INVESTIGATION ON MAGNETOHYDRODYNAMIC FLUID FLOW IN A SLIDER BEARING

MAURINE MARAKA WAFULA

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Investigation on Magnetohydrodynamic fluid flow

in a slider Bearing

Maurine Maraka Wafula

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DECLARATION

This thesis is my original work and has not been presented for a degree in any other University.

Signature:________________________________Date:______________

Maurine Maraka Wafula

This thesis has been submitted for examination with our approval as the university supervisors

Signature:________________________________Date:______________

Prof. Jackson K. Kwanza

JKUAT, Kenya

Signature:________________________________Date:______________

Dr. Jeconia A. Okelo

JKUAT, Kenya

DEDICATION

This thesis is dedicated to my husband Moses, my daughter Melisa and my son Morgan for their love, encouragement and commitment when I was pursuing this research.

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To my parents, I say thank you very much.

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LIST OF ABBREVIATIONS

JKUAT	Jomo Kenyatta University of Agriculture and Technology
NCST	National Council of Science and Technology
MHD	Magenetohydrodynamics
MATLAB	Matrix Laboratory
MFD	Magneto Fluid Dynamics
CGM	Conjugate Gradient Method

NOMENCLATURE

ROMAN SYMBOLS

Symbol	**Quantity**
A	Dimensionless function
B	Width of the bearing, m
B	Magnetic flux density vector, Wbm^{-2}
B_0	Applied magnetic field, Wbm^{-2}
C_f	Friction parameter, $C_f = f.L/h_L$
D	Electric displacement vector, cm^{-2}
D/Dt	Material derivative
E	Electric field intensity vector, vm^{-1}
E_x, E_z	Electric field in x-direction and z-direction, vm^{-1}
f	Friction coefficient, $f = \frac{F}{W}$
F^*	Dimensionless friction force, $F^* = \frac{Fh_L}{\mu ULB}$
g	Gravitational field vector, ms^{-2}
h	Film thickness, m

h^*	Dimensionless film thickness, $h^* = \frac{h}{h_L}$
h_L	Film thickness at the outlet, m
h_0	Film thickness at the inlet, m
H_p^*	Dimensionless friction power loss, $H_p^* = \frac{H_p h_L}{\mu U^2 LB}$
H	Magnetic field intensity vector, wbm^{-2}
J	Current density vector, Am^{-2}
L	Length of bearing, m
M	Hartmann number, $M = B_0 h_L \left(\frac{\sigma}{\mu}\right)^{\frac{1}{2}}$
p^*	Dimensionless film pressure, $p^* = \frac{p h_L^2}{\mu U L}$
t	Time, s
u, v, w	Velocity components in x-,y-,z-directions
u	Velocity field vector, ms^{-1}
U	Sliding velocity, ms^{-1}
W	Load carrying capacity

W^*	Dimensionless load carrying capacity, $W^* = \frac{Wh_L^2}{\mu UL^2 B}$
x,y,z	Rectangular coordinates
x^*, z^*	Dimensionless coordinates, $x^* = \frac{x}{L}, z^* = \frac{z}{B}$

GREEK SYMBOLS

Symbol	**Quantity**
α	Inlet-outlet film ratio, $\alpha = \frac{h_0}{h_L}$
β	Aspect ratio, $\beta = \frac{B}{L}$
μ	Fluid dynamic viscosity, $kgm^{-1}s^{-1}$
ρ	Fluid density, kgm^{-3}
σ	Fluid electrical conductivity, $\Omega^{-1}m^{-1}$
τ_s^*	Dimensionless shear stress at the lower surface, $\tau_s^* = \frac{\tau_s h_L}{\mu U}$
ϕ	Viscous dissipation function $\left(s^{-2}\right)$
∇	Gradient operator $\left[= i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right]$

ABSTRACT

In the present study an analysis of the performance characteristics of finite slider bearings with an electrically conducting fluid in presence of transverse magnetic field is made. A generalized two dimensional Reynolds-type equation is derived using the MHD motion equations with Maxwell's equations. The film pressure is numerically solved from the MHD Reynolds-type equation using iterative method of conjugate Gradient. The software MATLAB was utilized to achieve this approach. This pressure is used to evaluate the bearing characteristics such as load carrying capacity and friction parameter. These results were compared with those of the non-conducting lubricant and presented graphically. The application of magnetic field signifies an influence on the load carrying capacity and the friction parameter of the slider bearing depending upon the values of the Hartmann number, aspect ratio and the inlet-outlet film thickness ratio. It is clearly seen that the load carrying capacity increases considerably with an increase in the application of the magnetic field and with increase of the film thickness ratio.

CHAPTER ONE

INTRODUCTION

Slider bearings are often encountered in engineering applications. They support or guide the parts movably opposed to each other as well as absorb and transfer the occurring forces. This can be seen in shafts of motors and pulleys. Slider bearings necessitate various kinds of fluids as lubricants. The most important property of a liquid lubricant is its viscosity. To prevent the undesirable viscosity change with temperature, the use of electrically conducting fluid has received a great attention. These kinds of lubricants have a higher thermal and electrical conductivity, but lower viscosity than convectional lubricating oils. The high thermal conductivity means that heat generated by viscous friction can be readily conducted away. But the low viscous property would yield a reduced load-carrying capacity; this low load disadvantage can be improved by the application of external electromagnetic field. Since the motion of electrically conducting lubricant across the electromagnetic field induces electrical field intensity, it results in current density which interacts with the magnetic field to produce Lorentz force acting on the lubricant. This force may produce a component opposite to the direction of motion by properly orienting the applied magnetic field. As a result, the film pressure is increased. However, Ohmic heating due to electrical current decreases the viscosity of the lubricant and eventually a decrease may occur in film pressure. The magnetic field becomes strong and prevents this decrease in pressure. As a result of this the load carrying capacity increases. An effort has been made to study and analyze the performance of electrically conducting fluid lubricant in finite slider bearing.

In this chapter the main terms used in the thesis are defined. A review of literature related to the present work is given. The objectives and application of the work are stated at the end of the chapter.

1.1 FLUIDS

A fluid is a substance which is capable of flowing and undergoes deformation continuously under the action of shear stress. If a fluid is at rest, there can be no shearing forces acting and therefore, all forces in the fluid must be perpendicular to the planes upon which they act. Shear stresses are developed when the fluid is in motion, if the particles of the fluid move relative to each other so that they have different velocities. If on the other hand the velocity of the fluid is the same at every point, no shear stresses can be produced, since the fluid particles are at rest relative to each other. If the stress associated with the fluid motion depends linearly on the instantaneous value of the rate of deformation, the fluid is said to be *Newtonian fluid*. Suppose that two adjacent layers of the fluid at a distance *dy* apart are moving with velocities v_o and $v_o + \delta.v_o$ respectively, then the tangential force F acting on the layers is proportional to $\frac{dv_o}{dy}$, that is

$$F = \mu \frac{dv_o}{dy} \tag{1.1}$$

The constant of proportionality μ is the coefficient of viscosity of the fluid. Equation (1.1) is known as the Newtonian's law and any Newtonian fluid satisfies this condition. Fluids are further classified into compressible and incompressible fluids. Incompressible fluids are fluids whose density does not change significantly when subjected to change in pressure and temperature. Compressible fluids are those whose density changes when subjected to change in pressure and temperature. The study involving the analysis of fluid flows is known as hydrodynamics. If the flow variables for a given fluid flow are dependent on time, then the fluid flow is said to be unsteady, if the flow variables are independent of time the flow is said to be steady.

1.2 VISCOSITY

Viscosity is a scientific term that describes the resistance to flow of a fluid. This resistance is due to the friction produced by the fluid's molecule and affects both the extent to which a fluid will oppose the movement of an object through it and the pressure required to make a fluid move through a tube or a pipe.

Viscosity is affected by a number of factors, including the size and shape of the molecules, the interactions between them and the temperature.

1.3 LUBRICATION

Lubrication refers to the application of thin film of fluid of higher viscosity between two plates or two pieces of metal moving with a very small relative velocity to prevent friction. When two plates or two pieces of metal are in contact and moves with a very small relative velocity, they wear out due to high friction. To reduce this friction a very thin layer of a fluid of high viscosity is applied between the two plates. This type of arrangement is known as bearing and the fluid between the two plates is known as lubricant. The bearings which are commonly used include;

1.3.1 Step bearing

In this type of bearing, one of the plates is flat whereas the other plate is of a step type. The pressures in this bearing are generated in triangular pattern with their peak at the step. This helps to keep the plates in the entire bearing out of contact thus maintaining hydrodynamic lubrication type in the entire step bearing

1.3.2 Thrust bearing

This is the type of bearing that consist of two plates in which the upper plate rotates over the lower plate with a constant angular velocity. The lower plate has a small hole near the centre through which the fluid is injected.

1.3.3 Journal bearing

This is a type of bearing designed to reduce friction by supporting radial loads. Journal bearing are often used when the load is light and motion is relatively continuous.

1.3.4 Slider bearing

This is the type of bearing that consists of two plates for which the upper plate is inclined to the lower plate at a very small angle. The two plates are of infinite length so that the flow in between the two plates can be considered as a horizontal flow and therefore depends on the vertical distance alone. In this study, the upper plate is stationary while the lower plate is moving at a constant velocity u_{0}.

1.4 MAGNETO FLUID DYNAMICS

Magneto fluid dynamics is the study of flow of electrically conducting fluids in presence of a magnetic field. It unifies in a common frame work the electromagnetic and fluid dynamic theories to yield a description of the concurrent effects of the magnetic field. Magneto fluid dynamics (MFD) deals with electrically conducting fluids whereas Magneto-hydrodynamics (MHD) is specifically concerned with electrically conducting liquids. Magneto hydrodynamics (MHD) is an academic discipline which studies the dynamics of the flow of an electrically conducting fluid in presence of a magnetic field. The term “magneto” means magnetic field, “hydro” means liquids and “dynamic” means forces causing movement. Examples of electrically conducting fluids include plasmas, liquid metals and salty water.

The flow of an electrically conducting fluid in presence of a magnetic field in general gives rise to induced electric currents which interacts with magnetic field to produce Lorentz force acting on the lubricant. The induced current also generates its own magnetic field, which distorts the original magnetic field.

1.5 DIMENSIONAL ANALYSIS

Dimensional analysis is a mathematical system of using conversion factors to move from one unit of measurement to a different unit of measurement. It is the process of expressing the units of any given physical quantity in terms of the fundamental units, such as time, mass and length. It is built on the principle of dimensional homogeneity that states that an equation expressing a physical relationship between quantities must be dimensionally homogenous and proves to be a powerful tool in formulating problems that defy analytical solution and must be solved experimentally. In this study, dimensional analysis has been used to non-dimensionalize the governing equations by first selecting certain characteristic quantities and then substituting them in the equation.

1.6 BOUNDARY LAYER

The concept of boundary layer was first introduced by Prandtl (1904) and since then it has been applied to several fluid flow problems. The fluid layer in the neighborhood of the solid boundary where effects of fluid friction (viscous effects) are predominant is known as the boundary layer. Boundary layers are thin fluid layers adjacent to the surface of a body or solid wall in which strong viscous effects exist. Flow outside this layer is considered frictionless. The velocity near the boundary is affected by boundary shear stress. At low Reynolds number, viscous forces dominate over the inertial forces.

1.7 Literature Review

Hydrodynamic lubrication is and has been of great importance in modern and traditional technology. Some investigators have studied the Magneto-hydrodynamics (MHD) characteristics of slider bearing with electrically conducting fluids in the presence of magnetic fields. By supplying electrical power from the external source, Huges (1963) studied the effect of field geometries on the MHD Rayleigh step bearings. Significant increases in bearing load are found for a magnetic field both tangentially and transversely to the fluid film. In this study, the fluid is electrically conducting and there is no external current entering the film. In the study of the MHD one-dimensional inclined plane slider bearings by Synder (1963), load enhancement is predicted with liquid metal lubricants in the presence of a magnetic field. Liquid metal has low viscosity. In the study of the MHD wide slider bearing by Agrawal (1970), Anwar and Rodkiewicz (1972), inertia effect was found to increase the load capacity for small Hartmann numbers. In addition the nonuniform magnetic field gives a higher load capacity in comparison to uniform magnetic field. In the study of the MHD porous slider bearings, Gupta and Bhat (1979) established that the load capacity and friction of porous sliders increase markedly with increasing the Hartmann number. On the MHD slider bearing with non-Newtonian couple stress fluids, the problem of optimum load capacity was analyzed by Das (1998). It was found that both the maximum load and the corresponding film ratio depend upon the magnetic parameter, the couple stress parameter and the bearing shapes.

Lin *et al (2006)* studied the dynamic characteristics of a wide exponential film-shape bearing lubricated with a non-Newtonian couple stress fluid. He found out that the effects of non-Newtonian couple stress fluid characterized by the couple stress parameter signify a reduction in the friction parameter and the volume flow rate, as well as an increase in the load carrying capacity, the temperature, the dynamic stiffness coefficient and the dynamic damping coefficient for both type of bearing. Comparing those of the inclined slider bearing, the exponential shaped slider designed at larger profile parameters provides smaller volume flow rate, higher load carrying capacity and better dynamic stiffness and dumping characteristics for larger values of the profile parameter. These improvements of bearing dynamics are more pronounced with increasing values of the non-Newtonian couple stress parameter.

Chou *et al* (2003) studied the effects of externally applied magnetic fields on the squeeze film characteristics between a sphere and a flat plate with an externally conducting fluid. He presented the results of squeeze film characteristics such as load carrying capacity and time-height relationship for various values of Hartman number. As the value of Hartman number approaches zero the study reduced to the non-conducting lubricant case. The presence of externally applied magnetic fields signifies an increase in the magneto hydrodynamic squeeze film pressure. Comparing with the classical non-conducting lubricant case, the effects of externally applied magnetic fields characterized by Hartman number provide an enhancement to the magneto hydrodynamic load carrying capacity and lengthen the response time, especially for large values of Hartman number. On the whole, the squeeze film characteristics between a sphere and a plane surface are improved by the use of an electrically conducting fluid in the presence of transverse magnetic field.

Neminath and Gudadappagouda (2008) studied the rheological effect of micro polar fluid lubricants on the steady state and dynamic behavior of porous slider bearing by considering the squeezing action. The micro polar fluid lubricant provides an increased steady load carrying capacity and dynamic stiffness co-efficient which decreases the dynamic damping co-efficient. The adverse effects due to the presence of porous facing on the slider can be compensated by the proper choice of lubricants with appropriate additives.

Several researchers such as Kumar (1980) and Murti (1974) have analyzed the porous slider bearing by using Darcy's equation to model the flow of Newtonian lubricant in the porous matrix. All the studies assume the lubricant to be a Newtonian fluid.

Lin and Hung (2004) analyzed dynamic characteristics for wider slider bearing with an exponential film profile taking into account the bearing squeezing action. They concluded that both of the steady state performance and the dynamic characteristics are significantly affected by the inlet outlet film ratio of the slider bearing. Comparing with those of the inclined plane slider by Lin *et al* (2001) the exponential shaped slider provides higher load carrying capacity and better dynamic stiffness and damping characteristics at larger values of the inlet outlet film ratio.

Since slider bearing surfaces operate mainly upon the wedge-action principal, an understanding of the dynamic stiffness and damping behavior is helpful in designing bearing. Lin *et al* (2001) analyzed the dynamic of a wide inclined place slider bearing. It is found that higher dynamic stiffness and damping co-efficiency are predicted for the bearing with small values of profile parameter.

Lin and Lu (2010) investigated the dynamic characteristics of a wide exponential shaped slider bearing with an electrically conducting fluid in the presence of a transverse magnetic field on the basis of the magneto hydrodynamic thin film lubrication. Comparing with non-conducting lubricant case the magneto hydrodynamic exponential shaped bearing provides an increase in the steady load and the dynamic co-efficient. On the whole the effects of externally applied magnetic fields on the steady load and the dynamic stiffness co-efficient are more pronounced with larger values of Hartman number and profile parameter and small values of the minimum film thickness.

Patel *et al* (2010) analyzed the behavior of a magnetic fluid based squeeze film between transversely rough annular plates. A magnetic fluid was used as a lubricant and an external magnetic field oblique to the lower plate. The results obtained showed that magnetic fluid lubricant increases the load carrying capacity and that the load carrying capacity increased due to the negatively skewed roughness. Extending the MHD problems to annular squeeze-film disks by Lin (2001), the magnetic field effect provides an increase in the load capacity and the response time.

Most of the above studies dealt with electrically conducting fluid in presence of magnetic field, and how this affects the load carrying capacity. Some of these studies looked at the viscosity of electrically conducting fluids and that necessitated the present study to choose appropriate lubricant with proper additive. None of the above studies however studied the variation of aspect ratio with the magnetic field, and friction parameter. This prompted the present study.

In the presence of a transverse magnetic field, this study presents the MHD lubrication characteristics of finite slider bearings with an electrically conducting fluid. A generalized two-dimensional Reynolds-type equation, which is applicable to the study of the MHD finite slider bearings with different film shapes, is derived by using the MHD motion equations together with Maxwell equations to account for the Lorentz force on the lubricant film. The MHD bearing characteristics in terms of the load-carrying capacity, friction power, and friction parameter are presented for various values of the inlet-outlet thickness ratio, aspect ratio and the Hartmann number. Compared to the bearings using a nonconducting lubricant, the influence of magnetic fields on the performance characteristics is presented.

1.8 Statement of the Problem

An electrically conducting fluid flows between two plates of which the upper plate is stationary and the lower plate moves with constant velocity u_o. The bearing surfaces are made of insulating materials. Electrodes are assumed to be fastened to the side plates. The current is supplied to the bearing by connecting the electrodes to an external generator. The bearing is subjected to a transverse magnetic field $\mathbf{B_0}$ as shown. The film wedge generates a hydrodynamic pressure field that supports an applied load.

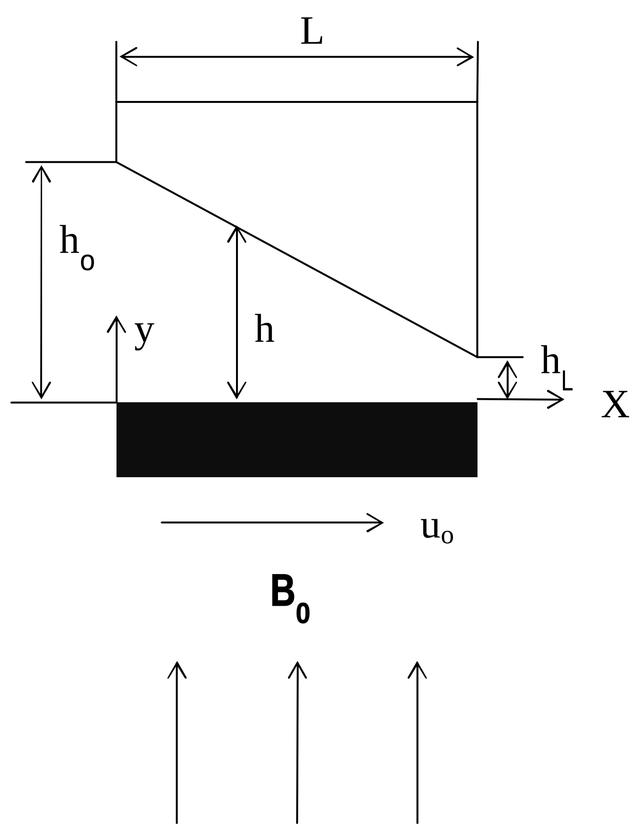

Figure 1.1 Physical configuration of a finite slider bearing in the presence of a uniformly transverse magnetic field.

The film thickens (h) has a linear taper along the direction of the surface velocity u.

$$h = h_0 - \left(\frac{h_0 - h_L}{L}\right).x \qquad 1.2$$

The length L is in the x-direction and width B in z-direction.

1.9 OBJECTIVES OF THE STUDY

(i) To determine whether the variation of magnetic field on an electrically conducting fluid lubricant increases the load carrying capacity.

(ii) To determine the variation of load carrying capacity with film thickness ratio.

(iii) To investigate whether electrically conducting fluid lubricant alter the friction.

1.10 JUSTIFICATION

Lubrication occurs in engines and machines to reduce friction between the moving plates. The basic functions of a lubricant are ;

(i) Friction reduction.

Friction is reduced by maintaining a film of lubricant between surfaces that are moving with respect to each other, thereby preventing the surfaces from going into contact and subsequently causing surface damage.

(ii) Heat removal

Lubrication acts as a coolant, removing heat generated by either friction or other sources of combustion or contact with high temperature substances.

Some of these contaminants include; water, acidic combustion products and particulate matter. The ability of lubricant to remain effective in the presence of outside contaminants is quite important.

Since the lubricant in this study is electrically conducting in presence of magnetic field, the heat generated is readily conducted away.

1.11 NULL HYPOTHESIS.

The null hypothesis of this study is that the load carrying capacity will not decrease when the applied magnetic field is increased.

Having defined the terms that are used in this study and stating the problem, we shall consider the governing equations in the next chapter.

CHAPTER TWO

THE EQUATIONS OF THE STUDY

2.0 INTRODUCTION

The equations governing the flow of an incompressible Newtonian electrically conducting fluid in the presence of transverse magnetic field are presented in this chapter. First, this chapter considers the assumptions made in this particular flow problem and the consequences arising due to these assumptions. The conservation equation of mass and momentum equations are considered. Later in this chapter, the equations governing the fluid flow are given in their general dimensional form. Non-dimensional parameters are then defined. A finite difference scheme to solve the resulting equations is described.

2.1 ASSUMPTIONS

The following assumptions are made in this study

1. The lubricant is incompressible isothermal electrically conducting fluid with electrical conductivity σ.

2. The induced magnetic field is negligible as compared to the applied magnetic field.

3. No slip boundary condition i.e. the velocity of the lubricant layer adjacent to the boundary is the same as that of the boundary.

4. The bearing surfaces are made of insulating materials.

5. The flow is steady and laminar.

6. The film is thin.

7. The fluid inertia is neglected.

8. The body force is negligible except for the Lorentz force.

2.2 THE GOVERNING EQUATIONS

2.2.1 Equation of continuity

The principle of continuity is based on two fundamental propositions namely;

(i) Mass can neither be created nor destroyed. i.e. the mass of the fluid is conserved.

(ii) The flow is continuous i.e. empty spaces do not occur between particles which are in contact. In this study, the fluid to be considered is incompressible fluid. i.e density is assumed constant. The equation of continuity in vector form therefore reduces to;

$$\nabla.\mathbf{u} = 0 \tag{2.1}$$

2.2.2 Equation of momentum The principle of conservation of momentum is basically an application of Newton's second law of motion to an element of fluid motion. The net rate of momentum flow must equal the net sum of forces acting on the fluid.

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}.\nabla\mathbf{u}\right) = -\nabla p + \mu\nabla^2\mathbf{u} + \mathbf{f} \tag{2.2}$$

On the left hand side of equation (2.2), the first term represent the temporal acceleration while the second term represent the convective acceleration. On the right hand side, the first term represent the pressure gradient, the second term represent the force due to viscosity and the third term is the body force.

Since the fluid is electrically conducting, in presence of magnetic field B_o, the MHD momentum equation governing the flow is given as;

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \mu \nabla^2 \mathbf{u} + \mathbf{J} \times \mathbf{B} \qquad 2.3$$

2.3 MAXWELL'S EQUATIONS are given as;

$$\left.\begin{aligned} &\nabla.\mathbf{D} = 0 \\ &\nabla.\mathbf{B} = 0 \\ &\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ &\nabla \times \mathbf{H} = \mathbf{J} \end{aligned}\right\} \qquad 2.4$$

2.4 OHM'S LAW

Ohm's law characterizes the ability of materials to transport electric charge under the influence of an applied electric field. The current density for an electrically conducting material at rest is given by;

$$\mathbf{J} = \sigma \vec{\mathrm{E}} \qquad 2.5$$

If an external magnetic field is present and the conductor is moving at velocity $\vec{u}$, then the current density is given as;

$$\mathbf{J} = \sigma\left(\mathbf{E} + \vec{u} \times \vec{B}\right) \qquad 2.6$$

2.5 LORENTZ FORCE

Underlying the MHD body force is the fact that free charges moving in a magnetic field experience a Lorentz force perpendicular to both their velocity and the magnetic field. If a current $\vec{J}$ passes through a conductor under a magnetic flux $\vec{B}$, the conductor experience Lorentz force perpendicular to both $\vec{J}$ *and* $\vec{B}$, this force is proportional to the magnitude of both $\vec{J}$ *and* $\vec{B}$.The Lorentz force experienced by a conductor is given as;

$$F = \vec{J} \times \vec{B}$$

2.6 NON-DIMENSIONAL NUMBERS

The dimensionless parameters allow one to apply the results obtained in a model to any other dynamically similar case. In this work there are two non-dimensional numbers that are used. These are;

- Reynolds number
- Hartmann number

2.6.1 The Reynolds Number, Re

The Reynolds number is important in analyzing any type of flow when there is substantial velocity gradient shear. The Reynolds number indicates the relative significance of the viscous effects compared to the inertia effect. The Reynolds number is proportional to inertial force divided by viscous force. It is expressed as;

$$\text{Re}=\frac{\rho VL}{\vartheta}=\frac{inertia\ \ forces}{viscous\ \ forces}$$

If the Reynolds number of the system is small, the viscous force is predominant and the effect of viscosity is important in the whole flow field. If Reynolds number is large, the inertia force is predominant and the effect of viscosity is important only in thin layer of the region near solid boundary.

2.6.2 Hartmann Number, M_o

It is the ratio of the magnetic force to viscous force and is defined as;

$$M^2 = \frac{\sigma \mu_e^2 H_o^2 v}{U^2}$$

2.7 NON-DIMENSIONALIZATION OF THE EQUATIONS

The MHD equations are expressed in dimensionless form so that the relative strengths of the different terms in the fluid flow equations can be inferred by the size of any multiplying factors. The non-dimensionalization of the equations is performed by first selecting a certain characteristic quantities, the independent variables may be non-dimensionalized according to the following definitions.

$$\left. x^* = \frac{x}{L},\ z^* = \frac{z}{B},\quad \beta = \frac{B}{L},\quad p^* = \frac{p h_L^2}{\mu U L},\ h^* = \frac{h}{h_L} = \alpha - (\alpha - 1)x^*,\quad \alpha = \frac{h_o}{h_L} \right\} \qquad 2.7$$

2.8 METHOD OF SOLUTION

In general, not all real-life problems have analytical solutions. Numerical techniques are now at the forefront of every kind of scientific research. The part of numerical analysis which has been most changed so far is the solution of partial differential equations by finite difference approximations.

2.8.1 Method of finite differences

The governing equation together with the boundary conditions are to be solved numerically using the finite difference method. This method is used because of its inherent advantages, which include: the finite difference programs take less memory, it converges and also it is stable.

2.8.2 Definition of a mesh

Let us assume a rectangular plane with the horizontal axis x and vertical axis z (this corresponds with the variables which will be used later in this chapter). Let x vary from 0-a and z from 0-b. Further let the intervals be divided into n and m subintervals respectively with each subinterval having width Δx and Δz respectively. We can now identify a point on the rectangular plane $(x_i z_j)$ such that:

$x_i = i\Delta x$ i=1, 2, 3----- and $z_j = j\Delta z$ j=1, 2, 3, --------

With the grid lines x_i *and* z_j intersecting at the mesh points. This is illustrated in the figure 2.1 below.

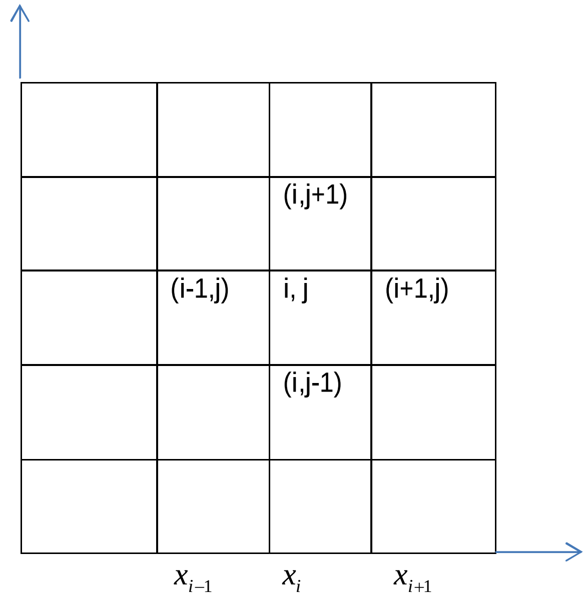

Figure 2.1 Illustration of the mesh

2.8.3 The finite difference approximations

From the mesh defined above, we define differences of du at point x_i using central difference as

$$\Delta u_i = u_{i+1} - u_{i-1} \qquad 2.8$$

the central difference approximation to the tangent slope at x_i is given as

$$\frac{\partial u}{\partial x} = \frac{\Delta u_i}{\Delta x_i} = \frac{u_{i+1} - u_{i-1}}{x_{i+1} - x_{i-1}} \qquad 2.9$$

Using Taylor's series expansion at point $x + \Delta x$ we have

$$N_{x+\Delta x} = N_x + \Delta x \frac{\partial N_x}{\partial x} + \frac{1}{2}\Delta x^2 \frac{\partial^2 N_x}{\partial x^2} + \frac{1}{6}\Delta x^3 \frac{\partial^3 N_x}{\partial x^3} + \frac{1}{24}\Delta x^4 \frac{\partial^4 N_x}{\partial x^4} + ------ \qquad 2.10$$

Taylor series expansion at point $x-\Delta x$ is given by

$$N_{x-\Delta x} = N_x - \Delta x \frac{\partial N_x}{\partial x} + \frac{1}{2}\Delta x^2 \frac{\partial^2 N_x}{\partial x^2} - \frac{1}{6}\Delta x^3 \frac{\partial^3 N_x}{\partial x^3} + \frac{1}{24}\Delta x^4 \frac{\partial^4 N_x}{\partial x^4} - \text{------} \qquad 2.11$$

Summing the Taylor series expansion of ($x+\Delta x$) and ($x-\Delta x$) we have

$$N_{x+\Delta x} + N_{x-\Delta x} = 2N_x + \Delta x^2 \frac{\partial^2 N_x}{\partial x^2} + \frac{1}{12}\Delta x^4 \frac{\partial^4 N_x}{\partial x^4} + ---- \qquad 2.12$$

On rearranging we have

$$\frac{\partial^2 N_x}{\partial x^2} = \frac{N_{x+\Delta x} - 2N_x + N_{x-\Delta x}}{\Delta x^2} + 0\left(\Delta x^2\right) \qquad 2.13$$

Truncating second order terms and higher,

$$0\left(\Delta x^2\right) = -\frac{1}{12}\Delta x^2 \frac{\partial^4 N_x}{\partial x^4} ---- \qquad 2.14$$

2^{nd} order central difference approximation of 2^{nd} derivative is given by

$$\frac{\partial^2 N_x}{\partial x^2} = \frac{N_{x+\Delta x} - 2N_x + N_{x-\Delta x}}{\Delta x^2} \qquad 2.15$$

Subtracting the Taylor series expansions we have

$$N_{x+\Delta x} - N_{x-\Delta x} = 2\Delta x \frac{\partial N_x}{\partial x} + \frac{1}{3}\Delta x^3 \frac{\partial^3 N_x}{\partial x^3} + --- \qquad 2.16$$

Rearranging we have

$$\frac{\partial N_x}{\partial x} = \frac{N_{x+\Delta x} - N_{x-\Delta x}}{2\Delta x} + 0\left(\Delta x^2\right) \qquad 2.17$$

With truncation error

$$0\left(\Delta x^2\right) = -\frac{1}{6}\Delta x^2 \frac{\partial^3 N_x}{\partial x^3} --- \qquad 2.18$$

2^{nd} Order central difference approximation of first derivative is given by

$$\frac{\partial N_x}{\partial x} = \frac{N_{x+\Delta x} - N_{x-\Delta x}}{2\Delta x} = \frac{N_{i+1} - N_{i-1}}{2\Delta x} \qquad 2.19$$

The central difference approximation to the m^{th} derivative across q nodes is given by

$$\frac{\partial^m N}{\partial x^m} \approx \sum_{i=1}^{q} \gamma_i N_i = \gamma_1 N_1 + \gamma_2 N_2 + ------ + \gamma_q N_q \qquad 2.20$$

Approximation to m^{th} derivative across q nodes is

$$\frac{\partial^m N}{\partial x^m} \approx \sum_{i=1}^{q} \gamma_i N_i \qquad 2.21$$

Taylor series expansion about each point x_* is given as

$$N_i = N_* + \left(x_i - x_*\right)\frac{\partial N_*}{\partial x} + \frac{1}{2}\left(x_i - x_*\right)^2 \frac{\partial^2 N_*}{\partial x^2} + \frac{1}{6}\left(x_i - x_*\right)^3 \frac{\partial^3 N_*}{\partial x^3} + ... \qquad 2.22$$

Combining (2.20) with (2.21) and gathering terms we have

$$\frac{\partial^m N}{\partial x^m} \approx \sum_{i=1}^{q} \gamma_i N_i = \sum_{i=1}^{q} \gamma_i N_* + \sum_{i=1}^{q} \gamma_i \left(x_i - x_*\right)\frac{\partial N_*}{\partial x} + \sum_{i=1}^{q} \gamma_i \frac{1}{2}\left(x_i - x_*\right)^2 \frac{\partial^2 N_*}{\partial x^2} + ... 2.23$$

Redefining we have

$$\sum_{i=1}^{q}\gamma_i N_i = B_0 N_* + B_1 \frac{\partial N_*}{\partial x} + B_2 \frac{\partial^2 N_*}{\partial x^2} + \ldots\ldots\ldots\ldots\ B_n = \sum_{i=1}^{q}\gamma_i \frac{1}{n!}(x_i - x_*)^n \qquad 2.24$$

For n=0………………….q-1

.

CHAPTER THREE

SOLUTION TO THE REYNOLD'S EQUATION

3.1 MATHEMATICAL FORMULATION.

The flow of the fluid is at steady state, the density does not change with time. The continuity equation (2.1) is therefore reduced to;

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad 3.1$$

The induced magnetic field is negligible as compared to the applied magnetic field.

Equation (2.6) is therefore substituted in equation (2.3) as follows;

Considering the momentum equation in x-direction,

$$\vec{u}\times\vec{B}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ u & 0 & 0\\ 0 & B_0 & 0\end{vmatrix}=\mathbf{i}(0)+\mathbf{j}(0)-\mathbf{k}(-B_{,0}u)$$

$\mathbf{J}=-\sigma(E_z+B_0u)$ and

$$\mathbf{J}\text{x}\mathbf{B}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 0 & \sigma(E_z+B_0u)\\ 0 & B_0 & 0\end{vmatrix}=\mathbf{i}(0-\sigma B_0(E_z+B_0u)-\mathbf{j}(0)+\mathbf{k}(0)=-\sigma B_0(E_z+uB_0)$$

Considering momentum equation in z-direction

$$\vec{u}\times\vec{B}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 0 & w\\ 0 & B_0 & 0\end{vmatrix}=\mathbf{i}(wB_0)+\mathbf{j}(0)-\mathbf{k}(0)$$

$\mathbf{J}=\sigma(E_x+wB_0)$

$$\mathbf{J}\times\mathbf{B}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ \sigma\left(E_x+wB_0\right) & 0 & 0\\ 0 & B_0 & 0\end{vmatrix}=\mathbf{i}(0)-\mathbf{j}(0)+\mathbf{k}(\sigma B_0(E_x+wB_0)=\sigma B_0(E_x+wB_0)$$

The momentum equation in x, y and z-direction reduces to;

$$\frac{\partial p}{\partial x}=\mu\frac{\partial^2 u}{\partial y^2}-\sigma B_o\left(\mathrm{E}_z+uB_o\right), \qquad 3.2$$

$$\frac{\partial p}{\partial y}=0 \qquad 3.3$$

$$\frac{\partial p}{\partial z}=\mu\frac{\partial^2 w}{\partial y^2}+\sigma B_o\left(E_x+wB_o\right) \qquad 3.4$$

The boundary conditions at the two surfaces of the bearing are

$$\left.\begin{array}{ll} u(x,0,z)=u, & w(x,0,z)=0\\ u(x,h,z)=0, & w(x,h,z)=0\end{array}\right\} \qquad 3.5$$

By applying the above boundary condition the velocity component u and w are solved from equations (3.2) and (3.4), respectively.

$$\begin{aligned} u=&\,U\left[\cosh\left(My/h_L\right)-\coth\left(Mh/h_L\right).\sinh\left(My/h_L\right)\right]+\\ &+\left(\frac{h_L^2}{\mu M^2}\frac{\partial p}{\partial x}+\frac{E_z}{B_o}\right)\left[\cosh\left(My/h_L\right)+\frac{1-\cosh\left(Mh/h_L\right)}{\sinh\left(Mh/h_L\right)}.\sinh\left(My/h_L\right)-1\right], \end{aligned} \qquad 3.6$$

$$w=\left(\frac{h_L^2}{\mu M^2}\frac{\partial p}{\partial z}+\frac{E_x}{B_o}\right)\left[\cosh\left(My/h_L\right)+\frac{1-\cosh\left(Mh/h_L\right)}{\sinh\left(Mh/h_L\right)}.\sinh\left(My/h_L\right)-1\right] \qquad 3.7$$

Where M denotes the Hartmann number

$$M = B_0 h_L (\sigma / \mu)^{1/2} \tag{3.8}$$

If the bearing surfaces are perfect insulators and there is no current external to the film, then the electric field may be approximated by requiring the net current flow to be zero

$$\int_{y=0}^{y=h} (E_z + B_o u) dy = 0, \tag{3.9}$$

$$\int_{y=0}^{y=h} (E_x - B_o w) dy = 0. \tag{3.10}$$

Substituting for u and w in Eqs. (3.9) and (3.10) and performing the integration, the electric fields E_z and E_x becomes

$$\begin{aligned} E_z &= \frac{U}{2} . \left[1 - \frac{\sinh(My/h_L) - \sinh(Mh/h_L)}{\sinh(Mh/h_L)} \right] + \\ &+ \frac{h_L h}{2\mu M} \frac{\partial p}{\partial x} \coth(0.5Mh/h_L) . \left[\frac{\sinh(Mh/h_L) + \sinh(Mh/h_L - My/h_L)}{\sinh(Mh/h_L)} - 1 \right], \end{aligned} \tag{3.11}$$

$$E_x = \frac{h_L h}{2\mu M} \frac{\partial p}{\partial z} \coth(0.5Mh/h_L) . \left[\frac{\sinh(Mh/h_L) + \sinh(Mh/h_L - My/h_L)}{\sinh(Mh/h_L)} - 1 \right]. \tag{3.12}$$

3.2 MHD REYNOLDS EQUATION

Integrating the continuity equation (3.1) across the film we have;

$$\int_{y=0}^{y=h} \frac{\partial u}{\partial x} \partial y + \int_{y=0}^{y=h} \frac{\partial v}{\partial y} \partial y + \int_{y=0}^{y=h} \frac{\partial w}{\partial z} \partial y = 0 \tag{3.13}$$

We then apply the Leibnitz rule for differentiating and integral whose limits are functions of the differentiation variable.

$$\frac{\partial}{\partial x}\int_{0(x,z)}^{h(x,z)} v_x(x,y,z)dy = \int_{0(x,z)}^{h(x,z)} \frac{\partial v_x}{\partial x}dy + v_x(x,h,z)\frac{\partial h}{\partial x} - v_x(x,0,z)\frac{\partial h}{\partial x} \tag{3.14}$$

$$\int_0^h \frac{\partial v_x}{\partial x}\partial y = \frac{\partial}{\partial x}\int_0^h v_x(x,y,z)dy - U_2\frac{\partial h}{\partial x} - U_1\frac{\partial h}{\partial x} \tag{3.15}$$

Where $U_1 = v_x(x, 0, z)$ and $U_2 = v_x(x, h, z)$

Similarly,

$$\int_0^h \frac{\partial v_z}{\partial z}\partial y = \frac{\partial}{\partial z}\int_0^h v_z(x,y,z)dy - W_1\frac{\partial h}{\partial z} - W_2\frac{\partial h}{\partial z} \tag{3.16}$$

Where $W_1 = v_z(x, 0, z)$ and $W_2 = v_z(x, h, z)$

Performing the integration by using the boundary conditions (3.5), and substituting the velocity components in (3.6) and (3.7) in the above integral, the general MHD Reynolds-type equation is derived as;

$$\frac{\partial}{\partial x}\left[a(M,h,h_L)\frac{\partial p}{\partial x}\right] + \frac{\partial}{\partial z}\left[a(M,h,h_L)\frac{\partial p}{\partial z}\right] = \frac{\mu U}{2}\frac{\partial h}{\partial x} \tag{3.17}$$

Where the function $a(M,h,h_L)$ is

$$a(M,h,h_L) = \frac{h_L^2 h}{2M^2}\left[\frac{Mh}{h_L}\coth\left(\frac{Mh}{2h_L}\right) - 2\right] \tag{3.18}$$

3.3 NON-DIMENSIOLIZATION OF THE REYNOLDS'S TYPE EQUATION

Using the non-dimensional quantities in equation (2.7), the Reynolds's equation (3.17) is non-dimensionalized as shown below;

$$\frac{\partial}{\partial x^{*} L}\left[a\left(M, h^{*}, h_{L} . \frac{h}{h^{*}}\right) \frac{\partial p^{*}}{\partial x^{*}} \frac{\mu U}{h_{L}^{2}}\right]+\frac{\partial}{\partial z^{*} B}\left[a\left(M, h^{*} h_{L} . \frac{h}{h^{*}}\right) \frac{\partial p^{*}}{\partial z^{*}}\left(\frac{\mu U L}{h_{L}^{2} B}\right)\right]=\frac{\partial h^{*}}{\partial x^{*}} \frac{\mu U}{2} \frac{h_{L}}{L}$$

$$\frac{\partial}{\partial x^{*} L}\left[a(M, h) \frac{\partial p^{*}}{\partial x^{*}} \frac{\mu U}{h_{L}^{2}}\right]+\frac{\partial}{\partial z^{*} B}\left[a(M, h) \frac{\partial p^{*}}{\partial z^{*}}\left(\frac{\mu U L}{h_{L}^{2} B}\right)\right]=\frac{\partial h^{*}}{\partial x^{*}} \frac{\mu U}{2} \frac{1}{L}$$

$$\frac{\partial}{\partial x^{*}}\left[a(M, h) \frac{\partial p^{*}}{\partial x^{*}} \frac{\mu U}{h_{L}^{2}}\right]+\frac{\partial}{\partial z^{*}} \frac{1}{\beta^{2}}\left(a(M, h) \frac{\partial p^{*}}{\partial z^{*}}\left(\frac{\mu U}{h_{L}^{2}}\right)\right)=\frac{\partial h^{*}}{\partial x^{*}} \frac{\mu U}{2}$$

$$\frac{\partial}{\partial x^{*}}\left[a(M, h) \frac{\partial p^{*}}{\partial x^{*}} \frac{1}{h_{L}^{2}}\right]+\frac{\partial}{\partial z} \frac{1}{\beta^{2}}\left[a(M, h) \frac{\partial p^{*}}{\partial z^{2}}\left(\frac{1}{h_{L}^{2}}\right)=\frac{1}{2} \frac{\partial h^{*}}{\partial x^{*}}\right]$$

$$\frac{\partial}{\partial x^{*}}\left[A\left(M, h^{*}\right) \frac{\partial p^{*}}{\partial x^{*}}\right]+\frac{1}{\beta^{2}} \frac{\partial}{\partial z^{*}}\left[A\left(M, h^{*}\right) \frac{\partial p^{*}}{\partial z^{*}}\right]=\frac{1}{2} \frac{d h^{*}}{d x^{*}} \qquad 3.19$$

Equation (3.19) is a dimensionless Reynolds type equation where

$$A\left(M . h^{*}\right)=\frac{h^{*}}{2 M^{2}}\left[M h^{*} \operatorname{coth}\left(\frac{M h^{*}}{2}\right)-2\right] \qquad 3.20$$

As M approaches zero, equation (3.16) reduces to the classical Reynolds-type equation Hamrock (1994). It is assumed that the lubricant is enough to flood the bearing, and the cavitation effect is neglected. Then the pressure boundary conditions are given by

$$p^{*}=0 \text{ at } x^{*}=0 \text{ and } p^{*}=0 \text{ at } x^{*}=1 \qquad 3.21a$$

$$p^{*}=0 \text{ at } z^{*}=0 \text{ and } p^{*}=0 \text{ at } z^{*}=1 \qquad 3.21b$$

3.4 NUMERICAL METHOD

The equations governing the MHD fluid flow are non-linear differential equations. This implies that it is not possible to get their analytical solutions. The approximate solution is found by applying numerical methods. In this study, finite difference method is used. These approximate solutions are computed subject to initial and boundary conditions which will take a numerical form.

3.5 NUMERICAL FORMULATION

The dimensionless MHD Reynolds-type equation with condition (3.21) is now solved numerically by using finite difference schemes. The film domain under consideration is divided by grid spacing as shown in figure 2. Using the central-difference approximation, the dimensionless MHD Reynolds-type equation is written in a finite difference format as

$$\gamma_0 p^*_{i,j} + \gamma_1 p^*_{i+1,j} + \gamma_2 p^*_{i-1,j} + \gamma_3 p^*_{i,j+1} + \gamma_4 p^*_{i,j-i} = \delta \qquad 3.22$$

Where

$$\gamma_0 = -\beta^2 b^2 \left(A_{i+\frac{1}{2},j} + A_{i-\frac{1}{2},j} \right) + A_{i,j+\frac{1}{2}} + A_{i,j-\frac{1}{2}},$$

3.23(a) $\gamma_1 = \beta^2 b^2 A_{i+\frac{1}{2},j}$ 3.23(b)

$$\gamma_2 = \beta^2 b^2 A_{i-\frac{1}{2},j} \qquad 3.23(c)$$

$$\gamma_3 = A_{i,j+\frac{1}{2}} \qquad 3.23(d)$$

$$\gamma_4 = A_{i,j-\frac{1}{2}} \qquad 3.23(e)$$

$$\delta = -0.5\beta^2 b^2 \left(\Delta x^*\right)^2 (\alpha - 1) \qquad 3.23(f)$$

$$b = \Delta z^* / \Delta x^* \qquad 3.23(g)$$

Equation (3.19) with boundary conditions (3.21a) and (3.21b) is numerically solved for pressure using the Conjugate Gradient Method (CGM). Based on a three-term recurrence, the CGM is an unconstrained optimization technique in which the search directions are conjugate (Decker *et al,* 1992)

The finite termination property implies that this method is guaranteed to terminate after a finite number of steps. Let the residual vector $r_{i,j}$ be

$$r_{i,j} = \gamma_0 p^*_{i,j} + \gamma_1 p^*_{i+1,j} + \gamma_2 p^*_{i-1,j} + \gamma_3 p^*_{i,j+1} + \gamma_4 p^*_{i,j-1} - \delta \qquad 3.24$$

Then the pressure values have converged according to the following convergent criteria

$$\left[\frac{\left|\sum r^{k+1}_{i,j} r^{k+1}_{j,i}\right|}{\left|\sum r^{k}_{i,j} r^{k}_{j,i}\right|}\right]^{\frac{1}{2}} \angle \quad 10^{-3} \qquad 3.25$$

3.6 MHD BEARING CHARACTERISTICS

With the film pressure obtained, the hydro-dynamic squeeze film force can be evaluated. The MHD load-carrying capacity is calculated by integrating the pressure over the film region

$$W = \int_{x=0}^{x=L} \int_{z=0}^{z=B} p dz dx \qquad 3.26$$

Introducing the dimensionless quantity

$$W^* = \frac{W h_L^2}{\mu U L^2 B} \tag{3.27}$$

The dimensionless load-carrying capacity can be expressed in a difference form

$$W^* = \Delta x^* \Delta z^* \sum_{i=0}^{m} \sum_{j=0}^{n} p_{i,j}^* \tag{3.28}$$

The shear stress which the film exerts on the lower surface is

$$\tau_s = \mu \frac{\partial u}{\partial y}\Big|_{y=0} \tag{3.29}$$

Expressed in a dimensionless form one has

$$\tau_s^* = -\frac{1}{2}\left[M \coth\left(0.5 M h^*\right) + h^* \frac{\partial p^*}{\partial x^*}\right] \tag{3.30}$$

The friction force acting on the slider is calculated by integrating the shear stress over the film region

$$F = \int_{z=0}^{z=B} \int_{x=0}^{x=L} \tau_s \, dx dz \tag{3.31}$$

Performing the integration and using in a dimensionless form, it results in

$$F^* = \frac{1}{\alpha - 1} \ln \frac{\sinh(0.5M)}{\sinh(0.5\alpha M)} - \frac{(\alpha - 1) W^*}{2} \tag{3.32}$$

and the coefficient of friction is

$$f = \frac{F}{W} \tag{3.33}$$

Using non-dimensional quantities, the friction parameter is given by

$$C_f = f.\frac{L}{h_L} = \frac{F^*}{W^*} \tag{3.34}$$

CHAPTER FOUR

4.0 DISCUSSION OF THE RESULTS

The mechanism of lubricating properties of fluid is such that when two solid surfaces wetted with fluid come into contact, the latter is forced out of a gap between them. Removal of a viscous fluid from a thin gap requires large pressure gradients and therefore a wedging pressure occurring in the fluid prevents the convergent of surfaces.

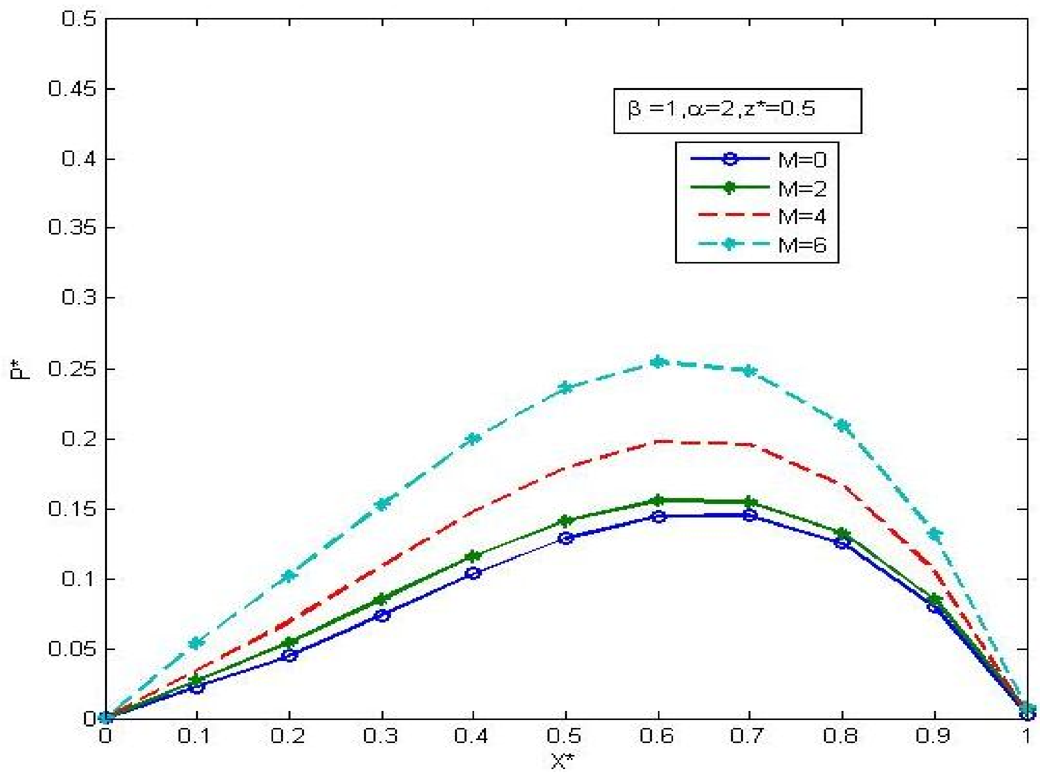

Figure 4.1: Dimensionless film pressure p* as a function of x* in the midplane with β=1 and α=2 for different M.

The graph of figure 4.1 depicts the dimensionless film pressure p^* as a function of dimensionless coordinate x^* in the midplane ($z^*=0.5$) with the aspect ratio $\beta=1$ and the inlet-outlet film thickness ratio α=2 for different values of the Hartmann number. The dark blue curve marked with circles represents the classical lubricant case in the absence of magnetic field. The green, red, and light blue curves show the results of the bearing lubricated with electrically conducting fluids for *M*=2, 4 and 6.The influence of magnetic fields is visibly apparent. The film pressure increases with increasing value of applied magnetic field. The fluid particles are retarded by magnetic forces which cause an additional increase of viscosity. Moreover, magnetic particles form a dense layer on the surface preventing friction surfaces from direct contact. Thus both the hydrodynamic and the boundary lubrication mechanisms are stronger when magnetic field is applied than in ordinary lubricants.

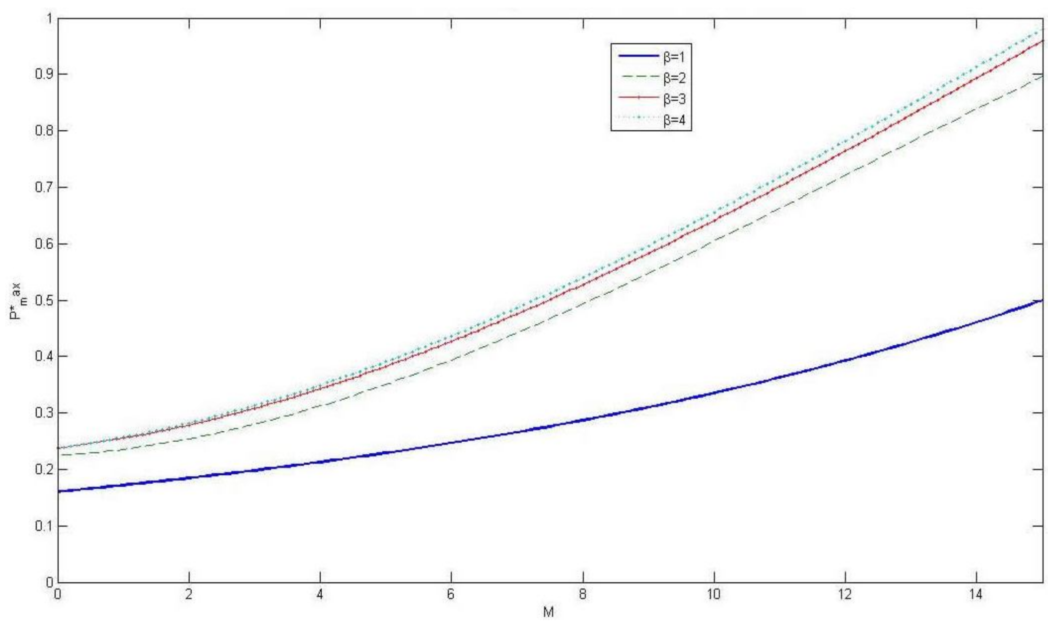

Figure 4.2 Dimensionless maximum pressure P* max versus Hartman Number M at α=2 for different values of β.

This graph of figure 4.2 shows th**e** dimensionless maximum film pressure $P^*_{\max}$ as a function of the Hartmann number M at film thickness ratio α=2 for different values of aspect ratio.

Compared with the no-conducting-lubricant case, the action of magnetic field signifies an increase in the value of $P^*_{\max}$ for the bearing with electrically conducting fluid. The wider the bearing width is, the more the magnetic field affects the maximum film pressure.

Totally, the Hartmann number dominates the effect of magnetic fields upon the maximum film pressure. Larger increments are observed for a wider bearing with large Hartmann number. This is because the magnetic field intensity increases.

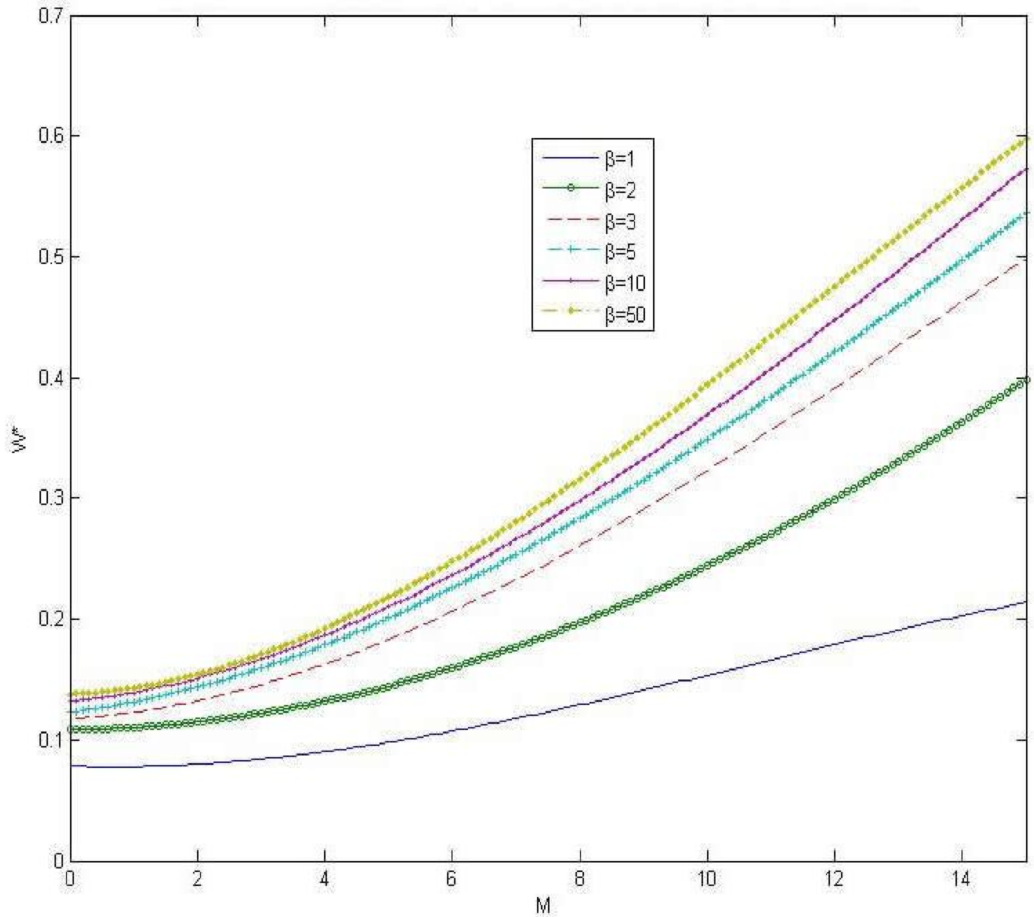

Figure 4.3 Dimensionless load carrying capacity W* versus the Hartman number M at α=2 for different β.

The graph of figure 4.3 displays the dimensionless load carrying capacity w^* verses the Hartmann number M at α=2 for different aspect ratios. Since the effect of magnetic fields give a higher film pressure as discussed above, the integrated load carrying capacity is similarly affected.

For the finite bearing with β=1, the load carrying capacity of the conducting fluid lubricated bearing is observed to increase with increasing Hartmann number. As the bearing tends to be wide (e.g., β=50), the load increases rapidly with the value of M.

This may be explained by considering that the magnetic field increases the fluid viscosity and therefore wider bearings has more magnetic field in it

Totally, the effect of magnetic fields on the bearing load is more pronounced for large values of β and *M*.

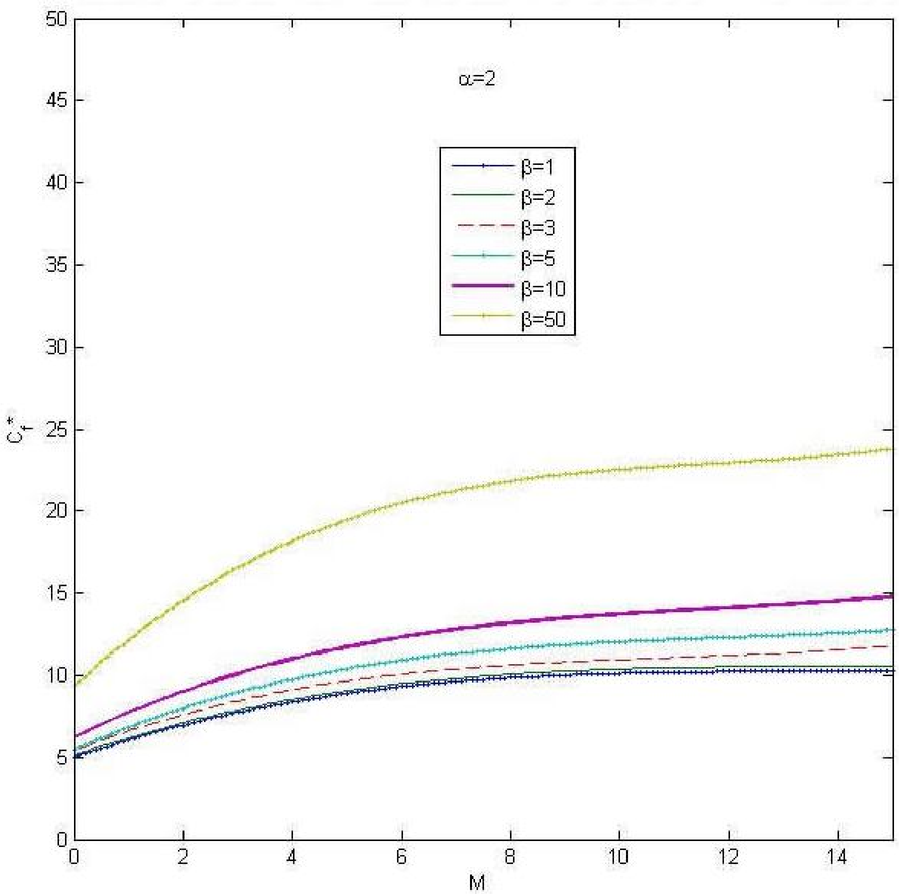

Figure 4.4 Dimensionless friction parameter C_f versus the Hartman number M at α =2 for different β

This graph of figure 4.4 shows the friction parameter c_f^* versus the Hartmann number *M* at α=2 for different aspect ratios. It shows that for a fixed β the effect of magnetic fields results in an increase in the friction parameter. When the magnetic field comes in contact with the fluid, it slows down the motion of the fluid particles causing the friction to increase.

However, for a fixed Hartmann number increasing the value of β yields a decrease in c_f^*.The wider the bearing width is, the more space the fluid moves. This reduces the friction.

CHAPTER FIVE

5.0 VALIDATION OF THE RESULTS.

According to the results discussed, we note that as the value of the Hartmann number approaches zero, the present study reduces to the classical non-conducting lubricant case.

Comparing with the results without magnetic fields, Taking the limit of the Hartmann number as $M \to 0$, the MHD bearing characteristics reduces to the nonconducting-lubricant case. The results of Taylor and Dowson (1974)], who considered the non-conducting lubricant they used the values;

$\alpha = 2$, $\beta = 1$: $W* = 0.0691$; and $\alpha = 3$, $\beta = 2$: $W* = 0.1042$. In this study

($M \to 0$), we used $\alpha = 2$, $\beta = 1$: $W* = 0.0686$; and $\alpha = 3$, $\beta = 2$: $W* = 0.1032$ .The difference is between the two methods when there is no magnetic field applied in the lubricant is small. However in this study we have also considered the application of Magnetic field ($M > 0$).

5.1 CONCLUSION.

The following conclusion can be drawn from this study;

1. The application of a transverse magnetic field signifies an influence upon the performance of slider bearings with an electrically conducting fluid depending on the Hartmann number M, the aspect ratio β and the inlet-outlet film thickness ratio α. For a fixed α, the MHD load capacity, increase with increasing value of M . This is because an increase in magnetic field increases the viscosity of the lubricant.

2. The load carrying capacity increases rapidly with increase in the film thickness ratio. This is because as the bearing widens, there is more lubricant leading to an increase in the load carrying capacity.
3. Under a fixed α and M, increasing the film thickness ratio results in an increase in the load capacity and a decrease in the friction parameter. This is because in wider bearing, the lubricant is able to flow without hindrance.

5.2 RECOMMENDATIONS

Some of the areas that need further research include

i. Considering variation of inlet-outlet film ratio.

ii. Fluid flow in the turbulent boundary layer.

iii. Fluid flow between two plates where the upper plate moves.

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APPENDICES

```
function A = Amh(hs,m)
%Reynolds Dimensionless Function
%   Gets the values h* and M(Hertman Number)
%Returns A values as vectors
A=(hs./(2*m^2)).*(m.*hs.*coth(m.*hs./2)-2);
%display(A');
end
function sig = sigma( B,b,Dx,al )
%Sigma constant
sig=0.5*B^2*b^2*Dx^2*al;


end
% a<x<b , c<z<d
% M = Hartmann number
% ho = height 0
% hl = height l
% h = height
% L = Length of bearing
% B=Width of bearing
%U=Sliding viscosity
%B0=applied magnetic field
%g=electrical conductivity
%u=fluid viscosity
%p=film pressure
```

```
% n = number of subintervals for x

% m = number of subintervals for z

% dx = delta x*

% dz = delta z*

a=0; b=1; c=0; d=1;

n=100; m=100; dx=(b-a)/n;num_iter=20000; dz=(d-c)/m;

M=2;al=2;beta=2;

B=4.0;L=2.0;ho=4;hl=2;g=0.5;u=0.2;U=0.101;beta=B/L;M=2;

x=a:dx:b;

z=c:dz:d;

h=ho-((ho-hl)/L)*x;


%p=6*u*U*sum(dx/h^2)*12*u*U*(ho*hl/(ho+hl))*sum(dx/h^3);

%p=ps/hl;

hs=h/hl;

al=ho/hl;

%As=(hs/2*M^2)*(M*hs*coth(M*hs/2)-2);

ps=zeros(n+1, m+1);

%

%As(1,1)=0;

%As(n+1,1)=0;

%As(1,m+1)=0;

%As(n+1,m+1)=0;

p=ps;

ps(1,1)=0;
```

```
ps(n+1,1)=0;
ps(1,m+1)=0;
ps(n+1,m+1)=0;
%ps=h/hl;al=ho/hl;
%As=hs/2*M^2*(M*hs*coth(M*hs/2)-2);
Dxs=dx/L;
Dzs=dz/B;
bi=Dxs/Dzs;
for it=1:num_iter
%wsave=w;
psave=p;
Rws=0;
Rps=0;
% Solution for w* & p*
Ai=Amh(hs,M);
display(Ai);
y0=yo(beta,bi,Ai,dx);
y1=yi(beta,bi,Ai,dx);
y2=yto(beta,bi,Ai,dx);
y3=ytr(beta,bi,Ai,dx);
y4=yf(beta,bi,Ai,dx);
for i=2:n
for j=2:m

 %ws(i,j)= c1*(ws(i,j+1)+ws(i,j-1)+ws(i+1,j)+ws(i-1,j))-c2*(ps(i+1,j)-ps(i-1,j))-c3;
```

```
ps(i,j)=   (y0(i-1)*ps(i,j)+y1(i-1)*(ps(i+1,j)+y2(i-1).*ps(i-1,j)+y3(i-1)*ps(i,j*1)+y4(i-
1)*ps(i,j-1)))-(-0.5*beta^2*(Dxs/Dzs)^2*(Dxs)^2*(al-1));

% Rps = Root mean square residuals for H*
%Rws=Rws+sqrt((w(i,j)-wsave(i,j))^2);
Rps=Rps+sqrt((p(i,j)-psave(i,j))^2);
%display( 'Y Os'); display(y0(1,j-1));
%display(y1(1,j-1));
end
end
%Rwss(it,1)=Rws;
Rpss(it,1)=Rps;
if (Rps<1e-8 )%& Rws<1e-8)
break
end
end
% t = Dimensionless temperature (theta)
t=zeros(n+1,m+1);
%B.C.'s at the four corners for theta (uniform surface temperature)
t(1,1)=0;
t(1,m+1)=0;
t(n+1,1)=0;
t(n+1,m+1)=0;
for i=2:n
t(i,m+1)=0;
```

```
t(i,1)=0;
end
for j=2:m
t(1,j)=0;
t(n+1,j)=0;
end
% Display solution with x from left to right
%ws=[ws'];
%w=[w'];
ps=[ps'];
p=[p'];
%display(ps);
%t=[t'];
x= a:dx:b; z=c:dz:d;
B0=0.2;
%p=0.5;
%w=0.2;
Hp=0.3;
F=0.4;
M=B0*hl*sqrt((g/u));
xs=x./L;zs=z./B;
m1=x./xs*u*U;
c1=p*hl^2;
%ps=c1./m1;
% a<x<b , c<y<d
```

```
% M = Hartmann number
% ho = height 0
% hl = height 1
% h = height
% L = Length of bearing
% B=Width of bearing
%U=Sliding viscosity
%B0=applied magnetic field
%g=electrical conductivity
%u=fluid viscosity
%p=film pressure
a=0; b=1; c=0; d=1; num_iter=20000; M=100;
n=100; m=100; h=(b-a)/n; k=(d-c)/m;
c1=1/4; c2=(h*M)/8; c3=(h^2)/4;

% ws = load carrying capacity (w*)
% Hs = friction power loss (H*)
%hs=film thickness (h*)
ws=zeros(n+1, m+1);
Hs=zeros(n+1, m+1);
%B.C.'s at the four corners for w*(no slip conditions) & H* (electrically insulated
surface)
ws(1,1)=0;
ws(n+1,1)=0;
ws(1,m+1)=0;
```

```
ws(n+1,m+1)=0;
Hs(1,1)=0;
Hs(n+1,1)=0;
Hs(1,m+1)=0;
Hs(n+1,m+1)=0;
%B.C.'s at the four sides for w*(no slip conditions) & H* (electrically insulated surface)
for i=2:n
ws(i,1)=0;
ws(i,m+1)=0;
Hs(i,1)=0;
Hs(i,m+1)=0;
end
for j=2:m
ws(1,j)=0;
ws(n+1,j)=0;
Hs(1,j)=0;
Hs(n+1,j)=0;
end
for it=1:num_iter
wsave=w;
Hsave=H;
Rws=0;
RHs=0;
% Solution for w* & H*
```

```
for i=2:n
for j=2:m
ws(i,j)= c1*(ws(i,j+1)+ws(i,j-1)+ws(i+1,j)+ws(i-1,j))-c2*(Hs(i+1,j)-Hs(i-1,j))-c3;
Hs(i,j)= c1*(Hs(i,j+1)+Hs(i,j-1)+Hs(i+1,j)+Hs(i-1,j))-c2*(ws(i+1,j)-ws(i-1,j));
% Rws = Root mean square residuals for w*
% RHs = Root mean square residuals for H*
Rws=Rws+sqrt((w(i,j)-wsave(i,j))^2);
RHs=RHs+sqrt((H(i,j)-Hsave(i,j))^2);
end
end
Rwss(it,1)=Rws;
RHss(it,1)=RHs;
if (RHs<1e-8 & Rws<1e-8)
break
end
end
% gamma = Non-dimensional pressure gradient
% w = Dimensionless axial velocity
% H = Dimensionless induced axial magnetic field
% f = friction factor
gamma=1/sum(sum(ws*h^2));
w=ws*gamma;
H=Hs*gamma;
f=-2*gamma;
% t = Dimensionless temperature (theta)
```

```
t=zeros(n+1,m+1);

%B.C.'s at the four corners for theta (uniform surface temperature)

t(1,1)=0;

t(1,m+1)=0;

t(n+1,1)=0;

t(n+1,m+1)=0;

%B.C.'s at the four sides for theta (uniform surface temperature)

for i=2:n

t(i,m+1)=0;

t(i,1)=0;

end

for j=2:m

t(1,j)=0;

t(n+1,j)=0;

end

for itt=1:num_iter

tsave=t;

Rt=0;

% Solution for theta

for i=2:n

for j=2:m

t(i,j)=(t(i-1,j)+t(i+1,j)+t(i,j-1)+t(i,j+1))/(4-4*h^2*nu*w(i,j));

% Rt = Root mean square residuals for theta

Rt=Rt+sqrt((t(i,j)-tsave(i,j))^2);

end
```

```
end
% thetam = Mean dimensionless temperature
% nu = Nusselt number (Nu)
thetam=(sum(sum(t.*w*h^2)))/(sum(sum(w*h^2)));
nu=-1/thetam;
Rtt(itt,1)=Rt;
if Rt<1e-8
break
end
end
% Display solution with x from left to right
ws=[ws'];
w=[w'];
Hs=[Hs'];
H=[H'];
t=[t'];
x= a:h:b; y=c:k:d;


function y4 = yf( B,b,A,h )
%Yo
%   1st coefficient of pij
[m,n]=size(A);
for i=1:m
   for j=2:n-1
   %As(i,j)=(A(i+1,j)-A(i-1,j))/2*h;
```

```
    As2(i,j)=(A(i,j+1)-A(i,j-1))/2*h;
    end
end
y4=As2;
end
function y1 = yi( B,b,A,h )
%Yo
%   1st coefficient of pij
[m,n]=size(A);
for i=1:m
    for j=2:n-2
    As(i,j)=(A(i,j+1)-A(i,j-1))/2*h;
        end
end
y1=B^2*b^2.*(As);
end
function y0 = yo( B,b,A,h )
%Yo
%   1st coefficient of pij
[m,n]=size(A);
for i=1:m
    for j=3:n-2
    As(1,j)=(A(1,j+1)-A(1,j-1))/2*h;
    As2(1,j)=(A(1,j+2)-A(1,j-2))/2*h;
    end
```

```
end
y0=B^2*b^2.*(As)+As2;
end

function y2 = yto( B,b,A,h )
%Yo
%   1st coefficient of pij
[m,n]=size(A);
for i=1:m
    for j=2:n-2

    As2(i,j)=(A(i,j+1)-A(i,j-1))/2*h;
    end
end
y2=B^2*b^2.*(As2);
end

function y3 = ytr( B,b,A,h )
%Yo
%   1st coefficient of pij
[m,n]=size(A);
for i=1:m
    for j=3:n-2

    As2(i,j)=(A(i,j+2)-A(i,j-2))/2*h;
```

```
    end
end
y3=(As2);
end
```