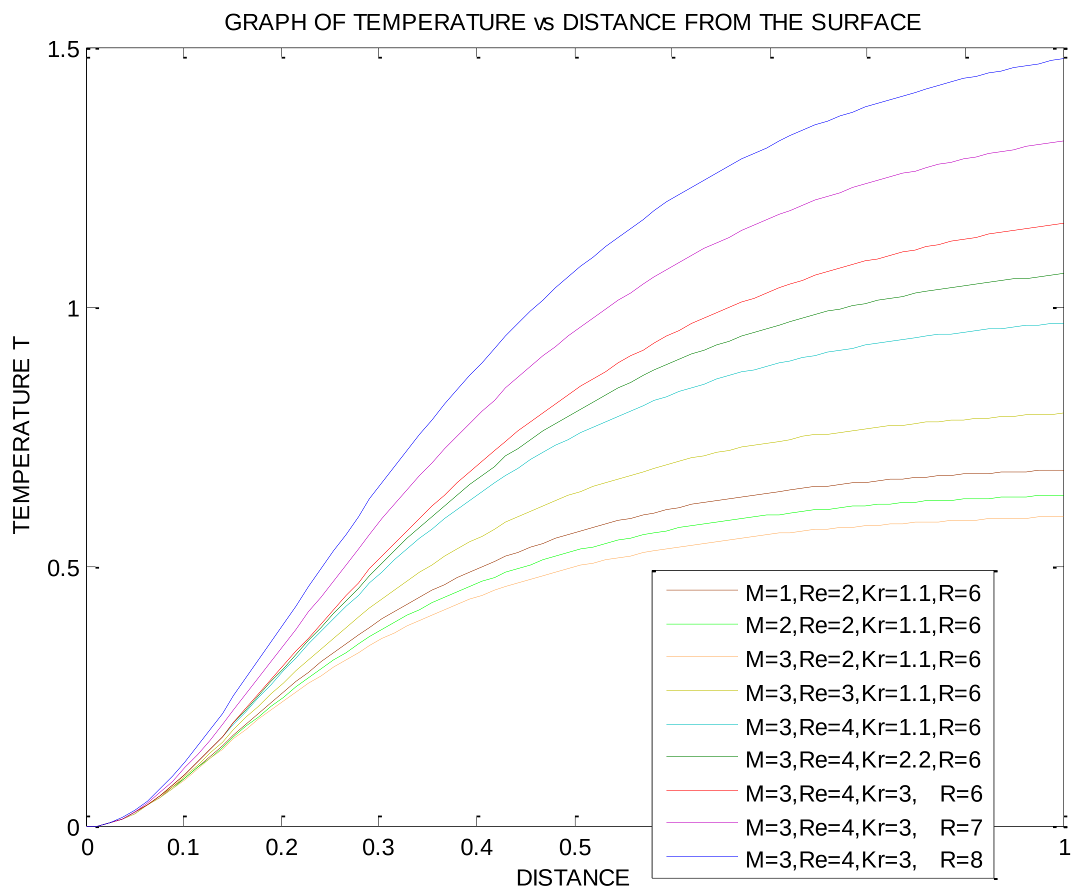

Figure 3.2: temperature fields for different magnetic number, Reynolds number, curvature and joules heating parameter.

The temperature profiles decreases with increase in M as observed in Figure 3.2 above. The reduced velocity by the frictional drag due to the Lorenz force is responsible for reducing thermal viscous dissipation in the fluid leading to a thinner thermal boundary layer. Magnetic field therefore can be used to control the velocity and temperature boundary layer characteristics.

From Figure 3.2, it was also observed that, an increase in Reynolds number causes a decrease in temperature profiles. When the value of Re is small it means that the viscous forces dominates over the inertia forces; these large viscous forces results to increased friction between the surface of the body and the fluid. This brings about the increased dissipation of heat within the boundary layer.

When Kr is increased, the heat dissipated in the boundary layer also increases. This is because increase in curvature increases velocity gradient, this in turn leads to an increase in shear stresses. These shear stresses bring about friction between the fluid and the surface and as a result this friction force leads to heat dissipation within the boundary layer region, this leads to the increased temperature profiles. This is deduced from the formula ,

which implies that when the velocity gradient is increased it leads to increased shear stress that lead to an increase in heat dissipation in the boundary layer.

From the Figure 3.2 above, it was observed that increase in the joule heating parameter R leads to increased temperature profiles. Increase in joule heating parameter leads to the heating of the fluid thereby increasing the velocity of the convection currents on the surface of the sheet.

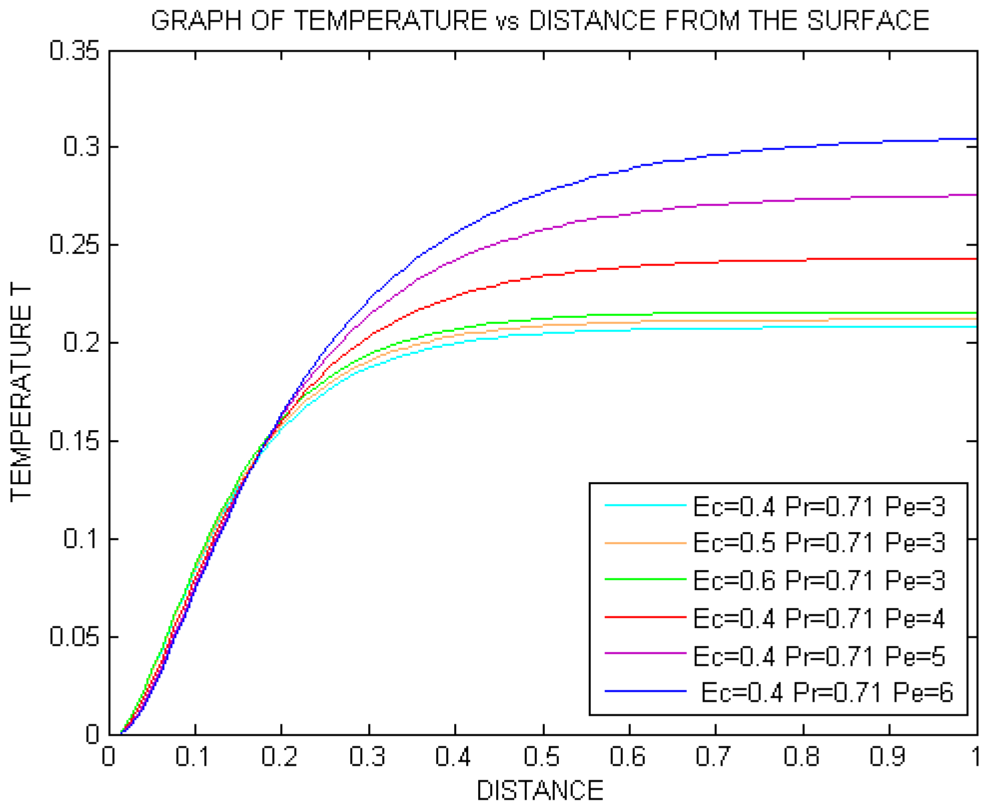

Figure 3.3: Temperature profile for different values of Eckert and Peclet numbers

From the Figure 3.3 above, it was observed that increase in Eckert number causes an increase in temperature profiles. The Eckert number expresses the relationship between kinetic energy in the flow and the enthalpy. It embodies the conversion of kinetic energy into internal energy by work done against viscous fluid stresses. Large Ec implies that kinetic energy is large and hence velocities are higher. When particles attain high velocity the vibration also increases leading to an increase in collision of particles. This leads to increased dissipation of heat in the boundary layer. A positive Eckert number implies cooling the surface of the body, implying heating the fluid. This causes a rise in temperature and the velocity of the fluid.

In the Figure 3.3 above, increase in Pe lead to an increase in temperature profile and a decrease in Pe lead to a low temperature profile in the boundary layer region. This is attributed by the fact that large Pe lead to an increase in velocities. The increased velocity of the fluid indicates an increase in fluid particles collisions which in turn causes an increase in amount of heat dissipated.

The effect of convective heat transfer on drag and lift is also discussed. The ratio of the shear stress to the quantity is known as local coefficient of drag, or local skin friction, denoted as C_D . The ratio of total drag force to the quantity is called average coefficient of drag; C_D = where is the density of the fluid, A is the area of the surface of the body, U is the free stream velocity. From the above it is deduced that $F_D = C_D$ is the formula for drag. The formula for lift is given by $L = C_L$ where L is lift and is the coefficient of lift. For symmetrical bodies the drag coefficient is 0.04 and the lift coefficient is 0.2. The convective heat transfer affects the fluid flowing around the body by varying the velocity of this fluid and hence affects the lift and drag.

CHAPTER FOUR

CONCLUSION AND RECOMMEDATION

4.1 Conclusion

The analysis of various parameters on unsteady hydromagnetic laminar boundary layer flow of an incompressible, electrical conducting fluid past an immersed axi-symmetrical body with curved surface was carried out. The direction of the applied magnetic field was considered to be normal to the direction of the flow. The equations governing the flow are highly non-linear and have been solved by finite difference method. The results obtained shows that convective heat transfer in electrical conducting fluid is influenced by magnetic field parameter, Reynolds number, Peclet number, curvature of the body, Eckert number and joule heating parameter.

This study reveals that introducing a transverse magnetic field to a flow slows down the velocity of the fluid. Decreasing the velocity of the fluid slows down the movement of the body thus decrease in frictional drag. Increase in Reynolds number causes a decrease in the magnitude of velocity and temperature profiles respectively. Re represent the ratio of inertial to viscosity forces. Increase in Re results into a larger inertia that in turn translate to lower velocities. When Re is large the inertia forces dominate over the viscous forces. This leads to reduced velocity in the boundary layer. When Re is small it implies that the viscous forces are dominated and hence temperature dissipation in the boundary layer occurs due to increased friction, hence increased drag. Due to this dissipation in the boundary layer, this result to decreased density of the fluid hence reduced lift.

When the radius of curvature Kr is increased the free stream velocity and temperature profiles of the fluid also increases. Body curvature was found to have a direct relationtionship with temperature and velocity profiles. It was also indicated that increase in the joule heating parameter R leads to increased temperature profiles. Increase in joule heating parameter lead to the heating of the fluid thereby boosting the velocity of the convection currents on the surface of the solid surface.

Increase in Pe leads to an increase in temperature profile and a decrease in Pe leads to a low temperature profile in the boundary layer region. It was observed that increase in Eckert number causes an increase in velocity profiles as well as temperature profiles. Thus increase in Ec number boosts both the velocity and the temperature of a fluid

4.2 Recommendations

This study has considered the analysis of convective heat transfer in electric- conducting fluid over an immersed axi-symmetry body with curved surface. The fluid is incompressible viscous and the flow is laminar. The present work can provide a basis for further research by including the following considerations.

- Hydromagnetic flow over an immersed axi-symmetrical body with curved surface and heat transfer in a compressible fluid
- Hydromagnetic fluid flow over an immersed axi-symmetrical body where the flow is turbulent.
- Hydromagnetic fluid flow over an immersed axi-symetrical body of this nature where the magnetic field is varied.
- Use of finite element method to solve magnetohydrodynamic flow over an immersed axi-symmetrical body.
- Hydromagnetic flow over an immersed axi-symmetrical moving body in the fluid

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APPENDICES

APPENDIX 1

The following computer program code in Matlab is used to solve the equations governing the fluid flow subject to the initial and boundary condition.

```
Y=0.1; tmax =1;  Kr = 1.1; R=0.5;
Pr = 0.71; Ec = 0.4; Re = 2.; Pe = 6; M = 2; H = 1.0; A=2.; R=8.; P=5.; Pt=1.; U_inf=1.;
ny = 80;
nt = 80;
% --- Compute mesh spacing and time step
delY= 0.2  ;
delT =0.0125  ;

%r = alpha*dt/dx^2; r2 = 1 - 2*r;
% --- Create arrays to save data for export
y = linspace(0,1,ny);
t = linspace(0,1,nt);
T= zeros(ny,nt);
U= zeros(ny,nt);
% --- Set IC and BC
U(1,nt) = 0; T(1,nt) = 0;  %U(nt,ny) = 1.; T(nt,ny) = 1.;  % initial and bound. conds/

SUFF1=1+delT/(Re*delY*delY);
SUFF2=1+delT/(Pe*delY*delY);
%for M= 1: 0.5:2 % Varying Magnetic parameter
    % for Ec=0.4:0.2:0.6  % Varying Eckert Number
        for J=2:nt-1
```

```
for I=2:ny-1
%Den=(4*Pe/(2*delY))*(U(I+1,J+1)-
U(I+1,J)+U(I,J+1)-U(I,J));

U(I+1,                 J)                  =(U(I,
J)+(delT*H*P*Pt)/(U_inf*U_inf)+(delT/(Re*2*delY*delY))*(U(I+
1, J+1) +U(I+1, J-1)+U(I, J+1)-2*U(I, J)+U(I, J-1))+ ...
             +(delT*Kr*H*U(I,    J)*U(I,    J))-
delT*M*U(I, J))/SUFF1;

T(I+1,                 J)                  =(T(I,
J)+(delT/(Pe*2*delY*delY))*(T(I+1,  J-1)  +T(I+1,  J+1)+T(I,
J+1)-2*T(I, J)+T(I, J-1)) + ...
             (delT*Ec/(Re*2*delY))*(U(I+1,   J+1)
-U(I+1,  J)+U(I,  J+1)-U(I,  J))*(U(I+1,  J+1)  -U(I+1,  J)+U(I,
J+1)-U(I, J)) + ...
             (delT*H*H*A/Pe)-(Kr*U(I,
J)*A*H*H*H*delT)/(4*Pe/(2*delY))*(U(I+1,J+1)-
U(I+1,J)+U(I,J+1)-U(I,J))+delT*R*U(I,J)*U_inf*H)/SUFF2;
 end

end

figure(1)
grid off
plot(y,U(:,40),'-.');
title('GRAPH OF VELOCITY vs DISTANCE FROM THE
SURFACE');
xlabel('DISTANCE');
ylabel('VELOCITY U');
hold on
```

```
        figure(2)
        grid off
        plot(t,T(:,40),'-.');
        title('GRAPH OF TEMPERATURE vs DISTANCE FROM THE SURFACE');
        xlabel('DISTANCE');
        ylabel('TEMPERATURE T');
        hold on
    %end
%end
```

APPENDIX II

Publication

A paper from this work has been accepted for publishing by IISTE (The International Institute of Science, Technology and Education.)

TITLLE: Hydromagnetic Fluid Flow over an Immersed Axi-Symmetrical Body with Curved Surface in Presence of Heat Transfer