Effects of Flow Parameters on Flow Variables of a Newtonian Fluid through a Cylindrical Collapsible Tube

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DECLARATION

This thesis is my original work and has not been presented for a degree in any other University.

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This thesis has been submitted for examination with our approval as University supervisors.

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DEDICATION

This thesis is dedicated to my parents and siblings.

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I am grateful to God for grace to carry out this research, to my family for the support and encouragement throughout this research study. I would like to thank my supervisors Dr. Kang'ethe Giterere and Prof. Mathew Kinyanjui for their intelligent supervision and inspiration. You have inspired me greatly through constant encouragement and objective analysis of my work. The work described in this thesis could not have been done without your generous and never ending help and guidance.

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NOMENCLATURE

Symbol	**Meaning**
T	Longitudinal tension, N
P	Internal pressure, Pa
$\mathbf{P_E}$	External pressure, Pa
$\mathbf{P_t}$	Transmural pressure ($P-P_E$), Pa
u	Fluid velocity, ms^{-1}
Q	Volumetric flow rate, m^3s^{-1}
A	Cross sectional area, m^2
$\mathbf{A_0}$	Area at the inlet, m^2
S	Peripheral length, *m*
$\mathbf{K_{PE}}$	Tube stiffness, Pa
$\mathbf{F_L}$	Skin friction co-efficient

Greek symbol

υ	Kinematic viscocity of the fluid, m^2s^{-1}
ρ	Fluid density, kgm^{-3}
μ	Coefficient of viscosity, $kgm^{-1}s^{-1}$

Abbreviations

FD	Finite Difference
PDE's	Partial Differential Equations

ABSTRACT

In this study the effects of longitudinal tension, tube stiffness and volumetric flow rate on the cross sectional area of a collapsible tube, flow velocity and internal pressure of a Newtonian fluid through a cylindrical collapsible tube have been determined. The tube was considered collapsible in the transverse direction, taken to be perpendicular to the main flow direction. Collapse happens when external pressure exceeds internal pressure and hence the tube results to a highly noncircular cross sectional area. The fluid flow in consideration was steady and incompressible. Equations governing the flow are non-linear and cannot be solved analytically. Therefore an approximate solution to the equations was determined numerically. In this case, finite difference method was used. A computer program then was used to generate the results which were presented in form of graphs. The results show that the longitudinal tension and tube stiffness are directly proportional to both the cross sectional area and internal pressure and inversely proportional to the flow velocity and that change in volumetric flow rate has no effect on the cross sectional area but it is directly proportional to the flow velocity and inversely proportional to the internal pressure.

CHAPTER ONE

1.0 INTRODUCTION

The study of flow through collapsible tubes is of utmost importance in biological studies as well as in industries. For instance, the dynamics of fluid flow in collapsible tube are vital in understanding the behavior and analysis of flow phenomenon in veins, arteries, airways, urethra, etc. Rosar and Charles (2001) explained that vessel collapse is most readily seen in the veins, such as in the veins of a hand raised above the level of the heart or in the jugular vein when a person is standing upright. Collapse also occurs in the arteries when high external pressures are applied, such as when an artery is compressed by a sphygmomanometer cuff during blood pressure measurement.

Jensen and Heil (2003) explained that under normal conditions arteries are under sufficiently large transmural (internal minus external) pressure and remain distended and stiff. Important exceptions are the coronary arteries, embedded in the muscular wall of the heart, which can be significantly constricted as the heart contracts and the brachial artery, which is compressed by a cuff inflated around the upper arm during blood-pressure measurement, in which case flow-induced instabilities generate clinically useful "Korotkoff sounds". Veins operate under much lower transmural pressures than arteries so that hydrostatic pressure variations (in systemic veins above the heart but outside the skull, or in the pulmonary circulation) can be sufficient to induce collapse (i.e. a significant reduction in cross-sectional area, but without complete occlusion), which can limit the flow of blood returning to the heart or passing through major organs such as the lungs. Venous collapse, however, is important during exercise, when muscular compression of leg veins is used to pump blood against gravity up to the heart, and in therapeutic compression of leg veins for the treatment of deep-vein thrombosis. Shapiro (1977) explained that the pulmonary system also displays collapse, for example, the airways of the lungs can show collapse during coughing or sneezing and during forced or rapid expiration.

Similarly, in the industry collapse may be experienced during cementing operations, trapped fluid expansion, or well evacuation, among many others. Most oilfield tubulars also experience collapse.

Fluid flow through collapsible tube is a complex phenomenon since it involves interaction between the flowing fluid and the tube wall. A mathematical model of physical phenomenon, often results in non-linear equations for some unknown function. Usually the problem cannot be solved analytically. Makinde (2005) noted that the nonlinear problems can be solved by expansion in powers of some small perturbation parameter. The advantage of this approach is that it reduces the original nonlinear problem to a sequence of linear problems. In this research study finite difference method was used because of its consistency, stability and convergence rate. This research work focused on investigating the effects of flow parameters on flow variables of a Newtonian fluid through a cylindrical collapsible tube thus expanding the understanding of fluid flow through collapsible tubes.

DEFINITIONS

In this study several terms have been used extensively and such terms are defined in this section.

1.1.1 Fluid

Fluid is a type of matter which undergoes continuous deformation when some external force is applied. It is said to undergo deformation if the distance between any two neighboring molecules change. Fluids are conventionally classified as liquids and gases.

Fluid flow may be termed as laminar or turbulent. The term laminar is used to refer to a fluid flow in which fluid particles move in an orderly manner in layers parallel to the solid boundary. Turbulent flow is characterized by eddies that cause mixing of layers of the fluid until the layers are no longer distinguishable. Quantities such as velocity and pressure show random variation with time and space.

1.1.2 Newtonian Fluid

A fluid is said to be Newtonian if it obeys the Newton's law of viscosity which states that the shear stress is proportional to the velocity gradient. The viscosity does not change with the rate of flow.

1.1.3 Collapsible tube

A collapsible tube is any tube with sufficiently flexible walls that it can elastically accommodate deformation to a highly noncircular cross section when the external pressure exceeds the internal pressure. In this study, the collapsible tube in consideration is cylindrical in shape.

1.1.4 Incompressible fluid

A fluid is said to be incompressible if changes of pressure cause practically no change in the fluid density. This means that, for an incompressible fluid, rate of change of density is assumed to be zero and hence the divergence of velocity is zero.

1.1.5 Steady flow

For a steady flow, all fluid properties for example velocity, temperature, pressure and density are independent of time. The properties however, may vary from point to point, which means that they could be functions of space coordinates.

LITERATURE REVIEW

Flow through collapsible tubes has been extensively studied in the laboratory. Pioneering work on collapsible tubes, explain that the veins play an important role in controlling the output of the heart. This control function of the veins is a passive one, and is as a result of their ability to collapse and inflate. Several experimental studies have been done on flexible tubing. Edward and Abraham (1972) explained that these experiments are based upon the assumption that the differences between them and veins are quantitative in nature rather than qualitative.

Bertram (1986) did an experimental study on collapsible and elastic tubes with finite-length and the upstream and downstream ends held open. Fluid, typically water or air was driven through the tube, either by applying a controlled pressure-drop between the ends of the rigid tubes or by controlling the flow rate. When the external pressure exceeded the fluid pressure by a sufficiently large amount, the tube buckled non-axi symmetrically, leading to a nonlinear relation between pressure-drop and flow rate. Bertram *et al.(*1990) explained that at sufficiently large Reynolds numbers, the system produces self-excited oscillations, and exhibits hysteresis in transitions between dynamical states, multiple modes of oscillations (each having distinct frequency range), rich and complex nonlinear dynamics.

A further study on self-excited oscillations was carried out by Luo and Pedley (1998) who investigated the effect of wall inertia on the self-excited oscillations in a collapsible channel flow. The effects of different values of mass ratio on the flow in a two-dimensional collapsible channel were studied. It was found that for mass ratio= 0.01, a value relevant to blood flow in arteries and veins, or experiments with water as the fluid, wall inertia had a negligible effect on the self-excited oscillations. This implied self-excited oscillations. For mass ratio= 0.1, a value more relevant to air flow in the lung, or experiments with air as fluid, flutter-type oscillations were found to develop and play an important role in destabilizing the system. The presence of wall inertia (with mass ratio=0.1) also increased the critical value of the tension below which the system became unstable.

Jensen and Heil (2003) also realized that nonaxisymmetrically collapsed vessels readily develop flow-induced, self excited oscillations. Physiological examples include wheezing during forced expiration and the development of korottkoff sounds during blood pressure measurement. This is in agreement with the study of Matthias and Sarah (2008). They studied finite-Reynolds-number flows in three-dimensional collapsible tubes whose walls perform prescribed high-frequency oscillations of finite amplitude. The analysis of the system's energy budget helped identify conditions under which the wall extracts energy from the mean flow. The main source of energy was shown to be the influx of kinetic energy generated by the axial sloshing flows that are driven by the oscillatory wall motion; the wall extracted energy from the flow if the net influx of kinetic energy exceeded the viscous dissipation in the flow. In a fully coupled fluid–structure interaction problem in which the wall motion is not prescribed, any energy extracted from the flow would be transferred to the wall's strain and kinetic energies, and therefore lead to an increase in the amplitude of the wall motion. He concluded that self-excited oscillations of collapsible tubes are much more likely to develop from steady-state configurations in which the tube is buckled non-axisymmetrically rather than from axisymmetric steady state.

Several studies have also been carried out on steady flow through a collapsible tube. Hazel and Heil (2003) investigated the steady flow through thin-walled elastic tubes for a finite Reynolds number. In their finite-element approach, they solved the steady 3-D Navier–Stokes equations simultaneously with the equations of geometrical nonlinear, Kirchhoff–Love thin-shell theory. One of the assumptions underlying thin-shell theory is that the wall thickness of the tube is some 20 or more times smaller than its radius.They showed how nonaxisymmetric buckling of the tube contributes to nonlinear pressure-flow relations that can exhibit flow limitation through purely viscous mechanisms.

Andrew and Matthias (2003) further investigated the steady flow of a viscous fluid through a thin-walled elastic tube mounted between two rigid tubes. The steady three-dimensional Navier–Stokes equations were solved simultaneously with the equations of geometrically nonlinear Kirchhoff–Love shell theory. He explained that if the transmural (internal minus external) pressure acting on the tube is sufficiently negative then the tube buckles non-axisymmetrically and the subsequent large deformations leads to a strong interaction between the fluid and solid mechanics. The main effect of fluid inertia on the macroscopic behavior of the system is due to the Bernoulli effect, which induced an additional local pressure drop when the tube buckles and its cross-sectional area is reduced. Thus, the tube collapses more strongly than it would in the absence of fluid inertia.

Mawasha *et al.* (2001) investigated the dynamic behavior of a collapsible tube system consisting of a reservoir with inlet and outlet flow conditions. The reservoir was subjected to a constant inlet flow rate and the outlet flow rate and the pressure–flow rate relation of the downstream collapsible regime was presented by a constitutive model containing a cubic nonlinearity. The relaxation oscillations observed for collapsible tube model were analogous to the behavior of the van der pol oscillator. The van der Pol oscillator is a canonical model for self-excited nonlinear oscillations exhibited in electrical circuits.

Brian (2003) formulated a mathematical model for a collapsible tube and developed a computational methodology to determine the best-fit values for parameters that describe constitutive behavior of the tube. Numerical results of varying multiple fluid flow parameters were presented for both an expanded and a collapsed tube. He realized that it is possible to estimate the best-fit tube stiffness parameters for a known fluid with experimental data about cross-sectional area ,external pressure and pressure at the inlet to the collapsible tube. He noted that the tube stiffness affected the cross sectional area of the tube and had no large influence on the internal pressure.

Makinde (2005) further described the fluid dynamics of a collapsible tube using a mathematical model. Numerical solutions were constructed for the problem using perturbation technique . The analysis of the resulting power series solutions were performed with a special type of Hermite-Padé approximants. He observed that the fluid axial velocity profile was parabolic with maximum value at centerline. He also noted that fluid axial velocity generally decreases with an increase in tube contraction due to the strong influence of the negative transmural pressure owing to marked reduction of rigidity.

Marzo *et al.* (2005) studied three-dimensional collapse of a steady flow through finite-length elastic tubes numerically. The Navier-Stokes equations coupled with large, nonlinear deformation of the elastic wall were solved using the finite-element software, FIDAP. Three-dimensional solid elements were used for the elastic wall, allowing the wall thickness to be specified. Previous findings by Hazel and Heil (2003) for thinner-walled tubing were confirmed and also they showed the existence of significant differences if a thick-walled tube is used.

Prashanta (2005) studied the problem of non- Newtonian and non-linear blood flow through a stenosed artery. Finite difference scheme was used to solve the unsteady nonlinear Navier-stokes equations in cylindrical coordinates assuming axial symmetry under laminar flow condition. The model was also employed to study the effects of the taper angle, wall deformation, severity of the stenosis within its fixed length, steeper stenosis of the same severity, nonlinearity and non-Newtonian rheology of the flowing blood on the flow field. Quantitative analysis was performed through numerical computations. He concluded that that the axial velocity profile assumed a flat shape in the presence of a converging tapering instead of a parabolic one for non-tapered artery when both are treated under stenotic conditions. He explained that if the tube is tapered, then inertial forces associated with the convective accelerations manifest themselves in an amount of the same order as viscous forces while the former compel the axial velocity profile to attain a flat shape.

Odejide *et al.* (2008) examined an incompressible viscous fluid flow and heat transfer in a collapsible tube. The non linear equations arising from the model were solved using perturbation series.It was noted that the fluid temperature increased with an increase in fluid Prandtl number with maximum value at the center.It was also noted that increase in Reynolds number led to an increase in the fluid temperature with maximum magnitude at the pipe center and minimum at the wall. The fluid velocity profile was noted to be parabolic in nature.

Andrew *et al.* (2008) described the role of venous valves in pressure shielding.A one-dimensional mathematical model of a collapsible tube, with the facility to introduce valves at any position, was used. It was found out that a valve decreased the dynamic pressures applied to a vein when gravity is applied by a considerable amount.

Liu *et al.* (2009) explained that the wall stiffness is dominated by the axial tension.If the tension is sufficiently small, the viscous pressure drop along the channel induces large-amplitude, steady wall deformations. In the absence of wall inertia, the steady configurations become unstable to relatively low-frequency self-excited oscillations when the Reynolds number is increased .

Emilie and Patrice (2010) developed a simple and effective numerical physiological tool to help clinicians and researchers in the understanding of flow phenomena. One-dimensional Runge–Kutta discontinuous Galerkin (RK-DG) method coupled with lumped parameter models for the boundary conditions was used.Various benchmark problems that showed the flexibility and applicability of the numerical method were presented. The emptying process in a calf vein squeezed by contracting skeletal muscle in a normal and pathological subject was also studied and the results compared with experimental simulations. After the comparison of the results it was noted that the efficiency of muscular calf pump is strongly dependent on the valves pathology and the walking frequency.

Eleuterio and Annunziato (2013) fomulated a one-dimensional time-dependent non-linear mathematical model for physiological fluid flow in collapsible tubes with discontinuous material properties, i.e. vessel wall thickness,equilibrium cross sectional area and Young's modulus. In particular,a mathematical model for blood flow in medium to large arteries and veins was studied. The resulting 6×6 hyperbolic system was analysed and the associated Riemann problem solved exactly. They explained that although the solution algorithm dealt with idealised cases, it is uniquely well-suited for assessing the performance of numerical methods intended for simulating more general situations.

This research work has presented a one dimensional mathematical model of fluid flow through collapsible tube. The fluid flow in consideration is steady. From the above literature review, we realize that a comprehensive study considering a combination of various flow parameters such as volumetric flow rate,longitudinal tension and tube stiffness and their effects on the cross sectional area of a collapsible tube, flow velocity and internal pressure of fluid in a collapsible tube has not been done such has been the motivation behind this research. This research therefore aimed at coming up with a more comprehensive model of flow through collapsible tubes.

STATEMENT OF THE PROBLEM

Several studies of fluid flow through collapsible tubes have been done experimentally so as to shed more light on the nature of the flow. However very few studies have been done numerically. This research covers a comprehensive study of the effects of flow parameters such as volumetric flow rate, longitudinal tension and tube stiffness on the flow variables, i.e. cross sectional area of a collapsible tube, flow velocity and internal pressure of a Newtonian fluid flow through a collapsible tube. The collapsible tube in this case is considered to collapse in the transverse direction perpendicular to the main flow direction as shown in Figure 1.1.

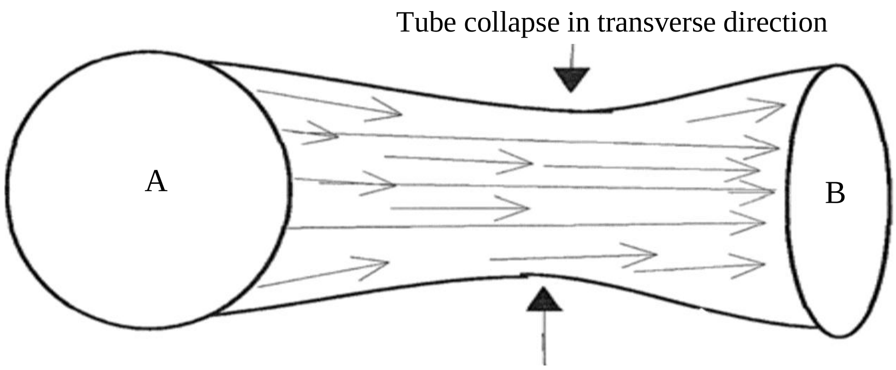

Figure 1.1 Collapsible tube under consideration. The ends A and B are fixed.

JUSTIFICATION

In the human body there are several fluid-conveying vessels that are elastic and collapsible. These vessels buckle nonaxisymmetrically when external pressure exceeds internal pressure.The study of flow through collapsible tubes is very useful for the study and prediction of many diseases, as the lung disease (asthma and emphysema), or the cardiovascular diseases (heart stroke).

In addition, the study is of great importance in prediction and prevention of possible collapse in industries during cementing operations, trapped fluid expansion, or well evacuation. Further applications are in the oil industry since most oilfield tubulars also experience collapse. The study also has applications such as in modelling submarine and aeronautical hydraullic system.

A study on flow parameters and flow variables is therefore necessary. This study aimed at coming up with a more comprehensive model of flow through collapsible tubes.The model described this work has the potential to increase understanding of fluid flow through collapsible tubes, and ultimately might be employed as part of an interventional planning tool.

NULL HYPOTHESIS

There is no effect of longitudinal tension and tube’s stiffness on cross sectional area of a collapsible tube, flow velocity and internal pressure of fluid in a collapsible tube.

OBJECTIVES

1.6.1 General objective

To determine the effect of the various flow parameters on the cross sectional area of a collapsible tube, the flow velocity and the internal pressure of a Newtonian fluid flow through a cylindrical collapsible tube

1.6.2 Specific objectives

1) To determine the effect of longitudinal tension on the cross sectional area, flow velocity and internal pressure.
2) To determine the effect of tube stiffness on the cross sectional area,flow velocity and internal pressure.
3) To determine the effect of volumetric flow rate on the cross sectional area, flow velocity and internal pressure.

In the next chapter, the general equations govening the flow through a collapsible tube are discussed. The assumptions for the flow are also outlined and specific equations governing the flow derived in reference to these assumptions. The method of solving these governing equations has also been discussed.

CHAPTER TWO

2.0 EQUATIONS GOVERNING THE FLOW

In this chapter the assumptions made for the flow through a collapsible tube have been outlined. The general governing equations which are used to derive the specific equations for flow through a collapsible tube are discussed.

2.1 ASSUMPTIONS

i. The flow in the tube is one-dimensional. The flow variables at a given instant in time only vary in the direction of flow.
ii. There is no flow separation occurring in a collapsed region of the tube.
iii. Tensile force throughout the collapsible tube is constant.
iv. The flow in the tube is steady meaning that the fluid properties are time independent.
v. The fluid is incompressible. This means that the density of the fluid is assumed to be a constant.

2.2 EQUATIONS GOVERNING THE FLOW

Flow through collapsible tube is basically governed by continuity equation, equation of conservation of momentum and the tube law. These equations have been discussed.

2.2.1 Equation of continuity

This equation arises from the fact that matter is neither created nor destroyed. The rate at which mass enters a system is equal to the rate at which mass leaves the system.

The differential form for a general continuity equation is given by;

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \vec{u} \right) = 0 \tag{2.1}$$

Where ρ is the fluid's density and $\vec{u}$ is the fluid's velocity.

For incompressible fluid flow, ρ is assumed to be a constant and hence equation (2.1) reduces to;

$$\nabla.\vec{u} = 0 \tag{2.2}$$

Equation (2.2) means that the divergence of velocity is zero.

In Cartesian co-ordinate form and considering a one dimensional fluid flow equation (2.2) is expressed as;

$$\frac{\partial \vec{u}}{\partial x} = 0 \tag{2.3}$$

The volumetric flow rate Q is given by area multiplied by velocity of the fluid, therefore equation (2.3) becomes;

$$\frac{\partial (A\vec{u})}{\partial x} = \frac{\partial Q}{\partial x} = 0 \tag{2.4}$$

Equation (2.4) is derived from the fact that mass is always conserved in fluid systems regardless of the pipeline complexity or direction of flow. The volumetric flow rate Q is constant but area and velocity of the fluid flow are variable so that if A decreases, $\vec{u}$ increases and vice versa.

2.2.2 Equation of conservation of momentum

The equation of conservation of momentum is derived from Newton's second law of motion, which states that the time rate change of momentum of a body matter is equal to the net external forces applied to the body. The external forces are divided into two, i.e. surface forces and body forces. The surface forces are exerted on the fluid element by its surroundings through direct contact at the surface. For example, forces due to static pressure and viscous stresses. The body forces are distributed over the entire mass or volume of the element usually expressed per unit mass of the element. Examples of body forces are gravitational force, centrifugal force and magnetic force. For this particular research problem, the body forces will be neglected since the collapsible tube is assumed to be horizontal. Of particular interest is to resolve the viscous forces for laminar fluid flow through a cylindrical collapsible tube.

The general equation is given by;

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - Ru + g \qquad (2.5)$$

where R>0 is a friction factor and g is the acceleration due to gravity when the tube is held vertically.

According to Shapiro (1977) and Hayashi *et.al* (1998) the the equation of conservation of momentum for steady laminar fluid flow is given by;

$$\rho u\frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x} - f_L\frac{\mu u}{D_e}\left(\frac{s}{A}\right), \qquad (2.6)$$

where s the peripheral length and f_L is the skin-friction coefficient for laminar flow.

Making the substitution $Q = Au$, equation (2.6) becomes;

$$\rho\frac{Q}{A}\frac{\partial\left(\frac{Q}{A}\right)}{\partial x} = -\frac{\partial p}{\partial x} - f_L\frac{\mu}{D_e}\frac{Q}{A}\frac{s}{A} \qquad (2.7)$$

which yields;

$$\frac{Q}{A}\frac{1}{A^2}\left(A\frac{\partial Q}{\partial x}-Q\frac{\partial A}{\partial x}\right)=-\frac{1}{\rho}\frac{\partial p}{\partial x}-\frac{1}{\rho}f_L\frac{\mu}{D_e}\frac{Q}{A}\frac{s}{A} \tag{2.8}$$

From equation (2.4), $\frac{\partial(Au)}{\partial x}=\frac{\partial Q}{\partial x}=0$, we have

$$\frac{A^3}{\rho}\frac{\partial p}{\partial x}=Q^2\frac{\partial A}{\partial x}-\frac{1}{\rho}f_l\frac{\mu}{D_e}\frac{s}{A}A^3 \tag{2.9}$$

Making $\frac{\partial \rho}{\partial x}$ the subject, equation (2.9) becomes;

$$\frac{\partial p}{\partial x}=\rho\frac{Q}{A^3}^2\frac{\partial A}{\partial x}-f_l\frac{\mu}{D_e}\frac{Q}{A}\frac{s}{A} \tag{2.10}$$

with the boundary condition

$P(0) = P_{o.}$ (2.11)

where P_o is the inlet pressure and P(0) is the pressure at the entrance of the tube.

2.2.3 The tube law

The tube law relates the transmural pressure ($P-P_E$) to the cross-sectional area of an ellastic tube. According to (Shapiro 1977), it is given by

$$p-p_E=\phi\left(\frac{A}{A_0}\right), \tag{2.12}$$

where the left hand side of the equation represents transmural pressure and the right hand side is a function of the ratio of the tube's cross-sectional area to the initial area. Putting into consideration the longitudinal tension in the tube law (Cancelli and Pedley (1985), equation (2.10) becomes

$$p-p_E=\phi\left(\frac{A}{A_o}\right)-T\frac{\partial^2 A}{\partial x^2}, \tag{2.13}$$

where T is the longitudinal tension the tube is subjected to. In the highly collapsed region the tube wall resembles two flattish membranes under tension, with the longitudinal curvature roughly propotional to $\frac{\partial^2 A}{\partial x^2}$. The function $\phi\left(\frac{A}{A_0}\right)$ is defined as

$$\phi\left(\frac{A}{A_0}\right) = \begin{cases} K_p\left(1-\frac{A}{A_o}\right)^{-\frac{3}{2}} if\ 0 < A \le A_0 \\ K_E\left(\frac{A}{A_0}-1\right)\ if\ A > A_0 \end{cases} \tag{2.14}$$

where K_E and K_p are the tube's stiffness when the tube is extended and collapsed respectively. The boundary conditions for equation (2.13) are as follows:

A(0) =A_0, A(L) = A_0

where A(0) and A(L) represent the area at the entrance and the end of the tube respectively, and A_0 represents area at the inlet.

However, according to Shapiro (1977**),** equation (2.14) reduces to the function

$$\phi(\alpha) = K_{PE}(\alpha^n - \alpha^{-\frac{3}{2}}) \tag{2.15}$$

where $\alpha = \frac{A}{A_0}$ and K_{PE} is the combined stiffness, which represents the overall stiffness of the tube, whether collapsed or distended. The value n=10 has been established as the standard, and has been used by many authours: among them Pedley (1980, 1996); Elad, *et al*. (1991); Brook and Pedley (2002).

Equation (2.13) reduces to

$$p - p_E = K_{PE}(\alpha^n - \alpha^{-\frac{3}{2}}) - T\frac{\partial^2 A}{\partial x^2} \tag{2.16}$$

with the boundary conditions: A(0) =A_0, A(L) = A_0

2.3 METHOD OF SOLUTION

The system of non-linear equations obtained for this particular flow problem, i.e. equations (2.10) and (2.16), will be solved using the numerical approximation method of finite differences. The derivatives in the governing equations will be replaced by their corresponding finite approximations.

Equation (2.10) will be discretized using forward differencing for area and pressure derivatives. One can approximate the term $\frac{df(x)}{dx}$ at discrete node points by first taking the Taylor expansion of $f(x)$ evaluated at the points x_i and x_{i-1} expanded about a node point $x = x_i$. Assuming that $f(x) \in c^2[x_{i-1}, x_i]$ we get that;

$$f(x_{i-1}) = f(x_i) - k\frac{df(x_i)}{dx} + \frac{k^2}{2}\frac{d^2 f(n_i)}{dx^2} + \ldots \text{ for some } n_i \in [x_{i-1}, x_i] \qquad (2.17)$$

Hence:-

$$f(x_i) - f(x_{i-1}) = k\frac{df(x_i)}{dx} - \frac{k^2}{2}\frac{d^2 f(n_i)}{dx^2} + \ldots . \qquad (2.18)$$

Thus the approximation,

$$\frac{df(x_i)}{dx} = \frac{f(x_i) - f(x_{i-1})}{k} \qquad (2.19)$$

which gives the general form for first order derivatives written in finite difference form. The higher powers are dropped because since their value tends to zero, *k* is the step size and its value when raised to a power tends to zero.

Discretizing equation (2.10) for area and pressure derivatives yields;

$$p_i = p_{i-1} + \frac{\rho Q^2 (A_i - A_{i-1})}{A_{im}(i)^3} - \Delta x f_L \frac{\mu}{D_e}\frac{Q}{A_{im}(i)}\frac{s_i}{A_{im}(i)} \qquad (2.20)$$

where s_i is the peripheral length and is expressed as $s_i = 2\pi\sqrt{\frac{A_i}{\Pi}}$, $A_{im}(i)$ is the area at pressure nodes and is expressed as $A_{im}(i) = \frac{A_i - A_{i-1}}{2}$ $i = 2...N$ and D$_e$ is the hydraulic diameter expressed as D$_e$ = 2 r .

Equation (2.16) will also be discretized for the cross sectional area by making use of the finite difference's central differencing scheme.At discrete points approximation to the term $\frac{\partial^2 A(x)}{\partial x^2}$ is needed. The Taylor expansion of A(x) about a node point x=x$_i$ evaluated at the nodes x_{i-1} and x_{i+1} with x_i - x_{i-1} =k for i=1…N is,

$$A(x_{i+1}) = A(x_i) + k\frac{\partial A(x_i)}{\partial x} + \frac{k^2}{2}\frac{\partial^2 A(x_i)}{\partial x^2} + \frac{k^3}{3!}\frac{\partial^3 A(x_i)}{\partial x^3} + ... \quad (2.21)$$

$$A(x_{i-1}) = A(x_i) - k\frac{\partial A(x_i)}{\partial x} + \frac{k^2}{2}\frac{\partial^2 A(x_i)}{\partial x^2} - \frac{k^3}{3!}\frac{\partial^3 A(x_i)}{\partial x^3} + ... \quad (2.22)$$

Summing these equations yields;

$$A(x_{i+1}) + A(x_{i-1}) = 2A(x_i) + k^2\frac{\partial^2 A(x_i)}{\partial x^2}. \quad (2.23)$$

Making the term $\frac{\partial^2 A(x_i)}{\partial x^2}$ the subject of the equation yields;

$$\frac{\partial^2 A(x_i)}{\partial x^2} = \frac{1}{k^2}\left(A(x_{i-1}) + A(x_{i+1}) - 2Ax_i\right) \quad (2.24)$$

Equation (2.16) is thus discretized with central differencing of $\frac{\partial^2 A}{\partial x^2}$ to yield;

$$T\frac{A_{i+1} - 2A_i + A_{i-1}}{\Delta x^2} = p_E - p_i + K_{PE}\left(\alpha^{10} - \alpha^{-\frac{3}{2}}\right). \quad (2.25)$$

The right hand side of equation (2.25) is linearized using the Taylor expansion of the term $K_{PE}\left(\alpha^{10} - \alpha^{-\frac{3}{2}}\right)$ expanded about the point A_i=c to get equation (2.26) as:

$$T\,\frac{A_{i+1} - 2A_i + A_{i-1}}{\Delta x^2} = p_E - p_i + K_{PE}\left[\left(\frac{c}{A_0}\right)^{10} - \left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right] + (A_i - c)K_{PE}\left[\frac{10}{c}\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2c}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right]$$

…………………. (2.26)

In the next chapter, equation (2.26) has been rearranged and written in Matrix form. A tridiagonal matrix has also been derived. The results obtained after solving these equations using MATLAB program have been presented and discussed.

CHAPTER THREE

3.0 RESULTS AND DISCUSSIONS

In this chapter the specific equations governing the flow have been presented and written in finite difference form. They have been further written in the equivalent matrix form. The effects of various flow parameters i.e. tube stiffness, longitudinal tension and volumetric flow rate on cross sectional area; flow velocity and internal pressure are presented in form of graphs.

3.1 EQUATIONS GOVERNING THE FLUID FLOW IN FINITE DIFFERENCE FORM.

The governing equations describing the steady, incompressible fluid flow through a cylindrical collapsible tube, i.e. equations (2.10) and (2.16), in finite difference form are given subject to their boundary conditions as:

$$p_i = p_{i-1} + \frac{\rho Q^2 \left(A_i - A_{i-1}\right)}{A_{im}(i)^3} - \Delta x f_L \frac{\mu}{D_e} \frac{Q}{A_{im}(i)} \frac{s_i}{A_{im}(i)} \tag{3.0}$$

Subject to the boundary condition;

$$P(0)=P_{o}. \tag{3.1}$$

where P_o is the pressure at the inlet.

and

$$T \frac{A_{i+1} - 2A_i + A_{i-1}}{\Delta x^2} = p_E - p_i + K_{PE}\left[\left(\frac{c}{A_0}\right)^{10} - \left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right] + \left(A_i - c\right)K_{PE}\left[\frac{10}{c}\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2c}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right] \tag{3.2}$$

Rearranging equation (3.2) in order to put the like terms together yields;

$$T\frac{A_{i+1}+A_{i-1}}{\Delta x^2}+A_i\left[\frac{-2T}{\Delta x^2}+K_{PE}\left(\frac{10}{c}\left(\frac{c}{A_0}\right)^{10}+\frac{3}{2c}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right)\right]=$$
$$p_E-p_i+K_{PE}\left[\left(\frac{c}{A_0}\right)^{10}-\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right]-K_{PE}\left[10\left(\frac{c}{A_0}\right)^{10}+\frac{3}{2}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right] \qquad (3.3)$$

Equation (3.3) is subject to the following boundary conditions;

$A_1 = A_0$, $A_N = A_0$

where A_0 is the area at the inlet.

Equation (3.3) is applied at all the nodes and a system of linear algebraic equations is obtained as shown below.

i=2:

$$T\frac{A_3+A_1}{\Delta x^2}+A_2\left[\frac{-2T}{\Delta x^2}+K_{PE}\left(\frac{10}{c}\left(\frac{c}{A_0}\right)^{10}+\frac{3}{2c}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right)\right]=$$
$$p_E-p_2+K_{PE}\left[\left(\frac{c}{A_0}\right)^{10}-\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right]-K_{PE}\left[10\left(\frac{c}{A_0}\right)^{10}+\frac{3}{2}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right] \qquad (3.4)$$

i=3:

$$T\frac{A_4+A_2}{\Delta x^2}+A_3\left[\frac{-2T}{\Delta x^2}+K_{PE}\left(\frac{10}{c}\left(\frac{c}{A_0}\right)^{10}+\frac{3}{2c}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right)\right]=$$
$$p_E-p_3+K_{PE}\left[\left(\frac{c}{A_0}\right)^{10}-\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right]-K_{PE}\left[10\left(\frac{c}{A_0}\right)^{10}+\frac{3}{2}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right] \qquad (3.5)$$

i=4:

$$T\,\frac{A_5 + A_3}{\Delta x^2} + A_4\left[\frac{-2T}{\Delta x^2} + K_{PE}\left(\frac{10}{c}\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2c}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right)\right] = \tag{3.6}$$

$$p_E - p_4 + K_{PE}\left[\left(\frac{c}{A_0}\right)^{10} - \left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right] - K_{PE}\left[10\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right]$$

.

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i=N-2:

$$T\,\frac{A_{N-1} + A_{N-3}}{\Delta x^2} + A_{N-2}\left[\frac{-2T}{\Delta x^2} + K_{PE}\left(\frac{10}{c}\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2c}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right)\right] = \tag{3.7}$$

$$p_E - p_{N-2} + K_{PE}\left[\left(\frac{c}{A_0}\right)^{10} - \left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right] - K_{PE}\left[10\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right]$$

i=N-1:

$$T\,\frac{A_N + A_{N-2}}{\Delta x^2} + A_{N-1}\left[\frac{-2T}{\Delta x^2} + K_{PE}\left(\frac{10}{c}\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2c}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right)\right] = \tag{3.8}$$

$$p_E - p_{N-1} + K_{PE}\left[\left(\frac{c}{A_0}\right)^{10} - \left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right] - K_{PE}\left[10\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right]$$

The values of the dependent variable A at i=1(the lower boundary) and at i=N (the upper boundary) are given, therefore, the first equation (3.4) and the last equation (3.8) can be written as;

$$T\frac{A_3}{\Delta x^2} + A_2\left[\frac{-2T}{\Delta x^2} + K_{PE}\left(\frac{10}{c}\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2c}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right)\right] =$$
$$p_E - p_2 + K_{PE}\left[\left(\frac{c}{A_0}\right)^{10} - \left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right] - K_{PE}\left[10\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right] - T\frac{A_0}{\Delta x^2} \qquad (3.9)$$

and

$$T\frac{A_{N-2}}{\Delta x^2} + A_{N-1}\left[\frac{-2T}{\Delta x^2} + K_{PE}\left(\frac{10}{c}\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2c}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right)\right] =$$
$$p_E - p_{N-1} + K_{PE}\left[\left(\frac{c}{A_0}\right)^{10} - \left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right] - K_{PE}\left[10\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right] - T\frac{A_0}{\Delta x^2} \qquad (3.10)$$

When these equations are represented in a matrix form, the coefficient matrix is tridiagonal meaning that it has non-zero elements only on the main diagonal, the first diagonal below this and the first diagonal above the main diagonal. The matrix system is of the form $B\vec{A} = \vec{R}$ where B is the tridiagonal matrix.

To write the equations in matrix form, let β represent

$$K_{PE}\left(\frac{10}{c}\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2c}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right) - \frac{2T}{\Delta x^2}$$

and λ represent $p_E + K_{PE}\left[\left(\frac{c}{A_0}\right)^{10} - \left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right] - K_{PE}\left[10\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right]$.

The matrix system of the form $B\,\vec{A} = \vec{R}$ is as shown below.

$$\begin{pmatrix} \beta & \frac{T}{\Delta x^2} & 0 & \cdot & \cdot & \cdot & \cdot \\ \frac{T}{\Delta x^2} & \beta & \frac{T}{\Delta x^2} & 0 & \cdot & \cdot & \cdot \\ 0 & \frac{T}{\Delta x^2} & \beta & \frac{T}{\Delta x^2} & 0 & \cdot & \cdot \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdot & \cdot & \cdot & 0 & \frac{T}{\Delta x^2} & \beta & \frac{T}{\Delta x^2} \\ \cdot & \cdot & \cdot & \cdot & 0 & \frac{T}{\Delta x^2} & \beta \end{pmatrix} \begin{pmatrix} A_2 \\ A_3 \\ A_4 \\ A_5 \\ \vdots \\ A_{N-3} \\ A_{N-2} \\ A_{N-1} \end{pmatrix} = \begin{pmatrix} -p_2 + \lambda - T\frac{A_0}{\Delta x^2} \\ -p_3 + \lambda \\ -p_4 + \lambda \\ \vdots \\ -p_{N-2} + \lambda \\ -p_{N-1} + \lambda - T\frac{A_0}{\Delta x^2} \end{pmatrix}$$

The tridiagonal matrix B is given by;

$$\begin{pmatrix} K_{PE}\left(\frac{10}{c}\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2c}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right) - \frac{2T}{\Delta x^2} & \frac{T}{\Delta x^2} & 0 & \cdot & \cdot & \cdot & \cdot \\ \frac{T}{\Delta x^2} & K_{PE}\left(\frac{10}{c}\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2c}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right) - \frac{2T}{\Delta x^2} & \frac{T}{\Delta x^2} & 0 & \cdot & \cdot & \cdot \\ 0 & \frac{T}{\Delta x^2} & K_{PE}\left(\frac{10}{c}\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2c}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right) - \frac{2T}{\Delta x^2} & \frac{T}{\Delta x^2} & 0 & \cdot & \cdot \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdot & \cdot & \cdot & 0 & \frac{T}{\Delta x^2} & K_{PE}\left(\frac{10}{c}\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2c}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right) - \frac{2T}{\Delta x^2} & \frac{T}{\Delta x^2} \\ \cdot & \cdot & \cdot & \cdot & 0 & \frac{T}{\Delta x^2} & K_{PE}\left(\frac{10}{c}\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2c}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right) - \frac{2T}{\Delta x^2} \end{pmatrix}$$

And vector R is given by:

$$\vec{R} = \begin{pmatrix} -p_2 + p_E + k_{PE}\left(\left(\frac{c}{A_0}\right)^{10} - \left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right) - k_{PE}\left[10\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right] - \frac{T}{\Delta x^2}A_0 \\ -p_4 + p_E + k_{PE}\left(\left(\frac{c}{A_0}\right)^{10} - \left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right) - k_{PE}\left[10\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right] \\ -p_4 + p_E + k_{PE}\left(\left(\frac{c}{A_0}\right)^{10} - \left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right) - k_{PE}\left[10\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right] \\ \vdots \\ -p_{N-2} + p_E + k_{PE}\left(\left(\frac{c}{A_0}\right)^{10} - \left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right) - k_{PE}\left[10\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right] \\ -p_{N-1} + p_E + k_{PE}\left(\left(\frac{c}{A_0}\right)^{10} - \left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right) - k_{PE}\left[10\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right] - \frac{T}{\Delta x^2}A_0 \end{pmatrix}$$

3.1.1 Data Representation

The tridiagonal matrix obtained along with vector R were used to obtain the following graphs using MATLAB program code. The values for the flow variables i.e. cross sectional area, flow velocity and internal pressure are taken at the point where the collapse of the tube is mostly felt, that being midway along the length of the tube.

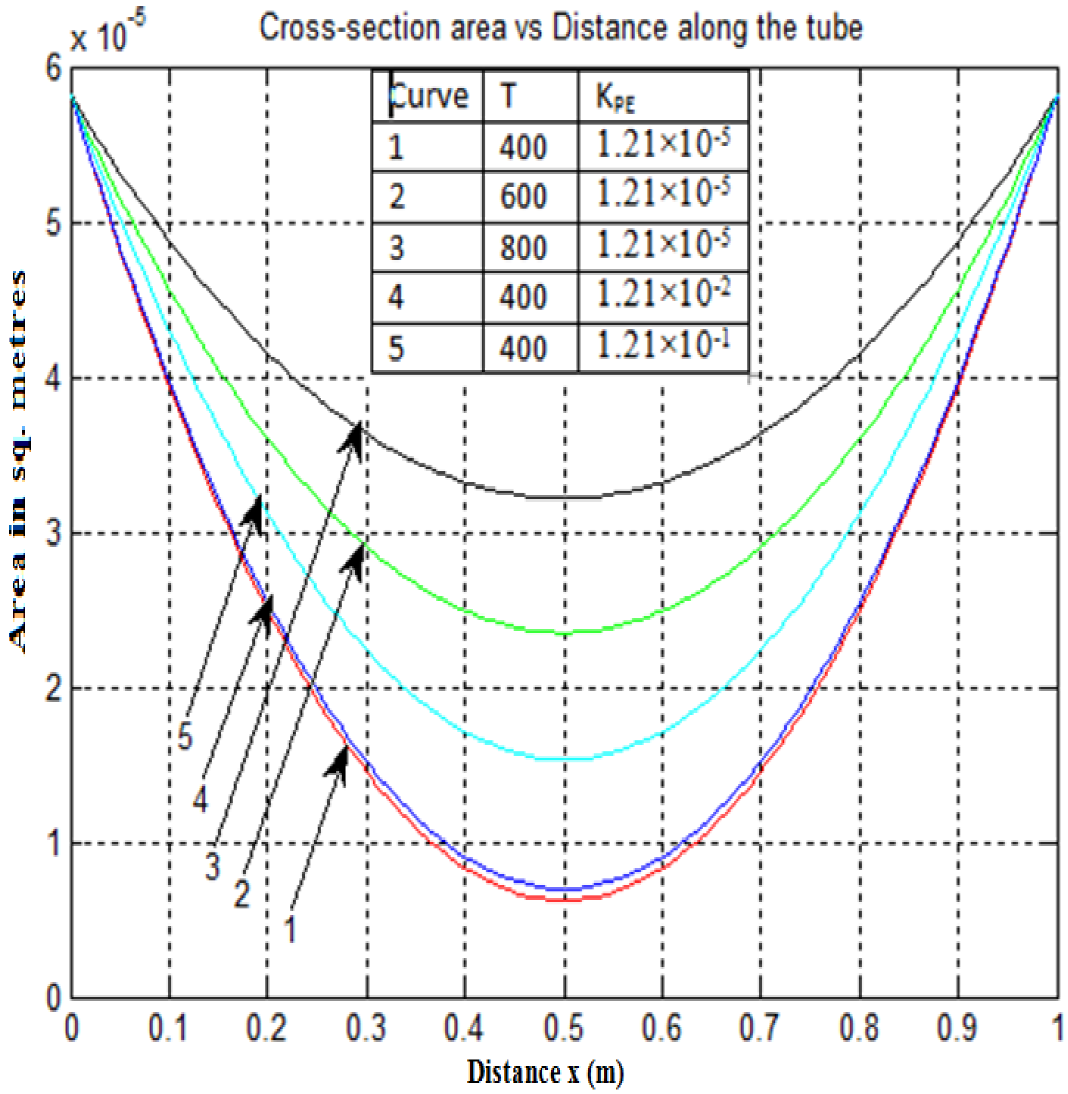
3.2. EFFECTS OF VARYING LONGITUDINAL TENSION AND TUBE STIFFNESS ON THE CROSS SECTIONAL AREA OF A COLLAPSIBLE TUBE.

3.2.0 Introduction

In order to determine the effect of longitudinal tension on the cross sectional area of the collapsible tube, the tube stiffness along with other parameters were held constant while the longitudinal tension was varied. Three curves were obtained. On the same axes, another two curves were plotted when the tube stiffness was varied while the other parameters were held constant. This is as shown in graph 3.1.

The effect of varying the longitudinal tension on the cross sectional area of the collapsible tube with tube stiffness held constant is shown by Curves 1,2 and 3 in Graph 3.1. On the same graph, the effect of varying tube stiffness on the cross sectional area of the collapsible tube with longitudinal tension constant is shown by Curves 1, 4 and 5.

GRAPH OF THE CROSS SECTIONAL AREA VERSUS DISTANCE ALONG THE COLLAPSIBLE TUBE WITH LONGITUDINAL TENSION AND TUBE STIFFNESS CHANGING.



Graph 3.1: Cross sectional area versus distance for $Q=5\times10^{-6}$, $\rho=1.0\times10^{3}$

$Pe=4.00\times10^{3}$ $r=4.3\times10^{-3}$

3.2.1 Results and Discussion

From Graph 3.1 it is observed that when the longitudinal tension was increased from 400N to 800N holding the tube stiffness constant (1.21×10^{-5}), the cross sectional area increased from 6.206×10^{-6} to 3.215×10^{-5}square meters. This is shown by Curves 1, 2 and 3.This can be explained by the reduction of the tube's tendency to collapse as the longitudinal tension increases which consequently leads to decrease in collapse hence increase in the cross sectional area.

From Graph 3.1, using Curves 1, 4 and 5, it is also noted that when the tube stiffness was increased from 1.21×10^{-5} to 1.21×10^{-1} holding the longitudinal tension constant, the cross sectional area increased from 6.206×10^{-6} to 1.531×10^{-5} square meters. This is because as the tube's stiffness increases, the tendency of the tube to collapse decreases hence the cross sectional area increases.

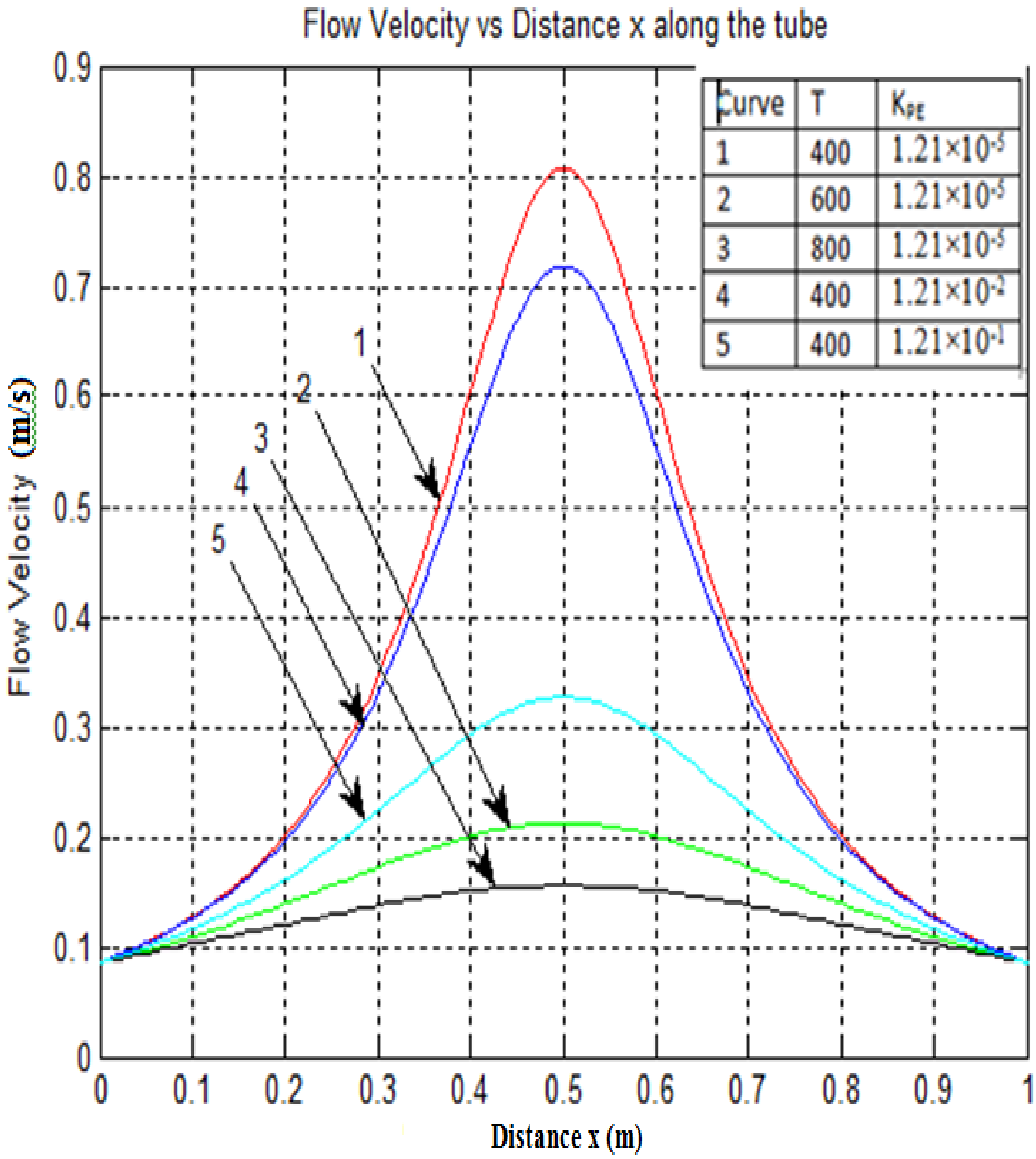
3.3 EFFECTS OF VARYING LONGITUDINAL TENSION AND TUBE STIFFNESS ON THE FLOW VELOCITY

3.3.0 Introduction

In this section the effects of the longitudinal tension and the tube stiffness on the velocity of the flow through a collapsible tube were investigated. The longitudinal tension was first varied while holding the tube stiffness constant and three curves were plotted. Similarly, when the tube stiffness was varied and longitudinal tension was held constant; two more curves were plotted on the same axes.

The resulting curves are as shown in Graph 3.2. The effect of varying the longitudinal tension on the flow velocity while holding the tube stiffness constant is shown by Curves1, 2 and 3. Curves 1, 4 and 5 on the same Graph 3.2, show the effect of varying the tube stiffness on the flow velocity when the longitudinal tension is held constant.

GRAPH OF FLOW VELOCITY VERSUS DISTANCE ALONG THE COLLAPSIBLE TUBE WITH LONGITUDINAL TENSION AND TUBE STIFFNESS CHANGING.



Graph 3.2: Flow velocity versus distance for $Q=5\times10^{-6}$, $\rho=1.0\times10^{3}$

$Pe=4.00\times10^{3}$ $r=4.3\times10^{-3}$

3.3.1 Results and Discussion

From Graph 3.2, it is observed that the longitudinal tension is inversely proportional to the flow velocity. As the longitudinal tension increases from 400N to 800N, the flow velocity decreases from 0.8051 to 0.1555 m/s. This is as shown by Curves 1, 2 and 3. The flow velocity decreases when the longitudinal tension increases because of the already increased cross sectional area. This happens in order to maintain a constant discharge.

Similarly, it is noted that as the tube stiffness increases, the flow velocity decreases. As the tube stiffness increases from 1.21×10^{-5} to 1.21×10^{-1}, the flow velocity decreases from 0.8051m/s to 0.3267 m/s. This is as shown by Curves 1, 4 and 5. The decrease in flow velocity is due to the already increase in cross sectional area which had resulted from a decrease in collapse. This helps maintain a constant volumetric flow rate. A decrease in collapse leads to a decrease in the flow velocity.

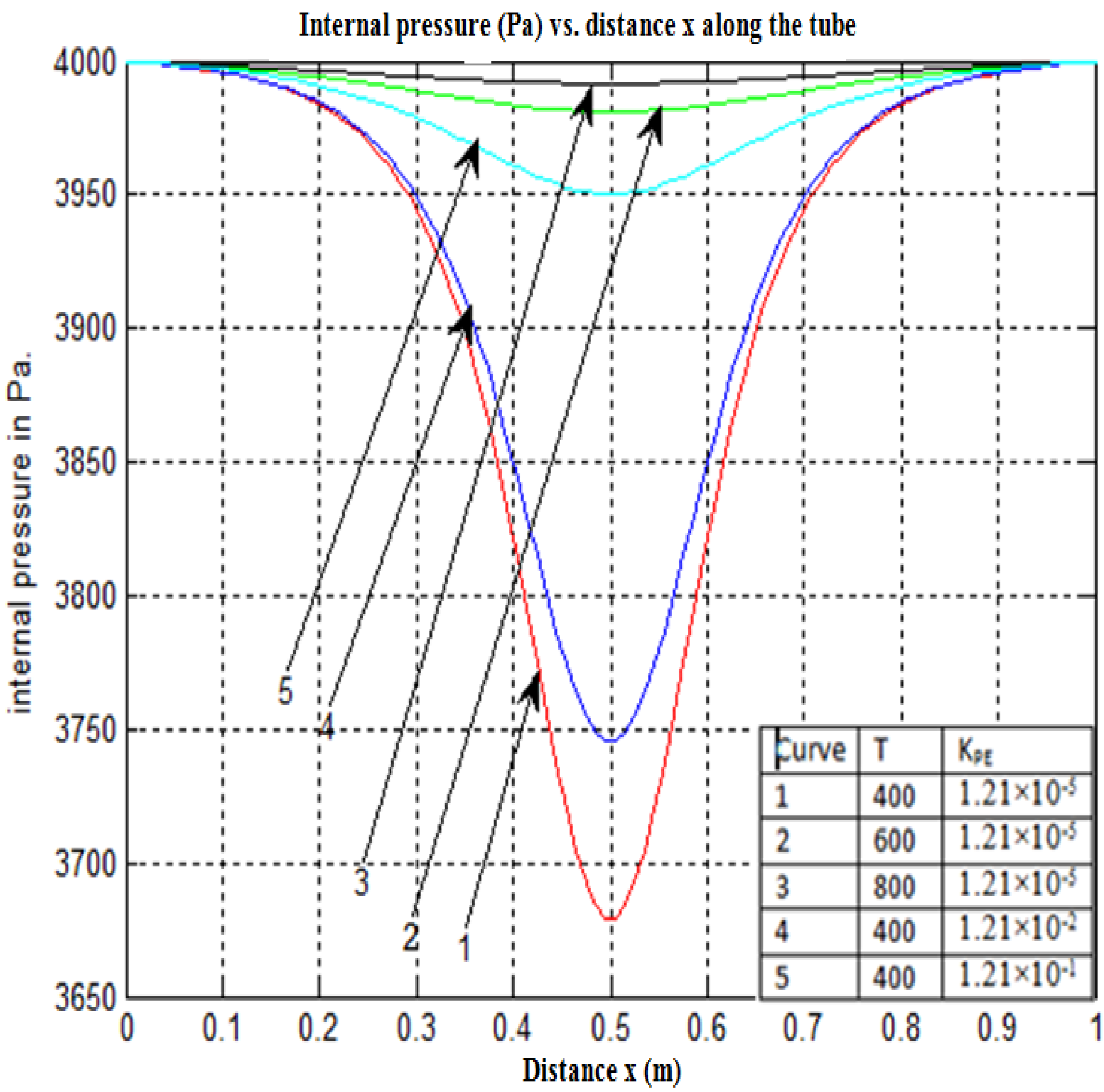
3.4 EFFECTS OF VARYING LONGITUDINAL TENSION AND TUBE STIFFNESS ON THE INTERNAL PRESSURE

3.4.0 Introduction

To investigate the effects of the longitudinal tension and the tube stiffness on the internal pressure of a collapsible tube, the tube stiffness was first held constant while the longitudinal tension was varied and three curves were obtained. The longitudinal tension was then held constant while the tube stiffness was varied. The five curves obtained were then plotted on the same axes as shown in Graph 3.3 below.

The effect of varying the longitudinal tension on the internal pressure while holding the tube stiffness constant is shown by Curves 1, 2 and 3, while the effect of varying the tube stiffness on the internal pressure when the longitudinal tension is held constant is shown by Curves 1, 4 and 5 in Graph 3.3.

GRAPH OF INTERNAL PRESSURE VERSUS DISTANCE ALONG THE COLLAPSIBLE TUBE WITH LONGITUDINAL TENSION AND TUBE STIFFNESS CHANGING



Graph 3.3: Internal pressure versus distance for $Q=5\times10^{-6}$, $\rho=1.0\times10^{3}$

$Pe=4.00\times10^{3}$ $r=4.3\times10^{-3}$

3.4.1 Results and Discussion

From Graph 3.3, it is observed that as the longitudinal tension increases the internal pressure increases. When the longitudinal tension is 400N the internal pressure is 3679 Pascals and when the longitudinal tension is increased to 800N, the internal pressure increases to 3992 Pascals. The increase in internal pressure as the longitudinal tension increases is due to the decrease in flow velocity. From Bernoulli principle, the sum of pressure energy at any part plus the kinetic energy per unit volume plus the potential energy per unit volume at that point is always constant and therefore a decrease in flow velocity leads to an increase in pressure.

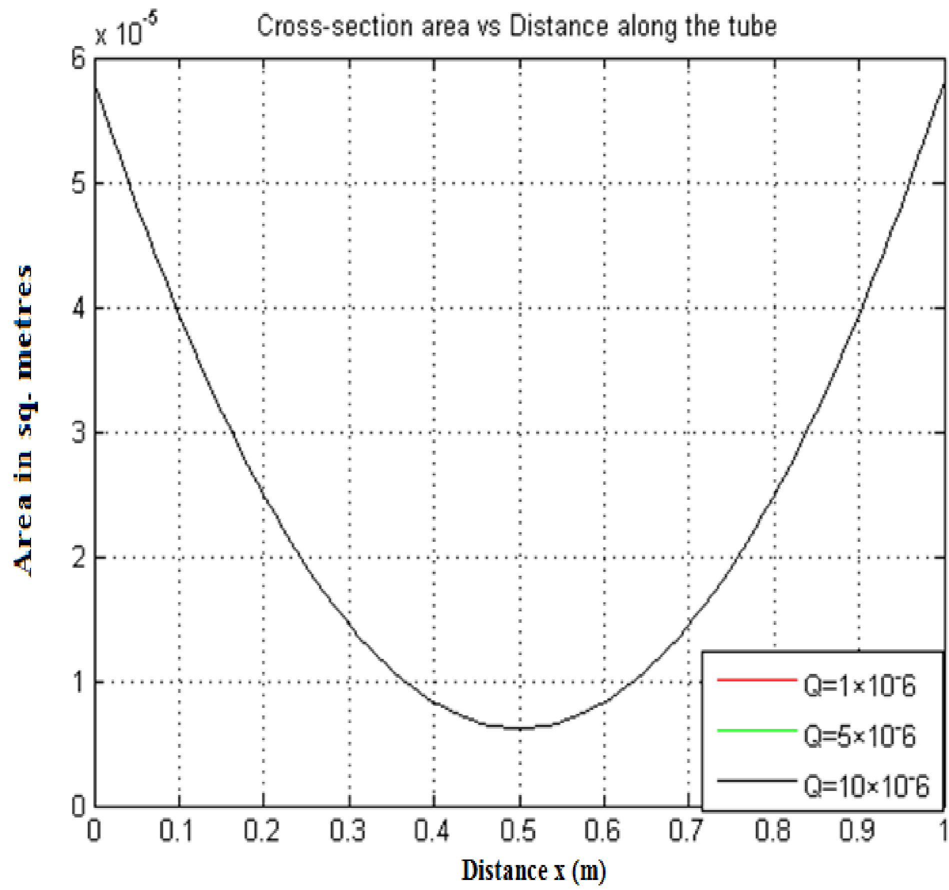
From the same Graph 3.3, it is also noted that as the tube stiffness increases the internal pressure increases. When the tube stiffness was increased from 1.21×10^{-5} to 1.21×10^{-1}, the internal pressure increased from 3679 Pascals to 3950 Pascals. This is as a result of the decrease in flow velocity. It is also observed that at the point where the collapse is mostly felt, the internal pressure is minimal.

3.5 EFFECTS OF VARYING VOLUMETRIC FLOW RATE ON THE CROSS SECTIONAL AREA

3.5.0 Introduction

To investigate the effects of volumetric flow rate on the cross sectional area of a collapsible tube, the longitudinal tension and the tube stiffness along with the other parameters were held constant while the volumetric flow rate was varied. The curves obtained were plotted on the same axes as shown in Graph 3.4 below.

GRAPH OF CROSS SECTIONAL AREA VERSUS DISTANCE ALONG THE COLLAPSIBLE TUBE WITH VOLUMETRIC FLOW RATE CHANGING



Graph 3.4: Cross sectional area versus distance for $T=4.0\times10^{2}$ $K_{PE}=1.21\times10^{-5}$

$\rho=1.0\times10^{3}$ $Pe=4.00\times10^{3}$ $r=4.3\times10^{-3}$

3.5.1 Results and Discussion

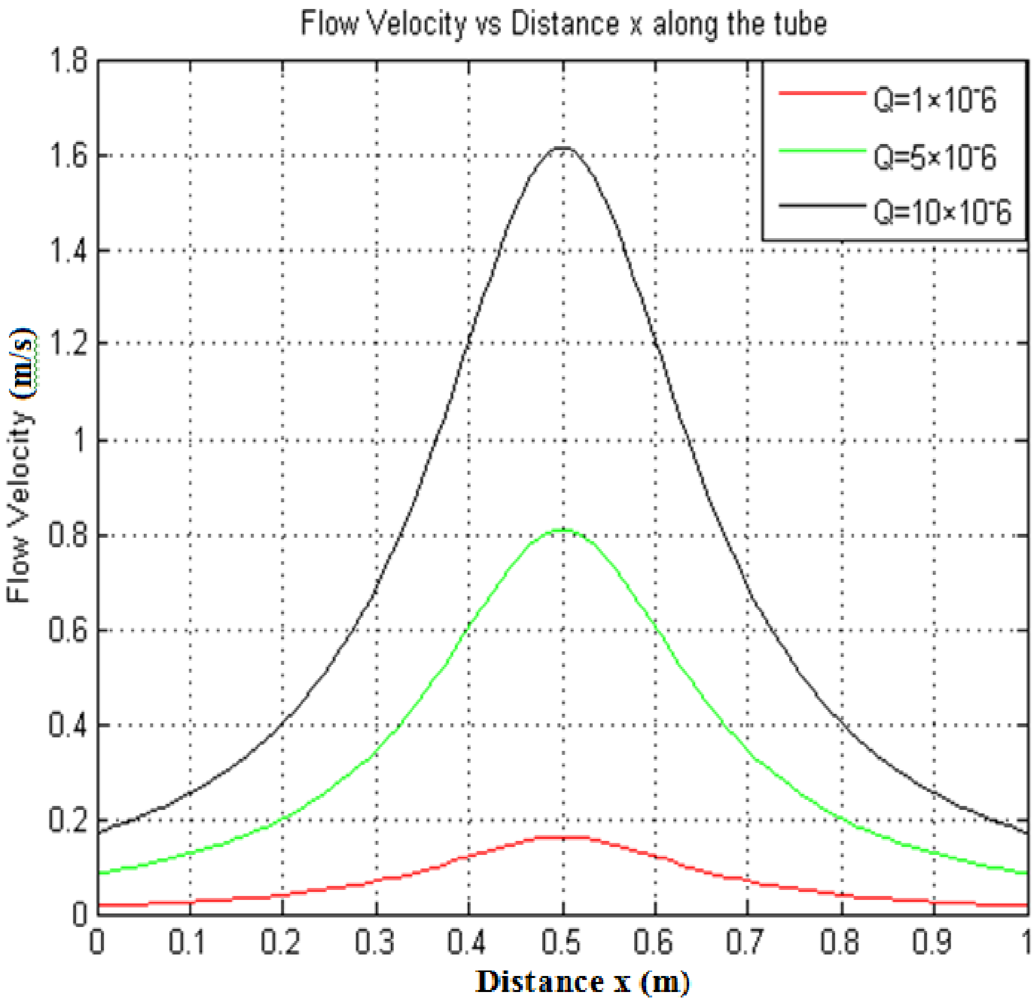
From Graph 3.4 it is observed that change in the volumetric flow rate does not affect the cross sectional area meaning that the cross sectional area is largely independent of the flow rate. The cross sectional area remains as 6.206×10^{-6} as the volumetric flow rate changes. This is because as the flow rate increases, the pressure drop increases in order to maintain the steady flow rate.

3.6 Effects of Varying Volumetric Flow Rate on the Flow Velocity

3.6.0 Introduction

In this study, the volumetric flow rate was varied while the other parameters were held constant. The curves obtained were then plotted on the same axes as shown in Graph 3.5 below.

GRAPH OF FLOW VELOCITY VERSUS DISTANCE ALONG THE COLLAPSIBLE TUBE WITH VOLUMETRIC FLOW RATE CHANGING



Graph 3.5: Flow velocity versus distance for $T=4.0\times10^{2}$ $Kp_E=1.21\times10^{-5}$

$\rho=1.0\times10^{3}$ $Pe=4.00\times10^{3}$ $r=4.3\times10^{-3}$

3.6.1 Results and Discussion

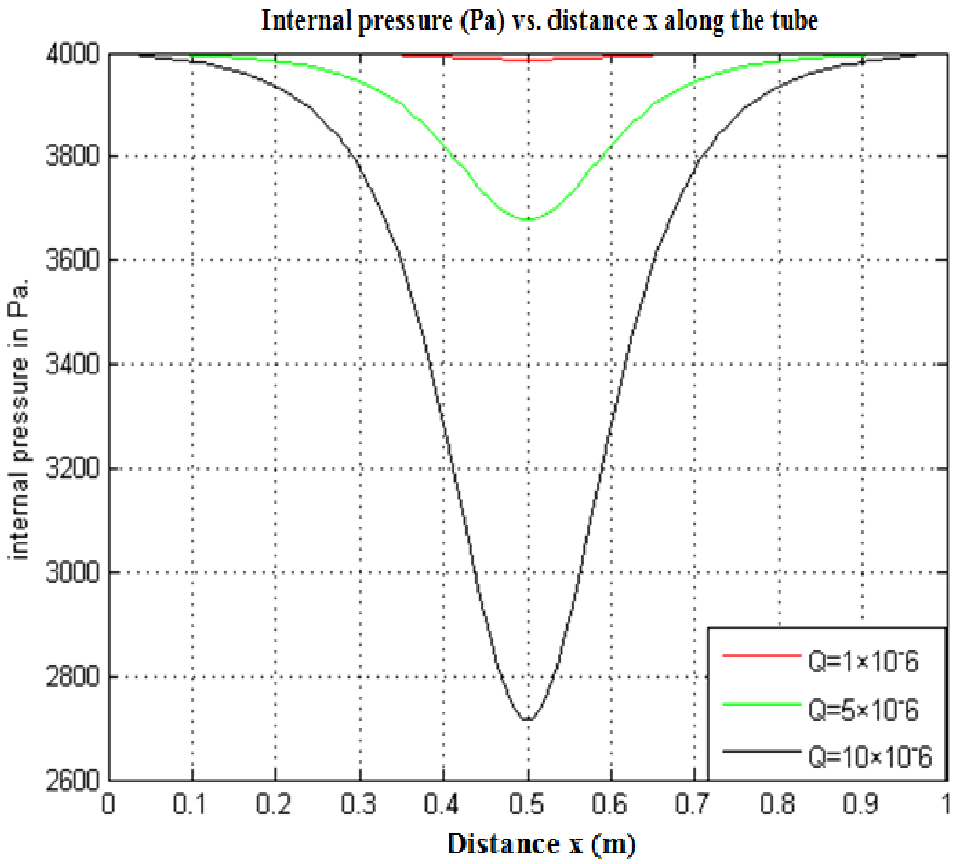
From Graph 3.5 it is observed that as volumetric flow rate increases, the flow velocity also increases. As the volumetric flow rate increases from 1×10^{-6} to 10×10^{-6}, the flow velocity increases from 0.1611 m/s to 1.611 m/s. This is because the volumetric flow rate is directly proportional to the flow velocity for a given cross sectional area. In this case the cross sectional area is constant hence an increase in volumetric flow rate translates to an increase in the flow velocity.

3.7 EFFECTS OF VARYING VOLUMETRIC FLOW RATE ON THE INTERNAL PRESSURE.

3.7.0 Introduction

To investigate the effects of volumetric flow rate on the internal pressure of a collapsible tube, different values of volumetric flow rate were used to plot curves while the other parameters were held constant. The curves obtained were plotted on the same axes as shown in figure 3.6 below

GRAPH OF INTERNAL PRESSURE VERSUS DISTANCE ALONG THE COLLAPSIBLE TUBE WITH VOLUMETRIC FLOW RATE CHANGING



Graph 3.6: Internal pressure versus distance for $T=4.0\times10^{2}$ $Kp_E=1.21\times10^{-5}$

$\rho=1.0\times10^{3}$ $Pe=4.00\times10^{3}$ $r=4.3\times10^{-3}$

3.7.1 Results and Discussion

From Graph 3.6, it is noted that as volumetric flow rate increases, the internal pressure decreases. As the volumetric flow rate increases from 1×10^{-6} to 10×10^{-6}, the internal pressure decreases from 3987 Pascals to 2718 Pascals. This is as a result of the already increased flow velocity. From Bernoulli principle an increase in flow velocity leads to a decrease in pressure.

3.8 DISCUSSION

Longitudinal tension describes how tight the tube is pulled out when attached at the edges. Tube's stiffness describes the tube's capability of resistance to elastic deformation in response to an applied force. The volumetric flow rate refers to the volume of fluid which passes through a given surface per unit time.

As fluid flows through the collapsible tube there is collision between molecules hence a decrease in kinetic energy. Pressure energy is converted into kinetic energy to maintain the flow velocity. This leads to a decrease in internal pressure. Since the external pressure remains constant, it exceeds the internal pressure causing the tube to collapse. The collapse leads to a decrease in cross sectional area and consequently the flow velocity increases in order to maintain a constant flow rate.

An increase in velocity of the fluid leads to an increase in the collision between the molecules hence greater loss in kinetic energy. This causes the internal pressure to decrease even more. In addition, according to Bernoulli principle, an increase in fluid velocity leads to a decrease in pressure.

The extent to which the tube collapses is dependent on the tube stiffness and the longitudinal tension. An increase in these parameters reduces the tube's tendency to collapse and therefore leads to an increase in the cross sectional area of the tube.

The volumetric flow rate is largely independent of the cross sectional area and therefore any change in the discharge affects the flow velocity and consequently the internal pressure.

3.8.1 Validation of Results

From this study it has been noted that when the discharge varies, the results obtained for the internal pressure and cross sectional area show the same trend as those obtained by Brian (2003).

The conclusions of this research study have been done in the next chapter. Finally, recommendations of areas on collapsible tubes that require further research have also been outlined.

CHAPTER FOUR

4.0 CONCLUSIONS AND RECOMMENDATIONS

This chapter presents conclusion of this research study and recommendations on areas that require further research.

4.1 CONCLUSION

i. Analysis of the effect of longitudinal tension on the cross sectional area, flow velocity and internal pressure of a collapsible tube has been carried out. The conclusion is that an increase in longitudinal tension leads to an increase in the cross sectional area, decrease in flow velocity and an increase in the internal pressure.

ii. The effect of tube stiffness on the cross sectional area, flow velocity and internal pressure indicate that the tube's stiffness is directly proportional to both the cross sectional area and internal pressure and inversely proportional to the flow velocity.

iii. Change in volumetric flow rate has no effect on the cross sectional area but it is directly proportional to flow velocity and inversely proportional to internal pressure.

4.2 RECOMMENDATIONS

It is recommended that further research should be carried out on;

i. Fluid flow through collapsible tube for a turbulent flow.

ii. Unsteady fluid flow through a collapsible tube.

iii. Flow through collapsible tube with tensile force varying

iv. Flow through other geometric shapes of collapsible tube.

v. Flow through porous collapsible tube.

REFERENCES

Andrew J.N., Patricia V. L. and David R. H. (2008), The role of venous valves in pressure shielding, *Journal of the American society for mass spectrometry,* **7**:8.

Andrew l. and Matthias Heil, (2003),Steady finite-Reynolds-number flows in three-dimensional collapsible tubes, *Journal of Fluid Mechanics,***486:**79-103.

Bertram C.D. (1986), Unstable equilibrium behavior in collapsible tubes, *Biomechanical Journal*, **19**:61-69.

Bertram C.D., Raymond C.J. and Pedley T.J., (1990), Mapping of instabilities for flow through collapsible tubes of differing lengths, *Journal of Fluid structures*, **4**:125-153.

Brian D.T. (2003), *Computational methods for flow in collapsible tubes*, MSc, Texas Tech University, Texas, USA.

Brook B. S. and Pedley, T. J. (2002), A model for time-dependent flow in (giraffe jugular) veins: Uniform tube properties. *Journal of Biomechanical Engineering.*

Edward K. and Abraham N. (1972), Cross-Sectional Shape of Collapsible Tube, *Biophysical Journal,***12** : 274-294.

Elad, D., Sahar, M., A vidor, J. M. and Eina v, S. (1992) Steady flow through collapsible tubes: measurements of flow and geometry . ASME *Journal of Biomechanical Engineering,* **114**: 84–91.

Eleuterio F. T. and Annunziato S. (2013), Flow in Collapsible tubes with Discontinuous mechanical properties: Mathematical model and Exact solutions, *Communications in Computational Physics Journal*, **13**: 361-385.

Emilie M. and Patrice F. (2010), Accurate modeling of unsteady flows in collapsible tubes, *Computer Methods in Biomechanics and Biomedical Engineering*,**13**: 279–290.

Hayashi S., Hayase T. and Kawamura K., (1998), Numerical Analysis for stability and self-excited oscillation in collapsible tube flow, *Journal of Biomechanical Engineering*, **120**:468-475.

Hazel A. and Heil M. (2003), Steady finite-Reynolds number flows in three dimensional collapsible tubes, *Journal of fluid Mechanics*, **486**:79-103.

Jensen O.E. and Heil M., (2003), High frequency self-excited oscillations in a collapsible channel flow, *Journal of Fluid Mechanics*, **481**:235-268.

Liu H.F., Luo X.Y., Cai Z.X and Pedley T.J. (2009), Sensitivity of unsteady collapsible channel flows to modeling assumptions, *Communications in Numerical Methods in Engineering*, **25**:483-504.

Luo X.Y.and Pedley T.J., (1998), The effects of wall inertia on flow in a two dimensional collapsible channel, *Journal of fluid mechanics,* **363**:253-280**.**

Makinde O. D. (2005), Collapsible Tube Flow: A Mathematical Model, *Romanian Journal of Physics,* **50**:493-506.

Marzo A., Luo X.Y. and Bertram C.D. (2005),Three-dimensional collapse and steady flow in thick-walled flexible tubes, *Journal of Fluids and Structures*, **20**:817–835.

Matthias Heil and Sarah L. Waters, (2008), How rapidly oscillating collapsible tubes extract energy from a viscous mean flow, *Journal of fluid Mechanics,* **601**:199-227.

Mawasha P.R., Ted A. Conway and Paul C. Lam. (2001), Investigations of dynamic behavior of a collapsible tube model through nonlinear analysis, *Cardiovascular Engineering: An International Journal,* **1:**183-189**.**

Odejide S.A, Aregbesola Y.A and Makinde O. D. (2008), Fluid Flow and Heat Transfer in a Collapsible Tube, *Romanian Journal of Physics*, **53**: 499–506.

Pedley, T. J. (1980), The fluid mechanics of large blood vessels. Cambridge: Cambridge University Press

Prashanta Kumar N. (2005), An unsteady analysis of non-Newtonian blood flow through tapered arteries with a stenosis, *International journal of Non-linear mechanics,* **40**:151-164.

Rosar M. E. and Charles S. P. (2001), Fluid Flow in Collapsible Elastic Tubes: A Three-Dimensional Numerical Model, *New York Journal of mathematics,* **7**: 281-302.

Shapiro A.H. (1977), Steady flow in collapsible tubes, *Journal of Biomechanical Engineering*, **99**:126-147.

Ulrich H. and Rodney K. L. (2001), Analysis of tracheal mechanics and applications, *Journal of Applied Physiology,* **91**: 290–297.

APPENDIX

COMPUTER CODE

The following is the MATLAB program code that was used to obtain the graphs labeled 3.1-3.6. The parameters varied were: Longitudinal tension (T), Tube's stiffness (K_{PE}) and Volumetric flow rate (Q).

```
clear; clc;
global  kpe n
outcount =1;
Q(outcount)=5*10^(-outcount -5);

n=100;  % number of nodes to calculate A at
kpe =1.21*10^-5;
T=4.0*10^2;          % tensile force
L=0.2;
r=4.3*10^(-3);     %tube radius
Pe=4.00*10^3;        % external pressure
rho=1.0*10^3;       % density of fluid
nu = 1.004*10^-6;      % kinematic viscosity of fluid
A0= 3.142*r*r;          % initial area
c=0.5*A0;
delx=L/n;                % spatial step size
P(1)  = 4.00*10^3;       % input pressure
epsilon = 10^(-15);        % error tolerance
% solve for Area
counter = 0;               % initialize counter for the iterations
error=10;
Aold=A0;
A(1:n)=A0;
V(1)=Q(outcount)/A0;

for m=2:n

    P(m)=P(1) ;
```

```
end

 R(1)=-P(2)+Pe+kpe*((c/A0)^10-(c/A0)^-1.5)                    -
kpe*10*(c/A0)^10+1.5*(c/A0)^(-1.5)- (T*A(1))/(delx^2);
for i=2:n-3

    R(i)=-P(i)+Pe+kpe*((c/A0)^10-(c/A0)^-1.5)                 -
kpe*10*(c/A0)^10+1.5*(c/A0)^(-1.5);
end

 R(n-2)=-P(n-1)+Pe+kpe*((c/A0)^10-(c/A0)^-1.5)                -
kpe*10*(c/A0)^10+1.5*(c/A0)^(-1.5)- (T*A(n))/(delx^2);

% matrix building (Tri-diagonal symmetric matrix)
Bmatrix=diag((-2*T/(delx^2)+kpe*((10/c)*(c/A0)^10+(1.5/c)*(c/A0)^(-
1.5) ) * ones(1,n-2)),0);% Diagonal of matrix

Bmatrix=Bmatrix+diag(ones(1,n-3)* ( T / (delx^2) ),1);% Superdiagonal

Bmatrix=Bmatrix+diag(ones(1,n-3)*(T/(delx^2) ),-1); %Subdiagonal

% correct the diagonal matrices
Bmatrix(1,1)=(- (2*T)/(delx^2)+kpe*(10/c)*(c/A0)^10+(1.5/c)*(c/A0)^(-
1.5));
Bmatrix(n-2,n-2)=(-
(2*T)/(delx^2)+kpe*(10/c)*(c/A0)^10+(1.5/c)*(c/A0)^(-1.5));
Bmatrix(1,2) =  T / (delx^2);
Bmatrix(n-2,n-3)= T / (delx^2);
%Solve for new Area values
A(2:n-1) = transpose(Bmatrix\R');

%figure Aim (area at staggered nodes)
Aim(1)=0;
for i=2:n
    Aim(i)=(A(i)+A(i-1))/2;
end
```

```
%figure Peripheral length, S
for i=1:n
    S(i)=2*pi*sqrt(A(i)/pi);

end
%   figure frictional coefficient

lambda=steadylamfun28(Q(outcount),r,A(i), nu);
for j=2:n
    P(j)=P(j-1)+(rho*Q(outcount)^2*(A(j)-A(j-1))) / (Aim(j)^3);
     P(j)=P(j)-(rho*nu*delx*Q(outcount))*(lambda(j)*S(j))            /
((Aim(j)^2)*2*r);
     V(j)=(Q(outcount))/A(j);



end          % of pressure solution loop

% check error
if (counter > 1)
    error = norm(Aold-A,inf);
end
Aold = A;
% ends iteration at each flow rate value
error;
counter;
x4graph=linspace(0,1,n);
figure(1)
grid on
plot(x4graph,A,'k:');
title('Cross-section area vs Distance along the tube');
xlabel('Distance x(m)');
ylabel('Area in Sq. metre');
hold on

figure(2)
```

```
grid on
plot(x4graph, P,'k:');
title('internal pressure (Pa) vs x');
xlabel('Distance x(m)');
ylabel('internal pressure in Pa.');
hold on
figure(3)
grid on
x4graph=linspace(0,1,n);
plot(x4graph,V,'k:');
title(' Flow Velocity vs Distance x along the tube');
xlabel('Distance x (m) along the tube');
ylabel('Flow Velocity (m/s)');
hold on
```