A STUDY ON THE CONVECTION HEAT TRANSFER IN A FLUID FLOW OVER IMMERSED CURVED SURFACE

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DECLARATION

This thesis is my original work and has not been presented for a degree in any other University.

Signature:……………………………… **Date:**………………………….

George Ochieng' Omboro

This thesis has been submitted for examination with our approval as the University Supervisors:

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JKUAT, Kenya

DEDICATION

To the topsy-turvy universe that marvels at its enormous wealth of knowledge.

ACKNOWLEDGMENT

Great honor, recognition, thanks and appreciation are to those undermentioned, whose assistance and contribution made this work to be true; -

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Above all, may I absolve the above of all errors of omission and commission that might be in this thesis.

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ABBREVIATIONS

e.g.	for example
i.e.	that is to say
LP	Low pressure
PX	Pressure term
rad	radian measure

NOMENCLATURE

Symbol	***Quantity***
a	real number
c_p	Specific heat at constant pressure, $Jkg^{-1}K^{-1}$
E_c	Eckert number
h	heat transfer coefficient, $h = q_w^{''}(T_w - T_\infty)$
L	reference length, m
m	real number
O	order
p	pressure, Pa
Pr	Prandtl number
Pe	Peclet number

Q	amount of heat added to the system. Nm.
q_s''	local wall heat flux.
Re	Reynolds number
T	temperature, K
τ_s	skin friction coefficient
u,v	fluid velocity components in x – and y – directions respectively
U	outer flow fluid velocity, ms^{-2}
V	reference fluid velocity, ms^{-2}
X, Y	body forces along x – and y - directions
x, y, z	Cartesian coordinates
i, j, k	unit vectors in the x, y and z directions respectively
y	distance away from the wall, m
$\vec{\nabla}$	Gradient operator $\left(\mathbf{i}\frac{\partial}{\partial x} + \boldsymbol{j}\frac{\partial}{\partial y} + \boldsymbol{k}\frac{\partial}{\partial z}\right)$
∇^2	Laplacian operator $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$

Greek Symbols

ϑ	Kinematic viscosity, m^2/s $\quad \vartheta = \frac{\mu}{\rho}$
μ	molecular viscosity (viscosity coefficient), kg/ms
π	$Pi \cong 3.141592654$
α	angle, rad.

ρ	fluid density, kg/m^3
ϕ	viscous dissipation function.
δ	boundary layer thickness.

Subscripts

∞	free stream function
s	surface condition
i,j	components along x – and y - directions

Superscripts

*	non dimensional quantity

ABSTRACT

In this study, the velocity distribution, the variation of temperature and effects of the convection heat generated within the boundary layer for a fluid flowing over an immersed curved surface were discussed. Most of the research investigations and findings always give emphasis to circulation that results to lift (as explained by Kutta-Joukowski hypothesis; Blasius and Kelvin's theorems), the effects of Shock waves and the formation of drag due to skin friction or as a result of the occurrence of separation at the trailing vortex/edge. This research study is on the extent to which mass and heat transfer have to both lift and drag, respectively on an immersed curved surface.

In this study, the continuity, the momentum and thermal energy equations were nondimensionalised and the solutions were approximated by use of the finite-difference method. From this research study, the convection heat generated due to the viscous effect on the curved surface is high within the boundary layer, thus affecting the lift and drag force..

The findings would go a long way in assisting Engineers in making necessary design and estimate improvements where such situations warrant, for instance in aerodynamics and thermal turbomachinery applications.

CHAPTER ONE

1.10 INTRODUCTION

In this chapter the main terms used in the thesis are defined and elaborated followed by a review of the literature related to the present work.

A figure showing velocity, thermal and concentration layer development, which is used as an important tool in vivid understanding of the background of this research problem, is also presented in this chapter. Towards the end of the chapter, the statement of the research problem, objectives and justification are precisely stated. The chapter ends by giving an outline of the entire thesis.

1.11 HEAT TRANSFER

Heat transfer involves the study of energy in transit as a result of temperature difference in a medium or between media. The temperature difference may arise from various causes such as radioactivity, absorption of thermal radiation and release of latent heat as fluid vapour condenses, or one due to viscous effect. Heat transfer takes place mainly in three modes: conduction, convection and radiation. This study is concerned with the convection heat transfer in a fluid flow over an immersed curved surface.

1.12 FLUID

Fluid is a general term used for matter in liquid or gas state. That is, fluid is any substance that flows and which offers no permanent resistance to changes of shape induced by pressure. Only a uniform isotropic pressure can be supported without distortion. Fluid cannot sustain a shear stress however small; that is, a fluid deform continuously with time under the influence of a shear stress.

A fluid may be compressible (as in gas, though air may behave as incompressible fluid if flow speed is less than that of sound), or assumed to be incompressible (as in liquid).

In this research, a Newtonian fluid (is one for which the shear stress τ_s is linearly proportion to the rate of angular deformation, $\tau_s = \mu \frac{\partial u}{\partial y}$) is considered.

A fluid flow over an object is steady if its velocity and thermodynamic properties at each point in the flow remains constant (this does not necessarily require that the velocity be the same at all points in the fluid), otherwise unsteady if the flow variables are dependent on time.

Fluid motion may be constrained by geometrical boundaries to be predominantly parallel to the sides. When the conditions (pressure, velocity, density and temperature) at all successive cross-sections are identical at any instant, the flow is termed *uniform* otherwise *non-uniform*.

Fluid flow may be termed as laminar or turbulent. The term *laminar* is used to mean a fluid flow in which fluid particles downstream of leading edge moves in an orderly manner in laminas or layers parallel to the solid boundary as opposed to *turbulent* whereby fluid velocity components have random turbulent fluctuations imposed upon their mean values. A fluid flow is determined to be laminar or turbulent by the velocity and channel configuration or size. Turbulent fluid motion is an irregular condition of flow in which various quantities like velocity and pressure show a random variation with time and space.

Turbulent flow is a type of flow characterized by eddies that causes mixing of layers of the fluid until the layers are no longer distinguishable. The mixing causes increase in heat transfer consequently, the greater the turbulence the larger the amount of heat transfer

A fluid is termed *ideal* if it is assumed that there exist no frictional effects between moving layers or between layers and the boundary walls.

1.13 CONVECTION

In the context of the mode of heat transfer, the term convection refers to the heat transfer that occurs on a surface and a moving fluid when they are at different temperatures.

Convection heat transfer is due to the superposition of energy transport by diffusion (random molecular motion) and by advection (the bulk, or macroscopic, fluid motion).

The contribution due to bulk fluid motion originates from the fact that boundary layer grows as the flow progresses. Convection laws rely on the fundamental principles of both heat transfer and fluid flow. These include the laws of mass conservation, momentum conservation and energy conservation.

Convection heat depends on viscosity, thermal conductivity, specific heat and density of the fluid. Viscosity also influences the velocity profile of the fluid flow.

Convection heat transfer may be categorized as forced convection, a situation whereby flow is caused by some external means such as by a fan, a pump, or atmospheric winds; or as free (normal) convection, a situation whereby the flow is induced by buoyancy forces resulting from density variations as a result of temperature variations in the fluid.

Mass transfer by convection is analogous to convection heat transfer. In mass transfer by convection, gross motion combines with diffusion to promote the transport of a species for which there exists a concentration gradient. A particular species may be transported by advection (i.e. with the mean velocity of the mixture) and by diffusion (relative to the mean motion) in the direction of each coordinate in question.

In this study, forced convection at a stabilized section is considered therefore convection mass transfer will not be of much significance since the gross or bulk fluid motion is due to mechanical work and not because of species in a mixture due to the presence of concentration gradient.

1.14 VISCOSITY

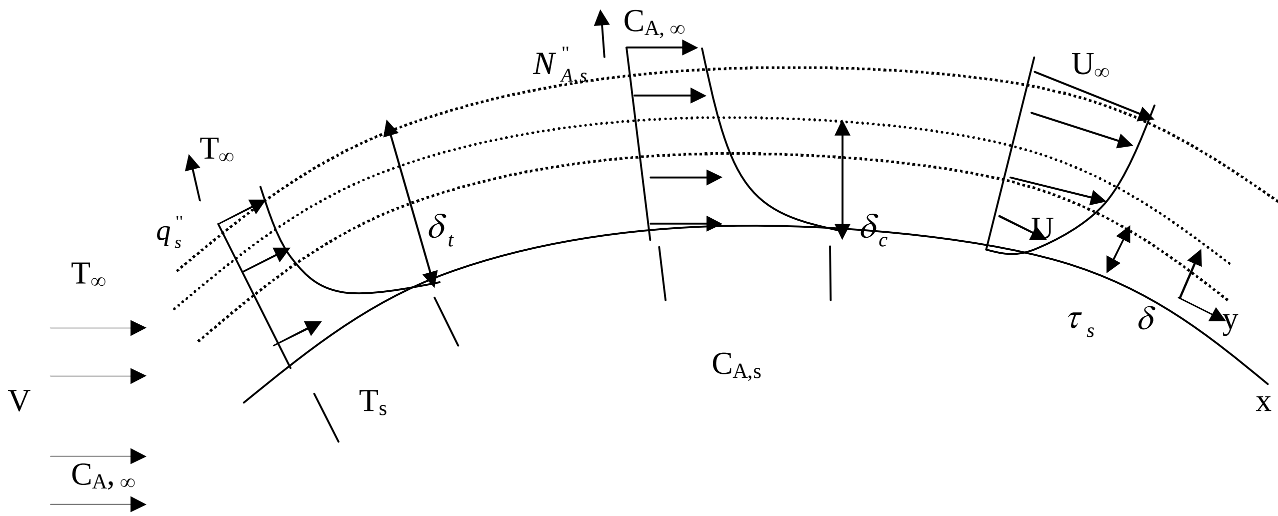
This is the resistance set up due to shear stresses within the fluid particles and the shear stresses between the particles and the solid surface for a fluid flowing around a solid body. As fluid exerts a shear stress on the boundary, the boundary exerts an equal and opposite force on the fluid called shear resistance (or frictional drag). Drag coefficient, C_d always depend on Reynolds number (R_e) and the body shape. The work done against the viscous effects usually causes fluid flow, consequently the energy spent in doing so is converted to heat.

At low values of Reynolds number, the fluid is highly viscous causing deformation drag. The fluid is deformed in a very wide zone around the body causing pressure force and frictional force.

At large values of Reynolds number, fluid is less viscous (as in water and air), and the viscous effect (deformation) is limited to the boundary layer thickness. In such a case, deformation drag is exclusively friction drag. The shear force exerted on the surface of the body due to the formation of boundary layer results into friction drag.

1.15 BOUNDARY LAYER

A boundary layer is a thin fluid layer of the fluid from the surface of a body or a solid wall in which viscous force is significant. Boundary layer thickness theory is important in analyzing flow problems involving convection transport. For fluid flows over any surface there may exist three boundary layers; velocity, thermal and concentration boundary layers. The fluid particles in contact with the stationary solid surface may assume zero velocity under no-slip condition more so at the leading edge. These particles act to retard the motion of the particles in the adjoining fluid layer, which again acts to retard the motion of particles in the next layer and so on until at a certain distance from the surface, where the effect becomes negligible. This region in which the velocity gradient is large is referred to as velocity boundary layer. If the fluid particles come into contact with an isothermal plate, they achieve thermal equilibrium at the plate's surface temperature. In turn the particles exchange energy with those in the successive fluid layer, and temperature gradients develop in the fluid. The region of the fluid in which these temperature gradients exists is the thermal boundary layer. And similarly if the concentration of species at the surface differs from that in the free stream, a concentration boundary layer develops. This is the region of the fluid in which concentration gradients exist. In our study we consider all the boundary layers in a laminar flow and investigate the surface friction, convection heat transfer and convection mass transfer. The velocity, thermal and concentration boundary layers are illustrated in figure 1 below.



δ_t - Thermal boundary layer

δ_c - Concentration boundary layer

Fig. 1: ***Development of Velocity, Thermal and Concentration boundary layer for an arbitrary surface.***

1.16 LIFT AND DRAG

Lift is the sum of all the forces on a body that makes it to move perpendicularly to the direction of flow. This effect occurs when a fluid moves over a stationary object.

Drag is an unavoidable consequence of an object moving through a fluid. Drag is the force generated parallel and in opposition to the direction of travel for an object moving through a fluid. Drag can be broken down into the following two components: Form drag (or pressure drag) – dependent on the shape of object moving through a fluid; and Skin friction – dependent on the viscous friction between a moving surface and a fluid

1.17 DIMENSIONAL ANALYSIS

It is built on the principle of dimensional homogeneity that states that an equation expressing a physical relationship between quantities must be dimensionally homogeneous i.e. dimensions of each side of equations must be the same. It affords a means of ascertaining the forms of physical equations from knowledge of relevant variables and their dimensions. Dimensional analysis therefore, is a method employed to obtain a single equation that relates all the physical factors of a problem to one another. By dimensional analysis, equations in dimensional form are reduced to non dimensional form by using dimensionless groups such as Prandtl number, Sherwood number e.t.c.

It proves a powerful tool in formulating problems that defy analytical solution and must be solved experimentally. Dimensional analysis gives results which only become quantitative from experimental analysis.

This method has applications in nearly all fields of engineering, in particular thermodynamics and hydrodynamics. It is an important tool for presenting experimental results in a concise form. It also gives a basis of mathematical models in solving fluid flow problems.

In this study, dimensional analysis has been used in the non-dimensionalisation of the governing equations by first selecting certain characteristic quantities and then substituting them in the equations.

1.18 LITERATURE REVIEW

The theory of Convective heat transfer strongly emerged by 20^{th} Century. By its nature, convective energy transfer has a close connection with the motion of fluid particles and therefore forms part of study in fluid mechanics. The evolutional changes in the latter (advent of hydrodynamics of non –Newtonian, electric current-conducting and magnetic media, the super- and hypersonic gas dynamics, dynamics of plasma, free molecular and heterogeneous flows, the hydro- and gas dynamic effects during physical and chemical transformations) have greatly affected the theory of heat and mass transfer in moving media.

The relation between the intensities of turbulent momentum and heat transfer process is one of the subtle problems of the convective heat transfer theory. The determination of Prandtl number, Pr is paramount. Its value is of order of unity beyond the viscous sublayer, but greater than one in the immediate vicinity of a solid body (at the depth of the viscous sublayer). At $Pr>100$, the turbulent thermal boundary layer is submerged in the viscous sublayer of the turbulent hydrodynamic boundary layer.

On the formation of boundary layer in a steady flow, Allen (1981) gave evidence to the effect that the location of transition from laminar to turbulent conditions in a boundary layer might be more closely dependent on local skin friction coefficient than Reynolds number.

Barenblatt et al (2002) in their study on a model of turbulent boundary layer with a non-zero pressure gradient observed that turbulent boundary layer at large Reynolds number consist of two separate layers upon which the structure of vorticity fields is different, although both exhibit similar characteristics.

In the first layer, vortical structure is common to all developed-bounded shear flows and the mean flows. The influence of viscosity is transmitted to the main body of flow via streaks separating from the viscous sublayer. The second layer occupies the remaining part of intermediate region of the boundary layer.

The upper boundary of the boundary layer is covered with statistical regularity by large-scale "humps" and that the upper layer is influenced by the external flow via the form drag of these humps as well as by the shear stress.

In their earlier works it is shown that the mean velocity profile is affected by the intermittency of the turbulence and as the hump affects intermittency, the two seeking regions are visible.

On the basis of these considerations, the effective Re, which determines the flow structure in the first layer (and is affected in turn by the viscous sublayer), was identified as one set of such parameters. The other parameters that influence the flow in the upper layer include pressure gradient, $\partial_x p$; dynamic (friction) viscosity, μ ; velocity, u ; fluid's kinematic viscosity, υ and density, ρ.

Another area that has been of great interest for the last three decades is the convective heat transfer through porous medium. Kim et al (1989) and Harris et al (1997) solved the problem of natural convection flow through porous medium past a plate.

Researchers such as Chandrasekhara et al (1992) and Panda et al (2003) made contribution on mass diffusion and natural convection flow past a flat plate. Recently, Magyari et al (2004), have discussed analytical solution for unsteady free convection in a porous media.

Solving the boundary layer equations has attracted many researchers in the present past. Smith et al (1963) in one of his papers presented a method for solving the complete incompressible laminar boundary layer equations; both for two-dimensional and axisymmetric laminar flow, in essentially full generality and with speed. In subsequent papers (1970, 1972) he discussed application potential flow and boundary layer theory in hydrodynamics. He also provided a solution technique of the laminar boundary layers by means of differential difference method. Wehrle (1986) presented an analytical shears for determination of separation point in laminar boundary layer flows.

The continuing interest in flows and heat transfer over flat plate, concave and convex surfaces stems from their possible effects in turbine blades of jet engines, vehicle aerodynamics, aircraft wings, submarines, spaceships, cooling lines of power plants etc. Flow phenomena are mainly subjected to pressure gradients (favorable or adverse), surface curvature and a wide range of Reynolds numbers.

There have been many previous investigations of flow and heat transfer on flat plate boundary layers with pressure gradients. For instance, investigations of Fukagata et al (2002) were concerned with transition to turbulent flow and Reynolds stress distribution, while those of Umur and Karagoz (1999) dealt with the augmentation of heat transfer with or without stream wise pressure gradients. Filippova and Hanel (1998) developed a curved boundary treatment using Taylor series expansion in both space and time for a single particle mass distribution near the wall. This boundary condition satisfies the no-slip condition to the second order in a space step and preserves the geometrical integrity of the wall boundary. Further still, Mei et al. (1999) and Bouzidi et al. (2001) proposed some other boundary treatment methods. In all those methods, the boundary conditions were treated separately for some specific space steps. When some variation occurs in the specified steps while dealing with curved boundaries, an abrupt change in the single particle mass distribution was caused. A unified scheme for curved wall was developed by Yu et al (2003).

In thermal turbomachinery applications; a variation in rate of heat transfer due to a small flow disturbance can lead to an increase in thermal stress and decrease the effective working life span of such a component. This is true for both hot gas side and coolant side of turbomachinery passages. On a highly curved wall, the change in heat transfer rate is mainly due to an increase or decrease of the turbulent mixing by the effect of the streamline curvature. It has been indicated in Von Karman’s stability argument (1934) that the convex wall has stabilizing effect on fluid particles, while the concave wall has a de-stabilizing effect with respect to an equivalent reference flat plate.

The measurement and prediction of rate of heat transfer for a two-dimensional boundary layer on a concave surface have been presented by Mayle et al (1979). It was found that the heat transfer on the convex surface was less than a flat surface having the same freestream, Re and turbulence. Concave surface heat transfer was augmented when compared to the flat surface.

Good agreement between numerical results and heat transfer experiments was noted for the convex surface when a two-dimensional differential boundary layer code was used with modified curvature model.

For the concave surface, the agreement with the measured heat transfer data was poor due to the uncertainties in the turbulence model. Camci (1985) obtained similar conclusions in heat transfer experiments performed on a gas turbine rotor blade under realistic free stream conditions.

One area of practical interest to researchers is on the degradation of the airfoils. Aerodynamic performance of the airfoils and wing plan form designs that are optimal for convectional, large-scale and high speed (therefore, high Reynolds number) would degrade significantly when they are used for low Re applications. The predominance of fluid viscosity effect for the low Reynolds number applications would result in 'boundary layers' growing rapidly and separating from the surfaces of the airfoils easily.

The behaviour of the laminar boundary layer on the low Reynolds number airfoils would affect the aerodynamic performance of airfoils significantly. Since laminar boundary layers are unable to withstand any significant pressure gradient, laminar flow separation is usually found on the low Reynolds number airfoils; and post-separation behaviour of the laminar boundary layers account for the deterioration in the hydrodynamic performance of low Reynolds number airfoils. The deterioration is exhibited in an increase in drag and decrease in lift.

The low-pressure turbines (LPT) airfoils are usually designed to have laminar flow along some parts of the suction surface and turbulent conditions in the rear part after going through laminar-turbulent transition. Often there exist small separation bubbles inside the boundary layer, where the flow reattaches after transition sets on.

There are quite many publications dealing with the development of boundary layers and separation bubbles in low – pressure turbines. Stratford (1957) investigated laminar separation phenomena in boundary layer. Tan and Auld (1991) performed hot wire tests on a turbine cascade in a low speed wind tunnel at various Reynolds number with transition and separation bubbles. In their paper they reported variation of different boundary layer parameters.

Scrivener et al (1991) observed losses connected with the appearance of separation bubbles on the suction surfaces of the LP turbines downstream of the blade row. Denton (1993) provides a very comprehensive overview over loss mechanisms in turbomachines with special emphasis on the concept of entropy generation. He gives a good classification of the different types of losses encountered in turbomachinery.

More experimental data on the flow and turbulence quantities in separated boundary layers with reattachment and transition were performed. For instance, Rivir (1996), Qui and Simon (1997), Sohn et al (1998), and Hatman and Wang (1998a, 1998b) conducted experiments on separated flow transition and used data to develop a model for transition in these flows.

From the above discussed research investigations and findings, it is clear that limited research study has been carried out to precisely give the extent to which the variations in velocity, temperature and convection heat transfer would have within the fluid boundary layer on a fluid flowing over immersed curved surface. This was the motivation of this research study.

1.19 STATEMENT OF THE RESEARCH PROBLEM

As fluid flow over any immersed curved surface, some work is done against viscous effects and energy spent is converted to heat. Also, vorticity formed in boundary layer due to high velocity gradient is swept outwards from the boundary layer.

Although much research has been carried out on mass and heat transfer, limited findings have been concluded on the extent to which temperature and velocity variations within the boundary layer would have in fluid flow over immersed curved surfaces. This thus forms the basis of this research study.

1.20 OBJECTIVES OF THE STUDY

This study is aimed at determining

- the velocity distribution of fluid flow past an immersed curved surface.
- the variation of temperature within the thermal boundary layer of fluid flow past an immersed curved surface due to the velocity variations.
- the effect of heat generated within the boundary layer of an immersed curved surface.

1.21 JUSTIFICATION

Rise in temperature decreases viscosity of fluid and vice versa, thus the need to design blades that would withstand such variations.

Heat injection or withdrawal on submerged curved surfaces enhance velocity variations in fluid flow thereby improving the maneuvers of such bodies in the fluid; as in the case of flying planes, pumps and fan blades etc, so more comprehensive research study is necessary.

Aquatic animals like fish that solely depends on their effective swimming ability, which is affected by variation in fluid physical conditions such as temperature.

1.22 THESIS OUTLINE

In chapter one of this thesis we have the introduction and definition of the basic terms. The literature review, statement of the research problem, objectives and the justification of the study are included.

In chapter two the outlines of the governing equations representing the conservation law of mass, momentum and energy – which are fundamental to the analysis of fluid flow phenomenon, are given. The problem is defined and the equations are non-dimensionalised. In this chapter, the non dimensional numbers are identified and mathematical formulations are outlined. Since the equations that arise are non – linear, the finite difference method is applied to compute the velocity and temperature profiles in accordance to the defined boundary conditions. Computer program, in this case JAVA is used to analyze the final set of the governing equations expressed in finite difference form with imposed boundary conditions.

In chapter three, the equations governing the flow are reorganized in finite difference form and the boundary conditions are given. The governing equations are then solved by using computer program. The various characteristic quantities used in solving the governing equations are described. The obtained results are then presented in graphical form, followed by discussion pertaining to velocity and temperature profiles; and the effects of heat generated within the boundary layer of an immersed curved surface.

In chapter four, the conclusion concerning the study and recommendations are summarized. At the end of the chapter, the list of references used in carrying out this study is arranged in alphabetical order.

CHAPTER TWO

2.10 OVERVIEW

In this chapter, mathematical formulation of equations governing a steady two-dimensional flow for an incompressible Newtonian fluid past an immersed curved surface is done. The fundamental equations to be considered in this research include the conservation equation of mass, conservation equation of momentum and equation of energy. Then a flow configuration of the fluid flow under consideration is described.

Later in the chapter, equations governing the fluid flow are given in their general dimensional forms. The non- dimensional parameters are then defined by using superscript star (*) in section 2.15; which are finally written in a finite difference form.

2.11 ASSUMPTIONS AND APPROXIMATIONS

To achieve the pre-stated objectives and to work with simplified forms of equations, the following assumptions are made: -

1 The fluid have negligible body forces (X = Y = 0). X and Yare body forces (such as the gravitational force, centrifugal force, magnetic and electric fields) along the x- and y- directions respectively.

2 Fluid is Newtonian.

3 Fluid is assumed incompressible.

4 The fluid is assumed to have constant thermal conductivity.

5 The fluid flow velocities are small compared to that of light ; i.e., $\frac{q^2}{c^2} \leq 1$

The boundary layer approximations are: -

1 $u \gg v$; Velocity component along surface is much larger than that normal to the surface.

2 $\frac{\partial u}{\partial y} \gg \frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial v}{\partial x}$; the gradients normal to the surface are much larger than those along the surface.

2.12 GOVERNING EQUATIONS

Essential equations of fluid dynamics are based on the following general laws of conservation: conservation of mass, momentum and energy

2.12.1 Equation of Continuity

This equation combines the law of mass conservation and that of the transport theorem; which is a Mathematical expression that provides a way of identifying a finite system (such as a control volume through which rate of change of any property or characteristics of most system is examined).

The equation arises from the fundamental prepositions that under normal conditions matter may neither be created nor destroyed and that the flow is continuous. Assuming unsteady flow condition, the general equation of continuity is given by

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{q}) = 0 \,, \tag{2.1}$$

where ρ is the fluid density, $\vec{q}$ is the fluid velocity and $\vec{\nabla}$ the gradient operator.

Since the time rate of change of density in the system goes to zero over time in steady incompressible flow, equation (2.1) reduces to:

$$\vec{\nabla} \cdot \vec{q} = 0 , \qquad (2.2a)$$

where $\vec{q} = u\vec{i} + v\vec{j}$ in two-dimensions.

or $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2.2b)$$

which is the equation of continuity for single species fluid in the velocity boundary layer in two dimension.

2.12.2 Equation of Conservation of Momentum

The equation of conservation of momentum arises when considering both the transport theorem and Newton's second Law of motion. In considering a control volume, Newton's second Law of motion states that the sum of all forces acting on the control volume must be equal to the net rate at which momentum leaves the control volume. For instance, the difference in the rate of momentum of outflow and inflow, i.e.

(Out flow – inflow) = [sum of all forces (Body forces X, Y plus surface forces F_s).

There are two kinds of forces that may act on the fluid; namely body forces in the x – and y – directions respectively (for example, gravitational, centrifugal, magnetic and/or electric fields) and surface forces (forces due to static pressure and viscous stresses). The viscous stresses at any point in velocity boundary layer may be resolved into two components – the normal stress (tensile stress), σ_{ii}, which vanishes at zero velocity gradient and shear stress, τ_{ij} . Viscous stress is a natural consequence of fluid motion and viscosity.

The momentum equation along x – axis becomes;

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\left(\frac{\partial \sigma_{xx}}{\partial x}+\frac{\partial \tau_{yx}}{\partial y}\right)+X \tag{2.3}$$

Along y-direction, the momentum equation becomes;

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\left(\frac{\partial \sigma_{yy}}{\partial y}+\frac{\partial \tau_{xy}}{\partial x}\right)+Y \tag{2.4}$$

The viscous stresses and shear stresses in two dimensions are defined by;

$$\sigma_{xx}=2\mu\frac{\partial u}{\partial x}-\frac{2}{3}\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) \tag{2.5a}$$

$$\sigma_{yy}=2\mu\frac{\partial v}{\partial y}-\frac{2}{3}\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) \tag{2.5b}$$

$$\tau_{xy}=\tau_{yx}=\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) \tag{2.5c}$$

Substituting equations (2.5a-c) into equation (2.3) and (2.4) we obtain momentum equation along the x- and y-directions respectively as

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left\{\mu\left[2\frac{\partial u}{\partial x}-\frac{2}{3}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right]\right\}+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+X \tag{2.6a}$$

and

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\frac{\partial}{\partial y}\left\{\mu\left[2\frac{\partial v}{\partial y}-\frac{2}{3}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right]\right\}+\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+Y \tag{2.6b}$$

In this study negligible body forces are assumed; and substituting (2.2b) into (2.6a) and (2.6b) we get

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+2\mu\frac{\partial^2 u}{\partial x^2}+\mu\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 v}{\partial x\partial y}\right) \tag{2.7a}$$

and

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+2\mu\frac{\partial^2 v}{\partial y^2}+\mu\left(\frac{\partial^2 u}{\partial y\partial x}+\frac{\partial^2 v}{\partial y^2}\right) \tag{2.7b}$$

Considering very small boundary layer thickness, δ to the extent that the velocity component in direction along the surface is much larger than that normal to the surface, (for instance $u \gg v$) and gradients normal to the surface being much larger than those along the surface ($\frac{\partial u}{\partial y} >> \frac{\partial u}{\partial x},\frac{\partial v}{\partial y},\frac{\partial v}{\partial x}$), then equation (2.7a) reduces to

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2} \tag{2.8}$$

and equation (2.7b) reduces to

$$\frac{\partial p}{\partial y}=0 \tag{2.9}$$

The pressure doesn't vary in the direction normal to the surface, so pressure in the boundary layer depends only on x and equals to that of the freestream outside the boundary layer; which depends on surface geometry therefore we have,

$$\frac{\partial p}{\partial x}=\frac{dp}{dx}\ , \tag{2.10}$$

which is obtained by applying Bernoulli equation

$$p + \frac{1}{2}\rho U^2 = constant \tag{2.11}$$

to the streamline at the wall with known potential flow.

In this particular case, we consider the accelerated and decelerated flows along the curved surface producing both favorable and adverse pressure gradient whose tangential component of the velocity of the outer flow reveals a power law dependence on streamwise distance x measured along the curved surface boundary as

$$U = cx^m , \tag{2.12}$$

where c is a positive velocity coefficient and m an integer obtained from the angle of inclination, α radians at any given point along the curved surface from a horizontal surface such that $m = \alpha/(2-\alpha)$. The fluid velocity gradient along the curved surface increases when m > 0, and decreases for m < 0.

Substituting equations (2.10) – (2.12) into equation (2.8), the boundary layer momentum equation reduces to

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = P_t + \nu\frac{\partial^2 u}{\partial y^2}, \tag{2.13}$$

where the pressure term in dimensional form is defined by

$$P_t = -\frac{1}{\rho}\frac{dp}{dx} \equiv c^2 m x^{2m-1}.$$

2.12.3 Equation of Conservation of Thermal Energy.

This equation is derived from the First Law of Thermodynamics that asserts the mutual equivalence between heat and mechanical work, which is stated as the amount of heat added to a system dQ equals the change in the internal energy dE plus the work done dW. In mathematical expression becomes

$$dQ = dE + dW = dE + pd\left(\frac{1}{\rho}\right) = dE + pdV \tag{2.14}$$

In most standard Fluid Mechanics text books this equation is written as follows:-

$$\rho\frac{Dh}{Dx} + h\left(\frac{D\rho}{Dx} + \rho\vec{\nabla}\cdot\mathbf{q}\right) = -\vec{\nabla}\cdot\mathbf{Q}'' + Q''' - P\vec{\nabla}\cdot\mathbf{q} + \mu\phi \tag{2.15}$$

Since ρ is assumed constant and from Equation of Continuity, $\vec{\nabla}\cdot\mathbf{q} = 0$ then (2.15) reduces to;

$$\rho\frac{Dh}{Dx} = -\vec{\nabla}\cdot\mathbf{Q}'' + Q''' + \mu\phi \quad , \tag{2.16}$$

where h is the enthalpy per unit mass of the mixture, ϕ is the dissipation function, Q_x'' and Q_y'' are the heat flux in the x- and y-directions, respectively. Q''' is the internal heat generated.

By considering steady incompressible flow in a control volume, the standard thermal energy equation for thermal boundary layer is given by

$$\rho u\frac{\partial h}{\partial x} + \rho v\frac{\partial h}{\partial y} = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y}\right) + \mu\emptyset + \dot{Q}, \tag{2.17}$$

where

$\mu\emptyset$ is the viscous dissipation, which accounts for the rate at which mechanical energy is converted to thermal energy due to viscous effects in the fluid.

$\dot{Q}$ is the rate at which energy is generated per unit volume.

Again, $h = E + p\left(\frac{1}{\rho}\right)$ (2.18)

which is the enthalpy per unit mass of species, and in differential form becomes

$$dh = dE + \frac{1}{\rho}dp + pd\left(\frac{1}{\rho}\right) \qquad (2.19)$$

By use of substantial or material derivative then

$$\frac{Dh}{Dx} = \frac{DE}{Dx} + \frac{1}{\rho}\frac{Dp}{Dx} - \frac{p}{\rho^2}\frac{D\rho}{Dx} \qquad (2.20)$$

From Maxwell's thermodynamic relations and the equation $dQ = TdS$, and from equation (2.14), we find

$$dE = TdS - pd\left(\frac{1}{\rho}\right). \qquad (2.21)$$

Substituting (2.21) into (2.19) and simplifying, we get

$$dh = TdS + \frac{1}{\rho}dp \qquad (2.22)$$

From Newton's Law of cooling, the local heat flux is defined by

$$q_s^{''} = \overset{*}{h}(T_s - T_\infty), \qquad (2.23)$$

where $\overset{*}{h}$ is the local convection coefficient.

Since flow conditions vary from one point to another on the curved surface, both q_s'' and $\overset{*}{h}$ also vary along the surface.

At any distance x from the leading edge of the curved body, local heat flux q_s'' is obtained by applying Fourier's Law to the fluid at $y = 0$ as

$$q_s'' = -k\frac{\partial T}{\partial y}\bigg|_{y=0} \qquad (2.24)$$

since by no slip-condition, the energy transfer is by conduction only.

The local convection heat coefficient is then expressed as

$$\overset{*}{h} = \frac{-k\frac{\partial T}{\partial y}\big|_{y=0}}{(T_s - T_{\infty})} \qquad (2.25)$$

For Newtonian fluid, the shear stress, τ_s is expressed as

$$\tau_s = \mu\frac{\partial u}{\partial y}\bigg|_{y=o} \qquad (2.26)$$

,

where μ is dynamic viscosity; and the local friction coefficient, C_f for external flow is given by $$C_f \equiv \frac{\tau_s}{\frac{1}{2}\rho U_{\infty}^2}$$

(2.27)

From equation (2.17), ϕ (for incompressible two dimensional flow) is defined by

$$\emptyset = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 \qquad (2.28).$$

The term $\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2$ in this equation originates from viscous shear stresses and $2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right]$ arises from viscous normal stresses, which by this flow problem is assumed negligible.

In the thermal boundary layer, the rate of heat conduction along the y – direction is larger than that along x – direction$\left(i.e.\ \frac{\partial T}{\partial y} \gg \frac{\partial T}{\partial x}\right)$. Further still, by assuming that $u\frac{\partial p}{\partial x}\ and\ v\frac{\partial p}{\partial y}$ are very negligible; and that the nature of substance is as of a perfect gas $(i.e. dh = c_p dT)$

whose specific enthalpy S = S(p, T) ; a function of pressure and temperature such that

$$dS = \left(\frac{\partial S}{\partial T}\right)_p dT + \left(\frac{\partial S}{\partial p}\right)_T dp. \tag{2.29}$$

Then the First Law of Thermodynamics, equation (2.17) reduces to

$$c_p \rho \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \right) = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \dot{Q}$$

or

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial^2 y} + \frac{v}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \ddot{Q} \tag{2.30}$$

where the term $\frac{v}{c_p}\left(\frac{\partial u}{\partial y}\right)^2$ is the viscous dissipation and the term $\ddot{Q} = \frac{Q}{c_p \rho}$ is the internal energy generated.

The term $\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)$ on the left hand side accounts for advection whereas $\alpha\frac{\partial^2 T}{\partial y^2}$ accounts for conduction.

Since the flow speed is neither sonic nor high speed motion of lubricating oils, for further simplicity, the viscous dissipation term may be ignored; thus the temperature – formulation of the First Law of Thermodynamics is expressed as

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial^2 y} + \ddot{Q} \tag{2.31}$$

2.13 DESCRIPTION OF THE FLOW

In this study, an external two – dimensional steady incompressible Newtonian fluid flowing past a submerged curved surface is considered. The immersed curved surface provide both concave and convex surfaces on its opposite sides.

In considering fluid flow over the convex surface, at point A (***Fig: 2***) the freestream fluid is brought to rest at the forward stagnation point; accompanied by rise in pressure. With points along the increasing streamline, pressure decreases and the boundary layer develops under the influence of a favourable pressure gradient ($\frac{\partial p}{\partial x} < 0$). Pressure eventually reaches a minimum at B. From B onwards, further boundary layer develops in the presence of an adverse pressure gradient ($\frac{\partial p}{\partial x} > 0$).Here upstream velocity U and freestream velocity U∞(x) are different since U∞(x) depends on distance x from stagnation point. U∞(x) = 0 at stagnation point and fluid accelerates due to favourable pressure gradient ($\frac{\partial U_\infty}{\partial x} > 0 \ when \frac{\partial p}{\partial x} < 0$); and it reaches maximum when $\frac{\partial p}{\partial x} = 0$. The fluid then decelerates due to adverse pressure gradient ($\frac{\partial U_\infty}{\partial x} < 0 \ when \frac{\partial p}{\partial x} > 0$). The velocity gradient at the surface $\left.\frac{\partial u}{\partial x}\right|_{y=0}$ eventually becomes zero at separation point (point where fluid near surface lacks sufficient momentum to overcome the pressure gradient and further downstream movement becomes impossible). Since oncoming fluid also inhibits flow back upstream, boundary layer separation must occur (i.e. the boundary layer detaches from the surface and a wake is formed in the downstream region. At this region, flow is characterized by vortex formation and is highly turbulent – in which case much energy is dissipated as heat.

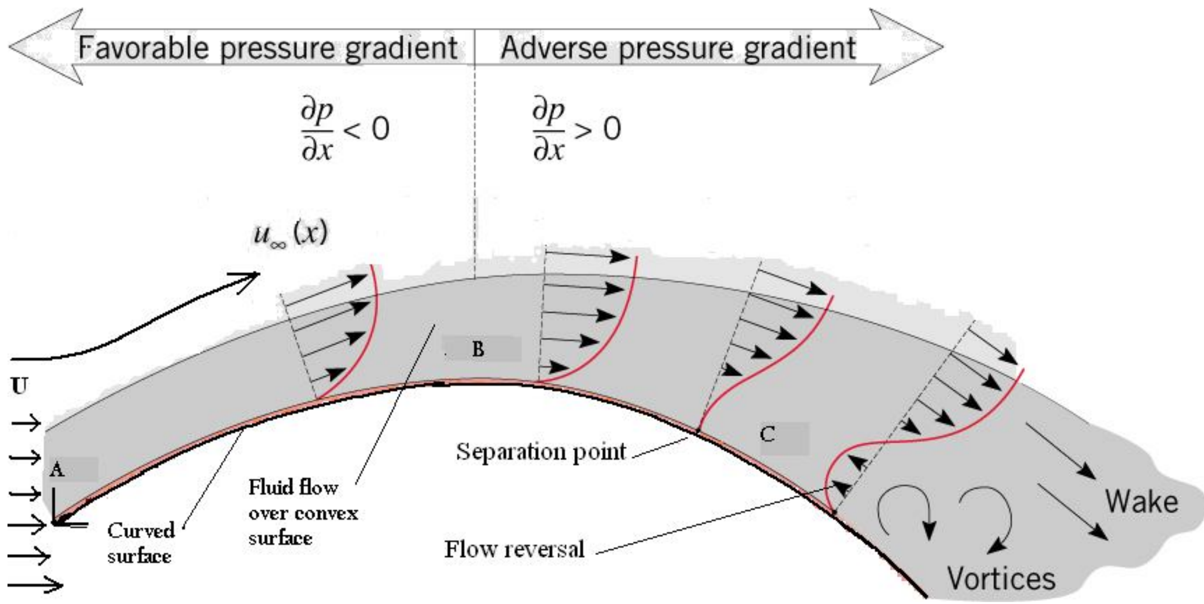


Fig. 2: ***Schematic diagram of the flow.***

The separation boundary layer tends to curl up in the reverse flow and the region of disturbed fluid extends for some distance downstream. Since energy eddies are dissipated as heat, pressure downstream remains approximately the same as at the separation point.

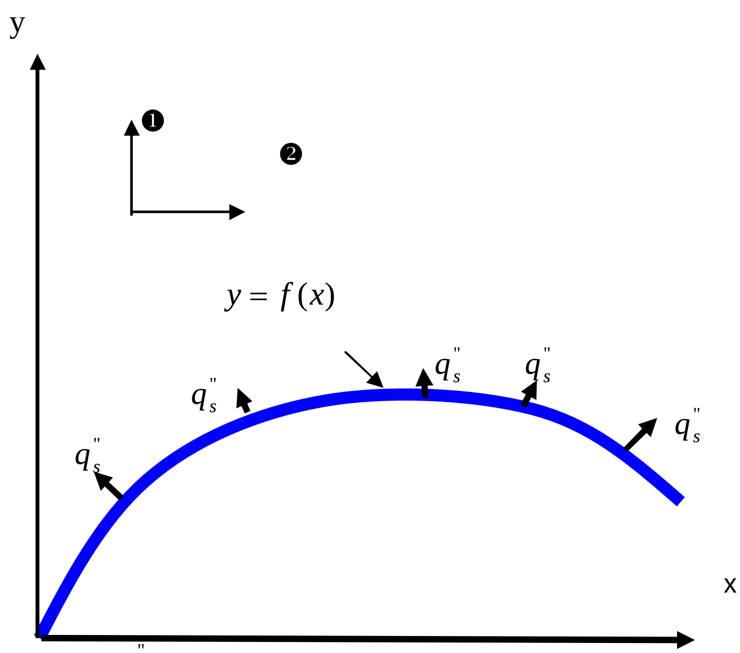
In such situation pressure acting at A is in excess of that around C resulting on a resultant force on the curved surface in direction relative to fluid motion, called pressure drag. The other force on the surface in direction relative to the fluid motion is the fluid shear stress at the surface called profile drag. Evident still is the induced drag due to the dissipation of the thin turbulent wake (for the streamlined body with finite span). The sum of profile drag and induced drag give the total fluid drag on the curved surface.

The point of boundary layer transition owes its determination from Reynolds number. It determines if the flow is laminar, transitional or turbulent.

2.14 MATHEMATICAL FORMULATION

In this study, a viscous incompressible flow past a curved surface submerged in a fluid is considered. A reasonable approximation for flow is done on a slightly contoured concave down surface.

The entire curved surface is defined by a trigonometric function of the form $y = a \sin x$ where $0 < a \leq 1$ and $0 \leq x < \pi$ for a reasonably curved surface. The end points are set at specific coordinate values when solving for a particular case upon which length of curvature is analytically obtained.



NB: q_s'' - the local heat flux.

Fig. 3: *Schematic diagram of a finite concave downward surface.*

The set boundary conditions are:-

On the plate's surface, the non- slip condition gives velocity as: $u(x,y) = v(x,y) = 0$.

When $y \geq \delta$ $then$ $u = u_\infty$

At the leading edge; $u(x_0, y_0) = v\ (x_0, y_0) = 0$; and at the trailing edge; $u(x_1, y_1) = u_\infty$, and $v(x_1, y_1) = 0$. The fluid satisfies the non-slip condition on the wall.

Temperature at the leading edge; $T(x_0, y_0) = T_\infty$, and at the trailing edge $T(x_1, y_1) = T_s$

2.15 NON-DIMENSIONALISATION OF THE EQUATIONS

In this study, all the dimensionless independent variables are expressed by using the superscript star (*)

In non-dimensionalization of the equations (2.2b), (2.13), 2.31), we first select certain characteristic quantities L, V, P and T to denote the characteristic length, velocity, pressure and temperature respectively.

Again, to normalize the boundary layer equations, the dimensionless independent variables are defined as;

$$x = x^* L, \quad y = y^* L, \quad u = u^* V, \quad v = v^* V, \quad p = p^* P, \quad T^* = \frac{(T - T_S)}{(T_\infty - T_S)},$$

On substituting these variables into the boundary layer governing equations we have the Continuity equation as

$$\frac{\partial (u^* V)}{\partial (L x^*)} + \frac{\partial (v^* V)}{\partial (y^* L)} = 0$$, which simplifies to

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \qquad (2.32)$$

The momentum equation (2.13) becomes

$$\frac{V^2}{L}u^*\frac{\partial u^*}{\partial x^*}+\frac{V^2}{L}v^*\frac{\partial u^*}{\partial y^*}=-\frac{P}{\rho L}\frac{dp^*}{dx^*}+\frac{\upsilon V}{L^2}\frac{\partial^2 u*}{\partial y^{*2}}$$, and by multiplying both sides by $\frac{L}{V^2}$ we have

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=P_T+\frac{\upsilon}{LV}\frac{\partial^2 u*}{\partial y^{*2}} \quad , \qquad (2.33)$$

where the pressure term P_T is given by $-\frac{p}{\rho V^2}\frac{dp^*}{dx^*}=Euc^2mx^{2m-1}$

The energy equation (2.31) becomes

$$u^*V\frac{\partial[(T_\infty-T_s)T^*+T_s]}{\partial(x^*L)}+v^*V\frac{\partial[(T_\infty-T_s)T^*+T_s]}{\partial(y^*L)}=\alpha\frac{\partial^2[(T_\infty-T_s)T^*+T_s]}{\partial(y^*L)^2}\ddot{Q}$$

Dividing through by $\frac{L}{V(T_\infty-T_s)}$ simplifies to

$$u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}=\frac{\alpha}{LV}\frac{\partial^2 T^*}{\partial y^{*2}}+\frac{L}{V(T_\infty-T_s)}\ddot{Q}$$

or

$$u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}=\frac{\alpha}{LV}\frac{\partial^2 T^*}{\partial y^{*2}}+\dddot{Q}\ , \qquad (2.34)$$

where $\dddot{Q}$ is the generated heat term defined by $\frac{L}{V(T_\infty-T_s)}\ddot{Q}$

2.16 NON-DIMENSIONAL NUMBERS

To be able to apply the results obtained in equations (2.32) – (2.34) to any other dynamically similar case, the following dimensionless parameters or numbers are considered:

The Prandtl Number, Pr

This number is named after Ludwig Prandtl-the German scientist who introduced the concept of boundary-layer theory. It is the parameter, which relates the relative thickness of the hydrodynamic and thermal boundary layers. The Prandtl number provide the link between the velocity field and the temperature field and it is expressed as

$$P_r = \frac{\upsilon}{\alpha} = \frac{\mu/\rho}{k/\rho c_P} = \frac{c_P \mu}{k}$$

That is, it is the ratio of momentum diffusivity to thermal diffusivity.

Reynolds number, Re

This is denoted by Re and is defined as the ratio of inertia forces (forces that there is no acceleration of a body $= \rho V^2$) to viscous forces (forces restricting motion of objects in fluids $= \frac{\mu V}{L}$). It is given by

$$\text{Re} = \frac{\rho VL}{\mu} = \frac{VL}{\upsilon}$$

Nusselt Number, Nu

This provides a dimensionless temperature gradient at the surface and it is given by

$$\text{Nu} \equiv \left(\frac{hL}{k}\right) = \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}$$

Eckert number, E_c

This is the measure of kinetic energy of the flow relative to the boundary layer enthalpy difference across thermal boundary layer, given by;

$$E_c = \frac{V^2}{C_p(T_s - T_\infty)}$$

It plays an important role in high speed flows for which dissipation is significant.

Peclet number, P_e

This is a dimensionless independent heat transfer parameter defined by

$$P_e = RePr \equiv \frac{VL}{\alpha}$$

Coefficient of friction, C_f.

This is a dimensionless surface shear stress, written as,

$$C_{f.} = \frac{\tau_s}{\rho V^2/2}$$

Skin friction coefficient, τ_s

This is given by;

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0}$$

2.17 FINAL SET OF EQUATIONS IN NON DIMENSIONAL FORM

Substituting dimensionless parameters in equations (2.33) – (2.36), the equations to be solved are

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \quad (2.35)$$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = PX + \frac{1}{Re} \frac{\partial^2 u^*}{\partial y^{*2}} \quad (2.36)$$

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Pe} \frac{\partial^2 T^*}{\partial y^{*2}} + \ddot{Q} \quad (2.37)$$

where PX is the pressure term, which is dependent on the x variable.

The boundary conditions in non dimensional form are:

The velocity at the leading edge $u^*(x_0^*, y_0^*) = 0$; $v^*(x_0^*, y_0^*) = 0$. At the trailing edge we have $u^*(x_1^*, y_1^*) = 1$; $v^*(x_1^*, y_1^*) = 0$.

At the wall, $u^*(x^*, 0) = 0$; $v^*(x^*, 0) = 0$ and at the free stream $u^*(x^*, \infty) = \frac{U_\infty(x^*)}{U}$

Temperature at the leading edge $T^*(x_0^*, y_0^*) = 1$; and at the trailing edge $T^*(x_1^*, y_1^*) = 0$.

At the wall $T^*(x^*, 0) = 0$ and at the free stream $T^*(x^*, \infty) = 1$

2.18 METHOD OF SOLUTION

The numerical analogue to solve the governing equations (2.35) to (2.37) is developed by discretization process, and the partial differential equations describing the flow are programmed for computer solution.

The method of numerical discretization chosen is the finite difference method, in which the value of the function is calculated at a large number of points on a structured

mesh with uniform spacing in Cartesian coordinates spanning the region of interest as shown below.

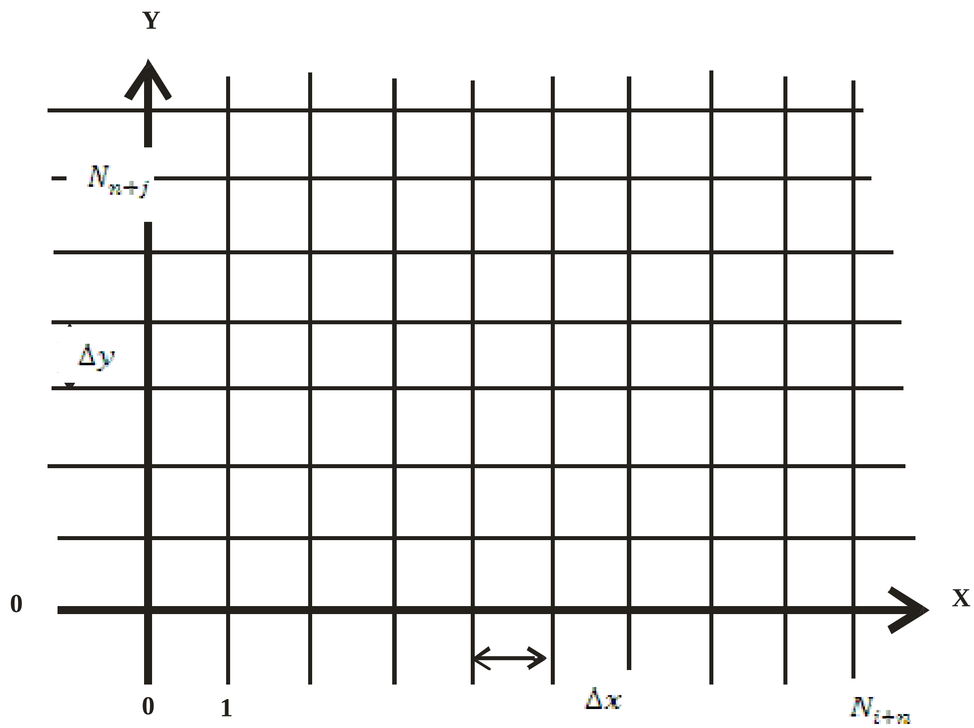


Fig. 4*: Definition of the Mesh*

The advantage of finite difference method, amongst many others is based on its stability, convergence and consistency.

In order to approximate the equations (2.35) to **(**2.37**)** by a set of finite difference equations, we first define a suitable mesh.

Let the x – y plane be divided into a network of uniform rectangular cells of width Δx and height Δy by means of the set of lines $x = i\Delta x \, ; i = 0,1,2,3, \ldots$ and $y = i\Delta y \; ; \; i = 0, 1, 2, 3, \ldots$.. respectively. Each corner of the cell forms the mesh or grid point, which is identified by a double index *i*, *j* defining its location with respect to x and y.

Defining a particular grid point variable as $\zeta(i,j) \equiv \zeta(i\Delta x, j\Delta y) = \zeta(x,y)$, both the previous and next grid points along the x-axis are defined as $\zeta(i\Delta x - 1, j\Delta y) \equiv \zeta(i-1,j) \quad and \quad \zeta(i\Delta x + 1, j\Delta y) \equiv \zeta(i+1,j)$ respectively. In a similar manner, the previous and the next grid points along the y-axis are $\zeta(i\Delta x, j\Delta y - 1) \equiv \zeta(i, j-1) \;\; and \;\; \zeta(i\Delta x, j\Delta y + 1) \equiv \zeta(i, j+1)$ respectively.

By using Taylor's series expansion we have

$$\left.\begin{aligned} \zeta(i+1,j) &= \zeta(i,j) + \Delta x \zeta'(i,j) + \frac{(\Delta x)^2}{2!}\zeta''(i,j) + \frac{(\Delta x)^3}{3!}\zeta'''(i,j) + \ldots \\ \zeta(i-1,j) &= \zeta(i,j) - \Delta x \zeta'(i,j) + \frac{(\Delta x)^2}{2!}\zeta''(i,j) - \frac{(\Delta x)^3}{3!} + \zeta'''(i,j) \ldots \end{aligned}\right\} \qquad (2.38)$$

On eliminating $\zeta''(i,j)$ from equation (2.41) we obtain the finite difference approximation to the first order partial derivative as

$$\zeta'(i,j) = \frac{\zeta(i+1,j) - \zeta(i-1,j)}{2\Delta x} + O(\Delta x) \qquad (2.39)$$

Similarly, by eliminating $\zeta'(i,j)$ from equation (2.38) we obtain the finite difference approximation to the second order partial derivative as

$$\zeta''(i,j) = \frac{\zeta(i+1,j) - 2\zeta(i,j) + \zeta(i-1,j)}{(\Delta x)^2} + O(\Delta x^2). \qquad (2.40)$$

Errors associated with higher order terms in equations (2.39) and (2.40) can be reduced by choosing Δx as small as possible, meaning the successive terms of the Taylor' series approximation are progressively growing smaller.

The central difference formulae that provides faster convergence for the first and second order partial differential equations with respect to y- axis are similarly expressed as

$$\left.\zeta'(i,j) = \frac{\zeta(i,j+1) - \zeta(i,j-1)}{2\Delta y} + O(\Delta y) \qquad \right\} \qquad (2.41)$$

$$\zeta''(i,j) = \frac{\zeta(i,j+1) - 2\zeta(i,j) + \zeta(i,j-1)}{(\Delta y)^2} + O(\Delta y^2)$$

The numerical difference formulae are always classified by geometrical relationship of the points (i.e. central, forward or backward differencing) or by the accuracy of the expression. The Central difference is second- order accurate whereas both the Forward and Backward differences are first - order accurate as higher order terms are neglected. The forward difference form of equations (2.39), (2.40) and (2.41) are given by

$$\left.\begin{aligned} \zeta'(i,j) &= \frac{\zeta(i+1,j) - \zeta(i,j)}{\Delta x} + O(\Delta x) \\ \zeta''(i,j) &= \frac{\zeta(i+1,j) - 2\zeta(i,j) + \zeta(i-1,j)}{(\Delta x)^2} + O(\Delta x^2) \\ \zeta'(i,j) &= \frac{\zeta(i,j+1) - \zeta(i,j)}{\Delta y} + O(\Delta y) \\ \zeta''(i,j) &= \frac{\zeta(i,j+1) - 2\zeta(i,j) + \zeta(i,j-1)}{(\Delta y)^2} + O(\Delta y^2) \end{aligned}\right\} \qquad (2.42)$$

Both types of the finite difference formulae are used because the equations governing the flow contain terms of unequal importance, so higher order approximation is used for significant terms such as the viscous dissipation and conduction terms. The one-sided difference (forward difference) is required for approximating the derivatives at the boundaries.

Using formulae (2.39) to (2.42) and by denoting u^*, v^*, T^*, by $u_{i,j}$, $v_{i,j}$, $T_{i,j}$, respectively, the finite difference analogues of the partial differential equations governing the flow in consideration are

$$\frac{u_{i+1,j} - u_{i,j}}{\Delta x} + \frac{v_{i,j+1} - v_{i,j}}{\Delta y} = 0. \qquad (2.43)$$

$$u_{i,j}\left\{\frac{u_{i+1,j} - u_{i,j}}{\Delta x}\right\} + v_{i,j}\left\{\frac{u_{i+1,j} - u_{i,j-1}}{2(\Delta y)}\right\} = PX_i + \frac{1}{Re}\left\{\frac{u_{i+1,j} - 2u_{i,j} + u_{i,j-1}}{(\Delta y)^2}\right\} \qquad (2.44)$$

$$u_{i,j}\left\{\frac{T_{i+1,j}-T_{i,j}}{\Delta x}\right\}+v_{i,j}\left\{\frac{T_{i+1,j}-T_{i,j-1}}{2(\Delta y)}\right\}=\frac{1}{Pe}\left\{\frac{T_{i+1,j}-2T_{i,j}+T_{i,j-1}}{(\Delta y)^2}\right\}+\dddot{Q} \qquad (2.45)$$

In Chapter three, the numerical differentiation by use of the Newton's interpolation formula is employed to compute the shear stress along the surface within the boundary layer thickness defined by $\delta = 5\sqrt{\frac{L\mu}{\rho U_{\infty}}}$. The finite difference shear stress is expressed as $\tau = \mu\left(\frac{u_{i,j+1}-u_{i,j}}{\Delta y}\right)$ The equations (2.43) to (2.45) governing the flow model expressed in finite difference form are reorganized and then solved using the computer program (JAVA) to obtain the temperature and velocity profiles of the flow within the boundary layer, which are graphically presented and the results of the velocity and temperature are discussed.

CHAPTER THREE

3.10 OVERVIEW

In this chapter the equations governing the flow problem in finite difference form are reorganized, computed by numerical method through iterations and the obtained velocity and temperature profiles within the boundary layer are graphically represented.

3.11 GOVERNING EQUATIONS IN FINITE DIFFERENCE FORM

The governing equations (2.43) to (2.45) describing the steady laminar incompressible flow past an immersed curved surface in finite difference form are reorganized and expressed as follows:

$$v_{i,j} = \frac{\Delta y}{\Delta x}\left(u_{i,j-1} - u_{i+1,j-1}\right) + v_{i,j-1} \tag{2.46}$$

$$u_{i+1,j} = \frac{\Delta x}{2\Delta y}\left\{\begin{matrix}\frac{1}{u_{i,j}}\left[\left(\frac{2}{R_e\Delta y} - v_{i,j}\right)u_{i,j+1} + \left(\frac{2}{R_e\Delta y} + v_{i,j}\right)u_{i,j-1}\right] \\ + \left(\frac{2\Delta y\,(PX_i)}{u_{i,j}} - \frac{4}{R_e\Delta y}\right)\end{matrix}\right\} + u_{i,j} \tag{2.47}$$

$$T_{i+1,j} = \frac{1}{u_{i,j}}\left\{\begin{matrix}\frac{\Delta x}{2\Delta y}\left[\left(\frac{2}{P_e\Delta y} - v_{i,j}\right)T_{i,j+1} + \left(\frac{2}{P_e\Delta y} + v_{i,j}\right)T_{i,j-1}\right] \\ + \left(\frac{P_e(\Delta y)^2 u_{i,j} - 2\Delta x}{P_e(\Delta y)^2}\right)T_{i,j} + \Delta x\ddot{Q}\end{matrix}\right\} \tag{2.48}$$

3.12 The Boundary Conditions

For the stability, consistency and convergence of the finite difference scheme or to ensure that the truncation error count go to zero, the computations are made by using small values of Δx and Δy, and in this case $\Delta x = \frac{\pi}{100}$ rad. along the curvature, $\Delta y = 0.001m$ for $i = 0, 1, 2, 3, \ldots, 100\ and\ j = 1, 2, 3, \ldots,$ 14 since the surface curvature of the immersed body is define by a trigonometric function and the flow under investigation is confined within the thin fluid boundary layer.

Considering the velocity boundary layer; at the leading edge, $v_{0,0} = 0$; $u_{0,0} = u_{0,1} = \cdots, u_{0,\infty} = u_{\infty} \equiv 5\ m/s$. At the surface contour (wall), $u_{i,0} = 0, v_{i,0} = 0$ and at the free stream $u_{i,\infty} = \frac{U_{\infty}}{U}$. The similarity parameter is Re.

For the Thermal boundary layer; at the wall $T_{i,0} = 0$ and at the free stream $T_{i,\infty} = 1$. The similarity parameter is Pe

3.13 DISCUSSION OF THE RESULTS

3.13.1 Profiles.

The system of equations (2.46 – 2.48) subject to the boundary conditions are solved numerically by the finite difference method. In order to determine velocity and temperature distribution within the boundary layer a JAVA program was run for some physical reference values in the governing equations of the flow. They were set or approximated at the following characteristic quantities : 3.13.1 $length\ L =$, 3.13.1 $velocity\ V = 4($,

3.13.1 $air\ density\ \rho = 0.1582\ l$,

3.13.1 $specific\ heat\ at\ constant\ pressure\ c_p = 1.417\ kJ$,

3.13.1 $momentum\ diffusivity\ of\ air\ \vartheta = 468\ X\ 10$,

3.13.1 $dynamic\ viscosity\ \mu = 7.98\ X\ 10^{-4}\ N$, 3.13.1 $wall\ temperature\ T_s =$,

3.13.1 ***free stream temperature*** $T_\infty =$. These are the reference values for which desirable results of the governing equations are obtained. Pr= 0.75 is for air (assumed to be incompressible) and Re = 50000 is within that for laminar flow. The fluid velocity used in the computer program is at speed greater than sonic speed but not at supersonic speed, since at supersonic speed shock waves emerges. The wall temperature is estimated at room temperature. The temperature and velocity variations are then plotted against - either the horizontal distance (x-axis) at grid points along the curved surface or vertical distance (y-axis) at the grid point from the solid surface within the fluid boundary layer.

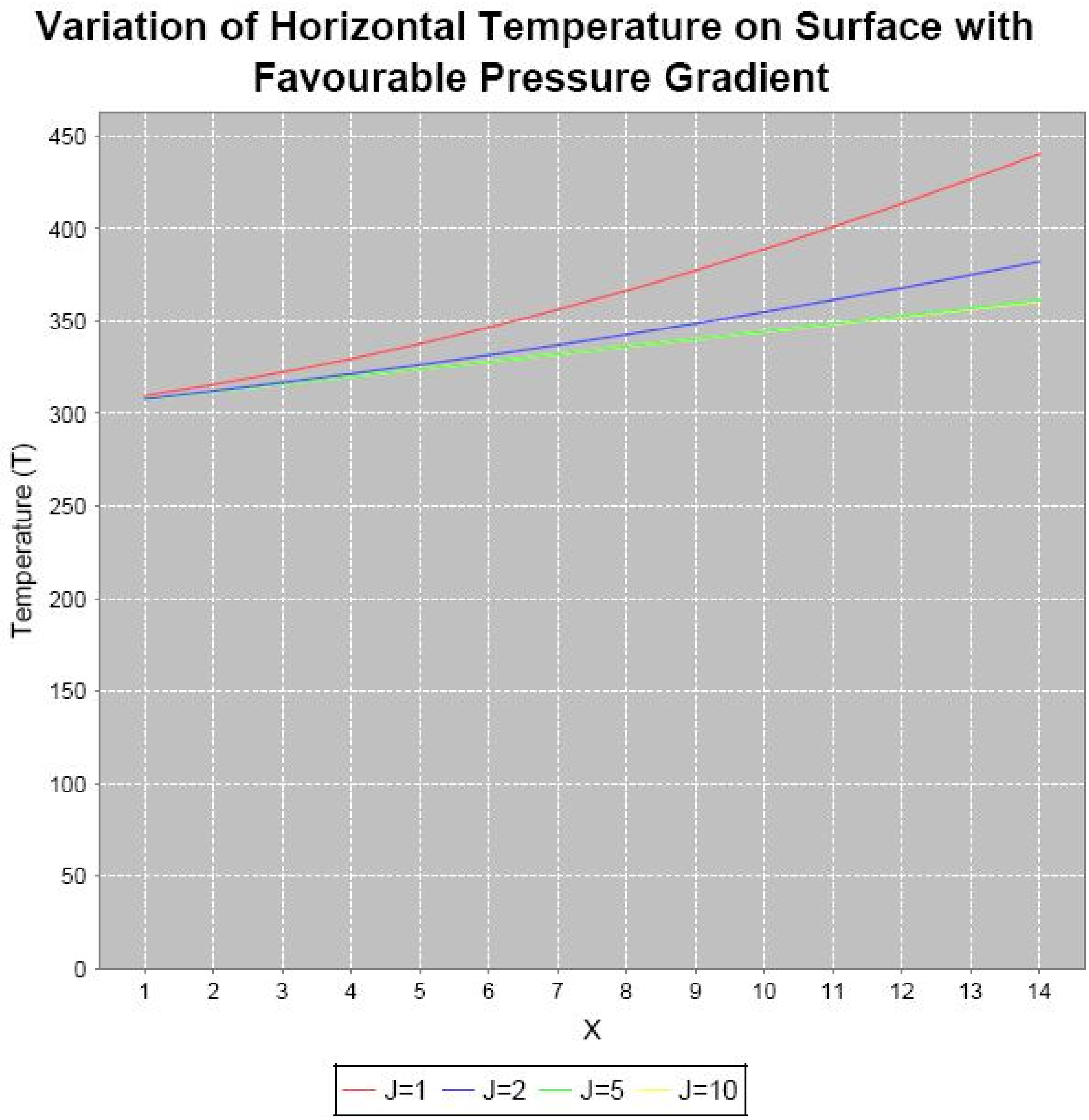
The pressure term in finite difference form is defined as $(PXi) = \left(\frac{L^m mc}{V}\right)^2 x_i^{2m-1}$, where m is an integer $\cong \pm 1, \pm 2, \pm 3$; $x_i = i\Delta x$ and c is a positive velocity coefficient ≅1 or 2 or 3 meant to vary free stream velocity at the leading edge of the curved surface; whereas the generated heat is represented by $\ddot{Q} = \frac{1}{2} E_c \frac{Lc_p \rho}{V}$

The velocity and temperature profiles are obtained by performing iterations for i and j as discrete variables for space interval variables x and y, e.g. x=1 is grid point at a distance equivalent to $\Delta x = \frac{\pi}{100}$ rad. from the leading edge of the immersed solid surface and for j=1 means a distance equivalent to $\Delta y = 0.001m$ from the wall of the solid body.

Both the temperature and velocity profiles are shown by use of different colours for selected points along the x-axis at various grid points along the solid surface.

The results are obtained for the parameters Pr, Re and the pressure term for the steady laminar flow, which are graphically presented below.

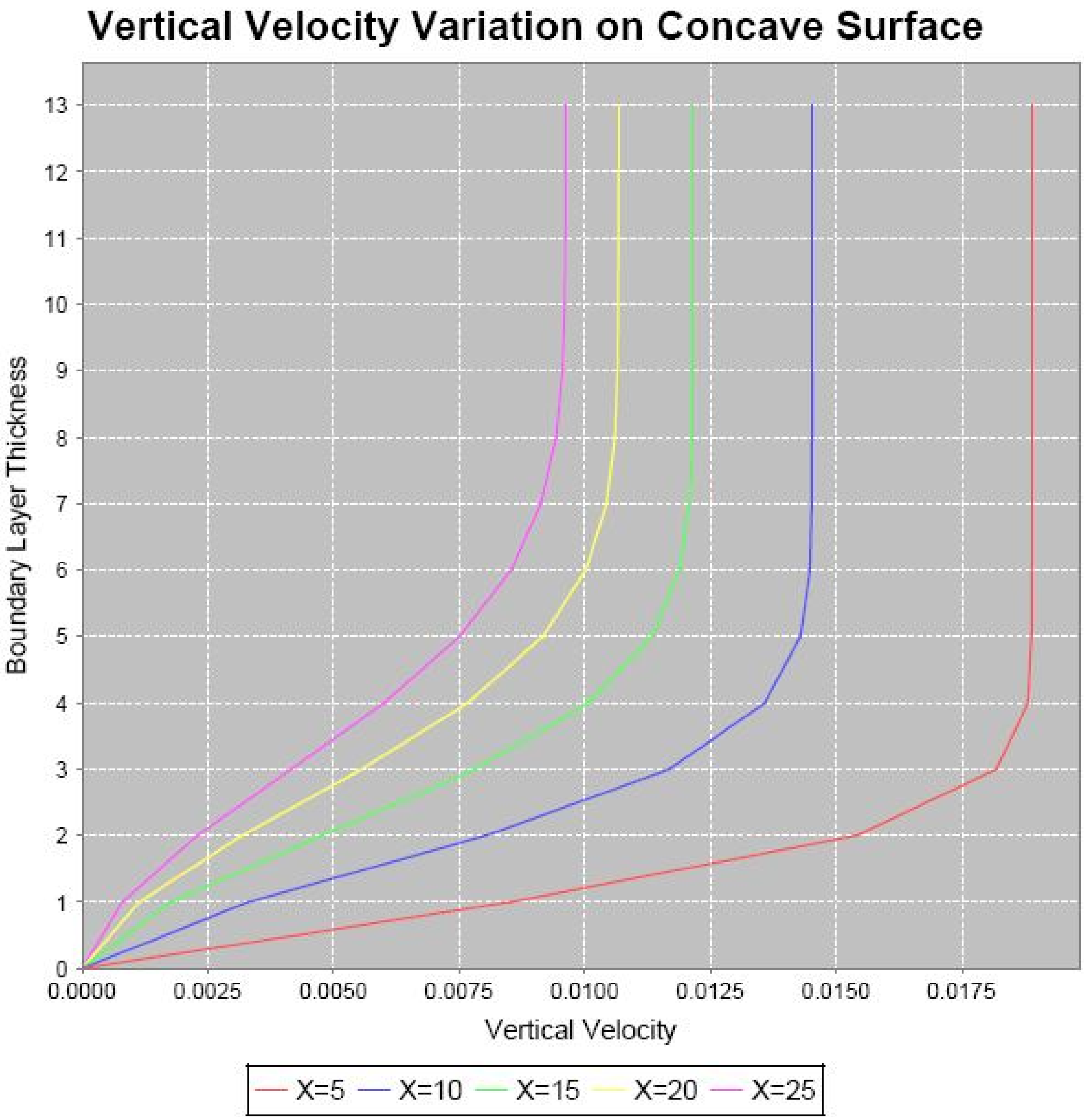
3.13.2 The Obtained Graphs of Fluid Flow



- chosen horizontal planes along the vertical distance from the solid surface

Re = 50000, Pr = 0.75 , Pe = RePr,

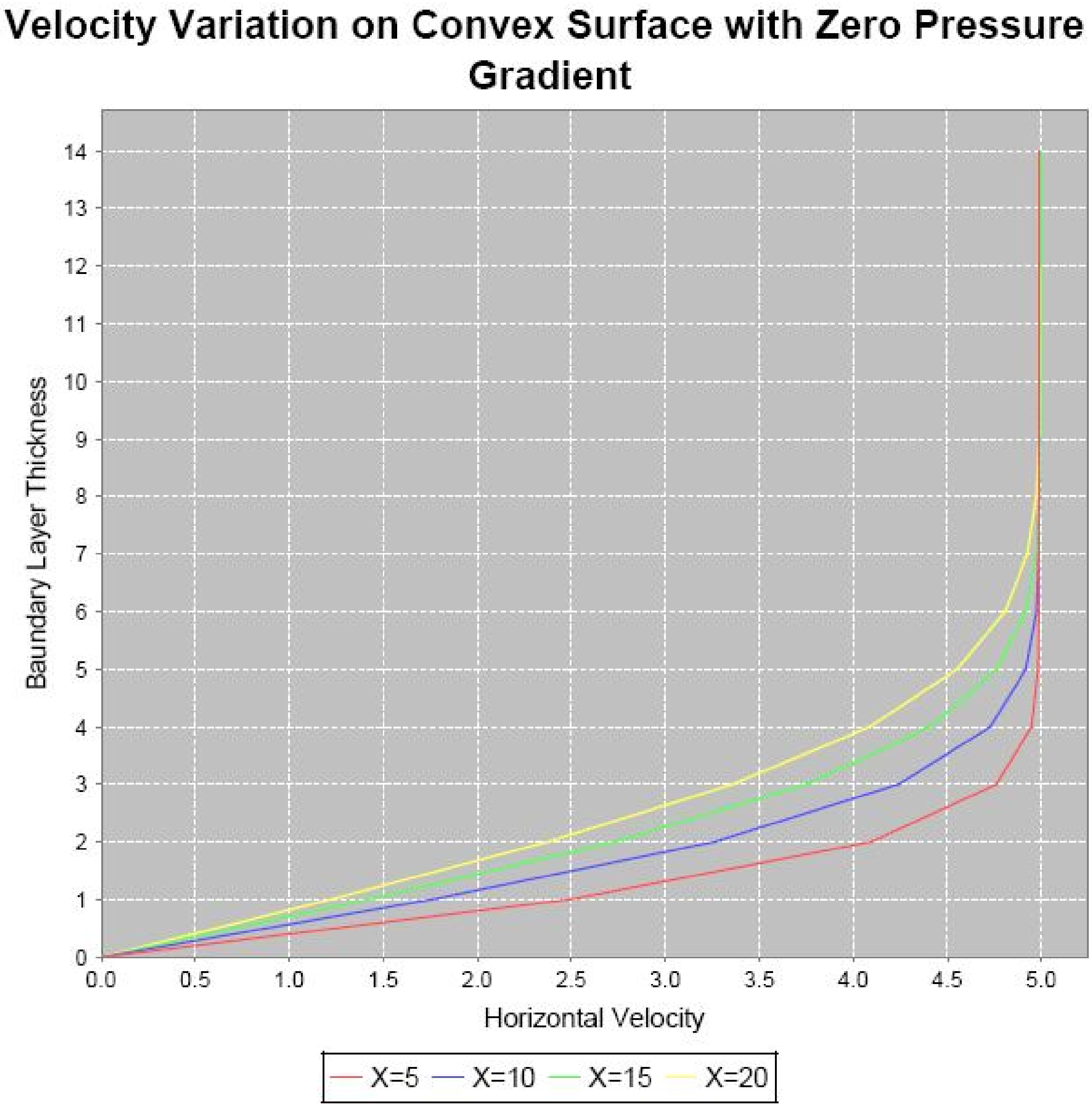
Fig. 5: Horizontal temperature profile on convex surface with favourable pressure gradient



- chosen vertical planes along the horizontal distance of the solid surface

Re = 50000, Pr = 0.75 , Pe = RePr,

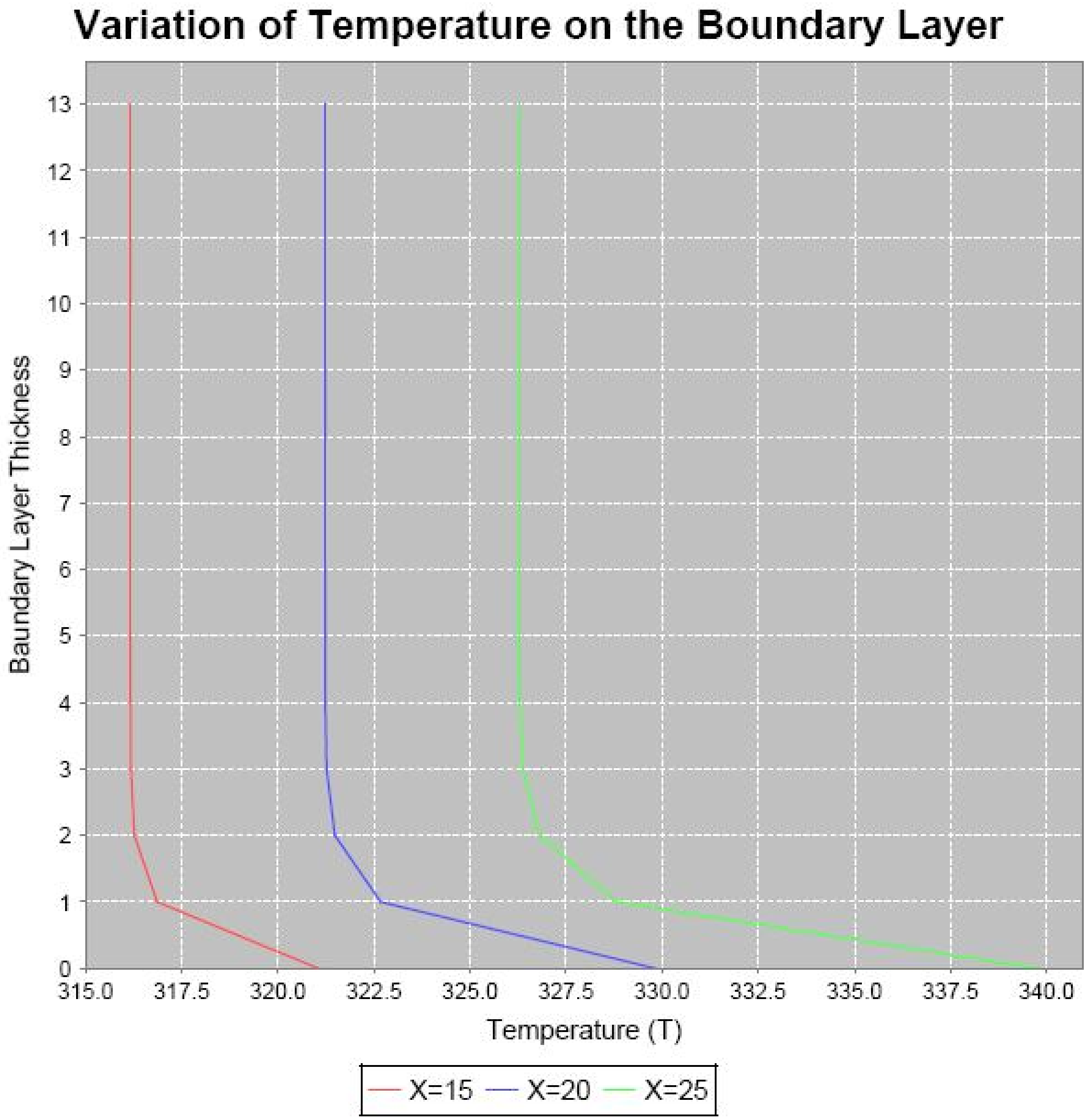
Fig. 6: Vertical velocity profile on concave surface within boundary layer.



- chosen vertical planes along the horizontal distance of the solid surface

Re = 50000, Pr = 0.75 , Pe = RePr,

Fig. 7: Horizontal velocity profile on convex surface with zero pressure gradients.



- chosen vertical planes along the horizontal distance of the solid surface

Re = 50000, Pr = 0.75 , Pe = RePr,

Fig. 8: Horizontal temperature profile within the thermal boundary layer thickness.

3.13.3 Discussions of the Profiles

i. From figure 5, temperature profile varies exponentially at region very close to the solid boundary within the thin boundary layer, though remains relatively linear away from the solid surface with a decreasing temperature gradient on the convex surface with favourable pressure gradient. This indicates that the rate of heat transfer is considerably high at points very close to the convex surface within the boundary layer thickness, consequently a decrease in fluid viscosity. Again this makes the boundary layer to reach separation condition sufficiently early. It then follows that heat generated due to viscous drag on the curved surface has an effect on the lift and drag forces.

ii. From figure 6, some specific points along the x axis are chosen and the variation of the vertical velocity component versus vertical distance within the boundary layer along the convex surface is such that as the gradient of the curvature increases, the vertical velocity component also increases to attain a constant magnitude away from the boundary layer thickness. Consequently the frictional drag on the curved surface is reduced. This effect causes fluid to detach from the surface causing early fluid separation at some distance from the leading edge in comparison to that of a flat surface. The vertical velocity component contributes significantly to transfer of momentum or energy through the boundary layer in which case would increase lift force.

iii. From figure 7, the horizontal velocity component within the boundary layer on convex surface with zero pressure gradients close to the leading edge slightly increases as distance along the surface increases downstream. The horizontal velocity gradient decreases upstream, which is clearly indicative that friction drag is significant at the region close to the surface and should not be ignored.

iv. In figure 8, the temperature profile on the boundary layer on the convex surface with favourable pressure gradient increases symmetrically as fluid flow downstream within the thin boundary layer; and viscous effects of fluid decreases with a decrease in density that occurs when temperature increases, consequently a low pressure fluid flow is experienced on the convex surface.

CHAPTER FOUR

4.10 CONCLUSION

An analysis on the velocity and temperature variation on fluid flow over an immersed curved surface has been done. After several iterative attempts in variation on Reynolds number, it has been observed that optimum values of characteristic quantities are achievable at Re = 50000, Pr = 0.75. The pressure term was evaluated at specific value of m and c; where m is an integer obtained from the angle of inclination, α radians at any given point along the curved surface from a horizontal surface such that $m = \alpha/(2-\alpha)$. The fluid velocity gradient along the curved surface increases when m > 0, and decreases for m < 0. c is the velocity coefficient that controls the outer flow fluid velocity.

Since equations governing the flow in the study are non linear, their solutions were obtained by use of finite difference scheme.

From the discussions on the temperature and velocity profiles within the thin thermal and velocity boundary layers, it is observed from the profiles that viscous effects are dominant at very close region to the curved surface within the boundary layer. At the boundary layer, heat energy generated enables molecules to overcome cohesive forces between them and so move freely causing a decrease in viscosity of the fluid. Due to this temperature rise, a slight decrease in density occurs; this results into increase on lift force, which does not waste energy.

In less dense fluid, fewer molecules are available per unit volume to transfer the motion from the moving layer; this in turn affects the spread of the different layers. As viscosity falls, the momentum is transferred less readily between the layers.

In a nutshell, both lift and drag forces are affected by the heat generated on the thermal boundary layer. Though it reduces viscous drag, it increases the lift on an immersed curved surface.

4.11 RECOMMENDATIONS

In this thesis, the meaningful results on effects of convection heat generated within the boundary layer to both lift and drag were obtained when most of the parameters used were put at certain specific constants. However, it is quite imperative to continue exploring the following areas:

- **i.** Compressible three-dimensional flow over immersed curved surface.
- **ii.** The effect of MHD flow over immersed curved metallic surface.
- **iii.** Experimental research on the effect of heat transfers on lift and drag force on an immersed curved surface.
- **iv.** Further study on effects of heat and mass transfer on turbulent fluid flow over immersed curved surface.

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