Unsteady Hydromagnetic Convective Flow past an Infinite Vertical Porous Plate through a Porous Medium

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DECLARATION

This thesis is my original work and has not been presented for a degree in any other University.

Signature …….. Date …....................…..........

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This thesis has been submitted for examination with our approval as University supervisors.

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DEDICATION

This thesis is dedicated to my loving husband Stephen Weru, my sons Collins Njega and Elvis Maina. To my dear parents Joseph Maina and Lucy Wangui and my siblings for their prayers, encouragement and support towards my education.

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NOMENCLATURE

ROMAN SYMBOL	MEANING
E	Electric intensity vector, [Vm^{-1}].
F_i	Body force, [N].
e	Unit charge, [C].
L	Characteristic length, [m].
J	Current density, [Am^{-2}].
P	Pressure force, [Nm^{-2}].
P^*	Dimensionless Pressure force.
u	Characteristic velocity,[ms^{-1}]
t	Dimensional time,[s]
N	Thermal conductivity, [W$m^{-1}k^{-1}$].
q	Velocity vector, [ms^{-1}].
B	Magnetic field vector, [Wbm^{-2}].
D	Electric displacement vector, [cm^{-2}].
H	Magnetic field intensity vector, [Wbm^{-2}].
i, j, k	Unit vectors in the x, y, and z directions
u, v, w	Components of velocity vector **q**
F_e	Electromagnetic force, [$kgms^{-2}$].
Q	Amount of heat added to a system, [Nm].
D_M	Molecular diffusion coefficient, [m^2s^{-1}]
C_p	Specific heat at constant pressure, [$Jkg^{-1}k^{-1}$]
T	Absolute free temperature of the fluid, [K].
T_∞	Characteristic free stream temperature,[K].
C_W	Concentration at the plate
C_∞	Concentration at infinity
u^*, v^*, w^*	Dimensionless fluid velocity
x^*, y^*, z^*	Dimensionless Cartesian coordinates
t^*	Dimensionless time
u^*	Dimensionless velocity
$P*$	Dimensionless pressure force

k	Darcy permeability, [m^2]
K_p	Permeability parameter
Pr	Prandtl number
Nu	Nusselt number
M	Hartmann parameter
Gr_θ	Grashof number for heat transfer.
Gr_c	Grashof number for mass transfer.
V_w	Injection parameter

GREEK SYMBOLS

μ	Coefficient of viscosity, [$kgm^{-1}s^{-1}$]
ν	Kinematic viscosity, [m^2s^{-1}]
ρ	Fluid density, [kgm^{-3}].
ρ_e	Electrical charge density, [cm^{-2}].
σ	Electrical conductivity, [$\Omega^{-1}m^{-1}$].
μ_e	Magnetic permeability, [Hm^{-1}].
$\Delta x, \Delta y, \Delta z$	Distance intervals [m]
Δt	Time interval [s]
ΔT	Temperature change, [K].
∇	Gradient operator, $\left[\mathbf{i}\frac{\partial}{\partial x}+\mathbf{j}\frac{\partial}{\partial y}+\mathbf{k}\frac{\partial}{\partial y}\right]$
Φ	Viscous dissipation function, [s^2].
β	Volumetric coefficient of thermal expansion, [K^{-1}]
β^*	Coefficient of thermal expansion due to concentration gradient, [K^{-1}]
τ	Skin friction

ABBREVIATIONS

FDM	Finite Difference Method
ODE	Ordinary differential equations
PDE	Partial Differential equations

MHD	Magnetohydrodynamics
SEM	Spectral Element Method
FEM	Finite Element Method

ABSTRACT

The effect of heat transfer on unsteady hydromagnetic free convective flow of a viscous incompressible electrically conducting fluid flow has been investigated. The fluid flows past an infinite vertical porous plate in presence of constant injection and heat source. The flow is subjected to transverse magnetic field. The partial differential equations governing the flow field has been derived and transformed to non-dimensional form. The equations and their respective initial and boundary conditions are then non-dimensionalized and solved numerically using finite difference method specifically, the Crank-Nicolson method. The effects of varying various flow parameters on the velocity, temperature and concentration profiles have been presented graphically. This has been done when Grashof number for heat transfer, $Gr_\theta>0$ (cooling of the plate) and also when $Gr_\theta<0$ (heating of the plate). It was observed that when the parameters are varied, there is an increase, decrease or no change in velocity, temperature, concentration, skin friction and rate of heat transfer on the surface of the plate. The results of variation of these parameters is very important especially in petroleum engineering where an engineer is able to make various decisions on how to extract fluids as they move through porous medium.

CHAPTER ONE

INTRODUCTION AND LITERATURE REVIEW

1.1 INTRODUCTION

Matter is classified into fluids and solids. A solid can resist shear stress by a static deformation but a fluid cannot. According to McCormack and Crane (1973), a fluid can either be a gas or liquid. Fluid flow may either be one dimensional, two dimensional or three dimensional. Individual particles move in the direction of the flow to constitute fluid flow. The fluid flow is said to be rotational when fluid particles moving in the direction of the flow rotate about their own axes, otherwise, the fluid flow is irrotational. If the flow variables and fluid properties do not change with time the flow is said to be steady, otherwise, the flow is unsteady.

Fluids are classified as compressible or incompressible. A fluid is said to be compressible if the fluid density varies with pressure whereas it's incompressible if change in density with pressure is so small to be negligible. In this study, the fluid is incompressible.

Shearing stress and the velocity gradient are given by a relationship of the form:

$$\tau = \mu \frac{du}{dy} \quad \text{……...……..………………………………} \quad (1.1)$$

Fluids in which the shearing stress is linearly related to the rate of shearing strain are referred to as Newtonian fluids whereas if they are not linearly related they are non-Newtonian fluids. In this study, a Newtonian fluid is considered.

1.1.1 Hydromagnetic flow

Hydrodynamics is the study of fluids in motion. Electromagnetism is the study of interaction between electric fields and magnetic fields. Hydromagnetics is a branch of science that deals with interaction of a magnetic field with an electrically conducting fluid. Hydromagnetic flow therefore occurs when an electrically conducting fluid flows in a magnetic field. According to Frank (1985), these fluids can be ionized gases commonly referred as plasma or liquid metals. The flow of an electrically conducting fluid in presence of a magnetic field gives rise to induced electric currents in which mechanical forces are exerted by the magnetic field. The induced electric current flows in the direction perpendicular to both the magnetic field and the direction of motion. This induced current also generates its own magnetic field which distorts the original magnetic field. The magnetic field generated needs to be taken into account in the flow field.

1.1.2 Heat Transfer

Siegel and Howell (1992), defined heat transfer as the energy transfer whenever there exists temperature gradient within a body or there is temperature difference between the body and its surrounding or two bodies at different temperatures are brought together. Heat transfer can be by convection, radiation or conduction. In heat transfer by convection, when there is temperature gradient as fluid flows inside a duct or over a solid body, heat transfer will take place between the fluid and solid surface. This is due to the motion of the fluid relative to the surface. If the fluid motion is by buoyancy effects resulting from density variation caused by the temperature difference in the fluid, the heat transfer is said to be free convection. On the other hand, if the fluid motion is artificially set up by means of an external source like blower or fan, this heat transfer is termed as forced convection. In this study heat transfer by free convection is considered.

1.1.3 Mass transfer

Mass transfer is the net movement of mass from one location to another. In the mass transfer by convection, gross motion combines with diffusion to promote the transport of a species for which there exists a concentration gradient.

1.1.4 Porous medium

Porous medium is a substance that contains pores or voids through which liquid or gas can pass. The void space consists of a complex tridimensional network as interconnected small empty volumes called pores with several continuous paths linking up the porous matrix. Some natural substances such as sand, soil, some rocks like sandstone and manmade materials such as sponge, reticulated foams or cement slabs are some examples of porous media. Porosity is a measure of the void spaces in a material. It is the fraction of the volume of voids over the total volume of material. Hydrodynamic permeability measures the ability of fluids to flow through porous media.

1.1.5 Injection

Injection of a fluid to a flow region occurs when a jet of fluid is injected or introduced into the flow field.

1.1.6 Thermal Boundary Layer

When temperature difference exists between the solid surface and the fluid in the freestream, a thermal boundary layer is formed. The fluid particles in contact with the solid surface acquire the temperature of the solid. If the temperature of the solid surface is higher than that of ambient fluid, the kinetic energy of the molecules of the adjacent fluid particles increases. These particles in turn exchange the acquired kinetic energy with those of the fluid particles in the adjacent fluid layers further away from the solid surface. This process continues in the adjacent fluid layers and temperature gradients develop in the fluid.

1.1.7 Concentration Boundary Layer

Concentration is a measure of how much of a given species is dissolved in a fluid per unit volume. Concentration boundary layer manifests itself when species concentration difference exists between the solid surface and the freestream region of the fluid. The region in which the species concentration gradient exists is known as Concentration boundary layer. The species transfer takes place through the process of diffusion and convection.

In this study the effect of heat transfer on unsteady free convective flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous plate with constant injection and heat source in presence of a transverse magnetic field will be analysed.

1.2 LITERATURE REVIEW

The phenomenon of hydromagnetic flow with heat and mass transfer in an electrically conducting fluid past a porous plate embedded in a porous medium has attracted attention of good number of investigators because of its various applications in engineering.

The study of magnetohydrodynamics (MHD) started with Faraday (1859), who carried out an experiment in which an electrically conducting fluid was passed between poles of magnet in a vacuum glass.

Subhas *et al.*, (2001) presented a numerical solution of two dimensional laminar boundary layer problem on free convection flow of an incompressible visco-elastic fluid through a porous medium over a stretching sheet. It was observed among other things that the introduction of chemical species diffusion (modified Grashof number) leads to an increase of horizontal velocity profile either when heating or cooling of the fluid. This observation was found to be true in the presence of porosity parameter but with reduced magnitude.

Rafael, (2005) investigated fluid flow and heat transfer in a porous medium over a stretching surface with internal heat generation and by presence of suction, blowing and impermeability of the surface. He observed that velocity decreases and temperature increases with increasing permeability parameter. He also observed that suction decreases velocity while injection increases the velocity.

Das *et al.*, (2006) estimated the effects of mass transfer on unsteady flow past a vertical porous plate with suction by using finite difference analysis. They observed that porosity parameter retards the velocity of the flow field at all points and also, the higher the suction parameter, the faster the reduction in the velocity of the flow field.

Alam *et al.*, (2006) studied the effects of mass transfer on steady two dimensional free convection flow past a continuously moving semi-infinite vertical porous plate in a porous medium. They observed that the temperature decreases with increase in suction parameter and modified Grashof number. They concluded from their analysis that temperature and concentration fields are influenced by the Dufour and Soret effects.

Naser, (2008) analysed numerically the effects of heat and mass transfer from an isothermal vertical flat plate to a non-Newtonian fluid through a porous medium. It was found that as time approaches infinity, the values of friction factor, heat transfer and mass transfer coefficients approach the steady state values.

Rajeswari *et al.*, (2009) studied the effect of heat and mass transfer on MHD flow through a vertical porous surface in presence of suction. They observed that in case of uniform magnetic field with suction at the wall of the surface and constant chemical reaction parameter, the velocity of the fluid decreases and the concentration of the fluid increases with increase in suction at the wall of the surface, due to combined effects of the buoyancy force and porosity of the plate.

Tamana *et al.*, (2009) analyzed heat transfer in a porous medium over a stretching surface with internal heat generation, suction or injection. They observed that velocity profile decreases with the increase of permeability parameter in both cases of injection and suction.

Palani and Srikanth, (2009) analyzed MHD flow of an electrically conducting, incompressible, viscous fluid past a semi-infinite vertical plate with mass transfer, under the action of transversely applied magnetic field. They observed among other things that the contribution of mass diffusion, increases the maximum velocity significantly and local shear stress gets reduced by the increasing value of magnetic field parameter.

Tania and Samad, (2010) studied the effects of radiation, heat generation and viscous dissipation on MHD free convection flow along a stretching sheet. They observed that suction stabilizes the boundary layer growth. They concluded that magnetic field can be used to control the flow characteristics and has significant effect on heat and mass transfer.

Elbasheshy *et al.*, (2010) studied unsteady boundary layer flow over a porous stretching surface embedded in a porous medium in presence of heat source. He observed that among other things, Nusselt number decreases with increase of porous parameter in the presence of heat source parameter and it also increases with suction.

Ferdows *et al.*, (2010) analyzed the problem of heat and mass transfer on natural convection adjacent to a vertical plate in a porous medium with high porosity. They observed that the velocity increases when porous parameter increases and the porous parameter has an increasing effect on temperature profiles.

Kang'ethe *et al.*, (2012) analyzed effects of various parameters on unsteady MHD laminar boundary layer flow of an incompressible, electrically conducting, viscous Newtonian fluids past a stretching sheet embedded in porous media in a rotating system with heat and mass transfer. They concluded among other things that the rate of heat transfer near a stretching surface in a rotating system is influenced more by magnitude of the primary velocity profiles rather than by the magnitude of the secondary velocity profiles. They also noted that absence of rotation leads to absence of secondary velocity profiles.

Das *et al.*, (2011) investigated the mass transfer effects on unsteady hydromagnetic convective flow past a vertical porous plate in a porous medium with heat source, where they observed that velocity of the flow field changes more or less with variation of flow parameters.

Chand *et al.*, (2012) analyzed an oscillatory hydromagnetic flow through a porous medium bounded by two vertical parallel porous plates where one plate of the channel is kept stationary and the other is moving with uniform velocity. He concluded among other things that the heat transfer is reduced with heat generation parameter and the rate of mass transfer is less for the fluid with large viscosity.

Anand *et al.*, (2012) analyzed the chemical reaction effects on an unsteady MHD free convection fluid flow past a semi-infinite vertical plate embedded in a porous medium with heat absorption. They observed that the velocity decreases with an increase in the magnetic parameter and it also increases with an increase in the permeability of the porous medium parameter.

Hussaini *et al.*, (2013) analysed MHD unsteady memory convective flow through porous medium of variable permeability bounded by an infinite vertical porous plate with variable suction. They observed that an increase in Grashof number rises the velocity of the flow due to enhancement of thermal buoyancy force.

Ashraf and Hassanin, (2013) studied MHD flow past a vertical porous plate through a porous medium under oscillatory suction. They concluded that the magnetic field parameter slows down the velocity of the flow field at all points due to magnetic pull of the Lorentz force acting on the flow field.

Sarada and Shanker, (2013) studied the effects of Soret and Dufour on unsteady MHD free convection flow past a vertical porous plate in the presence of suction or injection where they observed among other things that suction or injection parameter retards the velocity of flow field at all points.

From the foregoing studies MHD flows through porous medium is an area of interest to many researchers. However an analysis of effects of heat transfer on unsteady MHD free convective flow past a vertical porous plate in a porous medium with heat source and constant injection has not been fully investigated, hence the motivation behind this study.

1.3 STATEMENT OF THE PROBLEM

In the previous studies as cited above, the combined effects of constant injection and heat source on MHD flow past a porous plate in a porous medium have not been fully investigated. In this study, the effect of heat transfer on a viscous incompressible electrically conducting fluid flow, subjected to transverse magnetic field, past an infinite vertical porous plate in presence of constant injection and heat source through porous medium has been investigated.

1.4 JUSTIFICATION

Flows through porous media are very much prevalent in nature and therefore, the study of such flows has become of principal interest in many scientific and engineering applications. These types of flows have shown their great importance in petroleum engineering in the study of movement of natural gas, oil and water through oil reservoirs; in chemical engineering for the filtration and water purification processes. It's also applicable in MHD generators, plasma studies, nuclear reactors, oil exploration, flows in soil, control of pollutant in ground water, coolers, fuel and gas filters, geothermal energy extraction and in the boundary layer control in the field of aerodynamics.

1.5 HYPOTHESIS

The flow field variables and parameters on unsteady hydromagnetic free convective flow past a vertical porous plate through a porous medium with heat source and constant injection have no effect on the rate of heat transfer at the surface of the wall.

1.6 OBJECTIVES OF THE STUDY

1.6.1 General objective

To analyse the effects of unsteady hydromagnetic free convective flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous plate through a porous medium in presence of constant injection and heat source.

1.6.2 Specific objectives

(i) To determine the effects of variation of injection parameter, heat source parameter and permeability parameter on heat transfer and skin friction at the surface of the wall when heating or cooling of the plate.

(ii) To determine the effects of Prandtl number, permeability parameter, Schmidt number, Eckert number, heat source parameter, injection parameter and Grashof number for mass and heat transfer on the fluid velocity, temperature and concentration.

The general equations governing MHD flow past a vertical porous plate through porous medium, which include the equation of conservation of mass, momentum equations, equation of energy, species of concentration equation and Maxwell equations are given in the next chapter.

CHAPTER TWO

GOVERNING EQUATIONS

In this chapter, the equations of continuity, momentum and energy governing the MHD flow in a porous medium in presence of constant heat source and injection are stated. The equations governing the flow in porous medium have been discussed. Electromagnetic equations have also been given. The assumptions made in this study are also stated.

2.1 ASSUMPTIONS

In order to describe the phenomenon mathematically, the following assumptions are made:

i. The fluid is incompressible.
ii. The plate is considered to be infinite in x-direction, hence all physical quantities will be independent of x-direction.
iii. The density is a linear function of the temperature and species concentration so that the usual Boussinesq's approximation is applicable in the boundary layer flow.
iv. The velocity of the fluid is too small compared to that of light.
v. The fluid flow is laminar.
vi. Electrical conductivity, thermal conductivity and viscosity are constant.
vii. The porous medium is isotropic, homogeneous and non-magnetic; hence there is no magnetic induction.
viii. The fluid and the porous medium are in local thermodynamic equilibrium.

2.2 Darcy's Law

Darcy's law is a phenomenologicaly derived equation that describes flow of a fluid through a porous medium. According to Whitaker, (1986) the law was formulated by Henry Darcy in the year 1856, based on the results of experiments on the flow of water through beds of sand. It also forms the scientific basis of fluid permeability used in the earth sciences particularly in hydrogeology.

Darcy law states that the area-averaged velocity through a column of porous material between two points is directly proportional to the pressure gradient established along the column, and inversely proportional to both the fluid viscosity and the distance between the two points i.e.

$$u = -\frac{k}{\mu}\left(\frac{dP}{dx}\right) \qquad (2.1)$$

The pressure P is the intrinsic average pressure measured by a pressure gauge inside the fluid. The permeability k is an empirical constant that depends on the microstructure of the solid medium, and independent on the properties of the saturated fluid. The velocity u is the Darcian velocity.

The values of k are usually very small e.g. in a brick 10^{-15} to 10^{-13}, in a cigarette 10^{-9} and in sand 10^{-11} to 10^{-10} m^{2}.

2.3 Electromagnetic Equations

2.3.1 Maxwell's Equations

The following four equations represent the Maxwell equations. They give the relationship between the electric field intensity (**E**), the magnetic induction vector (**B**), the electric displacement (**D**), magnetic field intensity (**H**), the conduction current density vector (**J**) and the charge density ρ.

$$\nabla \cdot \mathbf{D} = \rho_e \qquad (2.2)$$

$$\nabla \cdot \mathbf{B} = \mathbf{0} \qquad (2.3)$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \qquad (2.4)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad (2.5)$$

Equation (2.5) is known as Faraday's law, named after Michael Faraday, who, in 1831, showed that a changing magnetic field within a loop gives rise to an induced current, which is due to a force or voltage within that circuit**.** From Faraday law, it can be concluded that a circulating electric field in space gives rise to a magnetic field changing in time.

Equation (2.4) according to Daniel, (2003) is the Ampere's law, named after Ampere Andre-Marie, who in 1826 was first to show that wires carrying electric currents attract and repel each other. From Ampere's law, it can be concluded that a flowing electric current **J** gives rise to a magnetic field that circles the current. A time changing electric flux density **D** gives rise to a magnetic field that circles the **D** field.

Equation (2.3) is the Gauss law for magnetic field which means that divergence of the B or **H** fields is always zero through any volume.

Equation (2.2) represents the Gauss law for electric charge which states that electric charge acts as sources or sinks for the electric field. If there exists electric charge, then the divergence of **D** at that point is non-zero otherwise it is equal to zero.

2.3.2 Forces on an electric charge

When a particle of unit charge e moves with velocity **q** in a region comprising both an electric field **E** and a magnetic field **B,** it experiences two types of forces: the electric force e**E,** and magnetic force e(**q** ×**B**)**.** The total electromagnetic force F_e on e is given by Lorentz's equation as the sum of these two forces and is given by the equation;

$$\mathbf{F}_e = e\,(\mathbf{E} + \mathbf{q} \times \mathbf{B}) \quad (2.6)$$

The force $\mathbf{F}_e$ is known as Lorentz force and is a force that acts on fluid particles. Experiments show that this force acts in a direction perpendicular to both **J** and **B** and is proportional to their magnitude. The generalized Ohm's law can be written as;

$$\mathbf{J} = \sigma\,(\mathbf{E} + \mathbf{q} \times \mathbf{B}) + e_e\mathbf{q} \quad (2.7)$$

The last term is the displacement current, which can be neglected since it is negligibly small at fluid velocity vector **q**.

2.4 Equation of Continuity

The equation of continuity is a statement of mass conservation. The law of conservation of mass states that mass can neither be created nor destroyed under normal circumstances. In fluid dynamics, the continuity equation is a mathematical statement that, in any steady state process, the rate at which mass enters a system is equal to the rate at which mass leaves the system and is based on the principle of conservation of mass.

The differential form of the continuity equation is:

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{u}) = 0 \qquad (2.8)$$

where ρ is fluid density, t is time, and **u** is fluid velocity.

For the case of incompressible flow, ρ is assumed to be a constant and the mass continuity equation simplifies to:

$$\nabla.\mathbf{u} = 0 \qquad (2.9)$$

2.5 Equation of Conservation of Momentum

This is derived from Newton's second law of motion which states that the sum of resultant forces is equal to rate of change of momentum of the flow. The law requires that the sum of all forces acting on a control volume must be equal to the rate of increase of the fluid momentum within the control volume i.e. the mass times acceleration in a given direction is equal to the external forces acting on the control volume in the same direction.

The momentum equation in tensor form is written as:

$$\frac{\partial \mathbf{u}_i}{\partial t} + \mathbf{u}_j \frac{\partial \mathbf{u}_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \mathbf{P}_i}{\partial x_j} + \nu \nabla^2 \mathbf{u}_i + \mathbf{F}_i \qquad (2.10)$$

where subscripts i = 0, 1, 2 and j = 0, 1, 2… are the variables along x, and y axes respectively. The term F_i represents body forces acting on the fluid. In this study, the body forces considered are the gravitational force and the electromagnetic force. The two terms on the left hand side of equation (2.10) represents the local acceleration $\frac{\partial u_i}{\partial t}$ and convective acceleration $u_j \frac{\partial u_i}{\partial x_j}$.

In the equation of the conservation of momentum (2.10), the body forces and surface forces balance with the rate of change of momentum. Body forces act on the entire control volume. The most common body force is the gravitational force. Surface forces act on only one particular surface of the control volume at a time.

From Maxwell's electromagnetic equations, the relation $\nabla.\mathbf{B} = 0$ equation (2.3) give:

$$\frac{\partial \mathbf{B}}{\partial z} = 0 \qquad \text{(2.11)}$$

When magnetic Reynolds number is small, induced magnetic field is negligible in comparison with applied magnetic field, so that

$B_x = 0$, $B_y = 0$ and $B_z = B_0$ (a constant)

The equation of conservation of charge $\nabla.\mathbf{J}=0$ gives J_z =constant where current density

$$\mathbf{J} = (J_x, J_y, j_z)\ .$$

Since the plate is non-conducting, this constant is zero, $J_z = 0$ at the plate and hence everywhere in the flow.

So $\mathbf{J} = (J_x, J_y, 0)$, $\mathbf{B} = (0, 0, B_0)$, $\mathbf{q} = (u, v, v_w)$

The generalised Ohms law, neglecting Hall effect, is given by;

$$\mathbf{J}= \sigma(\mathbf{E} + \mathbf{q}\times\mathbf{B}) \qquad \text{(2.12)}$$

The term $\mathbf{q}\times\mathbf{B}$ in the above equation yields:

$$\mathbf{q} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u & v & w \\ 0 & 0 & B_0 \end{vmatrix} = vB_0\mathbf{i} - uB_0\mathbf{j} \qquad \text{(2.13)}$$

from equation (2.6) and equation (2.7),

$$J_x = \sigma v B_0,\ \ J_y = -\sigma u B_0 \text{ and } J_z=0 \qquad \text{(2.14)}$$

The Lorentz force $\mathbf{J} \times \mathbf{B}$ becomes;

$$\mathbf{J}\times\mathbf{B}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ \sigma vB_0 & -\sigma uB_0 & 0\\ 0 & 0 & B_0\end{vmatrix}=-\sigma uB_0{}^2\mathbf{i}-\sigma vB_0{}^2\mathbf{j}\ldots\ldots\ldots\ldots(2.15)$$

The density has been assumed to be a linear function of the temperature and species concentration, and therefore the Boussinesq approximation has been used.

The latter assumes that the thermodynamic state of a fluid depends on the temperature, pressure and concentration. The pressure gradient term results from the change in elevation. To determine the pressure gradient term, the momentum equation is evaluated at the edge of the boundary layer. When the pressure gradient upwards is balanced by the pressure gradient downward due to fluid density,

$$\frac{\partial P}{\partial x}=-\rho_\infty g \ldots\ldots\ldots\ldots\ldots\ldots (2.16)$$

where ρ_∞ is density of fluid outside boundary layer.

Hence in the equation of momentum becomes,

$$-\rho g+\rho_\infty g=g(\rho_\infty-\rho)\ldots\ldots\ldots\ldots\ldots\ldots (2.17)$$

Using $\beta=-\frac{1}{\rho}\frac{\partial\rho}{\partial T}$ and $\beta^*=-\frac{1}{\rho}\frac{\partial\rho}{\partial C}$

this yields, $\Delta\rho=-\beta\rho\Delta T$ and $\Delta\rho=-\beta^*\rho\Delta C$

Hence total change in density as a result of temperature and concentration is then given as;

$$\Delta\rho=-\beta\rho\Delta T-\beta^*\rho\Delta C$$

$$g(\rho_\infty-\rho)=g\beta\rho(T_\infty-T)-g\beta^*\rho(C_\infty-C)\ldots\ldots\ldots\ldots (2.18)$$

Equation (2.18) represents the overall pressure gradient term in the momentum equation.

Considering the boundary layer approximations, the flow, heat and mass transfer of a fluid in porous medium with Darcian effects and applying the Boussinesq's approximation, the respective x and y momentum equations governing the flow in this study will appear as follows:

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\nu}{k}u-\frac{\sigma B_0{}^2 u}{\rho}+g\beta\left(T-T_\infty\right)+g\beta^*\left(C-C_\infty\right)\text{.........................}(2.19a)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial y}=\nu\frac{\partial^2 v}{\partial y^2}-\frac{\nu}{k}v-\frac{\sigma B_0{}^2 v}{\rho}+g\beta\left(T-T_\infty\right)+g\beta^*\left(C-C_\infty\right)\text{.........................}(2.19b)$$

2.6 Equation of energy:

The equation of energy is derived from the first law of Thermodynamics, which states that energy is conserved in any process involving a thermodynamic system and its surroundings.

It states that, heat added to a system, *dQ* is equal to change in internal energy, *dE* plus work done,

$$dW = p\,dv \text{..}(2.20)$$

$$dQ = dE + dW \text{..}(2.21)$$

$$dQ= dE + p\,dv \text{..}(2.22)$$

Considering flow of an incompressible fluid with constant thermal conductivity k, thermal energy equation is expressed as

$$\rho C_p\frac{DT}{Dt}=k\nabla^2 T+\mu\phi \text{ ....} (2.23)$$

From Boussinesq approximation, it is assumed that the fluid has a constant heat capacity per unit volume ρC_p implying that $\rho C_p\frac{DT}{Dt}$ is equal to the rate of heating per unit volume of a fluid particle. Thermal conductivity k of the fluid is the rate of flow of heat through the fluid per unit cross sectional area per unit temperature gradient, $\mu\phi$ is the internal heating due to viscous dissipation, ϕ is Viscous dissipation function and $\frac{DT}{Dt}$ is material derivative. Neglecting Ohmic heating, hall effect and ion-slip current, equation 2.23 becomes,

$$\rho c_p \frac{DT}{Dt} = \nabla.(k\nabla T) + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)^2 \quad \text{..(2.24)}$$

Considering the boundary layer approximations, the flow, heat and mass transfer of a fluid in porous medium with Darcian effects, injection, heat source and neglecting the induced magnetic field and Joule heat dissipation and applying the Boussinesq's approximation, the energy equation governing the flow in this study is given as:

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \frac{k}{C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 + Q(T - T_\infty) \text{..(2.25)}$$

2.5 The Equation for Species Concentration

The equation of species concentration is based on the conservation of mass. It is applicable when,

i. The porous medium is saturated with fluid.
ii. Flow is steady.
iii. Darcy law is applicable.

Convection is one of the major modes of heat transfer and mass transfer. Convective heat and mass transfer take place through diffusion i.e. the random Brownian motion of individual particles in the fluid and by advection in which dissolved substances or heat are carried along with bulk fluid flow. In advection, the species spread out from the path expected to be followed by the advection alone. The equation for species concentration is given as,

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = D_M \frac{\partial^2 C}{\partial y^2} \text{...(2.26)}$$

In the next chapter, mathematical formulation is done where the equations of momentum, energy and species concentration are non-dimensionalised and finite difference method is used to solve the equations numerically. The results are then generated using a computer program. The chapter concludes by discussion of the results.

CHAPTER THREE

MATHEMATICAL FORMULATION

Consider a two dimensional flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous plate through a porous medium in presence of uniform transverse magnetic field $B_{0,}$ constant heat source (Q) and injection (V_w). The vertical plate is taken to be the x-axis and y-axis normal to it. All the governing equations will therefore be independent of x-axis because it is infinite. . The physical sketch and geometry of the problem is shown in figure 3.1 below:

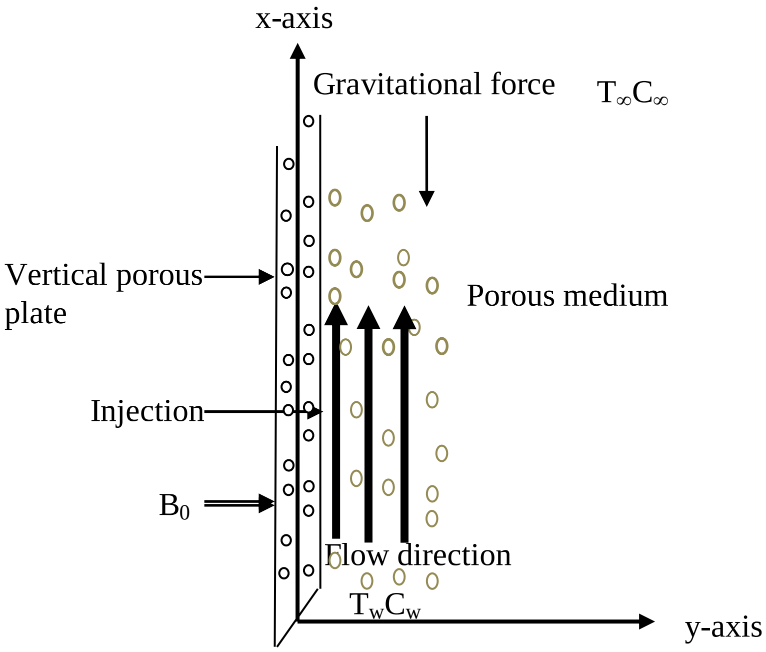

Figure 3.1: The Flow Configuration

The surface of the vertical plate is at uniform temperature T_w and concentration C_w. The temperature and concentration far away from the plate are T_∞ and C_∞ respectively. A magnetic field of strength B_o acts normal to the plate, i.e. along the y-axis. The density has been assumed to be a linear function of the temperature and species concentration, and therefore the Boussinesq approximation has been used.

The latter assumes that the thermodynamic state of a fluid depends on the temperature, pressure and concentration. The pressure gradient term results from the change in elevation. To determine the pressure gradient term, the momentum equation is evaluated at the edge of the boundary layer. When the pressure gradient upwards is balanced by the pressure gradient downward due to fluid density,

$$\frac{\partial P}{\partial x} = -\rho_\infty g \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.1)$$

where ρ_∞ is density of fluid outside boundary layer.

Hence in the equation of momentum becomes,

$$-\rho g + \rho_\infty g = g(\rho_\infty - \rho) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.2)$$

Using $\beta = -\frac{1}{\rho}\frac{\partial \rho}{\partial T}$ and $\beta^* = -\frac{1}{\rho}\frac{\partial \rho}{\partial C}$

this yields, $\Delta\rho = -\beta\rho\Delta T$ and $\Delta\rho = -\beta^*\rho\Delta C$

Hence total change in density as a result of temperature and concentration is then given as;

$$\Delta\rho = -\beta\rho\Delta T - \beta^*\rho\Delta C$$

$$g(\rho_\infty - \rho) = g\beta\rho(T_\infty - T) - g\beta^*\rho(C_\infty - C) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.3)$$

Equation (3.3) represents the overall pressure gradient term in the momentum equation.

Considering the boundary layer approximations, the equations of momentum, energy and species concentration governing the flow in this study are as follows:

Equation of momentum along the x-axis is

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\nu}{k}u - \frac{\sigma B_0{}^2 u}{\rho} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.4)$$

Equation of momentum along the y-axis is

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial y} = \nu\frac{\partial^2 v}{\partial y^2} - \frac{\nu}{k}v - \frac{\sigma B_0{}^2 v}{\rho} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.5)$$

The equation of energy is:

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \frac{k}{C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 + Q(T - T_\infty) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.6)$$

The equation of species concentration is:

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = D_M \frac{\partial^2 C}{\partial y^2} \qquad (3.7)$$

The boundary conditions of the problem are as follows:

At $y=0$: $u=0$, $v=v_w$, $T=T_w$, $C=C_w$

As $y\to\infty$, $u\to 0$, $T\to T_\infty$, $C\to C_\infty$

The initial conditions of the problem are as follows:

At $t=0$, $u=u_\infty$, $v=v_w$, $T=T_\infty$, $C=C_\infty$

Where v_w is the velocity of injection at the wall of the plate, T_w and C_w are temperature and concentration at the wall respectively, T_∞ and C_∞ are temperature and concentration away from the surface of the wall respectively.

3.1 Non-dimensionalisation

The process of non-dimensionalisation starts with selecting a suitable scale against which all dimensions in a given physical model are based. It is basically aimed at ensuring that the final results are applicable to other geometrically similar configurations under a similar set of flow conditions.

In this study boundary layer equations are non-dimensionalised so as to make the solution bounded and to vary between 0 and 1 inclusive. The free stream velocity U_∞ is taken as characteristic velocity.

To non-dimensionalise equations (3.4), (3.5), (3.6) and (3.7) and their respective boundary conditions the following non-dimensional parameters and variables defined as follows:

$$y^* = \frac{yU_\infty}{\nu}, \quad t^* = \frac{tU_\infty{}^2}{\nu}, \quad v^* = \frac{v}{U_\infty}, \quad u^* = \frac{u}{U_\infty}, \quad T^* = \frac{T - T_\infty}{T_w - T_\infty},$$

$$C^* = \frac{C - C_\infty}{C_w - C_\infty},$$

In order to transform the equations of momentum, energy and species concentration into their non-dimensional form, the following analysis is first carried out:

Momentum equation

$$\frac{\partial u}{\partial t}=\frac{\partial u}{\partial u^*}\cdot\frac{\partial t^*}{\partial t}\cdot\frac{\partial u^*}{\partial t^*}=\frac{U_\infty{}^3}{\nu}\frac{\partial u^*}{\partial t^*} \quad \text{..................................... (3.8)}$$

$$v\frac{\partial u}{\partial y}=v^*U_\infty\frac{\partial u}{\partial u^*}\cdot\frac{\partial y^*}{\partial y}\cdot\frac{\partial u^*}{\partial y^*}=\frac{v^*U_\infty{}^3}{\nu}\frac{\partial u^*}{\partial y^*} \quad \text{..................................(3.9)}$$

$$\nu\frac{\partial^2 u}{\partial y^2}=\nu\frac{\partial}{\partial y^*}\left(\frac{\partial u}{\partial u^*}\cdot\frac{\partial y^*}{\partial y}\cdot\frac{\partial u^*}{\partial y^*}\right)\frac{\partial y^*}{\partial y}=\frac{U_\infty{}^3}{\nu}\frac{\partial^2 u^*}{\partial y^{*2}} \quad \text{..........................(3.10)}$$

$$\frac{\partial v}{\partial t}=\frac{\partial v}{\partial v^*}\cdot\frac{\partial t^*}{\partial t}\cdot\frac{\partial v^*}{\partial t^*}=\frac{U_\infty{}^3}{\nu}\frac{\partial v^*}{\partial t^*} \quad \text{...(3.11)}$$

$$u\frac{\partial v}{\partial y}=u^*U_\infty\frac{\partial v}{\partial v^*}\cdot\frac{\partial y^*}{\partial y}\cdot\frac{\partial v^*}{\partial y^*}=\frac{u^*U_\infty{}^3}{\nu}\frac{\partial v^*}{\partial y^*} \quad \text{...(3.12)}$$

$$\nu\frac{\partial^2 v}{\partial y^2}=\nu\frac{\partial}{\partial y^*}\left(\frac{\partial v}{\partial v^*}\cdot\frac{\partial y^*}{\partial y}\cdot\frac{\partial v^*}{\partial y^*}\right)\frac{\partial y^*}{\partial y}=\frac{U_\infty{}^3}{\nu}\frac{\partial^2 v^*}{\partial y^{*2}} \quad \text{................................(3.13)}$$

Substituting equations (3.8), (3.9), and (3.10) in equation (3.4);

$$\frac{U_\infty{}^3}{\nu}\frac{\partial u^*}{\partial t^*}+\frac{v^*U_\infty{}^3}{\nu}\frac{\partial u^*}{\partial y^*}=\frac{U_\infty{}^3}{\nu}\frac{\partial^2 u^*}{\partial y^{*2}}-\frac{\nu}{k}u^*U_\infty-\frac{\sigma B_0{}^2u^*U_\infty}{\rho}+g\beta(T-T_\infty)+g\beta^*(C-C_\infty)$$

then, multiplying by $\frac{\nu}{U_\infty{}^3}$ the equation now becomes:

$$\frac{\partial u^*}{\partial t^*}+v^*\frac{\partial u^*}{\partial y^*}=\frac{\partial^2 u^*}{\partial y^{*2}}-\frac{\nu^2}{kU_\infty{}^2}u^*-\frac{\sigma B_0{}^2\nu}{\rho U_\infty{}^2}u^*+\frac{g\beta\nu(T-T_\infty)}{U_\infty{}^3}+\frac{g\beta^*\nu(C-C_\infty)}{U_\infty{}^3}$$

$$\frac{\partial u^*}{\partial t^*}+v^*\frac{\partial u^*}{\partial y^*}=\frac{\partial^2 u^*}{\partial y^{*2}}-\left(\frac{\sigma B_0{}^2\nu}{\rho U_\infty{}^2}+\frac{\nu^2}{kU_\infty{}^2}\right)u^*+\frac{\nu g\beta(T-T_\infty)}{U_\infty{}^3}+\frac{\nu g\beta^*(C-C_\infty)}{U_\infty{}^3}$$

The x-momentum equation now becomes:

$$\frac{\partial u^*}{\partial t^*}+v^*\frac{\partial u^*}{\partial y^*}=\frac{\partial^2 u^*}{\partial y^{*2}}-\left(M+\frac{1}{K_P}\right)u^*+Gr_\theta T^*+Gr_c C^* \quad \text{................................(3.14)}$$

where

$$M = \frac{\sigma B_0{}^2 \nu}{\rho U_\infty{}^2}, \quad K_p = \frac{U_\infty{}^2 k}{\nu^2}, \quad Gr_\theta = \frac{\nu g \beta (T_w - T_\infty)}{U_\infty{}^3}, \qquad Gr_c = \frac{\nu g \beta^* (C_w - C_\infty)}{U_\infty{}^3}$$

,

Substituting equations (3.11), (3.12), and (3.13) in equation (3.5);

$$\frac{U_\infty{}^3}{\nu}\frac{\partial v^*}{\partial t^*} + \frac{u^* U_\infty{}^3}{\nu}\frac{\partial v^*}{\partial y^*} = \frac{U_\infty{}^3}{\nu}\frac{\partial^2 v^*}{\partial y^{*2}} - \frac{\nu}{k} v^* U_\infty - \frac{\sigma B_0{}^2 v^* U_\infty}{\rho} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty)$$

then, multiplying by $\frac{\nu}{U_\infty{}^3}$ the equation now becomes:

$$\frac{\partial v^*}{\partial t^*} + v^* \frac{\partial v^*}{\partial y^*} = \frac{\partial^2 v^*}{\partial y^{*2}} - \left(\frac{\sigma B_0{}^2 \nu}{\rho U_\infty{}^2} + \frac{\nu^2}{k U_\infty{}^2} \right) v^* + \frac{\nu g \beta (T - T_\infty)}{U_\infty{}^3} + \frac{\nu g \beta^* (C - C_\infty)}{U_\infty{}^3}$$

The y-momentum equation now becomes:

$$\frac{\partial v^*}{\partial t^*} + u^* \frac{\partial v^*}{\partial y^*} = \frac{\partial^2 v^*}{\partial y^{*2}} - \left(M + \frac{1}{K_P} \right) v^* + Gr_\theta T^* + Gr_c C^* \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.15)$$

where,

$$M = \frac{\sigma B_0{}^2 \nu}{\rho U_\infty{}^2}, \quad K_p = \frac{U_\infty{}^2 k}{\nu^2}, \quad Gr_\theta = \frac{\nu g \beta (T_w - T_\infty)}{U_\infty{}^3}, \qquad Gr_c = \frac{\nu g \beta^* (C_w - C_\infty)}{U_\infty{}^3}$$

Equation of energy

$$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial T^*} \cdot \frac{\partial t^*}{\partial t} \cdot \frac{\partial T^*}{\partial t^*} = (T_w - T_\infty) \frac{U_\infty{}^2}{\nu} \frac{\partial T^*}{\partial t^*} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.16)$$

$$v \frac{\partial T}{\partial y} = v^* \, U_\infty \frac{\partial T}{\partial T^*} \cdot \frac{\partial y^*}{\partial y} \cdot \frac{\partial T^*}{\partial y^*} = (T_w - T_\infty) \frac{U_\infty{}^2}{\nu} v^* \frac{\partial T^*}{\partial y^*} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.17)$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{\partial}{\partial y^*} \left(\frac{\partial T}{\partial T^*} \cdot \frac{\partial y^*}{\partial y} \cdot \frac{\partial T^*}{\partial y^*} \right) \frac{\partial y^*}{\partial y} = (T_w - T_\infty) \frac{U_\infty{}^2}{\nu^2} \frac{\partial^2 T^*}{\partial t^{*2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.18)$$

$$\left(\frac{\partial u}{\partial y} \right)^2 = \frac{\partial u}{\partial u^*} \cdot \frac{\partial y^*}{\partial y} \cdot \frac{\partial u^*}{\partial y^*} = \left(\frac{U_\infty{}^2}{\nu} \frac{\partial u^*}{\partial y^*} \right)^2 = \frac{U_\infty{}^4}{\nu^2} \left(\frac{\partial u^*}{\partial y^*} \right)^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.19)$$

$$Q(T - T_\infty)=QT^*(T_w - T_\infty) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.20)$$

Substituting equations (3.16), (3.17), (3.18), (3.19) and (3.20) in equation (3.6);

$$\left(T_w - T_\infty\right)\frac{U_\infty{}^2}{\nu}\frac{\partial T^*}{\partial t^*} + \left(T_w - T_\infty\right)\frac{U_\infty{}^2}{\nu} v^* \frac{\partial T^*}{\partial y^*} = \left(T_w - T_\infty\right)k\frac{U_\infty{}^2}{C_p \nu^2}\frac{\partial^2 T^*}{\partial t^{*2}} + \frac{U_\infty{}^4}{C_p \nu}\left(\frac{\partial u^*}{\partial y^*}\right)^2 + QT^*(T_w - T_\infty)$$

then, multiplying by $\frac{\nu}{U_\infty{}^2(T_w - T_\infty)}$ the equation now becomes:

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{k}{C_p \nu}\frac{\partial^2 T^*}{\partial t^{*2}} + \frac{U_\infty{}^2}{C_p(T_w - T_\infty)}\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \frac{Q\nu}{U_\infty{}^2}T^*$$

the energy equation now becomes,

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{P_r}\frac{\partial^2 T^*}{\partial y^{*2}} + Ec\left(\frac{\partial u^*}{\partial y^*}\right)^2 + Q^* T^* \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.21)$$

where,

$$Ec = \frac{U_\infty{}^2}{C_p\left(T_w - T_\infty\right)}, \quad Q^* = \frac{Q\nu}{U_\infty{}^2}, \quad \Pr = \frac{\nu C_p}{k}$$

Concentration equation

$$\frac{\partial C}{\partial t} = \frac{\partial C}{\partial C^*}\cdot\frac{\partial t^*}{\partial t}\cdot\frac{\partial C^*}{\partial t^*} = \left(C_w - C_\infty\right)\frac{U_\infty{}^2}{\nu}\frac{\partial C^*}{\partial t^*} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.22)$$

$$v\frac{\partial C}{\partial y} = v^*$$

$$U_\infty \frac{\partial C}{\partial C^*}\cdot\frac{\partial y^*}{\partial y}\cdot\frac{\partial C^*}{\partial y^*} = v^*\left(C_w - C_\infty\right)\frac{U_\infty{}^2}{\nu}\frac{\partial C^*}{\partial y^*} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.23)$$

$$\frac{\partial^2 C}{\partial y^2} =$$

$$\frac{\partial}{\partial y^*}\left(\frac{\partial C}{\partial C^*}\cdot\frac{\partial y^*}{\partial y}\cdot\frac{\partial C^*}{\partial y^*}\right)\frac{\partial y^*}{\partial y} = \left(C_w - C_\infty\right)\frac{U_\infty{}^2}{\nu^2}\frac{\partial^2 C^*}{\partial y^{*2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.24)$$

Substituting equations (3.22), (3.23) and (3.24) in equation (3.7);

$$\left(C_w - C_\infty\right)\frac{U_\infty{}^2}{\nu}\frac{\partial C^*}{\partial t^*} + v^*\left(C_w - C_\infty\right)\frac{U_\infty{}^2}{\nu}\frac{\partial C^*}{\partial y^*} = \left(C_w - C_\infty\right)\frac{U_\infty{}^2 D_M}{\nu^2}\frac{\partial^2 C^*}{\partial y^{*2}}$$

Then multiplying by $\dfrac{\nu}{U_\infty{}^2(C_w - C_\infty)}$ the equation now becomes:

$$\frac{\partial C^*}{\partial t^*} + v^* \frac{\partial C^*}{\partial y^*} = \frac{D_M}{\nu} \frac{\partial^2 C^*}{\partial y^{*^2}}$$

The concentration equation now becomes,

$$\frac{\partial C^*}{\partial t^*} + v^* \frac{\partial C^*}{\partial y^*} = \frac{1}{Sc} \frac{\partial^2 C^*}{\partial y^{*2}} \quad \text{...(3.25)}$$

where : $\quad Sc = \dfrac{\nu}{D_M}$

The boundary conditions for equations (3.14), (3.15), (3.21), and (3.25) in non-dimensional form are computed as follows

At y = 0, $y^* = \dfrac{0U_\infty}{\nu} \Rightarrow y^* = 0$

$u = 0, u^* = \dfrac{0}{u_\infty} \Rightarrow u^* = 0$

$v = v_w$, $v^* = \dfrac{v_w}{u_\infty}$

$T = T_w, T^* = \dfrac{T_w - T_\infty}{T_w - T_\infty} \Rightarrow T^* = 1$

$C = C_w, C^* = \dfrac{C_w - C_\infty}{C_w - C_\infty} \Rightarrow C^* = 1$

Therefore, at $y^* = 0$; $\quad u^* = 0$, $v^* = \dfrac{v_w}{U_\infty}$, $T^* = 1$, $C^* = 1$………………(3.26a)

As $y \to \infty$, $y^* = \dfrac{\infty U_\infty}{\nu} = \infty, y^* \to \infty$

$u \to 0, u^* = \dfrac{0}{U_\infty} = 0, u^* \to 0$

$T \to \infty, T^* = \dfrac{\infty - T_\infty}{T_w - T_\infty} = 0, T^* \to 0$

$$C \to \infty, C^* = \frac{\infty - C_\infty}{C_w - C_\infty} = 0, C^* \to 0$$

Therefore, as $y^* \to \infty$; $u^* \to 0$, $T^* \to 0$, $C^* \to 0$, ……………..…(3.26)

At t=0, $t^* = \frac{0U_\infty{}^2}{\nu} \Rightarrow t^* = 0$

$$u = U_\infty, u^* = \frac{U_\infty}{U_\infty} = 1, u^* = 1$$

v = v_w, $v^* = \frac{v_w}{U_\infty}$

$$T = T_\infty, T^* = \frac{T_\infty - T_\infty}{T_w - T_\infty} = 0, T^* = 0$$

$$C = C_\infty, C^* = \frac{C_\infty - C_\infty}{C_w - C_\infty} = 0, C^* = 0$$

Therefore, at t^*=0; u^*=1, $v^* = \frac{v_w}{U_\infty}$, T^*=0, C^*=0………………...(3.27)

3.2 Dimensionless Numbers

After non-dimensionalising the three equations, the non-dimensional numbers contained in the non-dimensionalized equations are discussed below:

3.2.1 Prandtl number

The Prandtl number (Pr) is the ratio of fluid properties controlling the velocity and the temperature distributions. It is the ratio of viscous force to thermal force.

$$\mathrm{Pr} = \frac{\nu C_p}{k}$$ …………………………………...………(3.28)

3.2.2 Schmidt number

This is a dimensionless number that is a measure of relative effectiveness of momentum and mass transport by diffusion in the velocity and concentration boundary layers respectively. It embodies the ratio of the momentum to the mass diffusivity. According to Incropera and DeWitt (2002), the Schmidt number quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic (velocity) and the concentration (species) boundary layers.

$$Sc = \frac{\nu}{D_M} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.29)$$

3.2.3 The local mass Grashof number

The Grashof number for mass transfer Gr_c defines the ratio of the species buoyancy force to the viscous hydrodynamic force.

$$Gr_c = \frac{\nu g \beta^* (C_w - C_\infty)}{U_\infty^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(3.30)$$

3.2.4 The local temperature Grashof number

The Grashof number for heat transfer Gr_θ signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. If $Gr_\theta > 0$ i.e. positive, it corresponds to cooling of the plate or heating of the fluid. If $Gr_\theta < 0$ i.e. negative, it corresponds to heating of the plate or cooling of the fluid.

$$Gr_\theta = \frac{\nu g \beta (T_w - T_\infty)}{U_\infty^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.31)$$

3.2.5 Eckert number

The Eckert number (Ec), expresses the relation between the kinetic energy in the flow and the enthalpy. It represents the conversion of kinetic energy into internal energy by work that is done against the viscous fluid stresses.

$$Ec = \frac{U_\infty^2}{C_p (T_w - T_\infty)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.32)$$

3.2.6 Permeability parameter

Permeability is a measure of the ability of a porous material to allow fluids to pass through it. Permeability parameter (K_p) is directly proportional to the actual permeability (k).

$$K_p = \frac{U_\infty{}^2 k}{\nu^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.33)$$

3.2.7 Hartmann Number

Hartmann number (M) is the ratio of electromagnetic force to viscous force.

$$M = \frac{\sigma B_0{}^2 \nu}{\rho U_\infty{}^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3.33b)$$

3.3 Method of Solution

The three partial differential equations above governing MHD flow past an infinite porous plate through a porous medium in presence of constant heat source and injection are non-linear. It is therefore not possible to get an exact analytical solution. The equations are solved numerically using the FDM that applies Crank-Nicolson algorithm. This method was developed by John Crank and Phyllis Nicolson in 1947. It’s a second order method which is accurate and unconditionally stable and has less computational cost. According to Steven and Raymond (2010), Crank-Nicolson method provides an implicit scheme which is accurate in both space and time. To provide this accuracy, difference approximations are developed at the midpoint of the time increment. In this study FDM has been preferred because it’s simpler than some methods such as Spectral Element Method (SEM) or Finite Element Method (FEM).

3.3.1 Definition of the mesh

The flow domain is confined by the y and t axis. The y-axis values range from Y_{min} to Y_{max} so, delta y= (Y_{max} - Y_{min})/N The t-axis has values of t that range from 0 to T. The interval [Y_{min}, Y_{max}] is divided into (N + 1) intervals at y values indexed by j =

0, 1, ...N. The interval [0, T] is divided into (M + 1) equally spaced intervals at t values indexed by i = 0, 1,….M.

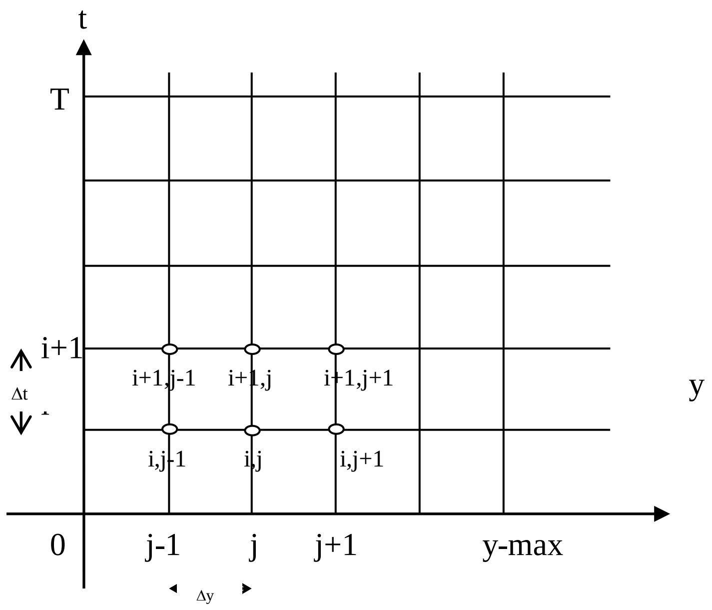

Fig 3.2: Mesh grid

3.3.2 The Finite Difference Method

The partial derivatives of U, V, T and C at each grid point are expressed using finite difference approximation.

The expressions for U, U_y, and U_{yy} are averages for times i and i +1 are:

$$\left.\frac{\partial U}{\partial t}\right|_{i+1,j} = \frac{U_j^{i+1} - U_j^i}{\Delta t} + O(\Delta t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.34)$$

$$\left.\frac{\partial U}{\partial y}\right|_{i,j} = \frac{1}{2}\left(\frac{U_j^{i+1} - U_{j-1}^{i+1}}{\Delta y} + \frac{U_j^i - U_{j-1}^i}{\Delta y}\right) + O(\Delta y)$$

$$= \frac{U_j^{i+1} - U_{j-1}^{i+1} + U_j^i - U_{j-1}^i}{2\Delta y} + O(\Delta y) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.35)$$

$$\left.\frac{\partial^2 U}{\partial y^2}\right|_{i,j} = \frac{1}{2}\left(\frac{U_{j-1}^{i+1} - 2U_j^{i+1} + U_{j+1}^{i+1}}{(\Delta y)^2} + \frac{U_{j-1}^i - 2U_j^i + U_{j+1}^i}{(\Delta y)^2}\right) + O(\Delta y)^2$$

$$= \frac{U_{j-1}^{i+1} - 2U_j^{i+1} + U_{j+1}^{i+1} + U_{j-1}^{i} - 2U_j^{i} + U_{j+1}^{i}}{2(\Delta y)^2} + O(\Delta y)^2$$
……………………….(3.36)

Similarly, the expressions for C, C_y & C_{yy} and T, T_y, & T_{yy} have the following finite difference approximations:

$$\left.\frac{\partial T}{\partial t}\right|_{i+1,j} = \frac{T_j^{i+1} - T_j^{i}}{\Delta t} + O(\Delta t)$$
……………………………………………………(3.37)

$$\left.\frac{\partial T}{\partial y}\right|_{i,j} = \frac{1}{2}\left(\frac{T_j^{i+1} - T_{j-1}^{i+1}}{\Delta y} + \frac{T_j^{i} - T_{j-1}^{i}}{\Delta y}\right) + O(\Delta y) = \frac{T_j^{i+1} - T_{j-1}^{i+1} + T_j^{i} - T_{j-1}^{i}}{2\Delta y} + O(\Delta y)$$
…...(3.38)

$$\left.\frac{\partial^2 T}{\partial y^2}\right|_{i,j} = \frac{1}{2}\left(\frac{T_{j-1}^{i+1} - 2T_j^{i+1} + T_{j+1}^{i+1}}{(\Delta y)^2} + \frac{T_{j-1}^{i} - 2T_j^{i} + T_{j+1}^{i}}{(\Delta y)^2}\right) + O(\Delta y)^2$$

$$= \frac{T_{j-1}^{i+1} - 2T_j^{i+1} + T_{j+1}^{i+1} + T_{j-1}^{i} - 2T_j^{i} + T_{j+1}^{i}}{2(\Delta y)^2} + O(\Delta y)^2$$
…………………………(3.39)

$$\left.\frac{\partial C}{\partial t}\right|_{i+1,j} = \frac{C_j^{i+1} - C_j^{i}}{\Delta t} + O(\Delta t)$$
…………………………………………………(3.40)

$$\left.\frac{\partial C}{\partial y}\right|_{i,j} = \frac{1}{2}\left(\frac{C_j^{i+1} - C_{j-1}^{i+1}}{\Delta y} + \frac{C_j^{i} - C_{j-1}^{i}}{\Delta y}\right) + O(\Delta y) = \frac{C_j^{i+1} - C_{j-1}^{i+1} + C_j^{i} - C_{j-1}^{i}}{2\Delta y} + O(\Delta y)$$
……..(3.41)

$$\left.\frac{\partial^2 C}{\partial y^2}\right|_{i,j} = \frac{1}{2}\left(\frac{C_{j-1}^{i+1} - 2C_j^{i+1} + C_{j+1}^{i+1}}{(\Delta y)^2} + \frac{C_{j-1}^{i} - 2C_j^{i} + C_{j+1}^{i}}{(\Delta y)^2}\right) + O(\Delta y)^2$$

$$= \frac{C_{j-1}^{i+1} - 2C_j^{i+1} + C_{j+1}^{i+1} + C_{j-1}^{i} - 2C_j^{i} + C_{j+1}^{i}}{2(\Delta y)^2} + O(\Delta y)^2$$
………………………(3.42)

$$\frac{\partial U}{\partial t} = \frac{U_j^{i+1} - U_j^{i}}{\Delta t}$$
……..………………………..……………………….……………..(3.43)

$$\frac{\partial U}{\partial y} = \frac{U_j^{i+1} - U_{j-1}^{i+1} + U_j^{i} - U_{j-1}^{i}}{2\Delta y}$$
……………….………………………..……………(3.44)

$$\frac{\partial^2 U}{\partial y^2} = \frac{U_{j-1}^{i+1} - 2U_j^{i+1} + U_{j+1}^{i+1} + U_{j-1}^{i} - 2U_j^{i} + U_{j+1}^{i}}{2(\Delta y)^2}$$
…………...(3.45)

$$\frac{\partial T}{\partial t} = \frac{T_j^{i+1} - T_j^{i}}{\Delta t}$$
…………………………...…………..(3.46)

$$\frac{\partial T}{\partial y}=\frac{T_j^{i+1}-T_{j-1}^{i+1}+T_j^i-T_{j-1}^i}{2\Delta y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.47)$$

$$\frac{\partial^2 T}{\partial y^2}=\frac{T_{j-1}^{i+1}-2T_j^{i+1}+T_{j+1}^{i+1}+T_{j-1}^i-2T_j^i+T_{j+1}^i}{2(\Delta y)^2} \quad \ldots\ldots\ldots\ldots(3.48)$$

$$\frac{\partial C}{\partial t}=\frac{C_j^{i+1}-C_j^i}{\Delta t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.49)$$

$$\frac{\partial C}{\partial y}=\frac{C_j^{i+1}-C_{j-1}^{i+1}+C_j^i-C_{j-1}^i}{2\Delta y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.50)$$

$$\frac{\partial^2 C}{\partial y^2}=\frac{C_{j-1}^{i+1}-2C_j^{i+1}+C_{j+1}^{i+1}+C_{j-1}^i-2C_j^i+C_{j+1}^i}{2(\Delta y)^2} \quad \ldots\ldots\ldots\ldots(3.51)$$

The finite difference equations corresponding to the governing equations and neglecting superscript * for clarity will be as given below:

Equation of momentum:

$$\frac{U_j^{i+1}-U_j^i}{\Delta t}+V_j^i\,\frac{U_j^{i+1}-U_{j-1}^{i+1}+U_j^i-U_{j-1}^i}{2\Delta y}=\frac{U_{j-1}^{i+1}-2U_j^{i+1}+U_{j+1}^{i+1}+U_{j-1}^i-2U_j^i+U_{j+1}^i}{2(\Delta y)^2}$$

$$-\left(M+\frac{1}{K_p}\right)\left(\frac{U_j^{i+1}+U_j^i}{2}\right)+Gr_\theta\left(\frac{T_j^{i+1}+T_j^i}{2}\right)+Gr_c\left(\frac{C_j^{i+1}+C_j^i}{2}\right)$$

then,

$$\frac{U_j^{i+1}}{\Delta t}+\frac{U_j^{i+1}}{2\Delta y}V_j^i+\frac{U_j^{i+1}}{(\Delta y)^2}+\frac{U_j^{i+1}}{2}\left(M+\frac{1}{K_p}\right)=\frac{U_j^i}{\Delta t}-\frac{V_j^i}{2\Delta y}\left(-U_{j-1}^{i+1}+U_j^i-U_{j-1}^i\right)$$

$$+\frac{1}{2(\Delta y)^2}\left(U_{j-1}^{i+1}+U_{j+1}^{i+1}+U_{j-1}^i-2U_j^i+U_{j+1}^i\right)-\frac{U_j^i}{2}\left(M+\frac{1}{K_p}\right)+\frac{Gr_\theta}{2}\left(T_j^{i+1}+T_j^i\right)+\frac{Gr_c}{2}\left(C_j^{i+1}+C_j^i\right)$$

multiplying through by Δt,

$$U_j^{i+1} + \frac{U_j^{i+1}}{2\Delta y}\Delta t V_j^i + \frac{U_j^{i+1}}{(\Delta y)^2}\Delta t + \frac{U_j^{i+1}}{2}\Delta t\left(M + \frac{1}{K_p}\right) = U_j^i - \frac{V_j^i}{2\Delta y}\Delta t\left(-U_{j-1}^{i+1} + U_j^i - U_{j-1}^i\right) + \frac{\Delta t}{2(\Delta y)^2}$$

$$\left(U_{j-1}^{i+1} + U_{j+1}^{i+1} + U_{j-1}^i - 2U_j^i + U_{j+1}^i\right) - \frac{U_j^i\Delta t}{2}\left(M + \frac{1}{K_p}\right) + \frac{Gr_\theta \Delta t}{2}\left(T_j^{i+1} + T_j^i\right) + \frac{Gr_c\Delta t}{2}\left(C_j^{i+1} + C_j^i\right)$$

making U_j^{i+1} the subject of the formula the equation of momentum along x-axis becomes;

$$U_j^{i+1} = \left[U_j^i - \frac{\Delta t}{2\Delta y}V_j^i\left(-U_{j-1}^{i+1} + U_j^i - U_{j-1}^i\right) + \frac{\Delta t}{2(\Delta y)^2}\left(U_{j-1}^{i+1} + U_{j+1}^{i+1} + U_{j-1}^i - 2U_j^i + U_{j+1}^i\right) - \frac{\Delta t}{2}\right.$$

$$\left.\left(M + \frac{1}{K_p}\right)U_j^i + \frac{\Delta t Gr_\theta}{2}\left(T_j^{i+1} + T_j^i\right) + \frac{\Delta t Gr_c}{2}\left(C_j^{i+1} + C_j^i\right)\right] \div \begin{bmatrix} 1 + V_j^i\dfrac{\Delta t}{2\Delta y} + \dfrac{\Delta t}{(\Delta y)^2} \\ + \dfrac{\Delta t}{2}\left(M + \dfrac{1}{K_p}\right)\end{bmatrix} \qquad \text{................(3.52)}$$

Hence equation of momentum along y axis is:

$$V_j^{i+1} = \left[V_j^i - \frac{\Delta t}{2\Delta y}V_j^i\left(-V_{j-1}^{i+1} + V_j^i - V_{j-1}^i\right) + \frac{\Delta t}{2(\Delta y)^2}\left(V_{j-1}^{i+1} + V_{j+1}^{i+1} + V_{j-1}^i - 2V_j^i + V_{j+1}^i\right) - \frac{\Delta t}{2}\left(M + \frac{1}{K_p}\right)V_j^i\right.$$

$$\left. + \frac{\Delta t Gr_\theta}{2}\left(T_j^{i+1} + T_j^i\right) + \frac{\Delta t Gr_c}{2}\left(C_j^{i+1} + C_j^i\right)\right] \div \left[1 + V_j^i\frac{\Delta t}{2\Delta y} + \frac{\Delta t}{(\Delta y)^2} + \frac{\Delta t}{2}\left(M + \frac{1}{K_p}\right)\right] \qquad \text{.........................(3.53)}$$

Equation of energy

$$\frac{T_j^{i+1} - T_j^i}{\Delta t} + V_j^i\left(\frac{T_j^{i+1} - T_{j-1}^{i+1} + T_j^i - T_{j-1}^i}{2\Delta y}\right) = \frac{1}{Pr}\left(\frac{T_{j-1}^{i+1} - 2T_j^{i+1} + T_{j+1}^{i+1} + T_{j-1}^i - 2T_j^i + T_{j+1}^i}{2(\Delta y)^2}\right)$$

$$+ Ec\left(\frac{U_j^{i+1} - U_{j-1}^{i+1} + U_j^i - U_{j-1}^i}{2\Delta y}\right)^2 + Q\left(\frac{T_j^{i+1} + T_j^i}{2}\right)$$

then,

$$\frac{T_j^{i+1}}{\Delta t} + V_j^i\frac{T_j^{i+1}}{2\Delta y} + \frac{T_j^{i+1}}{Pr(\Delta y)^2} - \frac{QT_j^{i+1}}{2} = \frac{T_j^i}{\Delta t} - \frac{V_j^i}{2\Delta y}\left(-T_{j-1}^{i+1} + T_j^i - T_{j-1}^i\right)$$

$$+ \frac{1}{2Pr(\Delta y)^2}\left(T_{j-1}^{i+1} + T_{j+1}^{i+1} + T_{j-1}^i - 2T_j^i + T_{j+1}^i\right) + Ec\left(\frac{U_j^{i+1} - U_{j-1}^{i+1} + U_j^i - U_{j-1}^i}{2\Delta y}\right)^2 + Q\left(\frac{T_j^i}{2}\right)$$

Multiplying through by Δt,

$$T_j^{i+1} + V_j^i \frac{\Delta t}{2\Delta y} T_j^{i+1} + \frac{\Delta t}{\Pr(\Delta y)^2} T_j^{i+1} - \frac{Q\Delta t}{2} T_j^{i+1} = T_j^i - \frac{\Delta t}{2\Delta y} V_j^i \left(-T_{j-1}^{i+1} + T_j^i - T_{j-1}^i\right)$$

$$+ \frac{\Delta t}{2\Pr(\Delta y)^2}\left(T_{j-1}^{i+1} + T_{j+1}^{i+1} + T_{j-1}^i - 2T_j^i + T_{j+1}^i\right) + Ec\Delta t\left(\frac{U_j^{i+1} - U_{j-1}^{i+1} + U_j^i - U_{j-1}^i}{2\Delta y}\right)^2 + Q\Delta t\left(\frac{T_j^i}{2}\right)$$

making T_j^{i+1} the subject of the formula;

$$\mathrm{T}_j^{i+1} = \Bigg[\mathrm{T}_j^i - \mathrm{V}_j^i \frac{\Delta t}{2\Delta y}\left(-\mathrm{T}_{j-1}^{i+1} + \mathrm{T}_j^i - \mathrm{T}_{j-1}^i\right) + \frac{\Delta t}{2\Pr(\Delta y)^2}\left(\mathrm{T}_{j-1}^{i+1} + \mathrm{T}_{j+1}^{i+1} + \mathrm{T}_{j-1}^i - 2\mathrm{T}_j^i + \mathrm{T}_{j+1}^i\right) +$$

$$+ \mathrm{Ec}\Delta t\left(\frac{\mathrm{U}_j^{i+1} - \mathrm{U}_{j-1}^{i+1} + \mathrm{U}_j^i - \mathrm{U}_{j-1}^i}{2\Delta y}\right)^2 + \mathrm{Q}\Delta t\left(\frac{\mathrm{T}_j^i}{2}\right)\Bigg] \div \left[1 + \frac{\mathrm{V}_j^i}{2\Delta y} + \frac{\Delta t}{\Pr(\Delta y)^2} - \frac{\mathrm{Q}\Delta t}{2}\right] \ldots\ldots\ldots\ldots(3.54)$$

Equation of concentration:

$$\frac{\mathrm{C}_j^{i+1} - \mathrm{C}_j^i}{\Delta t} + \mathrm{v}\frac{\mathrm{C}_j^{i+1} - \mathrm{C}_{j-1}^{i+1} + \mathrm{C}_j^i - \mathrm{C}_{j-1}^i}{2\Delta y} = \frac{1}{\mathrm{Sc}}\left(\frac{\mathrm{C}_{j-1}^{i+1} - 2\mathrm{C}_j^{i+1} + \mathrm{C}_{j+1}^{i+1} + \mathrm{C}_{j-1}^i - 2\mathrm{C}_j^i + \mathrm{C}_{j+1}^i}{2(\Delta y)^2}\right)$$

then,

$$\frac{\mathrm{C}_j^{i+1}}{\Delta t} + \mathrm{v}\frac{\mathrm{C}_j^{i+1}}{2\Delta y} + \frac{\mathrm{C}_j^{i+1}}{(\Delta y)^2} = \frac{\mathrm{C}_j^i}{\Delta t} - \mathrm{v}\left(\frac{-\mathrm{C}_{j-1}^{i+1} + \mathrm{C}_j^i - \mathrm{C}_{j-1}^i}{2\Delta y}\right) + \frac{1}{\mathrm{Sc}}\left(\frac{\mathrm{C}_{j-1}^{i+1} + \mathrm{C}_{j+1}^{i+1} + \mathrm{C}_{j-1}^i - 2\mathrm{C}_j^i + \mathrm{C}_{j+1}^i}{2(\Delta y)^2}\right)$$

Multiplying through by Δt

$$\mathrm{C}_j^{i+1} + \mathrm{v}\frac{\Delta t}{2\Delta y}\mathrm{C}_j^{i+1} + \frac{\Delta t}{(\Delta y)^2}\mathrm{C}_j^{i+1} = \mathrm{C}_j^i - \mathrm{v}\Delta\Delta\left(\frac{-\mathrm{C}_{j-1}^{i+1} + \mathrm{C}_j^i - \mathrm{C}_{j-1}^i}{2\Delta y}\right) + \frac{1}{\mathrm{Sc}}\Delta t\left(\frac{\mathrm{C}_{j-1}^{i+1} + \mathrm{C}_{j+1}^{i+1} + \mathrm{C}_{j-1}^i - 2\mathrm{C}_j^i + \mathrm{C}_{j+1}^i}{2(\Delta y)^2}\right)$$

making C_j^{i+1} the subject of the formula;

$$\mathrm{C}_j^{i+1} = \left[\mathrm{C}_j^i - \mathrm{v}\frac{\Delta t}{2\Delta y}\left(-\mathrm{C}_{j-1}^{i+1} + \mathrm{C}_j^i - \mathrm{C}_{j-1}^i\right) + \frac{\Delta t}{2\mathrm{Sc}(\Delta y)^2}\left(\mathrm{C}_{j-1}^{i+1} + \mathrm{C}_{j+1}^{i+1} + \mathrm{C}_{j-1}^i - 2\mathrm{C}_j^i + \mathrm{C}_{j+1}^i\right)\right]$$

$$\div\left[1 + \mathrm{v}\frac{\Delta t}{2\Delta y} + \frac{\Delta t}{(\Delta y)^2}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.55)$$

Equations 2.47, 2.48 and 2.49 are solved using a program code as shown in appendix page 55 where Matlab, a computer program has been used to generate the code.

3.3.3 Nusselt Number

The Nusselt number (Nu) is the ratio of the convective heat transfer to the conductive heat transfer. For instance if Nu =1, then there is no improvement of heat transfer by convection over conduction. If Nu=5, then rate of convective heat transfer is 5 times the rate of heat transfer if the fluid is stagnant. In our study this number is equal to the dimensionless temperature gradient at the surface and provides a measure of the convection heat transfer occurring at the surface.

$$Nu = \frac{\partial T}{\partial y}\Big|_{y=0} \quad \text{………………………………..(3.56)}$$

In non-dimensional form, the Nusselt number becomes,

$$\mathrm{Nu} = \frac{(\mathrm{T_w} - \mathrm{T}_\infty)\mathrm{U}_\infty}{\nu}\left(\frac{\partial \mathrm{T}^*}{\partial \mathrm{y}^*}\right)_{y^*=0} \quad \text{…………………………………………….(3.57)}$$

3.3.4 Skin friction

The skin friction represents the average rate of shear stress on the plate due to velocity. The skin friction at the wall is given by,

$$\tau_w = \left(\frac{\partial u}{\partial y}\right)_{y=0} \quad \text{……………………………………………………………..(3.58)}$$

In non-dimensional form, the skin friction at the wall becomes:

$$\tau_w = \frac{U_\infty{}^2}{\nu}\left(\frac{\partial u^*}{\partial y^*}\right)_{y^*=0} \quad \text{……………………………………………………….(3.59)}$$

In the next chapter, the results are analysed in form of tables and graphs and are hence discussed.

CHAPTER FOUR

RESULTS AND DISCUSSION

The effect of Mass and heat transfer on unsteady free convective flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous plate with constant injection and heat source in presence of a transverse magnetic field has been studied.

The governing equations of the flow field are solved using Finite Difference Method applying the Crank-Nicolson method. The effects of the pertinent parameters on the flow field are analysed and discussed with the help of velocity profiles, temperature profiles, and concentration distribution.

During numerical calculations, Sc = 0.62 has been chosen to represent water vapour, Pr = 0.71 to represent air at 20^0C. The results are analysed when $Gr_\theta<0$ (specifically $Gr_\theta=-30$) to represent heating of the plate and also when $Gr_\theta>0$ (specifically $Gr_\theta=30$) for cooling of the plate. This is because water vapour and air can represent humidity where temperatures inside and outside the plate may vary.

The figures below represent effects of variation of various parameters on velocity, temperature and concentration profiles.

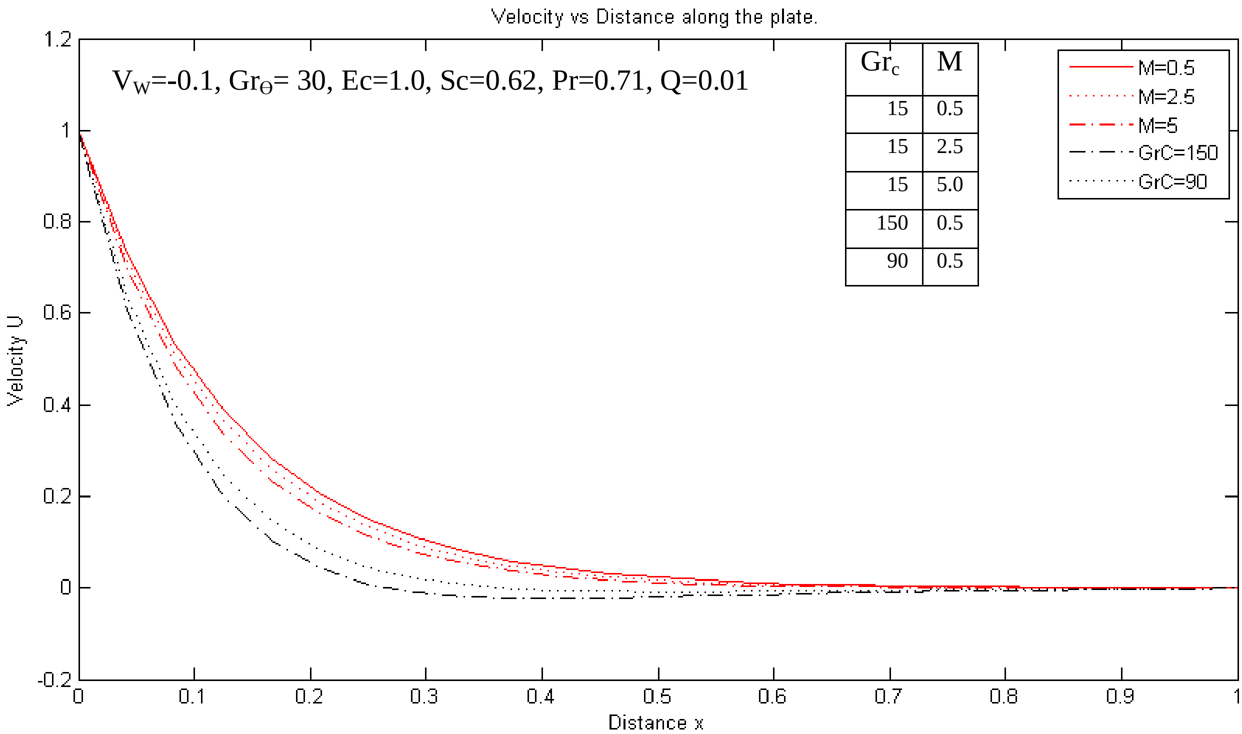

Fig 4.1: Velocity profile for Gr_θ>0 with variation of M and Gr_c

From figure 4.1, it is observed that, an increase in Hartmann number M, leads to a decrease in the velocity. This is because when M increases it means that, electromagnetic force increases and when transverse magnetic field is applied to an electrically conducting fluid, it gives rise to a force called the Lorentz force which acts against the flow if applied in the normal direction as in the present study. This resistive force has a tendency to slow down the motion of the fluid in the boundary layer. It's also observed that an increase in Grashof number for mass transfer (Gr_c) causes velocity to decrease. Gr_c defines the ratio of the species buoyancy force to the viscous hydrodynamic force. Hence its increase implies a reduction in viscous hydrodynamic force.

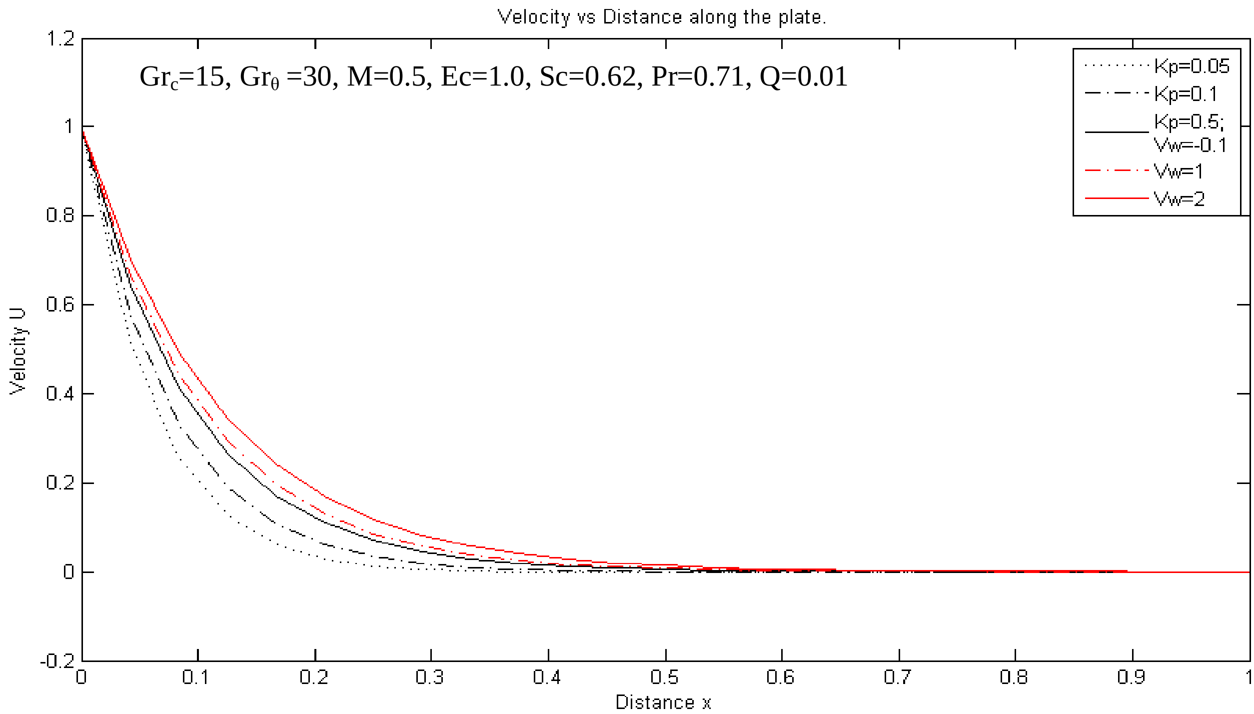

Fig 4.2: Velocity profile for Gr_θ>0 with variation of V_w and K_p

From figure 4.2, it is observed that an increase in permeability parameter K_p, increases the velocity of the flow. In this study, permeability parameter is directly proportional to the actual permeability k of the porous medium. Hence, increasing K_p decreases the resistance of the porous medium since permeability physically becomes more with an increase in K_p. This increases the velocity of the flow. It has also been observed that an increase in the magnitude of injection parameter (V_w) increases the velocity of the flow. This is because blowing destabilises the boundary layer, hence decreasing the viscous forces leading to an increase in the motion of the fluid.

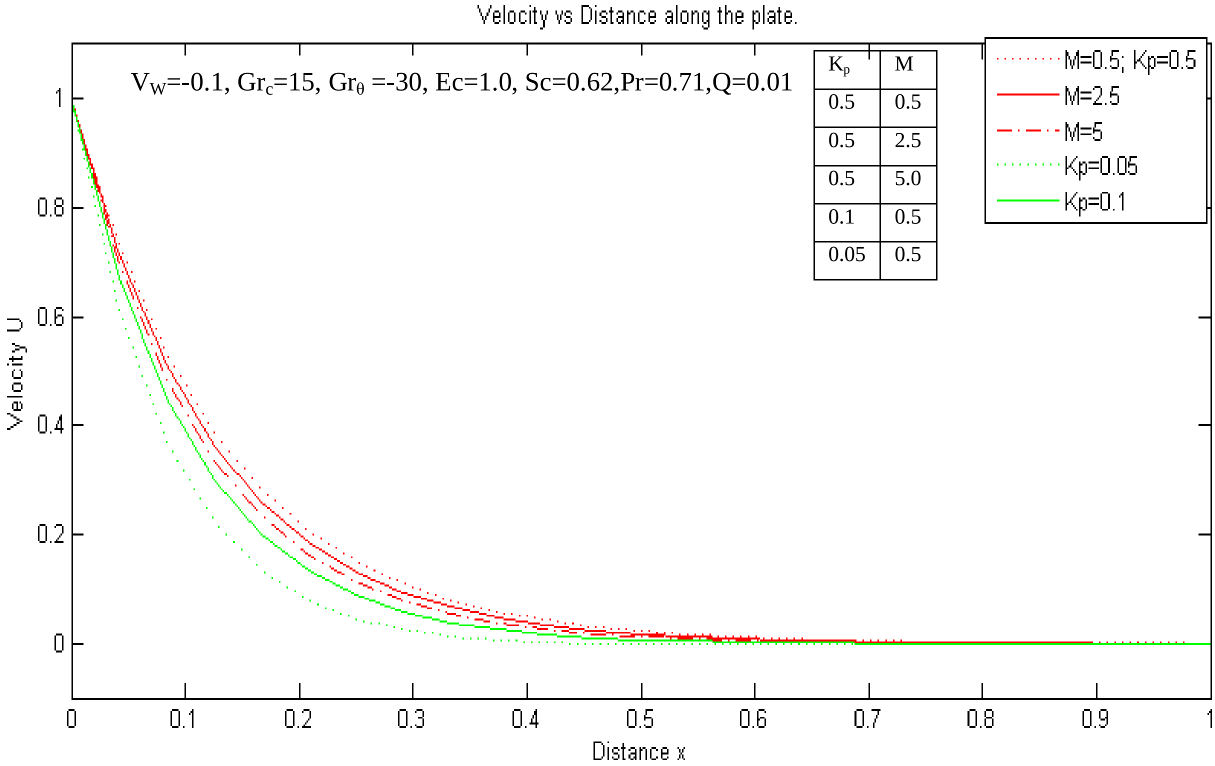

Fig 4.3: Velocity profile for Gr_θ<0 with variation of M and K_p

From figure 4.3, it is observed that increase in Hartmann number M, causes a decrease in the magnitude of the velocity profile. This is due to Lorentz force which acts against the flow hence decelerating the motion of the fluid. It has also been observed that an increase in permeability parameter K_p increases the velocity. Increasing K_p decreases the resistance of the porous medium since permeability physically becomes more with an increase in K_p. This increases the magnitude of the velocity of the flow.

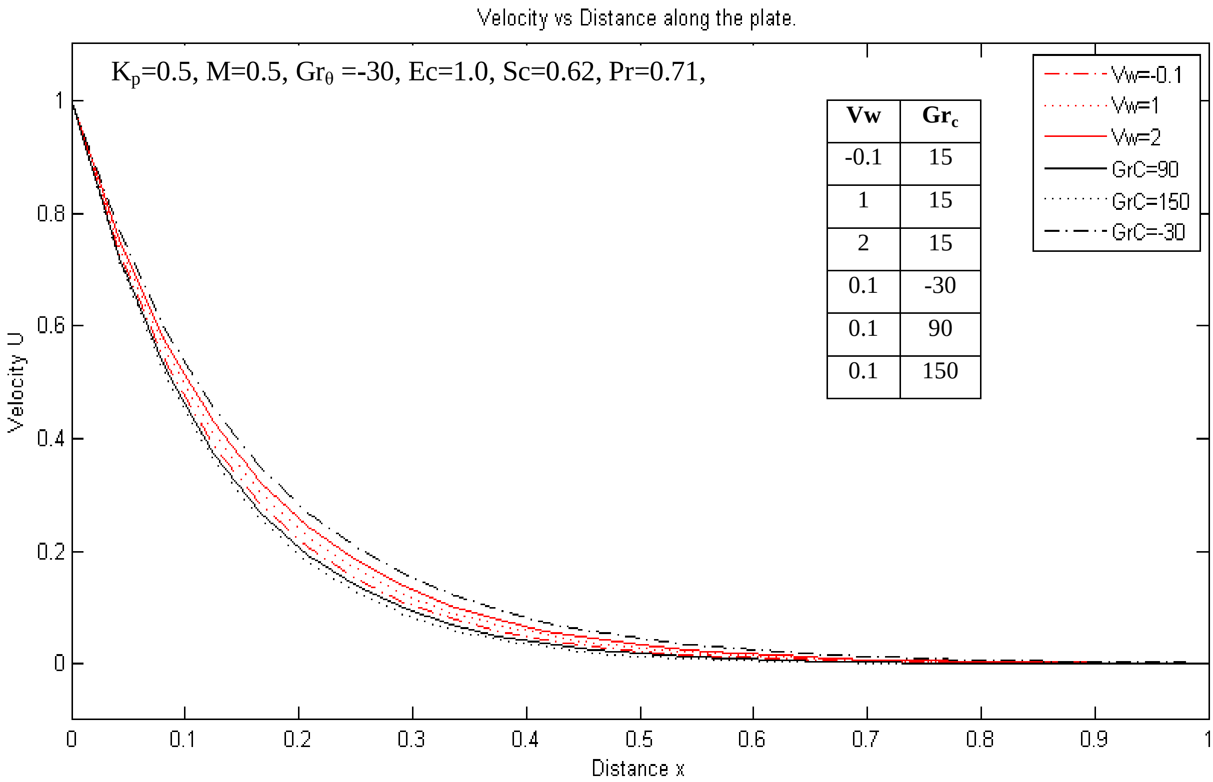

Fig 4.4: Velocity profile for $Gr_\theta<0$ with variation of V_w and Gr_c

From figure 4.4, it is observed that an increase in the magnitude of injection parameter (V_w) is found to increase the velocity of the flow. This is because injection accelerates the motion of a fluid. An increase in Grashof number for mass transfer (Gr_c) causes velocity to decrease. Gr_c defines the ratio of the species buoyancy force to the viscous hydrodynamic force. Hence its increase implies a reduction in viscous hydrodynamic force. This then causes a decrease in the velocity.

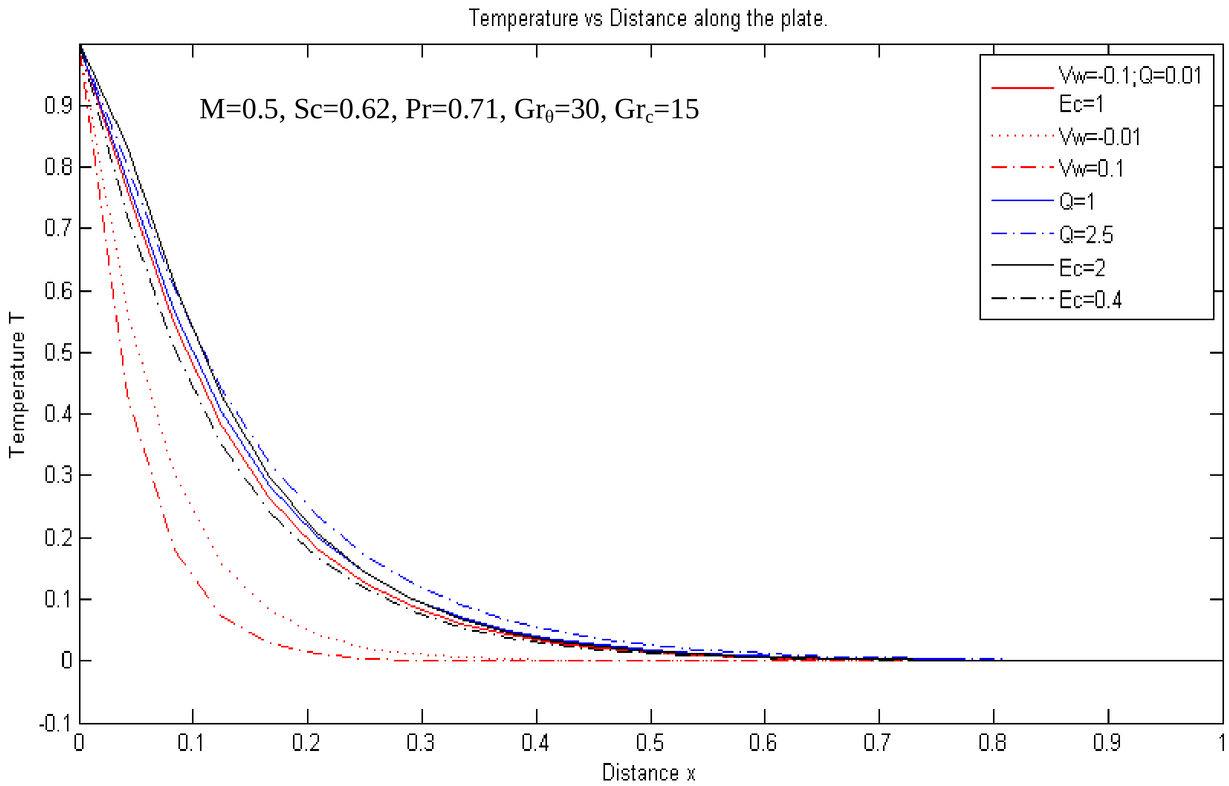

Figure 4.5: Temperature profile with variation of Q, V_w, and Ec

From figure 4.5 it is observed that an increase in the injection parameter makes the temperature of the flow to decrease. The results show that introducing injection destabilizes the temperature at the boundary layer and hence a reduction in the temperature. Also, as Eckert number (Ec) increases, temperature of the flow increases. This is because Ec expresses the relationship between the kinetic energy in the flow and the enthalpy. It embodies the conversion of kinetic energy into internal energy by work done against the viscous fluid stresses. The positive Eckert number implies cooling of the plate i.e., loss of heat from the plate to the fluid. Hence, greater viscous dissipative heat causes a rise in the temperature.

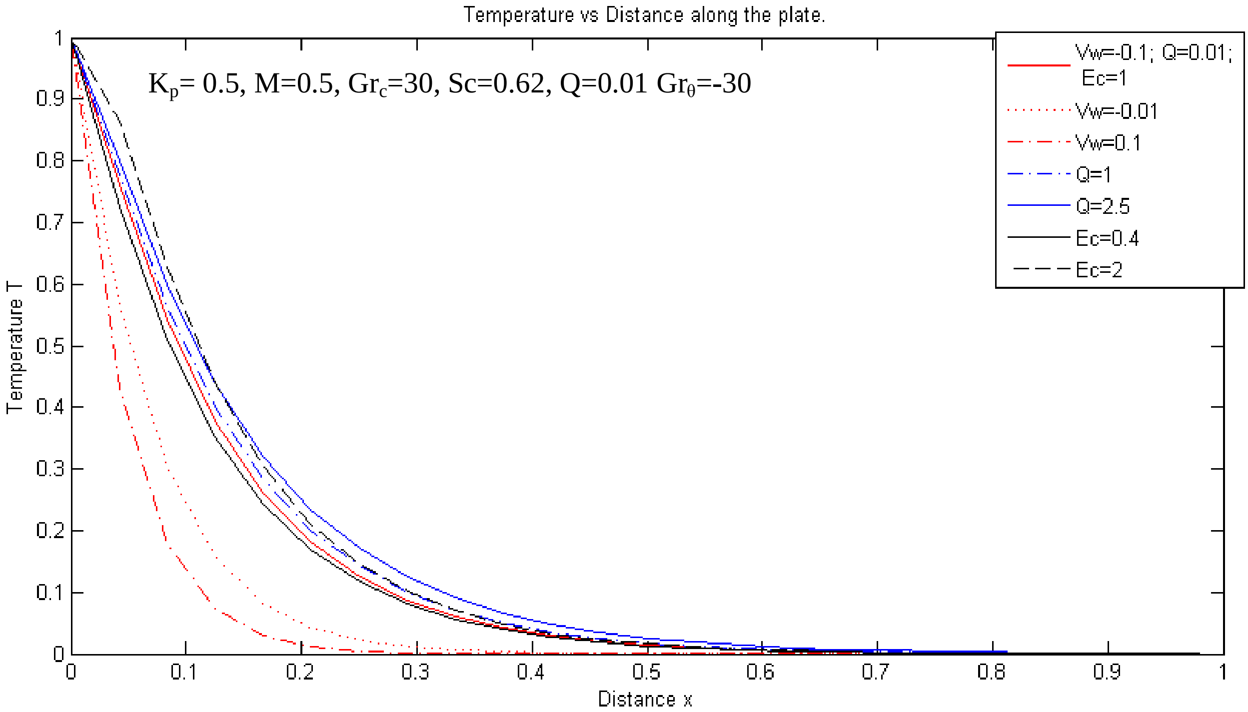

Figure 4.6: Temperature profile with variation of V_w, Q and Ec for Gr_θ<0

From figure 4.6, as heat source parameter (Q) increases the temperature of the flow increases. Increase in heat source produces a heating effect hence increase in the temperature. An increase in the injection parameter leads to a decrease in the temperature of the flow. Injecting fluid particles to a flow destabilises the temperature at the boundary layer hence, the reduction in the temperature of the flow. As Eckert number (Ec) increases, temperature of the flow increases. This is because Ec expresses the relationship between the kinetic energy in the flow and the enthalpy. It embodies the conversion of kinetic energy into internal energy by work done against the viscous fluid stresses. The positive Eckert number implies cooling of the plate i.e., loss of heat from the plate to the fluid. Hence, greater viscous dissipative heat causes a rise in the temperature.

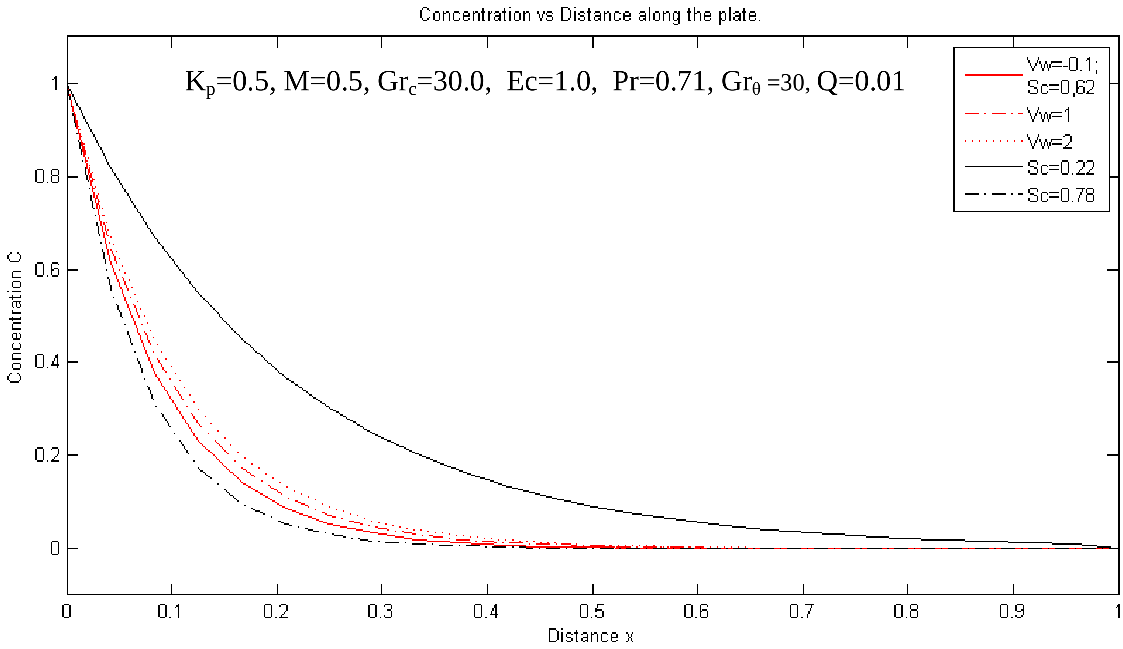

Figure 4.7: Concentration profile with variation of V_w and Sc when Gr_θ>0

From figure 4.7 it's observed that concentration field due to variation in Schmidt number (Sc) for the gases; hydrogen (Sc=0.22), water vapour (Sc=0.62) and ammonia (Sc=0.78). The concentration distribution is vastly affected by the presence of foreign species (Sc) in the flow field. The concentration distribution is found to decrease faster as the diffusing foreign species becomes heavier. Thus higher Sc leads to a faster decrease in concentration of the flow field. Also an increase in the injection parameter leads to an increase in the concentration profile.

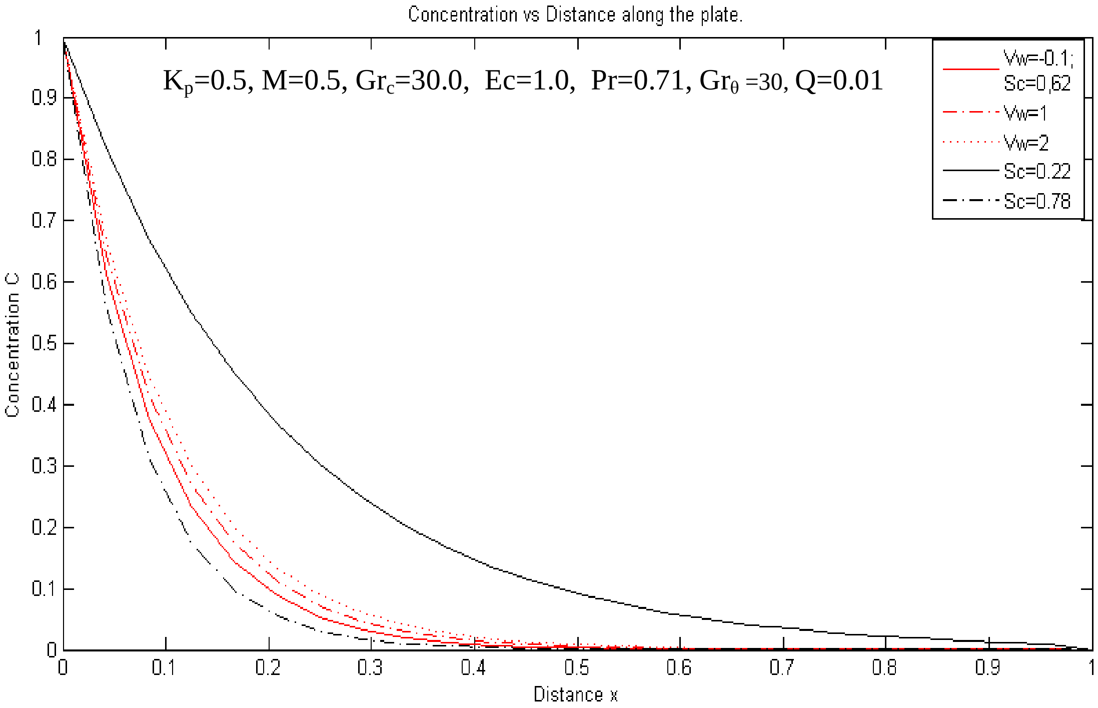

Figure 4.8: Concentration profile with variation of V_w and Sc when Gr_θ>0

From figure 4.8, it's observed that an increase in magnitude of the injection parameter leads to an increase in the concentration profile. This is so because injection destabilizes the flow of the fluid. It is also observed that an increase in Schmidt number (Sc) decreases the concentration boundary layer thickness of the flow field. The concentration distribution is vastly affected by the presence of foreign species (Sc) in the flow field. The concentration distribution is found to decrease faster as the diffusing foreign species becomes heavier. Thus higher Sc leads to a faster decrease in concentration of the flow field.

Rate of heat transfer and skin friction

The rate of heat transfer and skin friction at the wall are analysed in the tables below. This is done by varying the different flow parameters when Gr_θ is positive (heating of the fluid) and also when it's negative (cooling of the fluid)

Table 4.1: Values of skin friction (**٢)** and rate of heat transfer (Nu) at the wall for Gr_θ=30

M	Vw	Q	K_p	Pr	Ec	Sc	Gr_c	٢	**Nu**
0.5	-0.1	0.01	0.5	0.71	1.0	0.62	15	**7.480937**	**2.210481**
2.5	-0.1	0.01	0.5	0.71	1.0	0.62	15	**7.907399**	**2.062789**
5.0	-0.1	0.01	0.5	0.71	1.0	0.62	15	**8.412898**	**1.882545**
0.5	1.0	0.01	0.5	0.71	1.0	0.62	15	**6.480961**	**15.239438**
0.5	2.0	0.01	0.5	0.71	1.0	0.62	15	**5.794819**	**17.137171**
0.5	-0.1	1.00	0.5	0.71	1.0	0.62	15	**7.481550**	**1.996173**
0.5	-0.1	2.50	0.5	0.71	1.0	0.62	15	**7.482505**	**1.662497**
0.5	-0.1	0.01	0.1	0.71	1.0	0.62	15	**9.073508**	**1.639534**
0.5	-0.1	0.01	0.05	0.71	1.0	0.62	15	**10.705047**	**1.010938**
0.5	-0.1	0.01	0.5	0.015	1.0	0.62	15	**7.467723**	**6.743481**
0.5	-0.1	0.01	0.5	0.64	1.0	0.62	15	**7.479355**	**2.757300**
0.5	-0.1	0.01	0.5	0.71	0.4	0.62	15	**7.478331**	**3.089354**
0.5	-0.1	0.01	0.5	0.71	2.0	0.62	15	**7.485289**	**0.742829**
0.5	-0.1	0.01	0.5	0.71	1.0	0.22	15	**7.489135**	**2.206333**
0.5	-0.1	0.01	0.5	0.71	1.0	0.78	15	**7.478231**	**2.211746**
0.5	-0.1	0.01	0.5	0.71	1.0	0.62	-30	**7.334084**	**2.262127**
0.5	-0.1	0.01	0.5	0.71	1.0	0.62	90	**7.725698**	**2.122493**
0.5	-0.1	0.01	0.5	0.71	1.0	0.62	150	**7.921511**	**2.050380**

From the table 4.1 the following observations are made: An increase in Hartmann number (M) increases the skin friction at the wall. This is due to Lorentz force which has a tendency of slowing down the motion of the fluid in the boundary layer. Increase in M also reduces the rate of heat transfer. This is due to an increase in thickness of thermal boundary layer. An increase in Grashof number Gr_c for mass transfer increases the skin friction and decreases the rate of heat transfer. Gr_c is the ratio of the species buoyancy force to the viscous hydrodynamic force. Increase in Gr_c implies reduced viscous hydrodynamic forces that cause decrease in viscous dissipation. This translates to a decrease in the rate of heat transfer. An increase in injection Parameter (V_w) has an effect making skin friction to drop and rate of heat transfer to increase. Increase in V_w reduces the growth of the thermal boundary layer leading to an increased Nu. An increase in Eckert number (Ec) causes the skin friction to increase and Nusselt number (Nu) to drop. Increase in Ec translates to a lower value of temperature difference hence, to a reduced rate of heat transfer. Increase in Ec leads to an increase in velocity of a fluid hence increase in magnitude of skin friction. An increase in permeability parameter (K_p) causes the skin friction to drop and the rate of heat transfer to increase. In this study, permeability parameter is directly proportional to the actual permeability k of the porous medium. Therefore, increase in K_p leads to thinner temperature boundary layer, hence increase in the rate of heat transfer.

Increase in Schmidt number (Sc) causes the skin friction to decrease and the rate of heat transfer to increase. Sc embodies the ratio of momentum to the mass diffusivity. Physically it relates the relative thickness of the hydrodynamic layer and mass diffusivity. A larger value of Sc means presence of a heavier fluid hence as it increases, the rate of heat transfer will increase and skin friction to increase.

An increase in heat source parameter (Q) leads to an increase in the skin friction and Nusselt number (Nu) to drop. This is because increase in heat source enhances convection currents on the surface of the plate leading to increase in the skin friction. As Q increases, thermal boundary layer thickens hence temperature difference lowers, leading to a drop in the rate of heat transfer. An increase in Prandtl number (Pr) causes the skin friction to increase and Nusselt number (Nu) to drop. This is because Prandtl number is the ratio of viscous force to thermal force. Therefore the viscous force will increase holding the thermal force constant thus increasing the shear stress. An increase in Pr causes Nu to drop because smaller value of Pr is equivalent to an increase in the thermal conductivity of the fluid, and heat is able to diffuse away from the heated surface more rapidly for higher values of Pr. Hence in the case of smaller Prandtl numbers, the thermal boundary layer is thicker, and the rate of heat transfer is reduced.

Table 4.2: Values of skin friction (**۲**) and rate of heat transfer (Nu) at the wall for Gr_θ= -30

M	Vw	Q	K_p	Pr	Ec	Sc	Gr_c	۲	**Nu**
0.5	-0.1	0.01	0.5	0.71	1.0	0.62	15	**7.367835**	**2.251693**
2.5	-0.1	0.01	0.5	0.71	1.0	0.62	15	**7.707331**	**2.138159**
5.0	-0.1	0.01	0.5	0.71	1.0	0.62	15	**8.108252**	**2.001507**
0.5	1.0	0.01	0.5	0.71	1.0	0.62	15	**6.450017**	**15. 241675**
0.5	2.0	0.01	0.5	0.71	1.0	0.62	15	**5.776377**	**17. 137896**
0.5	-0.1	1.00	0.5	0.71	1.0	0.62	15	**7.367225**	**2.038474**
0.5	-0.1	2.50	0.5	0.71	1.0	0.62	15	**7.366275**	**1.706530**
0.5	-0.1	0.01	0.1	0.71	1.0	0.62	15	**8.629674**	**1.820225**
0.5	-0.1	0.01	0.05	0.71	1.0	0.62	15	**9.904830**	**1.364597**
0.5	-0.1	0.01	0.5	0.64	1.0	0.62	15	**7.369399**	**2.252936**
0.5	-0.1	0.01	0.5	0.015	1.0	0.62	15	**7.380927**	**6.743984**
0.5	-0.1	0.01	0.5	0.71	2.0	0.62	15	**7.363627**	**0.832242**
0.5	-0.1	0.01	0.5	0.71	0.4	0.62	15	**7.370365**	**3.105000**
0.5	-0.1	0.01	0.5	0.71	1.0	0.78	15	**7.365137**	**2.252936**
0.5	-0.1	0.01	0.5	0.71	1.0	0.22	15	**7.376010**	**2.247620**
0.5	-0.1	0.01	0.5	0.71	1.0	0.62	-30	**7.221291**	**2.302524**
0.5	-0.1	0.01	0.5	0.71	1.0	0.62	90	**7.612070**	**2.165076**
0.5	-0.1	0.01	0.5	0.71	1.0	0.62	150	**7.807453**	**2.094073**

From table 4.2, the following observations are made:

An increase in Grashof number Gr_c for mass transfer increases the skin friction and decreases the rate of heat transfer. Gr_c is the ratio of the species buoyancy force to the viscous hydrodynamic force. Increase in Gr_c implies reduced viscous hydrodynamic forces that cause decrease in viscous dissipation. This translates to a decrease in the rate of heat transfer.

An increase in Eckert number (Ec) causes the skin friction to decrease and the Nusselt number (Nu) to drop. This is because Eckert number represents conversion of kinetic energy into internal energy by work that is done against the viscous fluid stresses. When the fluid is heated, it becomes less viscous and hence less stress. A positive Eckert number implies cooling of the plate i.e. loss of heat from the plate to the fluid. Increase in Ec translates to a lower value of temperature difference. Hence, as it increases the rate of heat transfer will drop.

Increase in Schmidt number (Sc) causes the skin friction to decrease and the rate of heat transfer to increase. Sc embodies the ratio of momentum to the mass diffusivity. Physically it relates the relative thickness of the hydrodynamic layer and mass diffusivity. A larger value of Sc means presence of a heavier fluid hence decrease in skin friction, and increase in the rate of heat transfer.

An increase in Prandtl number (Pr) causes the skin friction to increase. An increase in Pr causes Nu to drop because smaller value of Pr is equivalent to an increase in the thermal conductivity of the fluid, and heat is able to diffuse away from the heated surface more rapidly for higher values of Pr. Hence in the case of smaller Prandtl numbers, the thermal boundary layer is thicker, and the rate of heat transfer is reduced.

As permeability parameter (K_p) increases, the rate of heat transfer increases and skin friction decreases. This observation is due to the fact that increase in K_p leads to thinner temperature boundary layer, hence increase in the rate of heat transfer. Increasing K_p decreases the resistance of the porous medium since permeability physically increases with an increase in K_p hence a drop in skin friction.

An increase in magnitude of injection parameter (V_w) causes the Nu to increase and skin friction to decrease. This is because V_w destabilizes the flow causing the rate of heat transfer to increase. Thermal boundary layer thickness decreases with increase in injection, leading to an increase in the rate of heat transfer.

An increase in heat source parameter (Q) causes the Nu to drop and also a decrease in skin friction. Thermal boundary layer thickens leading to lower rate of heat transfer. Since in this case there is heating of the fluid, hence when Q increases, velocity lowers leading to a decrease in skin friction.

Increase in Hartmann number (M) increases the skin friction at the wall. This is due to the magnetic pull of the Lorentz force acting on the flow field which has a tendency of slowing down the motion of the fluid in the boundary layer. Increase in M also reduces the rate of heat transfer due to an increase in thickness of thermal boundary layer.

CHAPTER FIVE

CONCLUSIONS AND RECOMMEDATIONS

5.1 CONCLUSION

A summary of effects of varying different flow parameters on the velocity, temperature and the concentration distribution of the flow field and also shear stress and the rate of heat transfer at the wall is given below.

An increase in Hartmann number (M) retards the velocity of the fluid, increases the skin friction at the wall and reduces the rate of heat transfer. Increasing Grashof number for mass transfer has an effect of retarding velocity of the flow field, increasing the skin friction and reducing the rate of heat transfer.

Increasing injection parameter accelerates the velocity of the flow field, increases the concentration and reduces the temperature of the flow field. It also increases the skin friction at the wall and decreases the rate of heat transfer. Increase in heat source parameter increases the temperature of the flow field, skin friction at the wall and reduces the rate of heat transfer.

An increase in permeability parameter accelerates the velocity and decreases the temperature of the flow field. It also reduces the skin friction and increases the rate of heat transfer at the wall. An increase in Prandtl number leads to a decrease in temperature of the flow field and it causes rate of heat transfer to increase.

An increase in Eckert number causes the temperature of the flow field to increase. The skin friction at the wall increases when $Gr_\theta>0$ and reverses when $Gr_\theta<0$ while the rate of heat transfer drops. Increase in Schmidt number has an effect of reducing the concentration of the flow field. It also increases the skin friction at the wall and the rate of heat transfer decreases.

5.2 RECOMMENDATIONS

This study has considered the effect of heat transfer on unsteady hydromagnetic free convective flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous plate in presence of constant injection and heat source. The present work can provide basis for further research by including the following considerations:

1. Flow that involves non-Newtonian fluids.
2. Flow subjected to variable magnetic field.
3. Flow with variable injection or suction.
4. Flow subjected to variable heat source.
5. A three dimensional flow.

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APPENDIX: CODES

```
Sc=0.62; Kp=0.5;
Pr = 0.71; Ec =1; M =0.5; Gr_c =15.0; Gr_theta =30.0; Vw=0.1; Q=0.01;
  ny = 25;
  nt = 500;
delY = 0.2;     %0.5
delT =0.0125 ;  %0.0125
y = linspace(0,1,ny);
U= zeros(nt,ny);
T= zeros(nt,ny);
C= zeros(nt,ny);

for J=1:ny
for I=1:nt

  T(I,1)= 1.;C(I,1)= 1. ;U(I,1) =1.; % init. conds/
   U(I,ny) = 0;  % bound. conds/
   T(I,ny) = 0;  % bound. conds/
   C(I,ny) = 0;  % bound. conds/
end
end

for J=2:ny-1
for I=2:nt-1

%C(I+1, J) =(C(I, J)-Vw*(delT/(2*delY))*(-C(I+1, J-1)+C(I, J)-C(I, J-
1))+(0.5*delT/(Sc*delY*delY))*(C(I+1, J-1) +C(I+1, J+1)+C(I, J-1)-2*C(I, J)+C(I,
J+1)))/(1+Vw*(delT/(2*delY))+delT/(delY*delY))
```

```
C(I+1, J) =(C(I, J)-Vw*(delT/(2*delY))*(-C(I+1, J-1)+C(I, J)-C(I, J-
1))+(0.5*delT/(Sc*delY*delY))*(C(I+1, J-1) +C(I+1, J+1)+C(I, J-1)-2*C(I, J)+C(I,
J+1)))/(1+Vw*(delT/(2*delY))+delT/(delY*delY));

U(I+1, J) =((U(I, J)-Vw*(delT/(2*delY))*(-U(I+1, J-1)+U(I, J)-U(I, J-1))
+0.5*delT/(delY*delY)*(U(I+1, J-1) +U(I+1, J+1)+U(I, J-1)-2*U(I, J)+U(I, J+1)) ...
        -0.5*delT*(M+1/Kp)*(U(I, J)+0.5*delT*Gr_theta*(T(I+1, J) +T(I,
J))+0.5*delT*Gr_c*(C(I+1, J) +C(I,
J))))/(1+Vw*(delT/(2*delY))+delT/(delY*delY)+0.5*delT*(M+1/Kp)));


T(I+1, J) =((T(I, J)-Vw*(delT/(2*delY))*(-T(I+1, J-1)+T(I, J)-T(I, J-1))
+0.5*delT/(Pr*delY*delY)*(T(I+1, J-1) +T(I+1, J+1)+T(I, J-1)-2*T(I, J)+T(I, J+1))
+ ...
        +0.5*delT*Ec*(1/(4*delY*delY))*(U(I+1, J)-U(I+1, J-1)+U(I, J)-U(I, J-1)
)*(U(I+1, J)-U(I+1, J-1)+U(I, J)-U(I, J-1))+ Q*delT*0.5*T(I,
J)))/(1+Vw/(2*delY)+delT/(Pr*delY*delY)-0.5*delT*Q);

 Tau_y=(5/6)*(25*U(40,1)-48*U(400,2)+36*U(400,3)-16*U(400,4)+3*U(400,5));
 %Nu =(5/6)*(25*T(40,1)-48*T(400,2)+36*T(400,3)-16*T(400,4)+3*T(400,5));
end
end

fprintf('%s,\t%s,\t%s,\t%s,\t%s,\t%s,\t%s,\t%s,\t%s,\t%s \n','Tau_y','Sc', 'Kp', 'Pr',
'Ec','M','Gr_c', 'Gr_theta', 'Vw', 'Q');
fprintf('%f,\t%d,\t%d,\t%d,\t%d ,\t%d,\t%d,\t%d,\t%d,\t%d\n',Tau_y,Sc,Kp, Pr,
Ec,M,Gr_c, Gr_theta, Vw, Q);          %shear stress
%fprintf('%s,\t%s,\t%s,\t%s,\t%s,\t%s,\t%s,\t%s,\t%s,\t%s \n','Nu','Sc', 'Kp', 'Pr',
'Ec','M','Gr_c', 'Gr_theta', 'Vw', 'Q');
%fprintf('%f,\t%d,\t%d,\t%d,\t%d ,\t%d,\t%d,\t%d,\t%d,\t%d\n',Nu,Sc,Kp, Pr,
Ec,M,Gr_c, Gr_theta, Vw, Q);          %shear stress
figure(1)
```

```
grid off
plot(y,U(400,:),'-.');
title('Velocity vs Distance along the plate.');
xlabel('Distance x');
ylabel('Velocity U');
hold on

figure(2)
grid off
plot(y,T(400,:),'-.');
title('Temperature vs Distance along the plate.');
xlabel('Distance x');
ylabel('Temperature T');
hold on

figure(3)
grid off
plot(y,C(400,:),'-.');
title('Concentration vs Distance along the plate.');
xlabel('Distance x');
ylabel('Concentration C');
hold on
```