Magneto-Hydrodynamics Analysis of Free Convection Flow between Two Horizontal Parallel Infinite Plates Subjected to Constant Heat Flux

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***Abstract*—**A steady MHD stokes free convection flow of an incompressible, electrically conducting fluid between two parallel infinite plates subjected to constant heat flux and pressure gradient has been studied. The flow is considered heat generating due to frictional heating of fluid particles. The upper plate is impulsively started at constant velocity while the lower plate is assumed to be porous and stationary. The two plates are separated by a distance h. A constant magnetic field is applied normal to the main flow direction and is found to affect velocity profiles considerably. Further, an investigation on how Prandtl number, Eckert Number and Hartman Number affect velocity profiles and temperature distribution has been carried out. The non-linear differential equations generated in this study are solved by finite difference method. The results that have been obtained on velocity profiles and temperature distribution are discussed in detail. An increase in Hartmann is found to cause a decrease in velocity profiles and an increase in temperature distribution. In addition, an increase in Eckert number causes an increase in temperature while an increase in Prandtl leads to a fall in temperature distribution. These results are found to merge with the physical situation of the flow.

***Keywords*—**Eckert Number; Hartmann Number; Heat Transfer; Prandtl Number; Pressure Gradient; Suction Velocity.

***Abbreviations*—**Department of Pure and Applied Mathematics (DPAM); Higher Order Terms (H.O.T); Jomo Kenyatta University of Agriculture and Technology (JKUAT); Magneto Hydro Magnetic (MHD); University of Nairobi (U.O.N).

I. INTRODUCTION

A fluid is a substance whose constituent particles may continuously change their positions relative to one another when shear force is applied to it. As fluid flows, heat is transferred from one point to another. Heat transfer in fluids is called convection. Fluids do not exist in isolation but with solids. Fluids flowing in engineering devices occur within magnetic field. Fluid flow in the presence a magnetic field is called hydro magnetic flow and the study of hydro magnetic flows is called Magneto Hydro Dynamics (MHD). The experimental and theoretical research on MHD flows is important to scientific and engineering fields. In particular the influence of a magnetic field on a viscous, incompressible flow of an electrically conducting fluid is encountered in engineering devices such as MHD generators, MHD fluid dynamos, flow meters, heat exchangers and pipes that connect system components.

II. RELATED WORKS

A number of scientists have carried out investigations in areas related to this study. Merkin & Mahmood (1990) investigated on free convection boundary layer on a vertical plate with prescribed surface heat flux while a study on Hydro magnetic free convection currents effects on boundary layer thickness was carried out by Marigi et al., (2010); Ram et al., (1995) solved magneto hydrodynamics stokes problem of convection flow for a vertical infinite plate in a dissipative rotating fluid with Hall current. They analyzed the effects of various parameters on the concentration velocity and temperature profiles. A steady MHD flow of an electrically conducting fluid between parallel infinite plates was done by Chandra (2005) while Kwanza et al., (2003) presented their work on MHD stokes free convection past an infinite vertical porous plate subjected to a constant heat flux with ion-slip and radiation absorption. They discussed their tabulated results on concentration, velocity and temperature distributions both theoretically and graphically. An investigation on an unsteady free convection of incompressible fluid past a semi-infinitive vertical porous plate in the presence of a strong magnetic field at an angle (α) to the plate with Hall ion-slip current effects was done by Okelo (2007). They discussed the effects of modified Grashof number, heat source parameter, Schmidt number, time, hall current, angle of inclination and Eckert number on a convectively cooled or convectively heated plate restricted to a laminar boundary layer. They found that an increase in mass diffusion parameter causes a decrease in concentration profiles while an increase in suction velocity leads to an increase in concentration profiles. Further, they realized that an increase in Eckert number results into an increase in temperature profiles whilst an increase in angle of inclination causes an increase in primary velocity profiles and a decrease in secondary velocity profiles. Palani & Abbas (2009) carried out an investigation on free convection MHD Flow with thermal radiation from an impulsively started vertical plate. They established that velocity increases with a decrease in magnetic field parameter. In addition they realized that dimensionless temperature decreases with an increase in thermal radiation. In addition Abuga et al., (2011) carried out an investigation on the effects of hall current and rotational parameter on dissipative fluid past a vertical semi-infinite plate. They found that an increase in hall parameter for both cooling of the plate by free convection currents and heating of the plate by free convection currents has no effect on temperature profiles but leads to an increase in velocity profiles. Similarly they found that an increase in rotational parameter led to a decrease in velocity profile when the Eckert number was 0.01 and an increase in velocity profile when Eckert number was 0.02. More over Manyonge et al., (2012) investigated on Steady MHD poiseuille flow between two infinite porous plates in an inclined magnetic field. They established that a high Hartmann flow (high magnetic field strength) decreases velocity whereas Gireesha et al., (2012) investigated the effect of Radiation on Hydro magnetic flow and heat transfer of a dusty fluid between two parallel plates. They established that thermal radiation parameter increases temperature of both the fluid and the dust. Similarly they found that velocities of both the dust and fluid decreases with increases in strength of the magnetic field and number density.

In spite of all these studies, the problem of MHD stokes free convection past an infinite horizontal stationary porous plate under constant heat flux has not received adequate attention. An assessment of the effect of uniform magnetic field perpendicular to the plates together with a constant pressure gradient on temperature distribution and velocity profile has been done. The fluid is viscous, incompressible and electrically conducting. It flows steadily between two horizontal parallel infinite plates. The upper plate impulsively started at constant velocity and the lower plate porous and stationary. In addition we have investigated on the effect of Prandtl, Eckert and Hartmann numbers on velocity profiles and temperature distribution.

III. Geometry of the Problem

A steady flow of a viscous, incompressible and electrically conducting fluid flowing between two horizontal plates separated at distance h is considered. The lower plate is assumed porous and stationary positioned along the x- axis of the Cartesian plane while the upper plate is impulsively started at a constant velocity. The y-axis is perpendicular to both plates. It is assumed that the plates are electrically non-conducting and a uniform magnetic field is applied normal to the plates. The Reynolds number is taken to be so small thus induced magnetic field is neglected. The flow configuration is as shown in figure 1.

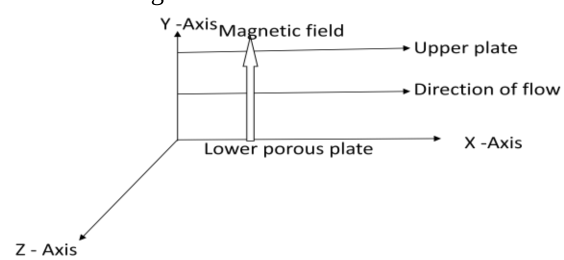

Figure 1: Geometry of Flow Configuration

IV. Governing Equations

The set of equations governing this flow comprise of Navier stokes and energy equations. These are given by

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho u) = 0 \text{ (Continuity equation)} \quad (1)$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2} - \rho g + \vec{J} X \vec{B} \quad (2)$$

(Momentum equation)

$$\rho C_p \frac{D_T}{D_t} = \nabla.(K\nabla.T) + \frac{1}{\sigma} j^2 + \mu\varphi \text{ (Energy equation)} \quad (3)$$

Where,

$\vec{J} \times \vec{B} = -\sigma B_0^2 U$ is lorentz force,

μ is Coefficient of viscosity, k is Thermal conductivity,

σ is Electrical conductivity,

g is Acceleration due to gravity,

T is temperature of the fluid,

ρ is Fluid density, P is Pressure,

$\frac{D}{DT}$ is Material derivative,

φ is Viscous dissipation function,

C_p is specific heat at constant pressure,

$\frac{1}{\sigma} j^2 = \sigma B_0^2 U^2$ is internal heat generation and

ϑ Kinematic viscosity.

Since the flow observes the no-slip conditions implies that velocity of the fluid at the upper plate equals the velocity of the plate while at the lower plate the velocity is zero.

Thus the initial and boundary conditions are; at y = 0, u = 0 and $T = T_0$ while at y = 1, $u = U$ and $T = T_1$.

Hence the momentum equation reduce to

$$\rho\left(-v_0\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} + P - \sigma\mu_e^2 H^2 u \tag{4}$$

while the energy equation reduces to

$$\rho C_p\left(\frac{\partial T}{\partial t}\right) = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma B_0^2 U^2 \tag{5}$$

On introducing the following non-dimensional parameters

$P_r = \frac{C_p\mu}{k}$ is Prandtl number, $E_c = \frac{U^2}{C_p(\Delta T)}$ is Eckert number and $M^2 = \frac{\sigma B_{0\vartheta}^2}{\rho U^2}$ is Hartmann number.

Equations (4) becomes,

$$\frac{\partial^2 u}{\partial y^2} + v_0\frac{\partial u}{\partial y} - M^2 u + c = 0 \tag{6}$$

whereas equation (5) reduces to,

$$\frac{\partial\theta}{\partial t} = \frac{1}{P_r}\cdot\frac{\partial^2\theta}{\partial y^2} + E_c\left(\frac{\partial u}{\partial y}\right)^2 + E_c M^2 u^2 \tag{7}$$

where $c = \frac{P}{\rho}$ is a constant, θ Dimensionless fluid temperature, T_0 and T_1 are temperatures for the lower and upper plates respectively.

The corresponding initial and boundary conditions in dimensional form are;

$$\text{when } t \le 0, u = 0 \text{ and } \theta = 0 \text{ at } y = 0 \text{ while when } t>0, u = 1 \text{ and } \theta = 1 \text{ at } y = 1 \tag{8}$$

V. Method of Solution

The non-linear differential equations (6) and (7) with initial and boundary conditions (8) are solved using finite difference approach. In this technique derivatives occurring in the generated differential equations have been replaced by their finite difference approximations. The resulting linear equations have then been solved by the central difference approximations. Equations (6) and (7) when expressed in finite difference form are as indicated by equations (9) and (10) respectively.

$$\frac{U_{ij+1} - 2U_{ij} + U_{ij-1}}{k^2} + v_0\frac{U_{ij+1} - U_{ij-1}}{2k} - M^2 u_{ij} + c + H.O.T = 0 \tag{9}$$

$$\frac{\theta_{ij+1} - \theta_{ij-1}}{k} = \frac{1}{p_r}\frac{\theta_{ij+1} - 2\theta_{ij} + \theta_{ij-1}}{k^2} + E_c\left(\frac{U_{ij+1} - U_{ij-1}}{2k}\right)^2 + E_c M^2 u_{ij}^2 + H.O.T \tag{10}$$

where i and j refer to y and t respectively.

The finite difference equations (9) and (10) are then solved using MATLAB software.

VI. Results and Discussions

The results on how Hartmann, Eckert and Prandtl Numbers affect velocity profiles and temperature distribution have been presented in tabular and graphical form. The results show that Eckert number, Hartmann number, Prandtl number affects greatly both velocity profiles and temperature distribution.

Table 1 below shows how the velocity of the fluid changes with varied Hartmann number. An increase in the Hartmann number leads into a decrease in velocity distribution. This is in agreement with the physical situation since Lorentz force generated due to the application of a constant magnetic field. This force being resistive, opposes the fluid motion hence decelerating the flow. Similarly, it is revealed that velocity of the fluid at the trailing edge is higher than that at the leading edge since the boundary layer formed as the fluid flow along the x-direction is thicker at the trailing edge and thinner at the leading edge.

Table 1: Effect of Different Values of Hartmann on Velocity Distribution at h = 0.1 and C = 2

At y	0.1	0.3	0.5	0.7	0.9
U for M = 0	0.9871	0.9110	0.7537	0.5144	0.1924
U for M = 0.5	0.9696	0.8724	0.7113	0.4824	0.1805
U for M = 1	0.9533	0.8371	0.6729	0.4536	0.1698
U for M = 1.5	0.9381	0.8048	0.6380	0.4276	0.1601

Plotting values of y in table 1 against corresponding values of velocity at different Hartmann numbers, give rise to graphs in figure 2 below. The graphs reveal a similar argument noted in table1.

Table 2 gives a summary of variation of Hartmann, Eckert and Prandtl Numbers on temperature distribution. It is noted that fluid particles flow at a low kinetic energy near the lower plate due to increased viscosity and thick boundary layer. This implies that collision of fluid particles is high in this region causing high heat generation. Consequently, temperature increases with increase in Hartmann number. When values of y are drawn against temperature for different Hartmann Numbers, graphs in figure 3 are obtained. This figure unfolds a similar occurrence noted in Table 2.

In addition a study on how varying Eckert number affect the distribution of temperature has been done. It is realized that an increase in Eckert number causes an increase in temperature distribution. This is so due stored heat energy of the fluid that results from frictional heating of fluid particles. For instance at y = 0.3 temperature changes from 0.7088 for Eckert = 0.5 to 0.7966 for Eckert = 1.5. However as the flow progress in the x-direction, the fluid flow is retarded hence collision of particles is reduced. This cause a gradual fall in temperature for all Eckert numbers as distance from the trailing edge increases. When different values of distance y are plotted against corresponding temperature values, figure 4 below is obtained. The figure agrees with the physical situation of the flow.

Further an investigation on how varying Prandtl number affect the distribution of temperature has been carried out. The results collected have been represented in table 2 and have a good agreement with figure 5. Thermal diffusion is higher near the upper plate and is slowed as distance increases towards the lower plate. In addition, boundary layer grows thicker at the trailing edge and thinner at the leading edge due to viscosity. Thus thermal boundary layer thickness reduces in the flow direction. It is therefore noted from table

2 that a rise in Pandtl number lead to a decrease in temperature distribution because an increase in prandtl number means a slow rate in thermal diffusion.

Table 2: Temperature Distribution at Various Values of Hartmann, Eckert and Prandtl Numbers

			Temperature Distribution at Different Values of y				
P_r	**E_c**	**M**	**0.1**	**0.3**	**0.5**	**0.7**	**0.9**
0.71	0.2	0.5	0.9444	0.7580	0.4991	0.2266	0.0304
		1.0	0.9931	0.8651	0.6091	0.2931	0.0412
		1.5	1.0377	0.9678	0.7179	0.3611	0.0526
0.71	0.5	0.2	0.9221	0.7088	0.4490	0.1966	0.0256
	1.0		0.9476	0.7613	0.4991	0.2244	0.0298
	1.5		0.9638	0.7966	0.5334	0.2438	0.0327
0.4	0.5	0.2	0.9467	0.7658	0.5053	0.2287	0.0305
0.7			0.9297	0.7305	0.4714	0.2097	0.0277
1.0			0.9130	0.6961	0.4387	0.1915	0.0249

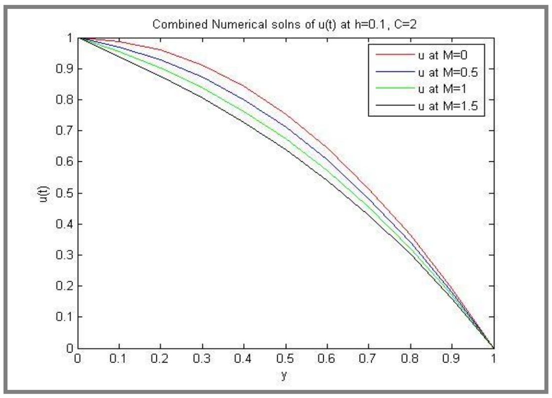

Figure 2: Effect of Hartmann Number on Velocity Profiles

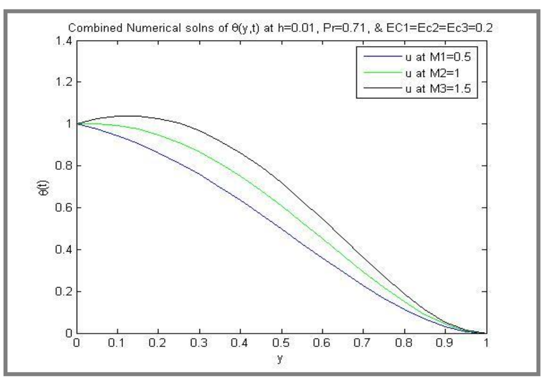

Figure 3: Effect of Hartmann Number on Temperature

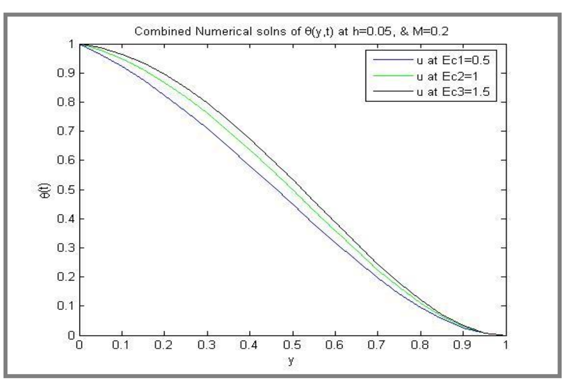

Figure 4: Effect of Eckert Number on Temperature

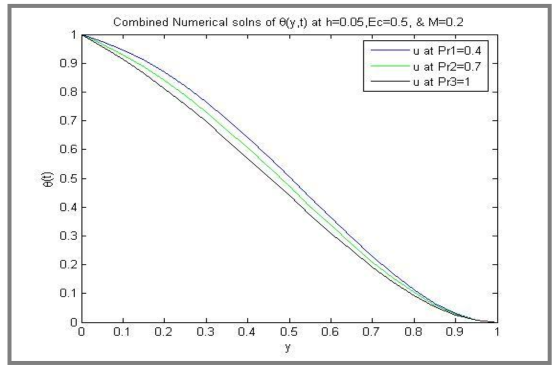

Figure 5: Effect of Prandtl Number on Temperature

VII. CONCLUSION

Velocity profiles and temperature distribution on a steady flow of an incompressible, viscous and electrically conducting fluid in parallel horizontal plates in the presence of a uniform magnetic field have been investigated. In the flow a constant pressure gradient is considered. Hartmann, Eckert and Prandtl Numbers are found to have a great effect on velocity profiles and temperature distribution. Results obtained for various values of these flow parameters have been found to suitably agree with the physical situation of the flow. It has been noted that an increase in Hartmann number causes a decrease in velocity profiles. However an increase in Hartmann number leads into an increase in temperature distribution. It has also been revealed that an increase in values of Eckert results into an increase in temperature distribution between the plates. Further an increase in Prandtl Number leads to a fall in temperature distribution.

VIII. FUTURE WORKS

It is recommended that an extension be done on the following areas in future;

- Two parallel plates inclined at an angle one porous and the other impulsively started under a transverse magnetic field.
- Fluid flow in an inclined infinite Annulus under a radial magnetic field.

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