Magneto-Hydrodynamics Analysis of Free Convection flow Between Two Horizontal Parallel Infinite Plates Subjected to Constant Heat Flux

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DECLARATION

This project is my original work and has not been presented for the award of any degree in any other institution of higher Learning.

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This project has been dedicated to my wife Damaris and my two daughters Sylvia and Sabrinah.

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ABBREVIATIONS

MHD	Magneto-hydrodynamics
FDM	Finite Difference Method
HOT	Higher Order Terms

NOMENCLATURE

$\vec{B}$	Magnetic flux density, Wb/m^2
$\vec{E}$	Applied electric fied strength with components Ex, Ey, Ez
g	Acceleration due to gravity, N/m^2
H	Magnetic field with components H_x, H_y, H_z
i, j, k	Unit vectors in x, y, z directions respectively
k	Thermal conductivity, $Wm^{-1}K^{-1}$
L	Characteristic length, m
$\vec{J}$	Current density with components $J_x, J_y, J_z (Am^2)$
P	Pressure, Nm^{-2}
Q	Amount of heat added to the system(Nm)
T	Temperature of the fluid, K
C_p	Specific Heat Capacity at constant pressure
h	Dimensional distance between plates(m)
q	Velocity vector with components u, v, w in the x, y, z directions respectively
t	Time(s)
U	Characteristic velocity, m/s
P_r	Prandtl number
E_c	Eckert number

R_e	Magnetic Reynolds number
M	Hertmann number
T_o	Reference temperature, K
T_1	Temperature of the upper plate
T_∞	Free stream temperature, K
$\frac{D}{DT}$	Material or Substantive derivative
ρ	Fluid density , kgm^{-3}
β	Co − efficient of volume expansion K^{-1}
μ	Coefficient of viscosity, $Kgm^{-1}s^{-1}$
μ_e	Magnetic permeability, Wm^{-1}
σ	Electrical conductivity, $\Omega^{-1}m^{-1}$
∇	Gradient operator $i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$
Δ	Laplacian operator $\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$
φ	Viscous dissipation function
θ	Dimensionless fluid temperature
ϑ	Kinematic viscosity, m^2

ABSTRACT

In this study we have investigated MHD stokes free convection of an incompressible, electrically conducting fluid between two horizontal parallel infinite plates subjected to a constant heat flux and pressure gradient. A uniform magnetic field is applied normal to the plates. The flow is steady and considered heat generating due to frictional heating of fluid particles. This implies that flow variables are independent of time. The upper plate is impulsively started at constant velocity while the lower plate is assumed to be porous and stationary. The two plates are separated by a distance h. An analysis of velocity profiles and temperature distribution that have been obtained has been done. In addition, an investigation on how Prandtl number, Eckert Number and Hartman Number affect velocity profiles and temperature distribution has been carried out. Differential equations that have been generated from this study are non-linear. The equations have been solved by finite difference method. The results that have been obtained are discussed in detail and presented graphically. It has been noted that an increase in Hartmann number causes a decrease in velocity profiles. However an increase in Hartmann number leads into an increase in temperature distribution.It is also revealed that an increase in values of Eckert results into an increase in temperature distribution between the plates. Further an increase in Prandtl Number leads to a fall in temperature distribution.

CHAPTER ONE

1.0 INTRODUCTION

The word magneto hydrodynamics is derived from magneto-meaning magnetic field, hydro-meaning liquid and dynamics-meaning movement. Therefore, magneto hydrodynamics refers to the study of flow of an electrically conducting fluid in the presence of magnetic field.

A fluid is a substance whose constituent particles may continuously change their positions relative to one another when shear force is applied to it. The fluid undergoes deformation irrespective of the magnitude of the force. As fluid flows, it transfers heat from one point to another. Heat transfer in fluids is called convection. Fluids do not exist in isolation but with solids. Heat transfer in solids is called conduction whereas heat transfer in a vacuum is called radiation. Convection as a mode of heat transfer is of great interest in relation to conduction and radiation since it has a wide range of applications in engineering and other scientific fields. Convection is categorized into:

(a) Natural (free) convection – refers to fluid flow due to density variations resulting from temperature differences within the fluid. Buoyant forces cause denser fluid to move downwards replacing less dense fluid that rises upwards resulting to free convectional currents.

(b) Forced convection – refers to fluid flows caused by external forces or urgencies such as fans, mixers or pumps.

A fluid can flow in an enclosure, in a pipe, in a channel or over a plate. When fluid flows, fluid particles interact with these surfaces resulting into two flow regions namely;

(i) **Boundary layer region**- is a region in which fluid particles are in contact with solid surface. In this region viscous effects of the fluid are abundant.

(ii) **Free stream region-** is a region in which fluid particles experience negligible viscosity. Consequently boundary layer region is more significant than free stream region. Fluid flow in boundary layer can be;

(a) **Turbulent flow-** Flow is turbulent if fluid particles move in a disorderly manner in a channel. This flow is said to be unsteady and occurs due to boundary roughness or variation in physical properties of fluid moving in straight path or porous walls.

(b) **Laminar flow-**Flow is laminar if fluid particles are orderly and do not mix with particles of adjacent layers. This flow is said to be steady. It is characterized by low Reynolds number.

Fluid flows in engineering devices occur within magnetic field. Fluid flow in the presence a magnetic field is called hydromagnetic flow and the study of hydro magnetic flows is called **magneto hydro dynamics** (MHD).

1.1 LITERATURE REVIEW

Magneto hydrodynamics is the study of hydro magnetic flows. It was first detected by Michael Faraday in 1931. Faraday performed an experiment to study the behavior of currents in circuits placed in time varying magnetic field. He used mercury as an electrically conducting fluid and allowed it to flow in a glass tube in a magnetic field. He observed that the voltage was induced in a direction perpendicular to both the direction of flow and the magnetic field. Hartmann (1938) discussed theoretically and experimentally the flow of a conducting fluid between two parallel plates while Stokes (1951) concentrated on the flow of an incompressible and viscous fluid past impulsively started infinite flat plates. Further, Ram et al (1995) solved magneto hydrodynamics stokes problem of convection flow for a vertical infinite plate in a dissipative rotating fluid with Hall current. This is an analysis of the effects of various parameters on the concentration velocity and temperature profiles while Kwanza et al (2003)presented their work on MHD stokes free convection past an infinite vertical porous plate subjected to a constant heat flux with ion-slip and radiation absorption. They discussed their tabulated results on concentration, velocity profiles and temperature distributions both theoretically and graphically. Sigey et al (2004) presented an investigation on a numerical study on natural convection turbulent heat transfer in an enclosure while Chandra B.S(2005) studied a steady MHD flow of an electrically conducting fluid between two parallel infinite plates when the upper plate is made to move with constant velocity while the lower plate is stationary.Okelo et al (2007) investigated unsteady free convection of incompressible fluid past a semi-infinitive vertical porous plate in the presence of a strong magnetic field at an angle (α) to the plate with Hall ion-slip current effects. They discussed the effects of modified Grashof number, heat source parameter, Schmidt number, time, hall current, angle of inclination and Eckert number on a convectively cooled or convectively heated plate restricted to a laminar boundary layer. They found that an increase in mass diffusion parameter causes a decrease in concentration profiles while an increase in suction velocity leads to an increase in concentration profiles. In addition they realized that an increase in Eckert number results into an increase in temperature profiles whilst an increase in angle of inclination causes an increase in primary velocity profiles and a decrease in secondary velocity profiles. Abbas I.A and Palani G. (2009) carried out an investigation on Free Convection MHD Flow with Thermal Radiation from an Impulsively Started Vertical Plate. They established that velocity increases with a decrease in magnetic field parameter. In addition they realized that dimensionless temperature decreases with an increase in thermal radiation.MarigiE.M et al(2010) carried out an investigation on hydro magnetic free convectional currents effects on boundary layer thickness while Abuga et al (2011) carried out an investigation on the effects of Hall current and Rotational Parameter on dissipative fluid past a vertical semi-infinite plate. They found that an increase in Hall parameter for both cooling and heating of the plate by free convection currents has no effect on temperature profiles but leads to an increase in velocity profiles. Similarly they found that an increase in Rotational parameter led to a decrease in velocity profile when the Eckert number was 0.01 and an increase in velocity profile when Eckert number was 0.02. Furthermore they realized that an increase in time led to an increase in both primary and secondary velocity profiles in case of cooling of the plates by convection currents but led to a decrease in velocity profiles in case of heating the plates by convection currents. Rajput U.S and Sah P.K (2011) conducted an investigation on unsteady transient free convection MHD flow between two long vertical parallel plates with constant temperature and variable mass diffusion. They established that velocity and skin friction of the fluid increase with increase with the value of time but decrease with increasing the value of the prandtl number, Schmidt number and magnetic parameter. More overManyonge et al (2012) investigated on Steady MHD Poiseuille flow between two infinite porous plates in an inclined magnetic field. They established that a high Hartmann flow (high magnetic field strength) decreases velocity whereasBhaskarkalita (2012) carried an investigation on unsteady free convection MHD flow and heat transfer between two heated vertical plates with heat source. It was found that an increase in Hartmann number caused high velocity profiles near the walls and low velocity profiles at the centre between the walls. Similarly, an increase in Prandtl number to $P_r = 7$ caused a remarkable change in temperature distribution.

In spite of all these studies, the problem of MHD stokes free convection past an infinite horizontal porous plate under constant heat flux has not received adequate attention. In this study we have assessed the effect of a uniform magnetic field perpendicular to the plates together with a constant pressure gradient on temperature distribution and velocity profiles. The flow is steady and the fluid considered is viscous, incompressible and electrically conducting. The two plates are parallel and infinite in extent with the upper plate impulsively started at constant velocity and the lower plate porous and stationary. In addition we have investigated on the effect of Prandtl, Eckert and Hartmann numbers on velocity profiles and temperature distribution. The lower plate is along the x- axis of the Cartesian plane whereas the y-axis is perpendicular to both plates.

1.2 **STATEMENT OF THE RESEARCH PROBLEM**

When an electrically conducting fluid flows between two horizontal parallel infinite plates in the presence of uniform magnetic field, fluid motion is interfered with hence velocity and temperature changes occurred .The flow is steady and in the x-direction. It is therefore necessary to obtain an approximation solution for velocity profile and temperature distribution.

1.3 **GEOMETRY OF THE PROBLEM**

A uniform magnetic field was applied normal to the plates. The upper plate is impulsively started at constant velocity in direction of the flow parallel to the x-axis while lower plate porous and stationary as shown.

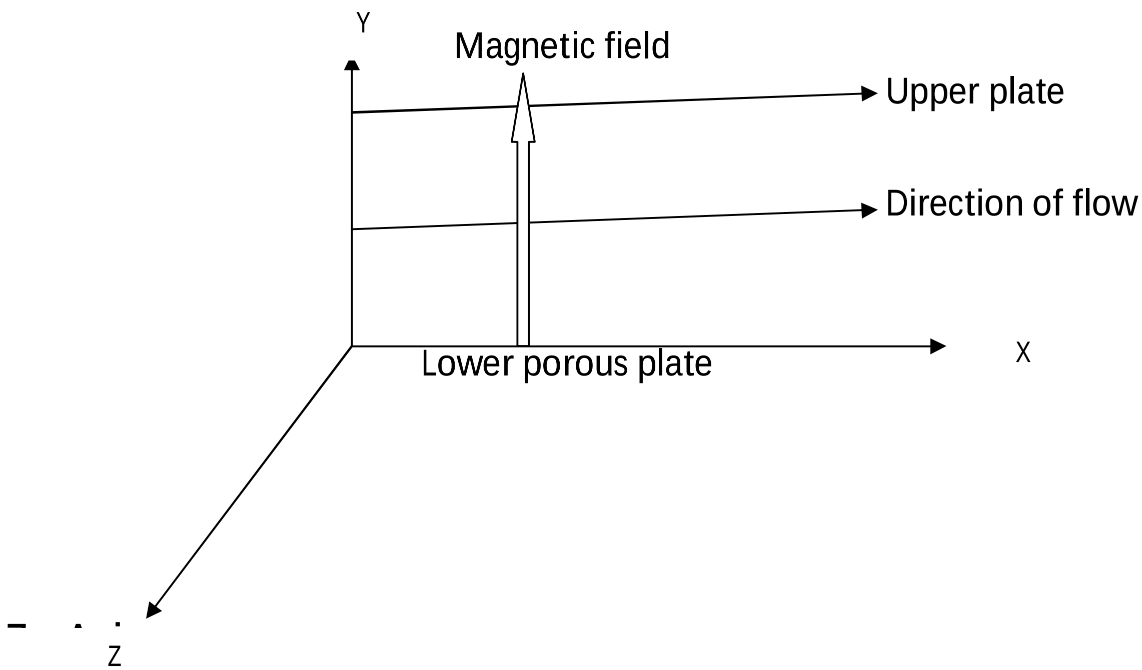

Figure 1: Geometry of flow Configuration

1.4 OBJECTIVES OF THE STUDY

The objectives of the study are;

1. To determine the velocity profiles and temperature distribution of hydro magnetic free convection in a steady and laminar fluid flow over a porous stationary plate.

2. To investigate how Eckert number, Prandtl number and Hartmann number affect flow variables.

1.5 JUSTIFICATION

MHD laminar flow through a porous medium has become very important in recent years because of its wide range of applications in many branches of science and technology. Some of the applications of this research area include;

In engineering: most engineering machines involve fluid flow. Melted metals are applied in technology, casting and as well in liquid metal cooling hoops of nuclear reactors.

Heat transfer devices: The results of this study can enable us design heat transfer devices with higher efficiency since most of heat transfer devices operate within an electric field which induce magnetic field.

In general operation of MHD devices: Most MHD devices have a channel conveying an electrically conducting fluid that passes between the poles of a magnetic material. There are electrodes in the channel that are in contact with the fluid. These electrodes lie in the plane perpendicularly to the magnetic field. These include MHD fluid dynamos, power generators, flow meters pump etc.

In addition it is used in agricultural engineering to study underground water resource, seepage of water in riverbeds while in petroleum technology it is to study movement of natural gas, oil and water through oil reservoirs.

In the following chapter, assumptions made and a detailed discussion of equations governing the flow is presented.

CHAPTER TWO

2.0 Assumptions

The following assumptions have been taken into account in the research problem:

(i) The fluid is assumed to be incompressible and with constant density.

(ii) The plate is electrically non-conducting.

(iii) Fluid velocity is too low and its Reynolds number is small.

(iv) There is no external applied force that is $\vec{E} = 0$.

(v) Thermal conductivity electrical conductivity and co-efficient of viscosity are constant.

(vi) Magnetic flux density is $\vec{B} = \mu_e \vec{H}$

(vii) The fluid does not undergo any chemical reaction.

(viii) Hall current is ignored since the magnetic field applied is weak.

2.1 Governing Equations

In this section equations governing the flow in the research problem have been generated well. The equations are derived using laws of conservation of mass, momentum and energy.

Governing equations can be categorized into;

2.2 Magneto hydrodynamics equations

Magneto hydrodynamics combines electromagnetic and fluid mechanics. Fluid mechanics is the study of the behavior of fluids whether at rest or in motion. The study of fluids at rest is called fluid statics whereas fluid dynamics is the study of fluids in motion.

Thus the general equations governing MHD flows have been derived from electromagnetic theory and fluid mechanics.

These equations are:

2.2.1 **Ohm's Law**

A material has an ability to transport electric charge under the influence of applied electric field. In moving electrically conducting fluids within a magnetic field, the magnetic field induces a current in the conductors of magnitude $\vec{q} \times \vec{B}$.

The current density is

$$\vec{J} = \sigma(\vec{E} + \vec{q} \times \vec{B}) \quad 2.2.1\ (1)$$

2.2.2 **Maxwell's equations**

These equations govern electric and magnetic fields .They include

$$\nabla.\vec{B} = 0$$

$$\nabla \times \vec{H} = \vec{J} \quad 2.\ 2.2(1)$$

$$\mu \frac{\partial H}{\partial t} = -\nabla \times \vec{E}$$

$$\nabla.\vec{D} = \rho_e$$

However, in this research area Maxwell's equations have been regarded redundant due to a small Reynolds number hence not considered.

2.3 Equations governing flow of an incompressible fluid in presence of magnetic field

Under this category, the equations include:

2.3.1 Continuity equation

The equation is based on the principle of conservation of mass of the fluid that is mass of fluid is conserved and that the flow is continuous.

The equation is:

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho u) = 0 \quad (1)$$

In three dimensions

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (2)$$

Since there is no flow along the z-axis, that is, the flow is stagnant in z- direction then thecontinuity equation reduces to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3)$$

Similarly in the flow velocity u depends on y only. Thus

$\frac{\partial u}{\partial x} = 0$ Hence

$$\frac{\partial v}{\partial y} = 0 \quad (4)$$

Integrating the equation(4) gives

$$V = -v_0 \qquad (5)$$

where v_0 is the suction velocity.

This velocity is maintained for a steady flow against suction of the fluid.

2.3.2 **Equation of momentum conservation**

The equation is based on the Newton's second law of linear motion. The equation is also called Navier-stokes equation. In vector notation the equation of motion considers the body force due to gravity and electromagnetic force only.

It is written as

$$\frac{\partial q}{\partial t} + (q.\nabla)q = -\frac{1}{\rho}\nabla p + \vartheta\nabla^2 q + F \qquad (1)$$

where

$\frac{\partial q}{\partial t} -$ is temporal acceleration

$(q\nabla q)$ Is convective acceleration and allows for acceleration even when the flow is steady.

$-\frac{1}{\rho}\nabla p -$ Is pressure gradient

$\vartheta\nabla^2 q -$ Is force due to viscosity

$F = \vec{J} \times \vec{B} -$ Is body force

The Navier-Stokes equation is also written in the form

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2} - \rho g + \vec{J} X \vec{B} \quad (2)$$

Since the flow is steady then $\frac{\partial u}{\partial t} = 0$ and $u\frac{\partial u}{\partial x} = 0$ because u depends on y only. Thus equation (2) reduces to

$$\rho\left(v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2} - \rho g + \vec{J} X \vec{B} \quad (3)$$

Differentiating equation (3) with respect to x gives $\frac{\partial^2 P}{\partial x^2} = 0$ which on integrating results into

$\frac{\partial p}{\partial x} = -P$(constant) that is pressure gradient is constant.

The negative sign shows that fluid pressure decreases in the direction of flow.

Hence equation (3) becomes

$$\rho v\frac{\partial u}{\partial y} = P + \mu\frac{\partial^2 u}{\partial y^2} - \rho g + \vec{J} X \vec{B} \quad (4)$$

In theflow, the effect of force of gravity is insignificant since the two plates are horizontal hence it is ignored. Further upon substituting equation 2.3.1(5) in equation 2.3.2(4), it reduces to

$$\rho\left(-v_0\frac{\partial u}{\partial y}\right) = P + \mu\frac{\partial^2 u}{\partial y^2} + \vec{J} X \vec{B} \quad (5)$$

Replacing $\vec{J} X \vec{B}$ in equation (5) gives

$$\rho\left(-v_0\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} + P - \sigma\mu_e^2 H^2 u \quad (6)$$

$$\text{where } (\vec{J} \times \vec{B}) = \sigma(\vec{q} \times \vec{B}) \times \vec{B}$$

$$\text{Thus } \vec{J} \times \vec{B} = -\sigma B_0^2 U$$

$$= \sigma \mu_e^2 H^2 \mu$$

2.3.3**Energy equation**

The equation is based on conservation of energywhich states that energy is neither created nor destroyed but can be transformed from one form to another. It is derived from the first law of thermodynamics which states that the amount of energy added to a system dQ equals to change of internal energy dE plus work done dWi.e

dQ = dE +dW (1)

The first law of thermodynamics requires that

$$\rho \frac{D_e}{D_t} + e\left(\frac{D\rho}{Dt} + \rho \nabla q\right) = -\nabla Q' + Q'' - p\nabla . q + \mu\varphi (2)$$

Since the flow is incompressible, then the density is assumed to be constant. The term in brackets in equation (2) above represent the equation of continuity and hence should be equated to zero.

Since

$$h = e + \left(\frac{1}{\rho}\right) p \ (3)$$

Then, the substantive derivative of enthalpy term in (3) is given as

$$\frac{D_h}{D_t} = \frac{D_e}{D_t} + \frac{1}{\rho}\frac{D_p}{D_t} - \frac{p}{\rho^2}\frac{D_\rho}{D_t} \quad (4)$$

where $\frac{D\,\rho}{D_t} = 0$ since density of the fluid is assumed to be constant.

Thus equation (4) reduces to

$$\rho\frac{D_e}{D_t} = \rho\frac{D_h}{D_t} - \frac{D_p}{D_t} \quad (5)$$

Substituting equation2.3.3 (5) in equation 2.3.3(2) above gives

$$\rho\frac{D_h}{D_t} = \nabla(K\nabla T) + \sigma B_0^2 U^2 + \frac{D_p}{D_t} + \mu\varphi - p\nabla q \quad (6)$$

From mass conservation equation the last term of equation (6) is zero hence it reduces to

$$\rho\frac{D_h}{D_t} = (K\nabla T) + \sigma B_0^2 U^2 + \frac{D_p}{D_t} + \mu\varphi \qquad (7)$$

The equation can be expressed in terms of temperature by replacing the specific enthalpy on the left hand side with an equivalent relation for h given by

$$dh = Tds + \frac{1}{\rho}dp \qquad (8)$$

T - is absolute temperatureand

ds –specific entropy change.

$$ds = \left(\frac{\partial s}{\partial T}\right) dT + \left(\frac{\partial s}{\partial p}\right) dp \qquad (9)$$

Thus

$$\left(\frac{\partial s}{\partial p}\right) = -\left(\frac{\partial \frac{1}{\rho}}{\partial T}\right) = \frac{1}{\rho^2}\left(\frac{\partial s}{\partial p}\right) = -\frac{\beta}{\rho} \qquad (10)$$

where

$\beta = -\frac{1}{\rho}\left(\frac{\partial s}{\partial p}\right)$. It represents the coefficient of thermal expansion.

while$\left(\frac{\partial s}{\partial T}\right) = \frac{C_p}{T}$

Hence equation 2.3.3(8) can be rewritten as

$$dh = C_p dT + \frac{1}{\rho}(1 - \beta T)dp \qquad (11)$$

That is in substantive derivatives equation (11) becomes

$$\rho\frac{D_h}{D_t} = \rho C_p \frac{D_T}{D_t} + (1 - \beta T)\frac{D_P}{D_t} \qquad (12)$$

Substituting equation 2.3.3(12) in equation 2.3.3(7) results into

$$\rho C_p \frac{D_T}{D_t} = \nabla.(K\nabla T) + \sigma B_0^2 U^2 + \beta T\frac{D_P}{D_t} + \mu\varphi \qquad (13)$$

Now for a fluid flow with constant fluid conductivity, k and negligible compressibility effect$\beta T \frac{D_P}{D_t}$, the energy equation reduces to

$$\rho C_p \frac{D_T}{D_t} = \nabla.(K\nabla.T) + \sigma B_0^2 U^2 + \mu\varphi \quad (14)$$

Energy equation is also expressed as

$$\rho C_p \frac{D_T}{D_t} = \nabla.(K\nabla.T) + \frac{1}{\sigma} j^2 + \mu\varphi \qquad (15)$$

where

$$Q' = -k\nabla T = -k\left(\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} + \frac{\partial T}{\partial w}\right). \quad \text{Hence} \quad -\nabla Q' = k\nabla^2 T$$

$k\nabla^2 T$ –This term is due to conduction of heat

Q'' - is the internal heat generation defined as $Q'' = \frac{1}{\sigma} j^2 = \sigma B_0^2 U^2$

$\frac{DT}{Dt}$ - is the material derivative and is expressed as

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + q\nabla T$$

Or $\frac{DT}{Dt} = \frac{\partial T}{\partial t} + (u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z})$

φ - is the dissipative heat which is expressed as

$$\varphi = \left\{2\left(\left[\frac{\partial u}{\partial x}\right]^2 + \left[\frac{\partial v}{\partial y}\right]^2\right) + \left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right]^2\right\}$$

In the flow both temperature T and velocity u are functions of y only. Hence it is evident that $\frac{\partial}{\partial x} = \frac{\partial}{\partial z} = 0$ for all quantities except pressure gradient $\frac{\partial p}{\partial x}$ which is assumed to be constant.

Thus the term due conduction of heat, material derivative, internal heat generation and dissipative heat reduces to

$k\nabla^2 T = k\frac{\partial^2 T}{\partial y^2}$, $\frac{DT}{Dt} = \frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y}$, $Q'' = \sigma B_0^2 U^2$ and $\varphi = \left(\frac{\partial u}{\partial y}\right)^2$ respectively.

Substituting these results in equation 2.3.3(15) gives

$$\rho C_p\left(\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma B_0^2 U^2 \quad (16)$$

Since fluid motion along the normal axis is very small, transverse velocity component v is approximated to be zero hence equation (16) becomes

$$\rho C_p\left(\frac{\partial T}{\partial t}\right) = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma B_0^2 U^2 \quad (17)$$

Since the flow observes the no-slip conditions implies that velocity of the fluid at the upper plate equals the velocity of the plate while at the lower plate the velocity is zero. Thus the initial and boundary conditions are; at y=0, u=0 and $T = T_0$ while at y=1, $u = U$ and $T = T_1$

2.4 **Dimensional analysis**

It is a mathematical technique that helps in analysis of fluid flow problems. It is built in the principle of homogeneity. It helps formulate fluid problems that defy analytical solutions and that must be solved experimentally. In this research dimensional analysis has been used to non-dimensionalize governing equations. Non-dimensional analysis refers to partial or full removal of units from an equation involving physical quantities by suitable substitution ofvariables.

2.5 Non-Dimensional parameters

These are non-dimensional numbers. The numbers are introduced into the governing equations to ensure that given solutions of natural phenomenon hold for all units. Some of theparametersused in this research area include:

2.5.1 Reynolds number R_e

Is the ratio of inertial force to viscous force. It gives the relative significance of inertial force in fluid flow problems.

Mathematically it is expressed as

$$R_e = \frac{UL}{\vartheta}$$

A large Reynolds number of a fluid, inertia forces predominates and viscosity effects are negligible. Similarly when the Reynolds number is small, viscous force dominates and inertia force can be neglected.

2.5.2 Eckert number, E_c

Is the ratio of kinetic energy of the flow to thermal energy.It is expressed by

$$E_c = \frac{U^2}{C_p \Delta T}$$

2.5.3 Prandtl number, P_r

Is the ratio of viscous force to thermal force.

It is also defined as the ratio of momentum diffusivity and thermal diffusivity. It is expressed as

$$P_r = \frac{\mu/\rho}{k/\rho_{C_p}} = \frac{C_p\mu}{k} = \frac{\vartheta}{\alpha}$$

where

$\vartheta = \frac{\mu}{\rho}$ – is kinematic viscosity and $\alpha = \frac{k}{\rho C_p}$ – is the thermal diffusibility

2.5.4 **Hartmann Number, M**

Is the ratio of magnetic force to viscous force.It is expressed as

$$M^2 = \frac{\sigma\mu_e^2 H^2}{\rho U^2}$$

$$= \frac{\sigma B_{0\vartheta}^2}{\rho U^2}$$

Or

$$M = \left(\frac{\sigma\mu_e^2 H^2}{\rho U^2}\right)^{\frac{1}{2}}$$

$$= \left(\frac{\sigma B_{0\vartheta}^2}{\rho U^2}\right)^{\frac{1}{2}}$$

2.6 **Non-dimensionalisation of velocity and energy equations**

Taking the characteristic length, velocity and pressure to be L, U and P respectively, the following non-dimensional variables are used.

$$t = t^* \frac{U^2}{\vartheta}, \quad y = y^* \frac{U}{\vartheta}, \quad \theta = \frac{T^* - T_\infty^*}{T_0^* - T_\infty^*}, \quad P_r = \frac{C_p\mu}{k}$$

$$u = \frac{u^*}{U}, \quad v_0 = \frac{v_0^*}{U}, \quad E_c = \frac{U^2}{C_p(T_w^* - T_\infty^*)}, \quad x^* = \frac{x}{l}$$

In dimensional form theequation 2.3.2(6) is written as

$$\rho\left(-v_0^*\frac{\partial u^*}{\partial y^*}\right) = P + \mu\frac{\partial^2 u^*}{\partial y^{*2}} - \sigma\,\mu_e^2 H^2 u^* \quad 2.6(1)$$

But $v_0^*\frac{\partial u^*}{\partial y^*} = v_0\frac{U^3}{\vartheta}\frac{\partial u}{\partial y}$ and

$$\mu\frac{\partial^2 u^*}{\partial y^{*2}} = \mu\frac{U^3}{\vartheta^2}\frac{\partial^2 u}{\partial y^2}$$

Then upon dropping the stars in equation (1) by replacing them by their respective non-dimensional parameters, the equation(1) becomes

$$\rho\left(-v_o\frac{U^3}{\vartheta}\frac{\partial u}{\partial y}\right) = P + \mu\frac{U^3}{\vartheta^2}\frac{\partial^2 u}{\partial y^2} - \sigma\mu_e^2 H^2 Uu \qquad (2)$$

Dividing each term containing u in (2)by $\rho\frac{U^3}{\vartheta}$and the constant P by ρ results into

$$\frac{\mu}{\rho\vartheta}\frac{\partial^2 u}{\partial y^2} + v_0\frac{\partial u}{\partial y} - \sigma\mu_e^2 H^2\frac{u}{\rho U^2} + \frac{P}{\rho} = 0 \qquad (3)$$

Substituting $\rho = \frac{\mu}{\vartheta}$ in equation (3) we get

$$\frac{\partial^2 u}{\partial y^2} + v_0\frac{\partial u}{\partial y} - M^2 u + c = 0 \quad (4)$$

where $M^2 = \frac{\sigma\mu_e^2 H^2}{\rho U^2}$ and$c = \frac{P}{\rho}$ (constant)

Similarly equation 2.3.3(17) is written in dimensional form as

$$\rho C_p\left(\frac{\partial T^*}{\partial t^*}\right) = k\frac{\partial^2 T^*}{\partial y^{*2}} + \mu\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \sigma B_0^2 U^{*2} \quad 2.7(1)$$

However applying chain rule we have

$$\frac{\partial T^*}{\partial \theta} = T_0^* - T_\infty^* \quad (2)$$

$$\frac{\partial T^*}{\partial y^*} = \frac{\partial T^*}{\partial \theta}.\frac{\partial \theta}{\partial y}.\frac{\partial y}{\partial y^*} \quad = \frac{U(T_0^* - T_\infty^*)}{\vartheta}.\frac{\partial \theta}{\partial y}$$

$$\frac{\partial^2 T^*}{\partial y^{*2}} = \frac{\partial}{\partial y^*}\left[\frac{\partial T^*}{\partial y^*}\right] = \frac{U^2(T_0^* - T_\infty^*)}{\vartheta^2}.\frac{\partial^2 \theta}{\partial y^2} \quad (3)$$

$$\frac{\partial T^*}{\partial t^*} = \frac{\partial T^*}{\partial \theta}.\frac{\partial \theta}{\partial t}.\frac{\partial t}{\partial t^*} = \frac{U^2(T_0^* - T_\infty^*)}{\vartheta}.\frac{\partial \theta}{\partial t} \quad (4)$$

$$u^{*2} = u^2 U^2 \quad (5)$$

Substituting equations (3), (4) and (5) in equation 2.7(1) gives

$$\frac{U^2(T_0^* - T_\infty^*)}{\vartheta}.\frac{\partial \theta}{\partial t} = \frac{k(T_0^* - T_\infty^*)U^2}{\rho C_p \vartheta^2}.\frac{\partial^2 \theta}{\partial y^2} + \frac{\mu U^4}{\rho C_p \vartheta^2}.\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma u^2 U^2 B_0^2}{\rho C_p} \quad (6)$$

Equation (6) can be rewritten as

$$\frac{\partial \theta}{\partial t} = \frac{k}{\mu C_p}.\frac{\partial^2 \theta}{\partial y^2} + \frac{U^2}{C_p \Delta T}.\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{U^2}{C_p \Delta T}\right)\left(\frac{\sigma \vartheta B_0^2}{\rho U^2}\right) u^2 \quad (7)$$

Equation (7) simplifies to

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r}.\frac{\partial^2 \theta}{\partial y^2} + E_c\left(\frac{\partial u}{\partial y}\right)^2 + E_c M^2 u^2 \quad (8)$$

The corresponding initial and boundary conditions in dimensional form are; when $t \leq 0$, $u = 0$ and $\theta = 0$ at $y = 0$ while when $t > 0$, $u = 1$ and $\theta = 1$ at $y = 1$.

In the chapter to follow, equations 2.6(4) and 2.7(8) are expressed in finite difference form.

CHAPTER THREE

3.0 **METHOD OF SOLUTION**

The problem under investigation hasgenerated non-linear differential equations. Being a boundary value problem, its solution has been determined using finite difference approach. In this technique derivatives occurring in the generated differential equations have been replaced by their finite difference approximations. The resulting linear equations have then been solved by the central difference approximations which involves selecting a uniform mesh that consists of a network of rectangles of width Δt and height Δy as shown

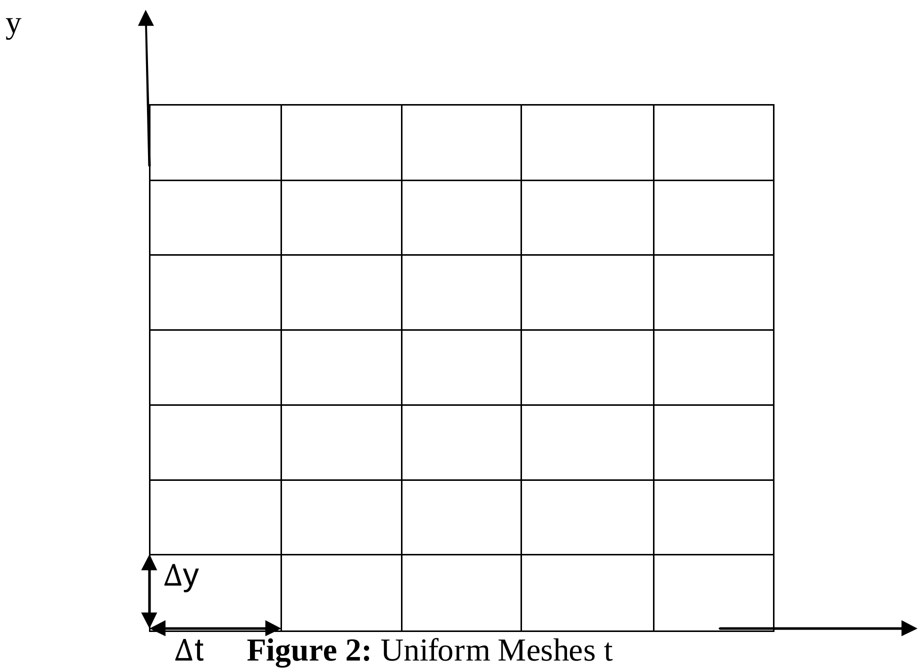

Figure 2: Uniform Meshes t

Central difference approximation is defined by obtaining Taylor's series expansions of y(x+h) and y(x-h) respectively as

$$y(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \frac{h^3}{3!}f'''(x) \quad (1)$$ and

$$y(x-h) = f(x) - hf'(x) + \frac{h^2}{2!}f''(x) - \frac{h^3}{3!}f'''(x) \quad (2)$$

To obtain first order differences subtract (2) from (1) giving

$$f'(x) = \frac{y(x+h)-y(x-h)}{2h} + H.O.T \quad (3)$$

Similarly to obtain second order differences add (1) to (2) to get

$$f''(x) = \frac{y(x+h)-2f(x)+f(x-h)}{h^2} + H.O.T \quad (4)$$

By taking $f''(x) = \frac{\partial^2 u}{\partial y^2} = U''$, $\frac{\partial^2 \theta}{\partial y^2} = \frac{\partial^2 T}{\partial y^2} = \theta''$, $f'(x) = \frac{\partial u}{\partial y} = U'$, $\frac{\partial \theta}{\partial y} = \frac{\partial T}{\partial t} = \theta$ and

setting $\Delta t = \Delta y$ where $\Delta t = h$ and $\Delta y = k$ then $h = k$

thus we have

$$\frac{\partial^2 u}{\partial y^2} = U'' = \frac{y(x+h) + y(x-h) - 2f(x)}{h^2} + H.O.T$$

$$= \frac{U_{ij+1} - 2U_{ij} + U_{ij-1}}{k^2} H.O.T \quad (5)$$

$$\frac{\partial u}{\partial y} = U' = \frac{U_{i,j+1} - U_{i,j-1}}{2k} + H.O.T \quad (6)$$

$$\frac{\partial^2\theta}{\partial y^2} = \theta'' = \frac{\theta_{ij+1} - 2\theta_{ij} + \theta_{ij-1}}{k^2} + H.O.T \quad (7)$$

$$\frac{\partial\theta}{\partial y} = \theta' = \frac{\theta_{ij+1} - \theta_{ij-1}}{2k} + H.O.T \quad (8)$$

Substituting directlydifferent values of M, P_r and E_c in equations 2.6(4) and 2.7(8) make it too difficult to obtain their exact solutions analytically. Hence the equations are solved using central finite difference by substituting equations 3.0(5) and 3.0(6) in equation 2.6(4) to give

$$\frac{U_{ij+1} - 2U_{ij} + U_{ij-1}}{k^2} + v_0 \frac{U_{ij+1} - U_{ij-1}}{2k} - M^2 u_{ij} + c + H.O.T = 0 \quad (9)$$

Similarly substituting equations 3.0(7), and 3.0(8) in equation 2.7(8) give

$$\frac{\theta_{ij+1} - \theta_{ij-1}}{k} = \frac{1}{p_r} \frac{\theta_{ij+1} - 2\theta_{ij} + \theta_{ij-1}}{k^2} + E_c \left(\frac{U_{ij+1} - U_{ij-1}}{2k}\right)^2 + E_c M^2 u_{ij}^2 + H.O.T \quad (10)$$

where i and j refer to y and t respectively.

The results generated from equations 3.0(8) and 3.0(10) using a central finite difference scheme are discussed and analyzed using tables and graphs.

CHAPTER FOUR

4.0 **RESULTS AND DISCUSSIONS**

The nonlinear differential equations 2.6(4) and 2.7(8) together with boundary conditions have been expressed in finite difference form and then solved using MATLABsoftware. The results on how Hartmann, Eckert and Prandtl Numbers affect velocity profiles and temperature distribution have been presented in tabular and graphical form.

Table 1 below shows how the velocity of the fluid changes with distance away from the lower plate. It is noted that at y = 0.3, the values velocity change from 0.9110 for$M = 0$ to 0.8048 for $M = 1.5$ measured normally from the lower plate. When M = 0 means that magnetic force is so small compared to viscous force. An increase in the Hartmann number results due to the fact that magnetic force increases as viscous force decreases. This is so because heat flux is subjected normally from the lower plate. Consequently, asthe distance from the upper plate is increased downwards magnetic force also increases implying thatLorentz force is generated. This force opposes the fluid motion hence decelerating the flow. In turn adecrease in velocity profiles as Hartmann number increases is noted. Similarly, the boundary layer formedas the fluid flow along the x-direction is thicker at the trailing edge and thinner at theleading edge. As the fluid flows, some of it is sucked by the lower porous plate hence stabilizing the laminar and steady flow. It is clear that velocity of the fluid at the trailing edge is higher than that at the leading edge.

When values of distance y in table 1 are drawn against corresponding values of velocity at different Hartmann numbers, the graphs in figure 3 below are obtained. The graphs reveal a similar trend as noted in table1.This implies that an increase in Hartmann number leads to a decrease in velocity profiles.

Table 1: Effect of different values of Hartmann on velocity distribution at h = 0.1 and C = 2

At y	0.1	0.3	0.5	0.7	0.9
U for M = 0	0.9871	0.9110	0.7537	0.5144	0.1924
U for M = 0.5	0.9696	0.8724	0.7113	0.4824	0.1805
U for M = 1	0.9533	0.8371	0.6729	0.4536	0.1698
U for M = 1.5	0.9381	0.8048	0.6380	0.4276	0.1601

The graphs corresponding to velocity profiles as per table 1 above are as shown in figure 3.

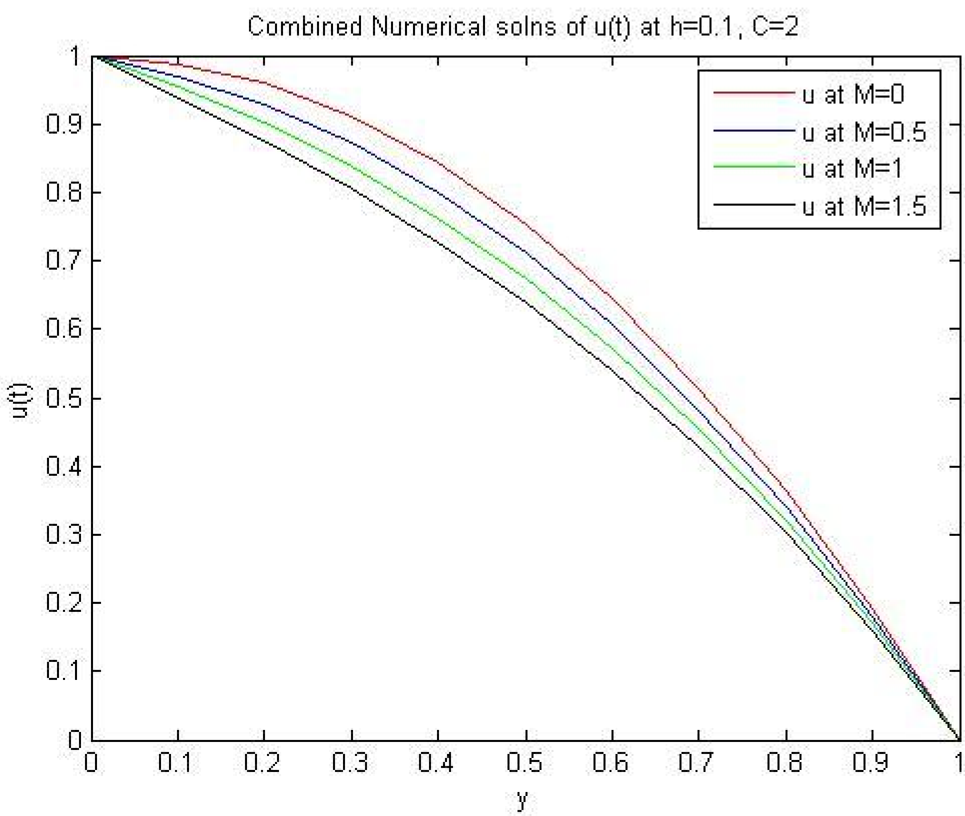

Figure 3: velocity profiles at different values of Hartmann

Further an investigation on how Hartmann Number affect temperature distribution has been carried out. At different values of y, respective values of temperature have been obtained when Hartmann number change from M=0.5 to M=1.5.The results are in table 2.It is noted that fluid particles flow at a higher kinetic energy near the upper plate and the energy decrease as distance is increased normally downwards. This implies that collision of fluid particles is higher near the upper plates thus causing high heat generation. As distance is increased downwards to the lower plate, kinetic energy decrease due increased viscosity hence reduces the heat generation. This means that temperature increase with increase in Hartmann number. For instance at y=0.3 the values of temperature change from 0.7580 to 0.9678 as Hartmann number change from M=0.5 to M=1.5respectively. It is noted that temperature changes in the same way for other values of y.This result indicates that temperature at any value of y increase with increase in Hartmann number. However, values of temperature reduce significantly as the distance from the trailing edge increases. This is due to the presence of Lorentz force that results from the application of a uniform transverse magnetic field normal to the plates.This force is resistive to the flow hence decelerates it.

When temperature distribution is plotted against vertical distance y,a similar occurrence is noted in figure 4 as revealed in table2.

Table 2: Effect of different values of Hartmann on temperature distribution when h = 0.01, $P_r = 0.71$ and $E_c = 0.2$

At y	0.1	0.3	0.5	0.7	0.9
θ for $M = 0.5$	0.9444	0.7580	0.4991	0.2266	0.0304
θ for $M = 1.0$	0.9931	0.8651	0.6091	0.2931	0.0412
θ for $M = 1.5$	1.0377	0.9678	0.7179	0.3611	0.0526

The figure 4 below corresponds to results in table 2.It shows variation of temperature with distance at h = 0.01, $P_r = 0.71$ and $E_c = 0.2$

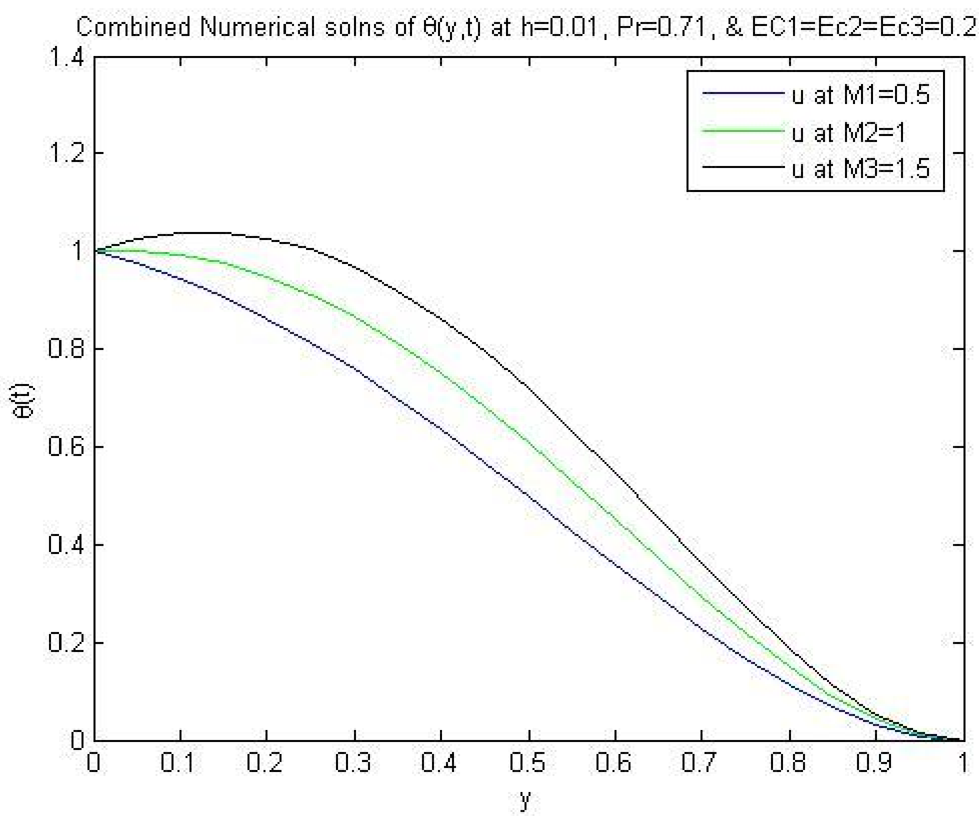

Figure 4: Effect of Hartmann on temperature distribution

In addition a study on how changing Eckert number affect the distribution of temperature has been done. The results obtained have been presented in table3 and corresponding figure 5. As already sited kinetic energy of fluid particles is higher near the upper plate in relation to thermal energy difference across the boundary layer. However kinetic energy reduces as distance increases in a vertical direction whereas thermal energy difference across the boundary layer increases as distance increase in the same direction. This implies that there is more heat generation near the upper plate. This outcome reveals an increase in temperature with an increase in Eckert number.For instance at y = 0.3 temperature changes from 0.7088 for $E_c = 0.5$ to 0.7966 for$E_c = 1.5$. However as the flow progress in the x-direction, the fluid flow is retarded hence collision of particles is reduced. This cause a gradual fall in temperature as distance from the trailing edge increases.

When different values of distance y in table 3are plotted against corresponding temperature values, figure 5 below is obtained. It is evidenced that at each Eckert number, temperature is higher near the origin and fallsgraduallythereafter. It is therefore generally clear that an increase in Eckert number results into an increase in temperature distribution.

Table 3: Variation of Eckert on temperature distribution when h = 0.05, M = 0.2 and $P_r = 0.71$

At y	0.1	0.3	0.5	0.7	0.9
θ for E_c= 0.5	0.9221	0.7088	0.4490	0.1966	0.0256
θ forE_c=1.0	0.9476	0.7613	0.4991	0.2244	0.0298
θ forE_c =1.5	0.9638	0.7966	0.5334	0.2438	0.0327

The figure 5 below represents the table3 findings. In these graphs temperature varies with distance at h = 0.05, M = 0.2 and $P_r = 0.71$ for different values of Eckert numbers

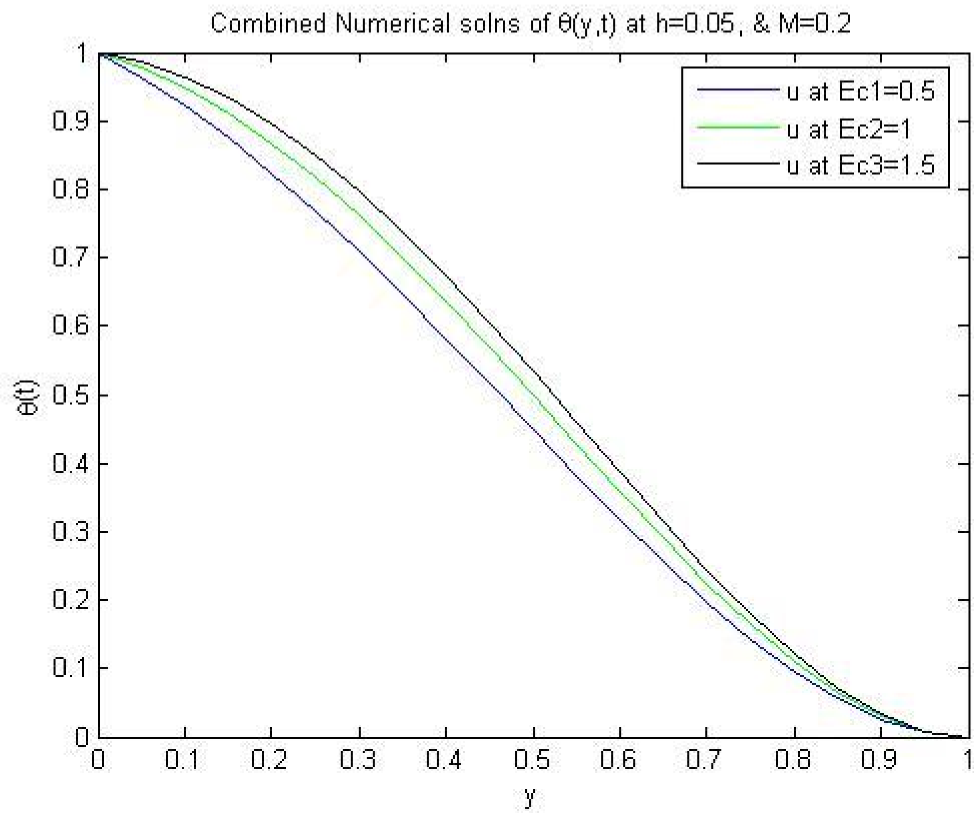

Figure 5: Effect of different values of Eckert number on Temperature

Further an investigation on how varying Prandtl number affect the distribution of temperature has been carried out. The results collected have been represented in table4 and figure 6 respectively. It is realized that temperature drops from 0.7658 when $P_r = 0.4$ to 0.6961 when$P_r = 1.0$ at y = 0.3 as revealed in table 4. Near the upper plate thermal diffusion is higher since temperature is high. As vertical distance from the upper plate increases downwards, kinetic energy reduces hence slowed rate of thermal diffusion in relation to viscous force. In addition, boundary layer grows thicker at the trailing edge and thinner at the leading edge due to viscosity. Thus thermal boundary layer thickness reduces in the flow direction. It is therefore noted from table 4 that a rise in Pandtl number lead to a decrease in temperature distribution because an increase in prandtl number means a slow rate in thermal diffusion.

Graphs in figure 6 suitably agree with table 4 results that an increase in prandtl number leads to a fall in temperature distribution because thermal diffusion is gradually overcome by momentum diffusibility.

Table 4: Variation of Prandtl Number on temperature distribution for h = 0.05, $E_c = 0.5$ and M= 0.2

At y	0.1	0.3	0.5	0.7	0.9
θ for $P_r = 0.4$	0.9467	0.7658	0.5053	0.2287	0.0305
θ for $P_r = 0.7$	0.9297	0.7305	0.4714	0.2097	0.0277
θ for $P_r = 1.0$	0.9130	0.6961	0.4387	0.1915	0.0249

The corresponding graphs for table4 above are as drawn in figure 6below. In the graphs temperature vary with a change in prandtl number at h=0.05, $E_c = 0.5$ and M =0.2

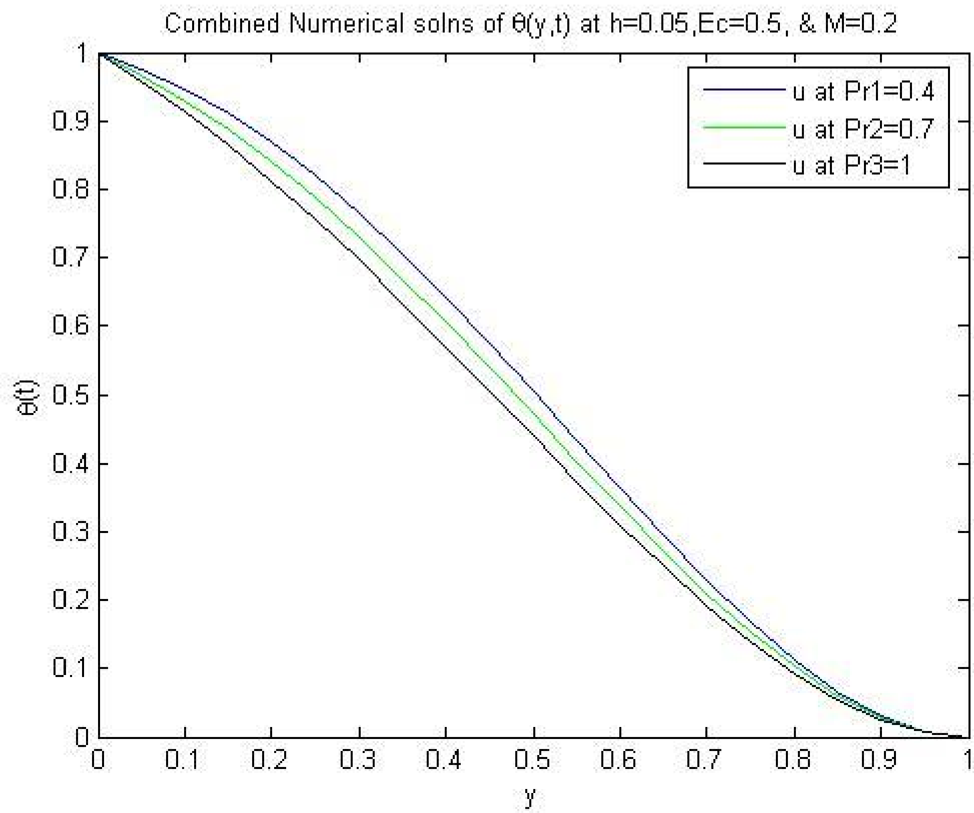

Figure 6: Effect of different values of Prandtl Number on temperature distribution

A summary of all these results is represented in the chapter to follow.

CHAPTER FIVE

5.0 CONCLUSION

Velocity profiles and temperature distribution on a steady flow of an incompressible, viscous and electrically conducting fluid in parallel horizontal plates in the presence of a uniform transverse magnetic field have been investigated. The effect of Hartmann, Eckert and Prandtl Numberson velocity profiles and temperature distribution has been carried out. It has been noted that an increase in Hartmann numbercauses a decrease in velocity profiles. However an increase in Hartmann number leads into an increase in temperature distribution. It is also revealed that an increase invalues of Eckert results into an increase in temperature distribution between the plates. Further an increase in Prandtl Number leads to a fall in temperature distribution.The $p_r$number is taken to be 0.71corresponding to air.

5.1 RECOMMENDATIONS

It is recommended that an extension of this work be done on the following areas;

1. Two parallel plates inclined at an angle one porous and the other impulsively started under transverse magnetic field.
2. Two parallel verticalplates one porous and the other impulsively started under constant heat flux.

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