ECONOMIC APPLICATIONS OF CO-INTEGRATION IN TIME SERIES

(A Case Study of the Kenyan Exchange and Interest Rates)

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DECLARATION

This project is my original work and has not been presented for a degree award in any other University.

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DEDICATION

Dedicated to my mum, Mrs. Evelyn Ng'etuny, my sisters: Mrs. Betsy Kosgei, Ms. Carolyn Ng'etuny and My brother Mr. Nathan Rotich.

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Contents

List of Tables

List of Figures

Nomenclature

ACF	Auto Correlation Function
ADF	Augmented Dickey Fuller test
AIC	Akaike Information Criterion
ARCH	Auto Regressive Conditional Heteroskedasticity
ARMA	Auto Regressive Moving Average Process
CBK	Central Bank of Kenya
CI	Co-integrated
COLS	Classical Ordinary Least Squares
ECM	Error Correction Model
GARCH	Generalized Autoregressive Conditional Heteroskedastic
GDP	Gross Domestic Product
I(k)	Integrated of order k
LIBOR	London Interbank Offered Rate
MSCI	Morgan Stanley Capital International
OLS	Ordinary Least Squares
RSE	Residual Standard Error

SSR	Sum of Squared Residuals
X	Exchange rate
Y	Interbank lending rate

DEFINITION OF TERMS

Co-integration

Given two series say A_t and B_t where both series are unit root non-stationary while a linear combination of the two series are stationary, then A_t and B_t are co-integrated (CI).

Definition 1. According to Sorensen (2005), x_t and y_t are said to be co-integrated if there exists a parameter α such that

$$u_t = y_t - \alpha x_t \tag{0.0.1}$$

is a stationary process.

Spurious Regressions

When a non stationary time series is regressed on another non stationary series, or Ordinary Least Squares (OLS) calculated for the data, then a high correlation might be seen to exist while there is no relationship. In such a case, we say that a spurious regression has been fitted. This problem is always fixed by first testing for stationarity and then differencing the series till it attains stationarity. Differencing is the most common way of converting a non stationary time series into mean-stationary. The stationary series can hence be regressed on each other. This results is explained as follows:

Definition 2. Suppose the series A_t and B_t are non stationary. Suppose we fit a regression line of the form

$$\lambda = A_t + \alpha B_t \tag{0.0.2}$$

Then Equation (0.0.2) is said to be a spurious regression for the two series.

Further suppose that the two series are integrated of first order ($I\left(1\right)$) such that the first difference $\triangle A_t = A_t^{'}$ and $\triangle B_t = B_t^{'}$ are stationary. Then the the two series are said to be co-integrated if the differenced series can be expressed as a linear combination of each other, say

$$\lambda^{'} = A_t^{'} + \alpha B_t^{'} \tag{0.0.3}$$

And the linear combination in Equation (0.0.3) is not a spurious regression. Differencing thus corrects spurious regression.

Super-consistent Estimators

An estimator for a parameter, say $\widehat{\alpha}$, is said to be super-consistent if the original series is unit root non stationary while the differenced series is stationary. That is, if we have a co-integrated series, then the OLS leads to a super consistent estimators of the variables. This leads to the following results:

Definition 3. Let A_t and B_t be two non stationary time series. Also, let these two series be co-integrated such that

$$\Omega = A_t - \alpha B_t \tag{0.0.4}$$

Then , if we choose the estimate of α, $\hat{\alpha}$, such that Equation (0.0.4) holds, then the estimate $\hat{\alpha}$ is said to be a super-consistent estimator of α.

Swap Spread

Definition 4. In financial sense, swap spread can be defined as the difference between the fixed rate of a swap and the newly negotiated rate.

ABSTRACT

The work presented in this thesis is done by both a simulation and empirical study. Two series of data are simulated using the Generalized Autoregressive Conditional Heteroskedastic (GARCH) model due to its ability to capture volatility and heteroskedasticity, which gives a guide to the empirical study. One main proposition is made that if two time series follow GARCH(1,1), the two series are cointegrated, a proposition first proved using a simulation study. In the empirical study, the U.S. dollar exchange rate and the interbank lending rate in Kenya are analyzed. Co-integration and OLS are used; and the model parameters tested for adequacy. The proposition in the simulation study is proved by a case study of the Kenyan market. Both the exchange and lending rates returned non-stationarity in all the tests. Differencing is applied to attain stationarity. Co-integrating factor is then estimated to be -0.490747, with its residuals being stationary. Relatively same R^2 and adjusted R^2 values indicates adequacy of the model which ascertains the proposition. Granger causality tests were as well done and only the exchange rate granger caused interbank lending rate. This can be explained by the instability in the exchange market. A linear Error Correction Model (ECM) is also fitted and there is evidence that a short-term relationship exists between the lending and the exchange rates. A high threshold value exists at the second lag, an indication of simple smoothing in the data. The residual deviance is greater than the degrees of freedom confirming that the model perfectly fit to the data, supported by the high R^2 value of 0.9308. It is recommended that a close track of exchange rates may lead to prediction of interbank lending rate movements. Further study should be conducted on tail clustering analysis, as well as on the factors influencing exchange rate movements and analysis of tail clustering. Also, a similar study should be undertaken with a combination of Auto Regressive Moving Average Process (ARMA) and GARCH models to capture both conditional variance and conditional expection properties.

Chapter 1

INTRODUCTION AND LITERATURE REVIEW

1.1 Background of the Study

Over the recent decades, there have been considerable research on time series analysis, with several scholars building different models for inter-data movements; most of which are volatility models which attempts to describe the movements with time. These models give an insight into the stochastic nature of the underlying data. Unfortunately, very few of these models have exhaustively discussed the source and existence of these volatilities. Nevertheless, Leykam (2008), Lin (2009), Musyoki et al. (2012), amongst others, concur that volatility posses a great challenge in forecasting of time series data, and that there exists a great need to analyze these shocks in two phases, the short and long-term, in order to capture the movements exhaustively; where imputation of the two relationships can be analyzed.

Volatility can be defined as the continuous displacement of a time series from its long-term mean-level. Volatility in financial markets can either be historical or implied. Historical volatility describes the changes in prices of financial instruments based on past price levels. Implied volatility considers the prices of these instruments from current to a future date (Mostly the expiration date, in case of options).

From the definition, volatility is displayed in a series of distortions from the long-term mean-level equilibrium; which occurs in the short-term. Unfortunately, most of the models put across by different schollars have not analyzed the short and long-term of a series independently. It has been assumed that the persistence of these shocks spread evenly to the future.

Interest and exchange rates are the most common examples of volatile series. Aside from their schollarly imputation, exchange and interest rates have economic consequence. Among others, interest rates and exchange rates form the economic indicators. Economic indicators are any statistics about an economy which can be used to predict future economic behaviour, based on some statistical procedures.

Exchange rates have been widely used for volatility analyses, including analyzing economic performance. Models developed have tried to capture the volatility experienced and incorporate shock trend in its forecasts. However, most of these models have not fully appreciated the fact that the relationship need to be analyzed in two fold; the short-run and the long-run. Also, despite exchange rates being one of the economic indicators, it does not directly affect the financial markets as much as interest lending rates. The most direct impact is felt particulary in the interbank lending rates as it determines the rate at which financial institutions will charge. Hence, if a relationship can be established between the exchange rates and the lending rates, then modeling of lending rates becomes more easier.

1.1.1 Introduction to Co-integration

Co-integration is a procedure which seeks to investigate the relationship between one time series and another. An important feature investigated is usually where they share a common drift. The technique measures the equilibrium of the time series in the long run. It utilizes the concept of relationship between non stationary time series, Engle and Granger (1987). Co-integration had been introduced earlier by Granger (1981) in his work where it was showed that if two time series are unit-root non-stationary and their first difference is stationary such that a linear combination of the original non-stationary series is stationary, then the two series are said to be co-integrated. Granger (1981) was analyzing a balance in an ECM where it was established that, there was an imbalance in a stationary ($I(0)$) and an integrated of order one ($I(1)$) series, Granger (2010). On analysis, it was appreciated that a linear combination of non-stationary series formed a stationary one. This result was then termed co-integration. The procedure involves estimation of a long-term and short-term relationship in existence. For these purposes and gaps in knowledge, a cointegration analysis procedure has been proposed.

Though co-integration may seem to be related to correlation, they are two different statistical properties of time series. Two or more time series can be strongly correlated but show very low co-integration, and even at some other circumstances, though minimal, no co-integration at all. Therefore, strong correlation does not necessarily imply an existence of co-integration, and vice versa.

1.1.2 Correlation

Correlation is a numerical value which indicates how two random variables are associated. It is normally computed as a coefficient called the correlation coefficient which ranges from $+1$ to -1. A value of $+1$ indicates a perfect positive correlation, a value of -1 indicates a perfect negative correlation while a value of zero means no correlation. It thus measures the strength of association between random variables.

The main difference between co-integration and correlation is that in correlation, the data used is assumed to be stationary. That is, we assume that the data does not have a large or a consistent deviance from the mean value. On the other hand, co-integration appreciates the fact that time series data are not stationary. Several procedures for measuring correlation have been put forth. The most commonly used measures are the Pearson's correlation coefficient and the Rank correlation coefficients.

1.1.2.1 Pearson's Product-Moment Coefficient

This is the most commonly used measure of correlation. It measures a linear relationship between two random variables. It attempts to fit a straight regression line to the data elements. It is normally denoted as r, measuring the distance of the data elements from the regression line. The value of r ranges from $+1$ to -1.

1.1.2.2 Rank Correlation Coefficient

This is a measure of correlation which is an improvement of the Pearson's correlation coefficient. It measures the extend of response of a variable upon a unit change in another variable. The relationship need not be a linear one. It is thus commonly used as an alternative to the Pearson's correlation. There exist several measures of rank correlation which has been developed; the common ones are the Spearman's rank correlation coefficient and the Kendall's rank correlation coefficients. Just like the Pearson's correlation, Rank correlation coefficient ranges from $+1$ to -1.

1.1.3 Why Co-integration

Co-integration best suites this analysis because:

1. In co-integration, we can analyze a series in the short-term and long-term. The short-term gives us the expected displacements of the series from its mean-level. It also captures the information which could have been lost in the long-term. The long-term gives a general trend relationship between the data.

2. If we can model the exchange rates based on previous studies, a plausible explanation on the interest rates can be made based on the exchange rates by building a linear model between the two rates.

3. Finally, in co-integration, the two series are considered simulteneously without converting each series to stationarity hence preserving important information contained in the series.

1.1.3.1 Spurious Regressions

The concept of spurious regressions can be dated back to a study by Granger and Newbold (1974) on spurious regressions in econometrics. They made the following observation;

Definition 5. If we have two series, say y_t and x_t, where the two series are simulated from a random walk for example, and we fit a regression line of the form;

$$y_t = \alpha_0 + \alpha_1 x_t + \varepsilon_t \tag{1.1.1}$$

where the series y_t and x_t are independent. Then by intuition, the value of α_1 in Equation (1.1.1), should equal to zero. but to the contrary, the limiting distribution of $\widehat{\alpha_1}$ is such that it approaches a family of functions of Brownian motions.

This study by Granger and Newbold (1974) further found out that there was a divergence of the same, with its distribution following a t-distribution with a divergence of the t-statistic at a rate $\sqrt{T}$. This posses a big challenge during estimation as it will be very hard to curb this or even detect using the OLS estimates.

In co-integration, instead of fitting the OLS model in Equation (1.1.1), we fit an OLS of a differenced series as in Equation (1.1.2) below:

$$\triangle y_t = \alpha_0 + \alpha_1 \triangle x_t + \varepsilon_t \tag{1.1.2}$$

The main aim being to get most consistent estimators of the regression coefficients. According to Sorensen (2005), these estimators are defined as follows:

Definition 6. Assume that Equation (1.1.1) holds with a stationary error term. This is exactly the case where x and y are co-integrated. In this case, $\hat{\alpha_1}$ is not only consistent, but it converges to the true value at rate T. We say that the OLS estimator is super-consistent.

1.1.3.2 Brownian Motion Representation

Suppose we have the model

$$y_t = \alpha_0 + \alpha_1 x_t + u_t \tag{1.1.3}$$

such that x_t is a simple random walk and u_t are serially uncorrelated. Then, according to Sorensen (2005) on his paper on co-integration, $T \times (\hat{\alpha_1} - \alpha_1)$ is asymptotically distributed as

$$\frac{\int_0^1 B_2 dB_1}{\int_0^1 B_2^2 dt} \tag{1.1.4}$$

where B_1 and B_2 are independent Brownian motions. From the theory of Brownian motion, it is clear that the limiting distribution has a mean zero and the resultant standard t-test is consequently asymptotically normally distributed.

1.2 Review of Previous Studies on the Subject of Study

Co-integration is among the most powerful tools in analyzing a time series. The capability of co-integration to explain a combined variation in two or more time series has attracted more scholars. Loosely speaking, co-integration can be seen as a strike between the basic stochastic processes and the standard inferential statistics.

1.2.1 Brief History

Granger (1981) is the first one to introduce the concept of co-integration. In his study, he showed that if two time series are unit-root non-stationary and their first difference is stationary such that a linear combination of the original non-stationary series is stationary, then the two series are said to be co-integrated. In his work, he was analyzing a balance in an ECM where he realized that there was an imbalance in $I(0)$ and $I(1)$ series, (Granger (2010)). On analysis of the series, he realized that a linear combination of the non-stationary series formed a stationary series. He then termed this result co-integration. Initially, Granger (1981) had been analyzing a $I(1)$ pair of time series. But there arose a need to be able to analyze time series of higher integration orders. This led to the extrapolation of the concept by Engle and Granger (1987) on their study which showed that the co-integration explained earlier by Granger (1981) could be extended to $I(p)$ integrated series.

1.2.2 OLS Estimation of Co-integration and Spurious Regression

Maurer (2008) applied co-integration techniques to estimate a co-integration model to track index and compare the co-integration approach to the time portfolio optimization approach. Some indices (FTSE100, DJ Industrial, DJ Composite Average) were used. A carefully done empirical analysis revealed that both the approaches yielded correct results. However, the co-integration approach was preferred due to its ability to capture data even with high volatility. Nevertheless, no particular model was preferred for index tracking analysis.

Later, Kazi (2009) used co-integration techniques to analyze the risk factors that emanates from the Australian stock market. He identifies five key risk factors; interest rates, dividend yield, corporate profitability, industrial production and global market influences. Kazi (2009) suggests the inclusion of these factors into the risk management portfolio for all investors. It was noted that regressing non-stationary time series often led to spurious regression, with an exception of co-integration indicating a long-run relationship.

Earlier, Korir et al. (2003) had applied these procedures in the analysis of beans markets in Tanzania and Kenya, the main aim being to establish if there was any integration relationship within the markets, and if so, the impact of this integration. Pearson's correlation coefficients were used. The occurrence of spurious regression was as well appreciated and co-integration was opted as a correction mechanism.

These concepts have also been applied in the study of real exchange rate equilibrium and misalignment by Musyoki et al. (2012). In their study, the Johansen's co-integration test was applied and ECM computed. OLS technique was used to estimate co-integration regression parameters, then the model residuals tested for unit root. Based on the tests, Musyoki et al. (2012) concluded that there existed enough evidence which showed that the real exchange rate maintained a level which was above its equilibrium. Nevertheless, within the study period, the country experienced sky-rocketing in the exchange rates market.

1.2.3 Residuals Unit-root test and Parameter Estimation

Vuranok (2009), in his term project, examines the relationship between the two elements in Turkey by investigating Gross Domestic Product (GDP) as an economic indicator, on a quarterly basis. The theoretical Johansen's test procedure for analyzing time series is discussed in details, but the results presented were analyzed using Schwarz Criteria. Residuals did not show any auto-correlation. There existed no long run association between the two variable. Vuranok (2009) explains this as a failure of existence of a strong financial systems with developed financial markets. It was suggested that, if the same procedure is done in a well developed financial markets, then a perfect positive relation will be observed.

In financial development and economic growth, Murinde (2012) puts to light the importance of unit-root tests for co-integration in analyzing the inconsistencies arising from recent empirical studies in the field. In his view, though multivariate methods and not bi-variate have been applied, there still existed a lot of inconsistencies and bias in their estimation. According to Murinde (2012), unit-root tests were fundamental irrespective of method, whether co-integration or causality analysis. Further, there exists a need for unit root and co-integration tests before causality analysis can be done. Meaning, the results of the causality analyses were pegged on the unit root and co-integration tests. Therefore, according to Murinde (2012), unit-root test is a powerful fundamental test.

1.2.4 Unit-root Stability and Granger Causality Model

Earlier, Fung and Hsieh (2004) had used co-integration on their study on hedge funds. They criticized the conventional approaches of model constructions for asset-class indices to be applied in hedging. Seven factors were identified from which a model was built. On analysis of parameter stability, Fung and Hsieh (2004) applies the cumulative recursive residual method and plots on a time scale to investigate the reversion of the model parameter in the risk factor model. The factors are co-integrated and hence influence each others' performance. Fung and Hsieh (2004) finally proposed a seven factor model to be applied for hedging.

Later, Leykam (2008) in his master's thesis in economics, analyzes the existence of co-integration in prices on different natural gas hubs in the European region. The study focused on four major hubs; NBP, Zeebrugge, TTF and Bunde.Leykam (2008) estimates a co-integration model to investigate the pairwise relationships. Of these hubs, NBP and Zeebrugge were found to have the strongest relation due to their direct pipeline connecting the two hubs. This result was further ascertained by the ECM which indicated an exogenous connection between these two hubs.

The study by Fung and Hsieh (2004) was extrapolated by Lin (2009) who analyzed the same hedge funds in view of further examining the validity of the method used in deriving the seven factors which had been suggested by Fung and Hsieh (2004) for inclusion in an hedging portfolio. In his research, he reports that Fung and Hsieh (2004) did not provide enough evidence to proof that the procedure used in choosing the factors is quite different from the Sharpe and Fama-French which only relies on one characteristics of the entire market. Contrary to Fung and Hsieh (2004), Lin (2009) bases his parameter stability on the adjusted R^2 statistic. Lin (2009) does not mention the reason for his selection of R^2 statistic instead of the cumulative recursive residual. He identifies nine hedge indices which can be included in the hedging strategy. A full rank co-integration in the industry was as well established, and an eight factor model to be used for hedging strategies as the most powerful model, is proposed.

Initially, Rashid (2004) had applied Granger Causality in agriculture in a study on spatial integration of maize markets undertaken in the post-liberalized Uganda. Different markets were identified. Causality tests were conducated on different pairs of markets, and the results indicated that all pairs which included Kampala and Jinja failed to reject the causality null. This was an indication of a uni-directional causality, implying that the regional maize prices Granger caused the prices in these two large cities. Also, a two directional causality effect was established between Mbale and Hoima indicating a dependence behavior; that is, all deviances in one market affects the other.

Korir et al. (2003)proposes the use of Granger Causality model in the analysis of the beans markets in Tanzania and Kenya. The existence of a co-integration relationship implies that there exists atleast one causal relationship, (Korir et al. (2003)). Causality test is an indication of the direction in which the variables affect each other. It regresses an explanatory variable (A) against its differenced series and another variable (B). If B is significant, it explains the variation in A and we say that B dynamically causes A.

In Kenya, there existed a crisis between May, 2011 and November 2011 where the shilling depreciated consistently against other currencies. During this period, there were also alot of movements in interest rates charge in the market. The amount of risk attached to this movement was very high with many companies over estimating their risk. This implied an over exploitation of customers due to imposed policies such as increased reserve and unstable interbank lending rate. Granger Causality techniques could have well addressed this issue. It is therefore clear that, there is an existing need for such a model as it will ease the financial institutions' tension and may be reduce their risk averseness.

1.2.5 Error Correction Model

Huang and Neftci (2004)investigated co-integration relationship that existed between the swap spreads and various rates such as the LIBOR rates, US corporate credit spreads and the treasury yield curve; which found evidence of co-integration existence. In their study, they showed that under the ECM framework, the daily swap spreads reacted to the corrective long-run forces except from the short-term fluctuations in the variables. They concluded that the swap spread had a negative effect only on one measure, the treasury yield curve, but positive in all the other rates.

Leykam (2008)in his master's thesis in economics, analyzes the existence of co-integration in prices on different natural gas hubs in the European region. The study focused on four major hubs; NBP, Zeebrugge, TTF and Bunde. Leykam (2008) estimates an ECM to investigate the pairwise short and long-run relations. Of these hubs, NBP and Zeebrugge were found to have the strongest relation due to their direct pipeline connecting the two hubs. This result was further ascertained by the ECM which indicated an exogenous connection between these two hubs.

An examination of the relationship among African stock markets (Adjasi and Biekpe (2006)) indicates existence of causal effects and significant feedbacks from small to large markets, under short-run ECM framework. Various stock markets were analyzed with the view of establishing an integration relationship and its influence on each other. Major relations were found in the South African stock market and minor in the Ghanaian market. South African market is considered larger and more active while Ghanaian the least active and smallest amongst the sampled countries. Interestingly, an upward causal effect was established. That is, the larger market influenced to a greater extend the smaller market but not vice versa. On the other hand, both the smaller and larger markets had a direct influence on the other sampled markets. Adjasi and Biekpe (2006) concludes that it is sufficient to monitor the two markets on the overall economic performance.

Later on, Petrov (2011) applies ECM in evaluating a pairwise co-integration strategy between the South African equity market and other emerging and developed markets; using the price indices rather than MSCI index, as used by Adjasi and Biekpe (2006). He gives two reasons; one, that price index is raw and two, it enabled comparisons across different markets. Petrov (2011) shows that all the markets were responding slowly to any long term disequilibrium. The integration proved to be high between most markets and hence portfolio selection was the most sensitive task to undertake. Petrov (2011) applies an ECM to analyze different portfolios of different sizes. He finds out that USA dominated in all the portfolios in which it was introduced. It was then recommended that such portfolios should be considered the most favorable for investors.

Credit risk is one of the most important type of risk which a bank will be keen to assess to ensure that it remains in business. On the other hand, most of the bank advances are made on a collateral basis. Karumba and Wafula (2012) studies this collateral lending characteristic of the lending institutions and their implication on the general financial equilibrium. They investigated its implication on the level of credit risk faced by the banks. Karumba and Wafula (2012) applies co-integration and error correction techniques to investigate long-run relationship. The study found over-reliance on collateral in institutional lending. A negative ECM adjustment coefficient was found indicating that advances in loans and collaterals had a short-term adjustment. With the introduction of credit referencing, the study concludes a general reduction in credit risk.

In Basel III accord, the main challenge is addressing rates volatility. Its evolution over time makes credit risk analysis more complex. In this research, it is contributed to the bank of literature by investigating existence of short-term and long-term relation between lending and interest rates. This contributes to mitigation of credit risk analyzed by Karumba and Wafula (2012). A procedure for modelling interbank lending rates can be seen as a milestone to the mitigation strategy. It will make the work proposed in Basel III accord much easier.

1.3 Problem Statement

Exchange and interest rates plays an important role in an economy. They are among the backbone of economic indicators. While the interbank lending rates are determinants of the interest rates imposed by banks on their customers thus influencing a country's economy, the exchange rate takes the larger perspective. Aside of influencing a country's economy, it is also, to some extent, an indicator of world wide economy performance because the dollar has been conventionally accepted as the denominator of international transactions. Therefore, its fluctuation influences transactions across borders. On the other hand, the interbank lending rates is determined by the central bank. It is much influenced locally unlike the exchange rates. Consequently, the interbank lending rates tend to be relatively unstable compared to the exchange rate. A model for exchange rates might be considered more stable between the two. Nevertheless, modeling of interbank lending rates is of more importance to a country as it directly affects its citizens. Therefore, if we can be able to establish a relationship between the two, it will be possible to model the lending rates based on the exchange rates.

1.4 Justification

Co-integration is a recent development in time series analysis. It has been applied widely in finance to model related time series data and explain the relationship in existence. Vast of these procedures and researches have been done on the developed countries with minimal on the African economies, despite its importance to an economy. This presents a need for extensive research to be conducted on an African economy. It is with this need that this project has been put forward to address this gap in knowledge. The study focuses on the Kenyan market. A model is fitted which reflects the Kenyan economy, since model parameters are estimated using data obtained locally. Aside from the schollarly contribution, it also provides a procedure by which we can model the interbank lending rates using the interest rates. The study therefore puts across a model which is considered as reliable as possible to be used for modeling.

1.5 Hypotheses

The following hypotheses were made to aid in the study;

1. Ordinary Least Squares can not be used to estimate a co-integration model.
2. Co-integration regression parameters are inadequate.
3. Granger Causality does not exist.
4. Error Correction Model cannot be fitted to the data.

1.6 Objectives of the Study

1.6.1 General Objective

To investigate the existence of cointegration in Kenyan exchange rates and interbank lending rates and its economic application.

1.6.2 Specific Objective

The specific objectives of the study were;

1. To estimate a co-integration model using Ordinary Least Squares
2. To test regression parameters for their adequacy.
3. To build a Granger Causality model.
4. To fit an Error Correction Model.

1.7 Significance of the Study

A procedure for modeling of interest rates has been of particular interest to several scholars and to its users as well. This research has provided yet another source of reference to the financial institutions in the country and interested parties on modeling of lending rates. It provides an insight into the behavior of the economy and the response to a fall or a rise in the exchange rates through the study of its relationship with interest rates, particularly the interbank lending rates. This is because it is a benchmark from which banks, for example, determines the rates at which they lend to their customers. Since a reliable procedure has been built for banks to model the lending rates as accurate as possible, it is therefore much easier for them to advice their customers, especially those advancing for loans. Likewise, it is now possible to determine their future cash-flows, to some level of significance, and hence ease their determination of the level of solvency and liquidity, because they are required by law to maintain a certain level of liquidity to cater for their day to day transactions.

1.8 Limitations

Despite the claim of exchange rate stability, there are cases where the exchange rates fluctuates without an immediate change in the interbank lending rate, specifically when there is a worldwide turmoil in the economy. Since the dollar is a conventionally accepted denominator for across-border transactions, such a turmoil will affect all countries. Nevertheless, there might be no change in the local country's lending rates. This presents a big limitation to our main assumption that the exchange is more stable of the two, as evident in the data analysis section. Other limitations include the disturbance in the economy caused by currency hoarding which leads to a disguised shortage in currency and hence unrealistic fluctuations. Also politically influenced economies through the central bank will often suffer from disguised financial crisis in form of raising money and hence distortion of the true relationship between the series.

1.9 Thesis Summary

This thesis begins by laying a background on the concepts of co-integration and other related concepts. A review of previous work under the subject matter is then given, from which a statement of the problem is established. The problem is then justified, followed by hypotheses, objectives and significance of the study. The Chapter is then concluded by giving limitations of the study. The methodology to be used in the study is reviewed in Chapter 2, where the various tests to be done are put to perspective. A simulation study is then presented in chapter 3, where two sets of data are simulated using a GARCH(1,1) model. These data are applied on the theory developed in Chapter 2. It gives a classical road map to the analyses done on the Chapter that follow. Empirical data on U.S exchange and interbank lending rates in Kenya are then considered in Chapter 4. The methods developed based on the simulated data in Chapter 3 are now applied on empirical data. Finally, we have the conclusion and recommendations in Chapter 5 that ends with an highlight of areas of further research.

Chapter 2

METHODOLOGY

2.1 Overview

The main aim of the research is to show empirical economic applications of co-integration. We employ several tests and inferential statistics presented therein in the analysis of the data. Data from (CBK), for atleast a period of ten years and daily frequency, are used. Unit root tests such as Durbin-Watson, KPSS and Augmented Dickey-Fuller are used. The Engle-Granger test for co-integration is applied, which tests the null of no co-integration. An estimated OLS is then fitted and the parameters tested for adequacy. The two-step Engle-Granger test is applied to establish the order of co-integration. Residuals are then tested for unit-root. A Granger Causality model is build; followed by an ECM to mirror the short-term relations and hypothesis testing for mean-reverting property. A model considered appropriate is then recommended. Graphics are employed in presentation of results.

Definition 7. If we have two distinct sets of time series data, say A_t and B_t, and both of them are non-stationary. Let these two sets of data be integrated time series of order 1, represented as $I\,(1)$. Also, let there exist a linear combination of the two series such that;

$$A_t = \eta B_t + c \tag{2.1.1}$$

such that, the linear combination as shown in Equation (2.1.1) of the two series is now an $I(0)$ stationary series. Then, the series A_t and B_t are said to be co-integrated. This is as good as saying, while non of the two sets of time series data tend to oscillate around a constant common mean level, a linear combination of the two series tend to converge and oscillate around a common mean. This is implied by the fact that a linear relationship of the two series is an $I(0)$ stationary time series. This then brings us to the result that co-integration tries to describe a particular kind of long run relationship between the two time series.

Definition 8. Co-integration results discussed above can as well be applied to more that two time series data where the time series are treated as multivariate data vector whose columns are the individual time series data.

2.2 Some Theoretical Review

2.2.1 Review of Autoregressive Representation

For illustration purposes, lets consider a general case of the AR (1) process. Define Z_t as the AR(1) process given by

$$Z_t = \alpha Z_{t-1} + \varepsilon_t \tag{2.2.1}$$

It can be shown that under the null hypothesis of unit root, it can be reduced to a random walk process given by

$$(1 - B)Z_t = a_t \tag{2.2.2}$$

where a_t is a Gaussian white noise process. It then turns out that Z_t becomes the sum of independent and identically distributed random variables $\{a_i\}_{i=1}^{n}$. Now let these series form non-white noise stationary process X_t given by the transformation

$$X_t = \sum_{t=0}^{\infty} \psi_j a_{t-j} = \psi(B)\, a_t \tag{2.2.3}$$

where

$$\psi(B) = \sum_{j=0}^{\infty} \psi_j B^j, \quad \psi_0 = 0, \ and \ \sum_{j=0}^{\infty} j\,|\psi_j| < \infty. \tag{2.2.4}$$

Then, according to Wei (2006), to test for a unit root in this general case, we can fit the following OLS regression

$$Z_t = \phi Z_{t-1} + X_t \tag{2.2.5}$$

and consider the estimator

$$\hat{\phi} = \frac{\sum_{t=1}^{n} Z_{t-1} Z_t}{\sum_{t=1}^{n} Z_{t-1}^2} \tag{2.2.6}$$

under the null hypothesis, $\mathrm{H}_0 : \phi = 1$, we have

$$\hat{\phi} = \frac{\sum_{t=1}^{n} Z_{t-1} Z_t}{\sum_{t=1}^{n} Z_{t-1}^2} = 1 + \frac{\sum_{t=1}^{n} Z_{t-1} X_t}{\sum_{t=1}^{n} Z_{t-1}^2} \tag{2.2.7}$$

and

$$n\left(\hat{\phi} - 1\right) = \frac{n^{-1}\sum_{t=1}^{n} Z_{t-1} X_t}{n^{-2}\sum_{t=1}^{n} Z_{t-1}^2} \tag{2.2.8}$$

The estimates of the parameters can thus be obtained by recursive substitution.

2.2.2 Review of GARCH Representations

According to Bollerslev (1986), a GARCH model can generally be given by

$$\omega_t = v_t \sqrt{h_t} \tag{2.2.9}$$

in which

$$h_t = \alpha_0 + \alpha_1 \omega_{t-1}^2 + \beta_1 h_{t-1} \tag{2.2.10}$$

where v_t is a white noise with $\sigma_v^2 = var\,(v_t) = 1$, $\alpha_0, \alpha_1, \beta_1 \geq 0$ and $\alpha_1 + \beta_1 < 1$.

From Equation (2.2.10), the term $\alpha_1 \omega_{t-1}^2$ is the Auto Regressive Conditional Heteroskedasticity (ARCH) representation while $\beta_1 h_{t-1}$ is the GARCH representation. The heteroskedasticity comes from the h_t term which is the one period ahead forecast of the variances whereas v_t represents the shocks.

Suppose we introduce a transformation such that $\alpha_1 \omega_{t-1}^2 = \varepsilon_t$, then the series becomes a general case of the AR (1) process, discussed above.

2.2.3 Review of Unit Root tests

The first step in co-integration analysis is to test for stationarity of the two series. It is a condition that for the two series to be co-integrated, they must be non stationary. Three tests (ADF, KPSS and PP tests) are used.

2.2.3.1 Review of the Augmented Dickey Fuller test

This is a generalized form of the Dickey Fuller test (Dickey and Fuller (1979)). It relies on the assumption that the residuals are independent and identically distributed. For a series y_t, ADF uses the model

$$\Delta y_t = \alpha + \lambda t + \eta y_{t-1} + \delta_1 \triangle y_{t-1} + \cdots + \delta_{p-1} \Delta y_{t-p+1} + \varepsilon_t \qquad (2.2.11)$$

which reduces to a random walk when $\alpha = 0$ and $\lambda = 0$; and a random walk with a drift when $\lambda = 0$. The ADF test thus detrends the series before testing for unit root. It uses lagged difference terms to address serial correlation. The ADF test clearly depends on differenced series. This thus possess a need for another validating test.

An inspection of the p-value also determines whether the null hypothesis of non-stationarity will be accepted. A small p-value (less than 0.05 or 0.01 depending on the statistician) leads to the rejection of the null hypothesis. An inspection of the Dickey-Fuller value is as well important as this indicates the mean-reverting property. It is normally a negative value. The larger its absolute value, the lower the chance of occurrence of mean-reverting property.

2.2.3.2 Review of the Kwiatkowski Philips Schmidt Shin test

Contrary to ADF test, KPSS (Kwiatkowski et al. (1992)) tests for the null hypothesis of level or trend stationarity. It gives a way to specify whether to test with a trend or without, in its test statistic. A regression model with linear combination of a deterministic trend (if test statistic is with a trend), a random walk and a stationary residual series

$$Y_t = \alpha + \beta t + \lambda \sum_{i=1}^{t} \varepsilon_i + \delta_t \tag{2.2.12}$$

is used where δ_t is stationary, βt is the trend component while $\sum_{i=1}^{t} \varepsilon_i$ is the random walk. $\beta t = 0$ if we assume a without-trend regression. The series in Equation (2.2.12) will be stationary if $\lambda = 0$. Regression is used to obtain the estimate of δ_t, that is $\widehat{\delta}_t$, from which we compute

$$\Omega_{resid} = \sum_{i=1}^{t} \widehat{\delta}_i \tag{2.2.13}$$

The test statistic for KPSS test is then calculated as

$$R = \frac{\sum_{i=1}^{n} \Omega_i^2}{n^2 \widehat{\theta}_T^2} \tag{2.2.14}$$

where the spectral density function estimator

$$\widehat{\theta}_T^2 = \widehat{\sigma}_\delta^2 + 2 \sum_{k=1}^{T} \left(1 - \frac{k}{T-1}\right) \widehat{\omega}_t \tag{2.2.15}$$

is a linear combination of the variance estimator $\widehat{\sigma}_\delta^2$ and covariance estimator

$$\widehat{\omega}_t = \frac{\sum_{t=k+1}^{n} \delta_t \delta_{t-k}}{n} \tag{2.2.16}$$

The test turns to a prudential choice of T in Equation (2.2.15) above.

2.2.3.3 Review of the Phillip Perron test

The Phillips Perron approach (Phillips and Perron (1988)) applies a nonparametric correction to the standard ADF test statistic, allowing for more general dependence in the errors, including conditional heteroskedasticity. If there were strong concerns over heteroskedasticity in the ADF residuals, this might influence an analyst to go for PP. If the addition of lagged differences in ADF did not remove serial correlation, then this again might suggest PP as an alternative.

2.2.3.4 Review of the Durbin Watson test

It is an important tool in testing the presence of autocorrelation of residuals from a regression analysis. It tests the null of no correlation against existence of correlation. Its statistic is;

$$\Psi = \frac{\sum_{k=2}^{n} \left(\varepsilon_i - \varepsilon_{i-1}\right)^2}{\sum_{k=1}^{n} \varepsilon_i^2} \tag{2.2.17}$$

where ε_i are the residuals given by $x_i - \widehat{x}_i$. The lower the value of Ψ, the higher the autocorrelation. For different lag levels, the critical values (with levels of significance) of the statistic Ψ are tabulated. Let Ψ_U denote the upper and Ψ_L denote the lower critical values. If;

$\Psi < \Psi_L$, then reject the null $H_0 : \rho = 0$

$\Psi > \Psi_U$, then do not reject the null $H_0 : \rho = 0$

$\Psi_L < \Psi < \Psi_U$, then no conclusion can be made.

The test assumes that the regression errors follow an $AR(1)$ process

$$\varepsilon_t = \rho\varepsilon_{t-1} + \phi_t \tag{2.2.18}$$

Incase the value of Ψ is negative, we calculate $4 - \Psi$ and proceed with the test as above.

2.2.4 The Engle-Granger two-step Method for Testing Co-integration

Two series A_t and B_t are co-integrated if it can be written in the form

$$A_t + \lambda B_t = \Theta_t \tag{2.2.19}$$

where Θ_t is stationary.Engle and Granger (1987) proposes a two-step procedure for this estimation. In the first step, we ensure the individual series are $I(1)$. If not, apply differencing to attain $I(1)$. Estimate a linear relationship using OLS between the two $I(1)$ series. That is, we estimate λ in Equation (2.2.19) above. Secondly, we extract the residuals from the estimated OLS Equation and test for stationarity. Co-integration exists if the residuals obtained from the OLS estimation are stationary.

2.2.5 Granger Causality

Granger (1969), first proposed a procedure of investigating causality using lagged series and residuals. Suppose that there is a series or vector y_t from which one wants to obtain k ahead predictions, y_{t+k}, from an information matrix Λ. Let Λ be a vector of random variables/series $(a_t, b_t, a_{t-1}, b_{t-1}, \cdots, a_1, b_1)$. Obtaining the y_{t+k} using least squares involves calculation of the conditional mean $E[y_y/\Lambda]$. In time series, it involves regressing y_t on Λ where Λ in this case has the variables $(y_{t,}y_{t-1,}y_{t-2,} \cdots y_1)$. This is rather complex. An easier procedure is to consider its causality.

2.2.5.1 Theoretical Representation of Granger Causality

Let X_t and Y_t be two series. X_t is said to Granger-Cause Y_t if the lagged values of X_t has statistically important information about the future values of Y_t . It is calculated for stationary series. An appropriate procedure is chosen to determine the lag to be used, to obtain optimum results. Regression is used for estimation. T-tests are used to retain the significant variables in the regression and F-test determines jointly significant variables to be retained.

The procedure involves fitting a regression of lagged values of y_t such that;

$$y_t = \lambda_0 + \lambda_1 y_{t-1} + \lambda_2 y_{t-2} + \cdots + \lambda_k y_{t-k} + \varepsilon_t \tag{2.2.20}$$

where ε_t are the residuals. The k statistically significant lags of y_t is augmented with lagged values of X_t such that;

$$y_t = \lambda_0 + \lambda_1 y_{t-1} + \lambda_2 y_{t-2} + \cdots + \lambda_k y_{t-k} + \mu_a x_{t-a} + \cdots + \mu_b x_{t-b} \tag{2.2.21}$$

The lagged values of X_t in Equation (2.2.21) are retained if it adds an explanatory power to the regression Equation. F-tests are used to determine the retained lagged values of X_t. The shortest possible regression has a values where longest has b values. The null hypothesis of no Granger Causality is rejected if and only if there exists at least one lagged value of X_t retained in Equation (2.2.21).

2.2.6 Error Correction Model

When estimating a Granger Causality relationship, the requirement is to ensure the series is $I(1)$. Making a series $I(1)$ implies differencing. Differencing removes trend and hence loss of some important information about the time series behaviour in the short-run. Also, a co-integration relationship assumes a linear relationship; which might not be always the case due to random shocks. A displacement from the equilibrium relation implies a response from one of the variables to attain the equilibrium. The rate at which either variables re-attains equilibrium is modelled by an ECM. Simply, an ECM is a model which gives an estimated response behavior of a variable upon dis-equilibrium. An ECM can be estimated as

$$\triangle A_t + \lambda \triangle B_t = \Omega + \beta(\Lambda) + \varepsilon_t \tag{2.2.22}$$

where λ and β are coefficients, Ω an intercept which may or may not be included; ε_t random noise; and Λ an 'error correction component'.

2.3 Estimated Variance Function

Let Z_t be a stationary time series defined by

$$Z_t = \sum_{j=0}^{\infty} v_j e_{t-j} \qquad t = 0, \pm 1, \pm 2, \cdots \tag{2.3.1}$$

Where $\{v_j\}$ is absolutely summable and the e_t are independent $(0, \sigma^2)$ random variables with distribution function $F_t\,(e)$ such that

$$\underset{\delta \to \infty}{lim} \ \underset{t=1,2,\cdots}{sup} \int_{|e|>\delta} e^2 dF_t\,(e) = 0. \tag{2.3.2}$$

Then, according to Fuller (1996), the residuals can be used to investigate the nature of the variance of Z_t as well as to study the auto-correlation structure.

Chapter 3

SIMULATION STUDY

A GARCH(1,1) is used to simulate data to be used in the analysis. The choice of the simulation procedure is aimed at capturing the heteroskedasticity which has been noted to exist in a series of returns (particularly in price returns), where volatility clustering exists. The classical inferential property of a high-low series is violated and a series of high levels tend to cluster together. In which case, variances tend to be related across different periods and hence leading to the result

$$var\left(X_t\right) = E\left(X_t^2\right) \tag{3.0.1}$$

that is the variance of the series at a given time, say t, is the same as the the expectation of the square of the series. This is the basis for heteroskedasticity and hence an indication that auto-correlation still exists in the squares of the returns.

3.1 Granger Causality

The first step in Granger Causality analysis is to establish stationarity of the time series. A basic investigation of this property is by visual inspection of time plot. Figure (3.1.1) below shows the plot of the simulated series. The values of α_0, α_1 and β_1 are chosen arbitrarily to satisfy the conditions (i.e $\alpha_0, \alpha_1, \beta_1 \geq 0$ and $\alpha_1 + \beta_1 < 1$.) for a GARCH(1,1) model.

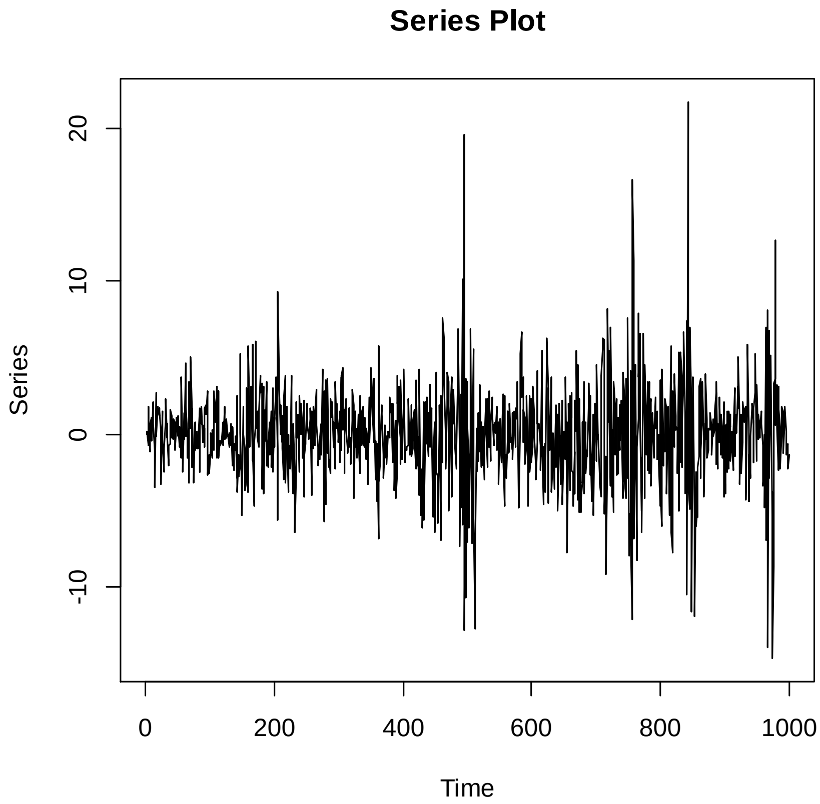

Figure 3.1.1: A Plot of the GARCH(1,1) Simulated Series.

Another GARCH(1,1) series is simulated and its plot superimposed (Figure 3.1.2) on the first series. An inspection of the superimposed plot suggest an existence of causality as in Figure (3.1.2) below.

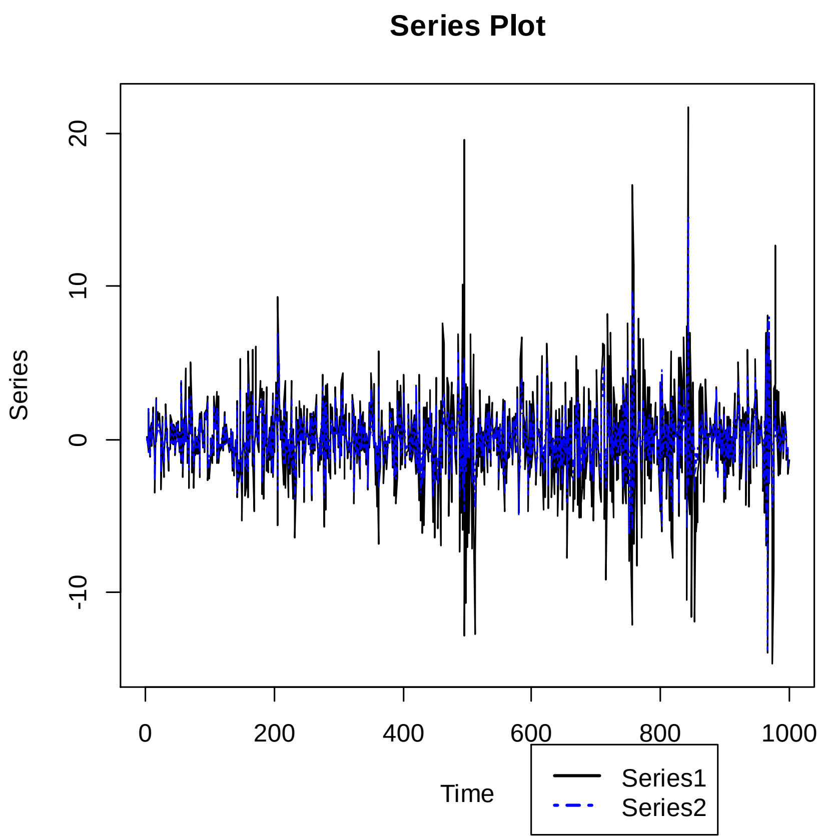

Figure 3.1.2: The Second Simulated GARCH(1,1) Series is Superimposed onto the Series Plot in Figure 3.1.1.

Before analysing the simulated series, an investigation of the GARCH properties should be done. All the properties must be met before the data can be used for any analysis. Being a GARCH model, it is expected to exhibit a high auto-correlation at lag one and insignificant correlation in higher orders. This translates to a spike at lag one in the Auto Correlation Function (ACF) plot which is evident in Figure (3.1.3). An investigation on the heteroskedastic property of the series can as well be investigated from Figure (3.1.3). An inspection of the ACF's of the squares of the two series indicate existence of serial correlation, an indication of heteroskedasticity property. The mean and variance properties of a GARCH model are all satisfied.

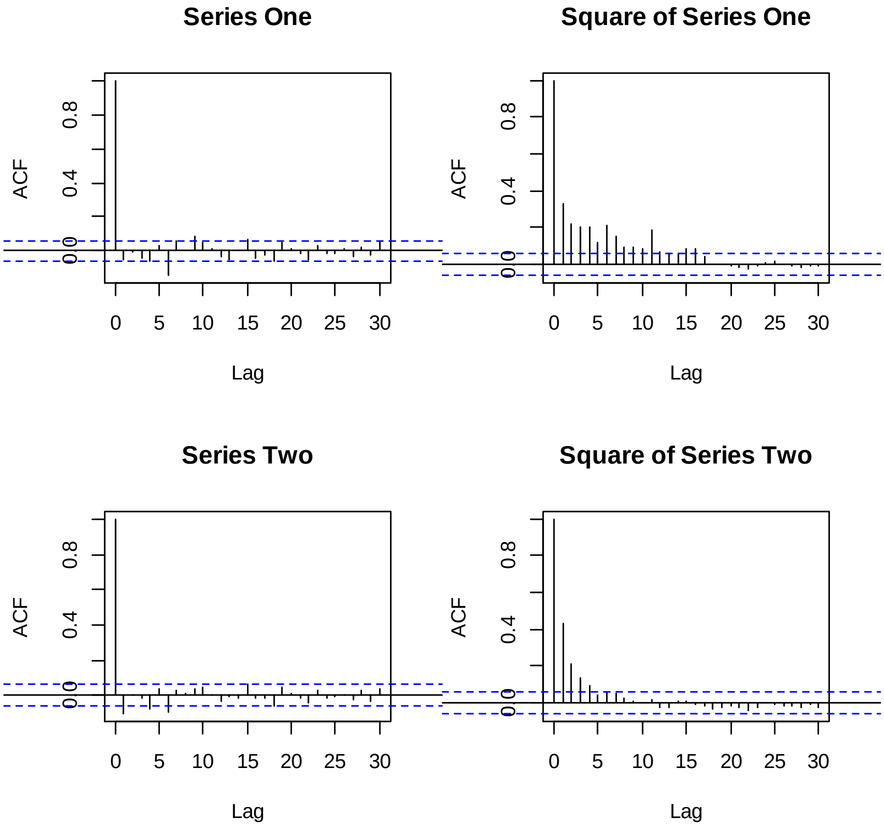

Figure 3.1.3: Auto Correlation Functions for the Series and Their Respective Squares, in that Order.

The model used for simulation should yield a stationary series, by definition. This is evident as the series plot resemble a mean zero white noise process. Nevertheless, unit root tests such as ADF, KPSS and PP tests discussed previously in section (2.2.3) are used to investigate stationarity in the series.

The ADF test tests the null hypothesis of non stationarity. The p-value indicates the amount of evidence against the null hypothesis. For the two simulated series, the ADF test output is as shown in table 3.1 below;

Table 3.1: Augmented Dickey Fuller test output for the two simulated series. Omega Represents the First Series Whereas Omega1 is the Second Series.

Data	Dickey-Fuller	Lag	P-value
Omega	-9.726	9	0.01
Omega1	-10.0075	9	0.01

From Table (3.1), the p-value is 0.01. At 5% level of significance, the classical probability rule dictates rejection of the null hypothesis. The decision rule is that there exists sufficient evidence that the series might be stationary. Also the absolute values of the Dickey-Fuller values are relatively low thus we may conclude both series may be mean-reverting. But ADF test has two downsides which has to be addressed.

1. The model for an ADF test uses the differenced series.

2. It assumes that the residuals are independent and identically distributed.

KPSS test addresses the differenced model in ADF test. It tests the null of stationarity with respect to an existing trend. It is similar to the ADF test but does not detrend the series. That is, the long term general movement of the series is preserved. Table (3.2) presents the output of this test.

Table 3.2: KPSS test output for the two simulated series

Data	KPSS Level	Lag	P-value
Omega	0.1328	7	0.1
Omega1	0.2012	7	0.1

The test yield a p-value of 0.1. Following the same decision rule, we fail to reject the null hypothesis at 5% significance level and conclude that the series might be stationary.

All the above tests are parametric. They all assume independence and identical ditribution of residuals. A non-parametric test need to be done, which can be used in presence of heteroskedasticity. Philip Perron test is the best alternative. To ascertain these decisions, a Philip Perron test is done whose results are presented in Table (3.3). This test, just like ADF, test the null of non stationarity. Having the same p-value of 0.01 as that in the ADF test, the same decision rule is followed.

Table 3.3: Phillip Perron test output for the two simulated series

Data	Dickey-Fuller Z_{α}	Lag	P-value
Omega	-955.7931	7	0.01
Omega1	-1025.337	7	0.01

It can finally be concluded that the two series are stationary and therefore satisfying all the underlying model assumptions. The requirement of the two series being integrated of order 1, $I(1)$, is fully satisfied since the two series were simulated from a GARCH(1,1) series, and all the tests above indicate stationarity satisfaction. It then remains to estimate the parameters and lag values of Equation (3.1.1) to estimate the granger causality.

Estimation of lag value to be used in the estimation of causal relation between the two series follow. Akaike Information Criterion (AIC) is the commonly used procedure in the estimation. The output of the estimation is presented in the following Table.

Table 3.4: Results for the AIC lag Estimation

	Degrees of Freedom	Sum of Squares	RSS	AIC
Null			5983	1792.1
Omega1	1	4819	10802	2380.3

NB : $rank = 2$

From the output, the second lag is the most appropriate for estimation. The model effects are therefore investigated and from Figure (3.1.4), it is clear that the model has a level effect except for tail values.

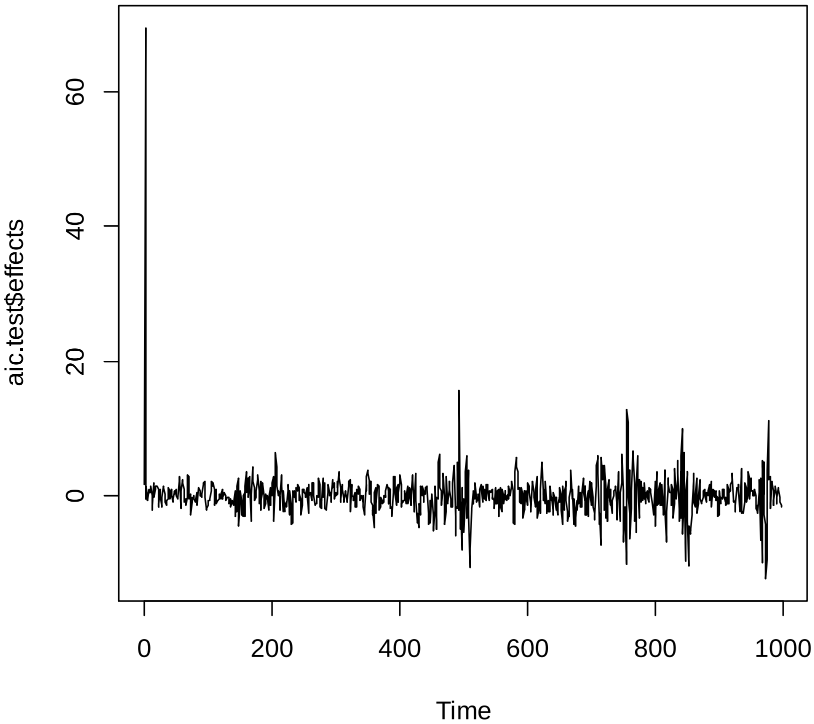

Figure 3.1.4: Model Effects for the Two Simulated Series

Since the model effects are level within the mean, the necessity of lag inclusion in the model is investigated. The results of this test is as shown in Table (3.5) below.

Table 3.5: Statistical Test Output on Lag Inclusion in the Model

Model	Inclusion	Residual Df	F-value	$pr(> F)$
One	$Lags(Y, 1:1) + Lags(X, 1:1)$	995		
Two	$Lags(Y, 1:1)$	996	0	0.9956

Y and X represents the two simulated series. The null hypothesis of the saturated model is rejected. Therefore, $\nexists$ sufficient evidence that the inclusion of the lagged values in causality estimation leads to the overall improvement in the model predictive ability. The coefficients for the Granger Causality model are presented in the table below.

Table 3.6: Granger Causality Model Coefficients

Intercept	X11	X12	X13
-0.05010	1.65084	-0.05791	0.01796

The first and fourth variables are found to be insignificant in the model, at 5% level of significance. The variables are therefore dropped. A linear model is thus fit with the series itself and its second lagged value. All the model parameters were found to be statistically significant at 5% level. From the output above, the Granger Causality model therefore becomes:

$$Y_t = 1.65084X_t - 0.05791X_{t-1} \qquad (3.1.1)$$

where Y_t represents the first simulated series while X_t is the second simulated series.

It therefore remains to check on the direction of the causality. The output is presented below.

Table 3.7: Direction of the Causality

	F-statistic	p-value
Omega -> Omega1	8.54254	1.0×10^{-7}
Omega1 -> Omega	7.6985	1.0×10^{-8}

It is clear that the two rates Granger Caused each other. Therefore, movements in one simulated series are more likely to be caused by the movements in the other series.

3.2 Error Correction Model

Once the Granger Causality model in sub-section (2.2.5) above has been built (Equation (3.1.1)), an ECM is easily built by considering the residuals of the model in Equation (3.1.1) above. ECM involves fitting a regression equation of the differenced series and the residuals of the fitted Granger Causality model. Due to the inclusion of the residuals, dynamic linear modeling is used. The fitted model will involve two main parts; the residual part which might be considered more stable than the differenced series part, hence the use of a dynamic linear model. The output of an estimated ECM is as shown in Table (3.8) below.

Table 3.8: An Estimation of an ECM for the Two Simulated Series

	Estimate	Std. Error	t value	*pr* (> |t|)
Intercept	-0.05787	0.06478	-0.893	0.372
Omega1	0.80422	0.02291	35.104	2.0×10^{-16}
Residual	1.11172	0.05338	20.826	2.0×10^{-16}
Residual Standard Error : 2.046 on 995 df				
Multiple R^2 : 0.6143				
Adjusted R^2 : 0.6135				
P-value = 2.2×10^{-16}				

NB : The fitted model is $A_t = \alpha + \beta B_t + \lambda \varepsilon_t$ where A_t =Omega (Series One), B_t =Omega1 (Series Two) and $\varepsilon_t = residual$

All the parameters, except the intercept, from the model are significant at 5% level. R^2 value of 0.6143 shows that the overall model fits well to the data. The fitted ECM therefore becomes;

$$Y_t = 0.80422 \triangle X_t + 1.11172\Lambda \tag{3.2.1}$$

where Y_t represents the first series, X_t the second series while Λ denotes the error correction component.

3.3 Cointegration

In this section, it is proposed that if two time series follow GARCH(1,1), then the two series are cointegrated.

3.3.1 The Main Proposition

In this subsection, the following proposition is made;

Lemma. *If two time series follow a GARCH(1,1) model, then the two series are cointegrated, and can simply be given as*

$$X_t = \alpha + \lambda Y_t \tag{3.3.1}$$

where the estimate of λ, $\widehat{\lambda}$, is the OLS estimate of Equation (3.3.1), and (X_t, Y_t) are the two time series under consideration. Further, the two series will not drift too much from each other.

3.3.2 Proof

Proof. The proof of the main proposition is done as a simulation study in this section, and proceeds as follows:

A GARCH(1,1) is used to simulate data to be used in the analysis. The choice of the simulation procedure is as discussed in the previous section. The plots of the simulated series are similar to the plots in the previous section. The values of α_0, α_1 and β_1 are chosen arbitrarily to satisfy the conditions for a GARCH(1,1) model. From the previous section, it was found that the two simulated series were stationary, and hence the two simulated series satisfy all the underlying model assumptions.

The requirement of the two series being integrated of order 1, $I(1)$, is fully satisfied since the two series were simulated from a GARCH(1,1) series, and all the tests above indicate stationarity satisfaction. A relatively low Dickey-Fuller absolute values is an indication of a possibility of co-integration. It then remains to estimate the parameter λ from Equation (2.2.19) to estimate co-integration relationship.

Equation (2.2.19) can as well be rearranged and written as

$$A_t = \Theta_t - \lambda B_t \tag{3.3.2}$$

which is a linear model in A_t and B_t. Classical Ordinary Least Squares (COLS) can therefore be used to estimate the parameter λ. A linear model is fitted which estimates λ to be 1.60477. This value remains an estimate till the residuals of the fitted model is tested for stationarity. Figure (3.3.1) below is a plot of the residuals. A visual inspection suggests stationarity as it has the form of a mean-zero white noise. Comparing it to Figure (3.1.1), the residual series is less random about the mean and hence its variance approaches unity.

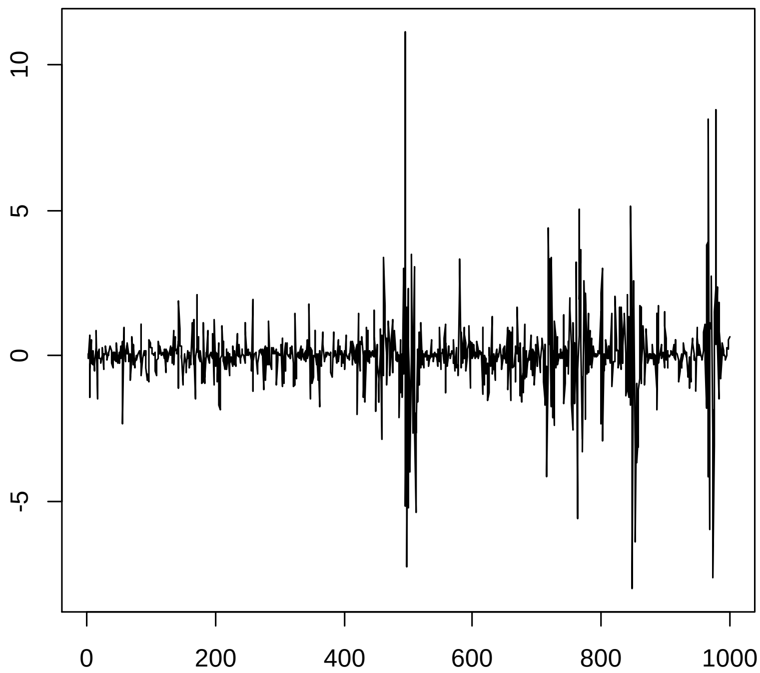

Figure 3.3.1: A Plot of the Residual Series. A Visual Inspection of the Plot Indicates a Stationary Process. It is a Replica of the Purely Random Process.

Similarly, stationarity tests are done on the residual series and Table (3.9) below contains the outputs from the three tests.

Table 3.9: Residual Series Tests for Stationarity

Test	Test Statistic	Lag	P-value
Augmented Dickey-Fuller	-12.338	9	0.01
KPSS	0.0728	7	0.1
Phillips-Perron Unit Root	-1011.148	7	0.01

Using the same decision rule, the residual series is stationary. Therefore a co-integration relation exists. It therefore remains to test the significance of the coefficient of co-integration. The hypotheses under consideration is

$$\begin{aligned} H_0 &: \lambda = 0 \\ &vs \\ &\lambda \neq 0 \end{aligned} \tag{3.3.3}$$

From the summary of the fitted model presented in Table (3.10), the p-value is infinitesimally small. The null hypothesis is rejected and concluded that the co-integration factor is different from zero.

Table 3.10: Summary of the Fitted Model

	Estimate	Std. Error	t value	$pr\,(>\lvert t\rvert)$
Intercept	-0.05039	0.03847	-1.31	0.191
Omega1	1.60477	0.02022	79.38	2.0×10^{-16}
Residual Standard Error : 1.217 on 998 df				
Multiple R^2 : 0.8633				
Adjusted R^2 : 0.8631				
P-value = 2.2×10^{-16}				

NB : The fitted model is $A_t = \alpha + \beta B_t$ where $A_t = Omega$ and $B_t = Omega1$

$\square$

3.3.3 The Model

The fitted model can be given by

$$A_t = -0.05039 - 1.60477B_t \tag{3.3.4}$$

or

$$-0.05039 = A_t + 1.60477B_t \tag{3.3.5}$$

The residual standard error is considerably small. It can be concluded that the co-integration coefficient is significant in the model. The R^2 value is 0.8633 and the adjusted R^2 is 0.8631. The two values are approximately the same, an indication that the sampled data characteristic does not differ much from the population data. The overall model is therefore significant. This therefore completes the proof.

3.4 Results and Discussions

Results of the Granger Causality indicate that the two rates Granger Causes each other. This is expected since the two series were simulated, and hence represent a very ideal scenario. Movements in series one can be used as an indication of the most probable movement in series two. The residual sum of squares from the Granger Causality model is quite low, an indication that the model optimally explains the variations in the data.

Finally, an ECM model is built and results presented in Table (3.8) in subsection (3.2). The model is presented in Equation (3.2.1). From the output, the R^2 value is 0.6135, an indication that the model well fits to the data. Further, the adjusted R^2 value is 0.6135, which is very close to the R^2 value, meaning the model can as well perfectly fit to any other data with similar characteristics. Therefore, the model can be used for predictive purposes. This arguement is ascertained by the very small Sum of Squared Residuals (SSR). The model can therefore be used to analyse data with similar characterisitcs as those used in the study. It can as well be adobted by institutions for their internal hedging and as a liquidity guard, as the ideal scenario.

On the other hand, there exists a co-integration relation between the two series. A change in the first series results in a change in the second series by 1.60477 units in the same direction. The series have a long-run equilibrium relationship. A negative small drift means that series A_t drifts in the opposite direction upon a movement in series B_t, and cannot drift too far apart from the equilibrium because economic forces will act to restore the equilibrium relationship . This therefore completes the proof of the first part of our proposition that the two series will not drift too far from each other.

Next, it is noted that at equilibrium the value of A is 1.60477 times the value of B. If A and B are prices, then when the price of A exceeds 1.60477 times the value of B, we expect either;

1. The level of A to decrease so as to reach the point of equilibrium in the near future, or
2. The level of B to be pushed up for it to balance at equilibrium with that of A.

A small value of the Residual Standard Error (RSE) indicates that most of the variability in the data is captured by the co-integration model. This is an indication of the model's ability to capture intra-data clustering. Future shocks which might be experienced are therefore easily captured in the forecasts. It therefore indicates a high level of significance in the forecasts of this model.

Notably, the R^2 and the adjusted R^2 values are almost equal. It is a good indication of high precision forecasts. It is an indication that the characteristic exhibited by series is persistent. A sample of the set of data will always have the same characteristic as the population of the data. This is in line with heteroskedasticity as data tend to cluster in a similar manner throughout the data. It is therefore an indication of the reliability of the forecasts obtained if the model was to be used. This completes the final proof of existence of a cointegration relationship.

It can be concluded therefore that if two series follow a GARCH(1,1) model, they are cointegrated and they do not drift too far from each other. It can as well be concluded that co-integration is a powerful tool in the analysis of time series data and can be used to obtain optimal forecasts. A co-integration relationship can therefore be used to explain the source of variability in one series if the variability in the other series is known. Finally, heteroskedasticity does not influence the predictability of a co-integration model. Therefore, highly significant forecasts can still be obtained from a highly heteroskedastic series. This wraps up the proof to the main proposition in Section (3.3.1).

Chapter 4

EMPIRICAL STUDY

Daily data on exchange rate of the Kenyan shilling against the dollar and the interbank lending rates are analysed in this section. All the analysis are done using the same procedure as used in simulation studies done in the previous Chapter. The data used in this research were obtained from CBK. The raw data were obtained in excel (.xls) format and a data cleaning excercise was done. Interpolation was applied to fill the missing gaps. The data were then imported to the Rgui platform for analysis, and required packages loaded. The analysis then proceeded as follows.

4.1 Granger Causality

The first step in Granger Causality analysis, as seen in the simulation study, is to establish stationarity of the time series. A basic investigation of this property is by visual inspection of time plot. A time plot is plotted, represented in Figure (4.1.1) below, and by inspection the series is non stationary.

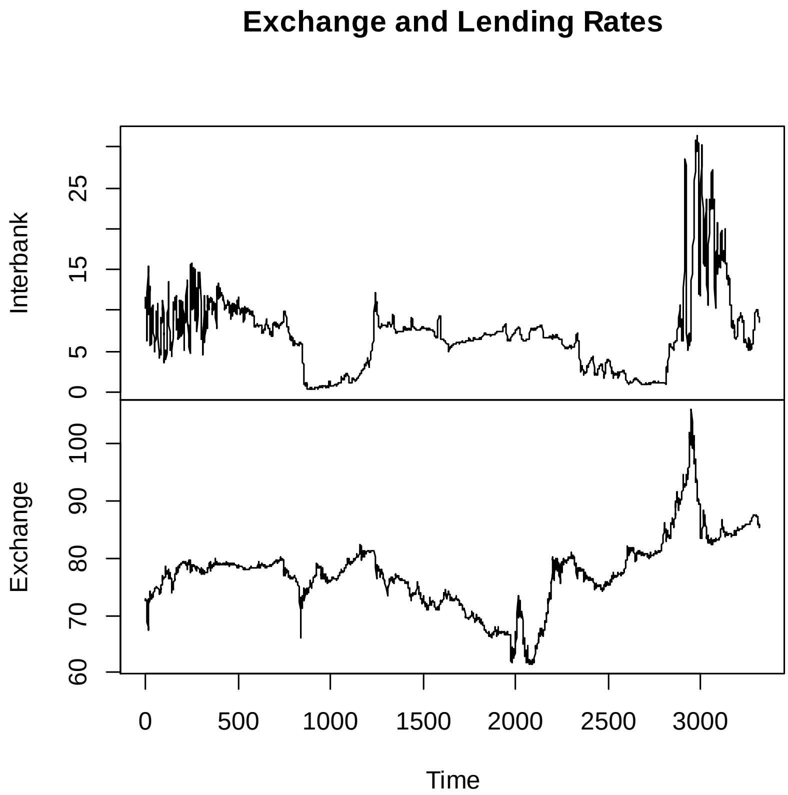

Figure 4.1.1: Time Plots of the Two Series, Dollar Exchange Rate and Interbank Lending Rate

ACF is conventionally used in time series analysis to inspect for stationarity. If the spikes tend to be constantly high close to a value of 1, the series is non stationary. The Figure below represents the respective ACF's of the two rates.

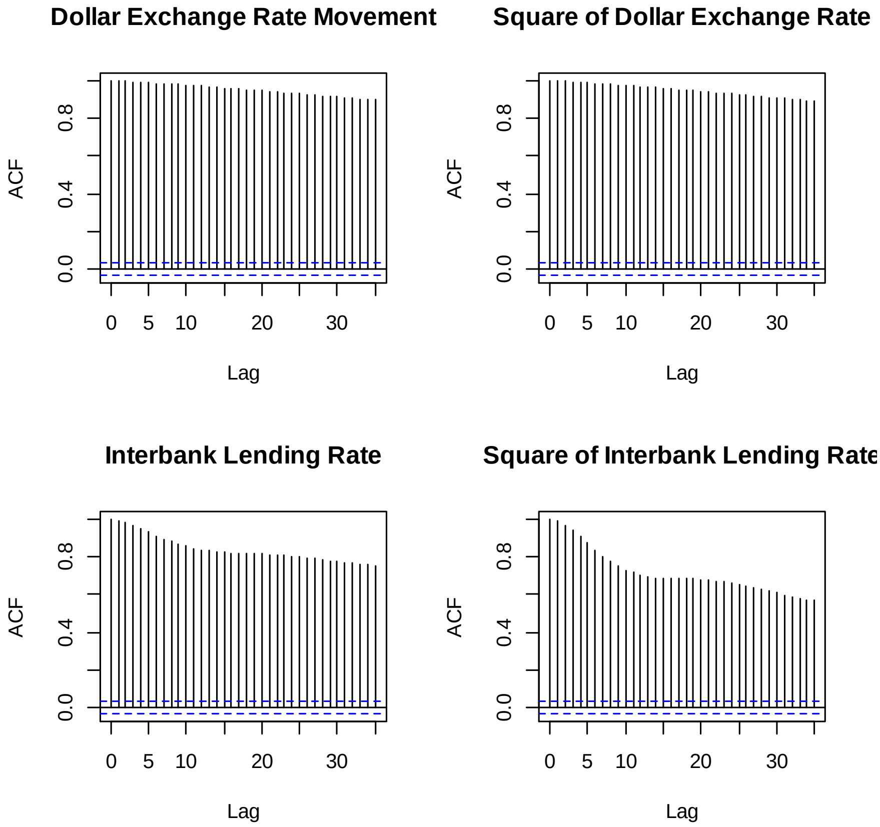

Figure 4.1.2: ACFs of the Exchange and Lending Rates

An inspection of the ACFs suggest non stationarity. The respective squares of the two series helps in the analysis of heteroskedasticity, which will determine the method used to investigate for causality. Nevertheless, mathematical tests such as KPSS, ADF and PP tests are necessary to ascertain non stationarity.

ADF test output of the two series is presented in the Table below.

Table 4.1: ADF Test Output for Exchange and Interbank Lending Rates

Data	Dickey-Fuller	Lag	P-value
Exchange Rate	-2.1937	14	0.4963
Interbank Rate	-3.6382	14	0.02897

The ADF test tests the null of non stationarity. From the ADF test output above, an inspection of the p-values indicates that the null hypothesis is not rejected at 5% level of significance for the dollar exchange rate. We reject the null hypothesis at 5% level of significance for the interbank lending rate. Conventionally, p-value indicates the amount of evidence we have against the null. It is therefore concluded that the exchange rate is non stationary while the interbank lending rate might be stationary. However, as discussed earlier, the ADF test has the following two weaknesses;

1. The model for an ADF test uses the differenced series.

2. It assumes that the residuals are independent and identically distributed.

These weakness call for the use of KPSS test. The KPSS test uses the series to test for non stationarity without differencing. The assumption on the distribution of the residuals is not required in this test. It is therefore a tentative alternative for stationarity test.

Table 4.2: KPSS Test Output for Exchange and Interbank Lending Rates

Data	KPSS Level	Lag	P-value
Exchange Rate	5.0182	13	0.01
Interbank Rate	1.3153	13	0.01

Contrary to the ADF test, the KPSS test tests for the null of stationarity. Therefore, from the output presented in Table (4.2) above, an inspection of the p-value calls for the rejection of the null hypothesis at 5% level of significance and conclude that the two series are not stationary. This is not in line with the results obtained from the ADF output in Table (4.1) above, where the interbank lending rate was established to be stationary.

To ascertain these results, we apply the PP test. The results of this test is presented in the Table below.

Table 4.3: PP Test Output for Exchange and Interbank Lending Rates

Data	Dickey-Fuller	Lag	P-value
Exchange Rate	-7.3937	9	0.6974
Interbank Rate	-48.5564	9	0.01

Just like the ADF, the PP test tests the null of non stationarity. An inspection of the p-values indicates that the Interbank lending rate is stationary while the exchange rate is non stationary. This result is contrary to the results from KPSS test in Table (4.2) above, but in line with the results obtained from Table (4.1). This therefore calls for an informed judgement on whether to assume stationarity of the interbank lending rates. In this study, we will assume that the interbank lending rates are stationary and its only the exchange rate which is non stationary. This is because two of the three tests performed supports this judgement.

Following the above results, it is necessary to inspect whether the two series might be having a causal relationship. This can be investigated by superimposing the two series onto each other and checking whether their movements are similar.

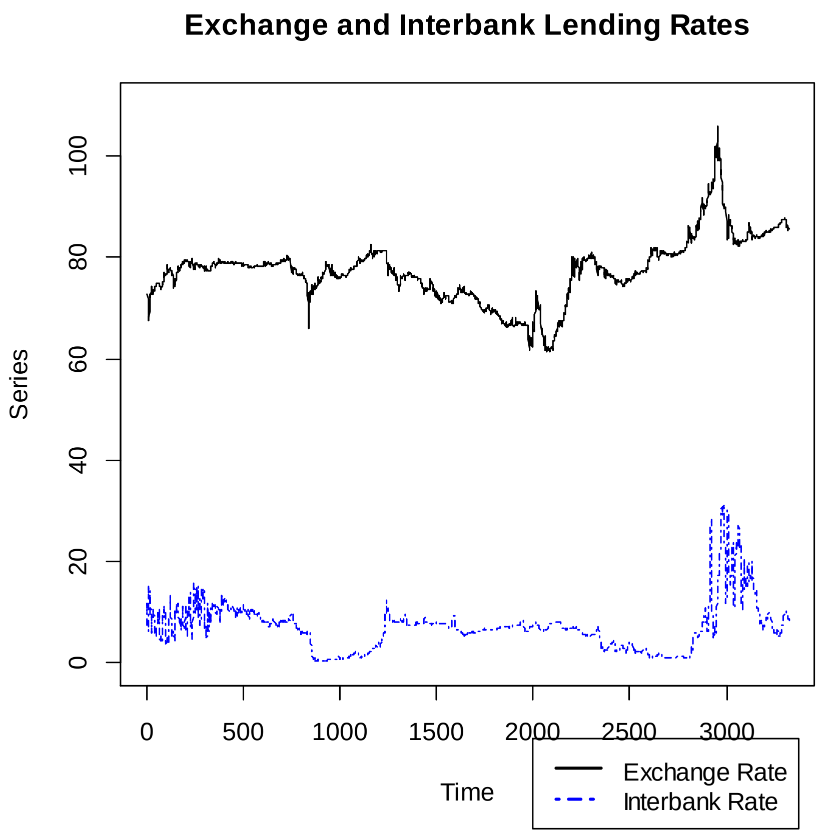

Figure 4.1.3: Superimposed Series of Exchange and Interbank Lending Rates

Clearly from Figure (4.1.3) above, the two series have causal relationship.

The exchange rate series is thus differenced and checked for stationarity. The same tests are applied.

Table 4.4: ADF Test Output for the Differenced Series of the Exchange Rate

Test	Test Statistic Value	Lag	P-value
ADF	-14.2119	14	0.01
KPSS	0.0153	13	0.1
PP	-1300.176	9	0.01

The null hypothesis is rejected at 5% level of significance. The exchange rate series is now stationary. Because of the weaknesses of the ADF test discussed above, KPSS test is done and the results presented in the same Table above. The results are same as for ADF test. We fail to reject the null hypothesis at 5% level of significance and conclude that the series is stationary. To ascertain these results, the PP test is done and results presented in that Table above.

This tests wraps up the unit root tests and we infer that the differenced series is stationary. It is therefore concluded that the exchange rate is *I(1)*.

Estimation of lag value to be used in the estimation of causal relation between the two series follow. AIC is the commonly used procedure in the estimation. The output of the estimation is presented in the following Table.

Table 4.5: Results for the AIC lag Estimation

	Degrees of Freedom	Sum of Squares	RSS	AIC
Null			74156	10319
Differenced Exchange Rate	1	247.4	74403	10328

NB : $rank = 2$

From the output, the second lag is the most appropriate for estimation. The model effects are therefore investigated and from Figure (4.1.4), it is clear that the model has a level effect except for tail values.

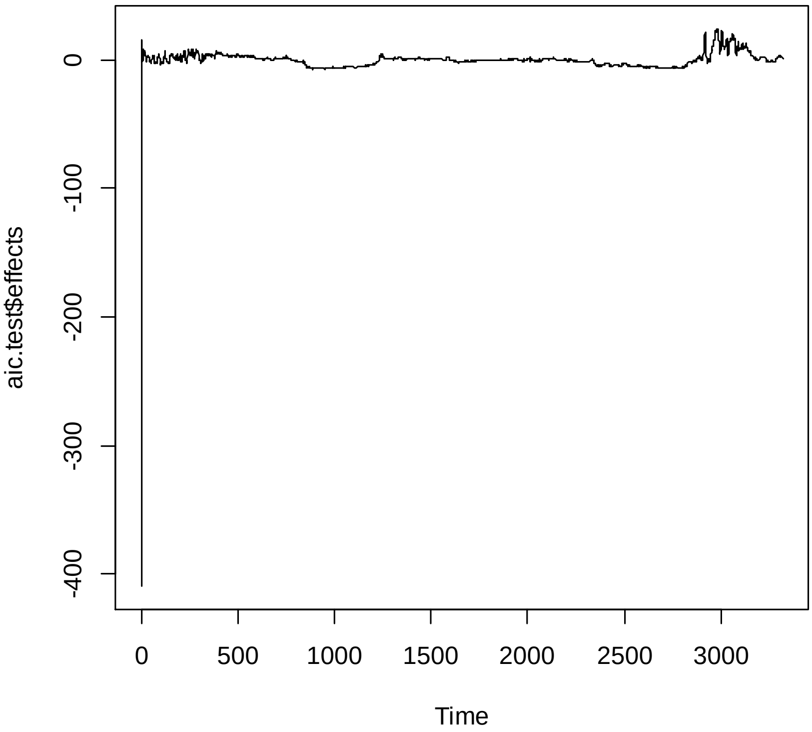

Figure 4.1.4: Exchange Rate verses

Since the model effects are level within the mean, the necessity of lag inclusion in the model is investigated. The results of this test is as shown in Table (4.6) below.

Table 4.6: Statistical Test Output on Lag Inclusion in the Model

Model	Inclusion	Residual Df	F-value	$pr(>F)$
One	$Lags(Y, 1:1) + Lags(X, 1:1)$	3317		
Two	$Lags(Y, 1:1)$	3318	-1	4.264×10^{-7}

Y and X represents the interbank lending rate and the exchange rate, respectively. The null hypothesis of the saturated model is not rejected. Therefore, $\exists$ sufficient evidence that the inclusion of the lagged values in causality estimation leads to the overall improvement in the model predictive ability. The coefficients for the Granger Causality model are presented in the table below.

Table 4.7: Granger Causality Model Coefficients

Intercept	X11	X12	X13
-7.9150651	0.1942408	0.1574291	-0.8701337

The third variable (first lag of the exchange rate) is found to be insignificant in the model, at 5% level of significance. The variable is dropped. A linear model is thus fit with the series itself and its second lagged value as follows;

Table 4.8: A Linear Model with Significant Lagged Values

Intercept	X11	X13
-7.9123922	0.1942063	-0.7914011

All the model parameters were found to be statistically significant at 5% level. From the output above, the granger causality model therefore becomes:

$$Y_t + 7.91239 = 0.19421X_t - 0.79140X_{t-2} \tag{4.1.1}$$

where Y_t represents the interbank lending rate while X_t is the dollar exchange rate.

It therefore remains to check on the direction of the causality. The output is presented below.

Table 4.9: Direction of the Causality

	F-statistic	p-value
Exchange -> Interbank	6.473963	0.00156268
Interbank -> Exchange	2.992947	0.05027497

It is clear that exchange rates Granger Causes the interbanking lending rates. Therefore, movements in interbank lending rates are more likely to be caused by the movements in the exchange rates. However, interbank lending rate does not Granger Cause the exchange rate, as could have been expected.

4.2 Error Correction Model

Once the Granger Causality model in sub-section (2.2.5) above has been built (Equation (4.1.1)), an ECM is easily built by considering the residuals of the model in Equation (4.1.1) above. ECM involves fitting a regression equation of the differenced series and the residuals of the fitted Granger Causality model. Due to the inclusion of the residuals, dynamic linear modeling is used. The fitted model will involve two main parts; the residual part which might be considered more stable than the differenced series part, hence the use of a dynamic linear model. The output of an estimated ECM is as shown in Table (4.10) below.

Table 4.10: An Estimation of an ECM for Exchange and Interbank Lending Rates

	Estimate	Std. Error	t value	$pr(>\|t\|)$
Intercept	7.108730	0.021622	328.78	2.0×10^{-16}
Differenced Exchange Rate	-0.775746	0.055974	-13.86	2.0×10^{-16}
Residual	1.000388	0.004744	210.85	2.0×10^{-16}
Residual Standard Error : 1.246 on 3317 df				
Multiple R^2 : 0.9308				
Adjusted R^2 : 0.9308				
P-value = 2.2×10^{-16}				

All the parameters from the model are significant at 5% level. R^2 value of 0.9308 shows that the overall model fits well to the data. The fitted ECM therefore becomes;

$$Y_t = 7.108730 - 0.775746 \triangle X_t + 1.000388 \Lambda \tag{4.2.1}$$

where Y_t represents the interbank lending rate, X_t the exchange rate while Λ denotes the error correction component.

4.3 Cointegration

In this section, it is proposed that if two time series follow GARCH(1,1), then the two series are cointegrated. The proposition is then proofed by a case study.

4.3.1 The Proposition

Lemma. *If two time series follow a GARCH(1,1) model, then the two series are co-integrated, and can simply be given as*

$$X_t = \alpha + \lambda Y_t \tag{4.3.1}$$

where the estimate of λ, $\widehat{\lambda}$, *is the OLS estimate of Equation (4.3.1), and* (X_t, Y_t) *are the two time series under consideration. Further, the two series will not drift too much from each other.*

4.3.2 Proof

Data on exchange and interbank lending rates are used in the analysis, which are investigated for GARCH(1,1) properties. The study is aimed at capturing heteroskedasticity property, a characteristic which commonly exist in time series data, where volatility clustering occur. In such a case, the statistical property of data gradually decreasing from high to low densities and vice versa does not apply. Rather, high densities tend to cluster together, in which case, variances tend to be related across different periods and hence leading to the result:

$$var\left(X_t\right) = E\left(X_t^2\right) \tag{4.3.2}$$

That is the variance of the series at a given time, say t, is the same as the the expectation of the square of the series. This is the basis for heteroskedasticity and hence an indication that auto-correlation still exists in the squares of the returns.

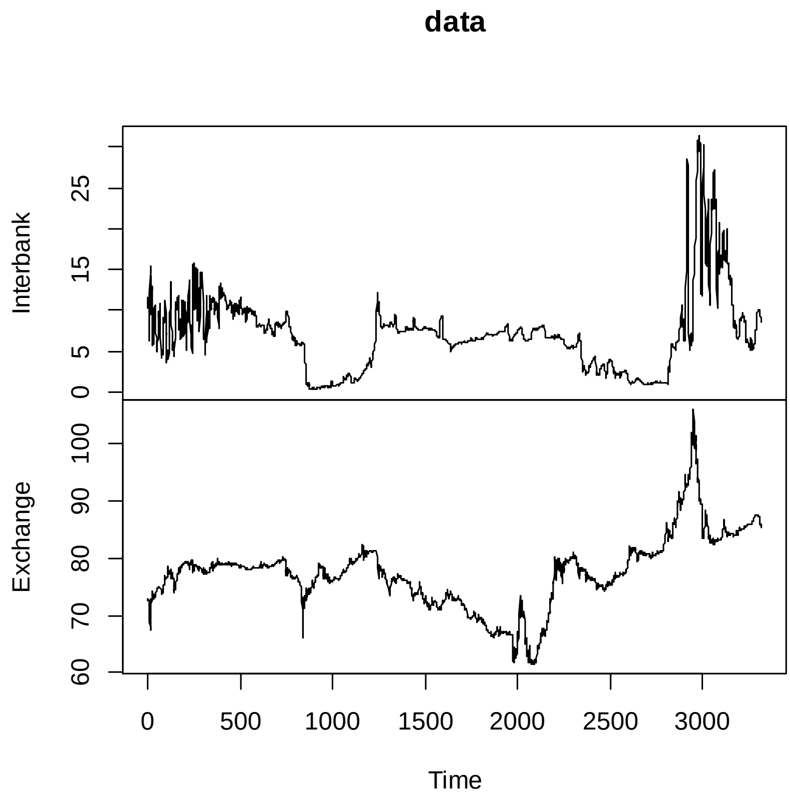

Figure 4.3.1: A Plot of the Exchange and Lending Rates. The Series is Plotted Against Time. It is Clearly a Non-stationary Series by Visual Inspection.

Figure (4.3.1) above shows the plot of the exchange and interbank lending rates. The movements of the two series seem to be similar, though it has been plotted on different scales.

The two series are plotted on the same scale for clearity in inspection and visual comparison. This is done by superimposing the lending rates series on the exchange rates series. A visual inspection of the superimposed plot suggest an existence of co-integration, as in Figure (4.3.2). Nevertheless, a visual inspection does not give any concrete judgment.

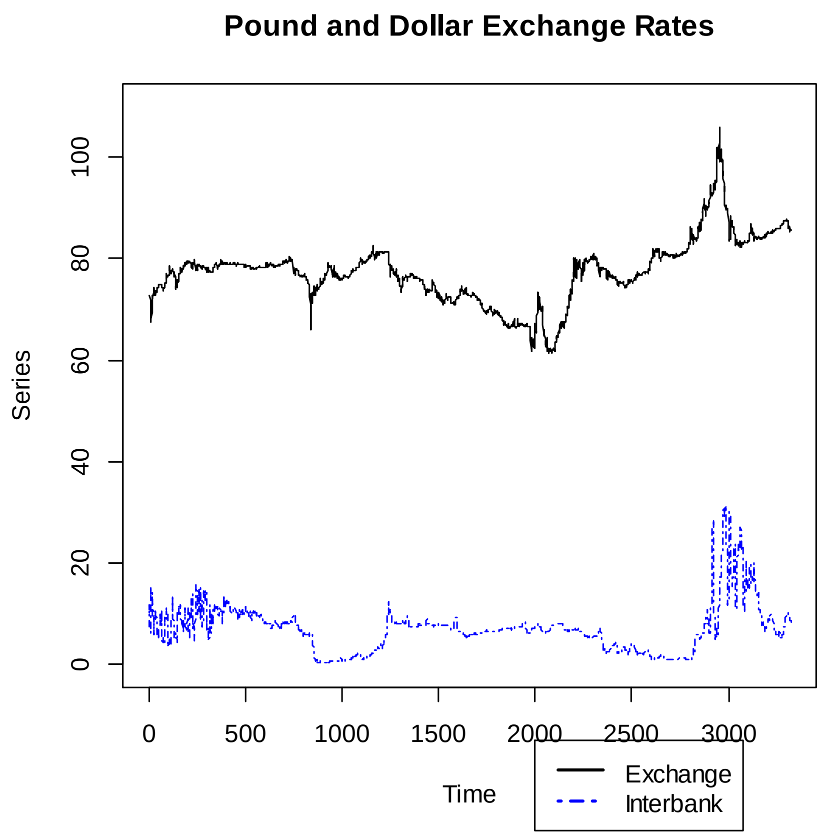

Figure 4.3.2: The Interbank Lending Rate is Superimposed onto the Exchange Rate Time Plot. A Visual Inspection Suggests a Co-integration Relationship

Since the series are non-stationary, differencing is applied to achieve stationarity. Also, following the proposition, an investigation of the GARCH properties should be done, to ensure all properties are satisfied before any other analysis can be done. Being a GARCH model, it is expected to exhibit a high auto-correlation at lag one and insignificant correlation in higher orders. This translates to a spike at lag one in the Auto-correlation Function (ACF) plot which is evident in Figure (4.3.3). An investigation on the heteroskedasticity property of the series can as well be investigated from Figure (4.3.3).

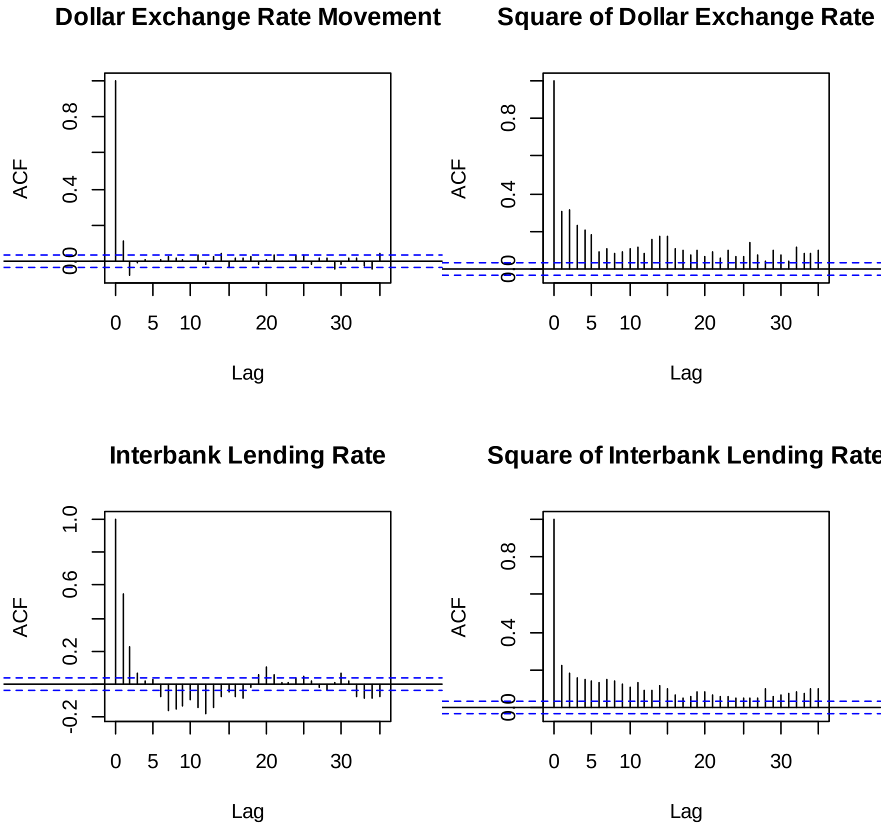

Figure 4.3.3: Auto Correlation Functions for the Series and Their Respective Squares.

An inspection of the ACF's of the squares of the two series indicate existence of serial correlation, an indication of heteroskedasticity property. The mean and variance properties of a GARCH model are satisfied. However, the ACF of the interbank lending rate does not seem to depict a classic GARCH(1,1) property.

The ACF for the interbank lending rates indicates higher order seasonal and serial correlation. It is important to remove high serial correlations in the data before further analysis. Logarithmic transformation removes this in the data. The ACF of the log-transformed series is given in Figure (4.3.4).

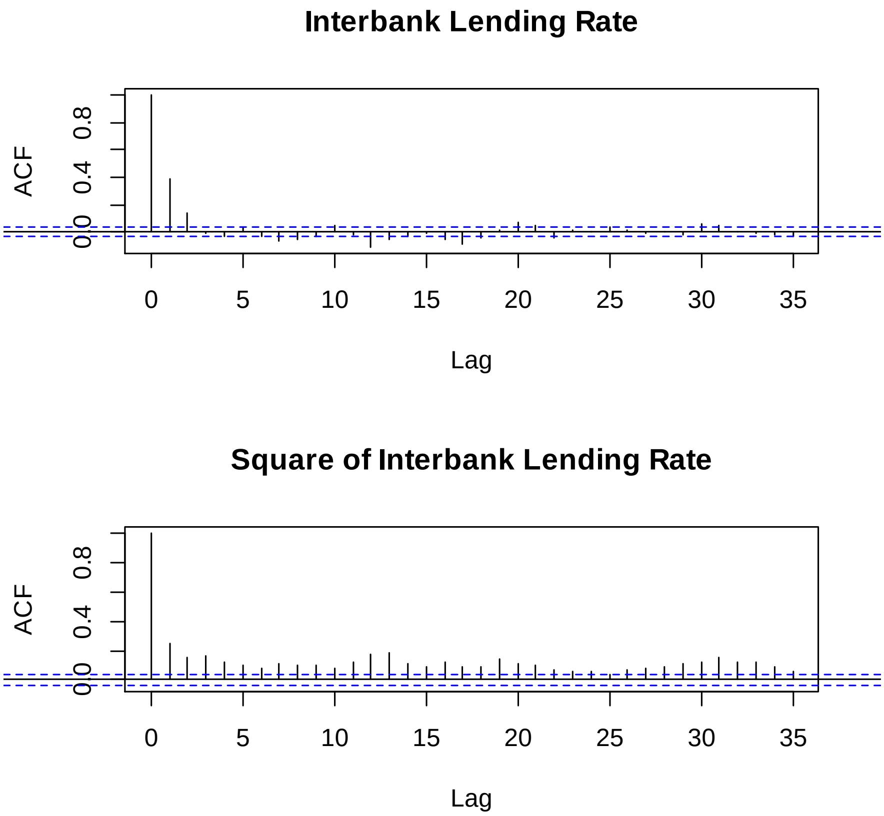

Figure 4.3.4: ACF of the Log-transformed Interbank Lending Rate

GARCH(1,1) conditions are satisfied. A visual inspection suggests that the two series are non-stationary. Nevertheless, unit root tests such as ADF, KPSS and PP tests discussed previously in section (2.2.3) are used to investigate stationarity in the series.

The ADF test tests the null hypothesis of non stationarity. The p-value indicates the amount of evidence against the null hypothesis. For the two time series, the ADF test output is as shown in Table (4.11).

Table 4.11: Augmented Dickey Fuller Test Output for the Two Series.

Data	Dickey-Fuller	Lag	P-value
Exchange Rate	-2.1937	14	0.4963
log of Interbank Rate	-2.5287	14	0.3545

From Table (4.11), the p-values are higher than 0.05. At 5% level of significance, the classical probability rule dictates failure to reject the null hypothesis. The decision rule is that there exists sufficient evidence that the series might be non-stationary. Differencing is the common procedure for stationarity attainment. The two series are differenced and ADF output for the differenced series presented in Table (4.12).

Table 4.12: ADF of the Differenced Series

Data	Dickey-Fuller	Lag	P-value
differenced Exchange Rate	-14.2119	14	0.01
differenced log-transformed Interest Rate	-16.364	14	0.01

From the p-values, the null hypothesis of non stationarity is rejected. Also the absolute values of the Dickey-Fuller values are relatively low thus we may conclude both series may be mean-reverting. In such a case, co-integration may exist. But ADF test has two downsides, as discussed earlier, which has to be addressed.

1. The model for an ADF test uses the differenced series.
2. It assumes that the residuals are independent and identically distributed.

KPSS test addresses the differenced model in ADF test. It tests the null of stationarity with respect to an existing trend. It is similar to the ADF test but does not detrend the series. That is, the long term general movement of the series is preserved. Table (4.13) presents the output of this test.

Table 4.13: KPSS Test Output for the Two Time Series

Data	KPSS Level	Lag	P-value
differenced Exchange Rate	0.0755	13	0.1
differenced log-transformed Interest Rate	0.0477	13	0.1

The test yield a p-value of 0.1. Following the same decision rule, we fail to reject the null hypothesis at 5% significance level and conclude that the series might be stationary.

All the above tests are parametric. They all assume independence and identical distribution of residuals. A non-parametric test need to be done, which can be used in presence of heteroskedasticity. Philip Perron test is the best alternative. To ascertain these decisions, a Philip Perron test is done whose results are presented in Table (4.14).

Table 4.14: Phillip Perron Test Output for the Two Series

Data	Dickey-Fuller	Lag	P-value
differenced Exchange Rate	-2765.71	9	0.01
differenced log-transformed Interest Rate	-1864.619	9	0.01

This test, just like ADF, test the null of non stationarity. Having the same p-value of 0.01 as that in the ADF test, the same decision rule is followed.

It can finally be concluded that the two differenced series are stationary and therefore satisfying all the underlying model assumptions. The requirement of the two series being integrated of order 1, $I(1)$, is fully satisfied since stationarity has been achieved by differencing the series once, indicated by the above tests. A relatively low Dickey-Fuller absolute values is an indication of a possibility of co-integration. It then remains to estimate the parameter λ from Equation (2.2.19) to estimate co-integration relationship.

Equation (2.2.19) can as well be rearranged and written as

$$A_t = \Theta_t - \lambda B_t \tag{4.3.3}$$

which is a linear model in A_t and B_t. COLS can therefore be used to estimate the parameter λ. A linear model is fitted which estimates λ to be -0.490747. This value remains an estimate till the residuals of the fitted model is tested for stationarity. Figure (4.3.5) below is a plot of the residuals. A visual inspection suggests stationarity as it has the form of a mean-zero white noise. The residual series is less random about the mean and hence its variance approaches unity.

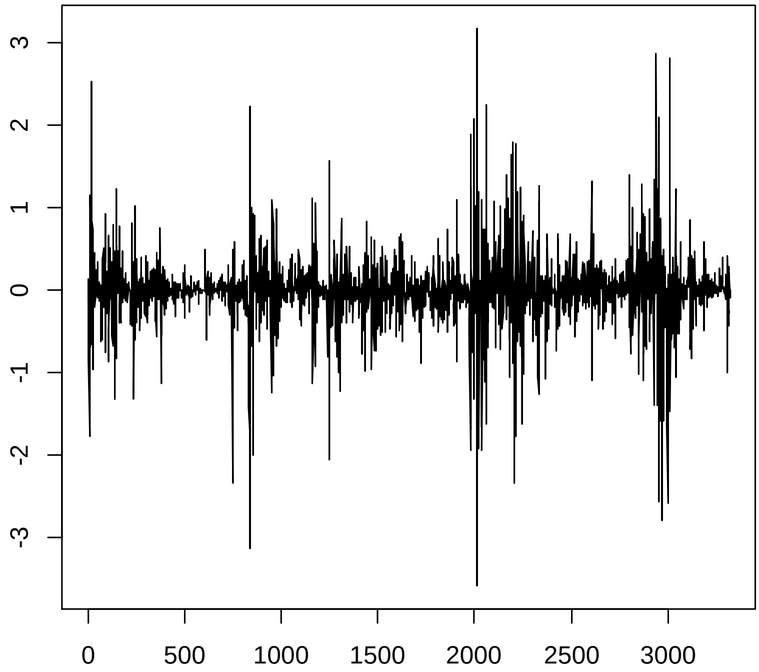

Figure 4.3.5: A Plot of the Residual Series. A Visual Inspection of the Plot Indicates a Stationary Process. It is a Replica of the Purely Random Process.

Similarly, stationarity tests are done on the residual series and Table (4.15) below contains the outputs from the three tests.

Table 4.15: Residual Series Tests for Stationarity

Test	Test Statistic	Lag value	P-value
ADF	-14.0361	14	0.01
KPSS	0.0794	13	0.1
PP	-2775.608	9	0.01

Using the same decision rule, the residual series is stationary. Therefore a co-integration relation exists. It therefore remains to test the significance of the coefficient of co-integration. The hypotheses under consideration is

$$
\begin{aligned}
&H_0 : \lambda = 0 \\
&vs \\
&H_1 : \lambda \neq 0
\end{aligned}
\tag{4.3.4}
$$

From the summary of the fitted model presented in Table (4.16), the p-value is infinitesimally small.

Table 4.16: Summary of the Fitted Model

	Estimate	Std. Error	t value	$pr\,(>\lvert t\rvert)$
Intercept	0.003863	0.006684	0.578	0.563
Differenced Interbank Lending Rate	-0.490747	0.110065	-4.459	8.52×10^{-6}
Residual Standard Error : 0.3852 on 3319 df				
Multiple R^2 : 0.005954				
Adjusted R^2 : 0.005655				
P-value = 8.519×10^{-6}				

.
NB : The fitted model is $A_t = \alpha + \beta B_t$ where $A_t = diff(Exchange)$ and $B_t = diff(log(Interbank))$

The null hypothesis is rejected and concluded that the co-integration factor is different from zero. Further, the intercept is not significant.

4.3.3 The Model

The fitted model can be given by

$$A_t = -0.490747B_t \tag{4.3.5}$$

or

$$A_t + 0.490747B_t = 0 \tag{4.3.6}$$

The residual standard error is considerably small. It can be concluded that the co-integration coefficient is significant in the model. However, despite the R^2 and the adjusted R^2 values being small, the two values are approximately the same, an indication that the sampled data characteristic does not differ much from the population data. The overall model is therefore significant.

Chapter 5

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

Contrary to time series theory, tests on interbank lending rates return stationarity. Nevertheless, the exchange rates seem to be consistently non stationary. The stationarity of the interbank lending rate can be attributed to the fact that;

- The interbank lending rates are controlled locally and are mainly set following fluctuation of world wide economic performance. The stability of interbank lending rates is mainly determined by the central bank.

- The exchange rate is normally controlled by the overall world wide economic performance. Its fluctuation is thus not influence locally by any country, i.e, it is not controlled monopolistically.

From the above arguements, we expect the interbank lending rates to be more stable of the two.

Results of the Granger Causality indicate that the exchange rates Granger causes the interbank lending rates. Movements in exchange rates can be used as an indication of the most probable movement in interbank lending rate. The residual sum of squares from the Granger Causality model is very low, an indication that the model optimally explains the variations in the data. From the superimposed plot of the two series in Figure (4.1.3), it is expected that the exchange rate and the interbank lending rate Granger Causes each other. However, contrary to this intuition, the interbank lending rate does not Granger Cause the exchange rate. This can as well be attributed to the local nature of the interbank lending rate.

On the other hand, an ECM model is built and results presented in Table (3.8) in sub-section (3.2). The model is presented in Equation (3.2.1). From the output, the R^2 value is 0.9308, an indication that the model well fits to the data. Further, the adjusted R^2 value is 0.9308, same as the R^2 value, meaning the model can as well perfectly fit to any other data with similar characteristics. Therefore, the model can be used for predictive purposes. This arguement is ascertained by the very small sum of squared residuals. The model can therefore be used to analyse data with similar characterisitcs as those used in the study. It can as well be adobted by institutions for their internal hedging and as a liquidity guard.

Also, there exists a co-integration relation between the two series. A unit change in exchange rate results in a change in the interbank lending rate by 0.490747 units in the opposite direction. Though the series has a long-run equilibrium relationship, a short term relationship is more useful as it is evident from the R^2 and the adjusted R^2 values. An intercept of zero indicates that the two series, the exchange and interbank lending rates do not have any drift, and if it occurs, they cannot drift too far apart from the equilibrium because economic forces will act to restore the equilibrium relationship . This therefore completes the proof of the first part of the proposition presented in Section (4.3.1) that the two series will not drift too far from each other.

Next, it is noted that at equilibrium the value of A is 0.490747 times the value of B. Since A and B are interbank and exchange rates respectively then when the price of A exceeds 0.490747 times the value of B, we expect either;

- The price of A to decrease so as to reach the point of equilibrium in the near future, or

- The price of B to be pushed up for it to balance at equilibrium with that of A.

A small value of the residual standard error indicates that most of the variability in the data is captured by the co-integration model. This is an indication of the model's ability to capture intra-data clustering. Future shocks which might be experienced are therefore easily captured in the forecasts. It therefore indicates a high level of significance in the forecasts of this model.

Notably, the R^2 and the adjusted R^2 values are almost equal. It is a good indication of high precision forecasts. It is an indication that the characteristic exhibited by series is persistent. A sample of the set of data will always have the same characteristic as the population of the data. This is in line with heteroskedasticity as data tend to cluster in a similar manner throughout the data. It is therefore an indication of the reliability of the forecasts obtained if the model was to be used. This completes the final proof of existence of a co-integration relationship.

It can be concluded therefore that if two series follow a GARCH(1,1) model, they are co-integrated and they do not drift too far from each other. It can as well be concluded that co-integration is a powerful tool in the analysis of time series data and can be used to obtain optimal forecasts. A co-integration relationship can therefore be used to explain the source of variability in one series if the variability in the other series is known. Finally, heteroskedasticity does not influence the predictability of a co-integration model. Therefore, highly significant forecasts can still be obtained from a highly heteroskedastic series. This wraps up the proof to the main proposition in section (3.3.1).

5.2 Recommendations

It is recommended that the same study be conducted with interbank lending rates being differenced to investigate whether their will be any change in the overall model. Also, an investigation on the stationarity of the interbank lending rates should be done to establish its cause and why it occurs. Further, a study on the tail values should be done to examine the cause of the tail clustering and its impact on the overall model. Given the Granger Causality model, it is recommended that a co-integration model be built to examine whether the two models differ from each other; and if so, examine the reasons for the difference.

As much as GARCH models captures heteroskedasticity, it is still a conditional variance model. GARCH models are therefore most appropriate for squared return series. On the other hand, ARMA models are built on conditional expectation. They are therefore perfect for a normal return series. Therefore, A combination of the two series will be most appropriate when the series exhibit correlation in both first and second order. It is therefore recommended that a similar study be undertaken with an investigation of a combination of ARMA and GARCH models. It is as well recommended that further study be done on the Kenyan interbank lending rate to investigate the source of very high serial correlations.

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Appendix A

Simulation Study

A.1 Granger Causality and ECM

Simulating GARCH(1,1)

```
set.seed(1)
```

Series One

```
a0<-0.5; a1<-0.3; b1<-0.65;
n<-1000
vt<-rnorm(n,0,1) #white noise
omega<-rep(0,n)
h<-rep(0,n)
for(i in 2:n)
{
h[i]<-a0+(a1*(omega[i-1]^2))+(b1*h[i-1])
omega[i]<-vt[i]*sqrt(h[i])
}
par(mfrow=c(1,1))
plot.ts(omega,main=" Series Plot ",ylab=" Series ")#a simulated se-
ries
```

Series Two

```
a0<-1; a1<-0.6; b1<-0.1
omega1<-rep(0,n)
```

```
for(i in 2:n)
{
h[i]<-a0+(a1*(omega1[i-1]^2))+(b1*h[i-1])
omega1[i]<-vt[i]*sqrt(h[i])
}
lines(omega1,type="l",col="blue",lty=4)
par(xpd=TRUE)
legend(600,-20,c("Series1","Series2"),lty=c(1,4),lwd=c(2.5,2.5),col=
c("black","blue"))
par(mfrow=c(2,2))
acf(omega,main="Series One")
acf(omega^2,main="Square of Series One")
acf(omega1,main="Series Two")
acf(omega1^2,main="Square of Series Two")
library(tseries)
```

Stationarity for Series One

```
adf.test(omega,alternative="stationary")
kpss.test(omega,null = c("Level", "Trend"))
pp.test(omega, alternative =c("stationary", "explosive"),type =
c("Z(alpha)", "Z(t_alpha)"), lshort = TRUE)
```

Stationarity for Series Two

```
adf.test(omega1,alternative="stationary")
kpss.test(omega1,null = c("Level", "Trend"))
pp.test(omega1, alternative = c("stationary", "explosive"),type =

c("Z(alpha)", "Z(t_alpha)"), lshort = TRUE)
```

AIC Tests

```
library(stats)
```

```
library(MASS)
model<-lm((omega[2:length(omega)])~diff(omega1))
aic.test<-stepAIC(model)
rank<-aic.test$rank par(mfrow=c(1,1))
rank
#Check fit appropriateness
plot.ts(aic.test$effects)
plot.ts(aic.test$model)
#Granger tests
library(lmtest)
#Check laged value inclusion
grangertest((omega[2:length(omega)])~diff(omega1))
#estimating granger causality model
omega2<-diff(omega1)
omega3<-diff(omega1,lag=2)
model1<-lm((omega[3:length(omega)])~(omega1[3:length(omega1)])+(omega2[
2:length(omega2)])+omega3)
summary(model1)
model2<-lm((omega[3:length(omega)])~(omega1[3:length(omega1)])+omega3)

summary(model2)
#creating differenced series matrix
omega4<-omega[2:length(omega)]
omega5<-diff(omega1)
dataset<-matrix(c(omega4,omega5),ncol=2)
#Naming Columns
colnames(dataset)<-c("Omega","Differenced Omega1")
#Check if causality exists
```

```
library(MSBVAR)
granger.test(c(omega,omega1),p=rank)
```

Estimating an ECM Model

```
#fit a dynamic model
library(dynlm)
resid<-model2$resid
omega6<-omega[3:length(omega)]
omega7<-omega5[2:length(omega5)]
fit<-dynlm(omega6~omega7+resid)
summary(fit)
#Extracting residuals fit.resid<-fit$resid
#plot for residuals
plot.ts(fit.resid,ylab="",xlab="")
#testing residuals for stationarity
adf.test(fit.resid,alternative="stationary")
kpss.test(fit.resid,null = c("Level", "Trend"))
pp.test(fit.resid, alternative = c("stationary", "explosive"),type
=
c("Z(alpha)", "Z(t_alpha)"), lshort = TRUE)
#Turn in the Coefficients
fit$coeff
```

A.2 Co-integration

Estimating Parameter Labda

```
fit<-lm(omega~omega1)
#Extracting residuals
fit.resid<-fit$resid
#plot for residuals
```

```
par(mfrow=c(1,1))
plot.ts(fit.resid,ylab="",xlab="")
#testing residuals for stationarity
adf.test(fit.resid,alternative="stationary")
kpss.test(fit.resid,null = c("Level", "Trend"))
pp.test(fit.resid, alternative = c("stationary", "explosive"),type
=
c("Z(alpha)", "Z(t_alpha)"), lshort = TRUE)
#Turn in the summary
summary(fit)
```

Appendix B

Case Study

B.1 Granger Causality and ECM

Reading in the Data

```
data<-read.table("F:\\GRADUATE\\Msc\\Thesis\\Analysis Data\\project
data.txt",header=T)
attach(data)
plot.ts(data,main=" Exchange and Lending Rates ")
par(mfrow=c(2,2))
acf(Exchange,main="Dollar Exchange Rate Movement")
acf(Exchange^2,main="Square of Dollar Exchange Rate")
acf(Interbank,main="Interbank Lending Rate")
acf(Interbank^2,main="Square of Interbank Lending Rate")
library(tseries)
par(mfrow=c(1,1))
plot.ts(Exchange,main=" Exchange and Interbank Lending Rates
",ylab=" Series ",ylim=c(0,110))
lines(Interbank,type="l",col="blue",lty=4)
par(xpd=TRUE)
legend(2000,-15,c("Exchange Rate","Interbank Rate"),lty=c(1,4),
lwd=c(2.5,2.5),col=c("black","blue"))
```

Stationarity for Exchange Rate

```
adf.test(Exchange,alternative="stationary")
kpss.test(Exchange,null = c("Level", "Trend"))
pp.test(Exchange, alternative = c("stationary", "explosive"),
type = c("Z(alpha)", "Z(t_alpha)"), lshort = TRUE)
```

Stationarity for Interbank Lending Rate

```
adf.test(Interbank,alternative="stationary")
kpss.test(Interbank,null = c("Level", "Trend"))
pp.test(Interbank, alternative = c("stationary", "explosive"),
type = c("Z(alpha)", "Z(t_alpha)"), lshort = TRUE)
#Diferenced Series
#Stationarity for Exchange Rate
adf.test(diff(Exchange),alternative="stationary")
kpss.test(diff(Exchange),null = c("Level", "Trend"))
pp.test(diff(Exchange), alternative = c("stationary", "explosive"),
type = c("Z(alpha)", "Z(t_alpha)"), lshort = TRUE)
#Stationarity for Interbank Lending Rate
adf.test(diff(Interbank),alternative="stationary")
kpss.test(diff(Interbank),null = c("Level", "Trend"))
pp.test(diff(Interbank), alternative = c("stationary", "explosive"),
type = c("Z(alpha)", "Z(t_alpha)"), lshort = TRUE)
```

AIC Tests

```
library(stats)
library(MASS)
model<-lm((Interbank[2:length(Interbank)])~diff(Exchange),data=data)
aic.test<-stepAIC(model)
rank<-aic.test$rank rank par(mfrow=c(1,1))
#Check fit appropriateness
plot.ts(aic.test$effects) plot.ts(aic.test$model)
```

```
#Granger tests
library(lmtest)
#Check laged value inclusion
Y<-Interbank[2:length(Interbank)]
X<-diff(Exchange)
grangertest(Y~X,data=data)
#estimating granger causality model
Exchange2<-diff(Exchange)
Exchange3<-diff(Exchange,lag=2)
Y1<-Interbank[3:length(Interbank)]
X11<-Exchange[3:length(Exchange)]
X12<-Exchange2[2:length(Exchange2)]
X13<-Exchange3
model1<-lm(Y1~X11+X12+X13,data=data)
model1$coeff
model2<-lm(Y1~X11+X13,data=data)
model2$coeff
#creating differenced series matrix
Interbank1<-Interbank[2:length(Interbank)]
Exchange1<-diff(Exchange)
dataset<-matrix(c(Interbank1,Exchange1),ncol=2)
#Naming Columns
colnames(dataset)<-c("Interbank","Differenced Exchange")
#Check if causality exists
library(MSBVAR)
granger.test(data,p=rank)
```

Estimating an ECM Model

```
#fit a dynamic model
```

```
library(dynlm)
resid<-model2$resid
Interbank3<-Interbank[3:length(Interbank)]
Exchange4<-Exchange1[2:length(Exchange1)]
fit<-dynlm(Interbank3~Exchange4+resid)
summary(fit)
#Extracting residuals
fit.resid<-fit$resid
#plot for residuals
plot.ts(fit.resid,ylab="",xlab="")
#testing residuals for stationarity
adf.test(fit.resid,alternative="stationary")
kpss.test(fit.resid,null = c("Level", "Trend"))
pp.test(fit.resid, alternative = c("stationary", "explosive"),
type = c("Z(alpha)", "Z(t_alpha)"), lshort = TRUE)
#Turn in the Coefficients
fit$coeff
```

B.2 Co-integration

Importing the data to R

```
data<-read.table("F:\\GRADUATE\\Msc\\Thesis\\Analysis Data\\
project data.txt",header=TRUE)
attach(data)
plot.ts(data)
par(mfrow=c(1,1))
plot.ts(Exchange,main=" Pound and Dollar Exchange Rates ",
ylab=" Series ",ylim=c(0,110))
lines(Interbank,type="l",col="blue",lty=4)
```

```
par(xpd=TRUE)
legend(2000,-15,c("Exchange","Interbank"),lty=c(1,4),
lwd=c(2.5,2.5),col=c("black","blue"))
par(mfrow=c(2,2))
acf(diff(Exchange),main="Dollar Exchange Rate Movement")
acf(diff(Exchange)^2,main="Square of Dollar Exchange Rate")
acf(diff(Interbank),main="Interbank Lending Rate")
acf(diff(Interbank)^2,main="Square of Interbank Lending Rate")
par(mfrow=c(2,1))
acf(diff(log(Interbank)),main="Interbank Lending Rate")
acf(diff(log(Interbank))^2,main="Square of Interbank Lending Rate")
library(tseries)
#Stationarity for Exchange Rate
adf.test(diff(Exchange),alternative="stationary")
kpss.test(diff(Exchange),null = c("Level", "Trend"))
pp.test(diff(Exchange), alternative = c("stationary", "explosive"),
type = c("Z(alpha)", "Z(t_alpha)"), lshort = TRUE)
#Stationarity for Interbank Rate
adf.test(diff(log(Interbank)),alternative="stationary")
kpss.test(diff(log(Interbank)),null = c("Level", "Trend"))
pp.test(diff(log(Interbank)), alternative = c("stationary",
"explosive"),type = c("Z(alpha)", "Z(t_alpha)"), lshort = TRUE)
#Estimating parameter labda
fit<-lm(diff(Exchange)~diff(log(Interbank)))
#Extracting residuals
fit.resid<-fit$resid
#plot for residuals
par(mfrow=c(1,1))
```

```
plot.ts(fit.resid,ylab="",xlab="")
#testing residuals for stationarity
adf.test(fit.resid,alternative="stationary")
kpss.test(fit.resid,null = c("Level", "Trend"))
pp.test(fit.resid, alternative = c("stationary", "explosive"),
type = c("Z(alpha)", "Z(t_alpha)"), lshort = TRUE)
#Turn in the summary
summary(fit)
```