THE EFFECT OF TEMPERATURE DEPENDENT VISCOSITY ON MAGNETOHYDRODYNAMIC FLOW PAST A CONTINUOUS MOVING SURFACE

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The Effect of Temperature Dependent Viscosity on Magnetohydrodynamic Flow Past a Continuous Moving Surface

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DECLARATION

This research thesis is my original work and has not in part or whole been presented in any other University for a degree award.

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This research thesis has been submitted for examination with our approval as University supervisors.

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DEDICATION

This thesis is dedicated to my husband and best friend Peter Ndiritu for his support both financially and emotionally, my children Yvonne Millie, Louis Ndebu and Wamaitha Ndiritu for their patience, encouragement as well as inspiration.

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LIST OF ABBREVIATIONS AND ACRONYMS

FDM	Finite Difference Method
MHD	Magneto hydrodynamics
HOT	Higher Order Terms

LIST OF NOMENCLATURE

$\vec{E}$	Electrical intensity vector (V/m)
$\vec{F}$	Body force vector (N)
E	Unit charge (C)
L	Characteristic length (m)
$\vec{J}$	Current density vector (Am^{-2})
P	Pressure force vector (Nm^{-2})
U	Characteristic velocity (ms^{-1})
t*	Dimensional Time (S)
K	Thermal conductivity ($Wm^{-1}k^{-1}$)
$\vec{q}$	Velocity vector (ms^{-1})
$\vec{B}$	Magnetic field vector (Wbm^{-2})
$\vec{D}$	Electric displacement vector (cm^{-2})
$\vec{H}$	Magnetic field intensity vector (Wbm^{-2})
$\vec{i}, \vec{j}, \vec{k}$	Unit vectors in the x, y and z directions respectively
u, v, w	Components of velocity vector q
$\vec{F_e}$	Electromagnetic force (kgm^{-2})
G	Acceleration due to gravity (ms^{-2})
$\frac{D}{Dt}$	Material derivative $\frac{D}{Dt}\left[\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}+w\frac{\partial}{\partial z}\right]$
Q	Amount of heat added to the system (Nm)

H	Dimensional distance of the plate(m)
T	General fluid temperature (K)
C_p	Specific heat at constant pressure (Jkg$-^1$ K$-^1$)
U_∞	Free stream fluid velocity (ms$-^1$)
T_∞	Characteristic free stream temperature (K)
θ	Dimensionless fluid temperature (K)
u, v, w	Dimensionless fluid velocity ms^{-1}
x, y, z	Dimensionless Cartesian coordinates
T	Dimensionless time
E_c	Eckert number $\left\{= \frac{U^2}{C_p(T-T_\infty)}\right\}$
Pr	Prandtl number $\left(= \frac{\mu C_p}{k}\right)$
Rm	Magnetic Reynolds number($=\sigma\mu_c Lu$)
Nu	Nusset number ($=\frac{hL}{k}$)
S	Magnetic force number $\left(= \frac{H_O}{L}\frac{\sqrt{\mu c}}{\rho}\right)$
M	Magnetic Parameter $\left(= \sqrt{\frac{\sigma H o^2 v}{\mu U_m^2/v}}\right)$
µ	Coefficient of viscosity, kg/ms
γ	Kinematic Viscosity ($m^2 s^{-1}$)
ρ	Fluid density, kg/m^3
ρ_e	Electrical charge density (cm^{-2})
σ	Electrical conductivity ($\Omega^{-1}m^{-1}$)

$\boldsymbol{\mu_e}$	Magnetic permeability (Hm^{-1})
$\Delta \boldsymbol{t}, \Delta \boldsymbol{y}, \Delta \mathbf{z}$	Time and distance intervals respectively (s, m)
$\Delta \boldsymbol{T}$	Temperature change (K)
$\boldsymbol{\nabla}$	Gradient operator $\left(= i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)$
$\boldsymbol{\nabla}^2$	Laplacian operator $\left(= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$
Ø	Viscous dissipation function (s^2)
Σ	Electrical conductivity (Ω^{-1} m^{-1})

ABSTRACT

The effect of temperature dependent viscosity on magneto hydrodynamic flow of a viscous incompressible fluid past a continuous moving surface has been studied. A steady, two dimensional laminar flow past a continuous moving surface with uniform surface temperature T_w and velocity U_w moving axially through an electrically conducting fluid has been considered. The x-axis runs along the continuous surface in the direction of the motion and the y-axis is perpendicular to it. The magnetic field is applied along the y-axis. The governing boundary layer equations have been transformed into non dimensional form using a set of dimensionless variables. The resulting non- linear system of partial differential equations governing the flow have been solved numerically by employing the finite difference method. The equations have been solved using Matlab software. The effect of varying various parameters on the velocity and temperature profiles has been obtained. This has been followed by graphical representation of the results. The observations have been discussed. A change in various parameters have been observed to increase, decrease or have no effect on skin friction coefficient, the rate of heat transfer, velocity and temperature profile on a continuous moving surface. It was observed that when magnetic parameter was increased there was a decrease in velocity. Temperature also decreased with increase of magnetic parameter.

CHAPTER ONE
INTRODUCTION AND LITERATURE REVIEW

1.1 Introduction

A fluid is a substance that is subject to deformation upon application of any shear force or stress. Fluid mechanics is the study of fluid motion and forces that cause the motion. Fluid flows are conventionally described as compressible only when the associated pressure and temperature changes are sufficiently large to cause density changes. When heat is applied to a fluid flow, then one seeks to study heat transfer. Heat transfer is the science that seeks to predict the energy transfer taking place between material bodies as a result of temperature difference.

From everyday experience, it is common knowledge that viscosity of fluids varies with temperature. Honey and syrups can be made to flow more readily by heating. Whenever there is a velocity gradient across the fluid's flow path frictional forces arise between adjacent fluid particles due to the viscosity of the fluid. The viscosity of the fluid depends on temperature and pressure. In general, the viscosity of a simple liquid decreases with increasing temperature and vice versa. As temperature increases, the averages speed of the molecules in a liquid increase. Thus as temperature increases, the average intermolecular forces decrease. The temperature dependence of liquid viscosity is the phenomenon by which liquid viscosity tends to decrease. Fluidity tends to increase as its temperature increases.

The theory of continuous surface heat transfer has had considerable progress in the last few decades. Important practical applications in industrial manufacturing process include various branches of engineering. For example MHD flows have applications in regulating fluid flow through reactors so as to maintain a uniform temperature throughout the bed of the reactors. The earth has its own magnetic field, therefore all activities taking place on earth involve an interaction with magnetic field. Many investigators have considered the flow problem of magnetohydrodynamics (MHD) of temperature dependent viscosity past a continuous moving surface Elbashbeshy and Bazid (2000).

1.2 Magneto-hydrodynamics

Magneto-hydrodynamics (MHD) is the science of the motion of electrically conducting fluids in presence of magnetic fields. MHD studies the dynamics of the interaction of electrically conducting fluids and the electromagnetic field. The fluids can be ionized gases commonly known as plasma. The flow of an electrically conducting fluid under a magnetic field in general gives rise to induced electric currents. The induced currents flow in the direction perpendicular to both the magnetic field and the direction of the motion of the fluid. However, the induced currents also generate their own induced magnetic field, which in turn affects the original magnetic field.

1.3 Velocity boundary layer

Velocity boundary layer arises as a result of the velocity difference between the fluid particles adjacent to a solid surface and those in the free stream. The fluid particles adjacent to the solid surface acquire the velocity of the surface. Hence, the assumption of the no slips condition. The latter is a physical requirement that the fluid and solid have equal velocity at that interface. Thus the flow velocity of a fluid is retarded by a fixed solid surface, and a finite slow moving boundary layer is formed. For a viscous fluid, velocity boundary layer thickness is defined as the perpendicular distance, measured away from the solid surface, where the velocity of the fluid becomes 0.99 of the free stream velocity. As the fluid moves past the surface of a solid body, collision of the fluid molecules within the fluid with those molecules adjacent to the solid surface reduce the kinetic energy of the molecules that are further away from the solid fluid interfaces. Thus a relatively thin layer of fluid is formed near the solid fluid interfaces in which there is usually a rapid change of velocity from zero to the freestream value. This is the layer referred to as the velocity boundary layer.

1.4 Thermal boundary layer

When temperature difference exists between the solid-fluid interface and the fluid in the free stream, a thermal boundary layer is formed. The fluid particles in contact with the plate's body acquire the temperature of the interface. If the temperature of the interface is higher than that of the ambient fluid, the kinetic energy of the molecules of the adjacent fluid particles increases. Those particles exchange heat energy with those fluid particles in the adjacent fluid layers and temperature gradients develop in the fluid. The region in the fluid in which this temperature gradient exists is the thermal boundary layer.

1.5 Significance of the boundary layer

The velocity boundary layer is associated with the presence of the velocity gradients and shear strength. Thermal boundary layer is associated with the temperature gradient and heat transfer. Fluid flowing through porous media may cause the formation of the velocity and temperature boundary layers. The physical significance of the boundary layer is that it is the region that determines the magnitude of the service friction and connective heat transfer.

1.6 Viscosity

Viscocity is the resistance set up due to shear stresses within the fluid particles and the shear stresses between the fluid particles and the solid surface of a fluid flowing past a solid body. As fluid exerts a shear stress on the boundary, the boundary exerts an equal and opposite force on the fluid called shear resistance (frictional drag). Drag coefficient always depends on the Reynolds number (Re) and the shape of the body. The work done against viscous effects usually causes fluid flow, consequently the energy spent in doing so is converted to heat. At low values of Reynolds number, the fluid is highly viscous causing deformation drag, the fluid is deformed in a wide zone around the body which brings about pressure force and frictional force. At large values of Reynolds number, the fluid is less viscous. For example in water and air, the viscous effect is limited to the boundary layer thickness. In this case deformation drag is exclusively friction drag. The shear force exerted on the surface of the body due to the formation of boundary layer results into friction drag.

1.7 Steady and unsteady flows

Fluid flow can be classified as either steady or unsteady. The flow is said to be steady if the fluid flow variables such as velocity, applied magnetic field and temperature are independent of time. If on the other hand the flow variables are dependent on time the flow is said to be unsteady.

1.8 Laminar and turbulent flows

Laminar fluid flow is the motion of the fluid particles in an orderly manner with all particles moving in straight lines parallel to the boundary walls. The particles do not encounter disturbance on their path. Turbulence in fluid flow occurs when a flowing fluid suddenly encounters a disturbance or a force. As a result the fluid particles move in a disorderly manner with different velocities and energies. The shape of the velocity curve (the velocity profile across any section of the flow channel) depends upon whether the flow is laminar or turbulent. For turbulent flow in a pipe, a fairly flat velocity distribution exists across the section of the flow field, with the result that the entire fluid flows at a given single value. If the flow is lamina, the shape is parabolic with the maximum velocity at the centre being about twice the average velocity in the pipe.

1.9 Model and prototype

A prototype is an actual or original object whereas a model is an imitation of the actual object constructed in such a way as to include all the technical characteristics of the actual object. For example, if a pump for corrosive liquids is being developed, several models of the actual pump are needed for accelerated life tests with different chemicals. The results of these tests are compared and the desired pump characteristics are compiled and used to construct the actual pump. Such imitations used for testing are called models. In order to analyse the governing equations in MHD flow, a method of developing the flow model is adopted. The fact that the fluid motion in the model and prototype flow can be compared using non-dimensional parameters is an important tool. These non-dimensional parameters are obtained by non-dimensionalising the governing equations. These non-dimensional parameters and the governing equations have been discussed in chapter two.

1.10 Literature review

The study of temperature distribution and heat transfer is of great importance to all branches of engineering and science for its almost universal occurrence. The study of MHD started in the early 1830s with Faraday. He experimented by passing an electrically conducting fluid between poles of a magnet in a vacuum glass.

Since the work of Sakiadis (1961) various aspects, of the problem of MHD and heat transfer have been investigated by many authors. Ali (1994) has reported flow and heat characteristics on a stretched surface subject to power-law velocity and temperature distributions. The direction and amount of heat flow were found to be dependent on the magnitude of n, temperature parameter and m, velocity parameter for the same Prandtl number. Nusselt number increases with increasing m. Mostafa (2009) analyzed the influence of radiation and temperature dependent viscosity on the problem of unsteady

MHD flow and heat transfer past an infinite vertical porous plate. His findings were that increase in Eckert number and decreasing viscosity leads to a rise in the velocity. Increase in the magnetic or radiation parameters is associated with a decrease in velocity.

Seddeek and Almushigeh (2010) he studied the effects of radiation and variable viscosity on MHD free convective flow and mass transfer over stretching sheet with chemical reaction. The findings were that the Magnetic parameter slows down the motion of the fluid in the boundary layer and increases its temperature and concentration. Velocity and concentration increase with decreasing the Schmidt number Sc. Temperature increases as Sc increases.

In all the above mentioned studies the viscosity of the fluid is assumed to be constant. However, it is known that this physical property may change significantly with temperature. To accurately predict the flow behavior, it is necessary to take into account this variation of viscosity. From result of Elbashbeshy and Ibrahim (1993) studied steady free convection flow with variable viscosity and thermal diffusivity along a vertical plate. It was observed that both the temperature and velocity of air increase as the thermal diffusivity increases. For water it was found that the temperature decreases as viscosity decreases. Elbashbeshy (2000) studied free convection flow with variable viscosity and thermal diffusivity along a vertical plate in the presence of the magnetic field. Numerical result show that the skin friction on the plate decreases by the effect of the magnetic field whereas the heat transfer increases by increasing the magnetic field.

Klemp *et al*. (1996) studied the effect of temperature dependent viscosity on the entrance flow in a channel in the hydrodynamic case. Attia and Kotb (1996) studied the steady MHD fully developed flow and heat transfer between two parallel plates with temperature-dependent viscosity. Later, Attia (1999) has extended the problem to the transient state.

Hazem *et al* (2006) studied the influence of temperature dependent viscosity on the MHD coulette flow of dusty fluid with heat transfer. The viscosity was assumed to vary exponentially with temperature and the joule and viscous dissipations were taken into consideration .The result indicated a crossover of the temperature curves due to the variation of the viscosity parameter and the influence of the magnetic field in the suppression of such crossover. Modathe *et al* (2012) studied variable viscosity effect on heat transfer over a continuous moving surface with variable internal heat generation in micro polar fluids. The result indicated that as the viscosity variation and heat source parameters increase the rate of heat transfer increases. This also leads to decrease in local skin friction coefficient for both gases and liquids. Elbashbeshy and Bazid (2000) investigated the effect of temperature dependent viscosity on heat transfer over a continuous moving surface. The results indicated that the skin friction decrease with increase in the viscosity parameter. Velocity increases with a decrease of viscosity parameter and the temperature increases with an increase of viscosity parameter. Raja *et al* (2012) studied the effects of variable viscosity and thermal conductivity on an unsteady two dimensional laminar flow of a viscous incompressible electrically conducting fluid past a semi-infinite vertical plate taking into account the mass transfer. The results indicated that the increase in the applied magnetics intensity contributes to the decrease in the velocity. The effect of Prandtl number on the velocity profiles is observed to increase as velocity decreases. It is also observed that as the Prandtl number increases, the temperature in the field medium decreases. Also skin friction decreases as magnetic parameter increases. Thiagarajan *et al* (2013) studied the nonlinear MHD boundary layer flow and heat transfer over a power-law stretching plate with free stream pressure gradient in the presence of variable viscosity, thermal conductivity and transverse variable magnetic field is considered. The results show that as magnetic parameter increases velocity decreases and temperature increases which physically conveys the fact that the effect of magnetic field is to enhance the temperature. Pradesh (2013) studied the steady boundary layer convective flow and heat transfer of an incompressible viscous electrically conducting fluid over a continuously moving vertical infinite plate with uniform suction and heat flux in porous medium, taking account of the effects of the variable viscosity. It was observed that the velocity increases as the viscosity of air or porous parameter increases whereas velocity decreases when Schmidt number increases. The present study investigates the effect of temperature dependent viscosity on magneto-hydrodynamic flow past a continuous moving surface. The flow is a steady two dimensional laminar flow on a continuously moving semi-infinite surface with uniform surface temperature and velocity. The fluid is considered to flow in the x-direction and the magnetic field is applied in the y-direction.

1.11 Statement of the problem

In the cited studies, the heat characteristics on a moving surface subject to velocity and temperature distribution in MHD flow has received little attention. A lot has been done with regard to heat transfer but less has been done in regard to the effects of viscosity/temperature parameters, skin friction coefficient, velocity and temperature gradient to MHD flow. The present study has investigated the effect of dimensionless parameters and generated heat on the rate of heat transfer, the velocity and temperature profiles of MHD flow problem past a continuous moving surface. This study also proposes to analyze the effect of temperature on the fluid viscosity.

1.12 Hypothesis

There exists no relationship between the effects of temperature dependent viscosity on velocity and temperature profiles.

1.13 Objectives

1.13.1 General Objective

To study MHD heat transfer characteristics of temperature dependent viscosity past a continuous moving surface.

1.13.2 Specific Objectives

1. To determine the velocity distribution of MHD fluid flow past a continuous moving surface.
2. To determine the effect of skin friction and dimensionless parameters on MHD fluid flow past a continuous moving surface.
3. To determine the effects of heat generation on the viscosity.

1.14 Justification

Clearly the variation of the viscosity with temperature is an interesting macroscopic physical phenomenon in fluid mechanics. In spite of its importance in many applications this effect has received rather little attention. The present work has studied the effects of temperature dependent viscosity on MHD flow past a continuous moving surface. Equipment and machines whose parts come in contact with fluid experience degradation. As a result, cost of maintenance is encountered on daily basis. Heat produced due to viscosity on the body surfaces has led to the degradation of equipments and high cost of maintenance. Changes in temperature affect the viscosity of the fluid, thus the need to design equipments with bodies that can withstand such variations. This study of the flow of a viscous incompressible electrically conducting fluid in he presence of a magnetic field is motivated by several important problems of geophysical and astrophysical interest and fluid engineering. Astrophysicists encounter MHD phenomena in the interactions of conducting fluids and magnetic fields that are present in and around heavenly bodies

CHAPTER TWO
GOVERNING EQUATIONS

2.1 Introduction

In this chapter, we have presented the equations governing a steady flow of an incompressible and electrically conducting fluid past a continuous moving surface in the presence of a magnetic field. The assumptions made in this problem are stated. The governing equations considered include mass conservation equation, momentum conservation equation and equation of energy. They have been subjected to the flow conditions of our problem and simplified. The description of the flow and dimensional analysis of equations that govern this fluid flow problem has been obtained. A method of solution using finite differences has then been discussed and the difference equations for this study presented.

2.2 Assumptions

The following assumptions were made for this research problem:

(i) All velocities are small compared with that of light i.e. ($\frac{V^2}{C^2} \ll 1$)

(ii) Flow is restricted to laminar domain i.e. the region being considered is the laminar boundary layer.

(iii) Fluid is incompressible

(iv) Fluid has constant thermal conductivity, constant electrical conductivity.

(v) There is no external applied electric field thus E=0

(vi) Fluid flow is steady (the fluid flow is time independent)

2.3 Governing Equations

The equations governing the fluid flows of any kind are based on general laws of conservation of mass, momentum and energy. They are modified to perfectly suit a particular fluid flow.

2.3.1 Maxwell's Electromagnetic Equations

These equations give the relationship between E the electric field intensity, B the magnetic induction vector, D the electric displacement, H magnetic field intensity, J the conduction current density vector and the charge density ρ_e these are;

$$\left.\begin{array}{l} \nabla \times \hat{H} = \hat{J} \\ \nabla \cdot \hat{B} = 0 \\ \nabla \times \hat{E} = -\dfrac{\partial \hat{B}}{\partial t} \end{array}\right\} \qquad (2.0)$$

2.3.2 Force on an electric charge

When a unit electric charge (e) moves with velocity (q) in a region comprising of an electric field E and magnetic field B, it experiences two types of forces; the electric force E, and magnetic forces $(\overrightarrow{q} \times \vec{B})$. The total electromagnetic force F_e on a unit electric charge is given by Lorentz's equation as the sum of these two forces, and is given by the equation:-

$$F_e = \vec{E} + \vec{q} \times \vec{B}$$

(2.1)

The force F_e is known as Lorentz's force and is a force that acts on the fluid particles.

Experiments show that this force acts in a direction perpendicular to both $\vec{J}$and $\vec{B}$ and is proportional to their respective magnitudes. The generalized Ohm's law can be written as:

$$J = \sigma[\vec{E} + \vec{q} \times \vec{B}] + \rho_e \vec{q}$$

(2.2)

2.2.3 Equation of continuity

The equation of continuity is a mathematical statement in any process where the rate at which mass transfer entering a system is equals to the rate at which mass leaves the system. This equation combines the law of mass conservation and the transport theorem. It arises from the fundamental preposition that matter is neither created nor destroyed under normal conditions and that the flow is continuous. The general expression representing mass conservation is given by

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}.(\rho\vec{u}) = 0$$

(2.3)

Where ρ and $\vec{u}$ are fluid's density and the fluid's velocity respectively.

For an incompressible two dimensional fluid flow,

w=0 and $\frac{\partial \rho}{\partial t}$ = 0 since density is assumed a constant hence equation (2.3) reduces to

$$\frac{du}{dx} + \frac{dv}{dy} = 0$$

(2.4)

2.3.4 Equation of conservation of momentum

The law of conservation of momentum postulates that the sum of all the resultant forces is equal to the rate of change of momentum. The momentum of a body is defined as the product of its mass and velocity. From this postulate it follows that on application of a force to an incompressible fluid mass, its velocity changes. The unit of momentum in SI units is a kilogram meter per second (kgm/s). When two bodies having different masses are acted upon by the same force for the same time, they attain different velocities but gain equal momentum. This important connection between force and momentum was recognized by Sir Isaac Newton and led to the formulation of the Newton's second law of motion which states that the rate of change of momentum of a body is proportional to the applied force and takes place in the direction in which the forces acts. In fluid flow, the rate of change of momentum is equal to the sum of the forces, acting on the fluid flow. For us to be able to consider all the forces taking effect in hydro magnetic flow, we first discuss electromagnetic force which acts on the fluid particles .The application of a magnetic field (B) to a conducting fluid in motion causes the formation of induced electric current (J). The induced current interacts with the externally applied magnetic field resulting in the damping of the flow field by the Lorentz force. An electric charge, e, moving in an electromagnetic field experiences an electric force E and a magnetic force q and B. The resultant force on the charge e, is the sum of the two forces and is given by Lorentz's equation which is expressed as

$$F_e = \rho(\vec{E} + \vec{J} \times \vec{B}) \tag{2.5}$$

The assumption is that there is no externally applied electric field and hence E=0, equation (2.5) is reduced to

$$F_e = \rho(\vec{J} \times \vec{B}) \tag{2.6}$$

The momentum equation governing the flow of an electrically conducting fluid flowing past a continuous moving surface can thus be written as:

$$\rho\left[\frac{\partial\vec{q}}{\partial t}+(\vec{q}\ .\ \nabla)\vec{q}\right]= -\nabla P+ \mu\nabla^2\vec{q}+\vec{J}\times\ \vec{B}+\ \rho\vec{f}$$

(2.7)

The first term on the LHS equation (2.8) represents the temporal acceleration and the second term the convective acceleration. The first term on left hand side vanishes since we are considering a steady flow. On the R.H.S, the first term is the pressure gradient force, which also vanishes because the fluid is stationary. The second term is the viscous force, and the third term is the Lorentz force and the last one is body force. Accordingly to Holman (1992), the latter two forces replace the body force. Since the flow is horizontal and gravitational force acts in a vertical direction downwards, the gravitational force term in equation (2.8) can be ignored and the equation becomes:-

$$(\vec{q}.\nabla)\vec{q}=\frac{1}{\rho}\left[\mu\nabla^2\vec{q}+\vec{J}\times\ \vec{B}\right]$$

(2.8)

2.3.5 Equation of conservation energy

Early in the twentieth century Albert Einstein put forward new ideas regarding the relationship between space, time and energy which have come to be known as the theory of relativity. It had long been accepted by scientists that matter cannot be destroyed. This assumption is expressed in the law of conservation of matter. Scientists have equally accepted that energy can neither be created nor destroyed but can be transformed from one form to another. There are at least six forms of energy namely mechanical, electrical, chemicals, nuclear, and electromagnetic and heat or thermal energy. The Mathematical formulation of equation of conservation of thermal energy is derived from the first law of thermo hydrodynamics. This law states that the amount of heat added to a system dQ is equal to the change in internal energy dE plus the work done dW and is expressed as.

$$dQ = dE + dW$$

(2.9)

Considering the flow of an incompressible fluid with constant thermal conductivity K, thermal energy equation is expressed as

$$\rho c_p \frac{DT}{Dt} = K\nabla^2 T + \mu\emptyset$$

(2.10)

In the spirit of Boussinesq approximation, it is supposed that the fluid has a constant heat capacity per unit volume ρc_p, implying that $\rho c_p \frac{DT}{Dt}$ is equal to the rate of heating per unit volume of a fluid particle. Thermal conductivity K of the fluid is the rate of flow of heat through the fluid per unit cross sectional area per unit temperature gradient. $\mu\emptyset$ is the internal heating due to viscous dissipation and $\frac{DT}{Dt}$ is the material derivative of the absolute temperatures T of the fluid B.

The viscous dissipation function $\emptyset$ for a two dimensional flow is given by Jalal M.J (2006)

$$\emptyset = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2$$

(2.11)

which reduces to $\emptyset = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{dx}\right)^2$ because the surface is semi infinite along the x-axis direction and hence $\frac{\partial u}{\partial x} = 0$ and $\frac{\partial u}{\partial y} = 0$. Consequently equation (2.11) becomes

$$\rho c_p \frac{DT}{Dt} = K\nabla^2 T + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2$$

(2.12)

2.3 Mathematical formulation

We consider a steady two dimensional MHD laminar boundary layer flow of an incompressible fluid past a continuous stretching semi-infinite surface with uniform surface temperature T_w and velocity $U_w$moving axially through a stationary fluid. The-axis runs along the continuous surface in the direction of the motion and the y-axis is perpendicular to it. Flow is horizontal, thus there is no pressure gradient in the fluid. Additionally, a magnetic field of strength B_0 is applied normal to the plate.

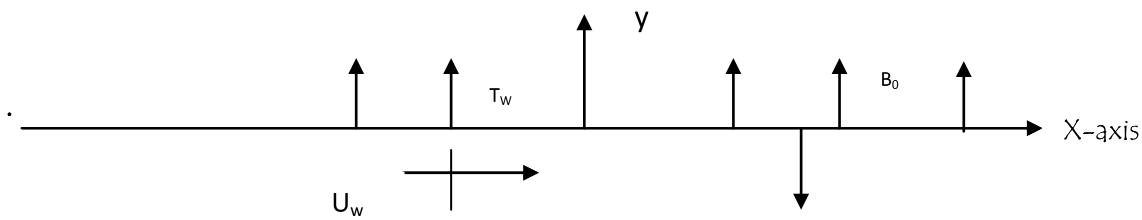
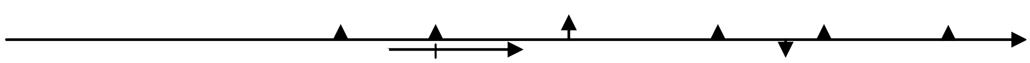


Figure 2.1: Flow configuration



The generalized Ohm's law, neglecting Hall effects is expressed as

$$\vec{J} = \sigma\,(\vec{E} + \vec{q} \times \vec{B})$$

(2.13)

Magnetic field is considered to be divergent, that is, there are magnetic flux sources and sinks within the field, and therefore

$$\nabla.\vec{B} = 0$$

The mathematical expression of equation of continuity in the case of conservation of electric charge is

$$\nabla.\vec{J} = -\frac{\partial \rho_e}{\partial t}$$
(2.14)

The term q x B in the equation 2.14 yields

$$\vec{q} \times \vec{B} = \begin{vmatrix} i & j & k \\ u & 0 & 0 \\ 0 & B_0 & 0 \end{vmatrix} = uB_0\boldsymbol{k}$$
(2.15)

Thus from equation (2.16), the current density becomes $j_z = uB_0k$

The Lorentz force $\vec{J} \times \vec{B}$ is given as

$$\vec{J} \times \vec{B} = \sigma \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & uB_0 \\ 0 & B_0 & 0 \end{vmatrix} = -\boldsymbol{i}\sigma uB_0^2$$
(2.16)

Neglecting joules heating the equations governing the flow become

2.4 Equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
(2.17)

2.5 Equation of momentum

Along the x-axis is

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \frac{\sigma u {B_0}^2}{\rho}$$
(2.18)

2.6 Equation of energy

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{k\partial^2 T}{\partial x^2}+\frac{k\partial^2 T}{\partial y^2}+\mu\phi$$

(2.19)

The initial and boundary conditions of the problem are;

$$y=0,\ \ u=U_\infty\ ,v=0\ ,T=T_w$$

$$y\to\infty\ ,\ \ u=0\ ,\ \ T=T_\infty$$

(2.20)

2.7 Non-dimensionalization

The subject of dimensional analysis considers how to determine the required set of scales for any given problem. Non-dimensionalization of the equations governing a particular fluid flow falls under abroad area of study known as dimensional analysis .A useful starting point is to emphasize that two similar flow patterns occur when the non – dimensional parameters are the same. We proceed to non-dimensionalize the momentum and energy equations with an objective of determining the important parameters necessary in analyzing our flow problem. Each parameter is a ratio of forces. The magnitude of the forces indicates the relative importance of the forces for the flow. The non-dimensional parameters allow for the application of results obtained for a boundary experiencing a given set of conditions to another boundary which is geometrically similar but experiencing totally different solutions.

2.8 Dimensionless coordinates

$$x=x^*H,\quad y=y^*H,\quad u=u^*U_\infty,\quad v=v^*U_\infty\quad T^*=\frac{T-T_\infty}{T_w-T_\infty}$$

(2.21)

Where H ,U_∞ and T are the characteristics length, velocity and temperature

2.9 Equation of momentum along the x-axis

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \frac{\sigma B_0^2 u}{\rho} \tag{2.22}$$

Taking $\mu = \frac{1}{\alpha(T - T_r)}$ Equation (2.22) reduces to;

$$v\frac{\partial u}{\partial y} + u\frac{\partial u}{\partial x} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\frac{1}{\alpha(T-T_r)}\frac{\partial u}{\partial y}\right) - \frac{\sigma B_0^2 u}{\rho}$$

(2.23)

$$v\frac{\partial u}{\partial y} + u\frac{\partial u}{\partial x} = \frac{1}{\rho}\left(\frac{1}{\alpha(T-T_r)}\frac{\partial^2 u}{\partial y^2} - \frac{1}{\alpha(T-T_r)^2}\frac{\partial T}{\partial y}\frac{\partial u}{\partial y}\right) - \frac{\alpha B_0^2 u}{\rho}$$

(2.24)

In order to transform the equation of momentum and energy into their respective non-dimensional form, the following analysis is first carried out;

$$T = T^*(T_w - T_\infty) + T_\infty, \quad \frac{\partial T}{\partial T^*} = T_w - T_\infty$$

(2.25)

$$\frac{\partial T}{\partial y} = \frac{\partial T}{\partial T^*}\frac{\partial T^*}{\partial y^*}\frac{\partial y^*}{\partial y} = \frac{T_w - T_\infty}{H}\frac{\partial T^*}{\partial y^*}$$

(2.26)

$$\frac{\partial^2 T}{\partial y^2} = \frac{\partial}{\partial y^*}\left(\frac{\partial T}{\partial y}\right)\frac{\partial y^*}{\partial y} = \frac{\partial}{\partial y^*}\left(\frac{T_w - T_\infty}{H}\frac{\partial T^*}{\partial y^*}\right)\frac{\partial y^*}{\partial y} = \frac{T_w - T_\infty}{H^2}\frac{\partial^2 T^*}{\partial y^{*2}}$$

(2.27)

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial u^*}\frac{\partial u^*}{\partial y^*}\frac{\partial y^*}{\partial y} = U_\infty \frac{\partial u^*}{\partial y^*}\frac{1}{H} = \frac{U_\infty}{H}\frac{\partial u^*}{\partial y^*}$$

(2.28)

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y^*}\left(\frac{\partial u}{\partial y}\right)\frac{\partial y^*}{\partial y} = \frac{\partial}{\partial y^*}\left(\frac{u_\infty}{H}\frac{\partial u^*}{\partial y^*}\right)\frac{1}{H} = \frac{U_\infty}{H^2}\left(\frac{\partial^2 u^*}{\partial y^{*2}}\right)$$

(2.29)

$$\frac{\partial T}{\partial x} = \frac{\partial T}{\partial T^*}\frac{\partial T^*}{\partial x^*}\frac{\partial x^*}{\partial x} = \frac{T_W - T_\infty}{H}\frac{\partial T^*}{\partial x^*}$$

(2.30)

$$\frac{\partial^2 T}{\partial x^2} = \frac{\partial}{\partial x^*}\left(\frac{\partial T}{\partial x}\right)\frac{\partial x^*}{\partial x} = \frac{\partial}{\partial x^*}\left(\frac{T_W - T_\infty}{H}\frac{\partial T^*}{\partial x^*}\right)\frac{\partial x^*}{\partial x} = \frac{T_W - T_\infty}{H^2}\frac{\partial^2 T^*}{\partial x^{*2}}$$

(2.31)

$$\frac{\partial v}{\partial y} = \frac{\partial v}{\partial v^*}\frac{\partial v^*}{\partial y^*}\frac{\partial y^*}{\partial y} = U_\infty \frac{\partial v^*}{\partial y^*}\frac{1}{H} = \frac{U_\infty}{H}\frac{\partial v^*}{\partial y^*}$$

2.32)

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial u^*}\frac{\partial u^*}{\partial x^*}\frac{\partial x^*}{\partial x} = U_\infty \frac{\partial u^*}{\partial x^*}\frac{1}{H} = \frac{U_\infty}{H}\frac{\partial u^*}{\partial x^*}$$

(2.33)

$$\frac{\partial v}{\partial x} = \frac{\partial v}{\partial v^*}\frac{\partial v^*}{\partial x^*}\frac{\partial x^*}{\partial x} = U_\infty \frac{\partial v^*}{\partial x^*}\frac{1}{H} = \frac{U_\infty}{H}\frac{\partial v^*}{\partial x^*}$$

(2.34)

$$\frac{\partial^2 v}{\partial x^2} = \frac{\partial}{\partial x^*}\left(\frac{\partial v}{\partial x}\right)\frac{\partial x^*}{\partial x} = \frac{\partial}{\partial x^*}\left(\frac{u_\infty}{H}\frac{\partial v^*}{\partial x^*}\right)\frac{1}{H} = \frac{U_\infty}{H^2}\left(\frac{\partial^2 v^*}{\partial x^{*2}}\right)$$

(2.35)

Substituting (2.26), (2.28), (2.29), (2.34) in equation (2.24)

$$v^* U_\infty \frac{U_\infty}{H}\frac{\partial u^*}{\partial y^*} + u^* U_\infty \frac{U_\infty}{H}\frac{\partial u^*}{\partial x^*} = \frac{1}{\rho\alpha\,(T-T_r)}\frac{U_\infty}{H^2}\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{1}{\rho\alpha(T-T_r)^2}\left(\frac{T_w-T_\infty}{H}\right)\frac{\partial T^*}{\partial y^*}\frac{U_\infty}{H}\frac{\partial u^*}{\partial y^*} - \frac{\sigma B_0^2 u^* U_\infty}{\rho}$$

(2.36)

On dividing by $\frac{{U_\infty}^2}{H}$, the equation (2.36) reduces to:

$$v^*\frac{\partial u^*}{\partial y^*} + u^*\frac{\partial u^*}{\partial x^*} = \frac{1}{\rho H U_\infty \alpha(T-T_r)}\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\Delta T}{(T-T_r)}\frac{1}{\rho H U_\infty \alpha(T-T_r)}\frac{\partial T^*}{\partial y^*} - \frac{\sigma B_0^2 u^* H}{\rho U_\infty}$$

(2.37)

where $\Delta T = T_w - T_\infty$

Equation (2.37) in non dimensional form is

$$v^*\frac{\partial u^*}{\partial y^*} + u^*\frac{\partial u^*}{\partial x^*} = \frac{1}{Re}\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{1}{\theta_r}\frac{1}{Re}\frac{\partial T^*\partial u^*}{\partial y^*\partial y^*} - Mu^*$$

(2.38)

Where

$Re = \frac{\rho\, U_\infty H}{\mu} = \frac{1}{\rho H\, U_\infty \alpha\,(T-T_r)}$, $\theta_r = \frac{T-T_r}{T_w - T_\infty} = \frac{T-T_r}{\Delta T}$ and $M = \frac{\sigma B_0^2 H}{\rho U_\infty}$, θ_r is the viscosity

Parameter and M is the magnetic parameter.

2.10 Equation of energy

$$v\frac{\partial T}{\partial y}+u\frac{\partial T}{\partial x}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial x^2}+\frac{1}{\alpha(T-T_r)\rho C_p}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2$$

(2.39) substituting equation (2.26), (2.27),(2.28),(2.30),(2.31),(2.34) in equation (2.39)

$$v^*U_\infty\left(\frac{T_w-T_\infty}{H}\right)\frac{\partial T^*}{\partial y^*}+u^*U_\infty\left(\frac{T_w-T_\infty}{H}\right) = \frac{k}{\rho C_p}\left(\frac{T_w-T_\infty}{H^2}\right)\frac{\partial^2 T^*}{\partial y^{*2}}+\frac{k}{\rho C_p}\left(\frac{T_w-T_\infty}{H^2}\right)\frac{\partial^2 T^*}{\partial x^{*2}}$$

$$+\frac{1}{\alpha(T-T_r)\rho C_p}\frac{U_\infty^2}{H^2}\left(\frac{\partial u^*}{\partial y^*}+\frac{\partial v^*}{\partial x^*}\right)^2$$

(2.40)

Dividing equation (2.40) by $\frac{U_\infty(T_W-T_\infty)}{H}$ or $\frac{U_\infty \Delta T}{H}$

$$\frac{v^*\partial T^*}{\partial y^*}+\frac{u^*\partial T^*}{\partial x^*}=\frac{k}{\rho C_p U_\infty H}\frac{\partial^2 T^*}{\partial y^{*2}}+\frac{k}{\rho C_p U_\infty H}\frac{\partial^2 T^*}{\partial x^{*2}}+\frac{\mu U_\infty}{\rho C_p H\Delta T}\left(\frac{\partial u^*}{\partial y^*}+\frac{\partial v^*}{\partial x^*}\right)^2$$

(2.41)

Hence equation (2.41) in non dimensional form

$$v^*\frac{\partial T^*}{\partial y^*}+u^*\frac{\partial T^*}{\partial x^*}=\frac{1}{Pe}\left(\frac{\partial^2 T^*}{\partial y^{*2}}+\frac{\partial^2 T^*}{\partial x^{*2}}\right)+\frac{Ec}{Re}\left(\frac{\partial u^*}{\partial y^*}+\frac{\partial v^*}{\partial x^*}\right)^2$$

(2.42)

Where Peclet number

$Pe = RePr = \frac{\rho U_\infty H C_p}{k} = \frac{U_\infty H}{\alpha}$, *Prandtl number* $Pr = \frac{C_p \mu}{k}$, Reynold's number $Re = \frac{\rho\, U_\infty H}{\mu}$ *andEckert number* $Ec = \frac{(U_\infty)^2}{C_p \Delta T}$

Equation (2.38) and (2.42) are then solved subject to initial and boundary conditions in non dimensional form

$$y^* = 0, u^* = 1, v^* = 0 \; and \; T^* = 1$$

$$y^* \to \infty, u^* = 0, v^* = 0 \; and \; T^* = 0 \tag{2.43}$$

2.11 Non-dimensional numbers

2.11.1 Prandtl number Pr

This number is a property of the fluid and was named after Ludwig. Prandtl, a German who was closely associated with the conception of boundary layer theory. It is the parameter which relates the relative thickness of the hydrodynamic and thermal boundary layers. The Prandtl number provided the link between the velocity field and the temperature field. It is expressed as

$$Pr = \frac{\mu/\rho}{\kappa/\ \rho C_P} = \frac{C_P\ \mu}{\kappa} \tag{2.44}$$

It is the ratio of the momentum diffusivity to the thermal diffusivity.

2.11.2 Magnetic Parameter M

It is defined as the ratio of the magnetic force to the viscous force. i.e

$$M = \sqrt{\frac{magnetic\ force}{inertia\ force}} = \sqrt{\frac{\sigma B_0{}^2 H}{\rho U_\infty}} \tag{2.45}$$

2.11.3 Reynolds number, R_e

The Reynolds number R_e given by

$$R_e = \frac{U_\infty H \rho}{\mu} \tag{2.46}$$

This is the ratio of inertia force to viscous force. A dimensionless combination of variables that is important in the study of viscous flow is called the Reynolds number. The Reynold number indicates the relative significance of the viscous effect compared to the inertia effect. When Re of the system is small, the viscous force is predominant and the effect of viscosity is important in the flow field. On the other hand if Re is large, the inertia force is predominant and the effect of viscosity is important only in the narrow area near the boundary or in any other region of large variation in velocity.

2.11.4 Eckert number, Ec

$$\text{Ec} = \frac{U_\infty{}^2}{C_P\,(T_W - T_\infty)}$$

(2.47)

It provides a measure of the kinetic energy of the flow relative to the enthalpy difference across the thermal boundary layer

2.11.5 Peclet number, Pe

This number is defined as the ratio of advection of a physical quantity by the flow rate of diffusion of the same quantity driven by an appropriate gradient. In the context of transport of heat, the Peclet number is equivalent to the product of the Reynolds number and the Prandtl number where:-

$$\text{Pe} = \text{RePr} = \frac{U_\infty \; H\,\rho\,C_p}{K}$$

(2.48)

The final form of the governing equations is:

Equation of momentum along the x-axis

$$v^*\frac{\partial u^*}{\partial y^*} + u^*\frac{\partial u^*}{\partial x^*} = \frac{1}{Re}\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{1}{\theta_r}\frac{1}{Re}\frac{\partial T^*\partial u^*}{\partial y^*\partial y^*} - Mu^*$$

(2.49)

Equation of energy

$$v^*\frac{\partial T^*}{\partial y^*} + u^*\frac{\partial T^*}{\partial x^*} = \frac{1}{\text{Pe}}\left(\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial x^{*2}}\right) + \frac{\text{Ec}}{\text{Re}}\left(\frac{\partial u^*}{\partial y^*} + \frac{\partial v^*}{\partial x^*}\right)^2$$

(2.50)

CHAPTER THREE
METHODOLOGY

3.1 Method of Solution

The governing differential equations (2.40) and (2.44) together with boundary conditions are solved numerically using an implicit finite difference method similar to that described by Patanker (1980). The differential equations are highly non-linear and therefore cannot be solved analytically but in an iterative manner. For this reason we have assigned pseudo time derivatives as follows:

On rearranging, and equating the equations above to pseudo time derivatives, $\frac{\partial u}{\partial t}\frac{\partial T}{\partial t}$ and bringing them to the implicit side i.e.

$$\frac{\partial u}{\partial t} = \frac{1}{Re}\frac{\partial^2 u}{\partial y^2}-\frac{1}{\theta_r}\frac{1}{Re}\frac{\partial T}{\partial y}\frac{\partial u}{\partial y} - u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - Mu \tag{3.1}$$

$$\frac{\partial T}{\partial t} = \frac{1}{Pe}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\frac{Ec}{Re}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2 - u\frac{\partial T}{\partial x} - v\frac{\partial T}{\partial y} \tag{3.2}$$

The solution has been determined by making use of finite difference method. Since a steady solution is desired the final solution has not been affected by the order of the pseudo time derivative. We have used a first order finite difference approximation for these pseudo time derivatives. This permits us to reduce the problem to the solution of a system of algebraic equations, which involves a number of unknowns .The following pseudo time derivatives are used.

$$\frac{\partial u}{\partial t} = \frac{U_{i,j}^{k+1} - U_{i,j}^{k}}{\Delta t} \tag{3.3}$$

$$\frac{\partial v}{\partial t} = \frac{V_{i,j}^{k+1} - V_{i,j}^{k}}{\Delta t} \tag{3.4}$$

$$\frac{\partial u}{\partial y} = \frac{U_{i,j+1}^{k+1} - U_{i,j}^{k+1} + U_{i,j+1}^{k} - U_{i,j}^{k}}{2\Delta y} \tag{3.5}$$

$$\frac{\partial u}{\partial x} = \frac{U_{i+1,j}^{k+1} - U_{i,j}^{k+1} + U_{i+1,j}^{k} - U_{i,j}^{k}}{2\Delta x}$$

(3.6)

$$\frac{\partial^2 u}{\partial y^2} = \frac{U_{i,j+1}^{k+1} - 2U_{i,j}^{k+1} + U_{i,j-1}^{k+1} + U_{i,j+1}^{k} - 2U_{i,j}^{k} + U_{i,j-1}^{k}}{2(\Delta y)^2} \tag{3.7}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{U_{i+1,j}^{k+1} - 2U_{i,j}^{k+1} + U_{i-1,j}^{k+1} + U_{i+1,j}^{k} - 2U_{i,j}^{k} + U_{i-1,j}^{k}}{2(\Delta x)^2} \tag{3.8}$$

$$\frac{\partial T}{\partial y} = \frac{T_{i,j+1}^{k+1} - T_{i,j}^{k+1} + T_{i,j+1}^{k} - T_{i,j}^{k}}{2\Delta y} \tag{3.9}$$

By representing time steps by k and distance along x by i and along y by j the momentum equation (3.1) along the x-axis in finite difference form becomes;

$$U_{i,j}^{k+1} - U_{i,j}^{k} =$$
$$\Delta t \left\{ \frac{1}{Re} \left(\frac{U_{i,j+1}^{k+1} - U_{i,j}^{k+1} + U_{i,j-1}^{k+1} + U_{i,j}^{k} + U_{i,j-1}^{k}}{2(\Delta y)^2} \right) - \right.$$
$$\frac{1}{\theta_r} \frac{1}{Re} \left(\frac{T_{i,j+1}^{k+1} - T_{i,j}^{k+1} + T_{i,j+1}^{k} - T_{i,j}^{k}}{2\Delta y} \right) \left(\frac{U_{i,j+1}^{k+1} - U_{i,j}^{k+1} + U_{i,j+1}^{k} - U_{i,j}^{k}}{2\Delta y} \right) -$$
$$\left. U_{i,j}^{k} \left(\frac{U_{i+1,j}^{k+1} - U_{i,j}^{k+1} + U_{i+1,j}^{k} - U_{i,j}^{k}}{2\Delta x} \right) - V_{i,j}^{k} \left(\frac{U_{i,j+1}^{k+1} - U_{i,j}^{k+1} + U_{i,j+1}^{k} - U_{i,j}^{k}}{2\Delta y} \right) - MU_{i,j}^{k} \right\}$$

(3.10)

earranging the equation and factoring out $U_{i,j}^{k+1}$ the equation simplifies to give;

$$U_{i,j}^{k+1} = U_{i,j}^{k} + \frac{\Delta t}{Re}\left(\frac{U_{i,j+1}^{k+1} + U_{i,j-1}^{k+1} + U_{i,j+1}^{k} - 2U_{i,j}^{k} + U_{i,j-1}^{k}}{2(\Delta y)^2}\right) - \frac{\Delta t}{Re\theta_r}\frac{T_s}{2\Delta y}\left(\frac{U_{i,j+1}^{k+1} + U_{i,j+1}^{k} - U_{i,j}^{k}}{2\Delta y}\right) -$$

$$-\Delta t U_{i,j}^{k}\left(\frac{U_{i+1,j}^{k+1} + U_{i+1,j}^{k} - U_{i,j}^{k}}{2\Delta x}\right) - \Delta t V_{i,j}^{k}\left(\frac{U_{i,j+1}^{k+1} + U_{i,j+1}^{k} - U_{i,j}^{k}}{2\Delta y}\right)$$

$$-\Delta t M U_{i,j}^{k}$$

$$/1 + \frac{2\Delta t}{2Re(\Delta y)^2} - \frac{\Delta t}{\theta_r}\frac{1}{Re}\frac{T_s}{4(\Delta y)^2} - \frac{\Delta t}{2\Delta x}U_{i,j}^{k} - \frac{\Delta t}{2\Delta y}V_{i,j}^{k}$$

where $T_s = T_{i,j+1}^{k+1} - T_{i,j}^{k+1} + T_{i,j+1}^{k} - T_{i,j}^{k}$ (3.11)

Similarly equation (3.2) becomes

$$T_{i,j}^{k+1} - T_{i,j}^{k} = \frac{\Delta t}{Pe}\left(\frac{T_{i+1,j}^{k+1} - 2T_{i,j}^{k+1} + T_{i-1,j}^{k+1} + T_{i+1,j}^{k} - 2T_{i,j}^{k} + T_{i-1,j}^{k}}{2(\Delta x)^2} + \frac{T_{i,j+1}^{k+1} - T_{i,j}^{k+1} + T_{i,j-1}^{k+1} + T_{i,j+1}^{k} - 2T_{i,j}^{k} + T_{i,j-1}^{k}}{2(\Delta y)^2}\right)$$

$$+\Delta t\frac{Ec}{Re}\left(\frac{U_{i,j+1}^{k+1} - U_{i,j}^{k+1} + U_{i,j+1}^{k} - U_{i,j}^{k}}{2\Delta y} + \frac{V_{i+1,j}^{k+1} - V_{i,j}^{k+1} + V_{i+1,j}^{k} - V_{i,j}^{k}}{2\Delta x}\right)^2$$

$$-\ \Delta t U_{i,j}^{k}\left(\frac{T_{i+1,j}^{k+1} - T_{i,j}^{k+1} + T_{i+1,j}^{k} - T_{i,j}^{k}}{2\Delta x}\right) - \Delta t V_{i,j}^{k}\left(\frac{T_{i,j+1}^{k+1} - T_{i,j}^{k+1} + T_{i,j+1}^{k} - T_{i,j}^{k}}{2\Delta y}\right) \qquad (3.12)$$

Rearranging the equation and factoring out $T_{i,j}^{k+1}$ equation simplifies to give;

$$T_{i,j}^{k+1} = T_{i,j}^{k} + \frac{\Delta t}{Pe}\left(\frac{T_{i+1,j}^{k+1} + T_{i-1,j}^{k+1} + T_{i+1,j}^{k} - 2T_{i,j}^{k} + T_{i-1,j}^{k}}{2(\Delta x)^2} + \frac{T_{i,j+1}^{k+1} + T_{i,j-1}^{k+1} + T_{i,j+1}^{k} - 2T_{i,j}^{k} + T_{i,j-1}^{k}}{2(\Delta y)^2}\right)$$

$$+\Delta t\frac{Ec}{Re}\left(\frac{U_{i,j+1}^{k+1} - U_{i,j}^{k+1} + U_{i,j+1}^{k} - U_{i,j}^{k}}{2\Delta y} + \frac{V_{i+1,j}^{k+1} - V_{i,j}^{k+1} + V_{i+1,j}^{k} - V_{i,j}^{k}}{2\Delta x}\right)^2$$

$$- \Delta t U_{i,j}^{k}\left(\frac{T_{i+1,j}^{k+1}+T_{i+1,j}^{k}-T_{i,j}^{k}}{2\Delta x}\right)-\Delta t V_{i,j}^{k}\left(\frac{T_{i,j+1}^{k+1}+T_{i,j+1}^{k}-T_{i,j}^{k}}{2\Delta y}\right)$$

$$/1+\frac{\Delta t}{Pe}\frac{1}{(\Delta x)^2}+\frac{\Delta t}{Pe}\frac{1}{(\Delta y)^2}-\frac{\Delta t}{2\Delta x}U_{i,j}^{k}-\frac{\Delta t}{2\Delta y}V_{i,j}^{k} \qquad (3.13)$$

3.2 Calculation of the Rate of Skin Friction and the rate of heat transfer

The quantities of main engineering interest in the problem at hand are the local Nusselt number and the shearing stress on the continuous moving sheet. The Nusselt number indicates the rate of heat transfer. The shearing stress on the surface of the continuous surface is defined as.

$$T_y = \mu \frac{\partial u}{\partial y} \qquad (3.14)$$

The skin friction is computed from the velocity profile using the equation

$$T_y^* = \frac{1}{R_e}\frac{\partial u}{\partial y} \qquad (3.15)$$

It is calculated by numerical differentiation using Newton's interpolation formula over the first five points.

$$T_y^* = \frac{5}{6}\left[25U(0,j) - 48U(1,j) + 36U(2,j) - 16U(3,j) + 3U(4,j)\right] \qquad (3.16)$$

The rate of heat transfer is calculated from the temperature profiles in terms of the Nusselt number which is given by

$$Nu = \frac{-1}{T_w - T_\infty}\frac{\partial T}{\partial y} \qquad (3.17)$$

The equations (3.11) and (3.13) are the final set of equations and were solved simultaneously using a computer code in MATLAB application software version 7.14.0.739.Next the velocity profiles obtained together with temperature profiles are graphically represented.

CHAPTER FOUR
RESULTS AND DISCUSSIONS

4.1 Results

Equations (3.11) and (3.13) were solved using the MATLAB version 7.14.0.739 computer code. Numerical solutions of the problem were obtained for various values of the physical parameters involved in the problem such as Reynolds number *Re*, Magnetic parameter *M*, Eckert number Ec, Peclet number *Pe*, and Prandtl number *Pr*. In the absence of magnetism the results were compared with that of Elbashbeshy and Bazid (2000). It was clearly seen that the results are in good agreement and correlation with that of Elbashbeshy and Bazid (2000). We thus discuss the behavior of the velocity and temperature profiles considering the effects of the variation of the physical parameters as stated herein in the presence of magnetism.

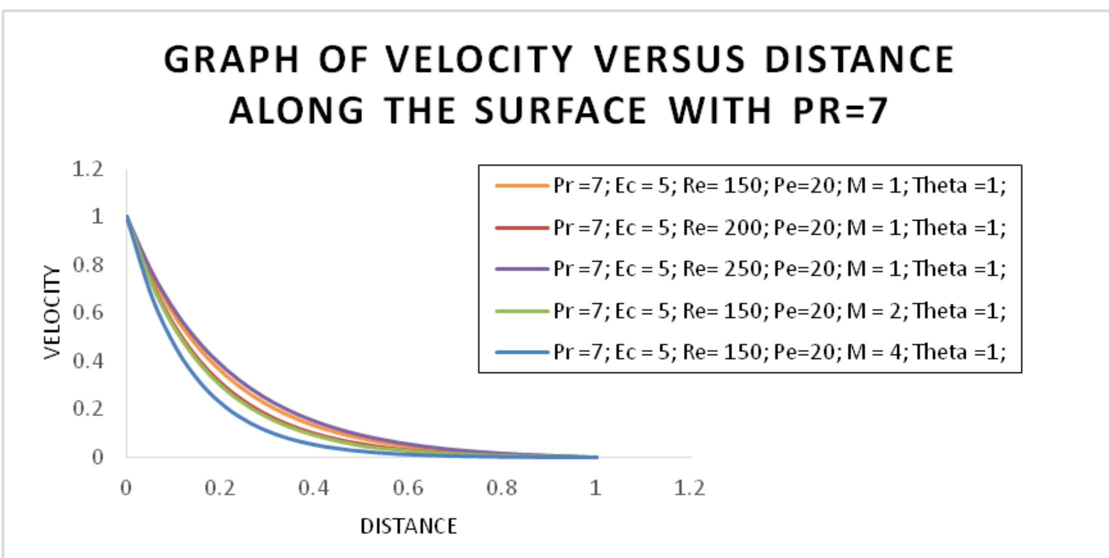


Figure 4.1: Velocity profiles for different values of Re and M.

Figure 4.1 displays the plot of dimensionless velocity profile for different values of Reynolds number (Re) and magnetic parameter *M*. The Reynolds number (Re) is the ratio of inertia force to viscous force. From the figure 4.1, it is observed that the dimensionless component of the velocity of the fluid increases with increase in the value of Reynolds number. When Reynolds number (Re) is small, it means that the viscous force is predominant and thus imposes drug in the fluid decreasing the velocity of the flow. When Reynolds number (Re) is large, the inertia force is predominant and the effect of viscosity is important only in the narrow area near the boundary or in any other region of large variation in velocity. It is also observed that as the magnetic parameter *M* increases, velocity decreases elucidating the fact that the effect of magnetic field is to decelerate the velocity. Application of a transverse magnetic field to an electrically conducting fluid gives rise to a resistive type of force called the Lorentz force. This force has the tendency to slow down the motion of the fluid in the boundary layer according to Kinyanjui *et al* (2001). The reduced velocity by the frictional drag due to the Lorentz force is responsible for reducing thermal viscous dissipation in the fluid leading to a thinner thermal boundary layer. Magnetic field can therefore be employed to control the velocity boundary characteristics of viscosity.

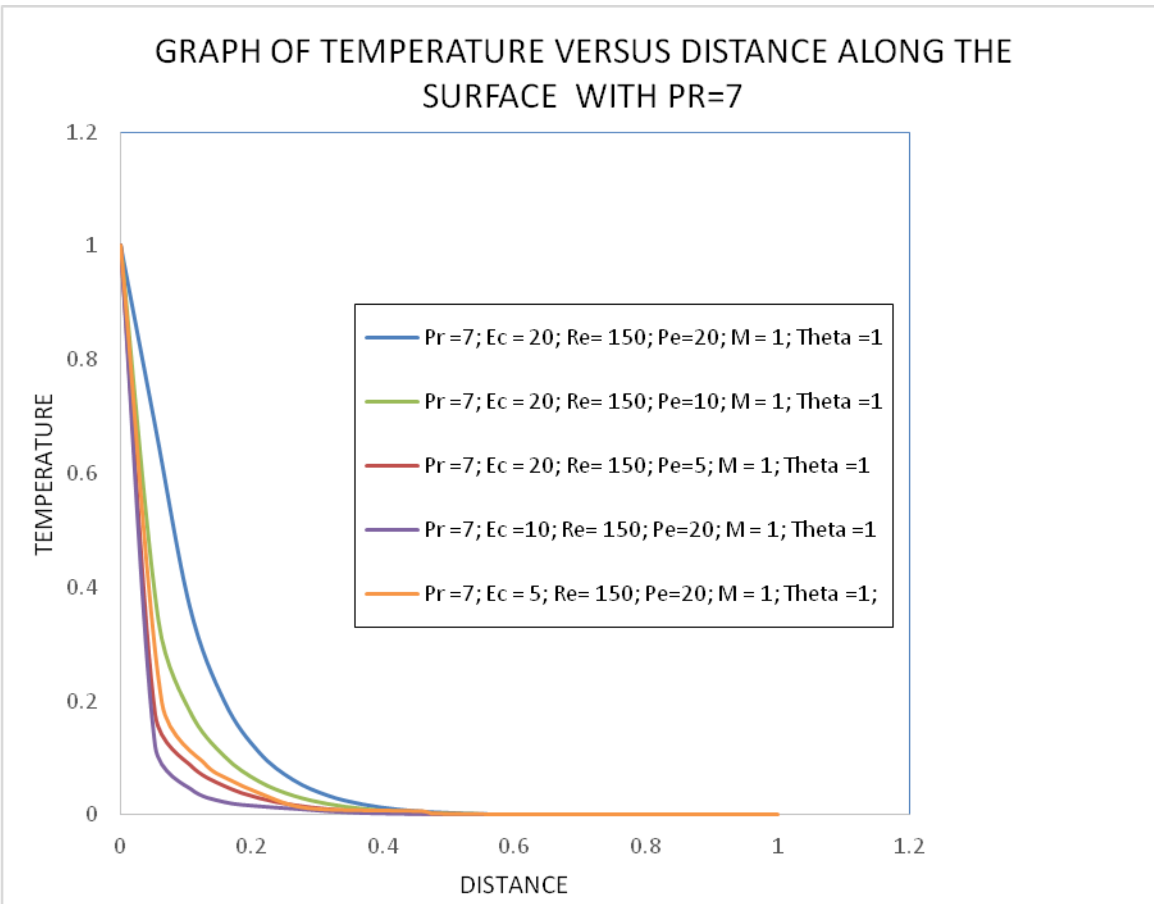


Figure 4.2: Temperature profiles for different values of Ec and Pe

Eckert number *Ec* provides a measure of the kinetic energy of the flow relative to the enthalpy difference across the thermal boundary layer. It is observed from figure 4.2 that decrease in Eckert number *Ec* causes an increase in the dimensionless temperature component. It embodies the conversion of kinetic energy into internal energy by work done against viscous fluid stress. A negative Eckert number implies heating of the fluid. It is clearly noted that decrease in Eckert number *Ec* in magnitude, leads to rise in temperature.

The figure 4.2 also shows that the Peclet number *Pe* variation with the dimensionless temperature. It can be observed that as Peclet number *Pe* increases so does the temperature. In the context of transport of heat, the Peclet number is equivalent to the product of the Reynolds number and the Prandtl number as the ratio of advection of a physical quantity by the flow rate of diffusion of the same quantity driven by an appropriate gradient.

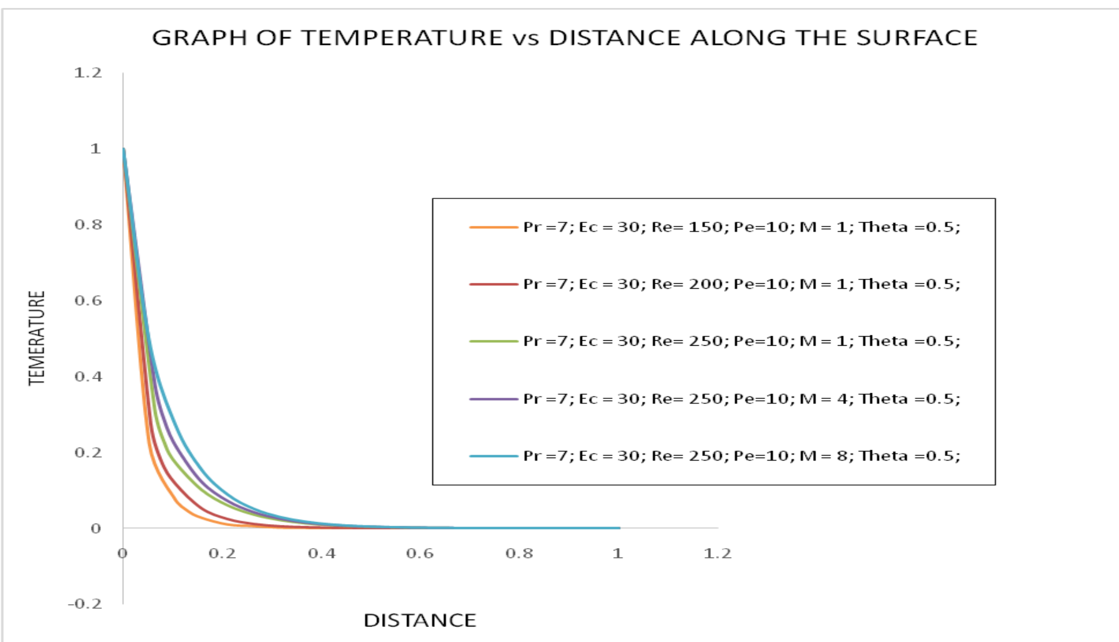


Figure 4.3: Temperature profiles for different values of Re and M.

Figure 4.3 shows the plot of dimensionless temperature for different values of Reynolds number (Re) and magnetic parameter *M*. It is observed that this component of the temperature of the fluid decreases with decrease in the values of Reynolds number. When Reynolds number (Re) is small, it means that the viscous force is predominant and thus the temperature of the flow is low. When Reynolds number (Re) is large, the inertia force is predominant and the effect of viscosity is important only in the narrow area near the boundary or in any other region of large variation in temperature. It is also observed that as magnetic parameter *M* increases, temperature decreases elucidating the fact that the effect of magnetic field is to decelerate the temperature, thus

magnetic parameter *M* is inversely varying with the temperature.

The table below gives the results as follows:-

Table 4.1: Variation of Coefficient of friction, τ_y^* and Nusselt numbers, Nu with various parameters: Ec, Re, Pe, M and   for Pr=7

Ec	Re	Pe	M	Theta	Nu	Tau
30	150	20	1	0.5	3.196713	2.663927
10	150	20	1	0.5	3.179666	2.649722
5	150	20	1	0.5	3.175397	2.646164
30	200	20	1	0.5	3.058965	2.549137
30	250	20	1	0.5	2.976269	2.480224
30	150	10	1	0.5	3.186813	2.655677
30	150	5	1	0.5	3.179036	2.649196
30	150	20	4	0.5	4.708973	3.924144
30	150	20	8	0.5	6.226197	5.188498
30	150	20	1	1	3.183018	2.652515
30	150	20	1	2	3.176158	2.646798
10	300	2	10	3	6.708017	5.590014

From table (4.1), we noted the following:

(i) Increase in Eckert number leads to increase in Nusselt number Nu.

(ii) Increase in theta leads to decrease in Nusselt number Nu.

(iii) Increase in Magnetic parameter number leads to increase in Nusselt number Nu.

(iv) Increase in Eckert number leads to increase in skin friction τ_y^*

(v) Increase in theta leads to decrease in skin friction τ_y^*.

(vi) Increase in Magnetic parameter number leads to increase in skin friction τ_y^*.

Increase in the magnetic parameter leads to an increase in the magnitude of the local skin friction coefficients due to the decreasing velocity. It also leads to a decrease in the Nusselt number. Thermal boundary layer thickness decreases with increase in Magnetic parameter leading to the observed increase in the Nusselt number.

Increase in the Eckert number leads to an increase in local skin friction coefficients but a decrease in the Nusselt number. A positive Eckert number implies increased velocity and the temperature of the fluid. The resulting thicker thermal boundary layer leads to a reduced rate of heat transfer.

The Peclet number here plays a role in heat transfer that is similar to that of the Reynolds number which contains the viscosity coefficient in fluid mechanics. The fact that the physical significance of the Reynolds number is that, it represents the ratio of inertial forces to viscous forces in the flow, or equivalently, the relative importance of convective transport of momentum compared with molecular transport of momentum. Thus, the Peclet number tells us the relative importance of convective transport of thermal energy when compared with molecular transport of thermal energy (conduction).

From the table 4.0 it also depicts the effects of the viscosity coefficient on the skin-friction coefficient and the Nusselt number Nu. It is observed that increase in the viscosity coefficient leads to decrease in the magnitude of the local skin friction coefficients and the heat transfer due to minute reduction in the velocity profiles and the negligible increases in the temperature profiles. This has the direct effect of increasing skin friction and decreasing Nusselt number Nu.

From the discussions of results above we have seen that the parameter in the governing equations affect the velocity and temperature profiles. Consequently their effects alter the skin friction and the rate of heat transfer.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

In this chapter, conclusion of the research carried out has been outlined and thereafter recommendations for future research made.

5.2 Conclusion

The analysis of various parameters on steady laminar boundary layer flow of an incompressible, electrically conducting, and viscous Newtonian fluid past a continuous electrically non conducting semi-infinite plate has been carried out. The numerical solutions of velocity and temperature fields are obtained by finite difference method with pseudo approximation.

The problem at hand has influence of constant magnetic field. In order to validate the present results, the boundary condition for the plate was changed so as there is effects of magnetic field. The results were compared with those of Elbashbeshy (2000); and agree. The main conclusions are as follows:

(i) Velocity profiles and temperature profiles decrease with increase in Magnetic parameter M

(ii) Increase in Reynolds number Re leads to increase in velocity profiles and temperature profiles.

(iii) Increase in Eckert number Ec leads to no change in velocity profiles but to an increase in temperature profiles

(iv) Increase in Peclet number *Pe* leads to increase in temperature profiles and no effects on velocity profiles.

(v) Increase in Eckert number leads to decrease in Nusselt number Nu.

(vi) Increase in theta leads to decrease in Nusselt number Nu.

(vii) Increase in Magnetic parameter number leads to increase in Nusselt number Nu.

(viii) Increase in Eckert number leads to increase in shearing stress τ_y^*.

(ix) Increase in theta leads to decrease in shearing stress τ_y^*.

(x) Increase in Magnetic parameter number leads to increase in shearing stress τ_y^*.

Thus varying magnetic parameter M, Reynolds' number Re, Eckert number Ec and Peclet number can be used to control the growth of velocity boundary layer and thermal boundary layer.

5.3 Recommendation

This work has considered the steady laminar boundary layer MHD flow of an incompressible, viscous, and electrically conducting Newtonian fluid over a continuous moving plate in a fluid. The present work can provide a basis for further research by including the following considerations:

(i) Flow of a compressible fluid.

(ii) Hydro-magnetic flow problem in three dimensional.

(iii) Turbulent flow over continuous moving surface.

(iv) Flow involving magnetic field applied at an angle.

(v) Flow subjected to variable magnetic field.

(vi) Unsteady flow.

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APPENDICES

Appendix 1: publication

The paper on the effect of temperature dependent viscosity on MHD past a continuous moving surface has been accepted for publication by International Institute of Science, Technology & Education (IISTE).

Appendix 2: CODES

```
Pr =7; Ec = 30; Re= 150; Pe=3; M = 1; Theta =0.5;
nx =90;
ny =20;
nt=1000;
% --- Compute mesh spacing and time step
delX=0.5;
delY=0.125;
delT=0.00125;



x = linspace(0,1,nx);
y = linspace(0,1,ny);
T= zeros(nx,ny,nt);
U= zeros(nx,ny,nt);
V= zeros(nx,ny,nt);

% --- Set IC and BC

for J=1:nt
 for I=1:nx
   % U(I,J,1) = 1.;    T(I,J,1) = 1.;    V(I,J,1) = 0;   %   init.
conds/
    %U(I,ny,K) = 0.;   T(I,ny,K) = 0.  ;  V(I,ny,K) = 0;     %
bound. conds/
end
end

for K=1 : nt
```

```
 for I=1:nx
    U(I,1,K) = 1.;    T(I,1,K) = 1.;    V(I,1,K) = 0;   %   bound.
conds/
    U(I,ny,K) = 0.;   T(I,ny,K) = 0.  ;     %  bound. conds/
end
end


for K=2 : nt-1
for J=2:ny-1
for I=2:nx-1

     U(I,J,K+1) =(U(I,J,K)+(delT/Re*(2*delY^2))*(U(I,J+1,K+1)
+U(I,J-1,K+1)+U(I,J+1,K)-2*U(I,J,K) +U(I,J-1,K))-
(delT/(4*delY*delY*Theta*Re))*(T(I,J+1,K+1) -T(I,J,K+1)+T(I,J+1,K)-
T(I,J,K))*(U(I,J+1,K+1)+U(I,J+1,K)-U(I,J,K))-
(delT/(2*delX))*U(I,J,K)*(U(I+1,J,K+1) +U(I+1,J,K) -U(I,J,K)) -
(delT/(2*delY))*V(I,J,K)*(U(I,J+1,K+1) +U(I,J+1,K) -U(I,J,K))-
M*delT*U(I,J,K))/(1+delT/(Re*delY*delY) -
(delT/(Theta*Re*4*delY*delY))*(T(I,J+1,K+1)-T(I,J,K+1)+T(I,J+1,K)-
T(I,J,K))- (delT/(2*delX))*U(I,J,K) -(delT/(2*delY))*V(I,J,K)) ;

    % V(I,J,K+1) =(V(I,J,K)+(delT/(2*delX^2))*(V(I+1,J,K+1) +V(I-
1,J,K+1)+V(I+1,J,K)-2*V(I,J,K) +V(I-1,J,K))-
(delT/(4*delX*delY*Theta*Re))*(T(I+1,J,K+1) -T(I,J,K+1)+T(I+1,J,K)-
T(I,J,K))*(V(I,J+1,K+1) +V(I,J+1,K)-V(I,J,K))-
(delT/(4*delX*delX))*U(I,J,K)*(V(I+1,J,K+1) +V(I,J+1,K) -V(I,J,K))
-(delT/(4*delY*delY))*V(I,J,K)*(V(I,J+1,K+1) +V(I,J+1,K) -
V(I,J,K)))/(1+delT/(Re*delY^2) -
(delT/(Theta*Re*4*delX*delY))*(T(I+1,J,K+1)-T(I,J,K+1)+T(I+1,J,K)-
T(I,J,K))- (delT/(2*delX))*U(I,J,K) +(delT/(2*delY))*V(I,J,K)) ;
```

```
     T(I,J,K+1)=(T(I,J,K)+(delT/2*delX*delX*(Pe))*(T(I+1,J,K+1)
+T(I-1,J,K+1)+T(I+1,J,K)-2*T(I,J,K) +T(I-
1,J,K))+(delT/2*delY*delY*(Pe))*(T(I,J+1,K+1) +T(I,J-
1,K+1)+T(I,J+1,K)-2*T(I,J,K) +T(I,J-
1,K))+((delT*Ec)/(Re))*((U(I,J+1,K+1) -U(I,J,K+1)+U(I,J+1,K)-
U(I,J,K))/(2*delY)+(V(I+1,J,K+1) -V(I,J,K+1)+V(I+1,J,K)-
V(I,J,K))/(2*delX))*((U(I,J+1,K+1) -U(I,J,K+1)+U(I,J+1,K)-
U(I,J,K))/(2*delY)+(V(I+1,J,K+1) -V(I,J,K+1)+V(I+1,J,K)-
V(I,J,K))/(2*delX)) -(delT/(2*delX))*U(I,J,K)*(T(I+1,J,K+1)
+T(I+1,J,K) -T(I,J,K)) -(delT/(2*delY))*V(I,J,K)*(T(I,J+1,K+1)
+T(I,J+1,K) -T(I,J,K)))/(1+delT/(Pe*delX*delX)
+delT/(Pe*delY*delY)-(delT/(2*delX))*U(I,J,K) -
(delT/(2*delY))*U(I,J,K)) ;

     %V(J,K+1) =(V(J,K) -(delT*V(J,K)/(delY))*V(J+1,K+1) +
(delT/(Re*delY*delY)*(V(J+1,K+1)+V(J-1,K+1)))...
       %   -(delT/(Theta*delY*delY))*(T(J+1,K+1)-
T(J,K+1))*V(J+1,K+1))/(1-(delT*V(J,K))/(delY)+
(2*delT)/(Re*delY*delY)-(delT/(Theta*delY*delY))*(T(J+1,K+1)-
T(J,K+1))) ;


 %Nu_*(1,K+1) =(T(1,K+1)-8*T(3,K+1)+8*T(5,K+1)-T(7,K+1))/(12*delY);
% Nu =(25*U(I,1,300)-48*U(I,2,300)+36*U(I,3,300)-
16*U(I,4,300)+3*U(I,5,300));
end
end
end

fprintf('%s,\t%s,\t%s,\t%s,\t%s \n','NU','Ec', 'Re', 'Pe', 'M');
```

```
fprintf('%f,\t%d,\t%d,\t%d,\t%d \n',Ec, Re, Pe, M);
%fprintf('%f,\t%d,\t%d,\t%d,\t%d \n',Nu_x(1,K+1),Ec, Re, Pe, M);
        figure(1)
         %grid on
        plot(y,U(10,:,800),'-.');
        title('GRAPH OF VELOCITY vs DISTANCE ALONG THE SURFACE');
        xlabel('DISTANCE');
        ylabel('VELOCITY U');
        hold on

         %figure(2)
          %grid on
        %plot(y,V(20,:,300),'-.');
        %title('GRAPH OF VELOCITY vs DISTANCE ALONG THE SURFACE');
        %xlabel('DISTANCE');
        %ylabel('VELOCITY V');
       % hold on

        figure(3)
         %grid on
        plot(y,T(10,:,800),'-.');
        title('GRAPH OF TEMPERATURE vs DISTANCE ALONG THE
SURFACE');
        xlabel('DISTANCE');
        ylabel('TEMPERATURE T');
        hold on
    %end
%end
```