MHD TURBULENT FLOW IN PRESENCE OF INCLINED MAGNETIC FIELD PAST A ROTATING SEMI-INFINITE PLATE

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DECLARATION

This thesis is my original work and has not been presented for a degree award in any other university.

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This thesis has been submitted for examination with our approval as University supervisors.

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DEDICATION

This thesis is dedicated to my parents Mr. and Mrs.Maswai, my siblings Rispa, Roy and Andrew and all my friends for their encouragement and support towards my education.

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NOMENCLATURE

Roman symbols

Symbol	Meaning
B	Magnetic flux density,Wb/m^2
C_p	Specific heat, J/kg K
D	Diffusion coefficient, m^2/s
e	Electric charge Coulomb, s/m^3
E	Electric field strength,V/m
g	Acceleration due to gravity, m/s^2
H_0	Magnetic field strength, A/m^2
$H_{0_x}, H_{0_y}, H_{0_z}$	Components of magnetic field strength, A/m^2
J	Electric current density, A/m^2
J_x, J_y, J_z	Components of the current density, A/m^2
k	Thermal conductivity, w/m. K
L	Characteristic length,m
P_e	The electric pressure, N/m^2
$\hat{u}$	Velocity of the fluid, m/s
u, v, w	Components of velocity, ms^{-1}
t	Dimensional time,s
T	Dimensional temperature, K

T_w	Temperature of the fluid at the plate, K
T_∞	Temperature of the fluid in the free stream, K
u_0	Dimensional injection velocity,m/s
x, y, z	Cartesian coordinates

Greek symbols

	Coefficient of volumetric expansion
~	Electrical conductivity, α/m
°	Fluid density, kg/m^3
«$_e$	Magnetic permiability,H/m
³	Kinematic viscosity, m^2/s
¢$_{ij}$	Kronector delta,Kg/sm
¦	Dynamic viscosity
Ω	Angular velocity,m/s

ABBREVIATIONS

MHD-Magnetohydrodynamics

ABSTRACT

A turbulent incompressible fluid flow past a semi-infinite vertical rotating plate has been investigated, the flow considered is in the presence of a strong inclined constant magnetic field. An induced electric current exists due to the presence of the constant magnetic field. The velocity distribution of the fluid flow past a semi-infinite vertical plate and it's temperature profiles have been determined. Finally, the effects of various parameters like non-dimensional numbers and the angle of inclination of the magnetic field on the flow variables have been determined. The equations governing this problem have been solved numerically using finite difference method because these equations are non-linear and there exists no analytical method of solving them. A sample result of the velocity profiles and temperature profiles have been obtained followed by a graphical representation of the same. It is noted that an increase in the Hall parameter, time and angle of inclination leads to an increase in the primary velocity while an increase in the rotational parameter Er and Eckert number leads to a decrease in the primary velocity profiles. An increase in the rotational parameter and Hall parameter leads to an increase in secondary velocity, Eckert number, time and angle of inclination leads to a decrease in secondary velocity. Increase in Eckert number, time, Hall parameter and rotational parameter leads to an increase in temperature profiles.

CHAPTER ONE
INTRODUCTION

1.1 Introduction to the study

Magnetohydrodynamics (MHD) is the study of dynamics of electrically conducting fluids. Examples of such fluids include plasmas, liquid metals, and salt water or electrolytes. The fundamental concept behind MHD is that magnetic fields can induce currents in a moving conductive fluid, which in turn changes the magnetic field itself and generates forces on the fluid. The set of equations which describe MHD are a combination of the Navier-Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism.

Free convection flows are of great interest in a number of industrial applications such as fiber and granular insulation and geothermal systems. MHD is attracting the attention of many authors due to its applications in geophysics; it is applied to study the stellar and solar structures, interstellar matter and radio propagation through the ionosphere. In some engineering devices, like MHD pumps, gas, can be ionized and so becomes an electrical conductor.

1.1.1 Definitions

In this study , several terms have been used and such terms are defined in this section.

1.1.2 Fluid

Fluid is a type of matter which undergoes continuous deformation when some external force is applied. Fluids are classified as liquids and gases.

1.1.3 Hydromagnetics

The term hydrodynamics is often applied to the science of incompressible fluids in motion whereas electromagnetism is the study of the interaction between electric and magnetic fields. Interaction of hydrodynamics and electromagnetism is known as hydromagnetics or Magnetohydrodynamics (MHD) which is the study of the motion of an electrically conducting fluid in presence of a magnetic field.

1.1.4 Unsteady Flow

When flow variables such as velocity and the thermodynamic properties at every point in space vary with respect to time, the flow is considered to be unsteady. If none of the fluid flow variables varies with respect to time the flow is steady. In the present study we consider an unsteady flow.

1.1.5 MHD Free Convection Flow

MHD free convection flow is very important because of its many applications ranging from engineering to the study of the universe. In free convection, fluid motion results when body forces act on the fluid in which density gradients exists. The density gradients may be due to temperature or concentration gradients existing in the fluid, while the body force is due to gravitational force. In our study we consider free convection flow due to temperature difference.

1.1.6 Turbulent flow

This is a flow regime characterized by chaotic and stochastic property changes. This includes low momentum diffusion, high momentum convection, and rapid variation of pressure and velocity in space and time. Turbulent flows are always unsteady i.e. it varies continuously with time even though there is a steady downstream motion of the fluid. Semi-infinite plate is whereby the plate is bound on one end and not bound on the other end i.e. the boundary condition imposed on the equations will be defined at one end and the other end tending to infinity.

1.1.7 Boundary layer

A boundary layer is a thin fluid layer adjacent to the surface of the body or a solid wall in which viscous forces affect the flow. Boundary layer theory is important in analyzing flow problems involving convection transport. For fluid flows over any surface there may exist three boundary layers namely velocity, thermal and concentration boundary layer. When fluid particles comes in contact with the surface, they assume the velocity of the solid surface. These particles retard the motion of the particles in the adjoining fluid layer, which in turn retards the motion of particles in the next layer and so on, until at a certain distance from the surface where the effect becomes negligible. This region in which the velocity gradient is large is referred to as velocity boundary layer. If the fluid particles come into contact with an isothermal plate, they achieve thermal equilibrium at the plate's surface. In turn these particles exchange energy with those of adjacent fluid layers and a temperature gradient develops in the fluid. The region of the fluid in which this gradient exists is the thermal boundary layer. Similarly if the concentration of the species at the surface differ from that in the free stream, a concentration gradient exists.

In this study the velocity and thermal boundary layers have been considered.

1.1.8 Hall current

Hall current is the production of a voltage difference (the Hall voltage) across an electrical conductor, transverse to an electric current in the conductor and a magnetic field perpendicular to the current.

1.2 Literature Review

Considerable progress has been made recently in the general theory of MHD flows due to its wide spread application on designing of cooling systems with liquid metals, petroleum industry, purification of crude oil and separation of matter from fluids.

The various people who have studied MHD taking into account effects of Hall current, include; Gupta (1975) discussed effect of Hall current and heat transfer on rotating fluid on a second grade fluid through a porous medium. Pop.I and Soundalgekar(1974) analyzed the Hall effect on the flow in rotating frame of reference. Katagiri (1969) discussed the effects of Hall current on the MHD boundary layer flow past a semi-infinite plate. Soundalgekar *et al*. (1979) studied free convection effects on MHD Stokes problem for a vertical plate and they discovered that skin friction increased owing to a greater heating of the plate. Chartuverdi (1996) studied the finite difference of MHD Stokes problem for a vertical infinite plate in a dissipative heat generating fluid with Hall and Ion-slip current. Soundalgekar *et al* (1979) analyzed the Finite difference analysis of free convection effects on Stokes problem for a vertical plate in a dissipative fluid with constant heat flux. Takhar and Soundalgekar, (1997) investigated the forced and free convective flow past a semi-infinite vertical plate and also did a study on MHD and heat transfer over a semi-infinite plate under a transverse magnetic field. Kinyanjui and Uppal (1998) studied the MHD Stokes problem for a vertical infinite plate in a dissipative rotating fluid with Hall current and they also investigated the effect of both Hall and Ion-slip currents on the flow of heat generating rotating fluid system. They observed that for an Eckert value of 0.02, there was a decrease in the primary velocity profile with an increase in rotational parameter but in the case of secondary velocity profiles, there was initially a decrease and as the distance from the plate increased, the secondary velocity profile increased. They also observed that an increase in Hall parameter has no effect on the temperature profile but an increase in time causes an increase in the temperature profiles. Kinyanjui *et al.* (1999) studied the Finite difference analysis of free convection effects on MHD problem for a vertical plate in a dissipative rotating fluid system with constant heat flux and Hall current. Kinyanjui *et al*. (2001) studied Magneto hydrodynamic free convection heat and mass transfer of a heat generating fluid past an impulsively started infinite vertical porous plate with Hall current and radiation absorption. MHD Stokes free convection flow past an infinite vertical porous plate subjected to constant heat flux with ion-slip current and radiation absorption was investigated by Kinyanjui, Kwanza and Uppal, (2003). Kinyanjui *et al.* (1999) investigated the Finite difference analysis of MHD Stokes problem for a vertical infinite plate in a dissipative fluid with constant heat and Hall current. Chamkha (2004) analyzed the unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. The presence of heat absorption (thermal sink) effects had the tendency to reduce the fluid temperature. Seth *et al* (2009) investigated MHD couette flow in a rotating system in the presence of an inclined magnetic field, they observed that there arises modified hydromagnetic Ekman boundary layer for large values of Rotational parameter and modified Hartmann boundary layer for large values of Hartman number near the moving plate and that the angle of inclination accelerates primary and secondary flows whereas it reduces primary and secondary induced magnetic fields. Rotation induces incipient reverse flow in primary flow direction near the stationary plate. Kinyajui *et al* (2012) analyzed the hydromagnetic turbulent flow of a rotating system past a semi-infinite vertical plate with Hall current, they observed that the parameters in the governing equations affects the velocity, temperature and concentration profiles. Consequently their effect alters the skin friction and the rate of mass transfer. Unsteady hydromagnetic Hartmann or Couette flow in a rotating system in the presence of an inclined magnetic field considering different aspects of the problem was investigated by Ghosh (1991).Seth *et al* (2012) presented their work on effects of Hall current and rotation on unsteady MHD Couette flow in the presence of an inclined magnetic field. They found out that Hall current and rotation tend to accelerate fluid velocity in both the primary and secondary flow directions. Magnetic field has retarding influence on the fluid velocity in both the primary and secondary flow directions. Angle of inclination of magnetic field has accelerating influence on the fluid velocity in both the primary and secondary flow directions. An investigation on Stokes problem of a convective flow past a vertical infinite plate in a rotating system in presence of variable magnetic field was carried out by Mutua *et al* (2013), they observed that all of the parameters affect the primary velocity, secondary velocity and temperature. Consequently their effect alters the rate of heat transfer and skin friction along the x and y axes. MHD turbulent flow problems with inclined magnetic field have received little attention and this was the motivation behind this study. In this study a turbulent flow of an incompressible fluid past a rotating semi-infinite plate with inclined magnetic field is considered.

1.3 Statement Of The Problem

In the studies cited above, turbulent flow problems with inclined magnetic field have not been investigated. A hydromagnetic turbulent fluid flow past a rotating semi-infinite plate is considered. A strong constant magnetic field H_0 is applied in a direction inclined to the flow at an angle ~ as shown in the figure 1.1. In the presence of a strong magnetic field Hall current significantly affect the flow. The induced magnetic field will be assumed to be negligible. The assumption is justified because the magnetic Reynolds number is very small. This research covers a study on the effects of non-dimensional numbers and the angle of inclination on the flow variables. As the partial differential equations governing this problem are non-linear they are solved numerically using finite difference method

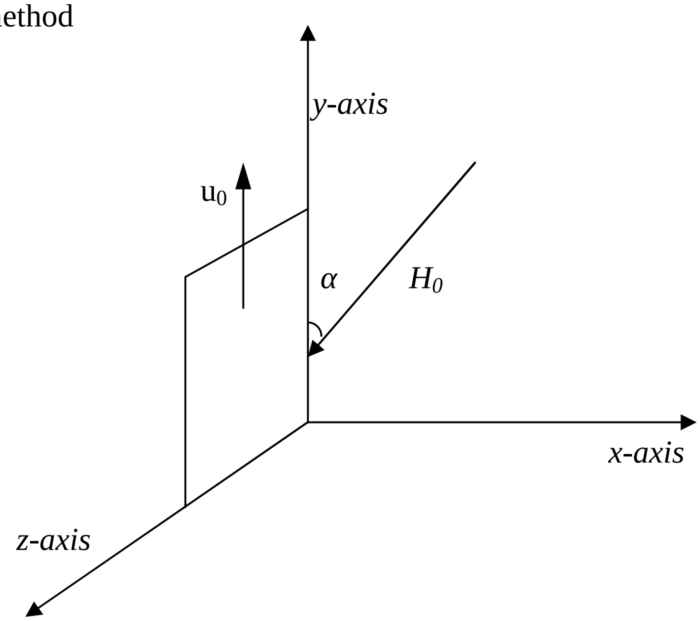


Figure 1.1 Geometry of the problem

1.4 Justification

Fluid mechanics has become an essential part of diverse fields such as Medicine, Meteorology, Astronomy and oceanography as well as traditional engineering disciplines. Many devices that use principles encountered in MHD do not have mechanical parts and hence the devices can be sealed completely and be used in hostile environments where human may not be able to operate for example in the presence of strong radioactive material and places with too high or too low temperatures where there is no oxygen. MHD offers the prospects of improved power stations efficiency and cheap light weight sources of power for space vehicles.The study of fluids past a rotating system has received considerable interest due to its application in practical situations like meteorology, geophysics and fluid dynamics also strong magnetic fields are used to confine rings or columns of hot plasma. Liquid metals are driven through a magnetic field in order to generate electricity.

The early works on fluid dynamics is mostly on laminar flows with little devotion on turbulent flows but most flows of engineering importance are turbulent, for instance when large objects such as ships, automobiles, aircrafts move through fluids and the flow of the fluid past them is always turbulent. Turbulence also occurs when a fluid moves past enclosures such as fans, pumps, Ducts and pipes.

Similarly, a transverse variable magnetic field is taken into consideration. Less emphasis has been given to the problem on turbulent fluid flow in presence of inclined magnetic field, hence, the main objective of the present study aimed at investigating the effects of non-dimensional numbers and the angle of inclination on the velocity profile and temperature profile.

1.5 Null Hypothesis

Non-dimensional numbers and the angle of inclination have no effects on the primary velocity profiles, secondary velocity profiles and temperature profiles .

1.6 Objectives

1.6.1 General objective

To determine the effects of various flow parameters on the flow variables of the hydromagnetic turbulent fluid flow in presence of inclined magnetic field.

1.6.2 Specific objectives

i. To determine the velocity distribution of the fluid flow past a semi-infinite vertical plate
ii. To determine the temperature profiles of the fluid flow past a semi-infinite vertical plate due to velocity variations.
iii. To investigate the effects of non-dimensional numbers and the angle of inclination of the magnetic field on the flow variables.

In the next chapter the general equations governing the flow are discussed. The assumptions for the flow are also outlined and the method of solving these governing equations has been discussed.

CHAPTER TWO

GOVERNING EQUATIONS

2.1 Introduction

In this chapter, equations governing the MHD turbulent flow of an incompressible fluid are discussed taking into account the assumptions made. The equations considered are mass conservation equation, momentum conservation equation and energy equation.

2.1.1 Assumptions

In order to describe the phenomenon mathematically the following approximations and assumptions are made.

1. The flow velocity is much smaller compared to that of light $\frac{q^2}{c^2} << 1$

2. Flow is incompressible

3. Fluid is of constant thermal conductivity, constant electrical conductivity and constant coefficient of viscosity.

4. There is no external applied electrical field that is E=0

5. Force $^{\circ}{}_{e}$ E due to electric field is negligible compared with the force J $\times$ B due to magnetic field.

6. There is no chemical reaction.

2.2 Equations governing the flow

The governing equations of MHD are obtained from the combination of two areas, electromagnetic theory and fluid mechanics. The equations governing the fluid flows of any kind are based on the general laws of conservation of mass, momentum and energy. The flow is subjected to a constant magnetic field inclined at an angle α to the flow. According to the configuration of the flow model, the physical variables governing the flow are functions of x , y and t.

2.2.1 Conservation Equations

The turbulent flows are irregular and there are rapid fluctuations of velocity in the flow variable with respect to time and location. Mean value provides a basis for studying the spatial variation.

For a general flow say v of a turbulent fluid motion, can be given as $v = \bar{v} + v'$ Where $\bar{v}$ is the mean value and v' the fluctuating component.

The Reynolds rules of averaging about varying quantities have been used. If f and g are two flow variables where $f = \bar{f} + f'$ and $g = \bar{g} + g'$ with $\bar{f}$ and $\bar{g}$ as mean values, f' and g' turbulent fluctuations then

$\bar{\bar{f}} = \bar{f}$, $\overline{f+g} = \bar{f} + \bar{g}$, $\overline{Cf} = C\bar{f}$ where C is a constant $\overline{\bar{f}\,g} = \bar{f}\,\bar{g}$,

$\overline{fg} \neq \bar{f}\,\bar{g}$, $\overline{\dfrac{\partial f}{\partial S}} = \dfrac{\partial \bar{f}}{\partial S}$ where S is an independent variable.

Mean value of fluctuation is equal to zero, i.e. $\overline{f'} = \overline{g'} = 0$

The Reynolds averaging rules have been used to transform equations governing laminar flow to turbulent flows.

2.2.2 Equation of continuity

Generally the equation of continuity is derived from the process where the rate at which mass enters a system is equal to the rate at which mass leaves the system. The continuity equation combines the law of conservation of mass and that of the transport theorem. The continuity equation originates from the assumption that mass under normal conditions is neither created nor destroyed and that the flow is continuous. Therefore the mass conservation equation is expressed as ;

$$\frac{\partial \circ}{\partial t} + \nabla.\left(\circ \hat{u}\right) = 0 \qquad \textbf{(2.1)}$$

where $\circ$ and $\hat{u}$ are the density and the velocity of the fluid respectively.

which can be expressed in tensor form as

$$\frac{\partial \circ}{\partial t} + \frac{\partial}{\partial x_i}\left(\circ u_i\right) = 0 \qquad \textbf{(2.2)}$$

Consider time average of equation (2.2)

$$\frac{\partial \circ}{\partial t} + \frac{\partial}{\partial x_i} \circ \left(\overline{u} + u'\right) = 0 \qquad \textbf{(2.3)}$$

$$\frac{\partial \circ}{\partial t} + \frac{\partial \circ \overline{u}}{\partial x_i} + \frac{\partial \circ u'}{\partial x_i} = 0 \qquad \textbf{(2.4)}$$

Applying Reynolds rule of averages

$$\frac{\partial \circ}{\partial t}+\frac{\partial \overline{\circ \overline{u}}}{\partial x_i}+\frac{\partial \overline{\circ u'}}{\partial x_i}=0$$

(2.5)

This yield

$$\frac{\partial \circ}{\partial t}+\frac{\partial}{\partial x_i}\left(\circ \overline{u_i}\right)=0$$

(2.6)

For incompressible flow

$\frac{\partial \overline{u}_i}{\partial x_i}=0$, since the density is a constant

(2.7a)

For this kind of flow the continuity equation is given as

$$\frac{\partial u}{\partial x}=0$$

(2.7b)

On integration Equation (2.7b) reduces to $u=-u_0$ which represents a constant injection in the negative direction of the x-axis.

2.2.3 Equation of motion

The equation of conservation of momentum is derived from the Newton's second law of motion, which states that the time rate of change of momentum of a body is equal to the external force applied to the body. This external force are surface forces (e.g. viscous force) and body forces (e.g. gravitational, centrifugal and magnetic force). Surface forces are due to interaction between the forces e.g. viscous forces and matter in contact with it and the body forces on the other hand are forces which act on the body from a distance.

The equation of motion in component form is given by;

$$\circ\left(\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}\right)=-\frac{\partial p}{\partial x_i}+\text{«}\nabla^2 u_i+F_i$$

(2.8)

The first term is the temporal acceleration while the second term is the convective acceleration. On the right hand side, the first term the pressure gradient, second term is the force due to viscosity and the third is the body force.

The equation (2.8) in terms of time average quantities yields;

$$\circ\left(\frac{\partial}{\partial t}\left(\overline{u}_i+u_i'\right)+\left(\overline{u}_j+u_j'\right)\frac{\partial}{\partial x_j}\left(\overline{u}_i+u_i'\right)\right)=-\frac{\partial}{\partial x_i}\left(\overline{p}+p'\right)+\text{«}\nabla^2\left(\overline{u}_i+u_i'\right)+F_i$$

(2.9)

Or

$$\circ\left(\frac{\partial\overline{u}_i}{\partial t}+\frac{\partial u_i'}{\partial t}+\overline{u}_j\frac{\partial\overline{u}_i}{\partial x_j}+\overline{u}_j\frac{\partial u_i'}{\partial x_j}+u_j'\frac{\partial\overline{u}_i}{\partial x_j}+u_j'\frac{\partial u_i'}{\partial x_j}\right)=-\left(\frac{\partial\overline{p}}{\partial x_i}+\frac{\partial p'}{\partial x_i}\right)+\text{«}\nabla^2\overline{u}_i+\text{«}\nabla^2 u_i'+F_i$$

(2.10)

Taking the time average on both sides

$$\circ\left(\overline{\frac{\partial \overline{u}_i}{\partial t}}+\overline{\frac{\partial u'_i}{\partial t}}+\overline{\overline{u}_j\frac{\partial \overline{u}_i}{\partial x_j}}+\overline{\overline{u}_j\frac{\partial u'_i}{\partial x_j}}+\overline{u'_j\frac{\partial \overline{u}_i}{\partial x_j}}+\overline{u'_j\frac{\partial u'_i}{\partial x_j}}\right)=-\left(\overline{\frac{\partial \overline{p}}{\partial x_i}}+\overline{\frac{\partial p'}{\partial x_i}}\right)+\text{«}\overline{\nabla^2\overline{u}_i}+\text{«}\overline{\nabla^2 u'_i}+\overline{F}_i$$

(2.11)

Simplifying equation (2.11) yields;

$$\circ\left(\frac{\partial \overline{u}_i}{\partial t}+\overline{u}_j\frac{\partial \overline{u}_i}{\partial x_j}+\overline{u'_j\frac{\partial u'_i}{\partial x_j}}\right)=-\frac{\partial \overline{p}}{\partial x_i}+\text{«}\nabla^2\overline{u}_i+\overline{F_i}$$

(2.12)

From continuity equation for incompressible flow;

$\frac{\partial u_i}{\partial x_i}=0$ Thus $u'_i\frac{\partial u'_j}{\partial x_j}=0$ and $\overline{u'_i\frac{\partial u'_j}{\partial x_j}}=0$

Adding $\overline{u'_i\frac{\partial u'_j}{\partial x_j}}$ on left sides of (2.12) and using $\overline{u'_j\frac{\partial u'_i}{\partial x_j}+u'_i\frac{\partial u'_j}{\partial x_j}}=\frac{\partial \overline{u'_i u'_j}}{\partial x_j}$ yields

$$\circ\left(\frac{\partial \overline{u}_i}{\partial t}+\overline{u}_j\frac{\partial \overline{u}_i}{\partial x_j}\right)=-\frac{\partial \overline{p}}{\partial x_i}+\text{«}\nabla^2\overline{u}_i-\circ\frac{\partial \overline{u'_i u'_j}}{\partial x_i}+\overline{F}_i$$

(2.13)

Since the magnetic force has been considered, then equation (2.13) becomes,

$$\circ\left(\frac{\partial \overline{u}_i}{\partial t}+\overline{u}_j\frac{\partial \overline{u}_i}{\partial x_j}\right)=-\frac{\partial \overline{p}}{\partial x_i}+\text{«}\nabla^2\overline{u}_i-\circ\frac{\partial \overline{u'_i u'_j}}{\partial x_i}+\overline{F}_i+\left(J\times B\right)$$

(2.14)

In free convectional fluid flow, the body force is given by $F_{ig} = \circ g$. The pressure gradient $\left(\frac{\partial \circ}{\partial x}\right)$ in the y-direction results from the change in elevation up the plate thus $\frac{\partial \circ}{\partial y} = - \circ_{\infty} g$. The electromagnet force may be written as $F_e = \circ_e E + \hat{J} \times \hat{B}$ in most flows problems the electrostatic force $\circ_e E$ is negligibly small as compared to the electromagnetic force $\hat{J} \times \hat{B}$ hence $F_e = \hat{J} \times \hat{B}$

$$\circ \left(\frac{\partial \bar{u}^*}{\partial t^*} + u_0{}^* \frac{\partial \bar{u}^*}{\partial x^*} + v^* \frac{\partial \bar{v}^*}{\partial y^*} \right) = \circ_{\infty} g - \circ g + \text{«} \left(\frac{\partial^2 \bar{u}^*}{\partial x^{*2}} + \frac{\partial^2 \bar{u}^*}{\partial y^{*2}} \right) - \circ \frac{\partial \overline{u^{*'}{}_i u^{*'}{}_j}}{\partial x^*{}_i} + (\hat{J} \times \hat{B})$$

(2.15)

Expressing the density difference terms $\circ_{\infty} - \circ$ of the volume coefficient of expansion

, where $= \frac{\circ - \circ_{\infty}}{\circ (T^* - T^*{}_{\infty})}$

therefore, the equation of momentum in component form is given by;

$$\frac{\partial \bar{u}^*{}_i}{\partial t^*} + \bar{u}^*{}_j \frac{\partial \bar{u}^*{}_i}{\partial x^*{}_j} = g \quad (T^* - T^*{}_{\infty}) + \frac{\text{«}}{\circ} \frac{\partial^2 \bar{u}^*{}_i}{\partial \bar{x}^*{}_j{}^2} - \frac{\partial}{\partial x_i} \bar{u}^*{}_i \bar{u}^*{}_j + \left(\frac{\hat{J} \times \hat{B}}{\circ}\right)$$

(2.16)

the y and z components are given by;

$$\frac{\partial \bar{v}^*{}_i}{\partial t^*} + \bar{u}^* \frac{\partial \bar{v}^*}{\partial x^*} + \bar{v}^* \frac{\partial \bar{v}^*}{\partial y^*} = g \quad (T^* - T^*{}_{\infty}) + \neg \left(\frac{\partial^2 \bar{v}^*}{\partial \bar{x}^{*2}} + \frac{\partial^2 \bar{v}^*}{\partial \bar{y}^{*2}} \right) - \frac{\partial}{\partial x^*} \overline{u^* v^*} + \frac{(\hat{J} \times \hat{B})_{y^*}}{\circ}$$

(2.17)

$$\frac{\partial \overline{w}^{*}{}_{i}}{\partial t^{*}}+\overline{u}^{*}\frac{\partial \overline{w}^{*}}{\partial x^{*}}+\overline{v}^{*}\frac{\partial \overline{w}^{*}}{\partial y^{*}}=\neg\left(\frac{\partial^{2}\overline{w}^{*}}{\partial \overline{x}^{*2}}+\frac{\partial^{2}\overline{w}^{*}}{\partial \overline{y}^{*2}}\right)-\frac{\partial}{\partial x^{*}}\overline{u^{*}w^{*}}+\frac{(\hat{J}\times\hat{B})_{z^{*}}}{\circ}$$

(2.18)

Where $(\hat{J}\times\hat{B})_{y^{*}}$ $(\hat{J}\times\hat{B})_{z^{*}}$ are the y and z components for $\hat{J}\times\hat{B}$

When the magnetic Reynolds number is small, induced magnetic field is negligible in comparison with the applied magnetic field, so that

$$\hat{B}_{x}=H_{0}\cos\sim \quad \hat{B}_{y}=H_{0}\sin\sim \quad \text{and}\ \hat{B}_{z}=0$$

(2.19)

If $(\hat{J}_{x},\hat{J}_{y},\hat{J}_{z})$ are components of electric current density **J,** the equation of conservation of electric charge $\nabla.\hat{J}=0$ gives,

$$\hat{J}_{x}=constant$$

(2.20)

Since the plate is electrically non-conducting , $\hat{J}_{x}=0$ at the plate and hence zero everywhere in the flow. $\hat{B}_{z}=0$ due to the geometric nature of this flow.

For electrically conducting fluid at rest the current density is given by;

$$\hat{J}=\pm\hat{E}$$

(2.21)

In moving electrically conducting fluids the magnetic field induces a voltage in the conductor of magnitude $\hat{u}\times\hat{B}$

Ohms Law neglecting hall effect yields,

$$\hat{J} = \pm(\hat{E} + \hat{u} \times \hat{B}) \tag{2.22}$$

Neglecting polarization effect, the electric potential $\hat{E}$ becomes $\hat{E} = 0$

therefore equation (2.22) reduces to $\hat{J} = \pm(\hat{u} \times \hat{B})$ and the components for ***J (***the electric current density), ***B* (**the magnetic induction) and ***u*** (velocity) are given as

$$\hat{J} = (0, \hat{J}_y, \hat{J}_z) \quad \hat{B} = (\hat{B}_x, \hat{B}_y, 0) \quad u = (0, v, w) \tag{2.23}$$

The term $\hat{q} \times \hat{B}$ in Equation (2.22) yields

$$\hat{u} \times \hat{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & v & w \\ B_x & B_y & 0 \end{vmatrix} = w\hat{B}_x \hat{j} - v\hat{B}_x \hat{k} \tag{2.24}$$

Thus from equation (2.24)

$$\hat{J}_y = \pm w\hat{B}_x \qquad \hat{J}_z = -\pm v\hat{B}_x \tag{2.25}$$

The Lorentz force becomes

$$\hat{J} \times \hat{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & J_y & J_z \\ B_x & B_y & 0 \end{vmatrix} = B_x J_z \hat{j} - B_x J_y \hat{k}$$

(2.26)

but $\hat{B}_x = «_e H_0 \sin \sim$

Therefore, substituting $(\hat{J} \times \hat{B})_{y^*}$ $(\hat{J} \times \hat{B})_{z^*}$ in equations (2.17) and (2.18) yields;

$$\frac{\partial \overline{v}^*}{\partial t^*} - \overline{u}_0{}^* \frac{\partial \overline{v}^*}{\partial x^*} + \overline{v}^* \frac{\partial \overline{v}^*}{\partial y^*} = g \quad (T^* - T^*{}_\infty) + ¬ \left(\frac{\partial^2 \overline{v}^*}{\partial \overline{x}^{*2}} + \frac{\partial^2 \overline{v}^*}{\partial \overline{y}^{*2}} \right) - \frac{\partial}{\partial x^*} \overline{u^* v^*} + \frac{«_e H_0 \sin \sim}{°} \hat{J}_z$$

(2.27)

$$\frac{\partial \overline{w}^*}{\partial t^*} - \overline{u}_0{}^* \frac{\partial \overline{w}^*}{\partial x^*} + \overline{v}^* \frac{\partial \overline{w}^*}{\partial y^*} = ¬ \left(\frac{\partial^2 \overline{w}^*}{\partial \overline{x}^{*2}} + \frac{\partial^2 \overline{w}^*}{\partial \overline{y}^{*2}} \right) - \frac{\partial}{\partial x^*} \overline{u^* w^*} - \frac{«_e H_0 \sin \sim}{°} \hat{J}_y$$

(2.28)

where g is the acceleration due to gravity, * is the volumetric coefficient of thermal expansion T^*, T^*_∞ are the temperature in the boundary layer and free- stream respectively, ° the fluid density, ¬ is the kinematic viscosity, $\hat{J}_y$ *and* $\hat{J}_z$ are the current density components and v^* *and* w^* are the components in the Y and Z direction.

The Coriolis effect is the apparent deflection of moving objects from a straight path when they are viewed from a rotating frame of reference. The Coriolis effect is caused by the Coriolis force, which appears in the equation of motion in a rotating frame of reference. Initially both the plates and the fluid are in a state of solid rotation with constant angular velocity Ω about the x-axis. The vector formula for the magnitude and direction of the Coriolis acceleration is given by;

$$2\Omega \times \hat{u} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2\Omega & 0 & 0 \\ u_0 & v & w \end{vmatrix} = 2\Omega w\hat{j} - 2\Omega v\hat{k}$$

(2.29)

From equation (2.29) the equation of momentum should now appear as follows;

$$\frac{\partial \bar{v}^*}{\partial t^*} - \bar{u}_0{}^* \frac{\partial \bar{v}^*}{\partial x^*} + \bar{v}^* \frac{\partial \bar{v}^*}{\partial y^*} + 2\Omega w^* = g \ \ (T^* - T^*{}_\infty) + \neg \left(\frac{\partial^2 \bar{v}^*}{\partial \bar{x}^{*2}} + \frac{\partial^2 \bar{v}^*}{\partial \bar{y}^{*2}} \right) - \frac{\partial}{\partial x^*} \overline{u^* v^*} + \frac{\text{«}_e H_0 \sin\sim}{\circ} J_z$$

(2.30)

$$\frac{\partial \bar{w}^*}{\partial t^*} - \bar{u}_0{}^* \frac{\partial \bar{w}^*}{\partial x^*} + \bar{v}^* \frac{\partial \bar{w}^*}{\partial y^*} - 2\Omega v^* = \neg \left(\frac{\partial^2 \bar{w}^*}{\partial \bar{x}^{*2}} + \frac{\partial^2 \bar{w}^*}{\partial \bar{y}^{*2}} \right) - \frac{\partial}{\partial x^*} \overline{u^* w^*} - \frac{\text{«}_e H_0 \sin\sim}{\circ} J_y$$

(2.31)

2.2.4 Energy Equation

The equation of conservation of energy is derived from the First Law of Thermodynamics, which states that energy is conserved in any process involving a thermodynamic system and its surroundings;

$$\circ c_p \left(\frac{\partial T^*}{\partial t^*} - u_0{}^* \frac{\partial T^*}{\partial y^*} + v^* \frac{\partial T^*}{\partial y^*} \right) = \copyright \frac{\partial^2 T^*}{\partial x^{*}} + Q^* + \frac{3}{\circ} \left[\left(\frac{\partial \bar{v}^*}{\partial x^*} \right)^2 + \left(\frac{\partial \bar{w}^*}{\partial x^*} \right)^2 \right]$$

(2.32)

In this study, all the variables with the superscript(*) star will represent the dimensional quantities.

In the next chapter, a turbulent flow of an incompressible electrically conducting fluid past a semi-infinite plate which is subjected to a constant magnetic field applied at an angle to the plate is considered. The mathematical analysis of the flow problem and the corresponding initial and boundary conditions are given. The dimensional equations are non-dimensionalised and then later solved by the finite difference method.

CHAPTER THREE

MATHEMATICAL FORMULATION AND METHODOLOGY

3.1 Introduction

In this chapter, a turbulent flow of an incompressible electrically conducting fluid past a semi-infinite plate which is subjected to a constant magnetic field applied at an angle to the plate is considered. In addition the effect of Hall current is taken into account. The mathematical analysis of the flow problem and the corresponding initial and boundary conditions are given. The non-linear equations are solved by the finite difference method. Expressions for the velocity and temperature at the plate have been obtained. The choice of the coordinates is such that the y-axis is taken along the plate in the vertical direction and the x-axis is taken normal to the plate.

Initially temperature of the fluid and the plate are assumed to be the same. At t>0 , the velocity of the fluid is u_0. The fluid is turbulent therefore there is a large magnetic field this implies Hall current affects the flow. The fluid in the plate is in a state of rigid rotation.

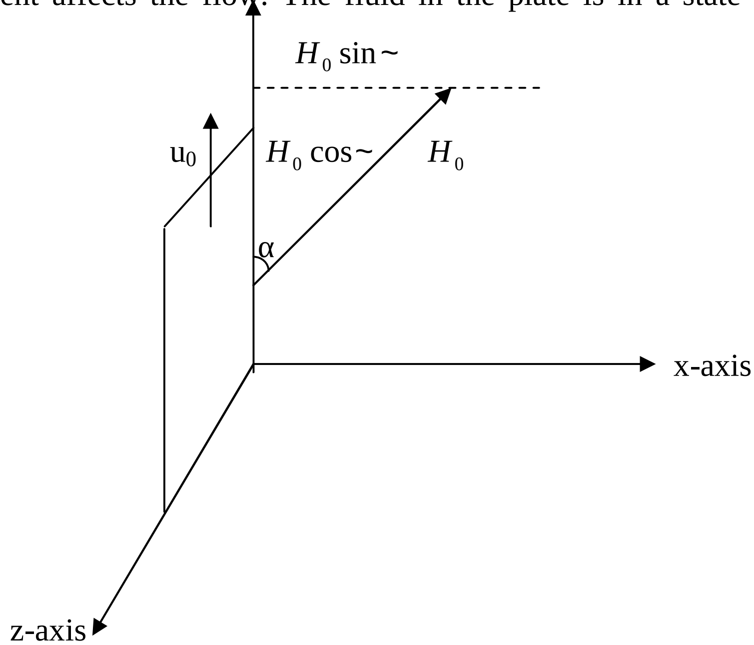


Figure 3.1 resolving the components of the inclined magnetic field

The Maxwell's equations are given as:

$$\nabla.\hat{u} = 0 \tag{3.1}$$

$$\nabla \times \vec{B} = \mu_e \vec{J} \tag{3.2}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{3.3}$$

$$\nabla.\vec{B} = 0 \tag{3.4}$$

Ohm's law for a moving conductor taking Hall current into account is given by

$$\hat{J} + \frac{\omega_e \tau_e}{H_0} \hat{J} \times \hat{H}_0 = \sigma \left(\hat{E} + \mu_e \hat{u} \times \hat{H}_0 + \frac{1}{e n_e} \nabla.p_e \right) \tag{3.5}$$

where $\sigma, \mu_e, \omega_e, \tau_e, e, n_e, p_e$ are the electrical conductivity, the magnetic permeability, the cyclotron frequency, the collision time, the electric charge, the number density of electron, the electron pressure respectively.

For partially ionized fluids the electron pressure gradient may be neglected. In this case, a short circuit problem in which the applied electric field E=0 is considered. Thus neglecting pressure equation (3.5) becomes ;

$$\hat{j} + \frac{\mu_e{}^{2}{}_e}{H}\left(\hat{j} \times \hat{H}_0\right) = ±«_e\left(\hat{u} \times \hat{H}_0\right)$$

(3.6)

$$\left(\hat{j}_y, \hat{j}_z\right) + \frac{m}{H_0}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & j_y & j_z \\ H_0 \sin\sim & H_0 \cos\sim & 0 \end{vmatrix} = ±«_e \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & v & w \\ H_0 \sin\sim & H_0 \cos\sim & 0 \end{vmatrix}$$

(3.7)

solving (3.7) and equating the y and z components yields;

$$\hat{j}_y + m(\hat{j}_z \sin\sim) = ±«_e(wH_0 \sin\sim)$$

(3.8)

$$\hat{j}_z - m(\hat{j}_y \sin\sim) = -±«_e(vH_0 \sin\sim)$$

(3.9)

calculating $\hat{j}_{y^+}$ and $\hat{j}_{z^+}$ we have;

$$\hat{j}_y = \frac{±«_e H_0 \sin\sim(w + mv\sin\sim)}{1 + m^2 \sin^2\sim}$$

(3.10)

$$\hat{j}_z = \frac{±«_e H_0 \sin\sim(mw\sin\sim - v)}{1 + m^2 \sin^2\sim}$$

(3.11)

where $m = ´\ {}_e^{2}{}_e$ is the Hall current

Replacing the values for equation (3.10) and (3.11) back to equation (2.30) and (2.31) respectively yields;

$$\frac{\partial v^*}{\partial t^*}-u_0^{\,*}\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}+2\Omega w^*=g\ \ (T^*-T^*{}_\infty)+\neg\left(\frac{\partial^2 v^*}{\partial x^{*2}}+\frac{\partial^2 v^*}{\partial y^{*2}}\right)-\frac{\partial}{\partial x^*}u^*v^*+\frac{\pm «_e^{\,2} H_0^{\,2}(\sin\sim)^2}{\circ}\left[\frac{(mw\sin\sim-v)}{1+m^2(\sin\sim)^2}\right]$$

(3.12)

$$\frac{\partial w^*}{\partial t^*}-u_0^{\,*}\frac{\partial w^*}{\partial x^*}+v^*\frac{\partial w^*}{\partial y^*}-2\Omega v^*=\neg\left(\frac{\partial^2 w^*}{\partial x^{*2}}+\frac{\partial^2 w^*}{\partial y^{*2}}\right)-\frac{\partial}{\partial x^*}u^*w^*-\frac{\pm «_e^{\,2} H_0^{\,2}(\sin\sim)^2}{\circ}\left[\frac{(w+mv\sin\sim)}{1+m^2(\sin\sim)^2}\right]$$

(3.13)

Energy equation for Turbulent flow

$$\circ\, c_p\left(\frac{\partial T}{\partial t}+\frac{\partial T}{\partial x_i}\right)=\frac{\partial}{\partial x_i}\left(k\frac{\partial T}{\partial x_i}\right)+\frac{J^2}{\pm}+¤$$

(3.14)

For incompressible flow the temperature and density fluctuations are negligible.

The initial and boundary conditions for this study take the form;

$$t^*\le 0 \qquad v^*(x^*,y^*,t^*)=0 \quad w^*(x^*,y^*,t^*)=0 \quad T^*(x^*,y^*,t^*)=0$$

$$t^*>0$$

$$v^*(0,y^*,t^*)=u_0 \qquad w^*(0,y^*,t^*)=0 \qquad T^*(0,y^*,t^*)=T_w$$

$$v^*(\infty,y^*,t^*)=0 \qquad w^*(\infty,y^*,t^*)=0 \qquad T^*(\infty,y^*,t^*)=0$$

(3.15)

In this study, all the variables with the superscript(*) will represent the dimensional quantities.

3.2 Non-Dimensionalisation

Dimensional analysis is a method by which the number of independent variablesn the problem is reduced into dimensionless groups. It therefore describes a natural phenomenon by a dimensionally correct equation with certain variables which affects the phenomenon. The following fundamental primary dimensions namely mass (m), length (l), time (t) and temperature (T) are used. The dimensions of all other physical variables in this study can be obtained in terms of these basic dimensions.

In this study non- dimensionalisation is based on the following non- dimensional quantities

$$t = \frac{t^{*}U^{2}}{\neg} \qquad x = \frac{x^{*}U}{\neg} \qquad y = \frac{y^{*}U}{\neg} \qquad z = \frac{z^{*}U}{\neg} \quad u_{0} = \frac{u_{0}^{*}}{U} \qquad v = \frac{v^{*}}{U} \qquad w = \frac{w^{*}}{U} \quad u = \frac{u^{*}}{U}$$

$$\Pr = \frac{\text{«}c_{p}}{k} \qquad Gr = \frac{{}^{3}g\left(q^{*3}\big/kU\right)}{U^{3}} \qquad E_{c} = \frac{U^{2}}{c_{p}\left(q^{*3}/kU\right)} \qquad T = \frac{T^{*} - T_{\infty}}{T_{w} - T_{\infty}}$$

$$E_{r} = \frac{\Omega^{3}}{U^{2}}$$

(3.16)

where $\neg$ is the characteristic length and U the free stream velocity.

3.2.1 Equation of continuity

For this particular fluid flow the equation of continuity is given by

$$\frac{\partial u^{*}}{\partial x^{*}} + \frac{\partial v^{*}}{\partial y^{*}} = 0 \qquad \textbf{(3.17)}$$

Non-dimensionalising the continuity equation yields

$$\frac{\partial u^*}{\partial x^*} = \frac{\partial u^*}{\partial u}\frac{\partial u}{\partial x}\frac{\partial x}{\partial x^*} = \frac{U^2}{\neg}\frac{\partial u}{\partial x} \qquad \frac{\partial v^*}{\partial y^*} = \frac{\partial v^*}{\partial v}\frac{\partial v}{\partial y}\frac{\partial y}{\partial y^*} = \frac{U^2}{\neg}\frac{\partial v}{\partial y} \tag{3.18}$$

$$\frac{U^2}{\neg}\frac{\partial u}{\partial x} + \frac{U^2}{\neg}\frac{\partial v}{\partial y} = 0$$

$$\frac{U^2}{\neg}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \quad , \frac{U^2}{\neg} = 0 \text{ therefore} \tag{3.19}$$

$$\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \tag{3.20}$$

3.2.2 Equation of conservation of momentum

The equation of conservation of momentum in this type of flow along the y-axis and the z-axis respectively is given by;

$$\frac{\partial v^*}{\partial t^*} - u_0\frac{\partial v^*}{\partial x^*} + v\frac{\partial v^*}{\partial y^*} + 2\Omega w^* = \neg\left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}}\right) + g\ (T - T_\infty) - \frac{\partial u^* v^*}{\partial x^*} + \frac{\pm\text{«}_e{}^2 H_0{}^2 (\sin\sim)^2}{\circ}\left[\frac{(mw\sin\sim - v)}{1 + m^2(\sin\sim)^2}\right] \tag{3.21}$$

$$\frac{\partial w^*}{\partial t^*} - u^*{}_0\frac{\partial w^*}{\partial x^*} + v^*\frac{\partial w^*}{\partial y^*} - 2\Omega v^* = \neg\left(\frac{\partial^2 w^*}{\partial x^{*2}} + \frac{\partial^2 w^*}{\partial y^{*2}}\right) - \frac{\partial u^* w^*}{\partial x^*} - \frac{\pm\text{«}_e{}^2 H_0{}^2 (\sin\sim)^2}{\circ}\left[\frac{(w + mv\sin\sim)}{1 + m^2(\sin\sim)^2}\right] \tag{3.22}$$

Adopting the Boussinesque approximation

$$^{2} = -{}^{\circ} v\overline{w} = A\frac{\partial \overline{v}}{\partial z}$$

(3.23)

Prandlt deduced that

$${}^{\circ} v\overline{w} = -{}^{\circ} l^{2}\left(\frac{\partial \overline{v}}{\partial z}\right)^{2}$$

(3.24)

Taking that $l = kz$ where k is the von Karman constant so we have

$${}^{\circ}\, \overline{vw} = -{}^{\circ}\, k^{2}z^{2}\left(\frac{\partial \overline{v}}{\partial z}\right)^{2}$$

(3.25)

Hence

$$\overline{vw} = -k^{2}z^{2}\left(\frac{\partial \overline{v}}{\partial z}\right)^{2}$$

(3.26)

Similarly we deduce

$$\overline{uv} = -k^{2}x^{2}\left(\frac{\partial \overline{v}}{\partial x}\right)^{2}$$

(3.27)

$$\overline{uw} = -k^{2}x^{2}\left(\frac{\partial \overline{w}}{\partial x}\right)^{2}$$

(3.28)

$$\frac{\partial v^*}{\partial t^*} - u_0{}^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} + 2\Omega w^* = \neg\left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}}\right) + g\ {}^*\left(T - T^{+}{}_{\infty}\right) + \frac{\partial}{\partial x^*}\left[k^2 x^2 \left(\frac{\partial v}{\partial x}\right)^2\right] + \frac{\pm\text{«}_e{}^2 H_0{}^2 (\sin\sim)^2}{\circ}\left[\frac{(mw\sin\sim - v)}{1+m^2(\sin\sim)^2}\right]$$

(3.29)

$$\frac{\partial w^*}{\partial t} - u_0{}^* \frac{\partial w^*}{\partial x^*} + v^* \frac{\partial w^*}{\partial y^*} - 2\Omega v^* = \neg\left(\frac{\partial^2 w^*}{\partial x^{*2}} + \frac{\partial^2 w^*}{\partial y^{*2}}\right) + \frac{\partial}{\partial x^*}\left[k^2 x^2 \left(\frac{\partial w}{\partial x}\right)^2\right] - \frac{\pm\text{«}_e{}^2 H_0{}^2 (\sin\sim)^2}{\circ}\left[\frac{(w + mv\sin\sim)}{1+m^2(\sin\sim)^2}\right]$$

(3.30)

or

$$\frac{\partial v^*}{\partial t^*} - u_0{}^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} + 2\Omega w^* = \neg\left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}}\right) + g\ {}^*\left(T - T^{+}{}_{\infty}\right) + 2k^2 x \left(\frac{\partial v}{\partial x}\right)^2 + 2k^2 x^2 \left(\frac{\partial^2 v}{\partial x^2}\right)\left(\frac{\partial v}{\partial x}\right) +$$
$$\frac{\pm\text{«}_e{}^2 H_0{}^2 (\sin\sim)^2}{\circ}\left[\frac{(mw\sin\sim - v)}{1+m^2(\sin\sim)^2}\right]$$

(3.31)

$$\frac{\partial w^*}{\partial t} - u_0{}^* \frac{\partial w^*}{\partial x^*} + v^* \frac{\partial w^*}{\partial y^*} - 2\Omega v^* = \neg\left(\frac{\partial^2 w^*}{\partial x^{*2}} + \frac{\partial^2 w^*}{\partial y^{*2}}\right) + 2k^2 x \left(\frac{\partial w}{\partial x}\right)^2 + 2k^2 x^2 \left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial w}{\partial x}\right) - \frac{\pm\text{«}_e{}^2 H_0{}^2 (\sin\sim)^2}{\circ}\left[\frac{(w + mv\sin\sim)}{1+m^2(\sin\sim)^2}\right]$$

(3.32)

Non-dimensionalising the momentum equation along the y -axis

$$\frac{\partial v^*}{\partial t^*} - u_0{}^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} + 2\Omega w^* = \neg \left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}} \right) + g \quad {}^*\left(T - T^+{}_\infty\right) + 2k^2 x \left(\frac{\partial v}{\partial x} \right)^2 + 2k^2 x^2 \left(\frac{\partial^2 v}{\partial x^2} \right)\left(\frac{\partial v}{\partial x} \right)$$
$$+ \frac{\pm\text{«}_e{}^2 H_0{}^2 (\sin\sim)^2}{\circ} \left[\frac{(mw\sin\sim - v)}{1 + m^2 (\sin\sim)^2} \right]$$

$$\frac{\partial v^*}{\partial t^*} = \frac{\partial v^*}{\partial v} \frac{\partial v}{\partial t} \frac{\partial t}{\partial t^*} = U \frac{\partial v}{\partial t} \frac{U^2}{\neg} = \frac{U^3}{\neg} \frac{\partial v}{\partial t} \qquad u_0{}^* \frac{\partial v^*}{\partial x^*} = u_0 U \frac{\partial v^*}{\partial v} \frac{\partial v}{\partial x} \frac{\partial x}{\partial x^*} = -u_0 \frac{U^3}{\neg} \frac{\partial v}{\partial x}$$

(3.33)

$$v^* \frac{\partial v^*}{\partial y^*} = Uv \frac{\partial v^*}{\partial v} \frac{\partial v}{\partial y} \frac{\partial y}{\partial y^*} = v \frac{U^3}{\neg} \frac{\partial v}{\partial y} \qquad 2\Omega w^* = 2\Omega U w \qquad T^* - T_\infty{}^* = -\frac{q^{*3}}{kU}$$

(3.34)

$$\frac{\partial^2 v^*}{\partial x^{*2}} = \frac{\partial}{\partial x^*} \left(\frac{\partial v^*}{\partial x^*} \right) = \frac{\partial}{\partial x^*} \left(\frac{\partial v^*}{\partial x^*} \right) \frac{\partial x}{\partial x^*} = \frac{\partial v^*}{\partial v} \frac{\partial v}{\partial x} \frac{\partial x}{\partial x^*} = \frac{U^3}{\neg^2} \frac{\partial^2 v}{\partial x^2}$$

(3.35)

$$\frac{\partial^2 v^*}{\partial y^2} = \frac{\partial}{\partial y} \left(\frac{\partial v^*}{\partial y^*} \right) = \frac{\partial}{\partial y} \left(\frac{\partial v^*}{\partial y^*} \right) \frac{\partial y}{\partial y^*} = \frac{\partial v^*}{\partial v} \frac{\partial v}{\partial y} \frac{\partial y}{\partial y^*} = \frac{U^3}{\neg^2} \frac{\partial^2 v}{\partial y^2}$$

replacing (3.33),(3.34) and (3.35) back to equation (3.31) yields

$$\frac{U^3}{\neg} \frac{\partial v}{\partial t} - u_0 \frac{U^3}{\neg} \frac{\partial v}{\partial x} + v \frac{U^3}{\neg} \frac{\partial v}{\partial y} - 2\Omega U w = \neg \frac{U^3}{\neg^2} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial x^2} \right) + \frac{U^3}{\neg} \left(2k^2 x \left(\frac{\partial v}{\partial x} \right)^2 + 2k^2 x^2 \left(\frac{\partial^2 v}{\partial x^2} \right)\left(\frac{\partial v}{\partial x} \right) \right) +$$
$$g \quad - \frac{q^{*3}}{kU} + \frac{\pm\text{«}_e{}^2 H_0{}^2 (\sin\sim)^2}{\circ} \left[\frac{(mw\sin\sim - v)}{1 + m^2 (\sin\sim)^2} \right]$$

(3.36)

dividing (3.36) through by $\frac{U^3}{\neg}$ yields

$$\frac{\partial v}{\partial t}-u_0\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+2E_r w=\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\left(2k^2x\left(\frac{\partial v}{\partial x}\right)^2+2k^2x^2\left(\frac{\partial^2 v}{\partial x^2}\right)\left(\frac{\partial v}{\partial x}\right)\right)+G_r\bar{}+\frac{\pm\ll_e{}^2H_0{}^2(\sin\sim)^2}{\circ}\frac{³}{U^3}\left[\frac{(mw\sin\sim-v)}{1+m^2(\sin\sim)^2}\right]$$

(3.37)

Non-dimensionalising the momentum equation along the z-axis

$$\frac{\partial w^*}{\partial t}-u_0{}^*\frac{\partial w^*}{\partial x^*}+v^*\frac{\partial w^*}{\partial y^*}-2\Omega v^*=\neg\left(\frac{\partial^2 w^*}{\partial x^{*2}}+\frac{\partial^2 w^*}{\partial y^{*2}}\right)+2k^2x\left(\frac{\partial w}{\partial x}\right)^2+2k^2x^2\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial w}{\partial x}\right)-\frac{\pm\ll_e{}^2H_0{}^2(\sin\sim)^2}{\circ}\left[\frac{(w+mv\sin\sim)}{1+m^2(\sin\sim)^2}\right]$$

$$\frac{\partial w^*}{\partial t^*}=\frac{\partial w^*}{\partial w}\frac{\partial w}{\partial t}\frac{\partial t}{\partial t^*}=U\frac{\partial w}{\partial t}\frac{U^2}{\neg}=\frac{U^3}{\neg}\frac{\partial w}{\partial t} \qquad u_0{}^*\frac{\partial w^*}{\partial x^*}=u_0U\frac{\partial w^*}{\partial w}\frac{\partial w}{\partial x}\frac{\partial x}{\partial x^*}=u_0\frac{U^3}{\neg}\frac{\partial w}{\partial x}$$

(3.38)

$$v^*\frac{\partial w^*}{\partial y^*}=Uv\frac{\partial w^*}{\partial w}\frac{\partial w}{\partial y}\frac{\partial y}{\partial y^*}=v\frac{U^3}{\neg}\frac{\partial w}{\partial y} \qquad 2\Omega v^*=2\Omega Uv$$

(3.39)

$$\frac{\partial^2 w^*}{\partial x^2}=\frac{\partial}{\partial x}\left(\frac{\partial w^*}{\partial x^*}\right)=\frac{\partial}{\partial x}\left(\frac{\partial w^*}{\partial x^*}\right)\frac{\partial x}{\partial x^*}=\frac{\partial w^*}{\partial w}\frac{\partial w}{\partial x}\frac{\partial x}{\partial x^*}=\frac{U^3}{\neg^2}\frac{\partial^2 w}{\partial x^2}$$

(3.40)

$$\frac{\partial^2 w^*}{\partial y^2}=\frac{\partial}{\partial y}\left(\frac{\partial w^*}{\partial y^*}\right)=\frac{\partial}{\partial y}\left(\frac{\partial w^*}{\partial y^*}\right)\frac{\partial y}{\partial y^*}=\frac{\partial w^*}{\partial w}\frac{\partial w}{\partial x}\frac{\partial y}{\partial y^*}=\frac{U^3}{\neg^2}\frac{\partial^2 w}{\partial y^2}$$

replacing(3.38), (3.39) and (3.40) back to the equation (3.32) yields

$$\frac{U^3}{¬}\frac{\partial w}{\partial t} - u_0\frac{U^3}{¬}\frac{\partial w}{\partial x} + v\frac{U^3}{¬}\frac{\partial w}{\partial y} - 2\Omega U v = ¬\frac{U^3}{¬^2}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) + \frac{U^3}{¬}\left(2k^2 x\left(\frac{\partial w}{\partial x}\right)^2 + 2k^2 x^2\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial w}{\partial x}\right)\right)$$
$$- \frac{±«_e{}^2 H_0{}^2(\sin ~)^2}{°}\left[\frac{(w + mv\sin ~)}{1 + m^2(\sin ~)^2}\right]$$

(3.41)

dividing (3.41)through by $\frac{U^3}{¬}$ yields

$$\frac{\partial w}{\partial t} - u_0\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} - 2E_r v = \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + 2k^2 x\left(\frac{\partial w}{\partial x}\right)^2 + 2k^2 x^2\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial w}{\partial x}\right) - \frac{±«_e{}^2 H_0{}^2(\sin ~)^2}{°}\frac{3}{U^3}\left[\frac{(w + mv\sin ~)}{1 + m^2(\sin ~)^2}\right]$$

(3.42)

Non-dimensionalising the energy equation

$$\frac{\partial T^*}{\partial t^*} = \frac{\partial T^*}{\partial \bar{}}\frac{\partial \bar{}}{\partial t}\frac{\partial t}{\partial t^*} = \frac{U^2}{3}\left(T_w^* - T_\infty^*\right)\frac{\partial \bar{}}{\partial t} \qquad u_0\frac{\partial T^*}{\partial x^*} = \frac{U^2}{3}u_0\left(T_w^* - T_\infty^*\right)\frac{\partial \bar{}}{\partial x}$$

(3.43)

$$v^*\frac{\partial T^*}{\partial y^*} = \frac{U^2}{3}v\left(T_w^* - T_\infty^*\right)\frac{\partial \bar{}}{\partial y} \qquad \frac{\partial^2 T^*}{\partial x^{*2}} = \frac{U}{3}\left(T_w^* - T_\infty^*\right)\frac{\partial^2 \bar{}}{\partial x^2}$$

(3.44)

Substituting (3.43) and (3.44) in (3.14) yields

$$\frac{U^2}{3}\left(T_w^*-T_\infty^*\right)\frac{\partial \text{¯}}{\partial t}+u_0\frac{U^2}{3}\left(T_w^*-T_\infty^*\right)\frac{\partial \text{¯}}{\partial x}+v\frac{U^2}{3}\left(T_w^*-T_\infty^*\right)\frac{\partial \text{¯}}{\partial y}=\frac{U^2}{3^2}\frac{\text{©}}{\text{°}c_p}\left(T_w^*-T_\infty^*\right)\frac{\partial^2 \text{¯}}{\partial x^2}$$

$$+\frac{\text{©}U^2\text{¢}}{3}+\frac{3}{c_p}\frac{U^4}{3^2}\left[\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial w}{\partial x}\right)^2\right]$$

(3.45)

Dividing through by $\frac{U^2}{3}\left(T_w^*-T_\infty^*\right)$ and Using non-dimensional quantities

$$\frac{\partial \text{¯}}{\partial t}-u_0\frac{\partial \text{¯}}{\partial x}+v\frac{\partial \text{¯}}{\partial y}=\frac{1}{\Pr}\left(\frac{\partial^2 \text{¯}}{\partial x^2}+\frac{\partial^2 \text{¯}}{\partial y^2}\right)+\frac{\text{¢}}{\Pr}\text{¯}+E_c\left[\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right]$$

(3.46)

The final set of non-dimensional equations are;

$$\frac{\partial v}{\partial t}-u_0\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+2E_r w=\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+2k^2x\left(\frac{\partial v}{\partial x}\right)^2+2k^2x^2\left(\frac{\partial^2 v}{\partial x^2}\right)\left(\frac{\partial v}{\partial x}\right)+G_r\text{¯}+M^2(\sin\text{~})^2\left[\frac{(mw\sin\text{~}-v)}{1+m^2(\sin\text{~})^2}\right]$$

(3.47)

$$\frac{\partial w}{\partial t}-u_0\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}-2E_r v=\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+2k^2x\left(\frac{\partial w}{\partial x}\right)^2+2k^2x^2\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial w}{\partial x}\right)-M^2(\sin\text{~})^2\left[\frac{(w+mv\sin\text{~})}{1+m^2(\sin\text{~})^2}\right]$$

(3.48)

$$\frac{\partial ¯}{\partial t} - u_0\frac{\partial ¯}{\partial x} + v\frac{\partial ¯}{\partial y} = \frac{1}{\Pr}\left(\frac{\partial^2 ¯}{\partial x^2} + \frac{\partial^2 ¯}{\partial x^2}\right) + \frac{¢}{\Pr} ¯ + E_c\left[\left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right]$$

(3.49)

$\frac{\pm B_0^{\ 2}}{\circ}\frac{¬}{U^3}$ is the magnetic parameter which is M^2.

The plate is impulsively started and therefore the velocity at $x=0$ changes from zero at $t < 0$ to 1

The initial and boundary conditions in non-dimensional form become

at $t \leq 0$ $\quad v(x, y, 0) = 0 \quad w\ (x, y, 0) = 0 \quad ¯(x, y, 0) = 0$

(3.50)

at $t > 0$ $\quad v(0, y, t) = 1 \quad w(0, y, t) = 0 \quad ¯\ (0, y, t) = 1$

$\quad v(\infty, y, t) = 0 \quad w(\infty, y, t) = 0 \quad ¯(\infty, y, t) = 0$ **(3.51)**

3.3 Non-dimensional numbers

The non-dimensionalised equations above contain the following dimensionless numbers.

3.3.1 Prandtl Number (Pr)

The Prandtl number (Pr) is the ratio of fluid properties controlling the velocity and the temperature distributions. It is the ratio of viscous force to thermal force.

$$\Pr = \frac{«c_p}{©} = \frac{¬}{\sim}$$

(3.52)

where « dynamic viscosity, ¬ kinematic viscosity, ~ thermal diffusivity, © thermal conductivity and c_p specific heat.

In heat transfer involving convection, warm and cool particles mix because of their pressure difference, local heat conduction occurs. This mixing also involves momentum transfer. Prandtl Number is a measure of the relative ability of the fluid to allow momentum diffusion and thermal diffusion.

3.3.2 Grashof number (Gr)

This number usually occurs in natural convection problems and is defined as

$$Gr = {}^{3}\, gB \frac{T - T_{\infty}}{U^{3}}$$

(3.53)

where g acceleration due to earth's gravity,   coefficient of thermal expansion, T surface temperature, T_{∞} bulk temperature, U vertical length and ¬ kinematic viscosity.

This gives the relative important of buoyancy force to viscous force.

3.3.3 Eckert Number (Ec)

It provides a measure of the kinetic energy of the flow relative to the enthalpy difference across the thermal boundary layer. It represents the conversion of kinetic energy into internal energy by work that is done against the viscous fluid stresses.

$$E_c = \frac{U^2}{c_p\left(T - T_\infty\right)} \tag{3.54}$$

where U flow velocity, c_p specific heat and $T - T_\infty$ difference between wall temperature and local temperature.

3.3 Method of solution

Equations (3.47), (3.48) and (3.49) that govern the MHD flow are non-linear and there exist no analytical method for solving them. Numerical solutions of the equations are generated by using the finite difference method. The equations are solved subject to the initial and boundary conditions.The derivatives are approximated with the implicit forward difference scheme.

3.3.1 The finite difference method

The stencil for the most common explicit method is shown in Figure 3.2.

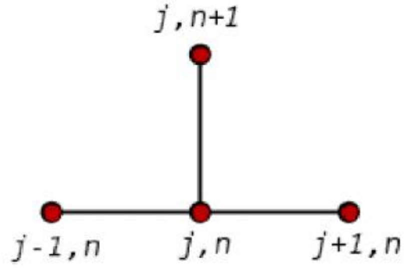


Figure 3.2 Stencil for the explicit method

The expressions for the averages are given as follows;

$$\frac{\partial v}{\partial t} = \frac{v_{i,j}^{k+1} - v_{i,j}^{k}}{\Delta t} \qquad \frac{\partial v}{\partial x} = \frac{v_{i+1,j}^{k+1} - v_{i,j}^{k+1}}{\Delta x} \qquad \frac{\partial v}{\partial y} = \frac{v_{i,j+1}^{k+1} - v_{i,j}^{k+1}}{\Delta y} \tag{3.55}$$

$$\frac{\partial^2 v}{\partial x^2} = \frac{v_{i+1,j}^{k+1} - 2v_{i,j}^{k+1} + v_{i-1,j}^{k+1}}{(\Delta x)^2} \qquad \frac{\partial^2 v}{\partial y^2} = \frac{v_{i,j+1}^{k+1} - 2v_{i,j}^{k+1} + v_{i,j-1}^{k+1}}{(\Delta y)^2} \tag{3.56}$$

$$\frac{\partial w}{\partial t} = \frac{w_{i,j}^{k+1} - w_{i,j}^{k}}{\Delta t} \qquad \frac{\partial w}{\partial x} = \frac{w_{i+1,j}^{k+1} - w_{i,j}^{k+1}}{\Delta x} \qquad \frac{\partial w}{\partial y} = \frac{w_{i,j+1}^{k+1} - w_{i,j}^{k+1}}{\Delta y} \tag{3.57}$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{w_{i+1,j}^{k+1} - 2w_{i,j}^{k+1} + w_{i-1,j}^{k+1}}{(\Delta x)^2} \qquad \frac{\partial^2 w}{\partial x^2} = \frac{w_{i,j+1}^{k+1} - 2w_{i,j}^{k+1} + w_{i,j-1}^{k+1}}{(\Delta y)^2} \tag{3.58}$$

$$\frac{\partial T}{\partial t}=\frac{T_{i,j}^{k+1}-T_{i,j}^{k}}{\Delta t}\qquad \frac{\partial T}{\partial x}=\frac{T_{i+1,j}^{k+1}-T_{i,j}^{k+1}}{\Delta x}\qquad \frac{\partial T}{\partial y}=\frac{T_{i,j+1}^{k+1}-T_{i,j}^{k+1}}{\Delta y}$$

(3.59)

$$\frac{\partial^2 T}{\partial x^2}=\frac{T_{i+1,j}^{k+1}-2T_{i,j}^{k+1}+T_{i-1,j}^{k+1}}{(\Delta x)^2}\qquad \frac{\partial^2 T}{\partial y^2}=\frac{T_{i,j+1}^{k+1}-2T_{i,j}^{k+1}+T_{i,j-1}^{k+1}}{(\Delta y)^2}$$

(3.60)

The finite difference equations corresponding to the governing equations will be given as;

Equation of momentum in the y direction:

$$\frac{v_{i,j}^{k+1}-v_{i,j}^{k}}{\Delta t}-u_0\left(\frac{v_{i+1,j}^{k+1}-v_{i,j}^{k+1}}{\Delta x}\right)+v_{i,j}^{k}\left(\frac{v_{i,j+1}^{k+1}-v_{i,j}^{k+1}}{\Delta y}\right)-2E_r w_{i,j}^{k}=$$

$$\left(\frac{v_{i+1,j}^{k+1}-2v_{i,j}^{k+1}+v_{i-1,j}^{k+1}}{(\Delta x)^2}\right)+\left(\frac{v_{i,j+1}^{k+1}-2v_{i,j}^{k+1}+v_{i,j-1}^{k+1}}{(\Delta y)^2}\right)+2k^2x\left(\frac{v_{i+1,j}^{k+1}-v_{i-1,j}^{k+1}}{2\Delta x}\right)$$

$$+2k^2x^2\left(\frac{v_{i+1,j}^{k+1}-2v_{i,j}^{k+1}+v_{i-1,j}^{k+1}}{(\Delta x)^2}\right)\left(\frac{v_{i+1,j}^{k+1}-v_{i,j}^{k+1}}{\Delta x}\right)+G_r\,\overline{\ }+M^2(\sin\sim)^2\,\frac{(w+mvH\sin\sim)}{1+m^2H^2\sin^2\sim}$$

(3.61)

Equation of momentum in the z direction:

$$\frac{w_{i,j}^{k+1}-w_{i,j}^{k}}{\Delta t}-u_0\left(\frac{w_{i+1,j}^{k+1}-w_{i,j}^{k+1}}{\Delta x}\right)+v_{i,j}^{k}\left(\frac{w_{i,j+1}^{k+1}-w_{i,j}^{k+1}}{\Delta y}\right)-2E_r v_{i,j}^{k}$$
$$=\left(\frac{w_{i+1,j}^{k+1}-2w_{i,j}^{k+1}+w_{i-1,j}^{k+1}}{(\Delta x)^2}\right)+\left(\frac{w_{i,j+1}^{k+1}-2w_{i,j}^{k+1}+w_{i,j-1}^{k+1}}{(\Delta y)^2}\right)+2k^2x\left(\frac{w_{i+1,j}^{k+1}-w_{i-1,j}^{k+1}}{2\Delta x}\right) \quad \textbf{(3.62)}$$
$$+2k^2x^2\left(\frac{w_{i+1,j}^{k+1}-2w_{i,j}^{k+1}+w_{i-1,j}^{k+1}}{(\Delta x)^2}\right)\left(\frac{w_{i+1,j}^{k+1}-w_{i,j}^{k+1}}{\Delta x}\right)-M^2(\sin\sim)^2\,\frac{(mw\sin\sim-v)}{1+m^2\sin^2\sim}$$

The energy equation in finite difference form is;

$$\frac{\bar{\ }_{i,j}^{k+1}-\bar{\ }_{i,j}^{k}}{\Delta t}-u_0\,\frac{\bar{\ }_{i+1,j}^{k+1}-\bar{\ }_{i,j}^{k+1}}{\Delta x}+v_{i,j}^{k}\,\frac{\bar{\ }_{i,j+1}^{k+1}-\bar{\ }_{i,j}^{k+1}}{\Delta y}$$
$$=\frac{1}{\Pr}\left(\frac{\bar{\ }_{i+1,j}^{k+1}-2\bar{\ }_{i,j}^{k+1}+\bar{\ }_{i-1,j}^{k+1}}{(\Delta x)^2}+\frac{\bar{\ }_{i,j+1}^{k+1}-2\bar{\ }_{i,j}^{k+1}+\bar{\ }_{i,j-1}^{k+1}}{(\Delta y)^2}\right)-\frac{\text{¢}}{\Pr}\bar{\ }$$
$$+E_c\left[\left(\frac{v_{i+1,j}^{k+1}-v_{i,j}^{k+1}}{\Delta x}\right)^2+\left(\frac{w_{i+1,j}^{k+1}-w_{i,j}^{k+1}}{\Delta x}\right)^2+\left(\frac{v_{i,j+1}^{k+1}-v_{i,j}^{k+1}}{\Delta y}\right)^2+\left(\frac{w_{i,j+1}^{k+1}-w_{i,j}^{k+1}}{\Delta y}\right)^2\right]$$

(3.63)

3.4 Equation governing the flow in finite difference form

The governing equations describing the unsteady MHD turbulent flow of an incompressible, electrically conducting and viscous Newtonian fluid past a rotating semi-infinite plate in finite difference form were given subject to their initial and boundary conditions as;

The equation of momentum along the y-axis is

$$
\frac{v_{i,j}^{k+1}-v_{i,j}^{k}}{\Delta t}-u_0\left(\frac{v_{i+1,j}^{k+1}-v_{i,j}^{k+1}}{\Delta x}\right)+v_{i,j}^{k}\left(\frac{v_{i,j+1}^{k+1}-v_{i,j}^{k+1}}{\Delta y}\right)-2E_r w_{i,j}^{k}=
$$

$$
\left(\frac{v_{i+1,j}^{k+1}-2v_{i,j}^{k+1}+v_{i-1,j}^{k+1}}{(\Delta x)^2}\right)+\left(\frac{v_{i,j+1}^{k+1}-2v_{i,j}^{k+1}+v_{i,j-1}^{k+1}}{(\Delta y)^2}\right)+2k^2x\left(\frac{v_{i+1,j}^{k+1}-v_{i-1,j}^{k+1}}{2\Delta x}\right)
$$

$$
+2k^2x^2\left(\frac{v_{i+1,j}^{k+1}-2v_{i,j}^{k+1}+v_{i-1,j}^{k+1}}{(\Delta x)^2}\right)\left(\frac{v_{i+1,j}^{k+1}-v_{i,j}^{k+1}}{\Delta x}\right)+G_r\,\bar{}\;+M^2(\sin\sim)^2\,\frac{(w+mvH\sin\sim)}{1+m^2H^2\sin^2\sim}
$$

(3.64)

making $v_{i,j}^{k+1}$ in Equation (3.64) the subject of the formula:

$$
v_{i,j}^{k+1}=\left(v_{i,j}^{k}+u_0\,\Delta t\,\frac{v_{i+1,j}^{k+1}}{\Delta x}-v_{i,j}^{k}\Delta t\,\frac{v_{i,j+1}^{k+1}}{\Delta y}+2E_r w_{i,j}^{k}\Delta t+\Delta t\left\{\begin{array}{l}\dfrac{v_{i+1,j}^{k+1}}{(\Delta x)^2}+\dfrac{v_{i-1,j}^{k+1}}{(\Delta x)^2}+\dfrac{v_{i,j+1}^{k+1}}{(\Delta y)^2}+\dfrac{v_{i,j-1}^{k+1}}{(\Delta y)^2}+\dfrac{k^2x}{(\Delta x)^2}\\ \left[\left(v_{i+1,j}^{k+1}\right)^2-2v_{i+1,j}^{k+1}v_{i-1,j}^{k+1}+\left(v_{i-1,j}^{k+1}\right)^2\right]+\dfrac{k^2x^2}{(\Delta x)^3}\left(\begin{array}{l}\left(v_{i+1,j}^{k+1}\right)^2\\ -\left(v_{i-1,j}^{k+1}\right)^2\end{array}\right)\\ +G_r\,\bar{}\,{}_{i,j}^{k}+M^2(\sin\sim)^2\,\dfrac{(w_{i,j}^{k}+mv_{i,j}^{k}\sin\sim)}{1+m^2\sin^2\sim}\end{array}\right\}\right)\div
$$

$$
\left(1+\frac{\Delta t u_0}{\Delta x}-\frac{\Delta t v_{i,j}^{k}}{\Delta y}+\Delta t\,\frac{2}{(\Delta x)^2}+\Delta t\,\frac{2}{(\Delta y)^2}\right)
$$

(3.65)

The equation of momentum along the z-axis is

$$\frac{w_{i,j}^{k+1}-w_{i,j}^{k}}{\Delta t}-u_0\left(\frac{w_{i+1,j}^{k+1}-w_{i,j}^{k+1}}{\Delta x}\right)+v_{i,j}^{k}\left(\frac{w_{i,j+1}^{k+1}-w_{i,j}^{k+1}}{\Delta y}\right)-2E_r v_{i,j}^{k}$$
$$=\left(\frac{w_{i+1,j}^{k+1}-2w_{i,j}^{k+1}+w_{i-1,j}^{k+1}}{(\Delta x)^2}\right)+\left(\frac{w_{i,j+1}^{k+1}-2w_{i,j}^{k+1}+w_{i,j-1}^{k+1}}{(\Delta y)^2}\right)+2k^2x\left(\frac{w_{i+1,j}^{k+1}-w_{i-1,j}^{k+1}}{2\Delta x}\right)$$
$$+2k^2x^2\left(\frac{w_{i+1,j}^{k+1}-2w_{i,j}^{k+1}+w_{i-1,j}^{k+1}}{(\Delta x)^2}\right)\left(\frac{w_{i+1,j}^{k+1}-w_{i,j}^{k+1}}{\Delta x}\right)-M^2(\sin\sim)^2\,\frac{(mw\sin\sim-v)}{1+m^2\sin^2\sim}$$

(3.66)

making $w_{i,j}^{k+1}$ in Equation (3.66) the subject of the formula:

$$w_{i,j}^{k+1}=\left(w_{i,j}^{k}+u_0\,\Delta t\,\frac{w_{i+1,j}^{k+1}}{\Delta x}-v_{i,j}^{k}\Delta t\,\frac{w_{i,j+1}^{k+1}}{\Delta y}+2E_r v_{i,j}^{k}\Delta t+\Delta t\left\{\begin{array}{l}\frac{w_{i+1,j}^{k+1}}{(\Delta x)^2}+\frac{w_{i-1,j}^{k+1}}{(\Delta x)^2}+\frac{w_{i,j+1}^{k+1}}{(\Delta y)^2}+\frac{w_{i,j-1}^{k+1}}{(\Delta y)^2}+\frac{k^2x}{(\Delta x)^2}\\ \left[\left(w_{i+1,j}^{k+1}\right)^2-2w_{i+1,j}^{k+1}w_{i-1,j}^{k+1}+\left(w_{i-1,j}^{k+1}\right)^2\right]+\\ \frac{k^2x^2}{(\Delta x)^3}\left(\left(w_{i+1,j}^{k+1}\right)^2-\left(w_{i-1,j}^{k+1}\right)^2\right)\\ -M^2(\sin\sim)^2\,\frac{(mw_{i,j}^{k}\sin\sim-v_{i,j}^{k})}{1+m^2\sin^2\sim}\end{array}\right\}\right)\div$$
$$\left(1+\frac{\Delta t u_0}{\Delta x}-\frac{\Delta t w_{i,j}^{k}}{\Delta y}+\Delta t\,\frac{2}{(\Delta x)^2}+\Delta t\,\frac{2}{(\Delta y)^2}\right)$$

(3.67)

Equation of Energy is

$$\frac{\bar{\ }^{k+1}_{i,j} - \bar{\ }^{k}_{i,j}}{\Delta t} - u_0 \frac{\bar{\ }^{k+1}_{i+1,j} - \bar{\ }^{k+1}_{i,j}}{\Delta x} + v^{k}_{i,j} \frac{\bar{\ }^{k+1}_{i,j+1} - \bar{\ }^{k+1}_{i,j}}{\Delta y}$$

$$= \frac{1}{\Pr}\left(\frac{\bar{\ }^{k+1}_{i+1,j} - 2\bar{\ }^{k+1}_{i,j} + \bar{\ }^{k+1}_{i-1,j}}{(\Delta x)^2} + \frac{\bar{\ }^{k+1}_{i,j+1} - 2\bar{\ }^{k+1}_{i,j} + \bar{\ }^{k+1}_{i,j-1}}{(\Delta y)^2} \right) - \frac{¢}{\Pr} -$$

$$+ E_c \left[\left(\frac{v^{k+1}_{i+1,j} - v^{k+1}_{i,j}}{\Delta x} \right)^2 + \left(\frac{w^{k+1}_{i+1,j} - w^{k+1}_{i,j}}{\Delta x} \right)^2 + \left(\frac{v^{k+1}_{i,j+1} - v^{k+1}_{i,j}}{\Delta y} \right)^2 + \left(\frac{w^{k+1}_{i,j+1} - w^{k+1}_{i,j}}{\Delta y} \right)^2 \right]$$

(3.68)

making $\bar{\ }^{k+1}_{i,j}$ in Equation (3.68) the subject of the formula:

$$\bar{\ }^{k+1}_{i,j} = \left[\begin{array}{l} \bar{\ }^{k}_{i,j} + u_0 \Delta t \frac{\bar{\ }^{k+1}_{i+1,j}}{\Delta x} - v^{k}_{i,j} \Delta t \frac{\bar{\ }^{k+1}_{i,j+1}}{\Delta y} + \frac{1}{\Pr} \Delta t \left(\frac{\bar{\ }^{k+1}_{i+1,j}}{(\Delta x)^2} + \frac{\bar{\ }^{k+1}_{i-1,j}}{(\Delta x)^2} + \frac{\bar{\ }^{k+1}_{i,j+1}}{(\Delta y)^2} + \frac{\bar{\ }^{k+1}_{i,j-1}}{(\Delta y)^2} \right) \\ - \frac{¢}{\Pr} (\Delta t) \bar{\ }^{k}_{i,j} + \\ (\Delta t) E_c \left(\left(\frac{v^{k+1}_{i+1,j} - v^{k+1}_{i,j}}{\Delta x} \right)^2 \left(\frac{w^{k+1}_{i+1,j} - w^{k+1}_{i,j}}{\Delta x} \right)^2 + \left(\frac{v^{k+1}_{i,j+1} - v^{k+1}_{i,j}}{\Delta y} \right)^2 + \left(\frac{w^{k+1}_{i,j+1} - w^{k+1}_{i,j}}{\Delta y} \right)^2 \right) \end{array} \right] \div$$

$$\left(1 + \frac{2\Delta t}{\Pr} \left(\left(\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2} \right) \right) + \frac{(\Delta t) u_0}{\Delta x} - \frac{(\Delta t) v^{k}_{i,j}}{\Delta y} \right)$$

(3.69)

Subject to the initial and boundary conditions

$$\left.\begin{array}{ll} \textbf{At } x = 0 & v^{0}(0, j) = 1 \quad w^{0}(0, j) = 0 \quad \overline{\ }^{0}(0, j) = 1 \\ \textbf{At } y = 0 & v^{0}(i, j) = 0 \quad w^{0}(i, j) = 0 \quad \overline{\ }^{0}(i, j) = 0 \\ \textbf{At } x = 0 & v^{k}(0, j) = 1 \quad w^{k}(0, j) = 0 \quad \overline{\ }^{k}(0, j) = 1 \end{array}\right\} \quad \textbf{(3.70)}$$

$$\textbf{At } y=0 \qquad v^{k}(i, j) = 0 \quad w^{k}(i, j) = 0 \quad \overline{\ }^{k}(i, j) = 0$$

The computations are performed using small values of Δt, in this research $\Delta t = 0.000125$and $\Delta x = 0.01$ $\Delta y = 0.01$. Fixing $x = 2.05$ that is $i = 41$as corresponding to $i = \infty$ because $v,\ w\ and\ \overline{\ }$ tend to zero at around $x = 2.0$.The velocities $v_{i,j}^{k+1}$,$w_{i,j}^{k+1}$ *and* $\overline{\ }_{i,j}^{k+1}$ are computed from equation (3.65), (3.67) and (3.69). This procedure is repeated until $k = 320$ that is $t = 0.5$. In the calculations the Prandlt number is taken as 0.71 which corresponds to air, magnetic parameter $M^2=50.0$ which signifies a strong magnetic field. Two cases are considered,

a) When the Grashof number, $Gr > 0(5.0)$ corresponding to convective cooling of the plate.

b) When the Grashof number, $Gr > 0(-5.0)$ corresponding to convective heating of the plate.

3.5 Stability and convergence

To ensure stability and convergence, a program is run using smaller values of Δt = 0.0001, 0.000125, 0.0003. It is observed that there were no significant changes in the results, which ensures that the finite difference method used in the problem will converge and is stable.

In the next chapter the results are obtained after solving equations (3.65), (3.67), (3.69) using JAVA program. These results are then presented and discussed.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Introduction

A program was run for various values of velocities and temperature for the finite difference equations. The velocities v and w at the end of each time step is computed from equation **(3.65 and 3.67)** in terms of velocity and temperature at earlier time steps. Similarly, theta is computed from equation **(3.69).**

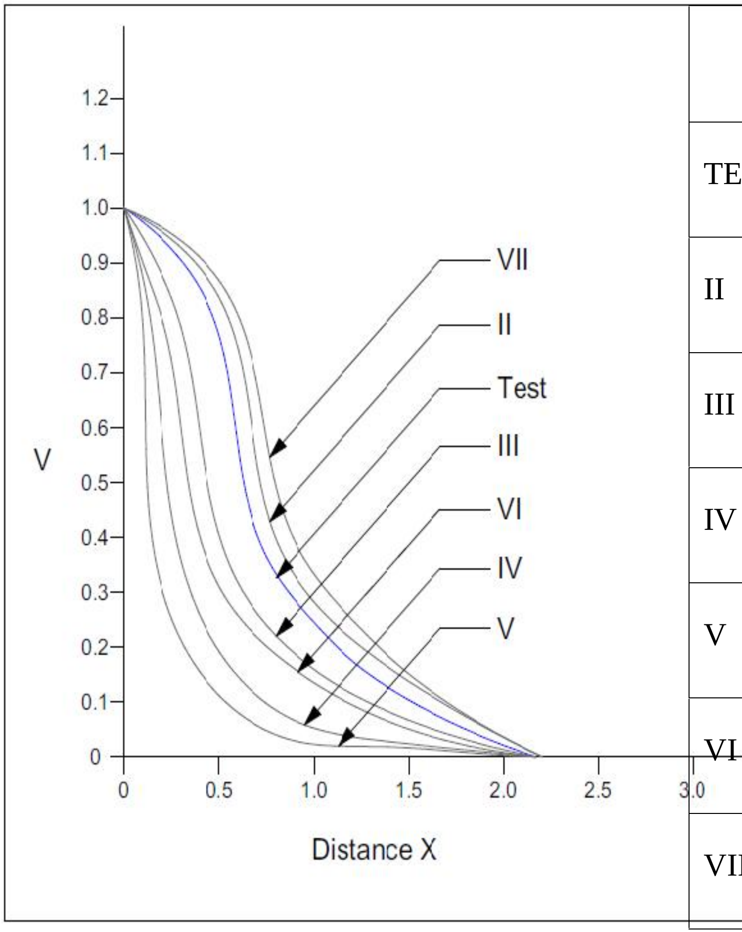
In order to get physical insight into the problem under study, the velocity field and temperature field are discussed by assigning numerical values to the parameters i.e. the angle of inclination and non-dimensional numbers (Hall parameter, Eckert number, Rotational parameter and heat parameter) encountered into the corresponding equations. To be realistic, the value of Eckert number is Ec=0.02. The velocities are classified as primary (v) and secondary (w) along the y and z axes respectively.

4.2 Case 1: Cooling at the Plate

In this case, the Grashof number Gr > 0. Hence the plate is at higher temperature than the surrounding and taking Gr = 5.0.

a) Primary Velocity

Figure 4.1: Primary Velocity Profile (Cooling at the plate)



	m	Er	Ec	¢	T	~
TEST	1.0	1.0	0.02	1.0	0.2	0.79
II	2.0	1.0`	0.02	1.0	0.2	0.79
III	1.0	2.0	0.02	1.0	0.2	0.79
IV	1.0	1.0	0.50	1.0	0.2	0.79
V	1.0	1.0	0.02	5.0	0.2	0.79
VI	1.0	1.0	0.02	1.0	0.25	0.79
VII	1.0	1.0	0.02	1.0	0.2	0.87

From Figure 4.1 we note that:

Increase in the rotational parameter Er leads to a decrease in the primary velocity. This is because the presence of the inclined magnetic field which creates a resistive force similar to the drag force that acts in the opposite direction of the fluid; thus causing the velocity of the fluid to decrease.

Increase in the Hall parameter m leads to an increase in the primary velocity. The Hall parameter increases with the magnetic field strength. Physically, the trajectories of electrons are curved by the Lorentz force. When the Hall parameter is low, their motion between the two encounters with heavy particles (neutral or ion) is almost linear. But if it is high, the electron movements are highly curved. Also, because effective conductivity decreases with an increase in Hall parameter which reduces magnetic damping force hence the increase in velocity.

Increase in the heat parameter ¢ leads to a decrease in the primary velocity. This is due to an increase in the internal heat generation and because the plate is cooling, the rate of energy transfer is increased therefore the velocity of the fluid will reduce.

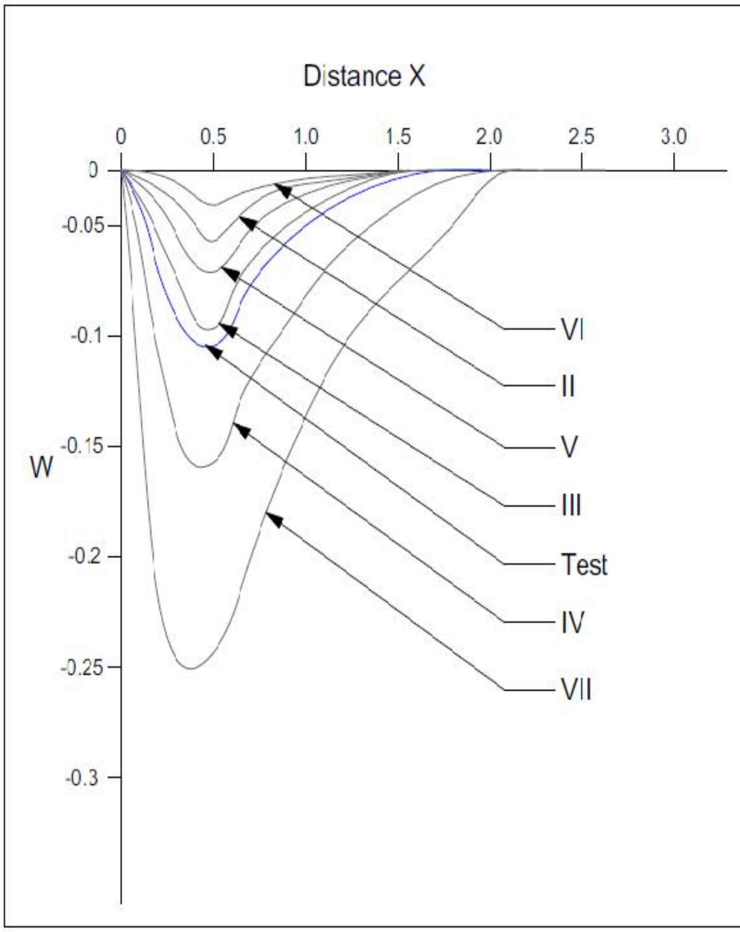
Increase in the Eckert number *Ec* leads to a decrease in the primary velocity. Increase in Ec means the fluid absorbs more heat energy that is released from internal viscous forces. This will in turn increase the velocity of the convection current.

Increase in time *t* leads to an increase in the primary velocity. With time the flow gets to the free stream and therefore its velocity increases.

Increase in the angle ~ leads to an increase in the primary velocity.

b) Secondary Velocity

Figure 4.2: Secondary Velocity Profile (Cooling at the plate)



	m	*Er*	*Ec*	¢	*T*	~
TEST	1.0	1.0	0.02	1.0	0.2	0.79
II	2.0	1.0`	0.02	1.0	0.2	0.79
III	1.0	2.0	0.02	1.0	0.2	0.79
IV	1.0	1.0	0.50	1.0	0.2	0.79
V	1.0	1.0	0.02	5.0	0.2	0.79
VI	1.0	1.0	0.02	1.0	0.25	0.79
VII	1.0	1.0	0.02	1.0	0.2	0.87

From figure 4.2 we note that:

Increase in the rotational parameter *Er* leads to an increase in the secondary velocity. This is because the presence of the inclined magnetic field creates a resistive force similar to the drag force that acts in the opposite direction of the fluid; thus causing the velocity of the fluid to increase.

Increase in the Hall parameter m leads to an increase in the secondary velocity , because effective conductivity decreases with an increase in Hall parameter which reduces magnetic damping force hence the increase in secondary velocity.

Increase in the heat parameter ¢ leads to an increase in the secondary velocity. This is due to an increase in the internal heat generation and because the plate is cooling, the rate of energy transfer is increased therefore the velocity of the fluid will reduce.

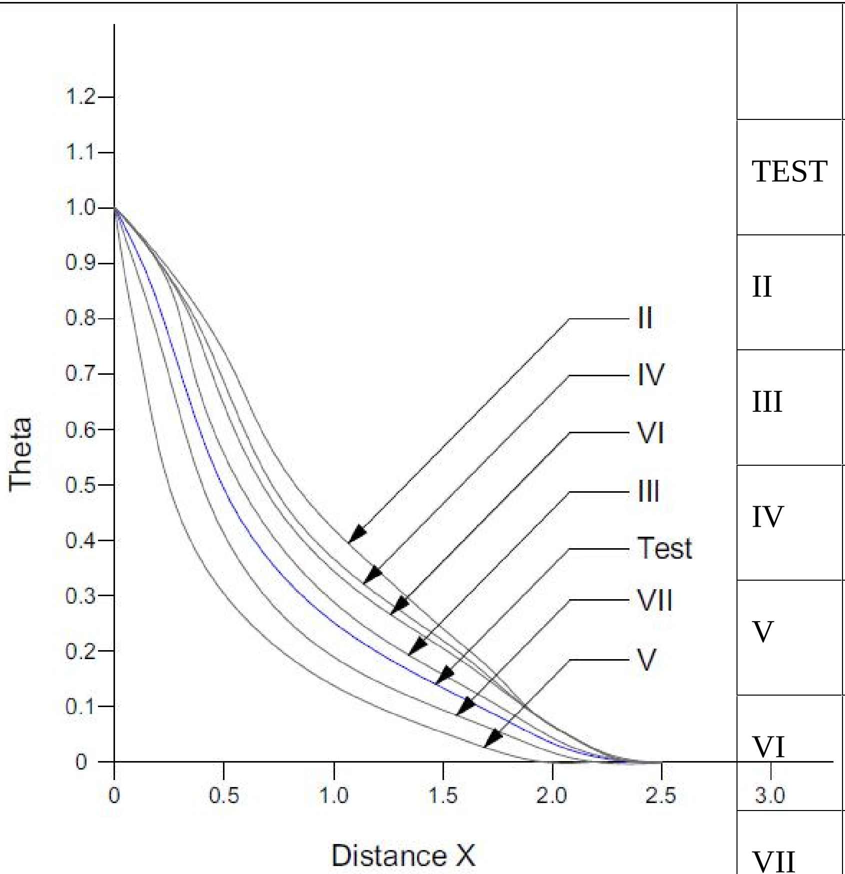
Increase in the Eckert number Ec leads to a decrease in the secondary velocity. Increase in Ec means the fluid absorbs more heat energy that is released from internal viscous forces. This will in turn increase the velocity of the convection current.

Increase in time t leads to a decrease in the secondary velocity. With time the flow in the free stream decreases.

Increase in the angle ~ leads to a decrease in the secondary velocity, increasing the angle of the magnetic field causes an increase in the Magnetic strength which retards the fluid motion by affecting the velocity .

b) Temperature profile

Figure 4.3: Temperature Profile (Cooling at the plate)



	m	*Er*	*Ec*	¢	*t*	~
TEST	1.0	1.0	0.02	1.0	0.2	0.79
II	2.0	1.0`	0.02	1.0	0.2	0.79
III	1.0	2.0	0.02	1.0	0.2	0.79
IV	1.0	1.0	0.50	1.0	0.2	0.79
V	1.0	1.0	0.02	5.0	0.2	0.79
VI	1.0	1.0	0.02	1.0	0.25	0.79
VII	1.0	1.0	0.02	1.0	0.2	0.87

From figure 4.3 we note that:

Increase in the rotational parameter *Er* leads to an increase in the temperature profile. Frequency of oscillation increase thus increasing the temperature of the fluid.

Increase in the Hall parameter *m* leads to a slight effect on the Temperature profiles, it tends to increase the temperature profile. This is due to the increase in the thermal boundary layer that is caused by an increase in Hall parameter. An increase in the thermal boundary layer decreases the temperature gradient and hence increases the temperature in the fluid.

Increase in the heat parameter ¢ leads to a decrease in the temperature profile. This is due to an increase in the internal heat generation and because the plate is cooling, the rate of energy transfer is increased therefore .

Increase in the Eckert number *Ec* leads to an increase in the temperature profile. Increase in Ec means the fluid absorbs more heat energy that is released from internal viscous forces. This will in turn increase the temperature.

Increase in time *t* leads to an increase in the temperature profile. With time as the flow gets to the free stream the velocity is increased hence there is increased rate of energy transfer and therefore the temperature will increase.

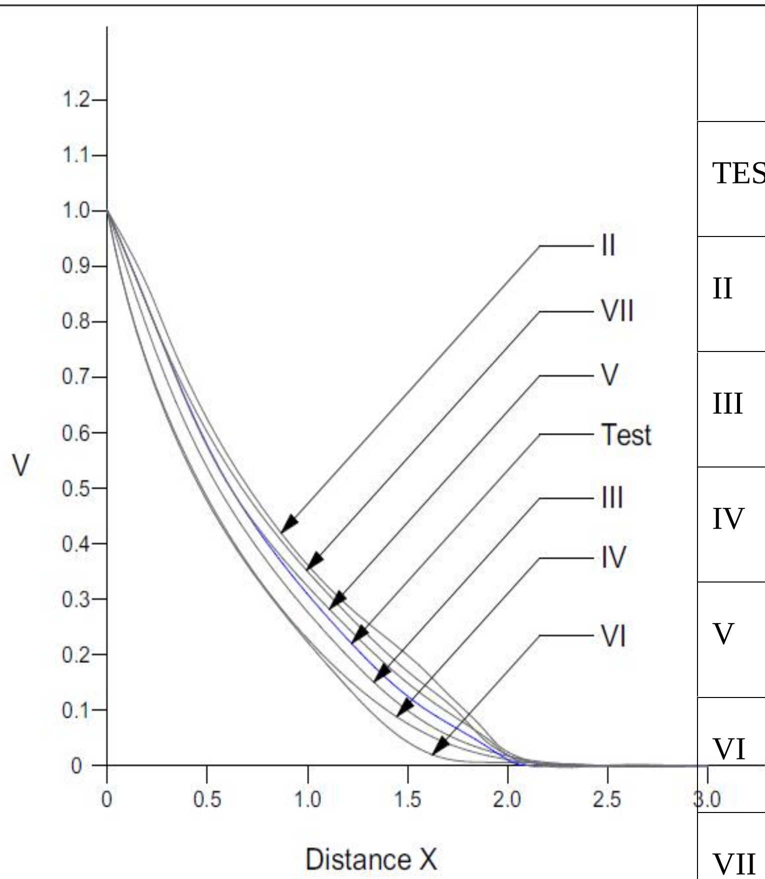
Increase in the angle ~ leads to a decrease in the temperature profile, increasing the angle of the magnetic field causes an increase in the Magnetic strength decreases the temperature.

4.3 Case 2: Heating at the Plate

In this case, the Grashof number Gr $<$ 0. Hence the plate is at a lower temperature than the surrounding and taking Gr = -5.0.

a) Primary Velocity

Figure 4.4 :Primary velocity(Heating at the plate)



	M	*Er*	*Ec*	¢	*t*	~
TEST	1.0	1.0	0.02	1.0	0.2	0.79
II	2.0	1.0`	0.02	1.0	0.2	0.79
III	1.0	2.0	0.02	1.0	0.2	0.79
IV	1.0	1.0	0.50	1.0	0.2	0.79
V	1.0	1.0	0.02	5.0	0.2	0.79
VI	1.0	1.0	0.02	1.0	0.25	0.79
VII	1.0	1.0	0.02	1.0	0.2	0.87

From figure 4.4 we note that:

Increase in the rotational parameter *Er* leads to a decrease in the primary velocity. This is because the presence of the inclined magnetic field which creates a resistive force similar to the drag force that acts in the opposite direction of the fluid; thus causing the velocity of the fluid to decrease.

Increase in the Hall parameter m leads to an increase in the primary velocity. The Hall parameter increases with the magnetic field strength. Physically, the trajectories of electrons are curved by the Lorentz force. When the Hall parameter is low, their motion between the two encounters with heavy particles (neutral or ion) is almost linear. But if it is high, the electron movements are highly curved. Also, because effective conductivity decreases with an increase in Hall parameter which reduces magnetic damping force hence the increase in velocity.

Increase in the heat parameter ¢ leads to an increase in the primary velocity. This is due to an increase in the internal heat generation and because the plate is heating, the rate of energy transfer is decreased therefore the velocity of the fluid will increase.

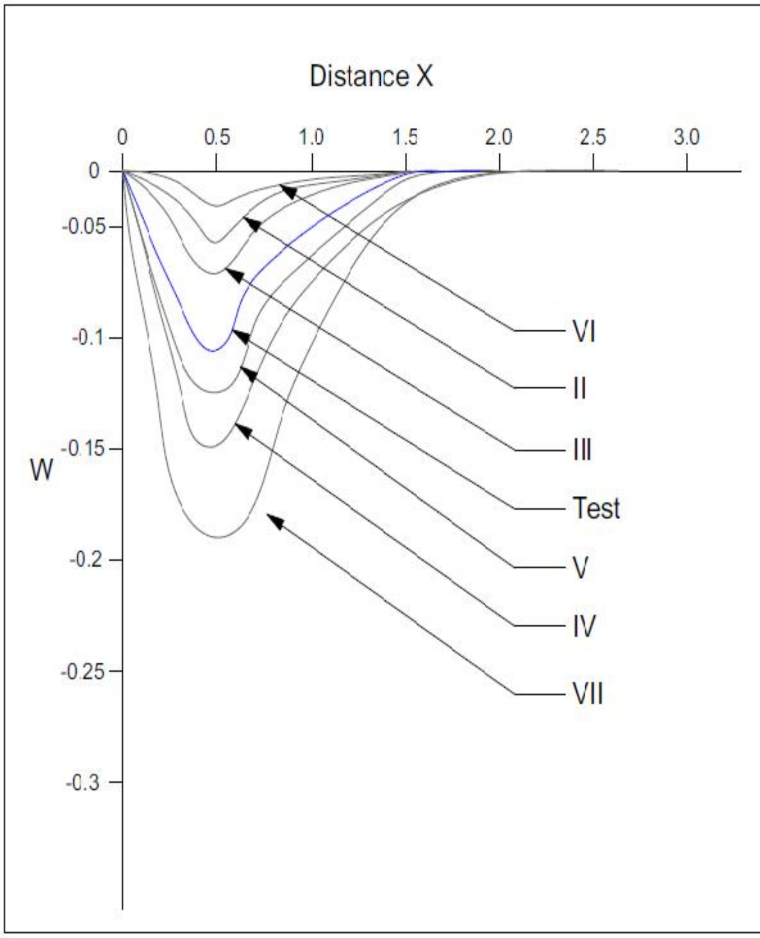
Increase in the Eckert number Ec leads to a decrease in the primary velocity. Increase in Ec means the fluid absorbs more heat energy that is released from internal viscous forces. This will in turn increase the velocity of the convection current.

Increase in time t leads to an increase in the primary velocity. With time the flow gets to the free stream and therefore its velocity increases.

Increase in the angle ~ leads to an increase in the primary velocity. This is due to a decrease in Lorentz force which reduces the magnetic damping force hence causing the increase in the primary velocity.

b) Secondary Velocity

Figure 4.5:Secondary velocity (Heating at the plate)



	m	*Er*	*Ec*	¢	*t*	~
TEST	1.0	1.0	0.02	1.0	0.2	0.79
II	2.0	1.0`	0.02	1.0	0.2	0.79
III	1.0	2.0	0.02	1.0	0.2	0.79
IV	1.0	1.0	0.50	1.0	0.2	0.79
V	1.0	1.0	0.02	5.0	0.2	0.79
VI	1.0	1.0	0.02	1.0	0.15	0.79
VII	1.0	1.0	0.02	1.0	0.2	0.87

From figure 4.5 we note that:

Increase in the rotational parameter *Er* leads to an increase in the secondary velocity. This is because the presence of the inclined magnetic field creates a resistive force similar to the drag force that acts in the opposite direction of the fluid; thus causing the velocity of the fluid to decrease.

Increase in the Hall parameter m leads to an increase in the secondary velocity , because effective conductivity decreases with an increase in Hall parameter which reduces magnetic damping force hence the increase in secondary velocity.

Increase in the heat parameter ¢ leads to an increase in the secondary velocity. This is due to an increase in the internal heat generation and because the plate is heating, the rate of energy transfer is decreased therefore the velocity of the fluid will increase.

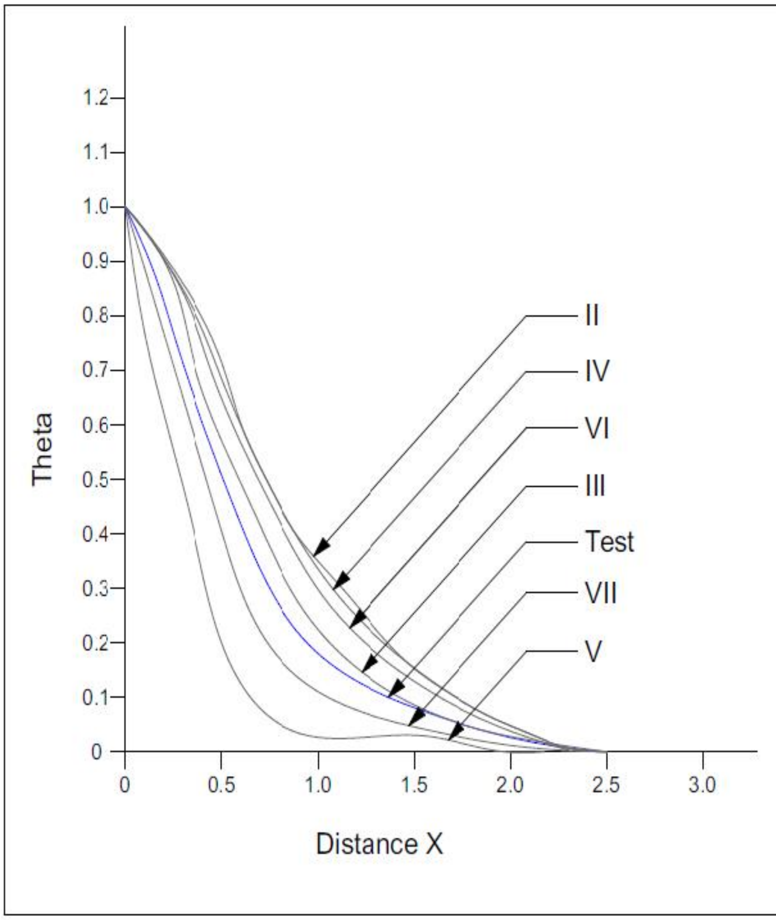
Increase in the Eckert number Ec leads to a decrease in the secondary velocity. Increase in Ec means the fluid absorbs more heat energy that is released from internal viscous forces. This will in turn increase the velocity of the convection current.

Increase in time t leads to a decrease in the secondary velocity. With time the flow in the free stream decreases.

Increase in the angle ~ leads to a decrease in the secondary velocity, increasing the angle of the magnetic field causes an increase in the Magnetic strength which retards the fluid motion by affecting the velocity .

c) Temperature profile

Figure 4.6: Temperature profile (Heating at the plate)



	m	*Er*	*Ec*	¢	*t*	~
TEST	1.0	1.0	0.02	1.0	0.2	0.79
II	2.0	1.0`	0.02	1.0	0.2	0.79
III	1.0	2.0	0.02	1.0	0.2	0.79
IV	1.0	1.0	0.50	1.0	0.2	0.79
V	1.0	1.0	0.02	5.0	0.2	0.79
VI	1.0	1.0	0.02	1.0	0.25	0.79
VII	1.0	1.0	0.02	1.0	0.2	0.87

From figure 4.6 we note that:

Increase in the rotational parameter *Er* leads to an increase in the temperature profile. Frequency of oscillation increase thus increasing the temperature of the fluid.

Increase in the Hall parameter *m* leads to a slight effect on the Temperature profiles, it tends to increase the temperature profile. This is due to the increase in the thermal boundary layer that is caused by an increase in Hall parameter. An increase in the thermal boundary layer decreases the temperature gradient and hence increases the temperature in the fluid.

Increase in the heat parameter ¢ leads to a decrease in the temperature profile. This is due to an increase in the internal heat generation and because the plate is heating, the rate of energy transfer is increased therefore the temperature decreases.

Increase in the Eckert number *Ec* leads to an increase in the temperature profile. Increase in Ec means the fluid absorbs more heat energy that is released from internal viscous forces. This will in turn increase the temperature.

Increase in time *t* leads to an increase in the temperature profile. With time as the flow gets to the free stream the velocity is increased hence there is increased rate of energy transfer and therefore the temperature will increase.

Increase in the angle ~ leads to a decrease in the temperature profile , increasing the angle of the magnetic field causes an increase in the Magnetic strength decreases the temperature.

The conclusions of this research have been done in the next chapter. Finally, recommendations have also been outlined .

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

This chapter presents conclusion of this research study and recommendation on areas that require further research.

5.1 CONCLUSION

In all cases considered, the applied magnetic field was resolved into components and the flow is considered turbulent. The equations governing the flows considered in our study are non-linear therefore in order to obtain their solutions, an efficient finite difference scheme has been developed. In order to validate the present results the angle of inclination is considered to be in 90 degrees. The results are compared to those of Kinyanjui *et al* and agree.

It was observed that an increase in *m* and ~ leads to an increase in the primary velocity profiles for both free convection cooling and heating at the plate while an increase in *Er*, *Ec* and *t* leads to a decrease in the primary velocity profiles for both free convection cooling and heating at the plate. ¢ leads to a decrease in the primary velocity profiles for the cooling of the plate and an increase at the heating of the plate. Increasing *Er, m* and *t* leads to an increase in the secondary velocity for both cooling and heating of the plate while, *Ec* and ~ leads to a decrease in the secondary velocity profile leads to a decrease in the secondary velocity profiles for the cooling of the plate and an increase at the heating of the plate .

Increase in *Er, Ec, t* and *m* leads to an increase in the temperature profiles for both free convection cooling and free convection heating. The effect of the magnetic field inclined at an angle is to retard the fluid motion by affecting the velocity and temperature profiles.

From this results, it's clear that the parameters in the governing equations affect the primary, secondary and temperature profile

5.2 RECOMMENDATIONS

It's recommended that this work be extended by considering the following:

- Variable inclined magnetic field.
- The effects of the parameters in the governing equations on skin friction and rate of mass transfer.
- Compressible fluid.
- Variable injection.
- Variable viscosity and thermal conductivity.

5.3 PUBLICATION

Rency, C.M, Kinyanjui, N.M & Kwanza, J.K. (2015). Mhd turbulent flow in presence of inclined magnetic field past a rotating semi-infinite plate, *International Journal of Engineering Science and Innovation,* ***4***(2),344-360.

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APPENDIX

Appendix I Computer code

The following is a JAVA program code that was used to obtain the graphs labeled 4.1-4.6.

```
import java.awt.Color;

//import javax.swing.JPanel;

import org.jfree.chart.ChartFactory;

import org.jfree.chart.ChartPanel;

import org.jfree.chart.JFreeChart;

import org.jfree.chart.axis.NumberAxis;

import org.jfree.chart.plot.PlotOrientation;

import org.jfree.chart.plot.XYPlot;

//import
org.jfree.chart.renderer.xy.XYLineAndShapeRenderer;

import org.jfree.data.xy.XYDataset;

import org.jfree.data.xy.XYSeries;

import org.jfree.data.xy.XYSeriesCollection;

import org.jfree.ui.ApplicationFrame;

import org.jfree.ui.RectangleInsets;
```

```
import org.jfree.ui.RefineryUtilities;


public class Equations extends ApplicationFrame {
	protected final double delX=0.01;
	protected final double delY=0.01;
	protected final double delT=0.000125;
	protected int K=320;
	protected int j=41;
	protected int i=41;
	protected final int squareM=50;
	protected final double U=0.5;
	protected double X=0.00;
	protected double Pr=0.71;
	//the user inputs
	protected double m=1.0;
	protected double Er=1.0;
	protected double Ec=0.01;
	protected double Gr=5.0;
```

```
     protected double sigma=1.5;

     protected double alpha=0.2;


     //primaryVelocity pv=new primaryVelocity();


    public Equations(String title) {

       super(title);




        XYDataset dataset = createDataset();

        JFreeChart chart = createChart(dataset);

        ChartPanel chartPanel = new ChartPanel(chart);

        chartPanel.setPreferredSize(new
java.awt.Dimension(500,800));

        setContentPane(chartPanel);

    }
```

```
    private static XYDataset createDataset() {
       XYSeries series1 = new XYSeries("First");



    /*   for (int i=-180; i<=180; i++){
         double j=Math.sin(i);
         series1.add(i,j);
       }*/
       XYSeriesCollection dataset = new
XYSeriesCollection();
      // dataset.addSeries(series1);
       //dataset.addSeries(series2);
       dataset.addSeries(series1);
       return dataset;
   }

   /**
    * Creates a chart.
    *
```

```
     * @param dataset the data for the chart.
     *
     * @return a chart.
     */
    private static JFreeChart createChart(XYDataset
dataset) {
// create the chart...
        JFreeChart chart = ChartFactory.createXYLineChart(
                "Line Chart Demo 2", // chart title
                "X", // x axis label
                "Y", // y axis label
                dataset, // data
                PlotOrientation.VERTICAL,
                true, // include legend
                true, // tooltips
                false // urls
                );
// NOW DO SOME OPTIONAL CUSTOMISATION OF THE CHART...
        chart.setBackgroundPaint(Color.white);
```

```
// get a reference to the plot for further customisation...
        XYPlot plot = (XYPlot) chart.getPlot();
        plot.setBackgroundPaint(Color.lightGray);
        plot.setAxisOffset(new RectangleInsets(5.0, 5.0,
5.0, 5.0));
        plot.setDomainGridlinePaint(Color.white);
        plot.setRangeGridlinePaint(Color.white);




        //XYLineAndShapeRenderer renderer =
(XYLineAndShapeRenderer) plot.getRenderer();
        //renderer.setShapesVisible(true);
        //renderer.setShapesFilled(true);
// change the auto tick unit selection to integer units
only...
        NumberAxis rangeAxis = (NumberAxis)
plot.getRangeAxis();
```

```
rangeAxis.setStandardTickUnits(NumberAxis.createIntegerTick
Units());

// OPTIONAL CUSTOMISATION COMPLETED.

        return chart;

    }



    /**

     * Starting point for the demonstration application.

     *

     * @param args ignored.

     */

    public static void main(String[] args) {

        Equations demo = new Equations("Line Chart Demo
2");

        demo.pack();

        RefineryUtilities.centerFrameOnScreen(demo);

        demo.setVisible(true);

    }
```

```
}



public class primaryVelocity extends Equations{


	//public primaryVelocity(){


	public primaryVelocity(String n) {
		super(n);
		// TODO Auto-generated constructor stub
	}


	//Equations eq=new Equations();
	secondaryVelocity sv=new secondaryVelocity(null);
	Temperature tEq=new Temperature();
	public void equation (int i, int j,int k){


		for(i=0; i<=41; i++){
```

```
            for(j=0; j<=41; j++){

                 for (k=0; k<=320; k++){

                      double eqn=(((-U)*(rule(i+1, j,
k)-2*rule(i-1, j, k))/(2*delX)) -

                      (rule(i, j, k)*(rule(i, j+1, k+1)-
rule(i, j-1, k)) / (2*delY)) +

                      (rule(i+1, j, k)-(2*rule(i, j,
k))+(rule(i-1, j, k))) / (Math.pow(delX, 2)) +

                      (2*Math.pow(k,
2)*delX)*((rule(i+1, j, k) - (rule(i-1, j, k))) /
(2*delX))*

                      ((rule(i+1, j, k) - (rule(i-1, j,
k))) / (2*delX)) +

                      (2*k*k*
((i*delX)*(i*delX))*(rule(i+1, j, k)-2*rule(i, j, k)+
rule(i-1, j, k))/ (delX*delX))*

                      ((rule(i+1, j, k)-rule(i-1, j,
k))/2*delX)+
```

```
                    2*Er*sv.rule(i, j, k) +Gr*rule(i,
j, k)-

     squareM*Math.sin(alpha)*Math.sin(alpha)*
                    ( (sv.rule(i, j, k)+ m*rule(i,j,
k)*Math.sin(alpha))/(1+(m*m*Math.sin(alpha)*Math.sin(alpha)
)) )  )+
                    rule(i, j, k);
               }
          }
     }
}
//lets set the conditions

public int rule (int a,int b, int c){
     int value=0;
     if (a==0 && b!=0){
          value=1;
     }else{
          value=0;
```

```
        }
        return value;
    }
}

public class secondaryVelocity extends Equations{
    public secondaryVelocity (String k){
        super(k);
    }

    public static void main(String args[]){
        secondaryVelocity sV=new secondaryVelocity(null);
        sV.equation();
    }
    Equations sEq=new Equations("");
    primaryVelocity pv=new primaryVelocity("");
    public void equation(){
```

```
            int i=0, j=0, k=0;

            for (i=0; i<=41; i++){

                  for (j=0; j<=41; j++){

                        for (k=0; k<=320; k++){

                              double eqn;

                              eqn=((-sEq.U)*(rule(i+1, j, k)-
rule(i-1, j, k))/(2*sEq.delX)-

                                    (rule(i, j, k)*(rule(i, j+1,
k+1)-rule(i, j-1, k))/(2*sEq.delY))+

                                    (rule(i+1, j, k)-2*rule(i, j,
k)+rule(i-1, j, k))/(sEq.delX*sEq.delX)+

                                    (2*k*k*i*sEq.delX)*(rule(i+1,
j, k)-rule(i-1, j, k)/(2*sEq.delX))*(rule(i+1, j, k)-
rule(i-1, j, k)/(2*sEq.delX))+
```

```
     (2*(k*i*sEq.delX)*(k*i*sEq.delX)*(rule(i+1, j, k)-
2*rule(i, j, k)+rule(i-1, j, k))/(sEq.delX*sEq.delX)) *

                              (rule(i+1, j, k)-rule(i-1, j,
k))/(2*sEq.delX)-

                              sEq.Er*pv.rule(i, j, k)-


     sEq.squareM*(Math.sin(sEq.alpha))*(Math.sin(sEq.alpha)
)*

                              ((sEq.m*rule(i, j,
k)*Math.sin(sEq.alpha))-rule(i, j,
k))/(1+sEq.m*sEq.m*(Math.sin(sEq.alpha))*(Math.sin(sEq.alph
a))))+

                              rule(i, j, k);

                         System.out.println(" "+eqn);

                    }

               }

          }

     }



     public int rule(int i, int j, int k){
```

```
            return 0;
        }
    }



    public class Temperature extends Equations{

        public Temperature(String k){
            super(k);
        }
        Equations tEq=new Equations("");
        primaryVelocity pV=new primaryVelocity(null);
        secondaryVelocity sV=new secondaryVelocity(null);
        public int rule(int i, int j, int k){
            int value=0;
            if (i==0 && j!=0){
                value=1;
            }else{
```

```
                value=0;
            }
            return value;
        }

        public void equations(){
            int i=0, j=0, k=0;
            double eq;



            eq=((-tEq.U)*(rule(i+1, j, k)-rule(i-1, j,
k))/(2*tEq.delX)-
                (rule(i, j, k)*(rule(i, j+1, k+1)-rule(i, j-
1, k))/(2*tEq.delY)) *
                (1/tEq.Pr)*( (rule(i+1, j, k)-2*rule(i, j,
k) + rule(i-1, j, k))/(tEq.delX*tEq.delX) ) -
                (tEq.sigma/tEq.Pr)*rule(i, j, k) +
```

```
                tEq.Ec*(((pV.rule(i+1, j, k)-pV.rule(i-1,j,
k))/(2*tEq.delX))*((pV.rule(i+1, j, k)-pV.rule(i-1,j,
k))/(2*tEq.delX)) +
                ((sV.rule(i+1, j, k)-sV.rule(i-1, j,
k))/(2*tEq.delX))*((sV.rule(i+1, j, k)-sV.rule(i-1, j,
k))/(2*tEq.delX))))+
                rule(i,j, k);




     }
}


public class mainClass{
     public static void main(String args[]){
          Equations eqn=new Equations(null);


     }
}
```